Modified Dirac equation and the holography=holomorphy hypothesis

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Abstract

The understanding of the modified equation as a generalization of the massless Dirac equation for the induced spinors of the space-time surface X^4 is far from complete. It is however clear that the modified Dirac equation is necessary.

Two problems should be solved.

1. It is necessary to find out whether the modified Dirac equation follows from the generalized holomorphy alone. The dynamics of the space-time surface is trivialized into the dynamics of the minimal surface thanks to the generalized holomorphy and is universal in the sense that the details of the action are only visible at singularities which define the topological particle vertices. Could holomorphy solve also the modified Dirac equation? The modified gamma matrices depend on the action: could the modified Dirac equation fix the modified gamma matrices and thus also the action or does not universality hold true also for the modified Dirac action?
2. The induction of the second quantized spinor field of H on the space-time surface means only the restriction of the induced spinor field to X^4. This determines the fermionic propagators as H-propagators restricted to X^4. The induced spinor field can be expressed as a superposition of the modes associated with X^4. The modes should satisfy the modified Dirac equation, which should reduce to purely algebraic conditions as in the 2-D case. Is this possible without additional conditions that might fix the action principle? Or is this possible only at lower-dimensional surfaces such as string world sheets?

In this article a proposal for how to meet these challenges is proposed and a holomorphic solution ansatz for the modified Dirac equation is discussed in detail.

Contents

1 Introduction

The understanding of the modified equation as a generalization of the massless Dirac equation for the induced spinors of the space-time surface X^4 [K1, K2] is far from complete. It is however clear that the modified Dirac equation is necessary [L2] and its failure at singularities, analogous to the failure of minimal surface property at them, leads to an identification of fundamental interaction vertices as 2-vertices for the creation of fermion pair in the induced classical electroweak gauge fields.

These singularities are lower-dimensional surfaces are related to the 4-D exotic diffeomorphic structures [A1, A2] and are discussed from the point of view of TGD in [L1]. They can be interpreted as defects of the standard diffeomorphic structure and mean that in the TGD framework particle creation is possible only in dimension $D = 4$.

A fermion-antifermion pair as a topological object can be said to be created at these singularities. The creation of particles, in the sense that the fermion and antifermion numbers (boson are identified as fermion-antifermion bound states in TGD) are not preserved separately, is only possible in dimension 4, where exotic differentiable structures are possible.

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   (a) Let's consider Dirac's equation in M^2 as a simplified example. Denote the light like coordinates (u,v) by $(z,\overline{z})$. The massless Dirac equation reduces to an algebraic condition if the modes are proportional to z^n or $\overline{z}^n$. $\gamma^z\partial_z$ *resp.* $\gamma^{\overline{z}}\partial_{\overline{z}}$ annihilates such a mode if γ^z *resp.* $\gamma^{\overline{z}}$ annihilates the mode.

   (b) These conditions must be generalized to the case of a 4-D space-time surface X^4. Now the complex and Kähler structure are 4-dimensional and holomorphy generalizes. γ^z is generalized to modified gammas Γ^{z_i}, determined by the action principle, which is general coordinate invariant and constructible in terms of the induced geometry. Modified gamma matrices $\Gamma^\alpha = \gamma^k T^\alpha_k$, $T^\alpha_k = \partial L/\partial(\partial_\alpha h^k)$ are contractions of the gamma matrices of H with the canonical impulse currents T^α_k determined by the action density L.

   Irrespective of action, field equations for the space-time surface reduce to the equations of a minimal surface, and are solved by the generalized holomorphy [L4]. The lower-dimensional singularities, at which the minimal surface equations fail, correspond to defects of the standard diffeomorphic structure and are analogs of poles and cuts to analytic functions [L1].

2. The induction of the second quantized spinor field of H on the space-time surface means only the restriction of the induced spinor field to X^4. This determines the fermionic propagators as H-propagators restricted to X^4. The induced spinor field can be expressed as a superposition of the modes associated with X^4. The modes should satisfy the modified Dirac equation, which should reduce by the generalized holomorphy to purely algebraic conditions as in the 2-D case. Is this possible without additional conditions that might fix the action principle? Or is this possible only at lower-dimensional surfaces such as string world sheets?

2 How to meet the challenges?

This section begins with an optimistic view of the solution of the problems followed by a critical discussion and detailed proposal for how the generalized holography would solve the modified Dirac equation.

