Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals

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Abstract

In this article the ideas inspired by the work of number theorist Minhyong Kim are applied to the construction of scattering amplitudes with finite cognitive precision in terms of cognitive representations and their orbits under subgroup S_D of symplectic group respecting the extension of rationals defining the adele. One could pose to S_D the additional condition that it leaves the value of action invariant: call this group $S_{D,S}$: this would define what I have called micro-canonical ensemble (MCE).

The obvious question is whether the simplest zero energy states could correspond to single orbit of S_D or whether several orbits are required. For the more complex option zero energy states would be superposition of states corresponding to several orbits of S_D with coefficients constructed of symplectic invariants. The following arguments lead to the conclusion that MCE and single orbit orbit option are non-realistic, and raise the question whether the orbits of S_D could combine to an orbit of its Yangian analog. A generalization of the formula for scattering amplitudes in terms of n-point functions emerges and somewhat surprisingly one finds that the unitarity is an automatic consequence of state orthonormalization in zero energy ontology.

Contents

1 Introduction

Number theorist Minhyong Kim has speculated about very interesting general connection between number theory and physics [A1, A2] (see http://tinyurl.com/y86bckmo). The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas [L5]. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, gives rise to what one can call identification problem. In TGD framework the imbedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem.

Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

This picture could be applied to the construction of scattering amplitudes with finite cognitive precision in terms of cognitive representations and their orbits under subgroup S_D of symplectic group respecting the extension of rationals defining the adele. One could pose to S_D the additional condition that it leaves the value of action invariant: call this group $S_{D,S}$: this would define what I have called micro-canonical ensemble (MCE).

The obvious question is whether the simplest zero energy states could correspond to single orbit of S_D or whether several orbits are required. For the more complex option zero energy states would be superposition of states corresponding to several orbits of S_D with coefficients constructed of symplectic invariants. The following arguments lead to the conclusion that MCE and single orbit orbit option are non-realistic, and raise the question whether the orbits of S_D could combine to an orbit of its Yangian analog. A generalization of the formula for scattering amplitudes in terms of n-point functions emerges and somewhat surprisingly one finds that the unitarity is an automatic consequence of state orthonormalization in zero energy ontology (ZEO).

2 Could zero energy states constructed in terms of orbits of S_D

The degrees of freedom at WCW level can be divided to zero modes, which do not contribute to WCW metric and correspond to symplectic invariants and to dynamical degrees of freedom which correspond to the orbits of symplectic group of $\delta M^4_\pm \times CP_2$. The assumption is that symplectic group indeed acts as isometries. The general proposal for the state construction in continuum case should have a discrete analog. There are good reasons to hope that the zero energy states in the degrees of freedom corresponding to the orbits of the the discrete variant S_D of the symplectic grop are analogous to spherical harmonics and are dictated completely by symmetry considerations.

2.1 Zero energy states

Quantum superposition of space-time surfaces - preferred extremals - defines zero energy state. The natural question is whether zero energy state could correspond to single orbit of S_D or whether several of them are needed.

1. Preferred extremal is fixed more or less uniquely by its ends, which are 3-surfaces at the opposite light-like boundaries of CD. The interpretation is in terms of holography forced also by general coordinate invariance requiring that one must be able to assign to a given 3-surface a unique space-time surface at which general coordinate transformations act. In ZEO 3-surface means union of 3-surface at opposite ends of CD.

   The idea about preferred extremals as analogs of Bohr orbits suggests that the 3-surface at the either end determines the 3-surface at the opposite end highly uniquely. The proposal that preferred extremals are minimal surfaces apart from singular 2-surfaces identifiable as string world sheet, means that they are separately extremals of both Kähler action and volume term supports this expectation as also the condition that sub-algebra of symplectic group Lie algebra isomorphic to it gives rise to vanishing Noether charges and also the Noether charges associated with its commutator with the full algebra vanish.

The condition that the zero energy state at the active boundary of CD is superposition of many-particle states with different particle number in topological sense suggests that this is not the case.

Even stronger form of holography would be that the data at string world sheets and partonic 2-surfaces determines the preferred extremal completely. In number theoretic vision one can consider even stronger number theoretic holography: if octonionic polynomials code for the space-time surfaces as $M^8 - H$ holography suggests [L1], cognitive representation consisting of discrete set of points with M^8 coordinates in extension of rationals would determine the preferred extremals.

