Symmetries and Geometry of the "World of Classical Worlds"

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Matti Pitkänen

orcid:0000-0002-8051-4364.
email: matpitka6@gmail.com,
url: http://tgdtheory.com/public_html/,
address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

The view of the symmetries of the TGD Universe has remained unclear for decades. The notion of "World of Classical Worlds" (WCW) emerged around 1985 but found its basic form around 1990. Holography forced by the realization of General Coordinate Invariance forced/allowed to give up the attempts to make sense of the path integral.

A more concrete way to express this view is that WCW does not consist of 3-surfaces as particle-like entities but almost deterministic Bohr orbits assignable to them as preferred extremals of Kähler action so that quantum TGD becomes wave mechanics in WCW combined with Bohr orbitology. This view has profound implications, which can be formulated in terms of zero energy ontology (ZEO), solving among other things the basic paradox of quantum measurement theory. ZEO forms also the backbone of TGD inspired theory of consciousness and quantum biology.

After the developments towards the end of 2023 leading to a discovery of explicit solution of field equations based on the 4-D geneneralization of holomorphy realizing holography, it seems that the extension of conformal and Kac-Moody symmetries of string models to the TGD framework is understood. What about symplectic symmetries, which were originally proposed as isometries of WCW? In this article this question is discussed in detail and it will be found that these symmetries act naturally on 3-D holographic data and one can identify conserved charges. By holography this is in principle enough and might imply that the actions of holomorphic and symplectic symmetry algebras are dual.

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1 Introduction

The view of the symmetries of the TGD Universe has remained unclear for decades. The notion of "World of Classical Worlds" (WCW) emerged around 1985 but found its basic form around 1990. Holography forced by the realization of General Coordinate Invariance forced/allowed to give up the attempts to make sense of the path integral.

A more concrete way to express this view is that WCW does not consist of 3-surfaces as particle-like entities but almost deterministic Bohr orbits assignable to them as preferred extremals of Kähler action so that quantum TGD becomes wave mechanics in WCW combined with Bohr orbitology. This view has profound implications, which can be formulated in terms of zero energy ontology (ZEO), solving among other things the basic paradox of quantum measurement theory. ZEO forms also the backbone of TGD inspired theory of consciousness and quantum biology.

WCW geometry exists only if it has maximal isometries: this statement is a generalization of the discovery of Freed for loop space geometries [A1]. I have proposed [K2, K1, K3, K4] that WCW could be regarded as a union of generalized symmetric spaces labelled by zero modes which do not contribute to the metric. The induced Kähler field is invariant under symplectic transformations of CP_2 and would therefore define zero mode degrees of freedom if one assumes that WCW metric has symplectic transformations as isometries. In particular, Kähler magnetic fluxes would define zero modes and are quantized closed 2-surfaces. The induced metric appearing in Kähler action is however not zero mode degree of freedom. If the action contains volume term, the assumption about union of symmetric spaces is not well-motivated.

Symplectic transformations are not the only candidates for the isometries of WCW. The basic picture about what these maximal isometries could be, is partially inspired by string models.

1. A weaker proposal is that the symplectomorphisms of H define only symplectomorphisms of WCW. Extended conformal symmetries define also a candidate for isometry group. Remarkably, light-like boundary has an infinite-dimensional group of isometries which are in 1-1 correspondence with conformal symmetries of $S^2 \subset S^2 \times R_+ = \delta M^4_+$.

2. Extended Kac Moody symmetries induced by isometries of δM^4_+ are also natural candidates for isometries. The motivation for the proposal comes from physical intuition deriving from string models. Note they do not include Poincare symmetries, which act naturally as isometries in the moduli space of causal diamonds (CDs) forming the "spine" of WCW.

3. The light-like orbits of partonic 2-surfaces might allow separate symmetry algebras. One must however notice that there is exchange of charges between interior degrees of freedom and partonic 2-surfaces. The essential point is that one can assign to these surface conserved charges when the dual light-like coordinate defines time coordinate. This picture also assumes a slicing of space-time surface by by the partonic orbits for which partonic orbits associated with wormrhole throats and boundaries of the space-time surface would be special. This slicing would correspond to Hamilton-Jacobi structure.

