About TGD counterparts of twistor amplitudes: part I

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Abstract

This article is the first part of the article devoted the construction of scattering amplitudes in the TGD framework based on twistor approach. The twistor lift of TGD, in which $H = M^4 \times CP_2$ is replaced with the product of twistor spaces $T(M^4)$ and $T(CP_2)$, and space-time surface $X^4 \subset H$ with its 6-D twistor space as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$, is now a rather well-established notion and $M^8 - H$ duality predicts it at the level of M^8.

Number theoretical vision involves $M^8 - H$ duality. At the level of H the quark mass spectrum is determined by the Dirac equation in H. In M^8 mass squared spectrum is determined by the roots of the polynomial P determining space-time surface and are in general complex. By Galois confinement the momenta are integer valued when p-adic mass is used as a unit and mass squared spectrum is also integer valued. This raises hope about a generalization of the twistorial construction of scattering amplitudes to TGD context.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless.

1. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was the realization that in H quarks propagate with well-defined chiralities and only the square of Dirac equation is satisfied. For a quark of given helicity the spinor can be identified as helicity spinor.
2. M^8 quark momenta are in general complex as algebraic integers. They are the counterparts of the area momenta x_i of momentum twistor space whereas H momenta are identified as ordinary momenta. Total momenta of Galois confined states have as components ordinary integers.
3. The M^8 counterpart of the Dirac equation in H would be an octonionic Dirac equation, which is algebraic as everything in M^8 and analogous to the massless Dirac equation. The natural guess is that in H quarks satisfy the Dirac equation $D(H)\Psi = 0$. There are however excellent reasons to ask whether H spinors satisfy $D(M^4)\Psi = 0$ so that M^8 spinors as octonionic spinors would correspond to off-mass shell states with mass squared values given by the roots of P.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW). Quarks are on-shell with H momentum p and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1} - x_i$.

Dirac operator $x_i^k\gamma_k$ restricted to fixed helicity L,R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. D is expressible in terms of the helicity spinors of given chirality and gives two independent holomorphic factors as in case of massless theories.

2. The scattering amplitudes would be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals.

3. The scattering amplitudes for a single 4-surface X^4 are determined by a polynomial. The integration over WCW is replaced with a summation of polynomials characterized by rational coefficients. Monic polynomials are highly suggestive. A connection with p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common p-adic prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D.

Contents

1 Introduction

The twistor program was originally introduced by Penrose [B24]. The application of twistors to gauge theories, in particular $\mathcal{N} = 4$ SUSY, led to a dramatic progress in the mathematical understanding of these theories. For beginners like me (still), the article of Elvang and Huang [B12] is an extremely helpful introduction to twistor scattering amplitudes.

I am not a specialist in the field. Therefore the following list of works that have had effect in my attempts to understand how twistors might relate to TGD, must look rather random in the eyes of a professional. It however gives some idea about the timeline of ideas.

> Witten's work (2003) [B10] on perturbative string theory in twistor space. The proof of Britto, Cachazo, Feng and Witten (2005) [B6] for tree level recursion relation (BCFW recursion) in Yang-Mills theory. The work of Hodges (2005) [B2] about twistor diagram recursion for gauge-theory amplitudes. The works of Mason and Skinner (2009) on scattering amplitudes and BCFW recursion in twistor space [B22] and on dual superconformal invariance, momentum twistors and Grassmannians (2009) [B23]. There is also the work of Bullimore, Mason and Skinner (2009) on twistor strings, Grassmannians and leading singularities [B7]. The work of Drummond, Henn and Plefka (2009) [B9] on Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ SUSY. The work of Goncharov et al (2010) [B19] on classical polylogarithms for amplitudes and Wilson loops. Nima Arkani-Hamed and colleagues have made impressive contributions. There is a work by Arkani-Hamed et al on S-Matrix in twistor space (2009) [B15, B14]; a work about unification of residues and Grassmannian dualities (2010) [B16]; a proposal for all-loop integrand for scattering amplitudes for planar $\mathcal{N} = 4$ SUSY (2011) [?] a work on scattering amplitudes and positive Grassmannian (2012) [B13]; the proposal of amplituhedron (2013) [B5] and work about positive amplitudes in amplituhedron [B4] (2014); a proposal of MHV on-shell amplitudes beyond the planar limit (2014) [B18] ; the notion of associahedron (2017) [B3].

The TGD approach to twistors [L1, L4] [L5, L12] has developed gradually during the last decade. The evolution of ideas began with the attempt to geometrize twistors in the same manner as standard model gauge fields are geometrized in TGD. Only quite recently, the number theoretic approach to twistors has started to evolve. The twistor lift of TGD geometrizes the notion of twistor by replacing the twistor field configurations with 6-D surfaces assigning to space-time surfaces analog of its twistor space obtained by inducing the twistor structure of the product $T(M^4) \times T(CP_2)$ of the twistor spaces of M^4 and CP_2. The construction requires that these twistor spaces have a Kähler structure. M^4 and CP_2 are unique in that only their twistor spaces allow a Kähler structure [A3]. Therefore TGD is mathematically unique: the same conclusion is forced by standar model symmetries and $M^8 - H$ duality. This gives strong motivation for an attempt to construct the TGD counterparts of the twistor scattering amplitudes. The number theoretic view about twistors based on $M^8 - H$ duality [L15, L16, L24] has developed during this year (2021) and this article tries to articulate this vision and leads to a proposal for how to construct twistor scattering amplitudes in the TGD framework.

1.1 Some background

In the following, the basic facts related to twistors are described. I cannot say anything about the technicalities of the twistorial computations and my basic aim is to clarify myself the contents of the notions involved and understand how the twistors diagrammatics might generalize to the TGD context.

1.1.1 Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a \tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see `http://tinyurl.com/y6bnznyn`).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\ \left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\tilde{\lambda}^{a'}\tilde{\mu}^{b'} \ , \\ p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \quad (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ . \end{aligned} \tag{1.1}$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\ \lambda\rangle} \ , \quad \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \quad \text{negative helicity} \ . \end{aligned} \tag{1.2}$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

3. What makes 4-D twistors unique is the existence of the index raising and lifting operations using antisymmetric ϵ tensors. In higher dimensions they do not exist and this causes difficulties. For octonionic twistors with quaternionic components possibly only in $D = 8$ the situation changes.

Also massive momenta and any point of M^4 can be expressed in terms of helicity spinors but momenta different by a light-like momenta on some light-like geodesic give rise to the same twistor.

1. One has $p_{a\dot{b}} = \mu_a\tilde{\lambda}_{\dot{b}}$. The spinors μ and λ are determined only modulo opposite complex scalings. One can say that the twistor line (sphere CP_1) determines a point of M^4. A possible interpretation is that the points of CP_1 correspond to the choices of spin quantization axis for momentum p and the scaling changes its direction.

2. The incidence relation $\mu^a = p^{a\dot{b}}\lambda_{\dot{b}}$ is also true for $p^{a\dot{b}} + k\lambda^a\lambda\dot{b}$, for any k, so that the points of a light-like line in M^4 are mapped to a point of the twistor space and therefore would correspond to the same direction of spin quantization axis. Physically this could be interpreted by saying that this is the case because the points with a light-like separation are not causally independent.

Twistors allow an elegant formulation of the kinematics and the Mandelstam variable $s_{ij} = (p_i - p_j)^2 = m_i^2 + m_j^2 - 2p_i\cdot p_j$ can be expressed in terms of twistors by expressing p as

$$p = |\mu\rangle[\tilde{\lambda}| + |\tilde{\mu}]\langle\lambda|$$

Since the states are massive, the inner product $p_1 \cdot 2$ can be expressed as

$$p_1\cdot p_2 = \langle\lambda_1\mu_2\rangle\ [\tilde{\lambda}_1\tilde{\mu}_2] \ ,$$

Since <> and [] are not complex conjugates of each other and can be regarded as independent complex variables. For massless case this is not case that the expression for $p_1 \cdot p_2$ reduces to modulus squared=

The notion of momentum twistor is nicely explained by Claude Durr in the slides of a talk "Momentum twistors, special functions and symbols" (`https://cutt.ly/AY7QYv3`). Momentum twistors are essential in the twistorial construction of the scattering amplitudes.

1. The notion makes sense for planar diagrams for which the momenta can be ordered. For non-planar diagrams this is not the case. Whether the embedding of non-planar diagrams to a surface with some minimal genus could allow the ordering (if two lines which cross in plane, the other line could go along the handle), is not clear to me.

2. One ends up with the momentum twistors Z_i, as opposed to ordinary twistors denoted by W_i, by performing a Fourier transform of a massless twistor amplitude, which is holomorphic in variables $\langle\lambda_i\lambda_j\rangle$ so that the relation of the helicity spinor μ to λ is essentially that of wave vector to a position vector. The helicity spinor pair $Z=(\omega,\lambda)$, where ω is essentially the complex conjugate of λ in massless case is replaced with (ω,μ). This transform makes sense also in the massive case.

   Momentum twistors correspond to what are called dual or area momenta. The ordinary momenta p_i can be expressed as their differences $p_i = x_{i+1} - x_i$ and area momenta in turn as as $x_i = \sum_{1\leq k\leq i} x_k$. The term area momentum comes from the observation that the planar diagrams divide the plane into disjoint regions and the area momenta can be assigned to these regions.

3. At the level of symmetries the possibility of momentum twistors means extension of the algebra of conformal symmetries of M^4 to a Yangian algebra whose generators are labeled by non-negative integers and which are poly-local so that the corresponding charges contain multilocal contributions (note that potential energy is bilocal and somewhat analogous notion). The generators generating conformal symmetries in the space of area momenta correspond to generators of conformal weight $h=1$ and whereas ordinary conformal generators have conformal weight $h=0$.

Remark: TGD suggests the interpretation of two kinds of twistors in terms of M^8-H duality. Area momenta and momentum twistors could correspond to M^8 level and ordinary momenta and twistors to H level. M^8 indeed has interpretation as analog of momentum space and M^8-H duality as the TGD counterpart of momentum-position duality having no generalization in quantum field theories where momentum and position are not dynamical variables.

1.1.2 MHV amplitudes as basic amplitudes

The following comments about MHV amplitudes sketch only the main points as I see them from my limited TGD perspective. One reason for this, besides my very limited practical experience with these amplitudes, is that it seems that The TGD approach in its recent form does not force their introduction.

The article of Elvang and Huang [B12] provides an excellent summary about the construction of twistor amplitudes explaining the important details (see also the slides by Claude Durr at `https://cutt.ly/AY7QYv3`). Maximally helicity violating (MHV) amplitudes with $k=2$ negative helicity gluons are defined as tree amplitudes of say $\mathcal{N}=4$ SUSY and involve gluons and their superpartners. It is convenient to drop the group theory factor $Tr(T_1T_2\cdots T_n)$ related to gluons.

NMHV amplitudes have $k>2$ and can be classified by the number of loops as also $k=2$ diagrams. NMHV diagrams are constructible in terms of MHV diagrams and the construction is known as BSFW construction which by recursion reduces these diagrams to $k=2$ diagrams, about which 3-gluon vertices is the simplest example. To my amateurish understanding, it is not yet clear whether also the planar Feynman diagrams allow twistorialization. The basic problem is that the area moment x_i with $p_i = x_{i+1}-x_i$ must be ordered and this is not possible for non-planar diagrams.