2. Also fermionic degrees of freedom at the ends are involved. Quantum classical correspondence (QCC) states that the classical charges in Cartan sub-algebra of symmetries are equal to the eigenvalues of quantal charges constructible in terms of fermionic oscillator operator algebra. Many-fermion states would correspond to preferred extremals and the fermionic statistics requires that one has superposition over corresponding 4-surfaces. The state at second end of CD is quantum entangled, and fermionic statistics suggests entanglement at both ends.

Symplectic isometries have subgroup with parameters in the extension of rationals defining the adele: call this subgroup S_D. Denote the subgroup of S_D leaving action invariant by $S_{D,S}$. The representations of S_D (or possibly $S_{D,S}$) are expected to be important concerning the construction of scattering amplitudes and on basic of zero energy state property one expects that the action of S_D ($S_{D,S}$) on the opposite ends of space-time surface compensate each other for zero energy states.

A reasonable looking question is whether simplest zero energy states could corresponds to single orbit of S_D. One expects that the number of points defining the cognitive representation is same for all preferred extremals at its orbit. There are several questions to be answered.

1. The existence of preferred extremals connecting given 3-surface with fixed topological particular number to 3-surface at the second end of CD having varying topological particle number looks rather plausible. Topological particle number can be identified either as number of disjoint 3-surfaces and number of disjoint partonic 2-surfaces carrying fermions.

   Can single orbit of S_D contain space-time surfaces with varying topological particle number at the other end of CD? If not , one must allow some minimal number of orbits of S_D in the definition of minimal zero energy state. This option looks the most realistic one.

2. What is the precise definition of cognitive representation?

3. Micro-canonical ensemble (MCE) hypothesis states that action is same for all space-time surfaces appearing in zero energy state. Can this hypothesis be consistent with the presence of many-particle states with different topological particle number? CP_2 type extremals represent particles and have non-vanishing actions. Also the action of symplectic group in general changes the Kähler action although the action is constant at co-dimension 1 surface of WCW so that the subgroup $S_{D,S}$ should act at this surface. It would seem that one must allow the variation of action and this is a challenge for number theoretic universality since the number theoretically non-universal part of action exponentials must be common to all space-time surfaces involves and must cancel in S-matrix.

What does one mean with cognitive representation? Is single orbit of S_D enough? Can one assume MCE? These are the key questions to be considered.

2.2 The action of symplectic isometries on cognitive representations

The action of S_D on cognitive representation defining the adele is straightforward. It is not however quite clear how to identify the cognitive representation.

1. Cognitive representation in question corresponds to a set of points of space-time surface with M^8 coordinates in extension of rationals defining the adele (a stronger condition is that also $M^4 \times CP_2$ coordinates satisfy the same condition).

2. Does cognitive representation contain only the points at the ends of CD, either end, or also interior points? Or does cognitive representation consists of singular points at which non-trivial subgroup of Galois group leaves the point invariant? The singular points could correspond to fundamental fermions at partonic 2-surfaces.

   Remark: If the fermionic lines are light-like geodesic they would correspond as cognitive representations exceptionally informative and easy ones containing infinite number of points of extensions essentially the number line defined by the extension. This raises the question whether the simplest string world sheets identifiable as planes M^2 could be the most interesting singularities of preferred extremals identified as singular minimal surfaces. Canonical imbedding of M^4 is also cognitively easy.

   The condition that the actions of symplectic group at opposite boundaries of CD compensate each other makes sense only if one restricts the cognitive representations at either boundary of CD. This would exclude interior points.

   Could one allow also points in the interior of space-time surface by generalizing the view about symplectic invariance of zero energy state? For instance, could the partonic 2-surface defining vertices in the interior contain points of the cognitive representation. Does the allowance of the points of cognitive representation in interior mean giving up strict determinism and does the variational principle with volume term allow it (mere 4-D Kähler action allows huge vacuum degeneracy).

3. When does the point of cognitive representation correspond to a fundamental fermion? I have proposed [L1] that this is the case if the point is critical in number theoretical sense meaning that there is subgroup of Galois group leaving it invariant: the sheets corresponding to different elements of Galois sub-group would co-incide at critical point. The number of singular points and thus number of fundamental fermions might vary.

