Minimal surfaces: comparison of the perspectives of mathematician and physicist

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Abstract

This article was inspired by a popular article telling about the work of mathematicians Fernando Coda Marques and Andre Neves based on the thesis of John Pitts about minimal surfaces but forgotten by mathematics community. Minimal surfaces are also central in TGD, and this motivated the following considerations representing minimal surfaces from the point of view of mathematician and physicist. I will discuss the basic ideas about minimal surfaces, summarize the basic mathematical results of Marques and Neves, and discuss minimal surfaces from TGD point of view.

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1 Introduction

The popular article "*Math Duo Maps the Infinite Terrain of Minimal Surfaces*" (see http://tinyurl.com/yyetb7c7) was an exceptional representative of its species. It did not irritate the reader with non-sense hype but gave very elegant and thought provoking representation of very abstract ideas in mathematics.

The article tells about the work of mathematicians Fernando Coda Marques and Andre Neves based on a profound and - as they tell - extremely hard-to-understand work of Jon Pitts forgotten by mathematics community. It is comforting that at least in mathematics good work is eventually recognized.

The results of Marques and Neves are about minimal hyper-surfaces imbedded in various spaces with dimension varying between 3 and 7 and clearly extremely general. These spaces have varying topologies and are called "shapes" in the popular article.

1.1 Some examples of minimal surfaces

To begin it is good to have some examples about minimal surfaces.

1. For mathematician any lower-dimensional manifold in some imbedding space is surface, even 1-D curve! Simplest minimal surfaces are indeed 1-D geodesic lines. In flat 3-space they are straight lines of infinite length but at the surface of sphere they are big circles.

2. Soap films are 2-D minimal surfaces spanned by frames and familiar for every-one. Frame is necessary for having minimal surface, which does not collapse to a point or extend to infinity and possibly self-intersect.

   Why minimal surfaces are not nice closed surfaces of finite size not intersecting themselves is due to the fact that the equations for minimal surface state the vanishing of the sum of external curvatures defined by the trace of so called second fundamental form defined by the covariant derivatives of tangent vectors of the minimal surface.

   One can say that for 2-D minimal surface the external curvatures in 2 orthogonal directions at given point of surface are of opposite sign. Surface looks locally like saddle rather than sphere. In n-dimensional case the sum of n principal curvatures - eigenvalues of second fundamental form as matrix- sum up to zero for each normal direction: more general saddle.

   In flat imbedding space this implies the saddle property always but in curved space it might happen that the covariant derivatives replacing the ordinary derivatives in the definition of second fundamental form - having interpretation as generalized acceleration - can change the situation and the question is whether non-flat closed imbedding space could contain closed minimal surfaces.

   Indeed, in compact spaces with non-flat metric minimal surface can be closed and there is a century old theorem by Birkhoff stating that sphere has always at least one closed geodesic independent of metric. In the case of ordinary sphere this geodesic is big circle, the equator. In complex projective space CP_2 there is infinite number of 2-D minimal surfaces which are closed: geodesic spheres are the simplest examples.

3. A good example about a non-closed 1-D surface is generic geodesic in torus with points labelled by two angles (ϕ_1, ϕ_2) in flat metric. The geodesic lines are of form $\phi_1 = \alpha\phi_2)$. For non-rational value of α the curve winds the torus infinitely many times and has infinite length. For $\alpha = m/n$ the curve winds m times around second non-contractible circles and n times around the second one. Note that now the geodesic line is absolute minimum: this is caused by the non-contractibility. It can be only shifted in both directions so that the minimum has 2-D degeneracy.

4. In spaces allowing Kähler structure - means that imaginary unit i satisfying $i^2 = -1$ has a representation as antisymmetric tensor - any complex algebraic surfaces representable as root for a set of polynomials, whose number is smaller than complex dimension of the space, is a minimal surface. This huge variety of minimal surfaces is due to the presence of complex structure.

1.2 What does minimal surface property mean?

Consider now what minimal surface property really means.

1. Strictly speaking, minimal surfaces are stationary with respect to the *local* variations of volume only. This is practically always true for physical variational principles defined by an action. For a great circle at sphere the minimality of length with respect to small variations is easy to understand by drawing to see what this variation means. With respect to non-local variations meaning shift toward North or East the area decreases so that one has maximum! This leads to the term Minimax principle used by Jon Pitts and his followers as a powerful guideline.