The construction gives a recursion formula allowing to express the amplitudes in terms of MHV tree amplitudes. Rather remarkably, all loop amplitudes are proportional to the tree level MHV amplitudes so that the singularity structure of the amplitudes is completely determined by the MHV amplitudes. A holography at the level of momentum space is realized in the sense that the singularities dictate the amplitudes completely.

1. The starting point is the observation that tree amplitude with $k = 0$ or $k = 1$ vanishes. The simplest MHV amplitudes have exactly $k = n - 2$ gluons of same helicity- taken by a convention to be negative - have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle \lambda_x, \lambda_y \rangle^4}{\prod_{i=1}^{n} \langle \lambda_i, \lambda_{i+1} \rangle} \tag{1.3}$$

   When the sign of the helicities is changed $\langle .. \rangle$ is replaced with [..].

2. It is essential that the amplitudes are expressible in terms of the antisymmetric bi-linears $\langle \lambda_i, \lambda_j \rangle = \epsilon^{ab} \lambda_{i,a} \lambda_{j,b}$. This implies holomorphy and homogeneity with respect to $\langle \lambda_i, \lambda_j \rangle$ follows for massless field theories by the cancellation of the []:s or ¡¿:s of spinor inner products with [] or ¡¿ appearing in $p_i \cdot p_j$ appearing in the massless propagator.

3. $k = 2$ MHV amplitudes take the role of vertices in the construction of amplitudes with $k > 2$ negative helicity gluons. These amplitudes are connected together by off-shell propagator factors $1/P^2$. MHV diagrams allow to develop expressions for the planar on tree amplitudes and also of loop amplitudes using recursion.

4. The treatment of off-mass shell gluons forces to introduce an arbitrary fixed spinor η such that η is not a complex conjugate of λ. η is not the helicity spinor μ assignable uniquely to a massive particle (now a virtual particle). This assumption makes sense for momentum twistors assignable to internal lines of the MHV diagrams since area momenta are in general off-mass-shell.

1.1.3 Yangian symmetry, Grassmannians, positive Grassmannians, and amplituhedron

The work by Nima Arkani Hamed [B14, B17, B13, B1, B5, B25] and other pioneers has led to a very beautiful vision in which the twistorial scattering amplitudes $A_{k,n}$ for $\mathcal{N} = 4$ SUSY are expressible as residue integrals over Grassmannians $Gr_{k,n}$ of integrands which depend on the twistors characterizing the external only via delta functions forcing the integration to surfaces of $Gr(k, n)$. BCFW diagrams and therefore the Grassmannian integrals as their representations are Yangian invariants.

The amplitudes are defined as residue integrals over $Gr(k, n)$ and contain data about momenta coded by twistors in the arguments of delta functions. The counterparts of the $< ij >$ or $ij]$ determining the integrand are the Plücker coordinates defined as the k-minors, that is determinants of the $k \times n$ matrices, characterizing the point of $Gr(k, n)$. The included minors are taken in cyclic order and contain subsequent columns [B12] (`https://cutt.ly/yY7QzQg`). One integrates over the k-planes, or equivalently, over $n - k$-planes, of C^n and the integral is residue integral. $Gr(n, k) = U(n)/U(k) \times U(n - k)$ has also an interpretation as a flag-manifold. The residues are located in the positive Grassmannian $Gr_{n,k}^{\geq 0}$. The integral reduces to a mere residue selecting a special k-plane of Grassmannian (note that a gauge fixing eliminating gauge degrees of freedom due to the $Gr(k)$ and $Gr(n)$ symmetries is performed). In the massless case, the delta function constraints state that the n-helicity spinors are orthogonal to $k - D$ and $n - k$-D planes of $GR_{k,n}$ and the conditions imply momentum conservation. In the massive case, the momentum conservation constraint states $\sum p_i = |\mu_i > [\tilde{\lambda}_i| + |\tilde{\mu}]_i < \lambda|_i = 0$. Also now, the interpretation as the inner product of n-helicity spinors is suggestive. A technically important detail is that the quadratic momentum delta function $\delta(\sum_i \lambda_i lam\tilde{b}da_i)$ is forced by a product of linear delta function constraints associated with part of $Gr_(k, n)$ to two parts corresponding to k and $n - k$ gluons with opposite helicities. The gauge invariance of these parts with respect to $Gl(k)$ and $Gl(n - k)$ allows a coordinate choice in $Gr(k, n)$ simplifying the calculation drastically.

This work has led to the notions of positive Grassmannian $Gr_{k,n}^{\geq 0}$ [B12] (https://arxiv.org/abs/2110.10856) defined as a sub-space of Grassmannian in which all Plücker coordinates defined by the $k \times k$ minors appearing in the expression of the twistor amplitude are non-negative. Any $n \times (k+m)$, whose minors are positive induces a map from $Gr_{k,n}^{\geq 0}$ whose image is the amplituhedron $\mathcal{A}_{\backslash,\|,\updownarrow\Updownarrow}$ (https://arxiv.org/pdf/1912.06125.pdf and https://en.wikipedia.org/wiki/Amplituhedron) introduced by Arkani-Hamed and Trnka. For $m = 4$ the BSFW recurrence relations for the scattering amplitudes can be used to produce collections of $4k$-dimensional cells in $Gr_{k,n}^{\geq 0}$, whose images are conjectured to sub-divide the amplituhedron. $\mathcal{A}_{\backslash,\|,\updownarrow\Updownarrow}$ generalizes the positive Grassmannian. Tree-level amplituhedron can be regarded as a generalization of convex hull of external data and the scattering amplitudes can be extracted from a unique differential form having poles at the boundaries of the amplituhedron.

1.2 How to generalize twistor amplitudes in the TGD framework?

Twistor approach works so beautifully in massless case such as $calN = 4$ SUSY because the scattering amplitudes for massless gluons can be written as holomorphic homogeneous functions of arguments constructed from the helicity spinors characterizing the momenta of the external massless particles.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless. In the massive QFT, one cannot write a simple twistorial expression of the amplitudes, which would be holomorphic homogeneous polynomials in the twistor components and involve only the twistor bilinears $< ij >$ or $[ij]$. The reason is that the external and internal particles are massive. For massive particles, the Mandelstam variables $s_{ij} = (p_i - p_j)^2$ do not factorize as $s_{ij} =< ij > [ij]$.

> The intuitive TGD based proposal has been that since quark spinors are massless in 8-D sense in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was however the realization that in H quarks propagate with well-defined M^4 chiralities and only the $D^2(H)$ of Dirac operator annihilates the spinors. M^8 quark momenta are in general complex as algebraic integers. They are identifiable as the counterparts of the area momenta x_i of the momentum twistor space whereas H momenta can be identified as ordinary momenta. The total momenta of Galois confined states have as components ordinary integers and the momentum spectra in H and M^8 are identical by $M^8 - H$ duality. The mass squared spectrum is quantized as integers for Galois confined states in accordance with super symplectic invariance implying "stringy" mass spectrum. The natural first guess is that in H the free quarks satisfy the Dirac equation $D(H)\Psi = 0$. There are however excellent reasons to ask whether H spinors satisfy $D(M^4)\Psi = 0$. If so, the M^8 spinors as octonionic spinors would correspond to off-mass shell states with mass squared values given by the roots $m^2 = r_n$ of P, which in general are complex. This conforms with an idea that the super-symplectic conformal weights have an imaginary part and conformal confinement forces total conformal weights to be integers. This would give rise to twistor holomorphy.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW).

2. Both M^8 and H quarks are on-shell with H momentum p_i and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1} - x_i$. Dirac operator $x^k\gamma_k$ restricted to a fixed helicity L, R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. M^8 quarks are effectively massless but off-shell but the helicity spinors μ and λ are independent unlike for massless particles.

3. The solutions of the octonionic Dirac operator $D(X^4)$ is expressible in terms of helicity spinors of given chirality and this gives two independent holomorphic factors: in the case

of massless theories they would be complex conjugates and the other one must cancel by a spinor contraction. Quark propagator is massless and the roots of P can be interpreted as virtual mass squared values of the polynomial characterizing the interaction region and identifiable as a functional composite of polynomials P_i assignable to external particles.

4. The scattering amplitudes would be rational functions in accordance with the number theoretic vision.

5. In the TGD framework the construction of the scattering amplitudes for a single space-time surface is not enough. One must also understand what the WCW integration could mean at the level of scattering amplitudes based on cognitive representations. WCW integration would be naturally replaced by a summation over polynomials such that the corresponding 4-surface correspond at the level of H maxima of the Kähler function. Monic polynomials are highly suggestive.

6. A connection with the p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common ramified prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D. This leads to the proposal that discriminant equals the exponent of Kähler function. This forces the identification of p-adic prime as ramified prime and fixes coupling constant evolution to a high degree.

1.3 Comparison with the gauge theory picture

There are several differences between the standard twistor approach applied in gauge theories and the TGD based vision.

1. Vertices involve external H line and two internal M^8 lines. If it indeed does not make sense to speak about internal on-mass-shell quark lines in H, the BCFW construction using MHV amplitudes as building bricks and utilizing now also internal H quark lines, is not needed. One can of course ask, whether the M^8 quark lines could be regarded as analogs of lines connecting different MHV diagrams replaced with Galois singlets. It seems that also Grassmannians, positive Grassmannians, and amplituhedron are unnecessary.

2. The identification of the twistor amplitudes as Yangian invariants is extremely attractive. The proposal has been that the super-symplectic algebra (SSA) and the extended half-Kac Moody algebra of isometries acting as symmetries of WCW extend to Yangians and that the higher charges of Grassmannisans with conformal weight $h > 0$ correspond to multiparticle contributions to conserved charges with potential energy as a very familiar 2-particle example.

   Hence the TGD based construction should produce the scattering amplitudes as Yangian invariants. One cannot of course exclude the possibility that the integration over the "world of classical worlds" having interpretation as a summation over polynomials in the TGD framework could produce analogs BCFW diagrams and their Grassmannian representations. Since ordinary particles correspond basically to massless Galois singlets with mass resulting from p-adic thermodynamics, it is very natural to expect that the QFT limit of TGD is a massless QFT. At this limit, the twistor Grassmannian approach would be very natural.

3. M^4 conformal invariance characterizes massless gauge theories and the ordinary twistor approach. M^4 conformal invariance is not a symmetry of TGD if the quark spinor modes are annihilated by the Dirac operator $D(H)$ so that they are massive. However, M^8-H duality [L15, L16, L24] and associativity as the basic dynamical principle at the level of $H = M^4 \times CP_2$, masslessness of quarks in QCD, and some facts about neutrino physics [L22] force to ask whether the Dirac operator $D(M^4)$ could determine the propagation of quarks as massless particles. This would also give rise to the twistor holomorphy. The mass squared values defined as roots of P would correspond to virtual masses. The Yangians would be associated with various super-symplectic algebras and for this option also with the M^4 conformal group.

4. In the TGD framework, the loop corrections are predicted to vanish and the scattering amplitudes for a given space-time surface would therefore be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals. Also this supports the view that Grassmannians are not needed.

2 TGD related considerations and ideas

The goal is to generalize twistorial construction of scattering amplitudes in the simplest possible manner to the TGD framework. One of the key challenges is the twistorial description of massivation. In this section I summarize briefly the ideas of TGD which seem to be relevant for the construction of the twistor amplitudes.