4. Fractal hierarchy of symmetry algebras with conformal weights, which are non-negative integer multiples of fundamental conformal weights, is essential and distinguishes TGD from string models. Gauge conditions are true only the isomorphic subalgebra and its commutator with the entire algebra and the maximal gauge symmetry to a dynamical symmetry with generators having conformal weights below maximal value. This view also conforms with p-adic mass calculations.

5. The realization of the symmetries for 3-surfaces at the boundaries of CD and for light-like orbits of partonic 2-surfaces is known. The problem is how to extend the symmetries to the interior of the space-time surface. It is natural to expect that the symmetries at partonic orbits and light-cone boundary extend to the same symmetries.

After the developments towards the end of 2023, it seems that the extension of conformal and Kac-Moody symmetries of string models to the TGD framework is understood. What about symplectic symmetries, which were originally proposed as isometries of WCW? In this article this question is discussed in detail and it will be found that these symmetries act naturally on 3-D holographic data and one can identify conserved charges. By holography this is in principle enough and might imply that the actions of holomorphic and symplectic symmetry algebras are dual.

2 The reduction of holography to a generalized holomorphy

The reduction of holography to generalized holomorphy reduced field equations to a ridiculously simple form. Field equations are satisfied because contractions of holomorphic tensors of type (1,1) with tensors of type (2,0)+(0,2) are identically vanishing. This ansatz works already for string sheets as minimal surfaces.

Preferred extremals as analogs of Bohr orbits are minimal surfaces irrespective of the action as long as it is a general coordinate invariant constructed using induced geometry and the minimal surface property fails only at lower-dimensional singularities analogous to the frames of a soap film.

At singularities the other parts of the action become visible by boundary conditions guaranteeing that conservation laws expressed by field equations are not violated. The other parts of action are visible only via the classical conservation laws and at interaction vertices [L1].

Twistor lift fixes the 4-D action to a sum of Kähler action and volume term emerging as a dimensional reduction of 6-surface in the Cartesian product of twistor spaces of M^4 and CP_2 to 6-D twistor space to twistor space as S^2 bundle over space-time surface. Only M^4 and CP_2 allow twistor space with Kähler structure so that TGD is unique from its mathematical existence [A2].

2.1 The conserved charges associated with holomorphies

Generalized holomorphy not only solves explicitly the equations of motion but, as found quite recently, also gives corresponding conserved Noether currents and charges.

1. Generalized holomorphy algebra generalizes the Super-Virasoro algebra and the Super-Kac-Moody algebra related to the conformal invariance of the string model. The corresponding Noether charges are conserved. Modified Dirac action allows to construct the supercharges having interpretation as WCW gamma matrices. This suggests an answer to a longstanding question related to the isometries of the "world of the classical worlds" (WCW).

2. Either the generalized holomorphies or the symplectic symmetries of $H = M^4 \times CP_2$ or both together define WCW isometries and corresponding super algebra. It would seem that symplectic symmetries induced from H are not necessarily needed and might correspond to symplectic symmetries of WCW. One would obtain a close similarity with the string model, except that one has half-algebra for which conformal weights are proportional to non-negative integers and gauge conditions only apply to an isomorphic subalgebra. These are labeled by positive integers and one obtains a hierarchy.

3. By their light-likeness, the light cone boundary and orbits of partonic 2-surfaces allow an infinite-dimensional isometry group. This is possible only in dimension four. Its transformations are generalized conformal transformations of 2-sphere (partonic 2-surface) depending on light-like radial coordinate such that the radial scaling compensates for the usual conformal scaling of the metric. The WCW isometries would thus correspond to the isometries of the parton orbit and of the boundary of the light cone! These two representations could provide alternative representations for the charges if the strong form of holography holds true and would realize a strong form of holography. Perhaps these realizations deserve to be called inertial and gravitational charges.

   Can these transformations leave the action invariant? For the light-cone boundary, this looks obvious if the light-cone is sliced by a surface parallel to the light-cone boundary. Note however that the tip of this surface might produce problems. A slicing defined by the Hamilton-Jacobi structure would be naturally associated with partonic orbits.

