Could metaplectic group have some role in TGD framework?

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Abstract

Metaplectic group appears as a covering group of linear symplectic group $Sp(2n, F)$ for any number field and its representations can be regarded as analogs of spinor representations of the rotation group. Since infinite-D symplectic group of $\delta M^4_+ \times CP_2$, where δM^4_+ is light-cone boundary, appears as an excellent candidate for the isometries of the "world of classical worlds" in zero energy ontology (ZEO), one can ask whether and how the notion of metaplectic group could generalize to TGD framework.

The condition for the existence of metaplectic structure is same as of the spinor structure and not met in the case of CP_2. One however expects that also the modified metaplectic structure exists if one couples spinors to an odd integer multiple of Kähler gauge potential. For triality 1 representation assignable to quarks one has $n = 1$. The fact that the center of $SU(3)$ is Z_3 suggests that metaplectic group for CP_2 is 3- or 6-fold covering of symplectic group instead of 2-fold covering.

Besides the ordinary representations of $SL(2, C)$ also the possibly existing analogs of metaplectic representations of $SL(2, C) = Sp(2, C)$ acting on wave functions in hyperbolic space H_3 represented as $a^2 = t^2 - r^2$ hyperbololoid of M^4_+ are cosmologically interesting since the many-sheeted space-time in number theoretic vision allows quantum coherence in even cosmological scales and there are indications for periodic redshift suggests tesselations of H_3 analogous to lattices in E^3 and defined by discrete subgroup of $Sl(2, C)$. In this case one could require that only the subgroup $SU(2)$ is represented projectively so that one would have an analogy with modular functions for discrete subgroup of $SL(2, Z)$ would be represented in this manner.

1 Introduction

Metaplectic group appears as a covering group of linear symplectic group $Sp(2n, F)$ for any number field and its representations can be regarded as analogs of spinor representations of the rotation group. Since infinite-D symplectic group of $\delta M^4_+ \times CP_2$, where δM^4_+ is light-cone boundary, appears as an excellent candidate for the isometries of the "world of classical worlds" in zero energy ontology (ZEO) [K3, ?, K5, K4, K2, K1], one can ask whether and how the notion of metaplectic group could generalize to TGD framework.

The condition for the existence of metaplectic structure is same as of the spinor structure and not met in the case of CP_2. One however expects that also the modified metaplectic structure exists if one couples spinors to an odd integer multiple of Kähler gauge potential. For triality 1 representation assignable to quarks one has $n = 1$. The fact that the center of $SU(3)$ is Z_3 suggests that metaplectic group for CP_2 is 3- or 6-fold covering of symplectic group instead of 2-fold covering.

Besides the ordinary representations of $SL(2, C)$ also the possibly existing analogs of metaplectic representations of $SL(2, C) = Sp(2, C)$ acting on wave functions in hyperbolic space H_3 represented as $a^2 = t^2 - r^2$ hyperbololoid of M^4_+ are cosmologically interesting since the many-sheeted space-time in number theoretic vision allows quantum coherence in even cosmological scales and there are indications for periodic redshift suggests tesselations of H_3 analogous to lattices in E^3 and defined by discrete subgroup of $Sl(2, C)$. In this case one could require that only the subgroup $SU(2)$ is represented projectively so that one would have an analogy with modular functions for discrete subgroup of $SL(2, Z)$ would be represented in this manner.

1.1 Heisenberg group, symplectic group, and metaplectic group

The following gives a brief summary of basics related to Heisenberg group, symplectic group, and metaplectic group.

1.1.1 Heisenberg group

1. The matrix representation of the simplest Heisenberg group `http://tinyurl.com/y2fomegs` is given by matrices

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \tag{1.1}$$

   A 3-D Lie group is in question. The multiplication for group elements (a_1, b_1, c_1) and (a_2, b_2, c_2) is given by $(a_1, b_1, c_1) \circ (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2 - a_1 b_2)$. The coefficients (a,b,c) can be belong to any ring sin the inverse can be expressed using only product and sum as $(-a, -b, ab - c)$. In particular, discrete variants of Heisenberg group such as those associated with extensions of rationals, exist. For odd primes one can define Heisenberg group modulo p as group of order p^3 in finite field F_p.

