

# Expanding Earth hypothesis, Platonic solids, and plate tectonics as a symplectic flow

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## Abstract

A FB discussion inspired by the recent findings of NASA suggesting the presence of life under the surface of Mars raised the question whether the TGD based Expanding Earth model is consistent with plate tectonics and with the motivating claim of Adams that the continents fit together nicely to cover the entire surface of Earth if its radius were one half of the recent radius. The outcome was what one might call Platonic plate tectonics.

1. The expansion would have started from or generated decomposition of the Earth's crust to an icosahedral lattice with 20 faces, which contain analogs of what is known as cratons and having a total area equal to that of Earth before expansion. The prediction for the recent land area fraction is 25 per cent is 4.1 per cent too low. The cause could be sedimentation or expansion continuing still very slowly.
2. The craton like objects (hereafter cratons) would move like 2-D rigid bodies and would fuse to form continents.
3. The memory about the initial state should be preserved: otherwise there would exist no simple manner to reproduce the observation of Adams by simple motions of continents combined with downwards scaling. This might be achieved if cratons are connected by flux tubes to form a network. For maximal connectivity given triangular face is connected by flux tube to to all 3 nearest neighbour faces. Minimal connectivity corresponds to an essentially unique dodecahedral Hamiltonian cycle connecting cratons to single closed string. At least for maximal connectivity this memory would allow to understand the claim of Adams stating that the reduction of radius by factor 1/2 plus simple motions for the continents allow to transform the continents to single continent covering the entire surface of the scaled down Earth.
4. The dynamics in scales longer than that of craton would be naturally a generalization of an incompressible liquid flow to area preserving dynamics defined by symplectic flow. The assumption that Hamilton satisfies Laplace equation and is thus a real or imaginary part of analytic function implies additional symmetry: the area preserving flow has dual. The flow has vanishing divergence and curl. Sources and sinks and rotation are however possible in topological sense if the tectonic plate has holes.

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## 1 Introduction

A FB discussion inspired by the evidence reported by Nasa for the existence of life in Mars coming from a generation of methane (see <http://tinyurl.com/y735g9kn>) (thanks to Nikolina Benedikovic for the link). It seems that it must originate below the surface of Mars - possibly from underground oceans. The emission of methane is periodic having the year of Mars as a period and has maximum during summer time. This suggests that solar radiation somehow serves as a source of metabolic energy. The TGD based explanation might be in terms of dark photons able to propagate through the crust to the underground oceans.

The finding provides support for TGD based Expanding Earth model [K2] explaining Cambrian explosion, which is one of the mysteries of recent day biology. According to this model life would have evolved in underground oceans where it was shielded from UV light, cosmic rays, and meteor bombardment, and burst to the surface of Earth during the period when Earth expanded and the crust developed cracks.

One can wonder whether Expanding Earth model is consistent with plate tectonics and with the motivating claim of Adams that the continents fit together nicely to cover the entire surface of Earth if its radius were one half of the recent radius. The outcome was what one might call Platonic plate tectonics.

1. The expansion would have started from or generated decomposition of the Earth's crust to an icosahedral lattice with 20 faces, which contain analogs of what is known as cratons and having a total area equal to that of Earth before expansion. The prediction for the recent land area fraction is 25 per cent is 4.1 per cent too low. The cause could be sedimentation or expansion continuing still very slowly.
2. Craton like objects (in the sequence briefly cratons) would move like 2-D rigid bodies and would fuse to form continents.
3. The memory about the initial state should be preserved: otherwise there would exist no simple manner to reproduce the observation of Adams by simple motions of continents combined with downwards scaling. This might be achieved if cratons are connected by flux tubes to form a network. For maximal connectivity given triangular face is connected by flux tube to to all 3 nearest neighbour faces. Minimal connectivity corresponds to an essentially unique dodecahedral Hamiltonian cycle connecting cratons to single closed string. At least for maximal connectivity this memory would allow to understand the claim of Adams stating that the reduction of radius by factor 1/2 plus simple motions for the continents allow to transform the continents to single continent covering the entire surface of the scaled down Earth.
4. The dynamics in scales longer than that of craton would be naturally a generalization of an incompressible liquid flow to area preserving dynamics defined by symplectic flow. The assumption that Hamilton satisfies Laplace equation and is thus a real or imaginary part of analytic function implies additional symmetry: the area preserving flow has dual. The flow has vanishing divergence and curl. Sources and sinks and rotation are however possible in topological sense if the tectonic plate has holes. This would suggest conformal invariance.