2.1 Optimistic view of how holomorphy solves the modified Dirac equation

Consider first the notations: the coordinates for the 4-surface X^4 are the light-like coordinate pair (u,v) and the complex coordinate pair $(z,\overline{z})$. To simplify the notation, we take the notation $(u,v) \equiv (z_1,\overline{z}_1)$ for the light-like coordinate pair (u,v), so that the coordinates of the space-time surface can be denoted by (z_1,z_2) and $(\overline{z}_1,\overline{z}_2)$. As far as algebra is considered, one can consider E^4 instead of M^4, from which Minkowski's version is obtained by continuing analytically.

1. Let us optimistically assume that the H spinor modes can be expressed as superpositions of conformal X^4 spinor modes, which in their simplest form are products of powers of two "complex" variables $z_i^{n_i}$ or $\overline{z}_i^{n_i}$. Only four different types of modes: $z_1^{n_1}z_2^{n_2}$, $\overline{z}_1^{n_1}z_2^{n_2}$, $z_1^{n_1}\overline{z}_2^{n_2}$ and $\overline{z}_1^{n_1}\overline{z_2}^{n_2}$ should appear.

   The spinor modes of H are plane waves if M^4 has no Kähler structure. Could this mean that the modes can be expressed as products of exponentials $exp(ik_iz_i), exp(ik_i\overline{z}_i)$, $i=1,2$. More general analytical functions and their complex conjugates can also be thought of as building blocks of modes. In some cases, the complex coordinate of CP_2 comes into question as well as the complex coordinate of the homologous geodesic sphere.

2. The fermionic oscillator operators associated with X^4 are linear combinations of contributions from different H modes. They satisfy anticommutation relations. It is not clear whether the creation (annihilation) operators for X^4 spinor modes are sums of only creation (annihilation) operators for H spinor modes or wheter for instance sums of the fermion creation operator and the antifermion annihilation operator apppear.

2.2 Objections

Consider now the objections against the optimistic view.

1. Also non-holomorphic modes involving $z_i^{n_1}\overline{z_i}^{n_2}$ could be present and in this case both Γ^{z_i} and $\Gamma^{\overline{z}_i}$ should annihilate the mode. This is not possible unless the metric is degenerate.

2. The spinor modes of CP_2 could make the 4-D holomorphy impossible in the proposed sense. The spinor modes of CP_2 are not holomorphic with respect to the complex coordinates of CP_2 and only the covariantly constant right-handed neutrino satisfies massless Dirac equation in CP_2. Could this imply the presence of X^4 spinor modes, which are not holomorphic (antiholomorphic) with respect to the given coordinate z_i ($\overline{z}_i$) so that the modes involving $z_i^m\overline{z}_i^n$ are possible?

3. The general plane wave basis for M^4 without Kähler form in the transversal degrees of freedom is not consistent with the conformal invariance. Here the sum over this kind of modes should give vanishing non-holomorphic modes.

   Note that the Kähler structure for M^4 adds to the M^4 Dirac equation of H a coupling to the Kähler gauge potential of M^4 and implies a transversal mass squared so that the transversal basis does not consist of plane waves but is an analog of harmonic oscillator basis. Also now the failure of holomorphy takes place.

4. For the massive modes of CP_2 spinors, massivation takes place in M^4 degrees of freedom. This would suggest that the plane waves in longitudinal M^4 degrees of freedom cannot be massless.

   However, M^8-H duality implies an important difference between TGD and ordinary field theories. The choice of $M^4 \subset M^8$ is not unique and since particles are massless at the level of H one can always choose $M^4 \supset CD$ in such a way that the momentum has only M^4 component and is massless in M^4 sense. Could the holomorphy at the space-time level be seen as the $M^8 - H$ dual of this at the space-time level?

2.3 How could one overcome the objections?

One can consider two ways to overcome these objections.

1. The sum of the contributions of products of M^4 plane waves and CP_2 spinor harmonics is involved and could simply vanish for the non-holomorphic modes. This would look like a mathematical miracle transforming the symmetry under the isometries of H to a conformal symmetry at the level of X^4. This mechanism would not depend on the choice of action although the modified Dirac equation might hold only for a unique action.

2. The 4-D conformal invariance for fermions could degenerate to its 2-D version so that only the modified Dirac equation at 2-D string world sheets would allow conformal modes. Indeed, a longstanding question has been whether this is the case for physical reasons. The restriction of the induced spinors to 2-D string world sheets is consistent with the recent view of scattering amplitudes in which the boundaries of string world sheets at the light-like orbits of partonic 2-surfaces, which are metrically 2-dimensional, carry point-like fermions. If this is really true, then the 4-D conformal invariance would effectively reduce to ordinary conformal invariance.

