Exotic smooth structures at space-time surfaces and master formula for scattering amplitudes in TGD

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Abstract

Exotic smooth structures appearing only in dimension $D = 4$ are problematic from the point of view of general relativity since they are accompanied by time loops and thus by causal anomalies. They are possible also in TGD, and I have already earlier considered the possibility that they might allow us to understand how fermion pair creation is possible in the TGD Universe, where space-times are 4-surfaces in $H = M^4 \times CP_2$.

In this article, the question is whether a master formula for scattering amplitudes, or equivalently for zero energy states in the zero energy ontology (ZEO) of TGD, is possible. The answer turned out to be affirmative provided that the dimension of the space-time surfaces is 4 and the creation of the pair corresponds to a defect of an ordinary smooth structure. The earlier result for the description of pair creation vertex for point-like fermions is generalized to the hierarchy of fermionic objects with dimension 1,2,3.

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1 Introduction

Gary Ehlenberger sent a highly interesting commentary related to smooth structures in R^4 discussed in the article of Gompf [A2] (https://cutt.ly/eMracmf) and more generally to exotics smoothness discussed from the point of view of mathematical physics in the book of Asselman-Maluga and Brans [A3] (https://cutt.ly/DMu0dYr). I am grateful for these links for Gary.

Only 4-D manifolds allow irreducible exotic smooth structures and they are therefore very interesting from the point of view of both general relativity and TGD. In general relativity smooth exotic structures involve time-like loops meaning causal anomalies. Thi raises the question of what happens in TGD and whether the exotic smoothness could have some deep physical implications in the TGD framework.

I wrote already earlier about the possible implications of exotic smoothness from the TGD point of view in the article "Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD" [L5]. Exotic smooth structure fails to be ordinary smooth structure in a set with a vanishing measure. The creation of a fermion pair in the classical field can be seen as a process in which fermion transforms to antifermion. Since fermion and antifermion can be said to move in opposite time directions, this process effectively represents a causal anomaly.

In the classical background defined by the induced gauge potential and metric, this corresponds to a V shaped curve turning around in time direction. This curve is not differentiable at the vertex. Could the vertex correspond to a defect of exotic smooth structure associated with a causal anomaly. In developed arguments in favor of this interpretation.

The motivation for this article came from the question of Marko Manninen: could one imagine a single formula characterizing TGD. To my view the best that one can achieve is a set of principles. However, in the case of classical TGD, that is dynamics space-time surfaces obeying holography realized in terms of 4-D generalization of holomorphy, one can say that thi is the case.

What about scattering amplitudes? Could one derive for the scattering amplitudes a master formula analogous to the corresponding formula in quantum field theories.

I ended this kind of proposal by starting from the question whether TGD allows at all pair creation and fermion boson vertices. This not at all obvious since fundamental fermions correspond to free second quantized spinor fields in $H = M^4 \times CP_2$ and there bosons are bound state of fermions and antifermions so that elementary bosons do not belong to the spectrum of the theory. It turns out that exotic smooth structures make the pair creation possible for 4-D space-time surfaces but not in other dimensions for the space-time surface. The earlier result for point-like fermions generalizes to 1-, 2-, and 3-dimensional fermionic objects predicted to correspond to topological vertices for particle reactions in TGD.

One indeed ends up with a master formula for scattering amplitudes, which has a high degree of resemblance with the standard QFT formula with the difference that the path integral is replaced with a functional integral of 4-D Bohr orbits of particles as 3-surfaces. The Kähler function defining WCW geometry is generalized to its sum with the modified Dirac action related by supersymmetry to the action defining Kähler function. If the Kähler function is cancelled by the normal ordering terms from the modified Dirac action the formulas for the scattering amplitudes are formally similar to those in QFT.

I have already earlier written about this topic and the discussion of [L5] is repeated as such in the first part of the article.

2 Could the existence of exotic smooth structures pose problems for TGD?

The article of Gabor Etesi [A1] (https://cutt.ly/2Md7JWP) gives a good idea about the physical significance of the existence of exotic smooth structures [A2, A3] and how they destroy the cosmic censorship hypothesis (CCH of GRT stating that spacetimes of GRT are globally hyperbolic so that there are no time-like loops.

2.1 Smooth anomaly

No compact smoothable topological 4-manifold is known, which would allow only a single smooth structure. Even worse, the number of exotics is infinite in every known case! In the case of non-compact smoothable manifolds, which are physically of special interest, there is no obstruction against smoothness and they typically carry an uncountable family of exotic smooth structures.

One can argue that this is a catastrophe for classical general relativity since smoothness is an essential prerequisite for tensory analysis and partial differential equations. This also destroys hopes that the path integral formulation of quantum gravitation, involving path integral over all possible space-time geometries, could make sense. The term anomaly is certainly well-deserved.