## 2 Summary of the model

### 2.1 Expanding Earth hypothesis in TGD framework

The TGD variant of Expanding Earth hypothesis [K2] (see <http://tinyurl.com/y75hku4x>) can be motivated by both cosmological and biological considerations.

1. The basic observation is that astrophysical objects seem to not take part of cosmic expansion but only to co-move. This leads to the idea that the corresponding space-time sheets experience cosmic expansion as relatively rapid jerks and have constant size between these jerks. Second motivation comes from the claim of Adams [F1] (see <http://tinyurl.com/fixsve>) that the continents would fit nicely together to form a single continent covering the entire surface of Earth if the radius of Earth were 1/2 its recent radius.
2. There is also a connection with biology. Cambrian explosion (see <http://tinyurl.com/ntvx38e>) is a poorly understood period in the history of life at Earth. Suddenly a burst of highly developed life forms emerged from some unknown source. TGD explanation would be in terms of rather rapid increase of the radius of Earth by factor of two from the recent size  $R_{Mars} \simeq R_E/2$  of Mars to the recent size  $R_E$  of Earth with the consequences that the stretching developed cracks. Since the radial scaling caused similar stretching everywhere, the decomposition to a lattice at some critical value of the scale parameter  $\lambda$  would have generated the cracks. The generation of a lattice in drying clay serves as an analogy.

The relatively highly developed underground life would had evolved below the surface of Earth, where it was shielded from the bombardment by meteors, cosmic rays, and UV radiation and was burst to the surface as the oceans were formed on the cracks.

The increase of the radius of Earth by factor 2 increased the duration of day by factor 4 and reduced the surface gravity by a factor 1/4. The genetically conserved features preceding the expansion would be still seen in biology. For instance, there might exist a 3 hour bio-rhythm if the underground life received solar radiation somehow. The reduction of gravity could explain the emergence of giant sized organisms such as dinosaurs.

Underground life must have some source of metabolic energy and photosynthesis should have developed already before the Cambrian expansion. This suggests that visible light from some source must have been present. I have considered possible sources in [K5]. The most science fictive proposal is that part of the photons of solar radiation transform to dark photons identified as a phase of ordinary photons residing at magnetic flux tubes. They would have had a non-standard value of Planck constant  $h_{eff} = n \times h_0$  and in absence of direct interactions with the ordinary manner would have managed to penetrate through the crust to the underground oceans.

In the recent biology bio-photons with energies in visible and UV range would emerge as energy conserving transformations of large  $h_{eff}$  photons to ordinary photons. The value of  $h_{eff}$  for charged particle of mass  $m$  would be by a generalization of Nottale's proposal equal to  $\hbar_{eff} = n \times \hbar_0 = h_{gr}GMm/v_0$ , where  $M$  could correspond to a dark mass assignable to Earth and  $v_0$  is a parameter having dimensions of velocity. This hypothesis implies that cyclotron energies of charged particles do not depend at all on the mass of the charged particle so that cyclotron photons can induce transitions of bio-molecules [K3, K4].

**Remark:**  $h_0$  is the minimal value of  $h_{eff}$ : the best guess for the ordinary Planck constant corresponds to  $n = 6$  [L2, L3].

This mechanism for the transfer of solar energy under the surface of Mars could explain the annual periodicity of the methane production in Mars. Magnetic fields serve as a shield against UV radiation and cosmic rays in the case of Earth. Mars has only weak and local magnetic fields above its surface. This gives a good reason why for the Martian life to stay below the surface. The strengthening of the Earth's magnetic field might have preceded or accompanied the proposed expansion of Earth.