4. What about Poincare symmetries? They would act on the center of mass coordinates of causal diamonds (CDs) as found already earlier [L4]. CDs form the "spine" of WCW, which can be regarded as fiber space with fiber for a given CD containing as a fiber the space-time surfaces inside it.

The super-symmetric counterparts of holomorphic charges for the modified Dirac action and bilinear in fermionic oscillator operators associated with the second quantization of free spinor fields in H, define gamma matrices of WCW. Their anticommutators define the Kähler metric of WCW. There is no need to calculate either the action defining the classical Kähler action defining the Kähler function or its derivatives with respect to WCW complex coordinates and their conjugates. What is important is that this makes it possible to speak about WCW metric also for number theoretical discretization of WCW with space-time surfaces replaced with their number theoretic discretizations.

2.2 Could generalized holomorphy allow to sharpen the existing views?

This picture is rather speculative, allows several variants, and is not proven. There is now however a rather convincing ansatz for the general form of preferred extremals. Could it help to make the picture more precise?

1. As explained, the explicit solution of field equations in terms of the generalized holomorphy is now known. The solution ansatz is independent of action as long it is general coordinate invariance depending only on the induced geometric structures.

   Space-time surfaces would be minimal surfaces apart from lower-dimensional singular surfaces at which the field equations involve the entire action. Only the singularities, classical charges and positions of topological interaction vertices depend on the choice of the action [L1]. Kähler action plus volume term is the choice of action forced by twistor lift making the choice of H unique.

2. The universality has a very intriguing implication. One can assign to any action of this kind conserved Noether currents and their fermionic counterparts (also super counterparts). One would have a huge algebra of conserved currents characterizing the space-time geometry. The corresponding charges can be made conserved by suitably modifying the form of holomorphic functions of the ansatz and therefore the time derivatives $\partial_t h^k$ at the 3-D end of space-time surface at the boundary CD. This need not be the case for all deformations of partonic orbits. In any case, the 3-D holographic data seem to be dual as the strong form of holography suggests. The discussion of the symplectic symmetries leads to the conclusion that they give rise to conserved charges at the partonic 3-surfaces obeying Chern-Simons-Kähler dynamics, which is non-deterministic.

3. Hamilton-Jacobi structures emerge naturally as generalized conformal structures of space-time surfaces and M^4 [L3]. This inspires a proposal for a generalization of modular invariance and of moduli spaces as subspaces of Teichmüller spaces.

4. One can assign to holomorphy conserved Noether charges. The conservation reduces to the algebraic conditions satisfied for the same reason as field equations, i.e. the conservation conditions involving contractions of complex tensors of type (1,1) with tensors of type (2,0) and (0,2). The charges have the same form as Noether charges but it is not completely clear whether the action remains invariant under these transformations. This point is non-trivial since Noether theorem says that invariance of the action implies the existence of conserved charges but not vice versa. Could TGD represent a situation in which the equivalence between symmetries of action and conservation laws fails?

   Also string models have conformal symmetries but in this case 2-D area form suffers conformal scaling. Also the fact that holomorphic ansatz is satisfied for such a large class of actions apart from singularities suggests that the action is not invariant.

5. The action should define Kähler function for WCW identified as the space of Bohr orbits. WCW Kähler metric is defined in terms of the second derivatives of the Kähler action of type (1,1) with respect to complex coordinates of WCW. Does the invariance of the action under holomorphies imply a trivial Kähler metric and constant Kähler function?

   Here one must be very cautious since by holography the variations of the space-time surface are induced by those of 3-surface defining holographic data so that the entire space-time surface is modified and the action can change. The presence of singularities, analogous to poles and cuts of an analytic function and representing particles, suggests that the action represents the interactions of particles and must change. Therefore the action might not be invariant under holomorphies. The parameters characterizing the singularities should affect the value of the action just as the positions of these singularities in 2-D electrostatistics affect the Coulomb energy.

Generalized conformal charges and supercharges define a generalization of Super Virasoro algebra of string models. Also Kac-Moody algebra assignable to the isometries of $\delta M^4_+ \times CP_2$ and light H generalizes trivially.