   In fact, minimal surfaces can be both minima and maxima for volume simultaneously. The general extremum as solution of equations defined by a general action principle is saddle. Minimum with respect to some variables and maximum with respect to others and minimal surfaces are this kind of objects in the general case.

2. There is a deep connection with Morse theory in topology (see http://tinyurl.com/ych4chg9). Morse function gives information about the topology of space. Morse function is a continuous monotonously increasing function from the space to real line and its extrema provide information about the topology of the space. Morse function can be seen as a kind of height function, a particular coordinate for the space.

   The height as z-coordinate for torus imbedded in 3-space gives a classical example of height function. As z varies on obtains 1-D intersections of torus. The minimum of z corresponds to a single point, above it one has circle, then circle decomposes to 2 circles at lower saddle, and circles fuse back to circle at upper saddle, which becomes a point at maximum. Therefore the extrema of heigh function tell about how the topology of the cross section of the torus varies with height: point-circle-2 circles-circle-point. The area of surface serves as a Morse function and minimal points are analogous to the points of the torus at which cross section changes its topology.

   A good guess is that the volume of the surface serves as a Morse function and thus gives information about the topology of rather abstract infinite-dimensional space: the space of surfaces. Minimal surfaces would be analogous to the critical points of height function at torus: points at which the cross section changes its topology.

3. Minimax property states the fact that minimal surfaces are in in generic situation saddle points in the space of surfaces. There would be a strange correspondence. The points of minimal surfaces are locally saddles in the finite-dimensional imbedding space H and minimal surfaces represent saddle points in the finite-dimensional space of surfaces in H. This strange local-global correspondence bringing in mind holography might be behind a general principle: saddle property could have representations at two levels: points of the surface and points of the space of surfaces.

   Are minimal surfaces a rare exception or could it be that for a general action principle the extremals are saddles locally and that the space of all field configurations (not only extremals) contains the extremals as saddle points?

   Remark: Minimal surfaces might be very special and related to what corresponds in physics to criticality implying that the dynamics in certain sense universal. The space of surfaces corresponds in TGD as the space of 3-surfaces and is analogous to Wheeler's superspace consisting of 3-metrics. By holography forced by 4-D general coordinate invariance 3-surfaces in question must be in one-one correspondence with 4-D surfaces identified as space-time surfaces. I have christened this space world of classical worlds (WCW). Space-time surfaces are 4-D minimal surfaces in 8-D $H = M^4 \times CP_2$ but possessing lower dimensional singularities having interpretation as orbits of string like objects and point like particles. Minimal surface property would be a correlate for quantum criticality so that minimal surface would be very special.

1.3 The question and the answer

The question that Marquez and Neves posed to themselves was under which conditions compact space allows a closed minimal surface not intersecting itself or whether all candidates intersect themselves or have infinite volume. In fact, Marquez and Neves restricted the consideration to hyper-surfaces. A possible good reason for this is that there is only one field like dynamical degree of freedom for co-dimension 1 - the coordinate in the normal direction- and this is expected to simplify the situation considerably. From the tone of the article - "-hyper" has been dropped away - one has a temptation to guess that the results are much more general.

The basic result of Marques and Neves was rather astonishing. In almost all closed spaces with dimension between 3 and 7 there exists an infinite series of imbedded *closed* minimal hyper-surfaces (imbedding means that there are no self-intersections). No frames needed! The irony was that they could not prove their result for spaces with roundest metrics (no bumps making metric positively curved, which in turn helps to have minimal surface property without local saddle property). Song however generalized this result to apply for arbitrary closed imbedding spaces [**?**] (see http://tinyurl.com/yycbw4lx).

What helped in the proof was a surprising result by Marques, Neves, and Liokumovich that the volume for these minimal hyper-surfaces depends on the volume of the compact imbedding space only [A3] (see http://tinyurl.com/y59pdawj)!

This dependence suggests that these closed minimal hyper-surfaces manage to visit a dense set of points of the imbedding space without intersecting themselves: in this manner they could "measure" the volume. Marques, Neves and Irie show that there is infinite set of imbedded minimal hyper-surfaces in spaces of dimension $3 \leq n \leq 7$ intersecting any given ball of the imbedding space [A2] (see http://tinyurl.com/y3u3bvnc). Even more, these minimal surfaces tend to fill space in some sense evenly.

A natural guess inspired by Minimax Principle is that minimals surfaces correspond to saddle points for the volume as functional of surface defining Morse function. The volume is analogous to action in TGD framework.

Two remarks are in order.