3. This vision profoundly modifies the ideas about what happened before Cambrian explosion. In particular, Snowball Earth hypothesis (see <http://tinyurl.com/prem7nj>) about the the climate evolution must be given up. The magnetic history of Earth allows to test the model.

## 2.2 Basic ideas of Platonic plate tectonics

The FB discussion raised the question whether the TGD based Expanding Earth model [K2] is consistent with plate tectonics and with the motivating claim of Adams that the continents fit

nicely to cover the entire surface of Earth if its radius were one half of the recent radius. The outcome was what one might call Platonic plate tectonics.

1. The expansion would have started from or generated decomposition of the Earth's crust to an icosahedral lattice with 20 faces, which contain what could be identified as cratons (see <http://tinyurl.com/y8juty2q>) having a total area equal to that of Earth before expansion. Cratons represent the stable part of the continental lithosphere and are found in the interiors of the tectonic plates. They consist of ancient crystalline basement rock and maybe be covered by younger sedimentary rock. They have a thick crust and deep lithospheric roots. The prediction 25 per cent for the recent land area is 4.1 per cent too low. The simplest explanation is that expansion still continues but very slowly. Also the formation of sedimentary rocks could have increased the area.
2. Cratons would move like 2-D rigid bodies and would fuse to form continents.
3. The memory about the initial state should be preserved: otherwise there would exist no simple manner to reproduce the observation of Adams by simple motions of continents combined with downwards scaling. This could be achieved if cratons are connected by flux tubes to form a network (for tensor networks in TGD Universe see [L1]). For maximal connectivity given triangular face is connected by flux tube to to all 3 nearest neighbour faces. Minimal connectivity corresponds to an essentially unique dodecahedral Hamilton's cycle [A5] (see <http://tinyurl.com/pf33vkt>) connecting cratons to single closed string. At least for maximal connectivity this memory would allow to understand the claim of Adams stating that the reduction of radius by factor 1/2 plus simple motions for the continents allow to transform the continents to single continent covering the entire surface of the scaled down Earth.
4. The dynamics in scales longer than that of craton would be naturally a generalization of an incompressible liquid flow to area preserving dynamics defined by symplectic flow. The assumption that Hamilton satisfies Laplace equation and is thus a real or imaginary part of analytic function implies additional symmetry: the area preserving flow has dual.

The proposal is that the expansion of Earth taking place as discrete jerks is basically a quantum phenomenon in astrophysical scales.

1. In TGD framework magnetic flux tubes are carriers of dark matter identified as phases of ordinary matter with non-standard value of Planck constant. As explained, the value of gravitational Planck constant  $h_{gr}$  would be enormous and imply quantum coherence in the size scale of Earth at the magnetic body forcing coherence at the level of ordinary matter [K4]. The transitions changing the value of  $h_{eff}$  would change the length of flux tubes and these transitions would be crucial for the dynamics of water [L5] (see <http://tinyurl.com/ydhknc2c>).
2. Also the ability of biomolecules to find each other in molecular soup would rely on the same mechanism. In biology also the formation of organs and organelles from cells would involve the shortening of flux tubes [L4] (see <http://tinyurl.com/y9pxr9dx>). In brain synchronously firing neuron groups would form dynamical networks. An interesting question inspired by the huge value of  $h_{gr}$  is whether cratons could be seen as analogs of cells and continents as analogs of organs of Mother Gaia. Note that the magnetic bodies of living systems with EEG would have layers with size scale of Earth [K1].

### 2.3 What happened in the expansion of Earth and after that?

One can try to imagine what happened during and after the expansion of Earth.

1. The spherical crust developed at least one hole as the radius increased by factor 2:  $R_f = 2R_i$ . The crust free regions became frozen magma covered by ocean. The total area of crust was preserved. A stronger condition is that only some minimal stretching required by the increase of the radius occurred. Too large a stretching would have generated the cracks.