6. An absolutely essential point is that generalized holomorphisms are *not* symmetries of Kähler function since otherwise Kähler metric involving second derivatives of type (1,1) with respect to complex coordinates of WCW is non-trivial if defined by these symmetry generators as differential operators. If Kähler function is equal to Kähler action, as it seems, Kähler action cannot be invariant under generalized holomorphies.

   Noether's theorem states that the invariance of the action under a symmetry implies the conservation of corresponding charge but does *not* claim that the existence of conserved Noether currents implies invariance of the action. Since Noether currents are conserved now, one would have a concrete example about the situation in which the inverse of Noether's theorem does not hold true. In a string model based on area action, conformal transformations of complex string coordinates give rise to conserved Noether currents as one easily checks. The area element defined by the induced metric suffers a conformal scaling so that the action is not invariant in this case.

There are several questions to be answered. Could also the symplectic symmetries act as isometries of WCW geometry? Could symplectic transformations act on 3-D holographic data without any continuation to the space-time interior and allow to assign conserved quantum charges with the 3-D data? Holographic generators act on 4-D space-time surfaces and can be associated with the boundary data at the space-like 3-surfaces at the boundaries of CD (at least). Could symplectomorphisms and generalized holomorphisms define algebras, which by holography are dual in some sense? This is possible since the quantum realizations of both algebras rely on second quantized free Dirac fields in H.

3 Challenging the existing view of symplectic symmetries in relation to WCW geometry

I have considered the possibility that also the symplectomorphisms of $\delta M^4 + \times CP_2$ could define WCW isometries. This actually the original proposal. One can imagine two options.

1. The continuation of symplectic transformations to transformations of the space-time surface from the boundary of light-cone or from the orbits partonic 2-surfaces should give rise to conserved Noether currents but it is not at all obvious whether this is the case.

2. One can assign conserved charges to the time evolution of the 3-D boundary data defining the holographic data: the time coordinate for the evolution would correspond to the light-like coordinate of light-cone boundary or partonic orbit. This option I have not considered hitherto. It turns out that this option works!

The conclusion would be that generalized holomorphies give rise to conserved charges for 4-D time evolution and symplectic transformations give rise to conserved charged for 3-D time evolution associated with the holographic data.

3.1 About extremals of Chern-Simons-Kähler action

Let us look first the general nature of the solutions to the extremization of Chern-Simons-Kähler action.

1. The light-likeness of the partonic orbits requires Chern-Simons action, which is equivalent to the topological action $J \wedge J$, which is total divergence and is a symplectic in variant. The field equations at the boundary cannot involve induced metric so that only induced symplectic

structure remains. The 3-D holographic data at partonic orbits would extremize Cherns-Simons-Kähler action. Note that at the ends of the space-time surface about boundaries of CD one cannot pose any dynamics.

2. If the induced Kähler form has only the CP_2 part, the variation of Chern-Simons-Kähler form would give equations satisfied if the CP_2 projection is at most 2-dimensional and Chern-Simons action would vanish and imply that instanton number vanishes.

3. If the action is the sum of M^4 and CP_2 parts, the field equations in M^4 and CP_2 degrees of freedom would give the same result. If the induced Kähler form is identified as the sum of the M^4 and CP_2 parts, the equations also allow solutions for which the induced M^4 and CP_2 Kähler forms sum up to zero. This phase would involve a map identifying M^4 and CP_2 projections and force induce Kähler forms to be identical. This would force magnetic charge in M^4 and the question is whether the line connecting the tips of the CD makes non-trivial homology possible. The homology charges and the 2-D ends of the partonic orbit cancel each other so that partonic surfaces can have monopole charge.

   The conditions at the partonic orbits do not pose conditions on the interior and should allow generalized holomorphy. The following considerations show that besides homology charges as Kähler magnetic fluxes also Hamiltonian fluxes are conserved in Chern-Simons-Kähler dynamics.

3.2 Can one assign conserved charges with symplectic transformations or partonic orbits and 3-surfaces at light-cone boundary?