2.1 The basic view about ZEO and causal diamonds

In the following are listed the ideas and concepts behind ZEO [K12] that seem to be rather stable.

1. General Coordinate Invarince (GCI) plays a crucial role in the construction of the Kähler geometry of WCW and implies holography, Bohr orbitology and zero energy ontology (ZEO) [L11, L24] [K12].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. A finite failure of determinism is however possible and is discussed in [L26]. Preferred extremals would be simultaneous extremals of both volume action and Kähler action outside singularities and thus minimal surfaces analogous to soap films spanned by frames. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD=$cd\times CP_2$) defined as intersection of future and past directed light cones (cds) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. ?? Fig. ??**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation - *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in the fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements (`https://cutt.ly/nURW3QE`). One could call this half-cone call as a passive half-cone. I have also talked about passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate identifiable as geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L8] as observed by Minev et al [L8].

7. h_{eff} hierarchy [K6, K7, K8, K9] implied by the number theoretic vision [L15, L16] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L9]: dissipation with reversed arrow of time looks like self-organization [L27].

2.2 Galois confinement

The notion of Galois confinement emerged originally in TGD inspired quantum biology [L27, L18, L19, L21]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L18, L21, L20]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L25] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L25]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

It seems that Galois confinement might define a notion, which is much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L10, L17, L3].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states? The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have some kind of reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

Also other quantum numbers, in particular color, could correspond to representations of the Galois group. In the case of angular momentum, Galois confinement would allow algebraic fractional angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L, the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential phase factors, which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers, the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

2.3 No loops in TGD

There are several arguments suggesting that there is no counterpart for loops of quantum field theories (QFTs) in TGD. Purely rational scattering amplitudes are required by number theoretic vision but the logarithmic corrections from loops would spoil the number theoretic beauty.

Loops however give rise to coupling constant evolution, which is a physical fact. What could be the TGD counterpart of coupling constant evolution?

1. The number theoretic and p-adic coupling constant evolutions, which are discrete rather than continuous, look natural. The effective coupling constant should be renormalized because the allowed momentum exchanges depend on the roots of a polynomial P or at least on their number. If the p-adic prime p corresponds to a ramified prime of extension, the dependence of the effective coupling parameters on the extension of rationals defined by P implies dependence on the prime p characterizing the p-adic length scale. The emerging picture will be described in more detail in the next section.

   In the scatterin amplitudes, a power of coupling g identifiable as Kähler coupling constant g_K appears. Also the factors from Galois singlets appear as well as the states, which correspond to the super-symplectic representations.

   It seems that for given external momenta a sum of several terms appear. If the number of momenta is small, a higher dimension of extension gives a larger number of diagrams and this could lead to number theoretic coupling constant evolution. If a given extension of rationals prefers some p-adic primes, mot naturally the ramified primes of the extension, number theoretic coupling constant evolution translates to a p-adic coupling constant evolution.

2. Does the integration over the WCW give Kähler coupling strength and various couplings or is Kähler coupling present at vertices from the beginning? The latter option would look natural. $M^8 - H$ duality strongly suggests that the exponent $exp(-K)$ of Kähler function K defining vacuum functional has a number theoretic counterpart. The unique counterpart would be the discriminant of the polynomial P and suggests that the value of $exp(-K)$ is equal to discriminant for maxima of K, which would naturally correspond to the space-time surface defining the cognitive representation.

2.4 Twistor lift of TGD

One could end up with the twistor lift of TGD from problems of the twistor Grassmannian approach originally due to Penrose [B24] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B11, B6, B18, B3]. For a very readable representation see [B12].

Twistor lift of TGD [L2, L13, L14] generalizes the ordinary twistor approach [L6, L7]. The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense. This gives hopes about massive twistorialization.

The basic recipe is simple: replace fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

he pleasant surprise was that the wistor space has the necessary Kähler structure only for M^4 and CP_2 [A3]: this had been discovered already when started to develop TGD! Since the Kähler structure is necessary for the twistor lift of TGD (the action principle is 6-D variant of Kähler action), TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory [L4]. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

2.5 Yangian of supersymplectic algebra

The notion of Yangian for conformal symmetry group of Minkowski space plays a key role in the construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY as Yangian invariants. There are excellent reasons to expect that also in TGD the scattering amplitudes are Yangian invariants.

2.5.1 Yangian symmetry

The notion equivalent to that of Yangian [A4] [B8, B9, B21] was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras.

The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [L1]. Besides ordinary product in the enveloping algebra there is co-product Δ, which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B8]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with the discrete index n being replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h.

Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2.5.2 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, there is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A1] and Virasoro algebras [A2] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras.

2. In the twistor approach conformal symmetries of M^4 are crucial. The isometries of H do not include scalings and inversions. The massless states of the super-symplectic representation would allow conformal invariance of M^4 as dynamical symmetries.

   There are however several alternatives.

   (a) The spectrum of the Dirac operator $D(H)$ contains only right-handed neutrino ν_R as a massless state and if M^4 Kähler structure is assumed it becomes tachyon.

   (b) The second option is that $D(M^4)$ annilates spinor modes. Dirac propagator would reduce to a delta function in CP_2 degrees of freedom. This option is favored by $M^8 - H$ duality and also by the associativity of the octonionic spinors implying that M^8 momenta reduce to M^4 momenta. This is actually achieved by a suitable choice of $M^4 \subset M^8$ always.

   (c) If $D(M^4)$ contains no coupling to M^4 Kähler gauge potential $A(M^4)$, on-mass-shell quarks are massless and realize M^4 conformal invariance. The appearance of roots polynomials as mass squared values in quark propagators would realize number theoretic breaking of M^4 conformal invariance at the level scattering amplitudes and allow twistor holomorphy.

   If $A(M^4)$ coupling is present, all quarks appear as spin doublets with positive and negative mass squared. M^4 conformal symmetry at the quark level is achieved only at long length scales when the spin term vanishes. The quark propagator in the scattering amplitudes would contain the coupling to $A(M^4)$ so that twistor holomorphy seems to be lost. M^4 gauge potential could explain small CP breaking, and one can imagine that the induced M^4 gauge potential appears only in the modified Dirac equation for the induced spinors.

3. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($cd \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones.

   The polygon with light-like momenta would be naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

4. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $cd \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups.

   This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

2.5.3 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

2.5.4 How could the Yangian structure of the super-symplectic algebra emerge?

The isometries of WCW should generalize conformal symmetries of string models and supersymplectic transformations of the light-like boundary of CD are a highly natural candidate in this respect.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K2].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

Consider next the general structure of the super-symplectic algebra (SSA).

1. The SSA and the TGD analogs of Kac-Moody algebras assignable to light-like partonic 3-surfaces have the property that the conformal weights assigned to the light-like coordinate r are non-negative integers. One can say that they are analogs of "half"-Kac-Moody algebras. Same holds true for the Yangian algebras, which suggests that these algebras could extend to Yangian algebras.

2. SCA (and also the Kac-Moody analogs) has fractal hierarchies of sub-algebras isomorphic to the algebra SSA itself at the lowest level. The conformal weights of sub-algebra SSA_n an n-multiplets of those of SSA: one obtains hierarchies of sub-algebras $SSA \supset SCA_{n_1} \supset SSA_{n_2 n_1}, ...$

3. This leads to the proposal that there is a hierarchy of analogs of "gauge symmetry" breakings. For the maximal "gauge symmetry", the entire SSA annihilates the states and classical Noether charges vanish. For SSA_n, only SSA_n and the commutator $[SSA_n, SSA]$ annihilate the physical states.

   One can ask whether these hierarchies could correspond to the hierarchies of extensions for rationals defined by the composition of polynomials defining 4-surfaces in M^8 and by M^8-H duality in H.

Cognitive representations play a key role and correspond to many quarks states.

1. Cognitive representations consist of the points of $X^4 \subset M^8$ with $M^4 \subset M^8$ coordinates belonging to an extension of rationals defined by a polynomial P defining X^4. It has become clear that here only the mass shells corresponding to the roots r_n of P need to be considered and that only algebraic integers defining the components of M^4 momenta need to be considered.

2. Cognitive representations consist of only those points which are "active", i.e. contain quark or antiquark. $M^8 - H$ duality maps the cognitive representations to H. The points of a given mass shell to the light-like boundary of CD. Momentum p as a point of $M^4 \subset M^8$

is mapped to a geodesic line starting from the center of CD and yields the image point as its intersection with the boundary of CD. The momenta at a given mass shell are actually mapped to the boundaries of all CDs forming a Russian doll hierarchy with common center points.

3. The cognitive representation codes for the physical states in quark degrees of freedom and should reflect themselves in the properties of the SSA state construction. The natural condition is that the Hamiltonians of SSA generate transformations leaving invariant the image points of cognitive representation at the boundaries of CD. This requires that the Hamiltonians vanish at the points of the cognitive representation. This is achieved if the Hamiltonians are obtained by multiplying the usual Hamiltonians, which can be chosen to define irreducible representations of $SU(2) \times SU(3)$, by a Hamiltonian H_{cogn}, which vanishes at the points of the cognitive representation.

   The condition that also the super-generators vanish at the points of cognitive representation implies that also the corresponding Hamiltonian vector field j vanishes so that at the points of cognitive representation all Hamiltonians vanish and are extrema. One would have a modification of the hierarchy of SSA_n but the gauge conditions would remain as such. These conditions could be regarded as a realization of quantum criticality.

4. The cognitive representation defined by the multi-quark states in M^8 would modify the SSA in H by multiplying its Hamiltonians with H_{cogn}. The level of WCW the role of the subalgebra SSA_{cogn} defined by cognitive representation would be similar to the algebra of isotropy group $SO(3)$ of particle momentum as a subgroup of $SO(3,1)$.

   This suggests that the induction procedure generating the irreducible representations for finite-dimensional Lie groups generalizes. The representations of $SO(3)$ have as an analog the representations of SSA_{cogn}. From these representations one would obtain by general symplectic transformations states analogous to the Lorentz boosts of a particle at rest. Note that for cognitive representations the Galois group acts non-trivially but one would have Galois singlet. One could have it in geometric sense so that the momenta would simply add up as vectors or in quantum sense as a many-quark state, with quarks at different points of the mass shell or at different mass shells.

How could one understand the generalization of the duality between momenta and area momenta?

1. The duality between ordinary momentum space and area momentum space means that dual conformal transformations act on area momenta x_i as symmetries of the scattering amplitudes. At the level of ordinary momenta this symmetry extends conformal symmetry algebra to a Yangian algebra.

2. Is this possible in the case of M^8-H duality? Does SSA realized at CD boundaries have a counterpart at the $M^4 \subset H$ mass shells? The counterparts of SSA transformations in M^8 must map the mass shells to itself and leave the points of the cognitive representation invariant. In the interior of $X^4 \subset M^8$ they would induce a deformation of X^4 consistent with the assumption that X^4 is obtained as a local element of $CP_2 = SU(3)/U(2)$, i.e. the deformation is induced by $SU(3)$ element $g(x)$ acting as octonionic automorphism such that $U(2) \subset SU(3)$ leaves the image point invariant. This would guarantee M^8-H duality.