The experimentation with toy models leads to the conclusion that minimal stretching is achieved if the crust decomposes into a spherical lattice - regular tessellation- having maximal number of cells. Platonic solids are the only regular tessellations of sphere. The dual  $P_D$  of platonic solid  $P$  has as its vertices the faces of  $P$  and vice versa. The list of Platonic solids (see <http://tinyurl.com/p4rwc76>) is short.

- Self-dual tetrahedron (4 faces and 4 vertices).
- Cube with 6 faces and 8 vertices faces and its dual octahedron.
- Icosahedron and its dual dodecahedron with 20 and 12 faces respectively. For icosahedron the number of faces is maximal and the size of the face minimal and the local stretching is therefore minimal. The faces of icosahedron correspond to the vertices of the dual dodecahedron and icosahedral tessellation is the best candidate to begin with. Note however that the 6 faces of cube could correspond to the 6 continents. One can of course image that the moving cratons later evolved to form an approximate cubical tessellation.

**Remark:** Surfaces with flat metric (plane and cylinder) allow warpings (see <http://tinyurl.com/ycyregve>) for which the induced metric remains flat so that the deformation can be regarded as an isometry with no stretching but non-trivial bending. For instance, for the surface  $z(x, y) = z_0$  one can have warping  $z = z_0 + f(x)$ . The dynamics for the page of book provides a good example of this kind of warping. Could this kind of warpings leading to one-dimensional deformations of the surface of Earth happen for continents in sufficiently short scales?

2. During subsequent evolution radius  $R_f$  remains (approximately) constant and the pieces of crust move along the surface of Earth. No stretching condition prevents the change of shape. If changes of shape are allowed, the first guess is that this evolution was area preserving and thus generated as by a Hamiltonian flow. This would be just classical Hamiltonian mechanics in 2-D phase phase associated with the piece of crust.

If distances inside cratons were preserved (no stretching and change of shape), the dynamics for small enough plates would reduce in a reasonable approximation to a rigid body rotation in the tangent plane at the center of mass of the plate and movement along a geodesic line along the Earth's surface plus collisions. If one accepts that the initial state was a tessellation defined by a Platonic solid, in particular icosahedron, the symplectic evolution trivializes in this manner. The faces contain cratons with area scaled down by factor 1/4. If craton like object is a disk with radius  $d$  one would have  $d = (1/2\sqrt{20})R_E \simeq .11R_E$ . Using  $R_E = 6371$  km this gives  $d = 1425$  km.

3. The first guess is that the expansion period is over now and one has  $R_f = 2 \times R_i$  exactly. As found, the predicted fraction of land area for  $R_f = 2 \times R_i$  is 4.1 per cent smaller than the actual value about 29.1 per cent. A possible explanation for 4.1 per cent is the generation of sedimentary rocks. This would give a probably testable prediction for the fractional area due to sedimentation. Subduction would increase this estimate.

One can also ask whether the expansion still continues slowly so that the radius is not yet quite equal to  $R_f = 2 \times R_i$  so that the fraction of land area is larger than 25 per cent. One would have  $R_f = 2xR_i$ ,  $x = .93$ . Subduction tends to increase and sedimentation to reduce the value of  $x$ . The separation of expansion period from the period during, which  $R_f$  stays constant would be a good approximation if the time scales for tectonics are considerably shorter than for the expansion.

## 2.4 Could flux tube network reproduce the claims of Adams?

The triangular faces can move around and can scale down their size scale by factor 1/2 to the size of craton so that a fusion of cratons to larger units forming continents becomes possible. If one takes the claim of Adams [F1] (see <http://tinyurl.com/fxsve>) seriously, the subsequent dynamics for the faces containing the cratons must be such that it is easy to see how to move continents in the scaling down of the radius of Earth to achieve the gluing together without overlaps and holes

(the mere scaling down does not allow to achieve this since the distances between scaled down continents would be 1/2 of the recent distances).

The dynamics must remember the initial regular icosahedral tessellation at  $S_i^2$ . In the ideal situation every face must “remember” its former nearest neighbours at  $S_i^2$  even when some of them can be faraway at  $S_f^2$ . This requires a network connecting the faces. If the faces are connected by a large enough number of flux tubes able to change their lengths this can be realized and as the radius is imagined to decrease by a factor 1/2, all faces combine to form a spherical crust without overlaps. One can consider two extreme situations.