4. Could the number of singular points vary for the 4-surfaces at the orbit so that the number of fundamental fermions would vary too? Could this allow to have superposition of many-particle states as active part of the zero energy state? This does not seem plausible since the number of points of cognitive representations must be S_D invariant. Several orbits of S_D seem to be required.

The role of Galois group of extension of rationals must be important.

1. Galois group act do not affect space-time surface but only inside the cognitive representation. Galois group can also have subgroup leaving invariant given point. A possible interpretation is as number theoretic correlate for fundamental fermion.

2. A natural hypothesis is that the sub-group of symplectic group leaving the cognitive representation invariant acts as Galois group. A goo analogous for Galois group is provide by the rotation group $SO(3)$ serving as isotropy group of time-like 4-momentum having vanishing 3-momentum in the rest system. For induced representations $SO(3)$ acts in spin degrees of freedom. In the recent case Galois group could act in number theoretic spin degrees of freedom. Could the action of Galois group be physically non-trivial. For instance, could the ordinary symmetries be represented as Galois transformations in fermionic degrees of freedom?

Symplectic invariants characterize the representation and Kähler fluxes for M^4 and CP_2 Kähler forms define this kind of invariants. Also higher fluxes are possible. The general state as superposition of states associated with the over orbits of S_D would have functions of these invariants as coefficients.

2.3 Zero energy states and generalization of micro-canonical ensemble

The space-time surfaces in micro-canonical ensemble (MCE) [L4] would have same action so that Kähler function would be constant. It is interesting to discuss this hypothesis in light of the idea that simplest zero energy state corresponds to a finite set of orbits of $S_{D,S}$.

2.3.1 Is micro-canonical ensemble consistent with zero energy state- S_D orbit correspondence?

The assumption that action is constant at the orbit is not problematic. Kähler function must vary in order to give rise to non-trivial Kähler metric. Kähler function is however constant at co-dimension 1 surfaces of WCW. For instance, the Kähler function of CP_2 is function of the radial coordinate invariant under subgroups invariant under $U(2)$ but not under $SU(3)$.

1. The simplest variant of MCE is that single space-time surface is involved. The action of $S_{D,S}$ would be essentially trivial - zero momentum would be more familiar Minkowski analogy. One would get rid of the action exponentials: this would solve the problems related to number theoretical universality caused by the fact that the exponential need not exist in various p-adic number fields.

2. A more realistic hypothesis is that $S_{D,S}$ has several 4-surfaces at its orbit. If the number of surfaces is N the sum of action exponentials is N-fold and the exponential disappears from the S-matrix elements in analogy with what happens in the full theory without discretization by cancellation of the exponential strong suggested by what happens in QFTs.

MCE has however problems.

1. It is not at all clear whether one can make restriction to a subgroup preserving the action. To gain some perspective, not that in the case of CP_2 this would mean restriction to $r = constant$ surface of CP_2 and this is not possible. In the case of rotation group this would mean restriction to to sphere.

   Physically it is also obvious that one should allow in the zero energy state all 4-surfaces which are allowed by the conditions posed by preferred extremal property and there seems no good reason to prevent final states with varying particle topological particle number.

2. Also the standard view about S-matrix suggests at active boundary of CD a superposition of final states with different topological particle numbers having different number disjoint 3-surfaces or same number of disjoint 3-surfaces but varying number of partonic 2-surfaces. That the action of S_D changes the number of the disjoint 3-surfaces is in conflict with naive intuitions but one must remember that number theoretic discretization loses information about connectedness.

3. If the zero energy state has at the active boundary 3-surfaces with a varying topological particle number identified as a number of CP_2 type extremals with unique maximal action, one expects that action exponential is not constant along the orbit of S_D. If the subgroup of S_D, call it $S_{D,S}$, preserves the value of the action, one must allow orbits of S_D with varying value of action. This would give superposition MCEs. Action preserving subgroup would be analogous to the little group of Poincare group preserving the momentum of particle. As notice, also several orbits of S_D must be allowed.

The conclusions seems to be that MCE is physically non-realistic.

2.3.2 Can one generalize micro-canonical ensemble to single orbit of S_D?

Suppose that the orbit of S_D contains many-particle states having in final state varying particle numbers measured as number disjoint 3-surfaces or partonic 2-surfaces. Is there any hope of understanding these many-particle states in terms of single representation of S_D?