1. As noticed, the popular article says that these results hold for minimal surfaces. The articles however restrict the consideration to minimal hyper-surfaces.

2. The theorem about the dependence of volume of hyper-surface on the volume of imbedding space was inspired by a result proven by Weyl for the high frequencies of drum defined as a boundary of some space: these frequencies depend on the volume of the space, not on the shape of drum! One can understand this intuitively by the fact that high frequency vibrations correspond to short wave lengths and therefore depend only on the *local* properties of the space and not on the global topology. The dependence on volume comes from boundary conditions at the boundaries of the volume.

   In the case of minimal hyper-surfaces the analogy would suggests that the addition of details to the minimal hyper-surface corresponds to the increase of the frequency for drum. Boundary conditions for drum would be replaced by the compactness of the imbedding space leading to the quantization of the volume analogous to that for frequency.

3. The infinite geodesic on flat torus described above is a rough analog for omni-presence although it is not closed. Also complex surfaces in CP_2 defined as zero loci of polynomials of complex coordinates (ξ^1, ξ^2) modified to contain irrational powers of ξ^i could define this kind of omni-present surfaces having however infinite area. There is however infinite number of minimal surfaces defined by complex polynomials, which are closed but not omni-present.

2 Minimal surfaces and TGD

In TGD framework surfaces satisfying minimal surface equations almost everywhere - play a central role.

2.1 Space-time surfaces as singular minimal surfaces

From the physics point this is not surprising since minimal surface equations are the geometric analog for massless field equations.

1. The boundary value problem in TGD is analogous to that defining soap films spanned by frames: space-time surface is thus like a 4-D soap film. Space-time surface has 3-D ends at the opposite boundaries of causal diamond of M^4 with points replaced with CP_2: I call this 8-D object just causal diamond (CD). Geometrically CD brings in mind big-bang followed by big crunch.

   These 3-D ends are like the frame of a soap film. This and the Minkowskian signature guarantees the existence of minimal surface extremals. Otherwise one would expect that the non-compactness does not allow minimal surfaces as non-self-intersecting surfaces.

2. Space-time is a 4-surface in 8-D $H = M^4 \times CP_2$ and is a minimal surface, which can have 2-D or 1-D singularities identifiable as string world sheets having 1-D singularities as light-like orbits - they could be geodesics of space-time surface.

Remark: I considered in [L1] the possibility that the minimal surface property could fail only at the reaction vertices associated with partonic 2-surfaces defining the ends of string world sheet boundaries. This condition however seems to be too strong. It is essential that the singular surface defines a sub-manifold giving deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. Singular points do not satisfy this condition.

String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations.

3. Geometrically string world sheets are analogous to folds of paper sheet. Space-time surfaces are extremals of an action which is sum of volume term having interpretation in terms of cosmological constant and what I call Kähler action - analogous to Maxwell action. Outside singularities one has minimal surfaces stationary with respect to variations of both volume term and Kähler action - note the analogy with free massless field. At singularities there is an exchange of conserved quantities between volume and Kähler degrees of freedom analogous to the interaction of charged particle with electromagnetic field. One can see TGD as a generalization of a dynamics of point-like particle coupled to Maxwell field by making particle 3-D surface.

4. The condition that the exchange of conserved charges such as four-momentum is restricted to lower-D surfaces realizes preferred extremal property as a consequence of quantum criticality demanding a universal dynamics independent of coupling parameters [L4]. Indeed, outside the singularities the minimal surfaces dynamics has no explicit dependence on coupling constants provided local minimal surface property guarantees also the local stationarity of Kähler action.

   Preferred extremal property has also other formulations. What is essential is the generalization of super-conformal symmetry playing key role in super string models and in the theory of 2-D critical systems so that field equations reduce to purely algebraic conditions just like for analytic functions in 2-D space providing solutions of Laplace equations.

5. TGD provides a large number of specific examples about closed minimal surfaces [K4]. Cosmic strings are objects, which are Cartesian products of minimal surfaces (string world sheets) in M^4 and of complex algebraic curves (2-D surfaces). Both are minimal surfaces and extremize also Kähler action. These algebraic surfaces are non-contractible and characterized by homology charge having interpretation as Kähler magnetic charge. These surfaces are genuine minima just like the geodesics at torus.

   CP_2 contains two kinds of geodesic spheres, which are trivially minimal surfaces. The reason is that the second fundamental form defining as its trace the analogs of external curvatures in the normal space of the surfaces vanishes identically. The geodesic sphere of the first kind is non-contractible minimal surface and absolute minimum. Geodesic spheres of second kind is contractible and one has Minimax type situation.