Note however that for 3-geometries appearing as basic objects in Wheeler's superspace approach, the situation is different since for $D < 3$ there is only a single smooth structure. If one has holography, meaning that 3-geometry dictates 4-geometry, it might be possible to avoid the catastrophe.

The failure of the CCH is the basic message of Etesi's article. Any exotic R^4 fails to be globally hyperbolic and Etesi shows that it is possible to construct exact vacuum solutions representing curved space-times which violate the CCH. In other words, GRT is plagued by causal anomalies.

Etesi constructs a vacuum solution of Einstein's equations with a vanishing cosmological constant which is non-flat and could be interpreted as a pure gravitational radiation. This also represents one particular aspect of the energy problem of GRT: solutions with gravitational radiation should not be vacua.

1. Etesi takes any exotic R^4 which has the topology of $S^3 \times R$ and has an exotic smooth structure, which is not a Cartesian product. Etesi maps maps R^4 to CP_2, which is obtained from C^2 by gluing CP_1 to it as a maximal ball B^3_r for which the radial Eguchi-Hanson coordinate approaches infinity: $r \to \infty$. The exotic smooth structure is induced by this map. The image of the exotic atlas defines atlas. The metric is that of CP_2 but $SU(3)$ does not act as smooth isometries anymore.

2. After this Etesi performs Wick rotation to Minkowskian signature and obtains a vacuum solution of Einstein's equations for any exotic smooth structure of R^4.

2.2 Can embedding space and related spaces have exotic smooth structure?

One can worry about the exotic smooth structures possibly associated with the M^4, CP_2, $H = M^4 \times CP_2$, causal diamond CD= $cd \times CP_2$, where *cd* is the intersection of the future and past directed light-cones of M^4, and with M^8. One can also worry about the twistor spaces CP_3 *resp.* $SU(3)/U(1) \times U(1)$ associated with M^4 *resp.* CP_2.

The key assumption of TGD is that all these structures have maximal isometry groups so that they relate very closely to Lie groups, whose unique smooth structures are expected to determine their smooth structures.

1. The first sigh of relief is that all Lie groups have the standard smooth structure. In particular, exotic R^4 does not allow translations and Lorentz transformations as isometries. I dare to conclude that also the symmetric spaces like CP_2 and hyperbolic spaces such as $H^n = SO(1,n)/SO(n)$ are non-exotic since they provide a representation of a Lie group as isometries and the smoothness of the Lie group is inherited. This would mean that the charts for the coset space G/H would be obtained from the charts for G by an identification of the points of charts related by action of subgroup H.

   Note that the mass shell H^3, as any 3-surface, has a unique smooth structure by its dimension.

2. Second sigh of relief is that twistor spaces CP_3 and $SU(3)/U(1) \times U(1)$ have by their isometries and their coset space structure a standard smooth structure.

   In accordance with the vision that the dynamics of fields is geometrized to that of surfaces, the space-time surface is replaced by the analog of twistor space represented by a 6-surface with a structure of S^2 bundle with space-time surface X^4 as a base-space in the 12-D product of twistor spaces of M^4 and CP_2 and by its dimension $D = 6$ can have only the standard smooth structure unless it somehow decomposes to $(S^3 \times R) \times R^2$. Holography of smoothness would prevent this since it has boundaries because X^4 as base space has boundaries at the boundaries of CD.

If exotic smoothness is allowed at the space-time level in the proposed sense ordinary smooth structure could be possible at the level of twistor space in the complement of a Cartesian product of the fiber space S^2 with a discrete set of points associated with partonic 2-surfaces.

3. *cd* is an intersection of future and past directed light-cones of M^4. Future/past directed light-cone could be seen as a subset of M^4 and implies standard smooth structure is possible. Coordinate atlas of M^4 is restricted to *cd* and one can use Minkowski coordinates also inside the *cd*. *cd* could be also seen as a pile of light-cone boundaries $S^2 \times R_+$ and by its dimension $S^2 \times R$ allows only one smooth structure.

4. M^8 is a subspace of complexified octonions and has the structure of 8-D translation group, which implies standard smooth structure.

The conclusion is that continuous symmetries of the geometry dictate standard smoothness at the level of embedding space and related structures.

2.3 Could TGD eliminate the smoothness anomaly or provide a physical interpretation for it?

The question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies.

What does the induction of a differentiable structure really mean? Here my naive expectations turn out to be wrong. If a sub-manifold $S \subset H$ can be regarded as an embedding of smooth manifold N to $S \subset H$, the embedding $N \to S \subset H$ induces a smooth structure in S (`https://cutt.ly/tMtvG79`). The problem is that the smooth structure would not be induced from H but from N and for a given 4-D manifold embedded to H one could also have exotic smooth structures. This induction of smooth structure is of course physically adhoc.

It is not possible to induce the smooth structure from H to sub-manifold. The atlas defining the smooth structure in H cannot define the charts for a sub-manifold (surface). For standard R^4 one has only one atlas.