1. The orbit of S_D must have 4-surfaces with varying value of action. This is possible if the action exponentials differ by a multiplicative rational number so that the number theoretically problematic part cancels out from the S-matrix since it appears in both denominator and numerator of the expression defining S-matrix element.

2. That cognitive representations at the orbit would have same number of points at all points of orbits is intuitively in conflict with varying topological particle number. If Galois group has a subgroup of order $m > 1$ acting trivially on points representing fundamental fermions,

the number of points in the representation is effectively reduced since m points are replaced by 1 point. This could allows to have a varying particle numbers identified as the number of points of cognitive representation.

If CP_2 type extremals in the final state serve as correlates for particles, one should understand how their addition is possible. Their addition to the state would require that some non-degenerate points of representation become degenerate. If the number N points is large, it is quite possible to have rather large number of fundamental fermions in the final state. The degeneration of these points would give rise to fermions. There is however an upper bound which also comes from infrared cutoff for energy.

3. It is not clear whether S_D can transform to each other points with different value of m. The problem is that idea that S_D maps some points to single point is in conflict with the idea that S_D action is bijective. It seems that this idea simply fails.

The conclusion seems to be that one must allow several orbits on basis of purely classical picture and QCC suggesting the possibility of finals states with varying topological particles number.

2.3.3 Could ZEO allow to understand the possibility of particle creation and annihilation?

The idea about quantum superposition of states with varying particle number in topological sense is natural if one believes in QFT based intuition. Just for fun one can ask whether ZEO could provide a loophole.

In ZEO "self" corresponds to a sequence of unitary time evolutions changing the state at active boundary. The active boundary itself becomes de-localized. "Small" state function reduction induces localization of the active boundary. This means measurement of clock time as temporal distance between CDs. The time increment ΔT between subsequent values of clock time varies, and one expects that particle number changes in each unitary evolution. The big state function reduction occurs at some time T, the lifetime of self, and one can assume that the value of T varies statistically.

Could one think that the particle number in topological is actually well-defined after each small reduction? The ensemble of detected particle reactions providing the data allowing to deduce the cross sections. Could the variation of intervals ΔT and the variation for the duration T gives rise to a variation of detected particle numbers in the final state. If this is the case the unitary time evolutions and "small" state function reductions would be very "classical". If so ZEO would simplify dramatically the structure of S-matrix.

To make this mechanism more detailed, one can add the existing wisdom about CP_2 type extremals as building bricks of particles.

1. The action is expected to depend on particle number and different numbers of CP_2 type extremals assignable to which fundamental fermions are assigned correspond to different values of actions. This is not a problem now since would not have have superposition over states with different number of CP_2 type extremals and even micro-canonical ensemble could make sense.

2. The addition of particle to the final state during the unitary evolution taking the active boundary farther away from the passive boundary would correspond to a creation of CP_2 type extremal. Simplest mechanism is 3-vertex defined by partonic 2-surface at which CP_2 type extremal replicates. The outgoing lines in the analogs of twistor diagrams would be unstable against replication. Replication is suggested to be universal process in TGD and the replication of magnetic body (MB) would induce DNA replication in TGD inspired quantum biology.

3. A possible interpretation would be in terms of quantum criticality. CP_2 type extremals would be unstable against decay. One could also interpret the analog of twistor diagram as a sequence of algebraic operations.

In this framework the scattering rates would be determined by a hierarchy of S-matrices labelled by different values of total durations $T_n \sum_{k=1}^{n} \Delta T_k$ for a sequence of unitary evolution followed by time localization. In standard picture they would correspond to single infinitely long time evolution. It would not be surprising if this difference could exclude the proposal as unrealistic.

2.3.4 Could one regard zero energy state involving several orbits of S_D as an orbit of Yangian analog of S_D?

QCC suggest strongly that one must allow zero energy states, which correspond to several orbits of S_D. An interesting possibility is that these orbits could be integrated to a representation of a larger group. What suggests itself is the possibly existing Yangian variant of S_D in which the group action is not local anymore even at the level of WCW. The Yangian of projective transformations of M^4 indeed appears in twistor Grassmannian approach and gives rise to huge symmetries behind the success of twistor Grassmannian approach. I have proposed that super-symplectic variant of Grassmannian indeed exists [K3, K4, K2, K5].

