

What could be the physical origin of Pythagorean scale?

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Abstract

The 12-note Pythagorean scale seems to be very fundamental for music experience. I have proposed a model for music harmony relying on icosahedral and tetrahedral geometries and surprisingly a connection with genetic code emerged. Music expresses and generates emotions and the natural idea was the the proposed bio-harmonies could serve as correlates for emotions. Geesink and Meijer propose that twelve note scale could be realized as frequency bands for oscillating square plate. This realization would be condensed matter realization. One can however ask whether the Pythagorean scale could be realized by some universal mechanism at the level of fundamental physics. Guitar string is the obvious heuristic guideline and leads to the proposal that magnetic flux tubes or even associated fermionic strings could provide a purely geometric realization in terms of fundamental lengths of of the string proportional to the inverses of the frequencies of the Pythagorean scale.

1 Introduction

I was contacted for a couple years ago by Hans Geesink and we had long discussions about consciousness and quantum biology. The discussion stimulated new ideas and this inspired me to write a chapter and article comparing our approaches (see http://tgdtheory.fi/public_html/articles/geesink.pdf). Now Hans sent me two prepublications by him and D. K. F. Meijer.

The first preprint “Bio-Soliton Model that predicts Non-Thermal Electromagnetic Radiation Frequency Bands, that either Stabilize or Destabilize Life Conditions” is in arXiv [I1] (see <http://tinyurl.com/zz3ew33>). The abstract reads as:

Solitons, as self-reinforcing solitary waves, interact with complex biological phenomena such as cellular self-organisation. Soliton models are able to describe a spectrum of electromagnetism modalities that can be applied to understand the physical principles of biological effects in living cells, as caused by electromagnetic radiation. A bio-soliton model is proposed, that enables to predict which eigen-frequencies of non-thermal electromagnetic waves are life-sustaining and which are, in contrast, detrimental for living cells. The particular effects are exerted by a range of electromagnetic wave frequencies of one-tenth of a Hertz till Peta Hertz, that show a pattern of twelve bands, if positioned on an acoustic frequency scale. The model was substantiated by a meta-analysis of 240 published papers of biological radiation experiments, in which a spectrum of non-thermal electromagnetic waves were exposed to living cells and intact organisms.

These data support the concept of coherent quantized electromagnetic states in living organisms and the theories of Davydov, Fröhlich and Pang. A spin-off strategy from our study is discussed in order to design bio-compatibility promoting semi-conducting materials and to counteract potential detrimental effects due to specific types of electromagnetic radiation produced by man-made electromagnetic technologies.

Second preprint “Phonon Guided Biology: Architecture of Life and Conscious Perception are mediated by Toroidal Coupling of Phonon, Photon and Electron Information Fluxes at Eigen-frequencies” is in Research Gate [I2] (see <http://tinyurl.com/j9rsyqd>). The abstract is following.

Recently a novel biological principle, revealing discrete life sustaining electromagnetic (EM) frequencies, was presented and shown to match with a range of frequencies emitted by clay-minerals as a candidate to catalyze RNA synthesis. The spectrum of frequency bands indicate that nature

employs discrete eigen-frequencies that match with an acoustic reference scale, with frequency ratios of 1:2, and closely approximated by 2:3, 3:4, 3:5, 4:5 and higher partials. The present study shows that these patterns strikingly resemble eigen-frequencies of sound induced geometric patterns of the membrane vibration experiments of E. Chladni (1787), and matches with the mathematical calculations of W. Ritz (1909).

We postulate that the spectrum of EM frequencies detected, exert a phonon guided ordering effect on life cells, on the basis of induction of geometric wave patterns. In our brain a toroidal integration of phonon, photon and electron fluxes may guide information messengers such as Ca^{2+} - ions to induce coherent oscillations in cellular macromolecules. The integration of such multiple informational processes is proposed to be organized in a fractal 4-D toroidal geometry, that is proposed to be instrumental in conscious perception. Our finding of an “acoustic life principle” may reflect an aspect of the implicate order, as postulated by David Bohm.

A very concise and very partial summary about the articles would be following.