   This deformation at the mass shell would induce in $X^4 \subset H$ an action having interpretation in terms of a local $SU(3)$ (CP_2) transformation, or possibly an symplectic transformation of CP_2 local with respect to light-cone. At the level of H one has group symplectic transformations of $S^2 \times CP_2$ expressible in terms of Hamiltonian in irreps of $SU(3)$.

3. Could the local $SU(3)/U(2) = CP_2$ transformations be representable as symplectic transformations as the duality would suggest? Does this somehow relate to the facts that both CP_2 and its twistor space $SU(3)/U(1) \times U(1)$ have Kähler structure [A3] and therefore also symplectic structure: this in fact makes CP_2 and M^4 completely unique.

4. What about the M^8 counterparts S^2 Hamiltonians. Could they somehow correspond to quaternionic automorphism group $SO(3)$. Could $SO(3)$ correspond to the allowed symplectic (contact) transformations for the mass shell itself whereas $SU(3)$ would act in the interior of $X^4 \subset M^8$?

The dual conformal transformations induce bilocal transformations in the ordinary Minkowski space and this leads to the notion of Yangian, which also implies higher multi-local actions. Why would be the physical origin of this multilocality?

1. Quantum group structure is involved and bi-local elements should correspond to tensor products $f_{abc}T^b \otimes T_c$ of Lie-algebra generators. This generalizes to higher multilocal states. Galois confinement is a multilocal phenomenon in M^8. $M^8 - H$ duality maps this multilocality to H. The simplest bi-local state is the quark-antiquark pair with total momentum which is an ordinary integer (necessarily non-tachyonic even if the roots r_n had negative real parts). Leptons would be tri-local states of quarks in CP_2 scale.

   The multilocality of the Galois confined many quark states in M^8 strongly suggests that the total charges include, besides the 1-local contributions, there are also multilocal contributions to Noether charges.

2. Galois confinement should force the multilocality of the symmetry generators. In particular, since the total momenta of quarks sum up to an ordinary integer, one cannot perform Lorentz transformations for them independently but one must transform several momenta simultaneously in order to guarantee that the total momentum changes in such a manner that Galois confinement condition is satisfied.

   The Galois group acts also on spinors which can have number theoretic analogs of spinor space assignable to algebraic extensions as linear spaces and providing a finite-D number theoretic counterpart for WCW spinors. Therefore the generators of Lorentz transformations must contain bi-local and also n-local terms. Same applies to scalings and conformal transformations and in fact to all other symmetries.

3. In the case of energy, these multilocal contributions could have an interpretation as binding energy or potential energy depending on the distance between the image points of different momenta at the boundary of CD. The question is how these multilocal contributions would emerge in H for the super-symplectic algebra having a representation as classical Noether charges and fermionic Noether charges.

4. The notion of gravitational coupling constant suggests strongly that conserved quantities have besides the local contribution also bilocal contribution for which gravitional Planck constant defines unit of quantization. A possible identification is as a bilocal Yangian contribution.

In $\mathcal{N} = 4$ SUSY, scattering amplitudes are invariants of the Yangian defined by conformal transformations of M^4 and its dual acting in the space of area momenta. Since SSA is proposed to act as isometries of the "world of classical worlds" (WCW), also zero energy states having interpretation as scattering amplitudes should be Yangian invariants.

2.6 $M^8 - H$ duality and twistorialization of scattering amplitudes

The precise formulation of twistor amplitudes has remained a challenge although I have considered several proposals in this direction. The progress made in the understanding of the details of M^8-H duality [L24] motivate the attempts to find more explicit formulation for the scattering amplitudes. The following tries to give a brief overall vision.

1. In its recent form $M^8 - H$ duality predicts the twistor spaces of M^4 and CP_2 and their map to each other having interpretation in terms of 6-D twistor spaces of space-time surfaces as 6-surfaces in the product of the twistor spaces of M^4 and CP_2 replacing space-time surfaces with their twistor spaces in the twistor lift of TGD [L24].

2. Momentum twistors and space-time twistors are related by M^8-duality. M^8 momenta are identified as area momenta different from M^4-momenta in H. The notion of area momentum makes sense only for planar diagrams (it is not clear to me whether the imbedding of diagrams genus g topology could allow a definition of area momentum).

3. In the usual twistor Grassmann approach to massless QFTs, the momenta of internal lines are massless and thus on-mass-shell but complex. The simplest option conforming is that both area momenta x_i and H-momenta p_i are on-mass-shell. Area momenta are indeed in general complex as algebraic integers. For a given polynomial P area mass squared spectrum of quarks is fixed as - in general complex - roots of polynomial P.

4. What looks first like a problem is that H momenta have naturally integer valued components (periodic boundary conditions) and mass squared is integer using a suitable unit determined by the p-adic length L_p for the CD. However, at the M^8 side the momenta have components which are algebraic integers in the extension determined by the polynomial P.

   A natural solution of the problem is provided by Galois confinement requiring that momentum components of confined states, which are Galois singlets, are integer valued rather than algebraic integers. This provides a universal mechanism for the formation of bound states. This allows also to have identical spectra for area momenta and ordinary momenta.

   In this picture, the particle would be a Galois singlet formed as a composite of quarks. This notion of a particle is extremely general as compared to the QFT view about elementary particles. The external lines of twistor diagrams carrying H quantum numbers would correspond to states in the representations of super-symplectic algebra (SSA) with Yangian structure.

5. The second quantization for quark fields of H means an enormous simplification. One avoids all problems related to quantization in a curved background. Here an essential role is played by the Kähler structure of M^4 forced by the twistor lift. The generators of supersymplectic algebra and generalized Kac-Moody algebras can be expressed in terms of quark oscillator operators.

6. For given H momenta, the momentum transfers are fixed by $p_i = x_{i-1} - x_i$. The twistor sphere S^2 characterizes the momentum directions. Momentum plus S^2 point s characterized by helicity spinor, defines a point in the twistor space and the geometric interpretation for s is that it characterizes the direction of spin quantization axis.

   The direction of quantization axies is defined only apart from a sign and for spin 1/2 particles the interpretation is as the sign of the spin projection. For massless states the spin axis is parallel to momentum.

7. Galois confinement is crucial. The conditions allow integer valued H momenta only if the area momenta correspond to Galois bound states of quarks. Entire composite of quarks at the same mass shell propagates as particle with total momentum which has integer components. By duality one can assign to the momentum p_i quantum numbers in supersymplectic representation.

   Clearly the notion of a particle as a Galois singlet is very general and corresponds to a multilocal state in both M^8 and H leading also to the notion of Yangian. In H, a particle is a state of a super-symplectic representation. At the level of M^8 it is a Galois confined state. These states correspond to each other.

The basic ideas related to the construction of scattering amplitudes are as follows.

1. $M^8 - H$ duality remains as such. $M^8 - H$ duality maps. Total area momenta X_i of Galois confined states to points at the boundary of corresponding CD with size determined by the total area momentum by $M^8 - H$ duality.

2. Basic vertices for Galois confined states involve many-quark Galois singlet in H with total momentum P_i and 2 many-quark Galois singlets in M^8 involving area momenta X_i and X_{i+1} satisfying $P_i = X_{i+1} - X_i$. The scattering amplitude reduces to quark level and one can say that quark lines connect different mass shells of $X^4 \subset M^8$.

3. 3-vertices are between two M^8 Galois singlets and super-symplectic Galois singlet in H at different M^8 mass shells and lines connecting them carrying momenta calculated at the level of H. Quarks in Galois singlets have collinear rational parts which are analogous to SUSY where monomials of theta parameters assignable to higher spin states are analogous to collinear many-fermion states.

3 Are holomorphic twistor amplitudes for massive particles possible in TGD?

Massive particles are believed to make twistorialization impossible. For instance, for a scalar field theory with Yukawa coupling to fermions, the part of scattering amplitude involving vertex with Yukawa coupling plus scalar propagator gives $g < 12 > \times 1/(p_1 - p_2)^2$. For massless particles, one has $(p_1 - p_2)^2 =< pq > [pq]$ and the expression reduces to $g/ < pq >$. This is essential for the holomorphy in twistor components in turn reflecting conformal invariance.

In MHV construction the MHV amplitudes with 2 negative helicities are used as building bricks of twistorial representations of more complex planar tree amplitudes and loop amplitudes connecting them with off-mass-shell lines involving propagators. The obvious question is whether this construction could be generalized.

The simplest MHV diagrams would be replaced with diagrams assignable to single CD and involving only on-mass-shell area momenta in M^8 and on-mass-shell area momenta in H as external particles. One would take several diagrams of this kind and connect them by a line carrying off-mass-shell M^8 momentum and quantum numbers of a state in SSA representation. In a given vertex involving this kind of virtual H-line, the on-mass-shell fermion momenta would be replaced by two 2 on-mass-shell area momenta and off-mass-shell momentum of the scalar particle would correspond to M^8 momentum.

The intuitive idea is that somehow 8-D massless at the level of H solves the problem but it is not at all clear whether it is possible to obtain twistor holomorphy somehow. One hint comes from the fact that twistors associated with massive particles involve two independent helicity spinors μ and λ? Could one have holomorphy with respect to both? A further hint comes from the observation that at the level of H tachyonic right-handed neutrino makes possible the construction of massless states. A further hint comes from Galois confinement: could the external particles be Galois confined states and could the propagating particles be quarks in M^8 having complex masses coming as roots of the polynomial P?

3.1 Is it possible to have twistor holomorphy for massive scalar and fermions?

Consider first the simple example of massive fermions and a massive scalar field. Assume that fermions are on-mass-shell with masses m_1 and m_2 and scalar off-mass-shell with mass m.

1. Assume Dirac spinors expressible in terms of left and right handed components. For massive scalar particle, the propagator factor reads as $(p_1 - p_2)^2 - m^2 = m_1^2 + m_2^2 - m^2 - 2(p_1 \cdot p_2)$.

2. The completeness relation for spinor modes reads in massive case as $p^k\gamma_k + m = O(p)$, $O(p) = |p\rangle [p| + |p[\langle p|$

   One can express $O(p)$ as $p^k\gamma_k = O(p) - m$. One obtains for Dirac spinor with left and right handed parts

   $$2p_1 \cdot p_2 = \frac{1}{4}Tr[(O(p_1) - m)(O(p_2) - m)] = -m^2 - \frac{1}{4}Tr[O(p_1)O(p_2)] \ .$$

   For

   $$m_1^2 + m_2^2 = 2m^2 \ ,$$

   the propagator factor reduces to $1/(Tr(O(p)O(q)) =< pq > [pq]$ as if the particles were massless. The part of the amplitude considered would reduce to $g < pq >$.

3. Could the masses for the generalized twistor diagram satisfy a generalization of the condition $m_1^2 + m_2^2 = 2m^2$ guaranteeing the holomorphy with respect to $< .. >$ or $[..]$? The prediction for spinors would be an effective prediction of massless QFT. Note that this result is also true when the masses are identical. This in turn might relate to SUSY. The additivity of mass squared values might in turn relate to 2-D conformal invariance in which mass squared operator is scaling generator and mass squared values are conformal weights. 2-D conformal invariance would generalize to its 4-D counterpart.

   Could this picture generalize to TGD in such a manner that external on mass states correspond to states constructed in H area momenta are off-mass-shell? It is easy to see that this generalization does not work as such.

3.2 Scattering amplitudes in a picture based on $M^8 - H$ duality

The basic assumptions are inspired by $M^8 - H$ duality, ZEO, and geometric view about helicity spinors.