2.3.1 Could holography of smoothness make sense in the general case?

The first trial to get rid of exotics was based on the holography of smoothness and did not involve TGD. Could a smooth structure at the boundary of a 4-manifold could dictate that of the manifold uniquely. Could one speak of holography for smoothness? Manifolds with boundaries would have the standard smooth structure.

1. The obvious objection is that the coordinate atlas for 3-D boundary cannot determine 4-D atlas in any way because the boundary cannot have information of the topology of the interior.

2. The holography for smoothness is also argued to fail (`https://cutt.ly/3MewYOt`). Assume a 4-manifold W with 2 different smooth structures. Remove a ball B^4 belonging to an open set U and construct a smooth structure at its boundary S^3. Assume that this smooth structure can be continued to W. If the continuation is unique, the restrictions of the 2 smooth structures in the complement of B^4 would be equivalent but it is argued that they are not.

3. The first layman objection is that the two smooth structures of W are equivalent in the complement $W - B^3$ of an arbitrary small ball $B^3 \subset W$ but not in the entire W. This would be analogous to coordinate singularity. For instance, a single coordinate chart is enough for a sphere in the complement of an arbitrarily small disk.

   An exotic smooth structure would be like a local defect in condensed matter physics. In fact it turned out that this intuitive idea is correct: it can be shown that the exotic smooth structures are equivalent with standard smooth structure in a complement of a set having co-dimension zero (`https://cutt.ly/7MbGqx2`). This does not save the holography of smoothness in the general case but gives valuable hints for how exotic smoothness might be realized in TGD framework.

2.3.2 Could holography of smoothness make sense in the TGD framework?

Could M^8-H duality and holography make holography of smoothness possible in the TGD framework?

1. In the TGD framework space-time is 4-surface rather than abstract 4-manifold. 4-D general coordinate invariance, assuming that 3-surfaces as generalization of point-like particles are the basic objects, suggests a fully deterministic holography. A small failure of determinism is however possible and expected, and means that space-time surfaces analogous to Bohr orbits become fundamental objects. Could one avoid the smooth anomaly in this framework?

   The 8-D embedding space topology induces 4-D topology. My first naive intuition was that the 4-D smooth structure, which I believed to be somehow inducible from that of $H = M^4\times CP_2$, cannot be exotic so that in TGD physics the exotics could not be realized. But can one really exclude the possibility that the induced smooth structure could be exotic as a 4-D smooth structure?

2. In the TGD framework and at the level of $H = M^4\times cP_2$, one can argue that the holography implied by the general coordinate invariance somehow determines the smooth structure in the interior of space-time surface from the coordinate atlas at the boundary. One would have a holography of smoothness. It is however not obvious why this unique structure should be the standard one.

3. One has also holography in M^8 and this induces holography in H by M^8-H duality. The 3-surfaces X^3 inducing the holography in M^8 are parts of mass shells, which are hyperbolic spaces $H^3\subset M^4\subset M^8$. 3-surfaces X^3 could be even hyperbolic 3-manifolds as unit cells of tessellations of H^3. These hyperbolic manifolds have unique smooth structures as manifolds with dimension $D<4$.

   The hypothesis is that one can assign to these 3-surfaces a 4-surface by a number theoretic dynamics requiring that the normal space is associative, that is quaternionic [L1, L2]. The additional condition is that the normal space contains commutative subspace makes it possible to parametrize normal spaces by points of CP_2. M^8-H duality would map a given normal space to a point of CP_2. M^8-H duality makes sense also for the twistor lift.

4. A more general statement would be as follows. A set of 3-surfaces as sub-manifolds of mass shells H^3_m determined by the roots of polynomial P having interpretation as mass square values defining the 4-surface in M^8 take the role of the boundaries. Mass-shells H^3_m or partonic 2-surfaces associated with them having particle interpretation could correspond to discontinuities of derivatives and even correspond to failure of manifold property analogous to that occurring for Feybman diagrams so that the holography of smoothness would decompose to a piece-wise holography.

   The regions of $X^4\subset M^8$ connecting two sub-sequent mass shells would have a unique smooth structure induced by the hyperbolic manifolds H^3 at the ends.

It is important to notice that the holography of smoothness does not force the smooth 4-D structure to be the standard one.

2.3.3 Could the exotic smooth structures have a physical interpretation in the TGD framework?

In the TGD framework, exotic smooth structures could also have a physical interpretation. As noticed, the failure of the standard smooth structure can be thought to occur at a point set of dimension zero and correspond to a set of point defects in condensed matter physics. This could have a deep physical meaning.

1. The space-time surfaces in $H = M^4 \times CP_2$ are images of 4-D surfaces of M^8 by $M^8 - H$-duality. The proposal is that they reduce to minimal surfaces analogous to soap films spanned by frames. Regions of both Minkowskian and Euclidean signature are predicted and the latter correspond to wormhole contacts represented by CP_2 type extremals. The boundary between the Minkowskian and Euclidean region is a light-like 3-surface representing the orbit of partonic 2-surface identified as wormhole throat carrying fermionic lines as boundaries of string world sheets connecting orbits of partonic 2-surfaces.

