Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD

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Abstract

The existence of exotic smooth structures even in the simplest possible 4-D space R^4 might have some relevance for TGD. The study of the smooth structures in 4-D case involves intersection form for 2-homology of the 4-manifold. However, the existence of smooth structures in the 4-D case is not the only reason to get interested in this topic.

The first reason is that in the TGD framework the intersection form describes the intersections of string world sheets and partonic 2-surfaces and therefore is of direct physical interest.

The second reason relates to the role of knots in TGD. The 1-homology of the knot complement characterizes the knot. Time evolution defines a knot cobordism as a 2-surface consisting of knotted string world sheets and partonic 2-surfaces. A natural guess is that the 2-homology for the 4-D complement of this cobordism characterizes the knot cobordism. Also 2-knots are possible in 4-D space-time and a natural guess is that knot cobordism defines a 2-knot.

Exotic smoothness could be anomalous in the TGD framework. Can one find any argument excluding the exotics? A reasonable expectation is that the metrics of Minkowski space M^4 and CP_2 fix completely the smooth structure of $H = M^4 \times CP_2$ but what about space-time surfaces $X^4 \subset H$. The smooth structure, unlike topology, of X^4 cannot be induced from that of H. In the case of Lie-groups, group structure implies the standard smooth structure: this is highly relevant for TGD.

In the TGD framework, but not generally (coordinate atlas cannot be extended from the boundary to the interior), one can consider the holography of smoothness, which in zero energy ontology (ZEO) implies that the X^4 and also the smooth structure in X^4 is uniquely induced from its boundary, that is from the ends of X^4 at light-like boundaries of causal diamond CD $\subset H$. It is known that exotic smoothness reduces to ordinary one in a complement of a set of arbitrary small balls of a manifold so that it is analogous to the existence of local defects in condensed matter physics.

The induced smooth structure need not be the standard one. The analogs of point defects would be associated with partonic 2-surfaces in the interior of space-time surfaces, and representing topological particle reaction vertices at which light-like parton orbits meet. Defect could correspond to points at which fermion pairs can be created. The smooth structure in the complement of the vertex would reduce to the ordinary smooth structure. One ends up with a concrete proposal in terms of a topological generalization of Feynman graphs.

Contents

1 Introduction

Gary Ehlenberger sent a highly interesting commentary related to smooth structures in R^4 discussed in the article of Gompf [A2] (`https://cutt.ly/eMracmf`) and more generally to exotics smoothness discussed from the point of view of mathematical physics in the book of Asselman-Maluga and Brans [A3] (`https://cutt.ly/DMu0dYr`). I am grateful for these links for Gary.

1.1 The role of intersection forms in TGD

The intersection form of 4-manifold (`https://cutt.ly/jMriNdI`) characterizing partially its 2-homology is a central notion in these consideration and it is expected to have a central role in TGD [K3, K6]. I am not a topologist but I had two good reasons to get interested.

1. In the TGD framework [L6], the intersection form describes the intersections of string world sheets and partonic 2-surfaces and therefore is of direct physical interest [K3, K6].

2. Knots have an important role in TGD. The 1-homology of the knot complement characterizes the knot. Time evolution defines a knot cobordism as a 2-surface consisting of knotted string world sheets and partonic 2-surfaces. A natural guess is that the 2-homology for the 4-D complement of this cobordism characterizes the knot cobordism. Also 2-knots are possible in 4-D space-time and a natural guess is that knot cobordism defines a 2-knot.

   The intersection form for the complement for cobordism as a way to classify these two-knots is therefore highly interesting in the TGD framework. One can also ask what the counterpart for the opening of a 1-knot by repeatedly modifying the knot diagram could mean in the case of 2-knots and what its physical meaning could be in the TGD Universe. Could this opening or more general knot-cobordism of 2-knot take place in zero energy ontology (ZEO) [L2, L5, L7] as a sequence of discrete quantum jumps leading from the initial 2-knot to the final one.

1.2 Why exotic smooth structures are not possible in TGD?

The existence of exotic 4-manifolds [A2, A3, A1] could be an anomaly in the TGD framework. In the articles [A2, A1] the term anomaly is indeed used. Could these anomalies cancel in the TGD framework?