The geometric picture is that symplectic symmetries are Hamiltonian flows along the light-like partonic orbits generated by the projection A_t of the Kähler gauge potential in the direction of the light-like time coordinate. The physical picture is that the partonic 2-surface is a Kähler charged particle that couples to the Hamilton $H = A_t$. The Hamiltonians H_A are conserved in this time evolution and give rise to conserved Noether currents. The corresponding conserved charge is integral over the 2-surface defined by the area form defined by the induced Kähler form.

Let's examine the change of the Chern-Simons-Kähler action in a deformation that corresponds, for example, to the CP_2 symplectic transformation generated by Hamilton H_A. M^4 symplectic transformations can be treated in the same way:here however M^4 Kähler form would be involved, assumed to accompany Hamilton-Jacobi structure as a dynamically generated structure.

1. Instanton density for the induced Kähler form reduces to a total divergence and gives Chern-Simons-Kähler action, which is TGD analog of topological action. This action should change in infinitesimal symplectic transformations by a total divergence, which should vanish for extremals and give rise to a conserved current. The integral of the divergence gives a vanishing charge difference between the ends of the partonic orbit. If the symplectic transformations define symmetries, it should be possible to assign to each Hamiltonian H_A a conserved charge. The corresponding quantal charge would be associated with the modified Dirac action.

2. The conserved charge would be an integral over X^2. The surface element is not given by the metric but by the symplectic structure, so that it is preserved in symplectic transformations. The 2-surface of the time evolution should correspond to the Hamiltonian time transformation generated by the projection $A_\alpha = A_k \partial_\alpha s^k$ of the Kähler gauge potential A_k to the direction of light-like time coordinate $x^\alpha \equiv t$.

3. The effect of the generator $j_A^k = J^{kl}\partial_l H_A$ on the Kähler potential A_l is given by $j_A^k \partial_k A_l$. This can be written as $\partial_k A_l = J_{kl} + \partial_l A_k$. The first term gives the desired total divergence $\partial_\alpha(\epsilon^{\alpha\beta\gamma} J_{\beta\gamma} H_A)$.

   The second term is proportional to the term $\partial_\alpha H_A - \{A_\alpha, H\}$. Suppose that the induced Kähler form is transversal to the light-like time coordinate t, i.e. the induced Kähler form does not have components of form $J_{t\mu}$. In this kind of situation the only possible choice for α corresponds to the time coordinate t. In this situation one can perform the replacement $\partial_\alpha H_A - \{A_\alpha, H\} \rightarrow dH_A/dt - \{A_t, H\}$ This corresponds to a Hamiltonian time evolution generated by the projection A_t acting as a Hamiltonian. If this is really a Hamiltonian time evolution, one has $dH_A/dt - \{A, H\} = 0$. Because the Poisson bracket represents a commutator, the Hamiltonian time evolution equation is analogous to the vanishing of a covariant derivative of H_A along light-like curves: $\partial_t H_A + [A, H_A] = 0$. The physical interpretation is that the partonic surface develops like a particle with a Kähler charge. As a consequence the change of the action reduces to a total divergence.

An explicit expression for the conserved current $J^\alpha_A = H_A \epsilon^{\alpha\beta\gamma} J_{\beta\gamma}$ can be derived from the vanishing of the total divergence. Symplectic transformations on X^2 generate an infinite-dimensional symplectic algebra. The charge is given by the Hamiltonian flux $Q_A = \int H_A J_{\beta\gamma} dx^\alpha \wedge dx^\beta$.

4. If the projection of the partonic path CP_2 or M^4 is 2-D, then the light-like geodesic line corresponds to the path of the parton surface. If A_l can be chosen parallel to the surface, its projection in the direction of time disappears and one has $A_t = 0$. In the more general case, X^2 could, for example, rotate in CP$_2$. In this case A_t is nonvanishing. If J is transversal (no Kähler electric field), charge conservation is obtained.

Do the above observations apply at the boundary of the light-cone?