2. These fermionic lines are counterparts of the lines of ordinary Feynman graphs, and have ends at the partonic 2-surfaces located at the light-like boundaries of CD and in the interior of the space-time surface. The partonic surfaces, actually a pair of them as opposite throats of wormhole contact, in the interior define topological vertices, at which light-like partonic orbits meet along their ends.

3. These points should be somehow special. Number theoretically they should correspond points with coordinates in an extension of rationals for a polynomial P defining 4-surface in H and space-time surface in H by $M^8 - H$ duality. What comes first in mind is that the throats touch each other at these points so that the distance between Minkowskian space-time sheets vanishes. This is analogous to singularities of Fermi surface encountered in topological condensed matter physics: the energy bands touch each other. In TGD, the partonic 2-surfaces at the mass shells of M^4 defined by the roots of P are indeed analogs of Fermi surfaces at the level of $M^4 \subset M^8$, having interpretation as analog of momentum space.

   Could these points correspond to the defects of the standard smooth structure in X^4? Note that the branching at the partonic 2-surface defining a topological vertex implies the local failure of the manifold property. Note that the vertices of an ordinary Feynman diagram imply that it is not a smooth 1-manifold.

4. Could the interpretation be that the 4-manifold obtained by removing the partonic 2-surface has exotic smooth structure with the defect of ordinary smooth structure assignable to the partonic 2-surface at its end. The situation would be rather similar to that for the representation of exotic R^4 as a surface in CP_2 with the sphere at infinity removed [A1].

5. The failure of the cosmic censorship would make possible a pair creation. As explained, the fermionic lines can indeed turn backwards in time by going through the wormhole throat and turn backwards in time. The above picture suggests that this turning occurs only at the singularities at which the partonic throats touch each other. The QFT analog would be as a local vertex for pair creation.

6. If all fermions at a given boundary of CD have the same sign of energy, fermions which have returned back to the boundary of CD, should correspond to antifermions without a change in the sign of energy. This would make pair creation without fermionic 4-vertices possible.

   If only the total energy has a fixed sign at a given boundary of CD, the returned fermion could have a negative energy and correspond to an annihilation operator. This view is nearer to the QFT picture and the idea that physical states are Galois confined states of virtual fundamental fermions with momentum components, which are algebraic integers. One can also ask whether the reversal of the arrow of time for the fermionic lines could give rise to gravitational quantum computation as proposed in [A3].

2.3.4 A more detailed model for the exotic smooth structure associated with a topological 3-vertex

One can ask what happens to the 4-surface near the topological 3-particle vertex and what is the geometric interpretation of the point defect. The first is whether the description of the situation is possible both in M^8 and H. Here one must consider momentum conservation.

1. By Uncertainty Principle and momentum conservation at the level of M^8, the incoming real momenta of the particle reaction are integers in the scale defined by CD. In the standard QFT picture, the momenta at the vertex of physical particles are at different mass shells.

In M^8 picture, the mass squared values of virtual fermions are in general algebraic and also complex roots of a polynomial defining the 3-D mass shells H^3_m of $M^4 \subset M^8$, determining 4-surface by associative holography.

In the standard wave mechanical picture assumed also in TGD, a given topological vertex, describable in terms of partonic 2-surfaces, would correspond to a multi-local vertex in M^8 in accordance with the representation of a local n-vertex in M^4 as convolution of n-local vertices in momentum space realizing momentum conservation.

2. M^8-H duality maps M^4 momenta by inversion to positions in $M^4 \subset H$. This encourages the question whether the topological vertex could be described also in M^8 as a partonic surface at single algebraic mass shell in M^8, mapped by M^8-H duality to a single $a = constant$ hyperboloid in $M^4 \subset H$.

   The virtual momenta at the level of M^8 are algebraic, in general complex, integers. The algebraic mass squared values at the mass shell of M^8 would be the same for all particles of the vertex. This kind of correspondence does not make sense if M^8-H duality applies to the full algebraic momenta. The assumption has been that it applies to the rational parts of the momenta.

3. The rational parts of the algebraic integer valued 4-momenta of virtual fermions are in general not at the same mass shell. Could this make possible a description in terms of partonic 2-surfaces at fixed mass *resp.* $a = consant$ shell at the level of M^8 *resp.* H?

   The classical space-time surface in H, partonic 2-surfaces and fermion lines at them are characterized by classical momenta by Noether's theorem. Quantum classical correspondence, realized in ZEO as Bohr orbitology, suggests that the classical 4-momenta assignable to these objects correspond to the rational parts of the momenta at M^8 mass shell. Could the rational projections of M^8 momenta at H^3_n correspond to different mass squared values at given H^3?