1. 12-note scale seems to be realized in good approximation as frequency bands (rather than single frequencies) for a membrane like system with the geometry of square obeying four-order partial differential equation studied numerically by Ritz. Since the boundary conditions are periodic this system has effective torus topology. This is rather remarkable experimental fact and extremely interesting from TGD point of view.
2. The papers also argue that also the octave hierarchy is realized. p-Adic length scale hierarchy indeed predicts that subset of powers of 2, and more generally of $\sqrt{2}$ defines a hierarchy of fundamental p-adic scales with p-adic prime p near to power of two.

In the following I will discuss first the condensed matter realization of 12-note scale and after that consider the significance and realization of 12-note scale from TGD point of view.

2 Condensed matter realization of 12-note scale in terms of oscillations of square plate

The article discusses a condensed matter physics based realization of 12-note. Acoustic waves are seen as fundamental. Certainly the sound waves are important since they couple to electromagnetic waves. My feeling is however that they provide a secondary realization.

1. The realization of 12-note system as 12 bands discussed in the articles is as eigen frequencies of deformations of square plate. Periodic boundary conditions imply that one can regard the system also as a torus. One has bands, not eigenfrequencies. I do not know whether one can pick up from bands frequencies, whose ratio to the fundamental would be rational and same as for Pythagorean scale. Since the system can be treated only numerically, it is difficult to answer this question.
2. So called Chladni patterns (see “An Amazing Resonance Experiment” at <http://tinyurl.com/kcbmrzz>) are associated with vibrating thin square plate and correspond to the node lines of the deformation of the plate in direction orthogonal to the plate. As one adds very small particles at the plate and if the vibrational acceleration is smaller than the gravitational acceleration the particles get to the node lines and form Chladni pattern. Hence the presence of gravitation seems to be essential for the Chladni patterns to occur. These patterns make visible the structure of standing wave eigenmodes of the plate. It is also possible to have patterns assignable to the antinodes at which the deformation is maximum but vibrational acceleration vanishes as in the harmonic oscillator at the maximum value of the amplitude.
3. The vibrations of square plate obey fourth order partial diff equation for the Chladni pattern having the general form

$$\partial_t^2 u = K(\nabla^2)^2 u \quad . \quad (2.1)$$

Here u is the small deformation in direction orthonormal to the plate. The equation can be deduced from the theory of elasticity about which I do not know much. For standing wave solutions the time dependence is separable to trigonometric factor $\sin(\omega t)$ or $\cos(\omega t)$, and one obtains eigenvalue equation

$$K(\nabla^2)^2 u = -\omega^2 u . \quad (2.2)$$

4. The natural basis for the modes is as products of 1-D modes $u_m(x)$ for string satisfying $\partial_x^2 u_m = 0$ at the ends of the string ($x = \{-1, 1\}$) this in both x and y directions. This must express the fact that energy and momentum do not flow out at boundaries.

The modes satisfy

$$\frac{d^4 u_m}{dx^4} = k_m^4 u_m . \quad (2.3)$$

Boundary conditions allow modes with both even and odd parity:

$$\begin{aligned} u_m &= \frac{[\cos(k_m)\cosh(k_m x) + \cosh(k_m)\cos(k_n x)]}{\cosh^2 k_m + \cos^2(k_m)} , \\ \tan(k_m) + \tanh(k_m) &= 0 , \quad m \text{ even} . \\ u_m &= \frac{[\sin(k_m)\sinh(k_m x) + \sinh(k_m)\sin(k_n x)]}{\sinh^2 k_m + \sin^2(k_m)} \\ \tan(k_m) - \tanh(k_m) &= 0 , \quad m \text{ odd} . \end{aligned} \quad (2.4)$$

5. The 2-D modes are not products of 1-D modes but sums of products

$$w_{mn}^\epsilon = u_m(x)u_n(y) + \epsilon u_m(y)u_n(x) , \quad \epsilon = \pm 1 . \quad (2.5)$$

Modern physicist would notice classical entanglement between x and y degrees of freedom. The first $\epsilon = 1$ mode is analogous symmetric two-boson state and second $\epsilon = -1$ mode to antisymmetric two-fermion state.

6. The variational ansatz of Ritz was superposition of these modes (this variational method was actually discovered by Ritz). Ritz minimized the expectation value of the Hermitian operator $(\nabla^2)^2$ in the ground state and obtained an approximation for the frequencies which holds true with 1 per cent accuracy.

