Hachimoji DNA from TGD perspective

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Abstract

Hoshika et al have discovered a genetic code realized in terms of 8 letters instead of 4. This code is highly interesting from the perspective of astrobiology and could have profound implications for artificial life. Hachimoji DNA can be transcribed to its RNA counterpart. TGD inspired quantum biology suggests two new realizations of the genetic code: for the first realization DNA codons correspond to dark proton triplets. For the second realization to 3-chords of bio-harmony. Ordinary genetic code could be seen as mimicry of the more fundamental dark proton code. The question considered in this article is whether hatsimoji codes can be realized in this manner.

The discovery of Hachimoji code relates interestingly to two realizations of the genetic code inspired by TGD based quantum biology.

1. The first realization is dark genetic code with codons realized as 64 3-proton states made of dark protons having non-standard value $h_{eff} = n \times h_0$ of Planck constant. The hierarchy of Planck constants is predicted by adelic physics providing physical correlates for correlation in terms of p-adic physics. Dark genetic code would be fundamental in TGD and bio-chemical realization would be kind of shadow or mimicry of it and not even complete in some cases. One cannot talk about letter decomposition for dark proton triplets since the 3-proton states are entangled.

2. Second realization relies on the notion bio-harmony: the realization of the genetic in terms of 3-chords of bio-harmony emerged as a by-product from a model of harmony.

The work with hachimoji code led to a discovery of new realization of bio-harmony as a fusion of 2 icosahedral harmonies with 20 chords and 2 dodecahedral harmonies with 12 chords produces genetic code with 20+20+12+12=64 codons. Icosahedral and dodecadedral harmonies correspond to dual tesselations of sphere so that bio-harmony can be represented as a bundle over sphere with two notes represented as points of the fiber. Hachimoji harmony is obtained by replacing 2-point fiber with 8×2-point fiber. The presence of the dual tesselations conforms with the fact that Eastern music uses micro-intervals, which rather naturally correspond to 20-note dodecahedral scale. Second discovery was that the quint scaling appearing in the earlier model can be replaced with a unique scaling, which is same for icosahedral and dodecahedral codes. One must however generalize the notion of Hamiltonian cycle by introducing the analog of gauge symmetry in a discrete bundle over sphere and allowing to generate new Hamiltonian cycles from given cycle by gauge transformations. In this manner one obtains extremely rich harmony from single basic chord transposable to $CEG\sharp$.

Contents

1 Introduction

The popular article "*Freaky Eight-Letter DNA Could Be the Stuff Aliens Are Made Of*" (see http://tinyurl.com/y5wb7cm8) tells about very interesting discovery related to astrobiology, where the possible existence of variants of DNA and other bio-molecules are of considerable interest. The article "*Hachimojii DNA and RNA: A genetic system with eight building blocks*" (see http://tinyurl.com/y2mcjb4r) published in Science tells about a discovery of a variant of DNA with 8 letters instead of 4 made by Hoshika et al [I2]. By using an engineered T7 RNA polymerase this expanded DNA alphabet could be transcribed into Hachimoji variant of RNA. The double strand structure of hachimoji DNA is similar to that of ordinary DNA and it is thermodynamically stable.

No amino-acid (AA) counterparts assigned to the hachimoji RNA were engineered: this would require the existence of translation machinery. The possible existence of also additional AAs leads to the speculation is that both alien life forms utilizing this kind of extended code could have evolved. One can also ask whether mere synthetic hachimoji RNA could be enough for synthetic life.

The abstract of the article gives a more technical description about what has been achieved.

> *We report DNA- and RNA-like systems built from eight nucleotide "letters" (hence the name "hachimoji") that form four orthogonal pairs. These synthetic systems meet the structural requirements needed to support Darwinian evolution, including a polyelectrolyte backbone, predictable thermodynamic stability, and stereoregular building blocks that fit a Schrödinger aperiodic crystal. Measured thermodynamic parameters predict the stability of hachimoji duplexes, allowing hachimoji DNA to increase the information density of natural terran DNA. Three crystal structures show that the synthetic building blocks do not perturb the aperiodic crystal seen in the DNA double helix. Hachimoji DNA was then transcribed to give hachimojii RNA in the form of a functioning fluorescent hachimoji aptamer. These results expand the scope of molecular structures that might support life, including life throughout the cosmos.*

If the additional code letters for DNA (8 code letters instead of 4) really carry information, the number of codewords is extended by factor $2^3 = 8$ giving $2^9 = 512$ code words. What the number of AAs would be, can be only guessed: the simplest guess is that also now the number is scaled up by factor 8 but this is only a guess.

In the sequel I consider hachimoji code from TGD perspective. The natural guess is that the hachimoji code corresponds to 8 copies of the ordinary genetic code in some sense. TGD predicts two basic realizations of the genetic code corresponding to dark genetic code and bio-harmony.