The first naive guess was that the exotic smooth structures are not possible in TGD but it turned out that this is not trivially true. The reason is that the smooth structure of the space-time surface is not induced from that of H unlike topology. One could induce smooth structure by assuming it given for the space-time surface so that exotics would be possible. This would however bring an ad hoc element to TGD. This raises the question of how it is induced.

1. This led to the idea of a holography of smoothness, which means that the smooth structure at the boundary of the manifold determines the smooth structure in the interior. Suppose that the holography of smoothness holds true. In ZEO, space-time surfaces indeed have 3-D ends with a unique smooth structure at the light-like boundaries of the causal diamond $CD = cd \times \mathrm{CP}_2 \subset H = M^4 \times CP_2$, where cd is defined in terms of the intersection of future and past directed light-cones of M^4. One could say that the absence of exotics implies that $D = 4$ is the maximal dimension of space-time.
2. The differentiable structure for $X^4 \subset M^8$, obtained by the smooth holography, could be induced to $X^4 \subset H$ by $M^8 - H$-duality. Second possibility is based on the map of mass shell hyperboloids to light-cone proper time $a = constant$ hyperboloids of H belonging to the space-time surfaces and to a holography applied to these.
3. There is however an objection against holography of smoothness (`https://cutt.ly/3MewYOt`). In the last section of the article, I develop a counter argument against the objection. It states that the exotic smooth structures reduce to the ordinary one in a complement of a set consisting of arbitrarily small balls so that local defects are the condensed matter analogy for an exotic smooth structure.

2 Intersection form in the case of 4-surfaces

Intersection form (`https://cutt.ly/jMriNdI`) for homologically trivial 2-surfaces of the space-time surface and 2-homology for the complement of these surfaces can be physically important in tGD framework.

2.1 Intersection form form 2-D manifolds

It is good to explain the notion of intersection form by starting from 1-homology. The intersection form for 1-homology is encountered for a cylinder with ends fixed. In this case, one has relative homology and homologically trivial curves are curves connecting the ends of string and characterized by a winding number.

In the case of torus obtained by identifying the ends of cylinder, one obtains two winding numbers (m, n) corresponding to to homologically non-trivial circles at torus. The intersection number for curves (m, n) and (p, q) at torus is $N = mq - np$ and for curves at cylinder one as $(m, n) = (1, n)$ giving $N = n - q$.

The antisymmetric intersection form is defined as 2×2 matrix defining intersections for the basis of the homology with $(m, n) = (1, 0)$ and $(n, m) = (0, 1)$ and is given by $(0, 1; -1, 0)$.

2.2 Intersection forms for 4-surfaces

In TGD, the intersection form for a 4-surface identified as space-time surface could have a rather concrete physical interpretation and the stringy part of TGD physics would actually realize it concretely.

1. $M^8 - H$ duality requires that the 4-surface in M^8 has quaternionic/associative normal space: this distribution of normal spaces is integrable and integrates to the 4-surface in M^8.

   The normal must also contain a commutative (complex) sub-space at each point. Only this allows us to parametrize normal spaces by points of CP_2 and map them to space-time surfaces in $H = M^4 \times CP_2$. The integral distribution of these commutative sub-spaces defines a 2-D surface. Physically, these surfaces would correspond to string world sheets and partonic 2-surfaces.

2. String world sheets and partonic 2-surfaces, regarded as objects in relative homology (modulo ends of the space-time surfaces at the boundaries of causal diamond (CD)), can intersect as 2-D objects inside the space-time surface and the intersection form characterizes them.

   There is an analogy with the cylinder: time-like direction corresponds to the cylinder axis and a homologically non-trivial 2-surface of CP_2 corresponds to the circle at the cylinder.

3. If the second homology of the space-time surface is trivial, the naive expectation is that the intersections of string world sheets are not stable under large enough deformations of the string world sheets. Same applies to intersecting plane curves. At the cylinder, the situation is different since the relative first homology is non-trivial and spanned by two generators: the circle and a line connecting the ends of the cylinder.

   The intersection form is however non-trivial as in the case of the cylinder for 2-surfaces having 2-D homologically non-trivial CP_2 projection. They would represent M^4 deformations of 2-D homologically trivial surfaces of CP_2 just like a helical orbit along a cylinder surface. A 2-D generalization of CP_2 type extremal would have a light-like curve or light-like geodesic as M^4 projection and could define light-partonic orbit.

4. The intersection of string world sheet and partonic 2-surface can be stable however. Partonic 2-surface is a boundary of a wormhole contact connecting two space-time sheets.