Unfortunately, 4-D geometry does not give rise to this kind of equations: time and space are not in democratic roles. TGD inspired vision would be different. The magnetic flux tubes and even strings could be the fundamental objects concerning biology and consciousness. The acoustic realization of the 12-note scale would be secondary one. Even genetic code would have fundamental realization at the level of dark nuclear physics [L2] and chemical realization of genetic code would be secondary realization.

3 Why 12-note scale?

Why I am convinced that 12-note scale should be so important?

1. The mysterious fact about music experience is that frequencies whose ratios come as rationals are somehow special concerning music experience. People with absolute pitch prefer the Pythagorean scale with this property as aesthetically pleasing. Pythagorean scale is obtained

by forming the 3^k multiples of fundamental and by dividing by a suitable power 2^m of 2 to get a frequency in the basic octave. This scale appears in TGD inspired model for music harmonies [L1] (see http://tgdtheory.fi/public_html/articles/harmonytheory.pdf), which as a byproduct led to a model of genetic code predicting correctly the numbers of DNA codons coding for given amino-acid. The appearance of powers of 2 and 3 suggest 3-adicity and 2-adicity. Furthermore, rationals correspond to the lowest evolutionary level defined by the hierarchy of algebraic extensions of rationals.

This gives excellent reasons to ask whether 12-note scale could be realized as some physical system. One might hope that this system could be somehow universal. Geometric realization in terms of wave equation would be the best that one could have.

2. The model of harmony is realized in terms of Hamilton cycles assignable to icosahedron and tetrahedron. Hamilton cycles at icosahedron are closed paths going through all 12 points of icosahedron and thus can define a geometric representation of the Pythagorean scale. The rule is that curve connects only nearest points of icosahedron and corresponds to scaling of frequency by $3/2$ plus reduction to basic octave by dividing by a suitable power of 2. The triangles of the icosahedron define allowed 20 chords for given harmony and one obtains 256 basic harmonies characterized by the symmetries of the cycle: symmetry group can be cyclic group Z_6 , Z_4 or Z_2 or reflection group Z_2 acting on icosahedron.

Bioharmonies are obtained by combining Z_6, Z_4 and Z_2 of either type. One obtains $20+20+20=60$ 3-chords defining the bio-harmony. One must add tetrahedral harmony with 4 chords in order to obtain 64 chords. It turns out that it corresponds to genetic code under rather mild assumptions. DNA codons with 3 letters could correspond 3-chords with letter triplets mapped to 3-chords. Amino-acids would correspond to orbits of given codon at icosahedron under one of the symmetry groups involved.

4 How to realize 12-note scale at fundamental level universally?

How could one realize 12-note scale at the fundamental level - that is in terms of 4-D geometry? The realization should be also universal and its existence should not depend on special properties of physical system. Vibrating strings provide the simplest manner to realize 12-note scale. Harmonics do not however allow its realization. They are in higher octaves and define only the color of the note. There are actually two realizations.

The simplest realization relies on the analogy with piano.

1. The string of piano corresponds to a magnetic flux tube/associated fermionic string and the frequency of the note would be determined by the length of the flux tube. The quantization for the length as certain rational multiples of p-adic length scale gives rise to the 12-note scale. Tensor network would be like piano with the flux tubes of the network with quantized lengths defining the strings of piano.
2. Why the length of the flux tube defining the fundamental frequency would correspond to a frequency of Pythagorean scale? Could this be due to the preferred extremal property realizing SH and posing very strong conditions on allowed space-time surface and 3-surfaces at their ends at boundaries of causal diamonds? If so, 12-note scale would be part of fundamental physics!

The rational multiples $f(m, n) = (m/n)f_0$, $m = 0, 1, ..n - 1$, of the fundamental f_0 with $m/n \leq 2$ (single octave) are in a preferred position mathematically since the superpositions of waves with these frequencies can be represented as superpositions of the suitable harmonics of the scaled down fundamental $f_1 = f_0/n$. For Pythagorean scale $m/n = 3^k/2^l$ the new fundamental is some "inverted" octave $f_1 = f_0/2^{k_{max}}$ of the fundamental and the allowed harmonics are of form $m = 2^r 3^l$.

Second realization would be dynamical and based on the analogy with string instruments.