The first guess is that area momenta x_i are assignable to M^8 quarks and are at complex mass shells $m^2 = r_n$. x_i algebraic integers in the extension determined by a polynomial P. Galois confinement implies that the quark momenta associated with mass shells belong to quark composites forming Galois singlets and have a total momentum, which is integer valued with respect to the p-adic mass scale assignable to the mass shell. Also mass squared values would be integers. For general Galois singlets the momenta are assignable to several mass shells $m^2 = r_n$ and thus multi-local entities in M^8, which suggests possible origin of the Yangian symmetry. The mass shells are mapped to the boundaries of corresponding CD in H by $M^8 - H$ duality mapping p-adic mass scale m to its inverse defining p-adic length scale $L = \hbar_{eff}/m$ implying multi-locality in H. CDs form a Russian doll-like structure.Assume that the incoming momenta p_i are H assignable to supersymplectic representations constructed from spinor harmonics in H for a second quantized quark field. $M^8 - H$ duality suggests that the momentum and mass squared spectra are identical at M^8 and H sides. This conforms with Galois confinement at M^8 side. Particles would be Galois confined multi-quark states. Assume that twistors and momentum twistors have a geometric interpretation so that helicity spinors do not represent fermions but points in the CP_1 fiber of CP_3 as a bundle and the states with given spin correspond to wave functions in CP_2 having also half-integer spins. Twistor amplitudes would be constructed as contractions of these wave functions with the scattering amplitudes that the basic scattering amplitude would be independent of spin. In this framework, the many-quark states constructed by elements of Clifford algebra would be analogous to components of a super-field. By Galois confinement, the rational parts of quark momenta would be collinear, which conforms with the basic idea of SUSY that n-monomials of theta parameters are analogous to states of p collinear fermions. The spin of a given state would correspond to a product of spin 1/2 sherical harmonics in the space defined by the helicity spinor. A huge generalization of the notion of particle would be in question. Particle would correspond to an arbitrary Galois singlet assignable to single CD. This would conform with the WCW picture in which physical states of the Universe correspond to WCW spinor fields identified as zero energy states. Vertices would correspond to the states of Yangian supersymplectic representation identifiable as mode of WCW spinor field and representing general fermionic state analogous to a component of super field but without Majorana condition. In the standard model, all couplings except the coupling of Higgs to itself and to fermions respect helicity conservation. Assume that this is true also in TGD so that one can decompose quark spinors to left and right handed parts and that they can be described by spin wave functions in the fiber of twistor space corresponding to the momentum of the quark. Note however that the helicity twistors would be purely geometric quantities rather than representing spinor basis of a fermion. At the level of the twistor space of H, spin states would be described by partial waves at the twistor sphere. At the level of M^8 twistor space, a completely geometric description as a point of twistor space characterizing momentum and spin quantization axis and the sign of the spin 1/2 projection is possible. Helicity spinors μ and $\tilde{\lambda}$ would characterize the direction of the spin quantization axis as a point twistor sphere S^2. This conforms with the fact that for massive particles the direction of helicity spinor is not unique since the spin μ is determined only apart from a spinor proportional to λ. For massless particles the direction of the quantization axis is unique. Since only quarks with spin 1/2 are fundamental fermions, the twistor sphere with a fixed radius is enough. This interpretation is similar to the interpretation of the twistor sphere of $SU(3)/U(1) \times U(1)$ as a characterizer of the color quantization axes. For many-quark states a common quantization axis would force the spins to be parallel or antiparallel. The sum of spins associated with different momenta as different points of twistor space would be the sum of these spins.

The special twistorial role of quarks as spin 1/2 particles supports the idea that the construction of scattering amplitudes should be reduced to quark level although the physical states are Galois singlets. The situation would be very similar to that in QCD, where the challenge is to understand how the scattering amplitudes between hadrons are constructible in terms of scattering amplitudes for quarks and gluons. The basic problem in QCD is that a mechanism for the formation of bound states is missing: in TGD it is provided by Galois confinement.

The basic assumption is therefore that the quarks in M^8 are on-mass-shell states with $m^2 = r_n$. If Galois singlets were regarded as fundamental objects, one would encounter problems with the description of spin degrees of freedom. Situation is essentially the same as in hadron physics.

One can speak about Galois singlet states as a generalization of super-field but without Majorana conditions with oscillator operator monomials replacing the components of superfield: Galois singlets having quark momenta with parallel rational components would in this sense propagate linearly. Each quark Dirac operator $p^k\gamma_k$ is added to the vertex and is expressible in terms of a pair of holomorphic quantities $< .. >$ and [..] which are independent for massive quarks.

3.3 Twistor amplitudes using only mass shell M^8 momenta as internal lines

The simplest proposal for the twistor amplitudes assignable to single 4-surface assumes that the physical particles correspond to Galois singlets with integer valued momentum components p_i and integer valued mass squared spectrum. The components of quark momenta in M^8 would be algebraic integers.

$M^8 - H$ duality requires that physical states in M^8 and H correspond to each other and have the same mass and momentum spectrum. A stronger form of $M^8 - H$ duality would force the identification of the quark momenta in M^8 and H. Quark momenta would be virtual momenta. If the coupling to M^4 Kähler potential is not present, the twistor holomorphy is achieved if spinor modes satisfy $D(M^4)\Psi = 0$.

3.3.1 What could be the basic assumptions?

The following summarize the assumptions, which look plausible.

1. All quark states in both H and M^8 are on-mass-shell states with momenta which are algebraic integers in the extensions determined by polynomial P determining the quark mass shells $m^2 = r_n$ as its roots. Momenta for Galois singlets could also be rationals but periodic boundary conditions allow only integers.

   The physical states are Galois singlets with integer valued momenta in a given p-adic length scale. Mass squared values are integers and one obtains a stringy mass squared spectrum. By $M^8 - H$ duality the spectra at M^8 and H sides are identical.

2. The analog of the idea that the scattering amplitudes are poles of residue integral in momentum space is adopted. This means that in M^8 the purely algebraic 4-D quark Dirac operators $D(M^4)$, rather than propagators as in Feynman diagrams, act on the vertex defined by the trilinear of 3 Galois singlets (particles do not propagate in momentum space as they do in x-space!). The Galois singlets have an interpretation as representations of super-symplectic algebra.

   The Galois singlet with total momentum $P_i = \sum p_{i,k}$ corresponds to H-state and the two other Galois singlets corresponds to states with area momenta X_i, X_{i+1} having similar decompositions $X_k =\sim x_{i,r}$ in terms of in general complex algebraic integer valued area momenta

x_i. The complex on-mass-shell area momenta are analogous to the complex on-mass- shell light-like virtual momenta in the twistor Grassmann approach.

3. The total momentum of the vertex is conserved and gives a constraint on the quark momenta associated with the 3 states. In each vertex one has sum over all possible quark momenta consistent with the Galois singlet property and the structure of the state. Momentum conservation at vertex does not make sense at quark level since fermion number conservation would fail unless one introduces fundamental bosons.

   Momentum conservation constraints $P_i = X_{i+1} - X_i$, which completely fixes the momentum exchanges as $2X_i \cdot X_j = P_i^2 - X_{i+1}^2 - X_i^2 - 2(X_i - X_j)^2$. Momentum conservation implies in ZEO that one can see scattering diagrams as polygons having momenta at mass shells at the half-light-cones of M^8.

4. An essential constraint is that the rational parts of the area momenta x_i are parallel to each other. This gives rise to an analogy with supersymmetry in which one could regard the higher components of the super field as parallelly propagating Majorana fermions.

5. The propagator lines correspond in M^8 to vertex factors with the analog of $D = x_i^k \gamma_k$ acting on Galois singlet i. This would mean that one has a residue of the Feynman propagator. By adding a multiplicative factor m^2, one could equally well use Feynman propagator $1/D = D/m^2$, where $m^2 = r_n$ is quark mass squared. The number of diagrams is limited by the number of roots and only the number of Galois singlets poses a limit to the summation if one considers only amplitudes for a single surface X^4.

   In principle all pairs of Galois singlets in M^8 with a non-vanishing trilinear overlap with a given Galois singlet in H are allowed in the vertex. Note that same Galois singlets can contain quarks assignable to different quark mass shells $m^2 = r_n$.

6. The details of the algebraic extension are not visible in the properties of Galois singlets as analogs of hadrons. The details of algebraic extension are however visible in the details of quark propagators and give rise to a number theoretic coupling constant evolution as will be found. Also the increase of the dimension of extension with the degree of P implies that the number of contributing diagrams increases.

   In principle, also roots r_n with negative rational parts are possible and one cannot exclude tachyonic states. From tachyonic states one can form non-tachyonic ones by requiring that the 3-momenta sum up to zero.

7. The big difference with respect to standard massive QFTs is that although the states are massive, they propagate with well-defined helicities. There is therefore a doubling of helicity spinors appearing as L-R degeneration. The division to positive and negative helicities corresponds to the presence of quarks and antiquarks.

8. It seems that quarks and antiquarks can correspond to the same CD and to the same diagram of the proposed kind. For a single space-time surface BCFW construction does not make sense since it would require an off-mass-shell H particle. One must however notice that the quark propagators bring in mind the $1/P^2$ lines connecting BCFW sub-diagrams and Galois singlets bring in mind the MHV diagrams.

   Can one construct Galois singlets from both quarks and antiquarks? It would seem that in this case the scattering amplitudes involve products of holomorphic and antiholomorphic monomials of the twistor variables. This option looks intuitively more plausible.

3.3.2 A possible solution of the mass problem

The basic problem of the twistor approach is that physical particles are not massless. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes.

1. The first key observation stimulated by the recent findings about right-handed neutrino candidate [L22] was that although neutrinos are massive, their right-handed component has not been observed. This leads to a proposal that in H quarks should propagate with well-defined chiralities so that only the square of Dirac equation $D^2(H)\Psi = 0$ is satisfied.

2. At the level of M^8 the octonionic M^4 quark spinor reducing to a quaternionic spinor corresponds to H spinors. A spinor with a given chirality can be identified as a helicity spinor λ_{dota} and is annihilated by the operator $p^{a\dot{b}} = \mu^a\lambda^{\dot{a}}$. This makes sense by the fact that in the TGD Universe quarks are the only fundamental particles implying that all other particles, including elementary particles, emerge as their many particle states as Galois singlets.

   The M^8 counterpart of the 8-D massless condition in H is the restriction of the quark momenta to mass shells $m^2 = r_n$ determined as roots of P. The M^8 counterpart of Dirac equation in H is octonionic Dirac equation, which is algebraic. The solution is a helicity spinor $\tilde{\lambda}$ associated with the massive momentum p.

3.3.3 What about tachyons?

Polynomials P allow also roots r_n, which are negative and correspond to tachyonic mass shells. Should one restrict the roots inside the future light-cone? Should one require that the mass squared values of the masses of Galois singlets are non-negative integers? In principle, one can have integer valued momenta with tachyonic mass squared. The sum of this kind of momenta however gives always a non-tachyonic state if the energies are of the same sign as they are for a given half-light-cone.

1. M^4 Kähler structure implies that covariantly constant right- handed neutrino in CP_2 is a tachyon [L22]. This gives rise to the highly desired tachyon required by p-adic mass calculations [K3, K1]: with it the scale of mass spectrum would be huge and given by CP_2 mass. Tachyonic property is not consistent with the unitarity and ν_R cannot appear as a free particle.