   $2n + 1$-D Heisenberg group consists of upper triangular with unit matrix at diagonal.

2. Continuous Heisenberg group is a nilpotent Lie group of dimension $d = 3$. Nilpotency means that it is Lie algebra elements are nilpotent. The Lie algebra is generated by upper-diagonal matrices and the commuation relations for the Lie algebra basis are $[X, Y] = Z$, $[X, Z] = 0$, $[Y, Z] = 0$. The coordinate $X = q$ and differential operator $Y = p = \hbar \partial_q$, $Z = i\hbar 1$satisfying $[p, q] = i\hbar Id$, define a concrete representation of the Lie algebra of the simplest 3-D Heisenberg group in the space of functions $f(q)$. By introducing n pairs of coordinates commuting to unit matrix one obtain $2n + 1$-D Heisenberg group.

1.1.2 Symplectic group

Symplectic group acts as automorphisms of Heisenberg group. Symplectic group leaves acts in function algebra of function $H(p, q)$ leaving invariant Poisson bracket $\{H_1, H_2\} = \partial_q H_1 \partial_p H_2 - \partial_q H_2 \partial_p H_1$. The Poisson bracket $\{p, q\} = 1$ giving the element of $J_{p,q} = 1$ symplectic form remaining invariant under symplectic transformations. Exponentiation of any Hamiltonian $H(p, q)$ acting as Hamiltonian generates symplectic flows. Symplectic group is infinite-D.

3-D linear symplectic group $Sp(2, F)$ is obtained as a special case. In continuous case Hamiltoniansare linear functions of p and q so that the action by Poisson bracket is linear. General linear symplectic group$Sp(2n, F)$ acts in $2n$-D space spanned spanned by the analogs of (q_p, p_i). When symplectic form is accompanied by complex structure and Kähler form symplectic isometries define a finite-D subgroup of symplectic group. For instance, in case of CP_2 symplectic isometries define group $SU(3)$.

1.1.3 Metaplectic group

Metaplectic group $Mp_m(2n, F)$ (see `http://tinyurl.com/y5mpswy8` and `http://tinyurl.com/y4kjys3e`) is an m-fold covering of the linear symplectic group $Sp(2n, F)$. Metaplectic group like also linear symplectic group metaplectic grop is defined for all number fields, in particular p-adic number fields and even adeles. All representations of the metapelectic group are infinite-D (non-compactness is not the only reason: even finite-D non-unitary matrix representations fail to exist).

$Sp(2, R)$ co-incides with a covering group the special linear group $Sl(2, R)$ acting as real Möbius transformations in upper half-plane. Metaplectic group does not allow finite-D matrix representations and all representations are infinite-dimensional. Metaplectic group can be regarded as m-fold cover of symplectic group and in Weil representation the cover can be chosen to be 2-fold cover.

The elements for the metaplectic group $M_2(2,R)$ as 2-fold covering of $Sp(2,R)$ have representation as pairs (g,ϵ) with g a Möbius transformation represented by matrix $(a,b;c,d)$ with unit determinant acting as $z \rightarrow (az+b)/(cz+d)$ and with $\epsilon(z)^2 = cz+d$. The product of group elements is given by $(g_1, epsilon_1)(g_2,\epsilon_2) = (g_1g_2,\epsilon)$, $\epsilon(z) = \epsilon_1(g_2(z))\epsilon_2(z)$. The entities transforming in this manner are not functions but analogous to spinors and one can speak of symplectic spinors.

3. One can generalize the notion of symplectic structure to that of metaplectic structure. The topological conditions (the second Stiefel-Withney class vanishes) for the existence of metaplectic structure for given symplectic manifold are same as for the spinor structure.

   Interestingly, in the case of CP_2 this condition is not satisfied and the problem is circumvented by coupling CP_2 spinors to an odd multiple of Kähler gauge potential giving rise to Kähler form: this is essential for obtain electroweak couplings correctly for the induced spinor structure at space-time surface. Since Kähler form relates so closely to symplectic structure, it is reasonable to expect that also in case of CP_2(CP_{2n}) symplectic spinors exist.