2.4 How to construct scattering amplitudes?

Lubos Motl (see `http://tinyurl.com/y5lndpn3`) told about two new hep-th papers, by Pate, Raclariu, and Strominger (see `http://tinyurl.com/yxqx237b`) and by Nandan, Schreiber, Volovich, Zlotnikov (see `http://tinyurl.com/y642yspf`) related to a new approach to scattering amplitudes based on the replacement of the quantum numbers associated with Poincare group labelling particles appearing in the scattering amplitudes with quantum numbers associated with the representations of Lorentz group.

Why I got interested was that in zero energy ontology (ZEO) the key object is causal diamond (CD) defined as intersection of future and past directed M^4 light-cones with points replaced with CP_2. Space-time surfaces are inside CD and have ends at its light-like boundaries. The Lorentz symmetries associated with ithe boundaries of CD could be more natural than Poincare symmetry, which would emerge in the integration over the positions of CDs of external particles arriving to the opposite light-like boundaries of the big CD defining the scattering region where preferred extremal describing the scattering event resides.

I did my best to understand the articles and - of course relate these ideas to TGD, where the construction of scattering amplitudes is the basic challenge. My technical skills are too limited for to meet this challenge at the level of explicit formulas but I can try to understand the physics and mathematics brought in by TGD.

While playing with more or less crazy and short-lived ideas inspired by the reading of the articles I finally realized that there is perhaps no point in starting from quantum field theories. TGD is not quantum field theory and I must start from TGD itself.

In TGD framework the picture inspired by adelic physics [L2, L3] is roughly following.

1. Cognitive representations realizing number theoretic universality of adelic physics consist of points of imbedding space with coordinates in the extension of rationals. The number of points is typically finite. Cognitive representation should contain as subset the points associated with n-point functions, which are essentially correlation functions.

   Fundamental fermions are building bricks of elementary particles, and a good guess is that fundamental fermions correspond to singular points for which the action of subgroup of Galois group of extension is trivial so that several points collapse together.

2. One must sum over the orbits of a subgroup S_D of symplectic group of light-cone boundary acting as isometries of both boundaries of CD. S_D consists of isometries with parameters in the extension of rationals defining the adele. All orbits needed to represent the pairs of initial and final 3-surfaces at the boundaries of CD allowed by the action principle must be realized so that single orbit very probably is not enough.

3. Correlations code for the quantum dynamics. In quantum field theories quantum fluctuations of fields at distinct points of space-time correlate and give rise to n-point functions expressible in terms of propagators and vertices: massless fields and conformal fields define the basic example. Operator algebra or path integral describes them mathematically.

In TGD correlations between imbedding space points belonging to the space-time surface result from classical deterministic dynamics: the points of 3-surface at opposite boundaries of CD are not independent.

This dynamics is non-linear geometric analog for the dynamics of massless fields: space-time sheets as preferred extremals are indeed minimal surfaces with string world sheets appearing as singularities. Minimal surface property is forced by the volume action implied by the twistor lift and having interpretation in terms of cosmological constant. The correlation between points at the same boundary of CD are expected to be independent since these 3-surfaces chosen rather freely as analogs of boundary values for fields.

Fermionic dynamics governed by modified Dirac action is dictated completely by super-symplectic and super-conformal symmetries. Second quantization of fermions at space-time level is necessary to realized WCW spinor structure: WCW gamma matrices are linear combinations of fermionic oscillator operators.

4. This suggests that the attempts to guess the conformal field theory producing the correlation functions makes things much more complex than they actually are. It should be possible to understand how these correlations emerge from the classical dynamics of space-time surfaces.

As the first brave guess one could try to calculate directly the correlations of spinor harmonics of imbedding space assigned with these points.

1. Sum over the symplectic orbits of cognitive representations must be involved as also vacuum expectation values in the fermionic sector for fermionic fields which must appear in vertices for external particles. At the level of cognitive representations anti-commutators for oscillator operators involve Kronecker deltas so that one has discretized variant of second quantization.

2. This could be achieved by expanding the restriction $\Psi^A_{|X^3}$ of the imbedding space harmonic Ψ^A restricted to 3-surface at end of space-time surface as sum of modes Ψ_n of the induced spinor field. This would be counterpart for the induction procedure. One can assign to singular points bilinear of type $\overline{\Psi}^A_{|X^3} D^{\leftrightarrow}\Psi$, where Ψ is second quantized induced spinor field expressible as sum over its modes multiplied by oscillator operators. D is modified Dirac operator. This gives as vacuum expectations propagators connecting fermions vertices at the opposite ends of space-time surface.