4. Note that this additional symmetry for complexified momentum space and position space descriptions would be analogous to the duality of twistor amplitudes position space and the space of area momenta.

How to describe the topological vertex in H? The goal is to understand how exotic smooth structure and its point defects could emerge from this picture. The physical picture applied hitherto is as follows.

1. 3 partonic orbits meet at a vertex described by a partonic 2-surface. Assume that they are located to single $a = constant$ $H^3 \subset M^4 \subset H$.

2. The partonic wormhole throats appear as pairs at the opposite Minkowskian space-time sheets. There are three pairs corresponding to 3 external particle lines and one line which must be a bosonic line describing fermion-antifermion bound state disappears: this corresponds to a boson absorption (or emission).

   The opposite throats carry opposite magnetic monopole charges. The only possibility, not noticed before, is that the opposite wormhole throats for the partoni orbit, which ends at the vertex, must coincide at the vertex. The minimal option is that the exotic smooth structure is associated with this partonic orbit turning back in time. The two partonic orbits, which bind 4-D Euclidean regions as wormhole throats, would fuse to a larger 4-D surface with an exotic smooth structure.

   Fermion-antifermion annihilation occurs at a point at which fermion and antifermion lines meet. The first guess is that this point corresponds to the defect of the smooth structure.

3. There is an analogy with the construction of Etesi [A1]in which a homologically non-trivial ball CP_1 glued to the C^2 at infinity to construct an exotic smooth structure. One dimension disappears for the glued 3-surface at infinity.

   In the partonic vertex, one has actually two homologically non-trivial 2-surfaces with opposite homology charges as boundaries between wormhole contact and Minkowskian regions and they fuse together in the partonic vertex. Also now, one dimension disappears as the partonic 2-surfaces become identical so that 3-D wormhole contact contracts to single 2-D partonic 2-surface.

4. The defect for the smooth structure associated with the fusion of the pair of wormhole orbits should correspond to a point at which fermion and antifermion lines meet.

   This suggests that the throats do not fuse instantaneously but gradually. The fusion would start from a single touching point identifiable asd the fermion-antifermion vertex, serving as a seed of a phase transition, and would proceed to the entire wormhole contact so that it reduces to a partonic 2-surface.

   One can argue that one has a problem if this surface is homologically non-trivial. Could the process make the closed partonic 2-surface homologically trivial. A simplified example is the fusion of two circles with opposite winding numbers ± 1 on a cylinder. The outcome is two homologically non-trivial circles of opposite orientations on top of each other. The phase transition starting from a point would correspond to a touching of the circles.

A couple of further comments are in order.

1. The connection of the pair of wormhole throats to the associative holography is an interesting question. The 4-D tangent planes of $X^4 \subset M^8$ mass shell correspond to points of CP_2. They would be different at the two parallel sheets.

   At the mass shell H^3_m the branches would coincide. The presence of two tangent planes could give rise to two different holographic orbits, which coincide at the initial mass shell and gradually diverge from each other just as in the above model for the fusion of partonic 2-surfaces. The failure of the strict determinism for the associative holography at the partonic 2-surface would make in TGD the analogy of fermion-antifermion annihilation vertex possible.

2. There is also an analogy with the cusp catastrophe in which the projection of the cusp catastrophe as a 2-surface in 3-D space with behavior variable x and two control parameters (a, b) has a boundary at which two real roots of a polynomial of degree 3 coincide. The projection to the (a, b) plane gives a sharp shape, whose boundary is a V-shaped curve in which the sides of V become parallel at the vertex. The vertex corresponds to maximal criticality. The particle vertex would be a critical phenomenon in accordance with the interpretation as a phase transition.

3 Is a master formula for the scattering amplitudes possible?

Marko Manninen asked whether TGD can in some sense be reduced to a single equation or principle is very interesting. My basic answer is that one could reduce TGD to a handful of basic principles but formula analogous to $F = ma$ is not possible. However, at the level of classical physics, one could perhaps say that general coordinate invariance $\rightarrow$ holography $\leftarrow$ 4-D generalization of holomorphy [L10, L8, L9] reduce the representations of preferred extremals as analogs of Bohr orbits for particles as 3-surfaces to a representation analogous to that of a holomorphic function.

Can one hope something analogous to happen at the level of scattering amplitudes? Is some kind of a master formula possible? I have considered many options, even replacing the S-matrix with the Kähler metric in the fermionic degrees of freedom [L3]. The motivation was that the rows of the matrix defining Kähler metric define unit vectors allowing interpretation in terms of probability conservation. However, it seems that the concept of zero energy state alone makes the definition unambiguous and unitarity is possible without additional assumptions.

1. In standard quantum field theory, correlation functions for quantum fields give rise to scattering amplitudes. In TGD, the fields are replaced by the spinor fields of the "world of classical worlds" (WCW), which can regarded as superpositions of pairs of multi-fermion states restricted at the 3-D surfaces at the ends of the 4-D Bohr orbits at the boundaries of CD.