2.4 Solution of the modified Dirac equation assuming the generalized holomorphy

Consider now the solution of the modified Dirac equation assuming that only holomorphic modes are present.

1. The modified Dirac equation reads a

   $$(\Gamma^{z_i} D_{z_i} + \Gamma^{\overline{z}_i} D_{\overline{z}_i})\Psi = 0 \ .$$

   Γ matrices are modified gamma matrices. D_{z_i} denotes covariant derivative. Generalized conformal invariance produces the equations of the minimal surface almost independently of the action. It is however not clear whether in the modified Dirac equation the modified gammas can be replaced by the induced gamma matrices $\Gamma^\alpha = \gamma_k \partial_\alpha h^k$ (action as 4-volume). At least at the singularities that determine the vertices, this does not apply [L2].

2. The solution of the modified Dirac equation should reduce to the generalized holomorphy. This is achieved if one of the operators $D_{\overline{z}_i}$, D_{z_i}, Γ^{z_i}, $\Gamma^{\overline{z}_i}$ annihilates the given mode on the space-time surface. It follows that $\Gamma^{z_i} D_{z_i}$ and $\Gamma^{\overline{z}_i} D_{\overline{z}_i}$ for each index separately annihilate the spinor modes. Either Γ^{z_i} ($\Gamma^{\overline{z}_i}$) or D_{z_i} ($D_{\overline{z}_i}$) would do this.

   Two gamma matrices in the set $\{\Gamma^{z_i}, \Gamma^{\overline{z}_i} | i = 1, 2\}$ must eliminate a given X^4 spinor mode. Since modified gammas depend on the action, this condition might fix the action.

3. There are two cases to consider. The generalized complex structure of the 4-surface X^4 is induced from that of H [L4] or if the space-time surface is a product of Lagrange manifolds $X^2 \times Y^2 \subset M^4 \times CP_2$, is induced from the complex structures of the 2-D factors associated with their induced metrics [L5].

4. I have proposed that M^4 allows several generalized Kähler structures, which I have called Hamilton-Jacobi structures [L3]. The 4-surface could fix the Hamilton-Jacobi structure from the condition that the modified Dirac equation is valid. Since the modified gammas depend on the action, the annihilation conditions for the modified gamma matrices might fix the choice of the action, and this choice could correlate with the generalized complex structure of X^4.

To sum up, the above considerations are only an attempt to clarify the situation and it is not at all obvious that the generalized holomorphy trivializes the solution of the modified Dirac action.

3 Fermionic oscillator operators in X^4 as fermionic supersymmetry generators acting as gamma matrices of the "world of classical worlds" (WCW)

The challenge is to construct the fermionic oscillator operators in X^4 assignable to the modes of the induced spinor field in X^4.

1. By holography and the experience with quantum field theories one expects that the oscillator operators are expressible in terms of data at $t = constant$ surface and do not depend on the value of t chosen. Therefore the X^4 oscillator operators should be conserved quantities and the identification as supercharges is natural. These supercharges in turn would define the gamma matrices of "world of classical worlds" (WCW).

2. Modified Dirac equation indeed is constructed so that it has supersymmetry in the sense that conserved fermionic Noether charges associated with the isometries of H and generalized conformal transformations of H appearing as symmetries in the holography= holomorphy ansatz gave super counterparts.

   If the conserved Noether current associated with this kind of symmetry is of form $\overline{\Psi} O^\alpha \Psi$, the corresponding conserved supercurrent associated with the c-number valued mode Ψ_n of the modified Dirac equation is $\overline{\Psi}_n O \Psi$. The form of O can be deduced from the change of the modified Dirac action under the symmetry.

3.1 The Noether currents associatd with the modified Dirac action

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3. The action density associated with the modified Dirac action is given by

$$
\begin{array}{ll}
L_D = \overline{\Psi} D \Psi \sqrt{g} \ , & D = D^{\rightarrow} - D^{\leftarrow} \ , \\
D^{\rightarrow} = \Gamma^\alpha D_\alpha^{\rightarrow} & D^{\leftarrow} = D_\alpha^{\leftarrow} \Gamma^\alpha \ , \\
\Gamma^\alpha = \gamma^k T_k^\alpha & T_k^\alpha = \frac{\partial}{\partial(\partial_\alpha h^k)} L_B \ .
\end{array}
\tag{3.1}
$$

   Here L_B denotes the bosonic action density defining space-time surfaces as preferred extremals satisfying holography (analogs of Bohr orbits). The replacement of the ordinary induced gamma matrices as projections of the gamma matrices of H with the modified gamma matrices guarantees the hermicity of the modified Dirac operator and implies supersymmetry so that the conserved Noether currents for L_D are accompanied by the fermionic super counterparts.