   These geodesic spheres are analogous to 2-planes in flat 3-space with vanishing external curvatures. For a generic minimal surface in 3-space the principal curvatures are non-vanishing and sum up to zero. This implies that minimal surfaces look locally like saddles. For 2-plane the curvatures vanish identically so that saddle is not formed.

2.2 Kähler action as Morse function in the space of minimal surfaces

It was found that surface volume could define a Morse function in the space of surfaces. What about the situation in TGD, where volume is replaced with action which is sum of volume term and Kähler action [L3, L2, L4]?

Morse function interpretation could appear in two manners. The first possibility is that the action defines an analog of Morse function in the space of 4-surfaces connecting given 3-surfaces at the boundaries of CD. Could it be that there is large number of preferred extremals connecting given 3-surfaces at the boundaries of CD? This would serve as analogy for the existence of infinite number of closed surfaces in the case of compact imbedding space. The fact that preferred extremals extremize almost everywhere two different actions suggests that this is not the case but one must consider also this option.

1. The simplest realization of general coordinate invariance would allow only single preferred extremal but I have considered also the option for which one has several preferred extremals. In this case one encounters problem with the definition of Kähler function which would become many-valued unless one is ready to replace 3-surfaces with its covering so that each preferred extremal associated with the given 3-surface gives rise to its own 3-surface in the covering space. Note that analogy with the definition of covering space of say circle by replacing points with the set of homologically equivalence classes of closed paths at given point (rotating arbitrary number of times around circle).

2. Number theoretic vision [K3, K5] suggests that these possibly existing different preferred extremals are analogous to same algebraic computation but performed in different manners or theorem proved in different manners. There is always the shortest manner to do the computation and an attractive idea is that the physical predictions of TGD do not depend on what preferred extremal is chosen.

3. An interesting question is whether the "drum theorem" could generalize to TGD framework. If there exists infinite series of preferred extremals which are singular minimal surfaces, the volume of space-time surface for surfaces in the series would depend only on the volume of the CD containing it. The analogy with the high frequencies and drum suggests that the surfaces in the series have more and more local details. In number theoretic vision this would correspond to emergence of more and more un-necessary pieces to the computation. One cannot exclude the possibility that these details are analogs for what is called loop corrections in quantum field theory.

4. If the action defines Morse action, the preferred extremals give information about its topology. Note however that the requirement that one has extremum of both volume term and Kähler action almost everywhere is an extremely strong additional condition and corresponds physically to quantum criticality.

   Remark: The original assumption was that the space-time surface decomposes to critical regions which are minimal surfaces locally and to non-critical regions inside which there is flow of canonical momentum currents between volume and Kähler degrees of freedom. The stronger hypothesis is that this flow occurs at 2-D and 1-D surfaces only.

2.3 Kähler function as Morse function in the space of 3-surfaces

The notion of Morse function can make sense also in the space of 3-surfaces - the world of classical worlds which in zero energy ontology consists of pairs of 3-surfaces at opposite boundaries of CD connected by preferred extremal of Kähler action [K1, K2, L3, L2]. Kähler action for the preferred extremal is assumed to define Kähler function defining Kähler metric of WCW via its second derivates $\partial_K \partial_{\overline{L}} K$. Could Kähler function define a Morse function?

1. First of all, Morse function must be a genuine function. For general Kähler metric this is not the case. Rather, Kähler function K is a section in a $U(1)$ bundle consisting of patches transforming by real part of a complex gradient as one moves between the patches of the bundle. A good example is CP_2, which has non-trivial topology, and which decomposes to

3 coordinate patches such that Kähler functions in overlapping patches are related bythe analog of U(1) gauge transformation.

Kähler action for preferred extremal associated with given 3-surface is however uniquely defined unless one includes Chern-Simons term which changes in U(1) gauge transformation for Kähler gauge potential of CP_2.

2. What could one conclude about the topology of WCW if the action for preferred extremal defines a Morse function as a functional of 3-surface? This function cannot have saddle points: in a region of WCW around saddle point the WCW metric depending on the second derivatives of Morse function would not be positive definite, and this is excluded by the positivity of Hilbert space inner product defined by the Kähler metric essential for the unitarity of the theory. This would suggest that the space of 3-surfaces has very simple topology if Kähler function.