1. Now the 3-surface is space-like and Chern-Simons-Kähler action makes sense. It is not necessary but emerges from the "instanton density" for the Kähler form. The symplectic transformations of $\delta M^4_+ \times CP_2$ are the symmetries. The most time evolution associated with the radial light-like coordinate would be from the tip of the light-cone boundary to the boundary of CD. Conserved charges as homological invariants defining symplectic algebra would be associated with the 2-D slices of 3-surfaces. For closed 3-surfaces the total charges from the sheets of 3-space as covering of δM^4_+ must sum up to zero.

2. Interestingly, the original proposal for the isometries of WCW was that the Hamiltonian fluxes assignable to M^4 and CP_2 degrees of freedom at light-like boundary act define the charges associated with the WCW isometries as symplectic transformations so that a strong form of holography would have been be realized and space-time surface would have been effectively 2-dimensional. The recent view is that these symmetries pose conditions only on the 3-D holographic data. The holographic charges would correspond to additional isometries of WCW and would be well-defined for the 3-surfaces at the light-cone boundary.

To sum up, one can imagine many options but the following picture is perhaps the simplest one and is supported by mathematical facts. The isometry algebra of $\delta M^4_+ \times CP_2$ consists of generalized conformal and KM algebras at 3-surfaces in $\delta M^4_+ \times CP_2$ and symplectic algebras at the light cone boundary and 3-D light-like partonic orbits. The latter symmetries give constraints on the 3-D holographic data. It is still unclear whether one can assign generalized conformal and Kac-Moody charges to Chern-Simons-Kähler action. The isomorphic subalgebras labelled by a positive integer and their commutators with the entire algebra would annihilate the physical states. The isomorphic subalgebras labelled by a positive integer and their commutators with the entire algebra would annihilate the physical states. These two representations would generalize the notions of inertial and gravitational mass and their equivalence would generalize the Equivalence Principle.

3.2.1 Objection against the idea about theoretician friendly Mother Nature

One of the key ideas behind the TGD view of dark matter is that Nature is theoretician friendly [L2]. When the coupling strength proportional to $\hbar_{eff}$ becomes so large that perturbation series ceases to converge, a phase transition increasing the value of h_{eff} takes place so that the perturbation series converges.

One can however argue that this argument is quantum field-theoretic and does not apply in TGD since holography changes the very concept of perturbation theory. There is no path integral to worry about. Path integral is indeed such a fundamental concept that one expects it to have some approximate counterpart also in the TGD Universe. Bohr orbits are not completely deterministic: could the sum over the Bohr orbits however translate to an approximate description as a path integral at the QFT limit? The dynamics of light-like partonic orbits is indeed non-deterministic and could give rise to an analog of path integral as a finite sum.

1. The dynamics implied by Chern-Simons-Kähler action assignable to the partonic 3-surface with light-one coordinate in the role of time, is very topological in that the partonic orbits is light-like 3-surface and has 2-D CP_2 and M^4 projections unless the induced M^4 and CP_2 Kähler forms sum up to zero. The light-likeness of the projection is a very loose condition and and the sum over partonic orbits as possible representation of holographic data analogous to initial values (light-likeness!) is therefore analogous to the sum over all paths appearing as a representation of Schrödinger equation in wave mechanics.

   One would have an analog of 1-D QFT. This means that the infinities of quantum field theories are absent but for a large enough coupling strength $g^2/4\pi\hbar$ the perturbation series fails to converge. The increase of h_{eff} would resolve the problem. For instance, Dirac equation in atomic physics makes unphysical predictions when the value of nucler charge is larger than $Z \sim 137$.

2. I have also considered a discrete variant of this picture motivated by the fact that the presence of the volume term in the action implies that the M^4 projection of the CP_2 type extremal is a light-like geodesic line. The light-like orbits would consist of pieces of light-like geodesics implying that the average velocity would be smaller than c: this could be seen as a correlate for massivation.

   The points at which the direction of segment changes would correspond to points at which energy and momentum transfer between the partonic orbit and environment takes place. This kind of quantum number transfer might occur at least for the fermionic lines as boundaries of string world sheets. They could be described quantum mechanically as interactions with classical fields in the same way as the creation of fermion pairs as a fundamental vertex [L1]. The same universal 2-vertex would be in question.