These 3-surfaces are extremely strongly but not completely correlated by holography implied by 4-D general coordinate invariance. The modes of WCW spinor fields at the 3-D surfaces correspond to irreducible unitary representations of various symmetries, which include supersymplectic symmetries of WCW and Kac-Moody type symmetries [K1, K4] [L4, L6, L10]. Hence the inner product is unitary.

2. Whatever the detailed form of the 3-D parts of the modes of WCW spinor fields at the boundaries of CD is, they can be constructed from ordinary many fermion states. These many-fermion state correspond in the number theoretic vision of TGD to Galois singlets realizing Galois confinement [L10, L7, L9]. They are states constructed at the level of M^8 from fermion with momenta whose components are possibly complex algebraic integers in the algebraic extension of rationals defining the 4-D region of M^8 mapped to H by $M^8 - H$ duality. Complex momentum means that the corresponding state decomposes to plane waves with a continuum of momenta. The presence of Euclidian wormhole contact makes already the classical momenta complex.

   Galois confined states have momenta, whose components are integers in the momentum scale defined by the causal diamond (CD). Galois confinement defines a universal mechanism for the formation of bound states. The induced spinor fields are second quantized free spinor fields in H and their Dirac propagators are therefore fixed. This means an enormou calculational simplification.

3. The inner products of these WCW spinor fields restricted to 3-surfaces determine the scattering amplitudes. They are non-trivial since the modes of WCW spinor fields are located at opposite boundaries of CD. These inner products define the zero energy state identifiable as such as scattering amplitudes. This is the case also in wave mechanics and quantum TGD is indeed wave mechanics for particles identified as 3-surfaces.

4. There is also a functional integral of these amplitudes over the WCW, i.e. over the 4-D Bohr orbits. This defines a unitary inner product. The functional integral replaces the path integral of field theory and is mathematically well-defined since the Kähler function, appearing in the exponent defining vacuum functional, is a non-local function of the 3-surface so that standard local divergences due to the point-like nature of particles disappear. Also the standard problems due to the presence of a Hessian coming from a Gaussian determinant is canceled by the square foot of the determinant of the Kähler metric appearing in the integration measure [K2, K4].

5. The restriction of the second quantized spinor fields to 4-surfaces and zero-energy ontology are absolutely essential. Induction turns free fermion fields into interacting ones. The spinor fields of H are free and define a trivial field theory in H. The restriction to space-time surfaces changes the situation. Non-trivial scattering amplitudes are obtained since the fermionic propagators restricted to the space-time surface are not anymore free propagators in H. Therefore the restriction of WCW spinors to the boundaries of CD makes the fermions interact in exactly the same way as it makes the induced spinor connection and the metric dynamical.

There are a lot of details involved that I don't understand, but it would seem that a simple "master formula" is possible. Nothing essentially new seems to be needed. There is however one more important "but".

3.1 Are pair production and boson emission possible?

The question that I have pondered a lot is whether the pair production and emission of bosons are possible in the TGD Universe. In this process the fermion number is conserved, but fermion and antifermion numbers are not conserved separately. In free field theories they are, and in the interacting quantum field theories, the introduction of boson fermion interaction vertices is necessary. This brings infinities into the theory.

1. In TGD, the second quantized fermions in H are free and the boson fields are not included as primary fields but are bound states of fermions and antifermions. Is it possible to produce

pairs at all and therefore also bosons? For example, is the emission of a photon from an electron possible? If a photon is a fermion-antifermion pair, then the fermion and antifermion numbers cannot be preserved separately. How to achieve this?

2. If fundamental fermions correspond to light-like curves at light-like orbit of partonic 2-surfaces, pair creation requires that that fermion trajectory turns in time direction. At this point velocity is infinite and this looks like a causal anomaly. There are two options: the fermion changes the sign of its energy or transforms to antiferion with the same sign of energy.

   Different signs of energy are not possible since the annihilation operator creating the fermion with opposite energy would annihilate either the final state or some fermion in the final state so that both fermion and antifermion numbers of the final state would be the same as those of the initial state.

   On the other hand, it can be said that positive energy antifermions propagate backwards in time because in the free fermion field since the terms proportional to fermion creation operators and antifermion annihilation operators appear in the expression of the field as sum of spinor modes.

   Therefore a fermion-antifermion pair with positive energies can be created and corresponds to a pair of creation operators. It could also correspond to a boson emission and to a field theory vertex, in which the fermion, antifermion and boson occur. In TGD, however, the boson fields are not included as primary fields. Is such a "vertex without a vertex" possible at all?

3. Can one find an interpretation for this creation of a pair that is in harmony with the standard view. Space-time surfaces are associated with induced classical gauge potentials. In standard field theory, they couple to fermion-antifermion pairs, and pairs can be created in classical fields. The modified Dirac equation [K3] and the Dirac equation in H also have such a coupling. Now the modified Dirac equation holds true at the fermion lines at the light-like orbits of the partonic 2-surface. Does the creation of pairs happen in this way? It might do so: also in the path integral formalism of field theories, bosons basically correspond to classical fields and the vertex is just this except that in TGD fermions are restricted to 1-D lines.