   The center of isometry group $SU(3)$ of CP_2 is Z_3 acting trivial on CP_2 coordinates. The action is analogous to that of Möbius transformations being induced by linear action oof $SU(3)$ on projective coordinates (z_1,z_2,z_3) and by the projective map such as $(z_1,z_2,z_3) \rightarrow (z_1/z_3, z_2/z_3, 1)$ in given coordinate patch defined by a choice of two complex coordinates (z_i,z_j) now $(z_1/z_3, z_2/z_3)$. Do symplectic spinors spinors transform like CP_2 spinors under metaplectic action of $SU(3)$?

   CP_2 spinors with unit coupling to Kähler gauge potential allow triality $t = \pm 1$ partial impossible without the coupling making possible spinor structure and presumabley also metaplectic struture. Does this mean that in the case of CP_2 the metaplectic group must be identified as 3-fold or possibly 6-fold covering of symplectic group. The holomy group is electroweak $U(2)$ and acts like $SU(2) \times U(1)$. Does holonomy group acts as double covering of $SO(3)$ and as 3-fold covering of $U(1)$ giving 6-fold covering of tangent space group $SO(4)$?

2 Possible role of metaplectic role in TGD

Since symplectic symmetries are fundamental in TGD, metaplectic group could have a role in TGD.

2.1 Symplectic group in TGD

In TGD the symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is light-cone boundary, and generated by Hamiltonian algebra, are central and act in the "world of classical worlds" (WCW) [K3, ?, K5, K4, K2, K1].

1. WCW is formed by pairs of 3-surfaces with members at opposite boundaries of causal diamond $CD = cd \times CP_2$ of imbedding space $H = M^4 \times CP_2$. cd is causal diamond of M^4 defined as intersection of future and past directed light-cones. The members of the pair are connected by preferred extremal of action defined by twistor lift of TGD: it is sum of Kähler action and volume term. Preferred extremal is analogous to Bohr orbit.

2. The obvious question is whether also infinite-D symplectic group of $\delta M^4_+ \times CP_2$ allows metaplectic variant. Second question is how symplectic spinors relate to ordinary spinors. Are ordinary spinors of H symplectic spinors as one might expect?

3. In TGD the spinors of "world of classical worlds" (WCW) [K3, ?, K5] should have interpretation as symplectic spinors. Spinors of WCW are fermionic Fock states created by quark oscillator operators replacing theta parameters in super-coordinates and in super-spinors of

super variant of imbedding space H. Their local composites appear as monomials with vanishing quark number in hermitian super-coordinates of super-variant of H and in super-quark-spinors of super-H containing only monomials with odd quark number. These super-fields differ from those of standard SUSY since monomials of theta parameters are replaced with monomials of quark oscillator operators and Majorana spinors are not in question.

Infinite-D metaplectic group $\delta M^4_+ \times CP_2$ should act on WCW spinor fields and the action should be induced from action in H.

2.2 Kac-Moody type approach to representations of symplectic/metaplectic group

Representations of the symplectic/metaplectic group. Kac-Moody type approach is strongly suggested physically. Kac-Moody group has Lie-algebra which is central extension of the Lie-algebra of local gauge transformation. Kac-Moody algebra elements are labelled by elements with conformal weight $n \in Z$ but also the variant $n \geq 0$ ("half-algebra" exists as sub-algebra is clear from the commutation relations.

1. Let r denote the radial light-like coordinate of light-cone boundary $\delta M^4_+ \times CP_2$. $\delta M^4_+ = S^2 \times R^+$ is metrically 2-sphere S^2 and this implies extension of usual conformal invariance for S^2 to conformal invariance localized with respect to r and explains why 4-D Minkowski space is physically unique.