   Consider a string arriving along space-time sheet A, going through the wormhole contact, and continuing along sheet B. The string has an intersection point with both wormhole throats. This intersection is stable against deformations. The orbit of this string intersects the light-like orbit of the partonic 2-surface along the light-like curve.

   One has a non-trivial intersection form with the number of intersections with partonic 2-surfaces equal to 1. In analogy with cylinder, also the intersections of 2-surfaces with 2-D homologically trivial CP_2 projection are unavoidable and reflect the non-trivial intersection form of CP_2.

2.3 About ordinary knots

Ordinary knots and 3-topologies are related and the natural expectation is that also 2-knots and 4-topologies are related.

2.3.1 About knot invariants

Consider first knot invariants (`https://cutt.ly/DMrgs14`)at the general level.

1. One important knot invariant of ordinary knots is the 1-homology of the complement and the associated first homotopy group whose abelianization gives the homology group.

2. The complement of the knot can be given a metric of a hyperbolic 3-manifold, which corresponds to a unit cell for a tessellation of the mass shell. M^8-H duality suggests that the intersection X^3 of 4-surface of M^8 with mass shell $H^3_m \subset M^4 \subset M^8$ is a hyperbolic manifold and identical with the hyperbolic manifold associated with the complement of a knot of H^3_a realized as light-cone proper time $a = constant$ hyperboloid of $M^4 \subset H$ and closed knotted and linked strings as ends of string world sheets at H^3_a.

   The evolution of the strings defined by the string world sheets would define a 1-knot cobordism. The 2-homology of the knot complement should characterize the topological evolution of the 1-homology of the knot.

2.3.2 Opening of knots and links by knot cobordisms

The procedure leading to the trivialization of knot or link can be used to define knot invariants and the procedure itself characterizes knot.

1. Ordinary knot is described by a knot diagram obtained as a projection of the knot to the plane. It contains intersections of lines and the intersection contains information telling which line is above and which line is below.

2. The opening of the knot or link to give a trivial knot or link, which is used in the construction of knot invariants, is a sequence of violent operations. In the basic step strings portions go through each other and therefore suffer a reconnection. This operation can therefore change the 1-homology of the 3-D knot complement.

   Knot or link can be modified by forcing two intersecting strands of the plane projection to go through each other. Locally the basic operation for two links is the same as for the pieces of knot. The transformation of the knot or link to a trivial knot or link corresponds to some sequence of these operations and can be used to define a knot invariants. This operation is not unique since there are moves which do not affect the knot.

   The basic opening operation can be also seen as a time evolution, knot cobordism, in which the first portion, call it A, remains unchanged and the second portion, call it B, draws a 2-D surface in E^3. A intersects the 2-D orbit at a single point.

3. The 2-homology for the string world sheets and partonic 2-surfaces as 2-surfaces in space-time serves as an invariant of knot cobordism and represents the topological dynamics of ordinary 1-knots of 3-surface and links formed by strings or flux tubes in 3-surface as cobordism defining the time evolution of a knot to another knot.

   In particular, the intersection form for the 2-homology of the complement of the cobordism defines an invariant of cobordism. This intersection form must be distinguished from the intersection form for the second homology of the space-time surface rather than the 2-knot complement.

4. One can also consider more general sequences of basic operations transforming two knots or links to each other as knot-/link cobordisms, which involve self intersections of the knots. Does this mean that the intersection form characterizes the knot cobordism. Could a string diagram involving reconnections describe the cobordism process.

2.3.3 Stringy description of knot cobordisms

$M^8 - H$ duality [L3, L4, L9, L8] requires string word sheets and partonic 2-surfaces. This implies that TGD physics represents the 2-homology of both space-time surfaces and the homology of the complement of the knotted links defined by them.

Although the "non-homological" intersections of string world sheets can be eliminated by a suitable deformation of the string world sheet, they should have a physical meaning. This comes from the observation that they affect nontrivially the 1-homology of the knot complement as 3-D time=constant slice.

The first thing that I am able to imagine is that strings reconnect. This is nothing but the trouser vertex for strings so that intersection form would define topological string dynamics in some sense. These reconnections play a key role in TGD, also in TGD inspired quantum biology.