1. In the case of the dark code it is possible to imagine an extension of the code based on the notion of dark nucleus and the number codons is multiplied by 8. In the case of bio-harmony fusion of 8 copies of bio-harmony allows to realize hachimoji code.

2. I have considered two basic realizations of bio-harmony [L1, L9] giving also realization of genetic code. The first realization is as a fusion of 3 icosahedral harmonies and tetrahedral harmony. Second realization is as a fusion of 2 icosahedral harmonies and 1 toric harmony. These constructions do not however allow any elegant geometric interpretation since two different geometries are involved in both cases.

During writing I was forced to reconsider this problem and realized that a fusion of 2 icosahedral harmonies with 20 chords and 2 dodecahedral harmonies with 12 chords produces genetic code with 20+20+12+12=64 codons. Icosahedral and dococadedral harmonies correspond to dual tesselations of sphere so that bio-harmony can be represented as a bundle over sphere with two notes represented as points of the fiber. Hachimoji harmony is obtained by replacing 2-point fiber with 8×2-point fiber. The presence of the dual tesselations conforms with the fact that Eastern music uses micro-intervals, which rather naturally correspond to 20-note dodecahedral scale.

3. The reason why for the hachimoji code could the basic problem of the music scale realized in terms of rational frequency ratios. Already Pythagoras was aware of this problem. The construction of the scale as powers of quint (3/2-fold scalings of the basic frequency) using octave equivalence produces with 12 iterations 7 octaves but only approximately: the 12th iterate does not quite correspond to the basic note in the octave equivalence. Performing the 12-fold iteration 8 times gives therefore a refined scale with each note replaced with 8 almost copies identifiable as hachimoji scale.

4. A further discovery was that the quint scaling appearing in the earlier model can be replaced with a unique scaling, which is same for icosahedral and dodecahedral codes. One must however generalize the notion of Hamiltonian cycle by introducing the analog of gauge symmetry in a discrete bundle over sphere and allowing to generate new Hamiltonian cycles from given cycle by gauge transformations. In this manner one obtains extremely rich harmony from single basic chord transposable to $CEG\sharp$.

2 Icosa-tetrahedral and icosa-dodecahedral bioharmonies as candidates for genetic code

Both the icosa-tetrahedral [L1] and icosa-dodecahedral harmony to be discussed below can be considered as candidates for bio-harmony as also the harmony involving fusion of 2 icosahedral harmonies and toric harmony [L6]. The basic reason is that the third harmony corresponds to doublets. One cannot exclude the possibility of several equivalent representations of the code.

2.1 Icosa-tetrahedral harmony

Icosahedral harmonies can be characterized by a subgroup of icosahedral isometries A_5 having 60 elements. If reflections are included the isometry group, oneas $A_5 \times Z_2$ with 120 elements. The group of symmetries is Z_6, Z_4, or Z_2. There are two choices for Z_2 and the interpretation has been that Z_2 correspond to either reflection or rotation by π. A_5 however allows also $Z_2 \times Z2$ as subgroup. AAs correspond to orbits of the symmetry group and DNA codons coding for the AA correspond to triangles (3-chords) at the orbit. In purely icosahedral model on obtains 20+20+20 codons. A fusion with tetrahedral harmony gives 64 codons.

1. Z_6 gives rise to 3 AAs coded by 6 codons each (leu,se,arg) and 2 AAs coded by 2 codons: the choice of the doublet would require additional conditions. One option is ile doublet.

2. Depending on whether one includes reflection or not, one can have either $Z_4 \subset A_5$ ($60 = 4 \times 15$) or $Z_4 = Z_{2,rot} \times Z_2 \subset A_5 \times Z_2$. I have assumed that $Z_4 = Z_{2,rot} \times Z_2$ but the recent argument suggests the first option. This does not have any implications for the earlier model. Icosahedral Z_4 gives rise to 5 AAs coded by 4 codons each ($5 \times 4 = 20$). This leaves 11 AAs and 3 "empty" AA formally coded by stop codons.

3. Icosahedral Z_2 gives rise to 10 dublets. These 4-plets would correspond to (phe, tyr, his, gln, asn, lys, asp, glu, cys, stop-doublet) This leaves (stop,trp) double and (ile,met) doublet with broken Z_2 symmetry.

   The fusion with tetrahedral code with 4- codons and 4 AAs should explain these 4 AAs. Tetrahedral isometries form group S_3 and reduce to group Z_3 for tetrahedral cycle.

(a) One could argue that ile-triplet and and met correspond to 3-element orbits with 1-element orbit. (stop,trp) would be formed by Z_2 symmetry breaking from trp doublet and there is no obvious mechanism for this.

(b) If one tetrahedral face is fixed as a face shared with icosahedron, the symmetry group of tetrahedral cycle reduces to Z_1. This would give 4 singlets identifiable as (ile,met) and (stop,trp) symmetry broken doubles. Since ile appears also in doublet, tetrahedral 1-orbit and icosahedral 2-orbit must have a common doubled triangle identifiable as the common face of icosahedron and tetrahedron. The doubling of the common triangle replaces ile-doublet with ile-triplet. This option looks rather reasonable.