1. Maximal connectedness requires that every face of icosahedron is connected to each of its 3 nearest neighbours. In this case the dynamics can only involve condensation of the cratons/faces of the network to form continents and for this option the claim of Adams seems trivial.
2. The minimally connected network would correspond to a string connecting the 20 faces to single non-self-intersecting closed string identifiable as a Hamiltonian cycle at dodecahedron. One identifies cycles differing only by an isometry of dodecahedron and already Hamilton discovered that dodecahedron allows only single cycle if one identifies cycles differing only by an isometry of dodecahedron. Given triangle would be connected by flux tube to 2 (rather than 3) nearest neighbors.

**Remark:** Hamilton’s cycles at icosahedron [A5, A2, A4, A1, A3] with 12 vertices play fundamental role in TGD inspired model for music harmony lead to a model of genetic code and of bio-harmony. In this case there is large number of harmonies [K6] [L6].

Whether this option is consistent with the claim of Adams is not clear. One can argue that without additional assumptions the dynamics of the Hamiltonian cycle can destroy the information about the initial icosahedral tessellation by permuting the faces. Could the condition that no self intersections of the flux tubes (strings) of the cycle take place, be enough to preserve the information about initial configuration? The (unique apart from isometries) Hamiltonian cycle can have a fold so that it turns back. The cratons of the antiparallel nearby portions of string can fuse together. The pairing induced by the folding can take place in several manners: say ... (1,6)-(2,5)-(3,4) or ... (-1,6)-(0,5)-(1,4)-(2,3). Here (a,b) corresponding fusion of cratons and - for the Hamiltonian link between neighbouring faces. The increase of the land area by 4.1 percent forces some overlap in the final state if the expansion period has ceased.

## 2.5 Plate tectonics as a symplectic flow in scales longer than the size of craton?

For the icosahedral model the short scale dynamics reduces to much simpler dynamics of 2-D rigid bodies at  $S^2$  having collisions leading to subductions. Cratons however fuse together to form continents having plate tectonics as their dynamics. Tectonic dynamics applies in length scales longer than craton size and cratons could be idealized as point like objects analogous to lipids in cell membrane.

The first guess for the dynamics after the expansion period is symplectic flow preserving the signed area of the continent defining an area preserving map for each value of the time parameter. The area preserving flow is analogous to an incompressible liquid flow in 3 dimensions and serves as a natural model for liquid crystals. For instance, cell membrane is liquid crystal. In this case lipids are idealized as point like objects with symplectic dynamics making sense in length scales longer than the thickness of lipid.

Symplectic flow would be therefore a natural model for plate tectonics (see <http://tinyurl.com/hmby9d4>), and the idealization of cratons as pointlike entities would allow to overcome the objection due to stretching. Symplectic flows could be also used to model the emergence of cracks using Hamiltonians discontinuous along cuts and to model “self-subductions” as flows, which become non-injective and generate mountains.

**Remark:** Symplectic flows could also be used to model the liquid magma in the outer core idealized as 2-D layer analogous to liquid crystal.

What conditions could one pose on the Hamiltonian defining the symplectic flow? The observation that Hamiltonians identified as real or imaginary parts of analytic functions have additional symmetry implying the existence of a dual flow for which flow lines are orthogonal to those for the flow. A good guess therefore that the local tectonics for a continent is defined by a Hamiltonian satisfying Laplace equation. There would be a nice connection between analytic functions and symplectic flows.

### 2.5.1 A model for the continuous time evolution of tectonic plate

The simplest model for a continuous local evolution of given tectonic plate in length scales longer than the size of craton after the expansion period and formation of continents assumes the conservation of signed area meaning that the evolution is symplectic flow generated by some Hamiltonian defined in the region defined by the continent. The symplectic flow would be a 2-D variant of incompressible hydrodynamics.