The dynamics of partonic 2-surfaces and string world sheets could relate to knot cobordisms, possibly leading to the opening of ordinary knot,

2.4 What about 2-knots and their cobordisms?

2-D closed surfaces in 4-D space give rise to 2-knots. What is the physical meaning of 2-knots of string world sheets? What could 2-knots for orbits of linear molecules or associated magnetic flux tubes mean physically and from the point of view of quantum information theory? One can try to understand 2-knots by generalizing the ideas related to the ordinary knots.

1. Intuitively it seems that the cobordism of a 1-knot defines a 2-knot. It is not clear to me whether all 2-knots for space-time surfaces connecting the boundaries of CD can be regarded as this kind of cobordisms of 1-knots.

2. The 2-homology of the complement of 2-knot should define a 2-knot invariant. In particular, the intersection form should define a 2-knot invariant.

3. The opening of 1-knot by repeating the above described basic operation is central in the construction of knot invariants and the sequence of the operations can be said to be knot invariant modulo moves leaving the knot unaffected.

   The opening or a more general cobordism of a 2-knot could be seen as a time evolution with respect to a time parameter t_5 parametrizing the isotopy of space-time surface. The local cobordism can keep the first portion of 2-knot, call it A, unchanged and deform another portion, call it B, so that a 3-D orbit at the space-time surface is obtained. For each value of t_5, the portions A and B of 2-knot have in the generic case only points as intersections.

   This would suggest that an intersection point of A and B is generated in the operation and moves during the t_5 time evolution along A along 1-D curve during the process. This process would be the basic operation used repeatedly to open 2-knot or to transform it to another 2-knot.

4. In quantum TGD, a sequence of quantum jumps, quantum cobordism, would have the same effect as t_5 time evolution. This brings in mind DNA transcription and replication as a process proceeding along a DNA strand parallel to the monopole flux tube as a sequence of SFRs involving direct contact between DNA strand and enzymes catalyzing the process and also of corresponding flux tubes. An interesting possibility is that these quantum cobordisms appear routinely in biochemistry of the fundamental linear bio-molecules such as DNA, RNA, tRNA, and amino-acids [K2, K4, K7, K1, K8, K5] [L1].

   The quantum cobordism of 2-knot is possible only in ZEO, where the quantum state as a time= constant snapshot is replaced with a superposition of space-time surfaces.

3 Could the existence of exotic smooth structures pose problems for TGD?

The article of Gabor Etesi [A1] (`https://cutt.ly/2Md7JWP`) gives a good idea about the physical significance of the existence of exotic smooth structures and how they destroy the cosmic censorship hypothesis (CCH of GRT stating that spacetimes of GRT are globally hyperbolic so that there are no time-like loops.

3.1 Smooth anomaly

No compact smoothable topological 4-manifold is known, which would allow only a single smooth structure. Even worse, the number of exotics is infinite in every known case! In the case of non-compact smoothable manifolds, which are physically of special interest, there is no obstruction against smoothness and they typically carry an uncountable family of exotic smooth structures.

One can argue that this is a catastrophe for classical general relativity since smoothness is an essential prerequisite for tensory analysis and partial differential equations. This also destroys hopes that the path integral formulation of quantum gravitation, involving path integral over all possible space-time geometries, could make sense. The term anomaly is certainly well-deserved.

Note however that for 3-geometries appearing as basic objects in Wheeler's superspace approach, the situation is different since for $D < 3$ there is only a single smooth structure. If one has holography, meaning that 3-geometry dictates 4-geometry, it might be possible to avoid the catastrophe.

The failure of the CCH is the basic message of Etesi's article. Any exotic R^4 fails to be globally hyperbolic and Etesi shows that it is possible to construct exact vacuum solutions representing curved space-times which violate the CCH. In other words, GRT is plagued by causal anomalies.

Etesi constructs a vacuum solution of Einstein's equations with a vanishing cosmological constant which is non-flat and could be interpreted as a pure gravitational radiation. This also represents one particular aspect of the energy problem of GRT: solutions with gravitational radiation should not be vacua.

1. Etesi takes any exotic R^4 which has the topology of $S^3 \times R$ and has an exotic smooth structure, which is not a Cartesian product. Etesi maps maps R^4 to CP_2, which is obtained

from C^2 by gluing CP_1 to it as a maximal ball B_r^3 for which the radial Eguchi-Hanson coordinate approaches infinity: $r \to \infty$. The exotic smooth structure is induced by this map. The image of the exotic atlas defines atlas. The metric is that of CP_2 but $SU(3)$ does not act as smooth isometries anymore.