2. Situation remains the same if the right-handed neutrino spinor mode is a good approximation for a Galois and color singlet of 3 quarks assignable to the same wormhole throat in H. ν_R as Galois singlet with tachyonic mass can be understood if tachyonic mass squared values are allowed for quarks.

   Could all quark masses could be tachyonic? Could this explain quark confinement? By generalizing slightly, also complex mass squared values for quarks could be seen as tachyonic so that Galois confinement would be essentially quark confinement.

3. A long-standing question has been whether ν_R could generate $N = 2$ SUSY. It seems that the tachyon property does not allow the analog of ordinary SUSY. States without ν_R would have huge masses of order CP_2 mass. One can also say that $calN = 2$ SUSY is broken in CP_2 scale.

3.3.4 Is the proposed picture consistent with coupling constant evolution?

Can one understand the discrete number theoretic coupling constant evolution in the proposed framework? As the number of roots of P increases, the number of scattering diagrams with N external particles with fixed momenta p_i increases since the number of Galois confined states characterized by mass shells $m_i^2 = n_i$ increases.

The number of diagrams contributing to the scattering increases and it becomes possible to speak about number theoretical coupling constant evolution. Otherwise the dependence on polynomials P is rather weak and brings in mind logarithmic coupling constant evolution replaced in TGD by discrete p-adic length scale evolution.

How does this relate to the p-adic coupling constant evolution and p-adic length scale hypothesis $p \simeq 2^k$, k some selected integer? For instance, could the p-adic primes preferred by a given extension correspond to the ramified primes of the extension dividing the product $\prod_i (r_i - r_j)$?

1. The dimensionless roots of $P(x)$ are of the form $r_n = R_n/M_p$, where R_n is the dimensional root of $P(M_p x)$. M_p would define the p-adic mass scale and the p-adic length scale of the corresponding CD. This would suggest that p-adic coupling constant evolution is not related to number theoretic coupling constant evolution.

2. On the other hand, the scattering amplitudes depend on the p-adic scale of the momenta. The reduction of scattering amplitudes to homogeneous functions of the factors $p_i \cdot p_j$ appearing in propagator denominators implies very simple dependence on momenta and the characteristic logarithmic dependence is absent. Does this mean that there should be a correlation between the p-adic length scale and algebraic extension? Why should a given extension prefer some p-adic primes, say ramified primes?

3. What about the vertices between Galois singlets, which involve a trilinear of an on-mass-shell state in H and two M^8 off-mass-shell states? How does the p-adic mass scale manifest itself in the properties of these Galois singlets? The conditions for Galois singlet property are scale invariant and the scale invariance is only broken by the condition that mass squared values are roots of polynomial P.

   $M^8 - H$ duality suggests the identification of the discriminant D of the polynomial as an exponent $exp(-K)$ of Kähler function defining vacuum functional and the identification of p-adic prime as a ramified prime dividing D. The real mass squared value would be determined by the canonical identification $\sum x_n p^n \to \sum x_n p^{-n}$ for ramified prime and depend on P.

4. p-Adic physics depends on the value of p-adic prime p. Could this bring in the p-adic coupling constant evolution and preferred p-adic primes number theoretically? The dimension of extensions of p-adics induced by a given extension of rationals depends on p since some roots exist as ordinary p-adic numbers. If p-adic physics as physics of cognition is essential also for real physics as p-adic mass calculations [K3, K1] suggest, it could force the natural selection of preferred p-adic primes and p-adic length scale evolution.

5. Only the identification of the preferred p-adic primes as ramified primes of extension comes into mind. What could make them so special? The p-adic variant of the polynomial P has a double root in order $O(p) = 0$ for a ramified prime. Double root is the mathematical counterpart of criticality and quantum criticality indeed is the basic dynamical principle of TGD. Could something which is of order $O(p^0)$ become order $O(p)$ for a ramified prime? The roots of P correspond to mass squared values: one would have $m_1^2 - m_2^2 = r_1 - r_2 = O(p)$ p-adically.

   For instance, could it be a generic mass squared scale defined by the difference $m_1^2 - m_2^2$ reduces from $M^2(CP_2)$ to $M(CP_2)^2/p$ for ramified primes or p-adic mass scale $M_p = M^2(CP)/p$ reduces to secondary $\mathrm{p}_a dicmassscale \mathrm{M}_{p,2} = M^2(CP)/p^2$. Could the interpretation be in terms of emergence of a massless excitation as counterpart of quantum criticality. Kind of number theoretic analog of Goldstone boson.

   There is some support for this idea. In the living matter, the 10 Hz biorhythm is fundamental. It corresponds to the secondary p-adic length scale of the electron characterized by Mersenne prime $M_{127} = 2^{127} - 1$ [K3]. 10 Hz biorhythm could correspond to a kind of Goldstone boson. This argument still leaves open the question why ramified primes near powers of 2 (or of a small integer such as 3 [I1, I2]) should be so special?

6. One can even speculate with the possibility that a kind of natural selection takes place already at this level. A high number of zero energy states could be possible for Galois singlet states associated with very special polynomials. In the functional composition $P_1 \circ P_2$ of polynomials conservation of roots takes place if the condition $P_i(0) = 0$ is satisfied. This could make possible evolutionary hierarchies in which conserved roots would be analogous to conserved genes.

An open challenge is to formulate a precise criterion fixing what diagrams are allowed. The intuitive picture is that the lines of the diagrams connecting mass shells $m_i^2 = n_i$ diagrams define convex polygons.

3.4 How can one include the WCW degrees of freedom?

The above consideration has been restricted to a single cognitive representation defined by a polynomial P. Already the inclusion of color degrees of freedom requires color partial waves in H and the superposition over space-time surfaces related by color rotation and therefore WCW spinor fields.

3.4.1 "Objective" and "subjective" representations of physics

The usual understanding of Uncertainty Principle (UP) requires that one has a WCW spinor field providing for instance the analogs of the plane waves in the center of mass degrees of freedom for 3-surface. This representation at the level of WCW might be called "objective" representation since one looks at the system from the H or WCW perspective. The localization of particles to the space-time surface violates UP in this "objective" sense.

Discrete cognitive representations define in ZEO what might be called a "subjective" representation of the Poincare and color group since one looks at the system from the perspective of a single space-time surface.

1. The "subjective" representations of isometries would be realized as flows inside X^4 rather than in H. The flows would be defined by the projections of Killing vectors on the space-time surface [L24].

2. The "subjective" representation is actually highly analogous to quantum group representation. For instance, for many-sheeted space-time surface, rotation by 2π would not bring the particle to a different space-time sheet and one would obtain charge fractionalization closely related to the hierarchy of many-sheeted structure corresponding to $h_{eff}/h_0 = n$ hierarchy were n is the dimension of the extension of rationals determined by the polynomial P. This representation could be restricted to Cartan algebra and does not require a 2-D system since the Cartan algebra effectively replaces the 2-D system.

3. The notion of "subjective" representation allows to generalize the gravitational and inertial mass to all conserved charges. Inertial charges would relate to the action in H and gravitational charges to the quantum group charges for flows restricted to $X^4 \subset H$. $M^8 - H$ duality indeed maps the momenta at mass shells associated with $X^4 \subset M^8$ to positions at the boundaries of CD and the action of Lorentz symmetries keeps the image points at the boundaries of CD.

3.4.2 Is WCW needed at the level of M^8?

The inclusion of WCW degrees of freedom is necessary for several reasons. WCW provides the "objective" perspective extending the "subjective" perspective provided by scattering amplitudes at a single space-time surface. Also the understanding of classical physics as an exact correlate of quantum physics requires WCW.

WCW has been introduced at the level of H and the question whether the notion of WCW makes sense also at the level of M^8, has remained open for a long time.

It is now clear that the polynomials P alone determine only the mass shells as their roots [L24]. Could the adelization and p-adization alone serve as the counterpart of WCW for M^8?

On the other hand, the interiors of 4-surfaces in M^8 involve the local CP_2 element and at the mass shells one has a local $S^2 = SO(3)/SO(2)$ element. Hence WCW might be realized at both sides as $M^8 - H$ duality suggests. An interesting conjecture is that by $M^8 - H$ duality, the two WCWs are one and the same thing. Therefore it would seem that adelization does not provide the counterpart of WCW in M^8.

3.4.3 Summation over polynomials as M^8 analog for the WCW integration

What could be the "cognitive" M^8 analog of WCW and integration over WCW?

1. The preferred extremal property of space-time surface $X^4 \subset H$ means that it is defined by its intersections with the boundary of CD. $M^8 - H$ duality requires that this is the case also

in M^8. This would mean that the polynomial P determines, not only the 3-D mass shells of selected M^4 as its roots contained in $X^4 \subset M_c^8$, but also the 4-surface as an $SU(3)/U(2)$ local deformation of M^4 containing them and mapped to H by $M^8 - H$ duality.

2. In the full theory, one has integration over WCW spinor fields. Number theoretical approach means number theoretically unique discretization using cognitive representation rather than its "active" points (containing quark) defining a representation of the Galois group.

   The natural proposal is that WCW integration reduces to a summation over some subset of polynomials and amplitudes associated with the corresponding cognitive representations for which the area momenta for quarks are algebraic integers. External momenta would be ordinary integers for a given p-adic prime p. Therefore the summation over polynomials of varying degree makes sense for amplitudes with fixed external momenta if one uses extension of rationals containing all extensions defined by the polynomials.

3. The rational coefficients of polynomials would serve as WCW coordinates for the polynomials. The assumption that they are rational, however, creates a problem since the summation over rationals defining the coefficients understood as real numbers does not define an analog of integration measure.

One can imagine two number theoretical solutions of the problem: both are inspired by p-adic thermodynamics [K5, K4].

1. One manner to overcome the problem would be a restriction of the coefficients of P to integers. This is natural if the polynomials are monic polynomials of the form $x^n + an - 1x^{n-1} + ...$ This would mean a loss of scaling invariance since $P(kx)$ is not a monic polynomial. The good news is that this might select preferred p-adic primes and explain even the p-adic length scale hypothesis.

2. For a monic polynomial of degree n, the summation would reduce to a summation over $n-1$ integers. The roots would be powers of a single generating root r_0 giving rise to a basis for algebraic integers, and one would have fractility since the quark mass shells correspond to the powers for the modulus of the generating root. The moduli for the differences of roots would be proportional to the power of the modulus of the root and it would be natural to assign p-adic prime to the root with the smallest modulus. This option is highly attractive both physically and mathematically.

3. One expects a rapid p-adic convergence in the sense that polynomials with coefficients, which differ by a large power of p give to scattering amplitudes p-adically very similar contributions. The sum over these contributions should converge rapidly.

   It would seem that the exponent of Kähler function must enter into the picture and give rise to something resembling p-adic thermodynamics with the Boltzmann weight $exp(-E/T)$ being replaced with p-adic number p^{S/T_p}, where the p-adic temperature T_p is inverse integer and S is integer valued. p-Adic number p^{S/T_p} would correspond to the exponent $exp(-K)$ of Kähler function for the H imasage of the surface associated with P. Canonical identification would map p^{S/T_p} to its p-adic norm p^{-S/T_p} identified with $exp(-K)$.