1. The dynamics would be dictated by the conservation of signed area element  $dS = R^2 \sin(\theta) d\theta \wedge d\phi$  defined by the symplectic form of  $J = J_{kl} ds^k \wedge ds^l$  of  $S^2$ . Symplectic transformations preserve the local area form and are generated by the exponentiation of Hamiltonian function  $H$  giving models for time evolutions as exponentiation of  $H$  defining a flow along the continent.
2. A model for the generation of cracks could be based on Hamiltonian function, which has line discontinuities completely analogous to discontinuities of imaginary or real part of an analytic function. The Hamiltonian flow would take the two sides of the cut to opposite directions in the Hamiltonian flow and crack would develop. The cracks would be filled with water and become oceans.
3. Hamiltonian time evolution defines symplectic map for each value of the time parameter  $t$ , which can cease to be injection at some moment of time at some point and give rise to growing regions into which two different regions of the continent are mapped. Cusp catastrophe with 3 sheets gives a standard topological description for what would have happened. The folding would have 3 plates above each other in the fold region. This “self-subduction” would produce regions analogous to those formed in subduction in which two continents drifting at the surface of magma collide and subduce. Also this process can generate mountains.

The signed area of the middle sheet of the cusp is negative if the area of the other sheets is positive. The formation of the cusp seems therefore to reduce the land area since the middle sheet and lowest sheet of the cusp are invisible. When plate subduces another plate visible land area is also lost. One can imagine two explanations for the missing 4.1 per cent: sedimentation has generated new land area or the expansion period has not yet ended.

One can formulate this picture in more detail as follows.

1. The area preserving symplectic time evolution obeys in general coordinates  $s^k$  for  $S^2$  the formula

$$\frac{ds^k}{dt} = j^k = J^{kl} \partial_l H \quad , \quad J_k^r J_r^l = -s_{kl} \quad . \quad (2.1)$$

where  $J_{kl}$  and  $s_{kl}$  are the symplectic form and standard metric of  $S^2$ . In spherical coordinates  $(\theta, \phi)$  one has  $J_{\theta\phi} = -J_{\phi\theta} = \sin(\theta)$ .  $H = H(\theta, \phi)$  is the function defining the Hamiltonian and subject to physical constraints.  $j^k$  has vanishing divergence:

$$D_k j^k = 0 \quad . \quad (2.2)$$

This equation codes for the local conservation of area.

2. The real or imaginary part of an analytic function having cut along curve can serve as a Hamiltonian in this case. Analyticity would give strong additional constraints on the discontinuity since Laplace equation would be satisfied meaning that not only the current  $j^k$  but also the dual current  $j_D = g^{kl}H_l$  is conserved:

$$D_k j_D^k = 0 \quad . \quad (2.3)$$

$j_D^k$  and  $j^k$  are orthogonal and correspond to real and imaginary parts of an analytic function. Also  $j_D^k$  defines an area preserving flow. This connection between conformal symmetries and symplectic symmetries for Hamiltonians satisfying Laplace equation does not seem to be very familiar to physicists. As a consequence the flow has vanishing divergence and curl. Sources and sinks and global rotation are possible in topological sense if the tectonic plate has holes. This would suggest conformal invariance in some sense.

The absence of sinks implies that one can express  $j_D^k$  as a curl of vector field orthogonal to  $S^2$ . A possible interpretation is as induced Kähler magnetic field or  $Z^0$  magnetic field. One of the first ideas related to the applications of TGD to condensed matter was that hydrodynamic flow could give rise to  $Z^0$  magnetic fields just like em currents give rise to magnetic fields and that vortices of the flow correspond to magnetic flux tubes. This picture makes sense for Kähler magnetic field as well - an option that seems more natural now. The different directions of rotational axis and magnetic dipole axis of Earth would correspond to different directions of the ordinary magnetic field and  $Z^0$  or Kähler magnetic field. These magnetic fields would be effective magnetic fields identified as sums of magnetic fields considered at different space-time sheets at quantum field theory limit of TGD. The flow dynamics could be essentially that of induced Kähler magnetic field orthogonal to  $S^2$ .