4. The conserved Noether current associated with the symmetry $h^k \rightarrow h^k + \epsilon j^k$ can be deduced from the variation of L_D

$$
\begin{aligned}
J_j^\alpha &= (X_1^\alpha + X_2^\alpha + X_3^\alpha + X_4^\alpha)\sqrt{g_4} \ , & X_1^\alpha &= d_\epsilon \delta\overline{\Psi}\Gamma^\alpha\Psi - \overline{\Psi}\Gamma^\alpha d_\epsilon \delta\Psi \ , \\
X_2^\alpha &= \overline{\Psi}(j_A^k T_{kl}^{\alpha\beta}(\gamma^l D_\beta^{\rightarrow} - D^{\leftarrow}\gamma^l)T_{kl}^{\alpha\beta} j_A^k \Psi \ , & T_{kl}^{\alpha\beta} &= \frac{\partial}{\partial(\partial_\alpha h^k)} T_l^\beta = \frac{\partial}{\partial(\partial_\alpha h^k)} \frac{\partial}{\partial(\partial_\beta h^l)} L_B \\
X_3^\alpha &= 2\overline{\Psi}\Gamma^\alpha A_k j_A^k \Psi \ , & X_4^\alpha &= L_D g^{\alpha\beta}\partial_\beta h^k h_{kl} j_A^l \ .
\end{aligned}
\tag{3.2}
$$

5. The super current associated with J_j^α is obtained by replacing in the above currents either $\overline{\Psi}$ (or Ψ) with its c-number valued mode $\overline{\Psi}_n$ (Ψ_n).

   $\Delta\Psi$ and $\delta\overline{\Psi}$ can be deduced from the action of the symmetry transformation on spin degrees of freedom. For instance, rotations and Lorentz transformations induce spin rotation. Only the operator D has a direct dependence on h^k and $\partial_\alpha h^k$.

6. The conserved supercharges

$$
Q_j = \int_{X^3} X^3 J_j d^3x \tag{3.3}
$$

   defines the fermionic oscillator operators for X^4. Note that J_j contains the $\sqrt{g_4}$ factor defining the integration measures. By general coordinate invariance and conservation of these charges it is enough that X^3 is deformable to a section of causal diamond with constant M^4 time or light-cone proper time.

   associated with J_j^α defines a gamma matrix for WCW and a fermionic oscillator operator for the space-time surface. The oscillator operators of H spinor modes can in this way be transformed to oscillator operators of the induced spinor modes.

The modes of CP_2 Dirac operator without M^4 Kähler form have mass scale of order CP_2 mass with one exception: covariantly constant right-handed neutrino. In the presence of M^4 Kähler form also this state has mass of order CP_2 mass. Both the color quantum numbers and mass squared depend on the electroweak spin.

Unless the M^4 plane corresponds to a state, which is nearly at rest in the the rest frame of CD, its large spatial momentum implies very rapid wiggling and the contribution to the super charge as analog of Fourier component of Ψ is expected to be very small. If the state is at rest, the restriction to $t = constant$ surface guarantees that the contribution to the super charge is non-vanishing and does not depend on time t.

It should be noticed that the Feynmann propagator for an arbitrary massive fermion between a pair of points of M^4 becomes independent of the mass as the distance becomes light-like [K3] so that H spinor modes with arbitrarily high mass behave like massless particles at the boundaries of the string world sheets located at light-like partonic orbits. This would correspond to the assignment Chern-Simons-Kähler (CSK) action to the partonic orbits. The presence of M^4 part in the CSK action would allow nonvanishing light-like M^4 momenta.

3.2 About the relationship between supercharges and spinor modes of H

What can one say about the behavior of the modes of the induced spinor field? The most natural choice for the basis for holomorphic modes is such that it is of the same form as the planewave modes for H. Therefore the products of imaginary exponentials $exp(ih_i z_i)$ of "complex" coordinates $\tau_i = exp(z_i)$ and their complex conjugates assignable to the Hamilton-Jacobi structure looks like a natural choice.

The conformal weights h_i could be analogous to conformal weights. M^4 momenta would be replaced with a pair of conformal weights h_1 and h_2. For single conformal weight the natural interpretation is as mass squared and the challenge is to generalize this picture. Physical intuition would suggest h_i are real for the physical states whereas for "virtual" states h_i would be (possibly) complex algebraic numbers (I have talked about conformal confinement as a consequence of Galois confinement). If this is the case, there would be only 2 real conformal weights as opposed to 4 components for M^4 momenta (restricted by mass shell conditions).