2. After this Etesi performs Wick rotation to Minkowskian signature and obtains a vacuum solution of Einstein's equations for any exotic smooth structure of R^4.

3.2 Can embedding space and related spaces have exotic smooth structure?

One can worry about the exotic smooth structures possibly associated with the M^4, CP_2, $H = M^4 \times CP_2$, causal diamond CD= $cd \times CP_2$, where *cd* is the intersection of the future and past directed light-cones of M^4, and with M^8. One can also worry about the twistor spaces CP_3 *resp.* $SU(3)/U(1) \times U(1)$ associated with M^4 *resp.* CP_2.

The key assumption of TGD is that all these structures have maximal isometry groups so that they relate very closely to Lie groups, whose unique smooth structures are expected to determine their smooth structures.

1. The first sigh of relief is that all Lie groups have the standard smooth structure. In particular, exotic R^4 does not allow translations and Lorentz transformations as isometries. I dare to conclude that also the symmetric spaces like CP_2 and hyperbolic spaces such as $H^n = SO(1,n)/SO(n)$ are non-exotic since they provide a representation of a Lie group as isometries and the smoothness of the Lie group is inherited. This would mean that the charts for the coset space G/H would be obtained from the charts for G by an identification of the points of charts related by action of subgroup H.

   Note that the mass shell H^3, as any 3-surface, has a unique smooth structure by its dimension.

2. Second sigh of relief is that twistor spaces CP_3 and $SU(3)/U(1) \times U(1)$ have by their isometries and their coset space structure a standard smooth structure.

   In accordance with the vision that the dynamics of fields is geometrized to that of surfaces, the space-time surface is replaced by the analog of twistor space represented by a 6-surface with a structure of S^2 bundle with space-time surface X^4 as a base-space in the 12-D product of twistor spaces of M^4 and CP_2 and by its dimension $D = 6$ can have only the standard smooth structure unless it somehow decomposes to $(S^3 \times R) \times R^2$. Holography of smoothness would prevent this since it has boundaries because X^4 as base space has boundaries at the boundaries of CD.

   If exotic smoothness is allowed at the space-time level in the proposed sense ordinary smooth structure could be possible at the level of twistor space in the complement of a Cartesian product of the fiber space S^2 with a discrete set of points associated with partonic 2-surfaces.

3. *cd* is an intersection of future and past directed light-cones of M^4. Future/past directed light-cone could be seen as a subset of M^4 and implies standard smooth structure is possible. Coordinate atlas of M^4 is restricted to *cd* and one can use Minkowski coordinates also inside the *cd*. *cd* could be also seen as a pile of light-cone boundaries $S^2 \times R_+$ and by its dimension $S^2 \times R$ allows only one smooth structure.

4. M^8 is a subspace of complexified octonions and has the structure of 8-D translation group, which implies standard smooth structure.

The conclusion is that continuous symmetries of the geometry dictate standard smoothness at the level of embedding space and related structures.

3.3 Could TGD eliminate the smoothness anomaly or provide a physical interpretation for it?

The question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies.

What does the induction of a differentiable structure really mean? Here my naive expectations turn out to be wrong. If a sub-manifold $S \subset H$ can be regarded as an embedding of smooth manifold N to $S \subset H$, the embedding $N \rightarrow S \subset H$ induces a smooth structure in S (https://cutt.ly/tMtvG79). The problem is that the smooth structure would not be induced from H but from N and for a given 4-D manifold embedded to H one could also have exotic smooth structures. This induction of smooth structure is of course physically adhoc.

It is not possible to induce the smooth structure from H to sub-manifold. The atlas defining the smooth structure in H cannot define the charts for a sub-manifold (surface). For standard R^4 one has only one atlas.

3.3.1 Could holography of smoothness make sense in the general case?

The first trial to get rid of exotics was based on the holography of smoothness and did not involve TGD. Could a smooth structure at the boundary of a 4-manifold could dictate that of the manifold uniquely. Could one speak of holography for smoothness? Manifolds with boundaries would have the standard smooth structure.

1. The obvious objection is that the coordinate atlas for 3-D boundary cannot determine 4-D atlas in any way because the boundary cannot have information of the topology of the interior.