   Radially local conformal transformations $z \rightarrow f(r,z)$of light-cone boundary with scaling $r \rightarrow |df(r,z,zbar)/dz|^{-1} \times r$ in light-cone radial coordinate r compensating for the conformal scaling factor $|df(r,z,zbar)/dz|^2$ as isometries of light-cone boundary as also color rotation local with respect to r. One has radially local $S = SO(3) \times SU(3)$ as isometries of light-cone boundary. This would serve as the TGD variant of color gauge symmetry.

2. Effective localization of the symplectic algebra of $S^2 \times CP_2$ with respect to the radial light-like coordinate r. Denote the radial conformal weight h.

   Option 1: Radial waves of form r^h, $h = -1/2 + iy$ (something to do with zeros of zeta) behave like plane waves with wave vector y for in inner product defined by integration measure dr. Orthogonal plane-wave basis effectively.

   Restriction to causal diamond CD defined as intersection of future and past directed light-cones implies $r \leq r_{max}$ defining the size of CD and periodic boundary conditions for a discrete basis r^h. If $h = -1/2 + iy$ corresponds to a zero of zeta, the size of CD determined by r_{max} is quantized. For instance, $sin(yln(r_{max})) = 0$ would imply $ln(r_{max}) = n \times \pi/y$. Also $cos(yln(r_{max})) = 0$ can be considered.

   Option 2: One can include the real part of h to the integration measure of inner product defined as $d\mu = dr/r$. This is dimensionless and very natural by scaling invariance. For this choice one has $h = iy$ and the connection with Riemann zeta is not anymore natural. $r_{max} = exp(n \times \pi/y)$ would give periodic boundary conditions.

   For $y = k\pi$ one would have $r_{max} = exp(1/k)$, k integer. This conforms with the adelic picture since the infinite-D extension of rationals generated by $e^{1/k}$ induces finite-D extension of p-adic numbers since e^p is ordinary p-adic number.

   $y = k\pi/log(p)$ gives $r_{max} = p^{n/k}$ and one can construct finite-D extensions of rationals allowing roots of p.

3. Super-symplectic algebra is assumed to have fractal structure . There is a hierarchy of isomorphic super-sympletcic sub-algebras SSA_n, $n = 1, 2...$, for which conformal weights n-multiples of the weights for the entire algebra.

   Option 1: One would have also conformal weights $n(-1/2 + iy)$ for these radial waves however inner product using dmu= dr as integration measures does reduce to inner product for plane waves but to $\int r^{-n+1} exp(in(y_1 - y_2))du$, $u = log(r/r_0)$. This leads out from the original state space. The modification of the integration measure to $d\mu = r^(n-1)dr$ does not seem plausible.

Option 2: Identify the conformal weight as $h = iy$ and include the real part -1/2 to the dimensionless integration measure $d\mu = dr/r$. This allows fractal hierarchy $h = niy$. This seems to be the only elegant option so that the connection with Riemann zeta seems artificial

This picture leads to some conjectures and questions.

1. Sub-algebra SSA_n and its commutator with entire algebra SSA represented trivially for physical states. Also classical Noether charges vanish: this gives strong conditions on preferred extremals and makes them analogs of Bohr orbits: only preferred pairs of 3-surfaces at opposite boundaries of CD are connected by preferred extremal. Hierarchy of state spaces is the outcome.

   This would be generalization of Super Virasoro conditions for which only the entire algebra would act trivially apart from the scaling generator L_0.

2. Could the hierarchies of extensions of rationals with dimensions $n_1|n_2|...$ (| is for "divides") correspond to hierarchies of inclusions of hyper-finite factors.

3. Could the hierarchies of SSA_n with $n_1|n_2|...$ correspond to hierarchies of extensions of extensions of.... of rationals with dimensions $n_1|n_2|....$

$\delta M^4_+ \times CP_2$ is metrically $S^2 \times CP_2$ and this leads to some questions.

1. Could one have Kac-Moody type representation of the symplectic algebra of $S^2 \times CP_2$, which is radially local and involves central extension? This is physically suggestive.

2. Symplectic isometries of $S^2 \times CP_2$ local with respect to r would define a sub-representation.

   Hamiltonians products of $\delta M^4_+ \times CP_2$ Hamiltonians for δM^4_+ and CP_2 labelled by angular momentum j and by the 2 Casimirs of triality $t = 0$ color representations.