4. The values of S/T_p correspond to the maxima of the Kähler function K for preferred extremals. These exponents exist p-adically only if the value of Kähler coupling strength α_K as an analog of inverse of a critical temperature satisfies strong number theoretic conditions reducing the exponent to an integer power of p (unless one assumes that also the roots of p can appear in the extension considered). These conditions would give rise to a p-adic coupling constant evolution for α_K and also to a coupling constant evolution as a function of algebraic extension.

5. One expects that these conditions can be satisfied only in a very restricted subset of preferred extremals so that one should assume a localization of WCW spinor field to a subset of maxima of the Kähler function. TGD is analogous to a complex square root of thermodynamics and this kind of localization takes place quite generally (spontaneous magnetization) in thermodynamics and also in quantum field theories (Higgs mechanism).

For spin glass discussed from the TGD point of view in [L23], this kind of localization occurs also and in the ultrametric topology of the spin glass energy landscape emerges naturally. p-Adic topologies represent basic examples about ultrametric topologies. The TGD inspired proposal indeed is that p-adic thermodynamics [K5, K3] allows the formulation of spin glass thermodynamics free of ad hoc assumptions.

TGD is Universe is indeed highly analogous to a spin glass in long scales, where the action approaches Kähler action having a huge vacuum degeneracy involving classical non-determinism as the length scale dependent cosmological constant Λ predicted by the twistor lift [L4, L5] approaches zero. An attractive proposal is that this kind of localization has a purely number theoretic origin making p-adic thermodynamics for a suitably chosen value of α_K possible [L23].

6. Also the summation over amplitudes associated with different polynomials of various degrees is in principle possible and could correspond to the summation appearing in perturbation theory and to the summation appearing in p-adic thermodynamics.

One cannot exclude a more general option in which there is a summation over all polynomials with rational coefficients analogous to the summation over the valleys of the energy landscape for spin glass phase.

1. For general rational polynomials, one would have a scaling invariance $P(x) \rightarrow P(kx)$. There would be a summation over scaled roots of P and rationally scaled mass shells. For monic polynomials the scaling invariance is lost and this seems the only realistic possibility.

2. One might hope that the summations over rationals assigned to the coefficients of P with fixed degree reduce to a p-adic integration and that a p-adic integration measure for this integral exists and reduces essentially to summation over p-adic integers with a given norm p^k plus to a summation over the norms p^k at the limit when the norm approaches infinity (`https://cutt.ly/UUbit6f`). Here the problem is that there is no natural lower bound on the p-adic norm of the coefficients as for monic polynomials and the integral need not converge.

The restriction to monic polynomials looks highly attractive. Another possible restriction is that polynomials are proportional to x so that the roots of P are also the roots of the functional composite $P \circ Q$. This restriction might be also an outcome of a number theoretical evolution.

3.4.4 M^8 analog of vacuum functional

The vacuum functional as an exponent of the Kähler function determines the physics at WCW level. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics requires that weight function is a power of p-adic prime and ramified primes of the extension are the natural candidates in this respect.

1. The discriminant D of the algebraic extension defined by a polynomial P with rational coefficients (`https://en.wikipedia.org/wiki/Discriminant`) is expressible as a square for the product of the non-vanishing differences $r_i - r_j$ of the roots of P. For a polynomial P with rational coefficients, D is a rational number as one can see for polynomial $P = ax^2 + bx + c$ from its expression $D = b^2 - 4ac$. For monic polynomials of form $x^n + a_{n-1}x^{m-1} + ...$ with integer coefficients, D is an integer. In both cases, one can talk about ramified primes as prime divisors of D.

   If the p-adic prime $p$is identified as a ramified prime, D is a good candidate for the weight function since it would be indeed proportional to a power of p and have p-adic norm proportional to negative power of p. Hence the p-adic interpretation of the sum over scattering amplitudes for polynomials P is possible if p corresponds to a ramified prime for the polynomials allowed in the amplitude.

   p-Adic thermodynamics [K3] suggest that p-adic valued scattering amplitudes are mapped to real numbers by applying to the Lorentz invariants appearing in the amplitude the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adics to reals in a continuous manner

2. For monic polynomials, the roots are powers of a generating root, which means that D is proportional to a power of the generating root, which should give rise to some power of p. When the degree of the monic polynomial increases, the overall power of p increases so that the contributions of higher polynomials approach zero very rapidly in the p-adic topology. For the p-adic prime $p = M_{127} = 2^{127} - 1 \sim 10^{38}$ characterizing electrons, the convergence is extremely rapid.

   Polynomials of lowest degree should give the dominating contribution and the scattering amplitudes should be characterized by the degree of the lowest order polynomial appearing in it. For polynomials with a low degree n the number of particles in the scattering amplitude could be very small since the number n of roots is small. The sum $x_i + p_i$ cannot belong to the same mass shell for timelike p_i so that the minimal number of roots r_n increases with the number of external particles.

3. $M^8 - H$ duality requires that the sum over polynomials corresponds to a WCW integration at H-side. Therefore the exponent of Kähler function at its maximum associated to a given polynomial should be apart from a constant numerical factor equal to the discriminant D in canonical identification.

   The condition that the exponent of Kähler function as a sum of the Kähler action and the volume term for the preferred extremal $X^4 \subset H$ equals to power of D apart from a proportionality factor, should fix the discrete number theoretical and p-adic coupling constant evolutions of Kähler coupling strength and length scale dependent cosmological constant proportional to inverse of a p-adic length scale squared. For Kähler action alone, the evolution is logarithmic in prime p since the function reduces to the logarithm of D.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminant D of the polynomial P is the natural guess. For monic polynomials D is integer having ramified primes as factors.

There are two options for the correspondence between $exp(-K)$ at its maximum and D assuming that P is monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/D$. For monic polynomials with high degree, D becomes large so that $exp(-K)$ is large.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of D, one would have naturally $exp(-K) = I(D)$, where one has $I(x) = \sum x_n p^n = \sum x_n p^{-n}$.

   If p is the largest ramified prime associated with D, this option gives the same result as the real option, which suggests a unique identification of the p-adic prime p for a given polynomial P. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

   This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding about why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition. Is this condition realized for preferred extremals as extremals of both Kähler action and volume term, or should one regard these conditions as additional conditions?

1. P fixes only the mass-shells as its roots r_n. The real parts of these roots belong to the same M^4. $M^8 - H$ duality is realized by assuming that the mass shells are connected by a 4-surface X^4, which is a deformation of M^4 by a local $SU(3)$ element $g(x)$ such that the subgroup $U(2)$ leaves the points of deformation invariant: this condition gives rise to an explicit form of $M^8 - H$ duality.

   P itself poses no conditions on the local CP_2 element. Could the condition $exp(-K) = I(D)$ for the image of $X^4 \subset M^8$ in H fix the $g(x)$ and thus $X^4 \subset H$?

2. The twistor lift should determine the surface $X^4 \subset H$. The counterpart of twistor lift is defined also at the level of M^8. It maps 6-D surface connecting 5-D mass shells of M^8 as roots of P identified as a local $SU(3)$ deformation of M^6 remaining invariant under $U(1) \times U(1)$ at each point. Hence a point of CP_2 twistor space is assigned to M^6 identified locally as a point of M^4 twistor space.

   One can assign to the twistor space of X^4 as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$ 6-D Kähler action reducing to 4-D Kähler action plus volume term by a dimensional reduction required by the bundle property. One can define the twistorial variant of WCW with the Kähler function K_6 defined by the 6-D Kähler action for X^6. The vacuum functional $exp(-K_6)$ would be the same as for WCW.

   Since S^2 degrees are non-dynamical, the two WCWs are more or less one and the same thing apart from delicacies of non-trivial windings numbers for the maps from the fiber S^2 of $T(X^4)$ to the fibers of $T(M^4)$ and $T(CP_2)$.

3. The $U(2)$ *resp.* $U(1) \times U(1)$ invariant points of the deformation of M^4 *resp.* M^6 would define X^4 *resp.* its twistor space $T(X^4)$. The condition that the image of the deformed M^6 is a preferred extremal of 6-D Kähler action, should determine $g(x)$. $I(D) = exp(-K)$ fixes the 6-D Kähler action action.

4. The formulation of the variational problem in H as a variational problem in $M^4 \subset M^8$ might provide some insight. The 6-D Kähler action for $X^6 \subset H$ naturally assigns an action to the deformed $M^6 \subset M^8$. At the level of M^8, the quantization condition $exp(-K) = I(D)$ plus the boundary conditions defined by the roots of P would select $X^6 \subset M^8$ as a preferred extremal of 6-D Kähler action. This condition could also induce a natural selection of p-adic primes explaining p-adic length scale hypothesis.

3.4.5 The evolution of α_K and of cosmological constant from number theory?

I have considered earlier the evolution of cosmological constant [L1, L4, L5] but it is interesting to look at it in a more detail from the number theoretic perspective.

1. There are three parameters involved: Kähler coupling strength α_K and the winding numbers n_1 and n_2 for the maps of the twistor sphere $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistor spaces $T(M^4)$ and $T(CP_2)$: these maps essentially identify the latter twistor spheres.

2. The 6-D Kähler action for $X^6 = T(X^4) \subset T(M^4) \times T(CP_2)$ is proportional to Kähler coupling strength and the scale factor $1/R^2$, which is equal to CP_2 radius squared. The recent interpretation is that CP_2 radius corresponds to the Planck length L_{Pl} scaled up by h_{eff}/h_0. So that for $h_{eff} = h_0$, the CP_2 radius would reduce to Planck length apart from a numerical constant.

3. Dimensional reduction is necessary in order that X^6 has the structure of the induced twistor bundle with $X^4 \subset H$ as a base-space. This requires maps of the twistor sphere S^2 of the twistor space $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$: this map identifies these twistor spheres locally.

4. Dimensional reduction gives rise to the usual 4-D Kähler action and a volume term with a cosmological constant Λ determined by the Kähler action for the S^2 part of 6-D Kähler action. The induced Kähler form in S^2 is the sum of the contributions from $S^2(M^4)$ and $S^2(CP_2)$.

   Unless the winding numbers of the maps differ from unit, the induced Kähler form is zero or twice the Kähler form of $S^2(CP_2)$ depending on the relative sign of the Kähler forms, whose normalization is fixed by the condition that the magnetic flux is quantized to unity. The form of the maps in spherical coordinates (θ, ϕ) for $S^2(X^4)$ is given by $\theta(M^4) = \theta(CP_2) = \theta$ and $\phi(M^4) = n_1\phi$ and $\phi(CP_2) = n_2\phi$.

5. If the winding numbers n_i are different and of opposite sign (assuming the same sign for Kähler forms), the induced Kähler form is given by $J = (n_2 - n_1)J(S^2(CP_2))$, where n_i are positive.