   At these points the minimal surface property would fail and the trace of the second fundamental form would not vanish but would have a delta function-like singularity. The CP_2 part of the second fundamental form has quantum numbers of Higgs so that there would be an analogy with the standard description of massivation by the Higgs mechanism. Higgs would be only where the vertices are.

3. What is intriguing, that the light-likeness of the projection of the CP_2 type extremals in M^4 leads to Virasoro conditions assignable to M^4 coordinates and this eventually led to the idea of conformal symmetries as isometries as WCW. In the case of the partonic orbits, the light-like curve would be in $M^4 \times CP_2$ but it would not be surprising if the generalization of the Virasoro conditions would emerge also now.

   One can write M^4 and CP_2 coordinates for the light-like curve as Fourier expansion in powers of $exp(it)$, where t is the light-like coordinate. This gives $h^k = \sum h^k_n exp(int)$. If the CP_2 projection of the orbits of the partonic 2-surface is geodesic circle, CP_2 metric s_{kl} is constant, the light-likeness condition $h_{kl}\partial_t h^k \partial_t h^l = 0$ gives $Re[h_{kl}\sum_m h^k_{n-m}\overline{h}^l_m] = 0$. This does not give Virasoro conditions.

   The condition $d/dt(h_{kl}\partial_t h^k \partial_t h^l = 0) = 0$ however gives the standard Virasoro condition in quantization condition stating that the operator counterparts of quantities $L_n = Re[h_{kl}\sum_m (n-m)h^k_{n-m}\overline{h}^l_m]$ annihilate the physical states. What is interesting is that the latter condition also allows time-like (and even space-like) geodesics.

   Could massivation mean a failure of light-likeness? For piecewise light-like geodesics the light-likeness condition would be true only inside the segments. By taking Fourier transform one expects to obtain Virasoro conditions with a cutoff analogous to the momentum cutoff in condensed matter physics for crystals.

4. In TGD the Virasoro, Kac-Moody algebras and symplectic algebras are replaced by half-algebras and the gauge conditions are satisfied for conformal weights which are n-multiples of fundamentals with with n larger than some minimal value. This would dramatically reduce the effects of the non-determinism and could make the sum over all paths allowed by the

light-likeness manifestly finite and reduce it to a sum with a finite number of terms. This cutoff in degrees of freedom would correspond to a genuinely physical cutoff due to the finite measurement resolution coded to the number theoretical anatomy of the space-time surfaces. This cutoff is analogous to momentum cutoff and could at the space-time picture correspond to finite minimum length for the light-like segments of the orbit of the partoic 2-surface.

3.2.2 Boundary conditions at partonic orbits and holography

TGD reduces coupling constant evolution to a number theoretical evolution of the coupling parameters of the action identified as Kähler function for WCW. An interesting question is how the 3-D holographic data at the partonic orbits relates to the corresponding 3-D data at the ends of space-time surfaces at the boundary of CD, and how it relates to coupling constant evolution.

1. The twistor lift of TGD strongly favours 6-D Kähler action, which dimensionally reduces to Kähler action plus volume term plus topological $\int J \wedge J$ term reducing to Chern Simons-Kähler action. The coefficients of these terms are proposed to be expressible in terms of number theoretical invariants characterizing the algebraic extensions of rationals and polynomials determining the space-time surfaces by $M^8 - H$ duality.

   Number theoretical coupling constant evolution would be discrete. Each extension of rationals would give rise to its own coupling parameters involving also the ramified primes characterizing the polynomials involved and identified as p-adic length scales.

2. The time evolution of the partonic orbit would be non-deterministic but subject to the light-likeness constraint and boundary conditions guaranteeing conservation laws. The natural expectation is that the boundary/interface conditions for a given action cannot be satisfied for all partonic orbits (and other singularities). The deformation of the partonic orbit requiring that boundary conditions are satisfied, does not affect X^3 but the time derivatives $\partial_t h^k$ at X^3 are affected since the form of the holomorphic functions defining the space-time surface would change. The interpretation would be in terms of duality of the holographic data associated with the partonic orbits *resp.* X^3.