The quantum numbers of H spinors are mapped to those of X^4. Could the conformal weights h_i correspond to the contributions of M^4 and CP_2 to the 8-momentum of M^8 and be identifiable as mass squared values for M^4 and CP_2? One cannot however assume that the M^4 and CP_2 mass squared values of H-spinors are mapped as such to h_i.

The identification $h_1 = m^2(M^4)$ and $h_2 = m^2(CP_2)$ combined with $m^2 = h_1 - h_2 = 0$ allows only massless states. $m^2 = h_1 - h_2 \geq 0$ for the physical mass squared is more plausible. p-Adic thermodynamics would give the physical mass as a thermodynamic expectation value so that positive values of $m^2 = h_1 - h_2$ are needed.

3.3 Does the presence of two conformal weights solve the tachyon problem of p-adic mass calculations

In p-adic mass calculations one assumes that physical fermion is created by the oscillator operator of H spinor mode. To this state super-Kac-Moody - or super-symplectic generator is applied to give a state with physical color quantum numbers.

One must also assume that the ground state is tachyonic with conformal weight $h = -3/2$ or $h = -5/2$. The action of Kac-Moody-/symplectic generators would compensate for the tachyonic conformal weight and give massless states as ground states. Their thermal excitations would give the physical mass as thermal mass squared. The challenge is to understand the origin of the tachyonic conformal weight.

1. For the 4-D generalization of conformal invariance, there would be two conformal weights h_1 and h_2 associated with longitudinal and transversal degrees of freedom of M^4 Hamilton-Jacobi structure [L3]. The conformal weights correspond physically to the mass squared and the identification $m^2 = h_1 - h_2 \geq 0$ for the physical mass squared could make sense. p-Adic thermodynamics would give the physical mass as a thermodynamic expectation value so that non-negative values of $m^2 = h_1 - h_2$ are needed. This would be the space-time analog for positive values of M^4 mass squared.

   Note that in the case of hadrons, longitudinal momenta of quarks are nearly massless but the transverse confinement gives rise to transversal momentum squared. The interpretation could be that the (dominating) contribution of the color magnetic body of the hadron mass makes the momentum of the state non-tachyonic.

2. In this framework, one could understand the construction of the physical states in the following way. The tachyonic ground state would correspond to a state having only the transversal contribution $-h_2$ to the mass squared and the action by Kac-Moody-/symplectic generators would add excitations with a nonvanishing h_1 and give a massless state as well as its excitations with positive mass squared. The replacement of 2-D string worlds sheets with 4-D space-time surface would solve the tachyon problem.

I have also considered an alternative approach to the tachyon problem and one can wonder if it is consistent with the proposed one.

1. As noticed, $M^8 - H$ duality involves a selection of $M^4 \subset M^8_c$. The octonionic automorphism group G_2 generates different choices of M^4. What could this freedom to choose $M^4 \subset M^8_c$ mean? How is it visible at the level of H? Since G_2 is an automorphism group, the states would be analogous to states differing by Lorentz boosts. Since these states are massless in M^8, it should be possible to find a choice of $M^4 \subset M^8_c$ for which the states are massless and thus also in $M^4 \subset H$. This choice is like going to the rest frame of a moving system in special relativity. How are these two states related at the level of H?

2. The natural proposal is that in $M^4 \subset M^8_c$ it is always possible to transform a given state with $m^2 \geq 0$ to a state with $m^2 = 0$. In the padic mass calculations this choice corresponds

to a construction of a massless state from a state which in absence of tachyons would have mass of order CP_2 mass.

The massless state would be obtained by an addition to the state of a transverse tachyonic contribution with a non-vanishing weight h_2 to give $h_1 = h_2$. The notion of mass defined as $m^2 = h_1 - h_2$ would be a relative notion like four-momentum in special relativity. Application of conformal generators would make it possible to generate states with different rest frames.

3. $SO(1,7)$ contains G_2 as a subgroup of the rotation group $SO(6) \subset SO(1,7)$. More general transformation of $SO(1,7)$ analogous to Lorentz boosts would not be allowed number-theoretically. The integer valued spectrum for m^2 allows only a discrete subgroup of G_2. In special relativity this would correspond to a discrete subgroup of the Lorentz group.

To sum up, the tachyon problem of the superstring models could be seen as the compelling reason for replacing string world sheets with 4-D space-time surfaces. The predicted two conformal weights would allow to get rid of tachyons, which also appeared in the p-adic mass calculations based on ordinary conformal invariance.

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