2. The holography for smoothness is also argued to fail (https://cutt.ly/3MewYOt). Assume a 4-manifold W with 2 different smooth structures. Remove a ball B^4 belonging to an open set U and construct a smooth structure at its boundary S^3. Assume that this smooth structure can be continued to W. If the continuation is unique, the restrictions of the 2 smooth structures in the complement of B^4 would be equivalent but it is argued that they are not.

3. The first layman objection is that the two smooth structures of W are equivalent in the complement $W - B^3$ of an arbitrary small ball $B^3 \subset W$ but not in the entire W. This would be analogous to coordinate singularity. For instance, a single coordinate chart is enough for a sphere in the complement of an arbitrarily small disk.

   An exotic smooth structure would be like a local defect in condensed matter physics. In fact it turned out that this intuitive idea is correct: it can be shown that the exotic smooth structures are equivalent with standard smooth structure in a complement of a set having co-dimension zero (https://cutt.ly/7MbGqx2). This does not save the holography of smoothness in the general case but gives valuable hints for how exotic smoothness might be realized in TGD framework.

3.3.2 Could holography of smoothness make sense in the TGD framework?

Could $M^8 - H$ duality and holography make holography of smoothness possible in the TGD framework?

1. In the TGD framework space-time is 4-surface rather than abstract 4-manifold. 4-D general coordinate invariance, assuming that 3-surfaces as generalization of point-like particles are the basic objects, suggests a fully deterministic holography. A small failure of determinism is however possible and expected, and means that space-time surfaces analogous to Bohr orbits become fundamental objects. Could one avoid the smooth anomaly in this framework?

   The 8-D embedding space topology induces 4-D topology. My first naive intuition was that the 4-D smooth structure, which I believed to be somehow inducible from that of $H = M^4 \times CP_2$, cannot be exotic so that in TGD physics the exotics could not be realized. But can one really exclude the possibility that the induced smooth structure could be exotic as a 4-D smooth structure?

2. In the TGD framework and at the level of $H = M^4 \times cP_2$, one can argue that the holography implied by the general coordinate invariance somehow determines the smooth structure in the interior of space-time surface from the coordinate atlas at the boundary. One would have a holography of smoothness. It is however not obvious why this unique structure should be the standard one.

3. One has also holography in M^8 and this induces holography in H by $M^8 - H$ duality. The 3-surfaces X^3 inducing the holography in M^8 are parts of mass shells, which are hyperbolic spaces $H^3 \subset M^4 \subset M^8$. 3-surfaces X^3 could be even hyperbolic 3-manifolds as unit cells of tessellations of H^3. These hyperbolic manifolds have unique smooth structures as manifolds with dimension $D < 4$.

   The hypothesis is that one can assign to these 3-surfaces a 4-surface by a number theoretic dynamics requiring that the normal space is associative, that is quaternionic [L3, L4]. The additional condition is that the normal space contains commutative subspace makes it possible to parametrize normal spaces by points of CP_2. $M^8 - H$ duality would map a given normal space to a point of CP_2. $M^8 - H$ duality makes sense also for the twistor lift.

4. A more general statement would be as follows. A set of 3-surfaces as sub-manifolds of mass shells H^3_m determined by the roots of polynomial P having interpretation as mass square values defining the 4-surface in M^8 take the role of the boundaries. Mass-shells H^3_m or partonic 2-surfaces associated with them having particle interpretation could correspond to discontinuities of derivatives and even correspond to failure of manifold property analogous to that occurring for Feybman diagrams so that the holography of smoothness would decompose to a piece-wise holography.

   The regions of $X^4 \subset M^8$ connecting two sub-sequent mass shells would have a unique smooth structure induced by the hyperbolic manifolds H^3 at the ends.

It is important to notice that the holography of smoothness does not force the smooth 4-D structure to be the standard one.

3.3.3 Could the exotic smooth structures have a physical interpretation in the TGD framework?

In the TGD framework, exotic smooth structures could also have a physical interpretation. As noticed, the failure of the standard smooth structure can be thought to occur at a point set of dimension zero and correspond to a set of point defects in condensed matter physics. This could have a deep physical meaning.