   There can of course exist deformations, which require the change of the coupling parameters of the action to satisfy the boundary conditions. One can consider an analog of renormalization group equations in which the deformation corresponds to a modification of the coupling parameters of the action, most plausibly determined by the twistor lift. Coupling parameters would label different regions of WCW and the space-time surfaces possible for two different sets of coupling parameters would define interfaces between these regions.

In order to build a more detailed view one must fix the details related to the action whose value defines the WCW Kähler function.

1. If Kähler action is identified as Kähler action, the identification is unique. There is however the possibility that the imaginary exponent of the instanton term or the contribution from the Euclidean region is not included in the definition of Kähler function. For instance instanton term could be interpreted as a phase of quantum state and would not contribute.

2. Both Minkowskian and Euclidean regions are involved and the Euclidean signature poses problems. The definition of the determinant as $\sqrt{-g_4}$ is natural in Minkowskian regions but gives an imaginary contribution in Euclidean regions. $\sqrt{|g_4|}$ is real in both regions. $i\sqrt{g_4}$ is real in Minkowskian regions but imaginary in the Euclidean regions.

   There is also a problem related to the instanton term, which does not depend on the metric determinant at all. In QFT context the instanton term is imaginary and this is important for instance in QCD in the definition of CP breaking vacuum functional. Should one include only the 4-D or possibly only Minkowskian contribution to the Kähler function imaginary coefficient for the instanton/Euclidian term would be possible?

3. Boundary conditions guaranteeing the conservation laws at the partonic orbits must be satisfied. Consider the $\sqrt{|g_4|}$ case. Charge transfer between Euclidean and Minkowskian regions. If the C-S-K term is real, also the charge transfer between partonic orbit and 4-D

regions is possible. The boundary conditions at the partonic orbit fix it to a high degree and also affect the time derivatives $\partial_t h^k$ at X^3. This option looks physically rather attractive because classical conserved charges would be real.

If the C-S-K term is imaginary it behaves like a free particle since charge exchange with Minkowskian and Euclidean regions is not possible. A possible interpretation of the possible M^4 contribution to momentum could be in terms of decay width. The symplectic charges do not however involve momentum. The imaginary contribution to momentum could therefore come only from the Euclidean region.

4. If the Euclidean contribution is imaginary, it seems that it cannot be included in the Kähler function. Since in M^8 picture the momenta of virtual fermions are in general complex, one could consider the possibility that Euclidean contribution to the momentum is imaginary and allows an interpretation as a decay width.

3.3 The TGD counterparts of the gauge conditions of string models

The string model picture forces to ask whether the symplectic algebras and the generalized conformal and Kac-Moody algebras could act as gauge symmetries.

1. In string model picture conformal invariance would suggest that the generators of the generalized conformal and KM symmetries act as gauge transformations annihilate the physical states. In the TGD framework, this does not however make sense physically. This also suggests that the components of the metric defined by supergenerators of generalized conformal and Kac Moody transformations vanish. If so, the symplectomorphisms $\delta M^4_+ \times CP_2$ localized with respect to the light-like radial coordinate acting as isometries would be needed. The half-algebras of both symplectic and conformal generators are labelled by a non-negative integer defining an analog of conformal weight so there is a fractal hierarchy of isomorphic subalgebras in both cases.

2. TGD forces to ask whether only subalgebras of both conformal and Kac-Moody half algebras, isomorphic to the full algebras, act as gauge algebras. This applies also to the symplectic case. Here it is essential that only the half algebra with non-negative multiples of the fundamental conformal weights is allowed. For the subalgebra annihilating the states the conformal weights would be fixed integer multiples of those for the full algebra. The gauge property would be true for all algebras involved. The remaining symmetries would be genuine dynamical symmetries of the reduced WCW and this would reflect the number theoretically realized finite measurement resolution. The reduction of degrees of freedom would also be analogous to the basic property of hyperfinite factors assumed to play a key role in thee definition of finite measurement resolution.

3. For strong holography, the orbits of partonic 2-surfaces and boundaries of the spacetime surface at δM^4_+ would be dual in the information theoretic sense. Either would be enough to determine the space-time surface.

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