1. String instruments allow to realize 12-note scale by varying the length of the vibrating string. The note of scale corresponds to the fundamental frequency for the portion of the shortened string, which is picked. Why the lengths of shortened strings should correspond to inverses of frequencies of 12-note scale? One should have powers of 3 divided by powers of 2 to get a frequency in fundamental octave. Could p-adic length scale hypothesis, which generalizes and length scales coming as powers of square roots of small primes help?
2. Strings bring in mind magnetic flux tubes connecting partonic 2-surfaces. They behaving in good approximation like strings and are actually accompanied by genuine fermionic strings and corresponding string world sheets. Flux tubes play a fundamental role in living matter in TGD Universe. Flux tubes carrying dark matter identified as large $h_{eff} = n \times h$ phases would serve as space-time correlates for negentropic entanglement and gives rise to tensor nets with partonic 2-surfaces as nodes and flux tubes connecting them [L3]. Could magnetic flux tubes or associated fermionic strings provide the instruments using Pythagorean scale?

Partonic 2-surfaces and string world sheets dictate space-time surface by strong form of holography (SH) implied by strong form of general coordinate invariance. It is quite possible that not all configurations of partonic 2-surfaces and string world sheets allow SH that is realization as space-time surface: perhaps only the flux tubes with length corresponding to Pythagorean scale allow it. For p-adic counterparts of space-time surfaces the possibility of p-adic pseudo-constants (failure of strict determinism of field equations) makes this possible: the interpretation is as imagined p-adic space-time surface which cannot be realized as real space-time surface.

How these flux tubes could behave like strings of guitar? When my finger touching the guitar string it dividing it to two pieces. The analogy for this is the appearance of additional partonic 2-surface between the two existing ones so that one has two flux tubes connecting the original partonic two-surface to the new one. A change of the topology of 3-space would be involved with this stringy music!

More precisely, the flux tubes would be closed if they carry monopole magnetic flux: they would begin from “upper” wormhole throat of wormhole contact A (partonic 2-surface), go along “upper” space-time sheet to the throat of wormhole contact B go the “lower” space-time sheet through it, return to the “lower” throat of wormhole contact A and back to the “upper” throat. Shortening of the string would correspond to a formation of wormhole contact at some point of this flux tube structure splitting the flux tube to two pieces.

3. Another realization could be in terms of the quantization of the distance between partonic 2-surfaces connected by flux tubes and associated strings in given p-adic length scale, which by p-adic length scale hypothesis would correspond to power of square root of 2 so that also octaves and possibly also half octaves would be obtained (note that half octave corresponds to tritonus, which was regarded by church as an invention of devil!). Also now the justification in terms of SH.

5 Could Chladni mechanism allow to realize morphogenesis?

Morphogenesis represents one of the basic unsolved problems of biology. Molecular biology and gene hypothesis have not allowed to understand what is involved. The probable reason is that biochemistry is local approach whereas morphogenesis is a non-local phenomenon. There have been attempts to understand morphogenesis using the catastrophe theory of Thom [A1]. Sheldrake has done highly interesting work with morphogenesis too. Robert Merrick’s article harmonic theory of evolution (see <http://tinyurl.com/qda9461>) suggests a connection between the notion of harmony as expressed by 12-note scale and morphogenesis.

The basic building bricks of TGD vision about morphogenesis would be following.

1. Macroscopic quantum coherence is to my view a necessary ingredient of morphogenesis and hierarchy of Planck constants allows to realize it. The notion of magnetic body (MB) is also necessary. MB would guide the morphogenesis. For instance, the replication of living system

would be induced by that for MB. The fundamental dynamics takes place at the level of MB and biochemical level is only a shadow of this dynamics. “Topological light rays” (“massless extremals”, MEs) is second key element. MB would use MEs to control visible living matter, in particular to guide morphogenesis. The challenge is to understand how MB achieves this.

2. The notion of harmony assignable to various musical scales realized as Hamiltonian cycles at Platonic solids is central. The TGD based model for harmony [L1] was actually inspired by the book of Merrick’s theory of music [J1]. The model for harmonies assignable to 12-note scale led to a model for genetic code in terms of so called Hamiltonian cycles on icosahedron and tetra-hedron predicting correctly the numbers of DNA codons coding for given amino-acid and also predicted two additional amino-acids Pyl and Sec appearing in Nature.
3. The fusion of real physics for sensory experience and various p-adic physics for cognition gives rise to adelic physics. In particular, one can speak about adelic variants of space-time surfaces and the notion of monadic geometry emerges. Geometric objects have discrete “spine” for which points have coordinate values in an algebraic extension of rationals for some preferred coordinate system dictated by the symmetries of the imbedding space $M^4 \times CP_2$. Space-time surfaces are also locally continuous and smooth so that classical partial differential equations defining space-time surfaces as preferred extremals of Kähler action or its twistor lift make sense.