   Isometry algebras $SO(3)$ and $SU(3)$ are sub-algebras of symplectic algebra determined by Hamiltonians at light-cone boundary in given representation to themselves. There are no higher-D sub-algebras so that one cannot consider hierarchy analogous to the hierarchy of sub-algebras labelled by radial conformal weights as n-multiples of weights of the entire algebra.

This in turn leads to a series of questions concerning what happens if one takes gauge symmetry and Kac-Moody symmetry as its analog as a physical guideline.

1. The metaplectic group of $SL(2, R)$ has only infinite-D representations but no matrix representations. Can this be true also for the metaplectic representation of infinite-D for $SO(3) \times SU(3)$ which is compact and allow finite-D unitary ordinary representations. $SO(3)$ must be lifted to $SU(2)$ and this is natural for quark spinors. SU(3) allows only triality $t = 0$ partial waves.

   Since $SU(3)$ has Z_3 as center one expects that the notion of metaplectic representation in this case generalizes so that one has 3-fold covering of function space instead of 2-fold one. Quark spinors indeed allow CP_2 partial waves which are in $t = 1$ representations. As already noticed CP_2 allows does not allow metaplectic structure in standard sense but the coupling to the Kähler gauge potential probably makes this possible since the condition for the existence of generalized metaplectic structure is same as for the existence of modified spinor structure.

2. Should one treat all S^2 Hamiltonians with $l > 1$ as gauge degrees of freedom? A possible interpretation would be in terms of finite measurement resolution and analog of Kac-Moody symmetry acting very much like gauge symmetry representing the finite measurement resolution. Symplectic group would effectively reduce to $SO(3) \times SU(3)$. If so, one would have $SO(3) \times SU(3)$ gauge theory with $l = 1$ states and spin 1/2 states with color as particles.

3. Only quark triplets and singlets of fermions and color octets of gluons are observed. Without any additional conditions TGD predicts infinite number of spinor harmonics. For CP_2 spinor harmonics there is a correlation for the color quantum numbers and electroweak quantum numbers of spinor harmonic. In QCD the color representation of quark does not however

depend on electroweak quantum numbers. Also the masses of spinor harmonics depend on electroweak quantum numbers and are typically very large.

Remark: One could of course ask whether quarks could move in different color partial waves but having $t = 1$. This however seems rather implausible.

The proposal is that Kac-Moody type generators can be used to build massless states with have correct correlation between color represented as angular momentum like quantum number and electroweak quantum numbers. Could the experimental absence of higher color partial waves be due to the fact the gauge nature of higher excitations of symplectic algebra making higher color partial waves of quarks and leptons gauge degrees of freedom?

4. What about $l = 1$ states assignable to $SO(3)$? Twistor lift of TGD predicts that also M^4 has analog of Kähler form and induced $U(1)$ gauge field analogous to induced Kähler form. The physical effects are weak and would be responsible for CP breaking and matter antimatter asymmetry. Could the $l = 1$ triplet correspond to this $U(1)$ gauge boson somewhat like $SU(3)$ octet corresponds to gluon (gluon is identified as pair of quark and antiquark at different positions)?

5. How does this relate to the analog of metaplectic group for $SO(3) \times SU(3)$? What about the central extension of $SO(3) \times SU(3)$ assignable to spinor representations with weight $n = 1/2$. If one adds to the Hamilton associated with rotation generator L_z around z-axis in $SO(3)$ and to hyper-charge generator Y of $SU(3)$ a constant, one obtains what looks like central extension at the level of Poisson brackets since right hand side of brackets receives an additive constant. In $SU(3)$ degrees of freedom one can have only $t = 0$ color partial waves for scalars but for spinors one obtains the $t = 1$ waves and can say that color partial waves possess and anomalous hyper-charge Y.

   The spectra of L_z and Y are shifted but Killing vector fields are not affected. The couplings of isometry generators are changed since there is coupling proportional to Hamiltonian. This does not seem to have have interpretation as a mere gauge transformation since it makes $t = 1$ color partial waves possible for quarks.