**Remark:** At fundamental level only the effects of classical fields on test particle touching several space-time sheets sum up, not the fields. At QFT limit induced fields from different space-time sheets sum up.

The equation for the flow can be integrated for a given flow line as

$$s^k(t) = \exp(tj^r \partial_r) s^k(0) \quad . \quad (2.4)$$

3. The model for the emergence of a crack requires Hamiltonian discontinuous along a 1-D cut. One has  $H = H_\pm$  at the two sides of the cut. The expression of  $s^k(t)$  for the flow lines beginning from the point  $s^k(0) = s_\pm^k(0)$  of the cut and continuing to the side  $\pm$  is given by

$$s_\pm^k(t) = \exp(tJ^{rl} \partial_l H_\pm) \partial_r s^k(0) \quad . \quad (2.5)$$

The model for the emergence of “self-subductions” and generation of mountains can be constructed using non-injective Hamiltonian evolutions in which regions having as pre-images two regions appear. These regions correspond to two continent plates above each other. Both self-subduction and subduction reduce the land area.

### 3 Appendix: Some mathematical details

The icosahedral model for the generation of continents was an outcome of experimentation. I started with a model inspired by the idea that an analog of super-continent Gondwana was generated as single cap during the expansion period but realized soon that it requires quite too large stretching unless one allows generation of cracks. Also a model with two gaps seemed non-realistic. Homogenous upwards scaling of the Earth’s radius suggests strongly lattice like structure and the minimization of stretching led to icosahedral model. I however decided to include these attempts as Appendix - a kind of confession. Hasty reader can skip these parts of the Appendix.



### 3.1 Generation of one or two caps requires too much stretching

The basic objection against single cap model is that the proposed model for expansion requires quite much stretching, which requires large energy. It is also clear that too much stretching leads to a generation of cracks. The following argument is more precise formulation of this observation in terms of a toy model.

1. The first option is that supercontinent analogous to Gondwana (see <http://tinyurl.com/hcgjnr>) was generated as an expanding hole in the crust of  $S_i^2$  emerged somewhere in what became Pacific Ocean - call this place "South pole". Gondwana hypothesis is consistent with Wegener's construction.
2. This period corresponds to a total area preserving map taking the spherical surface (crust) of  $S_i^2$  to a cap of  $S_f^2$  with the same area. The area of the cap should have been thus fraction  $S_f/S_i = R_i^2/R_f^2 = 1/4$  of the total area: this corresponds to 25 per cent of the area of Earth. The actual portion of continents from total area is 29.1 per cent. 4 per cent of new land area should have been generated later by some mechanism.
3. The expansion would take the crust covering entire  $S_i^2$  to a supercontinent covering part of  $S_f^2$ . The simplest map of this kind maps the surface of  $S_i^2$  to a cap of  $S_f^2$  defined by the condition  $\theta_f \in [0, \pi/3]$ : this corresponds to  $[0, 60]$  degrees.  $\theta_f = 0$  would correspond to the "North Pole". This model is certainly non-realistic since it requires large stretching at the bottom of the gap. The stretching is expected to cause cracks mainly in the direction of the coordinate lines of  $\theta_f$ .

For the cap at "North pole" the stretching along the coordinate circles of  $\phi_f$  would be very large near the bottom of the cap. One possibility is that cracks in direction of  $\theta_f$  were generated or that the boundary of cap or that the boundary was "wavy".

A slightly more plausible option reducing the stretching along coordinate circles of  $\phi_f$  would assume generation of 2 caps located at "South pole" and "North pole" as a crack along equator was generated. Also now a wavy crack would allow to minimize the stretching along the coordinate circles of  $\phi_f$ . There would be also stretching along coordinate lines of  $\theta_f$ . In this case one would have two separate super-continent from the beginning and fitting together along their boundaries of the gaps.

### 3.2 Cap models for the expansion period

The expansion period as generation of one or two caps is unrealistic since it produces too much stretching. In the following however the details of the model are given.