3.2 Fundamental fermion pair creation vertices as local defects of the standard smooth structure of the space-time surface?

Here comes the possible connection with a very general mathematical problem of general relativity that I have discussed in [L5].

1. Causal anomalies as time loops that break causality are more the rule than an exception in general relativity the essence of the causal anomaly is the reversal of the arrow of time. Causal anomalies correspond to exotic diffeo-structures that are possible only in dimension $D = 4$! Their number is infinite.

2. Quite generally, the exotic smooth structures reduce to defects of the usual differentiable structure and have measure zero. Assume that they are point like defects. Exotic differentiable structures are also possible in TGD, and the proposal is that the associated defects correspond to a creation of fermion-fermion pairs for emission of fermion pairs of of gauge bosons and Higgs particle identified in TGD as bound states of fermion-antifermion pairs. This picture generalizes also to the case of gravitons, which would involve a pair of vertices of this kind. The presence of 2 vertices might relate to the weakness of the gravitational interaction.

   The reversal of the fermion line in time direction would correspond to a creation of a fermion-antifermion pair: fermion and antiferion would have the same sign of energy. This would be a causal anomaly in the sense that the time direction of the fermion line is reversed so that it becomes an antifermion.

I have proposed that this causal anomaly is identifiable as an anomaly of differentiable structure so that emission of bosons and fermion pairs would only be possible in dimension 4: the space-time dimension would be unique!

3. But why would a point-like local defect of the differentiable structure correspond to a fermion pair creation vertex. In TGD, the point-like fermions correspond to 1-D light-like curves at the light-like orbit of the partonic 2-surface.

   In the pair creation vertex in presence of classical induced gauge potentials, one would have a V-shaped world line of fermion turning backwards in time meaning that antifermion is transformed to fermion. The antifermion and fermion numbers are not separately conserved although the total fermion number is. If one assumes that the modified Dirac equation holds true along the entire fermion worldline, there would be no pair creation.

   If it holds true only outside the V-shaped vertex the modified Dirac action for the V-shaped fermion libe can be transformed to a difference of antifermion number equal to the discontinuity of the antifermion part of the fermion current identified as an operator at the vertex. This would give rise to a non-trivial vertex and the modified gamma matrices would code information about classical bosonic action.

4. The 1-D curve formed by fermion and antifermion trajectories with opposite time direction turns backwards in time at the vertex. At the vertex, the curve is not differentiable and this is what the local defect of the standard smooth differentiable structure would mean physically!

3.3 Master formula for the scattering amplitudes: finally?

Most pieces that have been identified over the years in order to develop a master formula for the scattering amplitudes are as such more or less correct but always partially misunderstood. Maybe the time is finally ripe for the fusion of these pieces to a single coherent whole. I will try to list the pieces into a story in the following.

1. The vacuum functional, which is the exponential Kähler function defined by the classical bosonic action defining the preferred extremal a an analog of Bohr orbit, is the starting point. Physically, the Kähler function corresponds to the bosonic action (e.g. EYM) in field theories.

   Because holography is almost unique, it replaces the path integral by a sum over 4-D Bohr trajectories as a functional integral over 3-surfaces plus discrete sum.

2. However, the fermionic part of the action is missing. I have proposed a long time ago a super symmetrization of the WCW Kähler function by adding to it what I call modified Dirac action. It relies on modified gamma matrices modified gamma matrices Γ^α, which are contractions $\Gamma_k T^{\alpha k}$ of H gamma matrices Γ_k with the canonical momentum currents $T^{\alpha k} = \partial L/\partial_{\partial_\alpha h^k}$ defined by the Lagrangian L. Modified Dirac action is therefore determined by the bosonic action from the requirement of supersymmetry. This supersymmetry is however quite different from the SUSY associated with the standard model and it assigns to fermonic Noether currents their super counterparts.

   Bosonic field equations actually follow as hermiticity conditions for the modified Dirac equation. These equations also guarantee the conservation of fermion number(s). The overall super symmetrized action that defines super symmetrized Kähler function in WCW would be unambiguous. One would get exactly the same master formula as in quantum field theories, but without the path integral.

3. The overall super symmetrized action is sum of contributions assignable to the space-time surface itself, its 3-D light-like parton orbits as boundaries between Minkowskian regions and Euclidian wormhole contact, 2-D string world sheets and their 1-D boundaries as orbits of point-like fermions. These 1-D boundaries are the most important and analogous to the lines of ordinary Feynman diagrams. One obtains a dimensional hierarchy.