2.3 Relationship to modular functions

The metaplectic representations involve in basic form $Sp(2n, F)$, F any number field.

1. $n = 1$ is physically special: one has $Sp(2, C) = SL(2, C)$, which is double covering of Lorentz group. The so called modular representations giving rise to basic functions appearing in number theory are related to the representations of $SL(2, C)$ with the condition that SL(2,Z) or its discrete subgroup (there are infinite number of them) is represented either trivially or mere projective factor. In the representation realizing SL(2,C) as Möbius transformations $z \rightarrow (Az + B)/Cz + D)$ or upper half-plane one has $f(z) \rightarrow (Cz + D)^k f(z)$ when (A,B;C,D) represents element of SL(2,Z) or its subgroup G. k is integer or half integer. One has modular invariance apart from the projective factor.

   Although these nodularity conditions apply only to a discrete subgroup of $SL(2, R)$ they they imply projective invariance of the analytic functions involved so that projectively their support of the function reduces to $G\backslash H$, H upper complex plane analogous to unit cell. Could this kind of conditions correspond to the proposed analogs of Kac-Moody type gauge conditions proposed for symplectic symmetries of $\delta M^4_\pm \times CP_2$?

2. $SO(3, 1)$ acts as isometries of the hyperbolic space H_3 identifiable as the hyperboloid H_3 as $a^2 = t^2 - r^2 = constant$ surface of future light-cone M^4_+: a defines in TGD Lorentz invariant cosmic time and is natural imbedding space coordinate in ZEO. Since $SL(2, Z)$ has infinite number if discrete subgroups, one has infinite number of tesselations of H_3 analogous to lattices in 3-D Euclidian space.

   In TGD quantum coherence is possible in even cosmological scales since TGD predicts hierarchy of effective values of Planck constants. Could one have quantum coherent structures represented as tesselations of the hyperboloid? The prediction would be quantization of red-shift as reflection of quantization of distances from given point of tesselations to other points. Evidence for this kind of quantization has been observed.

3. Finite measurement resolution suggests consideration of tesselations as discretization of H_3 and assignable to extensions of rationals and also to subgroups of $SL(2,Z)$. This would mean discretized wave functions in the tesselation. This would be like wave function for particle in discrete lattice in E^3. On the other hand, modular functions with projective modular invariance would be analogs for wave functions of particles periodic symmetry implied by lattice but represented projectively.

   Could one decompose the representation to products of modular forms as projective representations in coset space $SL(2,C)/\Gamma$, Γ a discrete subgroup of $SL(2,C)$ and of representations of discrete subgroup corresponding to finite measurement resolution. This would be like representation of wave functions as products of discrete lattice wave function and wave functions in the space of momenta modulo lattice momenta: Fermi sphere would be replaced by the coset space $SU(2)/G$.

4. The projective factor $\epsilon^2(Z) = (Cz + D)^k$ is essential for the projective representation of $Sp(2,C)$. Is it possible to generalize this factor acting on upper complex plane to the case of H_3? If subgroup $SO(3)$ is represented projectively, then one can use for H_3 coordinates (r, θ, ϕ), such that r as radius of sphere S^2 remains invariant under r and $SO(3)$ actsob the complex coordinate of S^2 transforming linearly under $SO(1)$ as $z \rightarrow (Az + B)/(Cz + D)$ so that the projective factor can be identified. These representations would be analogous to modular representations: the discrete subgroup of $SL(2,C)$ would be replaced with $SU(2)$.

   It would seem that it must be replaced with $SU(2)$ as subgroup. Could one generalize the notion of modular form invariant under discrete subgroup of $SL(2,C)$ so that the discrete subgroup would become discrete subgroup of $SO(3)$ ($SU(2)$).

   Platonic solids are lattices at S^2 and their isometries and finite subgroups $D(2n)$ appear in McKay correspondence relating discrete subgroups of $SU(2)$ and ADE Lie groups. Finite measurement resolution as dual interpretation. What about infinite discrete subgroups. Does invariance mean projective SU(2) invariance (the case when $n = 0$)

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