Platonic solids represents unique monadic geometries since they correspond to finite discrete subgroups of the 3-D rotation group giving rise to 3-dimensional structures as their geometric representations. Also planar polygons represent this kind of realizations and can be assigned to the inclusion hierarchy of von Neuman algebras knowns hyper-finite factors of type II_1 and very probably also to the analogous fractal hierarchy of sub-algebras of super-symplectic algebra isomorphic to the full algebra.

4. The discrete points realizing monadic geometries could be accompanied by tensor networks having partonic 2-surfaces as notes connected by magnetic flux tubes serving as correlates for negentropic entanglement between the nodes at their ends would serve give rise to the emergence of proprioception - the experience about 3-space. The flux tubes would be analogous to strings of the music instruments with transverse oscillations defining the fundamental frequencies defining the notes of the scale. Tensor network could be regarded as music instrument having flux tubes as strings.

12-note scale could be by its special mathematical features and by preferred extremal condition fundamental from the point of view of morphogenesis. The lengths of flux tubes are quantized. One can imagine two options. The effective length of given flux tube can be varied as done in guitar or the tensor network would be like piano or harp: the lengths of flux tubes assignable to the tensor network would have quantized lengths coming as rational multiples of fundamental length in such a manner that a representation of the 12-note system would be obtained.

The model of music harmony and 12-note scale would be assignable to icosahedron which would also define a very natural monadic geometry. This harmony would also related to genetic code. Monadic geometry could in turn emerge naturally in morphogenesis so that genetic code could after all lurk behind morphogenesis but being realized in terms of 3-chords rather than triplets of DNA nucleotides. Morphogenesis could be a realization of genetic code in terms of interfering fields.

How morphogenesis could then be realized in this picture?

1. Chladni mechanism is a clever trick to make the nodal curves associated with standing waves visible. This mechanism could transcend to a basic mechanism of morphogenesis. The idea is very simple. Biomolecules could end up to the nodal surfaces for a standing waves of say electric field since the force on them would vanish at the nodal surfaces. This would give stationary structures. MB could control morphogenesis by using this kind of standing waves forcing the formation of various structures at their nodal surfaces.

2. The objection is that TGD does not allow single-sheeted realizations of standing waves. This objection is not lethal. In many-sheeted space-time one can realize effective sinusoidal standing waves as 2-sheeted structures from two MEs propagating to opposite spatial directions and carrying plane waves with a fixed frequency. These two-sheeted structures would serve as basic building bricks. The test particle having necessarily wormhole contacts to both MEs would experience the force caused by the sum of the induced gauge fields assigned to the two MEs. The force would be same as that caused by a standing wave with separable temporal and spatial dependence not realizable as preferred extremal: that is a product of trigonometric functions - say $\sin(\omega t)\sin(kz)\epsilon(x)$, $\omega = kc$. The force would vanish at nodal surfaces, which would thus define naturally the shape of a stationary structure defined by molecules. Now these surfaces would be zeros of $\sin(kz)$ and $\epsilon(z)$.

One can take several primitive MEs and allow them to have different directions but common frequency. One would obtain effective standing wave with common factorized time dependence and spatial dependence given by the sum of spatial parts of the sinusoidal waves. The nodal surface for this wave would correspond to the nodal surface for the sum of the spatial waves and one would obtain arbitrarily complex nodal surfaces.

The nodal surfaces for these waves would naturally associated with the nodes of the tensor network, where the flux tubes of MB indeed meet. Fractal structure with tensor networks with nodes of tensor networks can be assumed in TGD framework.

3. There is a connection with holography in which reference wave and the wave of same frequency reflected from the target interfere. Now all waves can be regarded as standing reference waves coming from different directions and generated by magnetic body and propagating along flux tubes of magnetic body. Bio-structures would be formed to the nodal surfaces of this hologram.

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