1. The space-time surfaces in $H = M^4 \times CP_2$ are images of 4-D surfaces of M^8 by $M^8 - H$-duality. The proposal is that they reduce to minimal surfaces analogous to soap films spanned by frames. Regions of both Minkowskian and Euclidean signature are predicted and the latter correspond to wormhole contacts represented by CP_2 type extremals. The boundary between the Minkowskian and Euclidean region is a light-like 3-surface representing the orbit of partonic 2-surface identified as wormhole throat carrying fermionic lines as boundaries of string world sheets connecting orbits of partonic 2-surfaces.

2. These fermionic lines are counterparts of the lines of ordinary Feynman graphs, and have ends at the partonic 2-surfaces located at the light-like boundaries of CD and in the interior of the space-time surface. The partonic surfaces, actually a pair of them as opposite throats of wormhole contact, in the interior define topological vertices, at which light-like partonic orbits meet along their ends.

3. These points should be somehow special. Number theoretically they should correspond points with coordinates in an extension of rationals for a polynomial P defining 4-surface in H and space-time surface in H by $M^8 - H$ duality. What comes first in mind is that the throats touch each other at these points so that the distance between Minkowskian space-time sheets vanishes. This is analogous to singularities of Fermi surface encountered in topological condensed matter physics: the energy bands touch each other. In TGD, the

partonic 2-surfaces at the mass shells of M^4 defined by the roots of P are indeed analogs of Fermi surfaces at the level of $M^4 \subset M^8$, having interpretation as analog of momentum space.

Could these points correspond to the defects of the standard smooth structure in X^4? Note that the branching at the partonic 2-surface defining a topological vertex implies the local failure of the manifold property. Note that the vertices of an ordinary Feynman diagram imply that it is not a smooth 1-manifold.

4. Could the interpretation be that the 4-manifold obtained by removing the partonic 2-surface has exotic smooth structure with the defect of ordinary smooth structure assignable to the partonic 2-surface at its end. The situation would be rather similar to that for the representation of exotic R^4 as a surface in CP_2 with the sphere at infinity removed [A1].

5. The failure of the cosmic censorship would make possible a pair creation. As explained, the fermionic lines can indeed turn backwards in time by going through the wormhole throat and turn backwards in time. The above picture suggests that this turning occurs only at the singularities at which the partonic throats touch each other. The QFT analog would be as a local vertex for pair creation.

6. If all fermions at a given boundary of CD have the same sign of energy, fermions which have returned back to the boundary of CD, should correspond to antifermions without a change in the sign of energy. This would make pair creation without fermionic 4-vertices possible.

   If only the total energy has a fixed sign at a given boundary of CD, the returned fermion could have a negative energy and correspond to an annihilation operator. This view is nearer to the QFT picture and the idea that physical states are Galois confined states of virtual fundamental fermions with momentum components, which are algebraic integers. One can also ask whether the reversal of the arrow of time for the fermionic lines could give rise to gravitational quantum computation as proposed in [A3].

3.3.4 A more detailed model for the exotic smooth structure associated with a topological 3-vertex

One can ask what happens to the 4-surface near the topological 3-particle vertex and what is the geometric interpretation of the point defect. The first is whether the description of the situation is possible both in M^8 and H. Here one must consider momentum conservation.

1. By Uncertainty Principle and momentum conservation at the level of M^8, the incoming real momenta of the particle reaction are integers in the scale defined by CD. In the standard QFT picture, the momenta at the vertex of physical particles are at different mass shells.

   In M^8 picture, the mass squared values of virtual fermions are in general algebraic and also complex roots of a polynomial defining the 3-D mass shells H^3_m of $M^4 \subset M^8$, determining 4-surface by associative holography.

   In the standard wave mechanical picture assumed also in TGD, a given topological vertex, describable in terms of partonic 2-surfaces, would correspond to a multi-local vertex in M^8 in accordance with the representation of a local n-vertex in M^4 as convolution of n-local vertices in momentum space realizing momentum conservation.

2. M^8-H duality maps M^4 momenta by inversion to positions in $M^4 \subset H$. This encourages the question whether the topological vertex could be described also in M^8 as a partonic surface at single algebraic mass shell in M^8, mapped by $M^8 - H$ duality to a single $a = constant$ hyperboloid in $M^4 \subset H$.

   The virtual momenta at the level of M^8 are algebraic, in general complex, integers. The algebraic mass squared values at the mass shell of M^8 would be the same for all particles of the vertex. This kind of correspondence does not make sense if $M^8 - H$ duality applies to the full algebraic momenta. The assumption has been that it applies to the rational parts of the momenta.