3. A more concrete picture must rely on a concrete model for elementary particles. Elementary particles have as building bricks pair of wormhole contacts with fermion lines at the light-like orbits of the throats at which the signature of the metric changes from Minkowskian to Euclidian. Particle is necessarily a pair of two wormhole contacts and flux tube connects them at both space-time sheets and forms a closed flux tube carrying monopole flux.

   All particles consist of fundamental fermions and anti-fermions: for instance gauge bosons involve fermion and anti-fermion responsible for the quantum numbers at the opposite throats of second wormhole contact. Second wormhole contact involves neutrino pair neutralizing electroweak isospin in scales longer than the size of the flux tube structure.

4. The topological counterpart of 3-vertex appearing in Feynman diagram corresponds to a replication of this kind of 3-surface highly analogous to bio-replication. In replication vertex, there is no singularity of 3-surface analogous to that appearing in the vertices of stringy diagrams but space-time surface is singular just like 1-D manifold is singular for at vertex of Feynman diagram.

   These singular replicating 3-surfaces and the partonic 2-surfaces give rise to the counterparts of interaction vertices. Fermionic 4-vertex is impossible and fermion lines can only be re-shared between outgoing partonic orbits. This is however not enough as will be found. It will be found that also the creation of fermion pair as effective turning of fermion lines entering along "upper" wormhole throat and turning back at Euclidian wormhole throat and continuing along the orbit of "lower" wormhole throat must be possible.

To see how this conclusion emerges consider the following problem. One should obtain also emission of bosons identified as fermion pairs from fermion line. One has incoming fermion and outgoing fermion and fermion pair describing boson which represents gauge boson or graviton with vanishing B and L. Fermionic 4-vertex is not allowed since this would bring in divergences.

1. The appearance of a sub-CD assignable to the partonic 2-surface is possible but does not solve the problem considered. There would be incoming fermion line at lower boundary and 1 fermion line and fermion and antifermion line associated with the boson at the "upper" boundary. There would be non-locality in the scale of the partonic 2-surface and sub-CD meaning that the lines can end to vacuum. Now one would encounter the same difficulty but only in shorter scale.

2. Could one say that fermion line turns backwards in time? A line turning back could be described as an annihilation of fermion pair to vacuum carrying classical gauge field, which is standard process. In QFT picture this would be achieved if a bilinear $\overline{\Psi}D\Psi$ is allowed in the vertex where annihilation takes place. Not in TGD: fermionic action vanishes identically by field equations expressing essentially the conservation of fermion current and various super currents obtained as contractions fermion field with modes.

   Could fermion-anti-fermion pair creation occur at singular points associated with partonic surfaces representing the turning of fermion line backwards in time. This looks still too singular.

   Rather, the turning backwards in time should mean that a fermion line arriving from future along the orbit of "upper" throat (say) goes through Euclidian wormhole throat and continues along the orbit of "lower" throat back to future than making discontinuous turn-around. Euclidian regions of space-time surface representing one key distinction between GRT and TGD would thus be absolutely essential for the generalized scattering diagrams. An exchange of momentum with classical field would be Feynman diagrammatic manner to say this.

   New oscillator operator pairs emerge at the partonic vertices and would correspond to the above described turn-around for fermion line at wormhole contact. Fermion pairs present at the "lower" boundary of CD could also disappear.

3. The anti-commutation relations fermions are modified due to the presence of vacuum gauge fields so that the anti-commutator of fermionic creation operators $a_m^\dagger$ and anti-fermionic creation operators $b_n^\dagger$ is non-vanishing. A proper formulation of the fermionic anti-commutation relations at the ends of space-time surface is needed and in discretization provided by cognitive representation this should be relatively straightforward.

   One can imagine that although standard anti-commutation relations at the lower end of space-time surface hold true, the time evolution of Ψ in the presence of vacuum gauge potentials implies that the vacuum expectations $\langle vac|a_m^\dagger b_n^\dagger|vac\rangle$ are non-vanishing. This would require that for instance $b_n^\dagger$ and a_n are mixed.

There are still questions to be answered.