   The induced line element is $ds^2 = d\theta^2 + sin^2(\theta)(n_1^2 + n_2^2)\phi^2$. The determinant $\sqrt{g}$ of the induced metric of S^2 is $\sqrt{g} = \sqrt{n_1^2 + n_2^2}\sqrt{g(CP_2)}$. The contravariant induced Kähler form is given by

$$J^{\theta\phi} = \frac{g^{\theta\theta}g^{\phi\phi}}{J}_{\theta\phi} = (n_1 - n_2)/n_1^2 + n_2^2 J^{\theta\phi}(CP_2) \ . \tag{3.1}$$

   The Kähler action for S^2 is given by

$$J^{\theta\phi}J_{\theta\phi}\sqrt{g} = \frac{n_1 - n_2}{\sqrt{n_1^2 + n_2^2}} J^{\theta\phi}(CP_2)J_{\theta\phi}(J(CP_2))\sqrt{g(CP_2)} \ .$$

   For small values of $n_1 - n_2$ and large values of $n_1 \sim n_2$ the contribution to action behaves like $\Delta n/n_1$ and can become arbitrarily small. This would predict that cosmological constant approaches to zero in long p-adic length scales.

   This poses a condition on the integers n_i depending on the p-adic prime p identified as a ramified prime: $\Delta n/n_1$ should behave like the inverse of the p-adic length scale scale. The p-adic length scale evolution of both α_K and integers n_i should follow from the condition that the total action equals to the discriminant D (also a polynomial of discriminant can in principle be considered but this seems artificial). The best one can hope is that $M^8 - H$ duality completely fixes both coupling constant evolutions.

6. For the cosmological constant Λ in cosmological scales, the dark energy density is parameterized as $\rho_{vac} = 1/L^4$, $L \sim L_{neuron}$, where $L_{neuron} \simeq 10^{-4}$ m corresponds to the size scale of neuron.

   This rough estimate follows from the identification $\Lambda/8\pi G = 1/L^4$ giving $L(8\pi G/\Lambda)^{1/4}$. Λ itself would correspond to an inverse of p-adic length square, which is of order of the horizon size (naturally the size of cosmological CD).

3.4.6 Do Grassmannians emerge at the QFT limit of TGD?

There is no obvious use for Grassmannians and related concepts in the construction of twistor amplitudes for a space-time surface associated with a given polynomial P.

However, a given scattering amplitude is a sum of contributions associated with monic polynomials P with an increasing number of roots such that a given p-adic prime p appears as their ramified prime. The discriminant D is assumed to play the role of the vacuum functional $exp(-K)$. This picture is highly analogous to a perturbation theory in a given p-adic length scale.

This suggests that QFT with massless particles is a reasonable approximation of TGD at the QFT limit and that the basic twistorial structures could appear at this limit.

Apart from masses given by p-adic thermodynamics [K3, K1], elementary particles, to be distinguished from fundamental quarks, correspond to massless states so that massless QFT is a good guess for the QFT limit.

The emergence of the massless states requires M^4 Kähler structure forced by the twistor lift [L22]. This breaks the Lorentz symmetry to that of $M^2 \times E2$ and the transversal degrees of freedom correspond to harmonic oscillator type degrees of freedom just as in string model and are characterized by two conformal weights. This spontaneous breaking of Lorentz symmetry characterizes massless particles and hadronic quarks. It makes possible the required tachyonic ν_R making it possible to construct massless ground states in p-adic mass calculations.

1. In M^8, the mass shells in general correspond to complex roots. It is possible to have tachyonic Galois confined states. Covariantly constant right-handed neutrino ν_R would be such a state and needed to construct massless Galois confined physical states in H.

2. In H, only the ν_R constructed from quarks is tachyonic in the approximation that H-spinor mode with Kähler charge $Q_K = 3$ describes leptons as 3-quark Galois singlets. $M^8 - H$ duality suggests that there are no other tachyonic quark states and that all Galois confined states are non-tachyonic so that the momenta belong to the interior of the light-cone in M^8.

3. If the amplitudes in the massless sector are indeed Yangian invariants, Grassmannians would emerge naturally at the QFT limit.

The following series of questions is an attempt to crystallize my ignorance.

1. Could a QFT based on twistorial amplitudes for massless Galois confined external particles in H provide a QFT limit of TGD?

2. Could the sum over amplitudes for different polynomials having a given p-adic prime p as a ramified prime correspond to structure resembling that produced in BCFW recursion?

3. Or could MHV structure emerge at the level of a single polynomial P: this is the case if the quark propagators connecting Galois singlets in the amplitudes be regarded as analogs of the propagators $1/P^2$ connecting parts of MHV amplitudes?

4. How the coupling constant evolution emerges at the QFT limit. Number theoretic approach does not allow logarithmic contributions coming from loops but it would not be surprising if the discrete p-adic coupling constant evolution would allow a logarithmic coupling evolution as a reasonable approximation.

   This is also suggested by the fact that the expression of α_K in terms of discriminant D involves logarithm of the p-adic length scale (, that is p). If $exp(-K)$ equals to the image $I(D)$ under canonical identification, one has $\alpha_K = S/log(I(D))$, where $S = K\alpha_K$ is the total action without the proportionality factor $1/\alpha_K$. For ramified primes α_K is proportional to $1/log(p)$.

3.5 What about the twistorialization in CP_2 degrees of freedom?

The proposed picture does not use CP_2 twistor space at all. One should understand why this is the case.

The treatment of color degrees of freedom involves several new aspects. First of all, color is not a spin-like quantum number in the TGD framework.

1. One can identify colored states as color partial waves in WCW degrees of freedom associated with the center of mass degrees of freedom of 3-surface. H spinor modes can be indeed regarded as color partial waves in H.

   It would seem that one cannot speak of color for a single space-time surface. This is indeed true for an "objective" view about the isometries of H. One can however define the "subjective" representations of the isometries by replacing them with flows defined by the projections of Killing vectors to the space-time surfaces [L24].

   For cognitive representations the "subjective" representations could in some situations be reduced to those for the discrete Galois group. One can wonder whether color confinement could reduce to Galois confinement.

2. "Subjective" representations are analogous to quantum group representations [L24]. Objective-subjective dichotomy coud also generalize the inertial-gravitational dichotomy. Note that one can also assign Noether charges to the projected flows. This applies also to supersymplectic symmetries.

The treatment of CP_2 degrees of freedom for the twistor amplitudes remains a challenge and in the following I can only try to clarify my thoughts.

1. Twistor lift strongly suggests that $M^8 - H$ duality defines a map of the twistor spaces of H and M^8 to each other. The M^8 counterparts of 6-D twistor space as a surface $X^6 \subset T(M^4) \times T(CP_2)$ would be 6-D surface with a commutative normal space defined by a

deformation of complexified Minkowski space M^6 by a local $SU(3)$ element, which is left-invariant under $U(1) \subset U(1)$. This would give a 6-surface Y^6 as a counterpart of the 6-surface. It would seem that M^6 should correspond to the twistor $T(M^4)$, perhaps via the identification with a projective space of M^8 by 2-D projective scalings (perhaps by hypercomplex numbers).

2. This map would preserve S^2 bundle structure so that the twistor spheres of $T(M^4)$ and $T(CP_2)$ would be mapped to each other. This looks strange at first but conforms with the general picture.

   At the level of $T(H)$ twistor wave functions at the twistor spheres S^2 of $T(M^4)$ and $T(CP_2)$, which have been identified, describe spin and color or electroweak quantum numbers (the holonomy group of the spinor connection of CP_2 defining weak gauge group can be identified as $U(2) \subset SU(3)$). This implies a correlation for spin and electroweak spin doublets defined quarks apart from the sign factors.

   In the algebraic picture a single point of M^8 does not define only the momentum of quark momentum: rather quark momentum and spin corresponds to a single point of $X^6 \subset M^8$. Fermi statistics boil down to the condition that each point of X^6 can contain only a single quark. Also now directions of the quantization axis characterize the sign of spin and electroweak spin.

3. Spin-isospin correspondence makes sense only because quarks are both spin and weak isospin doublets. The fact that spin value $\pm 1/2$ corresponds to the two directions of the quantization axis allows all possible pairings of spin and electroweak (or color) isospin.

   This map between $T(M^4)$ and $T(CP_2)$ can be understood at M^8 level and generalizes the mapping of M^4 to CP_2 for a space-time surface with 4-D M^4 projection. There are 4-suraces X^4 for which the dimensions of the projections M^4 or CP_2 projection are not maximal. These 4-surfaces correspond to singularities in which normal space at the points of the singularity is not unique [L26].

   It is enough that the twistor spheres of $T(M^4)$ and $T(CP_2)$ are mapped to each other by locally 1-to-1 projection to the twistor sphere of $T(X^4)$: the base space of the twistor space X^6 need not have 4-D projection to M^4 or CP_2.

4. CP_2 twistors can be regarded as functions of M^4 twistors for a given space-time surface with 4-D M^4 projection. The implications for the construction of scattering amplitudes remain to be understood.

How color degrees of freedom are described at M^8 level? There are two equivalent manners to understand the emergence of CP_2 in $M^8 - H$ duality.

1. The normal spaces of $X^4 \subset M^8$ define an integrable distribution. Normal space of X^4 is regarded as a CP_2 point characterizing the deformation of fixed M^4 [L24, L15, L16] so that one obtains $M^8 - H$ duality.

   This distribution contains an integrable distribution of commutative 2-surfaces in turn defining a 6-D surface X^6, which is a good candidate for the counterpart of twistor space. The assignment of the normal space defines a point of the twistor space $SU(3)/U(1) \times U(1)$.

2. Second view [L24, L15, L16], which emerged only quite recently from the detailed study of the surfaces determined by polynomials P, is that the element of local $SU(3)$ naturally defines a deformation of X^4, which is invariant under left or right action by $U(2) \subset SU(3)$ so that local element of $CP(2)$ is in question. This means that color $SU(3)$ corresponds to a subgroup of the automorphism group G_2 of octonions. P as such does not etermines the local CP_2 element. What determines P, will be discussed later.

   The counterpart for the distribution of commutative normal spaces of X^6 is a deformation of M^6, or its variant with some signature of metric, defined by a local element of $SU(3)$ such that the imagine point remains invariant by $U(1) \times U(1) \subset SU(3)$ so that it assigns a point of the twistor space $SU(3)/U(1)U(1)$ to each point of X^6.

3. The equivalence of these views is not rigorously proven. Note that the polynomial P itself defines only 3-D complex mass shells as its roots and the 4-surface connecting them is determined from the condition that $M^8 - H$ duality makes sense.

There is an objection against CP_2 type extremal as a blow-up of 1-D singularity of $X^4 \subset M^8$. Is it really possible to describe CP_2 type extremal as 1-D singularity of $X^{\subset}M^8$ using the $U(2)$ invariant map $M^4 \rightarrow CP_2$?

1. The line singularity can be identified as an 1-D intersection of 2 Minkowskian space-time sheets as roots of P. At H level, this leads to a generation of wormhole contact with an Euclidean signature of metric, CP_2 type extremal, connecting the space-time sheets. The Minkowskian space-time becomes Euclidean at the wormhole throats.

2. At each point of 1-D curve L the singularity should be 3-D surface in CP_2. This requires that the normal space is non-unique and the normal spaces at a point x of L form a 3-D surface in CP_2. If one however thinks about how this could be achieved, one ends up with a problem. One can think that the images of an arbitrarily small sphere S^2 around the point of L is a sphere of CP_2. At the limit one would obtain 2-D rather than 3-D surface of CP_2.

3. The $U(2)$ invariant local $SU(3)$ transformation as a deformation of M^4 defining a local CP_2 transformation is not quite enough to describe the situation. The solution is to consider its inverse as a map from CP_2 to M^4 having a singularity at which a 4-D region of CP_2 is mapped to a line of M^4.

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