   This is too hasty conclusion! WCW metric is expected to depend also on zero modes, which do not contribute to the WCW line element. What suggests itself is bundle structure. Zero modes define the base space and dynamical degrees of freedom contributing to WCW line element as fiber. The space of zero modes can be topologically complex.

There is a fascinating open problem related to the metric of WCW.

1. The conjecture is that WCW metric possess the symplectic symmetries of $\Delta M^4_+ \times CP_2$ as isometries. In infinite dimensional case the existence of Riemann/Kähler geometry is not at all obvious as the work of Dan Freed demonstrated in the case of loops spaces [A1], and the maximal group of isometries would guarantee the existence of WCW Kähler geometry. Geometry would be determined by symmetries alone and all points of the space would be metrically equivalent. WCW would be an infinite-dimensional analog of symmetric space.

2. Isometry group property does not require that symplectic symmetries leave Kähler action, and even less volume term for preferred extremal, invariant. Just the opposite: if the action would remain invariant, Kähler function and Kähler metric would be trivial!

3. The condition for the existence of symplectic isometries must fix the ratio of the coefficients of Kähler action and volume term highly uniquely. The physical interpretation is in terms of quantum criticality realized mathematically in terms of the symplectic symmetry serving as analog of ordinary conformal symmetry characterizing 2-D critical systems. Note that at classical level quantum criticality realized as minimal surface property says nothing outside singular surfaces since the field equations in this regions are algebraic. At singularities the situation changes. Note also that the minimal surface property is a geometric analog of masslessness which in turn is a correlate of criticality.

4. Twistor lift of TGD [**?**]eads to a proposal for the spectra of Kähler coupling strength and cosmological constant allowed by quantum criticality [L2]. What is surprising that cosmological constant identified as the coefficient of the volume term takes the role of cutoff mass in coupling constant evolution in TGD framework. Coupling constant evolution discretizes in accordance with quantum criticality which must give rise to infinite-D group of WCW isometries. There is also a connection with number theoretic vision in which coupling constant evolution has interpretation in terms of extensions of rationals [K3, K7, K6].

2.4 Can one apply the mathematical results about closed minimal surfaces to TGD?

The general mathematical thinking involved with the new results is applied also in TGD as should be clear from the above. But can one apply the new mathematical results described above to TGD? Unfortunately not as such. There are several reasons for this.

1. The dimension of $H = M^4 \times CP_2$ is $D = 8 > 7$. M^4 is non-compact and also the signature of M^4 metric is Minkowskian rather than Euclidian. Could one apply these results to special kinds of 4-surface such as stationary surfaces $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. No: the problem is that E^3 is non-compact.

2. In TGD one does not consider closed space-time surfaces but analogs of soap films spanned by a frame defined by the 3-surfaces at the opposite ends of CD. Note that the singular surfaces of dimension $D = 2, 1$ are analogous to frames with boundaries at the ends of space-time surface.

3. In TGD framework preferred extremal property requires that space-time surface is both minimal surface and extremal of Kähler action outside singularities. This is known to be the case for all known extremals. This poses very strong conditions on extremals and seems to mean the existence of a generalization of Kähler structure and conformal invariance to 4-D situation. This drops a large number of minimal surface extremals from consideration

4. Minimal surfaces filling space evenly do not have any reasonable physical interpretation. Maybe this could be used to argue that one must have $D = 8$ and that signature must be Minkowskian in order to have soap films rather than closed minimal surfaces.

   What about E^4 with Euclidian signature instead of M^4 and closed space-time surfaces in analogy with Euclidian field theories? Would the projections of closed minimal 4-surfaces in $E^4 \times CP_2$ which are also extremals of Kähler action reduce to a point in E^4 and complex 2-surfaces in CP_2: Euclidianized TGD would degenerate to an Euclidian version of string model. Also in $H = S^4 \times CP_2$ the situation might be same since the property of being extremal of Kähler action is very powerful. It is however essential that also M^4 has analog of Kähler structure: S^4 does not have it although it allows twistor structure so that this options drops out.

5. Can one apply the results of Marques, Neves and others about hyper-surfaces to TGD? What comes in mind is a minimal 4-surface, which is a Cartesian product of geodesic line $M^1 \subset M^4$ and 3-D hyper-surface $X^3 \subset CP_2$ visiting all points of CP_2 and having a finite volume. If the action would contain only the volume term, this extremal would be possible. The action however contains Kähler action and this very probably excludes this extremal.

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