1. There exists no isometry between the crust associated with  $S_i^2$  and connected crust associated with  $S_f^2$ . Isometry would require that curvature scalars are same and this is impossible since the radii of  $S_i^2$  and  $S_f^2$  are different.
2. The conservation of total area in the map  $S_i^2 \rightarrow S_f^2$  taking spherical crust to cap  $0 \leq \theta_f \leq \theta_{max}$  with same area:  $S_f = S_i$ .
3. If the expansion begins from an icosahedral lattice the dynamics of expansion period could reduce to simple scaling in a reasonable approximation. The fraction of land area is however 29.1 per cent rather than 25 per cent however that the expansion is still occurring albeit very slowly. Therefore one cannot separate expansion period completely from the tectonic dynamics. One can however think of time dependent scaling combined with the motion and collisions of cratons leading to their fusion.

Consider a more detailed definition of the cap models.

1. In the case of single-cap model the simplest manner to guarantee this is to require  $\cos(\theta_{f,max}) = \cos(\theta_{i,max})/4 + 3/4 = 1/2$  giving  $\cos(\theta_{f,max}) = 1/2$  and  $\theta_{f,max} = \pi/3$ , which corresponds to 60 degrees. As mentioned the large strength in  $\phi_f$  direction requires either a wavy boundary or generations of cracks in  $\theta_f$  direction.

- For the two-cap model the hemispheres  $\theta_i < \pi/2$  and  $\theta_i > \pi/2$  are contracted to caps when the crack at  $\theta_i = \pi/2$  is generated. The condition that no stretching occurs along the coordinate circles of  $\phi_f$  is guaranteed if one has

$$2\sin(\theta_f) = \sin(\theta_i) . \tag{3.1}$$

For small values of  $\sin(\theta_f)$  near poles this condition reduces approximately to the condition  $2\theta_f = \theta_i$ , which guarantees that the distances along coordinate lines of  $\theta_f$  are same as along those of  $\theta_i$  so that stretching is minimal also along this direction near poles.

This correspondence is well-defined only for  $\sin(\theta_f) \leq 1/2$ , which corresponds to  $|\cos(\theta_f)| \geq \sqrt{3}/2$ . On the other hand, the condition that the sum of the areas of the caps equals the area of  $S_i^2$  gives  $|\cos(\theta_f)| \geq 3/4 < \sqrt{3}/2$  so that one must have larger gaps than allowed by no-stretching condition along coordinate circles of  $\phi_f$ . A possible manner to solve the problem is to assume that the boundaries of the gaps are wave or that cracks are generated mainly in  $\theta_f$  direction.

One can model the expansion period  $t = (0, T)$  as a homotopy  $R = R(t)$ , [ $R(0) = R_i = R, R(T) = R_f = 2R$ ]. During this period the cap develops and  $\theta_{f,max}$  satisfies the formulas guaranteeing the conservation of distances along coordinate circles of  $\phi_i$  and of total area.

- For single-cap case one has

$$\frac{R(t)}{R_i} \sin(\theta_f) = \sin(\theta_i) , \quad \left(\frac{R(t)}{R_i}\right)^2 (1 - \cos(\theta_{f,max})) = 2 . \tag{3.2}$$

The first condition can be satisfied only for  $\cos(\theta_f) \geq \sqrt{1 - (R_i/R(t))^2}$ . This lower limit should be smaller than the limit given by the latter condition:  $R_i/R(t) \leq \sqrt{7}/4$ . For  $R(t)/R_i > 4/\sqrt{7} < 2$  the conditions are consistent with each other.

- The 2-gap case gives

$$\frac{R(t)}{R_i} \sin(\theta_f) = \sin(\theta_i) , \quad \left(\frac{R(t)}{R_i}\right)^2 (1 - \cos(\theta_{f,max})) = 1 . \tag{3.3}$$

Also for this option one must have  $\cos(\theta_f) \geq \sqrt{1 - (R_i/R(t))^2}$ . The condition  $\cos(\theta_{f,max}) = 1 - (R_i/R(t))^2$  implies that the first condition cannot be satisfied for all values of  $\cos(\theta_f)$ .

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