4. One can assign to these objects of varying dimension actions defined in terms of the induced geometry and spinor structure. The supersymmetric actions for the preferred extremals

analogous to Bohr orbit in turn give contributions to the super symmetrized Kähler function as an analogue of the YM action so that, apart from the reduction of path integral to a sum over 4-D Bohr orbits, there is a very close analogy with the standard quantum field theory.

However, some problems are encountered.

1. It seems natural to assume that a modified Dirac equation holds true. I have presented an argument for how it indeed emerges from the induction for the second quantized spinor field in H restricted to the space-time surface assuming modified Dirac action.

   The problem is, however, that the fermionic action, which should define vertex for fermion pair creation, disappears completely if Dirac's equation holds everywhere! One would not obtain interaction vertices in which pairs of fermions arise from classical induced fields. Something goes wrong. In this vertex total fermion number is conserved but fermion and antifermion numbers are changed since antifermion transforms to fermion at the V-shaped vertex: this condition should be essential.

2. If one gives up the modified Dirac equation, the fermionic action does not disappear. In this case, one should construct a Dirac propagator for the modified Dirac operator. This is an impossible task in practice.

   Moreover, the construction of the propagator is not even necessary and in conflict with the fact that the induced spinor fields are second quantized spinors of H restricted to the space-time surface and the propagators are therefore well-defined and calculable and define the propagation at the space-time surface.

3. Should we conclude that the modified Dirac equation cannot hold everywhere? What these, presumably lower-dimensional regions of space-time surface, are and could they give the interaction vertices as topological vertices?

The key question is how to understand geometrically the emission of fermion pairs and bosons as their bound states?

1. I have previously derived a topological description for reaction vertices. The fundamental 1 $\rightarrow$ 2 vertex (for example e$\rightarrow$e+ gamma) generalizes the basic vertex of Feynman diagrams, where a fermion emits a boson or a boson decays into a pair of fermions. Three lines meet at the ends.

   In TGD, this vertex can topologically correspond to the decomposition of a 3-surface into two 3-surfaces, to the decomposition of a partonic 2-surface into two, to the decomposition of a string into two, and finally, to the turning of the fermion line backwards from time. One can say that the n-surfaces are glued together along their $n-1$-dimensional ends, just like the 1-surfaces are glued at the vertex in the Feynman diagram.

2. In the previous section, I already discussed how to identify vertex for fermion-antifermion pair creation as a V-shaped turning point of a 1-D fermion line. The fermion line turns back in time and fermion becomes an antifermion. In TGD, the quantized boson field at the vertex is replaced by a classical boson field. This description is basically the same as in the ordinary path integral where the gauge potentials are classical.

   The problem was that if the modified Dirac equation holds everywhere, there are no pair creation vertices. The solution of the problem is that the modified Dirac equation at the V-shaped vertex cannot hold true.

   What this means physically is that fermion and antifermion numbers are not separately conserved in the vertex. The modified Dirac action for the fermion line can be transformed to the change of antifermion number as operator (or fermion number at the vertex) expressible as the change of the antifermion part of the fermion number. This is expressible as the discontinuity of a corresponding part of the conserved current at the vertex. This picture conforms with the appearance of gauge currents in gauge theory vertices. Notice that modified gamma matrices determined by the bosonic action appear in the current.

3. This argument was limited to 1-D objects but can be generalized to higher-dimensional defects by assuming that the modified Dirac equation holds true everywhere except at defects represented as vertices, which become surfaces. The modified Dirac action reduces to an integral of the discontinuity of say antifermion current at the vertex, i.e. the change of the antifermion charge as an operator.

What remains more precisely understood and generalized, is the connection with the irreducible exotic smooth structures possible only in 4-D space-time.

1. TGD strongly suggests that 0-dimensional vertices generalize to topological vertices representable as surfaces of dimension $n = 0, 1, 2, 3$ assignable to objects carrying induced spinor field. In the 1→ 2 vertex, the orbit of an $n < 4$- dimensional surface would turn back in the direction of time and would define a V-shaped structure in time direction. These would be the various topological vertices that I have previously arrived at, but guided by a physical intuition. Also now the vertex would boild down to the discontinuity of say antifermion current instead of the current itself at the vertex.

2. It is known that exotic smooth structures reduce to standard ones except in a set of defects having measure zero. Also non-point-like defects might be possible in contrast to what I assumed at first. If the defects are surfaces, their dimension is less than 4. If not, then only the direction of fermion lines could change.

   If the generalization is possible, also 1-D, 2-D, and 3-D defects, defining an entire hierarchy of particles of different dimensions, is possible. As a matter of fact, a longstanding issue has been whether this prediction should be taken seriously. Note that in topological condensed matter physics, defects with various dimensions are commonplace. One talks about bulk states, boundary states, edge states and point-like singularities. In this would predict hierarchy of fermionic object of various dimensions.

To summarize, exotic smooth structures would give vertices without vertices assuming only free fermions fields and no primary boson fields! And this is possible only in space-time dimension 4!

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