1. Is the first guess enough? It is not as becomes clear after a thought about the continuum limit. The WCW degrees of freedom are described at continuum limit in terms of super-symplectic algebra (SSA) acting on ground state are neglected. Imbedding space spinor modes characterizee only the ground staes of these representations. These degrees of freedom are essential already in elementary particle physics [K1].

   Sub-algebra SSA_m of SSA with conformal weights coming as m-multiples of those of SSA and its commutator with SSA annihilate the physical states, and one obtains a hierarchy. How to describe these states in the discretization? The natural possibility are the representations of S_D such that $(S_D)_m$ and the subgroup generated by the commutator algebra are represented trivially. One has non-trivial $(S_D)_m$ representations at both ends of WCW such that the action of S_D on the tensor product acts trivially.

   There are also fermionic degrees of freedom. The challenge is to identify among other things WCW gamma matrices as fermionic super charges and it would be nice if all charges were

Noether charges. The simplest guess is that the algebra generated by fermionic Noether charges Q^A for symplectic transformations $h^k \rightarrow h^k + j^{Ak}$ assumed to induce isometries of WCW and Noether supercharges Q_n and their conjugates for the shifts $\Psi \rightarrow \Psi + \epsilon u_n$, where u_n is a solution of the modified Dirac equation, is enough.

The commutators $\Gamma_n^A = [Q^A, Q_n]$ are super-charges labelled by (A, n). One would like to identify them as gamma matrices of WCW. The problem is that they are labelled by (A, n) whereas isometry generators are labelled by A only. There should be one-one correspondence. Do all supercharges Γ_n^A except Γ_0^A corresponding to $u_0 = constant$ annihilate the physical states so that one would have 1-1 correspondence. This would be analogous to what happens quite generally in super-conformal algebras.

The generators of this fermionic algebra could be used to generate more general states. One should also construct the discretized versions of the generators as sums over points of the cognitive representation at the ends of space-time surface. Note that this requires tangent space data.

2. What about the conservation of four-momentum and other conservation laws? This can be handled by quantum classical correspondence (QCC). The momentum and color labels defined by fermionic quantum numbers in Cartan algebra can be assumed to be equal to the corresponding classical Noether charges for particle-like space-time surfaces entering to CD. The technical problem is that if one knows only the discretization - even with tangent space data - one does not know the values of these charges! It might be that $M^8 - H$ correspondence in which M^8 side fixes space-time surfaces as roots for real or imaginary parts of octonionic polynomials from the data at discrete set of points is needed.

3. ZEO means deviations from ordinary description. S_D invariance of zero energy state forces sum over the 4-surfaces of the orbit with identical coefficients. Symplectic invariance implies time-like entanglement. One can describe this in terms of hermitian square root Ψ of density matrix satisfying $\Psi^\dagger \Psi = \rho$. The coefficients of different orbits need not be same and allows description in terms of dynamical density matrix. If there is Yangian symmetry also this entanglement is analogous to the entanglement due to statistics.

   Surprisingly - and somewhat disappointingly after decades of attempts to understand unitarity in TGD - unitarity is trivial in ZEO since state basis is defined essentially by the rows of matrices and orthogonality conditions their orthogonality and therefore unitarity. More concretely, for single state at the passive end state function normalization to unity defined by inner product as sum over 3-surfaces at active end would give conservation of probability. Orthogonality of the state basis with inner product as sum over surfaces passive boundary gives orthogonality for the coefficients defining rows of a matrix and therefore unitarity. In the case that single orbit or even several of them defines the states one obtains the same result.

   What then guarantees the orthogonality of zero energy states? In ordinary quantum mechanics the property of being eigenstates of some hermitian operator guarantees orthogonality. In TGD zero energy states would be solutions of the analog of massless Dirac equation in WCW consisting of pairs of 3-surfaces with members at the ends of preferred extremals inside CD. This generalizes Super Virosoro conditions of superconformal theories and would provide the orthonormal state basis.

The outcome would be amazingly simple. There would be no propagators, no vertices, just spinor harmonics of imbedding assigned with these $n = n_1 + n_2$ points at the boundaries of CD, and summation over the orbits of the symplectic group. All these mathematical objects would emerge from classical dynamics. The sum over the orbits for chosen spinor harmonics would produce n-point functions, vertices and propagators. It is difficult to imagine anything simpler and quantum classical correspondence would be complete.

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