3. The rational parts of the algebraic integer valued 4-momenta of virtual fermions are in general not at the same mass shell. Could this make possible a description in terms of partonic 2-surfaces at fixed mass *resp.* $a = consant$ shell at the level of M^8 *resp.* H?

   The classical space-time surface in H, partonic 2-surfaces and fermion lines at them are characterized by classical momenta by Noether's theorem. Quantum classical correspondence, realized in ZEO as Bohr orbitology, suggests that the classical 4-momenta assignable to these objects correspond to the rational parts of the momenta at M^8 mass shell. Could the rational projections of M^8 momenta at H^3_n correspond to different mass squared values at given H^3?

4. Note that this additional symmetry for complexified momentum space and position space descriptions would be analogous to the duality of twistor amplitudes position space and the space of area momenta.

How to describe the topological vertex in H? The goal is to understand how exotic smooth structure and its point defects could emerge from this picture. The physical picture applied hitherto is as follows.

1. 3 partonic orbits meet at a vertex described by a partonic 2-surface. Assume that they are located to single $a = constant$ $H^3 \subset M^4 \subset H$.

2. The partonic wormhole throats appear as pairs at the opposite Minkowskian space-time sheets. There are three pairs corresponding to 3 external particle lines and one line which must be a bosonic line describing fermion-antifermion bound state disappears: this corresponds to a boson absorption (or emission).

   The opposite throats carry opposite magnetic monopole charges. The only possibility, not noticed before, is that the opposite wormhole throats for the partoni orbit, which ends at the vertex, must coincide at the vertex. The minimal option is that the exotic smooth structure is associated with this partonic orbit turning back in time. The two partonic orbits, which bind 4-D Euclidean regions as wormhole throats, would fuse to a larger 4-D surface with an exotic smooth structure.

   Fermion-antifermion annihilation occurs at a point at which fermion and antifermion lines meet. The first guess is that this point corresponds to the defect of the smooth structure.

3. There is an analogy with the construction of Etesi [A1]in which a homologically non-trivial ball CP_1 glued to the C^2 at infinity to construct an exotic smooth structure. One dimension disappears for the glued 3-surface at infinity.

   In the partonic vertex, one has actually two homologically non-trivial 2-surfaces with opposite homology charges as boundaries between wormhole contact and Minkowskian regions and they fuse together in the partonic vertex. Also now, one dimension disappears as the partonic 2-surfaces become identical so that 3-D wormhole contact contracts to single 2-D partonic 2-surface.

4. The defect for the smooth structure associated with the fusion of the pair of wormhole orbits should correspond to a point at which fermion and antifermion lines meet.

   This suggests that the throats do not fuse instantaneously but gradually. The fusion would start from a single touching point identifiable asd the fermion-antifermion vertex, serving as a seed of a phase transition, and would proceed to the entire wormhole contact so that it reduces to a partonic 2-surface.

   One can argue that one has a problem if this surface is homologically non-trivial. Could the process make the closed partonic 2-surface homologically trivial. A simplified example is the fusion of two circles with opposite winding numbers ± 1 on a cylinder. The outcome is two homologically non-trivial circles of opposite orientations on top of each other. The phase transition starting from a point would correspond to a touching of the circles.

A couple of further comments are in order.

1. The connection of the pair of wormhole throats to the associative holography is an interesting question. The 4-D tangent planes of $X^4 \subset M^8$ mass shell correspond to points of CP_2. They would be different at the two parallel sheets.

   At the mass shell H^3_m the branches would coincide. The presence of two tangent planes could give rise to two different holographic orbits, which coincide at the initial mass shell and gradually diverge from each other just as in the above model for the fusion of partonic 2-surfaces. The failure of the strict determinism for the associative holography at the partonic 2-surface would make in TGD the analogy of fermion-antifermion annihilation vertex possible.

2. There is also an analogy with the cusp catastrophe in which the projection of the cusp catastrophe as a 2-surface in 3-D space with behavior variable x and two control parameters (a, b) has a boundary at which two real roots of a polynomial of degree 3 coincide. The projection to the (a, b) plane gives a sharp shape, whose boundary is a V-shaped curve in which the sides of V become parallel at the vertex. The vertex corresponds to maximal criticality. The particle vertex would be a critical phenomenon in accordance with the interpretation as a phase transition.

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