2.2 Dodecadedral harmony

Dodecahedral harmony correspond to the unique Hamilton cycle at dodecahedron. Dodecahedral harmony as 20 notes and and 12 5-chords. If one assumes that the octave divides to 20 notes, this brings in mind "eastern" view about harmony.

The obvious objection against dodecahedral harmony is that dodecahedral faces are pentagons so that dodecahedral chords would be 5- rather than 3-chords so that the correspondence between chords and DNA codons would be lost. The situation changes if 3 notes - 3-chord - determine the 5-chord completely and one can assign a unique 3-chord to each pentagon. This is indeed the case!

1. 3-edges meet in every dodecahedral vertex (this makes the dodecahedral cycle unique apart from rotations) and each edge pair in the vertex belongs to same pentagon (in the case of icosahedron there are 5 edges per vertex so that this is not true). Therefore each pentagon must contain at least 2 edges of Hamilton's cycle.

   The cycle must visit all vertices of pentagon, and the visit to the vertex means that the cycle shares at least one edge with pentagon. Since all vertices of the pentagon must be visited, there are two options. For option a) given pentagon shares with the cycle disjoint 2-edge with 3 vertices and 1-edge with two vertices. For option b) the pentagon shares with the cycle 4-edge with 5 vertices.

2. The numbers n_a of pentagons with 4-edges and $n_b = 12 - n_a$ 2-edge+ 1-edge (making 3 edges) can be deduced easily. Cycle has 20 edges. Pentagon of type a) shares 3 edges with the cycle and the edge is shared by 2 pentagons. This gives $3n_a/2$ edges. Pentagon of type b) shares 4 edges with the cycle. This gives $2n_b = 2(12 - n_a)$ edges. The total number of edges is $3n_a/2 + 2n_b = 20$, which gives $n_a = 8$ and $n_b = 4$. Dodecahedral Hamilton's cycle can be found from web (see `http://tinyurl.com/y5woajcb`). The structure is as deduced here.

   For case a) the 3-chords correspond naturally to the 3 vertices of the 2-edge shared with the cycle. Therefore it is possible to assign unique 3-chords to the dodecahedral harmony and to obtain connection with codons in this case. One however obtains also 12 2-chords: could they have some genetic counterpart?

   What about 5-chords for pentagons of type b)? Hamiltonian cycle can be oriented and this is induces orientation of the pentagons. One can say that the first vertex in the 4-edge is the vertex at which cycle arrives to the pentagon and identify the 3-chord as the first three vertices. It turns out that for the replacement of quint cycle this is not actually necessary.

2.3 Is icosa-dodecahedral harmony consistent with the genetic code?

One must check whether icosa-dodecahedral harmony is consistent with the degeneracies of the genetic code.

1. A fusion of 2 icosahedral harmonies and 2 copies of dodecahedral harmony would be in question. As in the case of icosahedral harmony already discussed, the two icosahedral harmonies would have symmetry groups Z_6 and Z_4 and give the codons coding for 3 6-plets and 1 doublet+ 5 4-plets + two copies of dodecahedral harmony.

2. Can the model predict correctly the numbers of codons coding for AAs? It is known that dodecahedral Hamilton cycle divides dodecahedron to two congruent pieces related by Z_2 symmetry (see `http://tinyurl.com/yy6pcogt`). Also the Hamiltonian cycle defining the common boundary has Z_2 symmetry. A good guess is that these Z_2:s corresponds to reflection symmetry and rotation by π but I am not able to exclude $Z_4 \subset G_0$, where G_0 consists of 60 orientation preserving isometries. In this case some orbits - presumably all 3 of them - could contain 4 pentagons. This is not consistent with the condition that one has doublets and singlets.

   If the second symmetry corresponds to reflection, it can be excluded by simply assuming that reflections change the orientation of the cycle.

3. Rotation by π has two fixed points corresponding to opposite poles so that one has 5 2-orbits and 2 1-orbits giving 12 triangles for each copy. Two copies of dodecahedral harmony would give 5+5=10 doublets and 2+2=4 singlets. A possible interpretation would be as (ile,met) and (stop,trp).

Consider now objections against dodecahedral harmony.

1. Why two copies of dodecahedral code? What distinguishes between them? If imirror symmetry leaves the cycle invariant apart from orientation the copies could be mirror images and consist of same faces. The second option is that they related by a rotation?

2. The number of dodecahedral AAs is 24 rather than 20. Could the additional 4 AAs as orbits have interpretation as AAs in some sense. Could the "empty" AAs coded by stop codons be counted as AAs exceptional in some sense. In TGD framework one can consider the possibility that although AA is "empty", there is analog of AA as physical signature for the end of protein telling what stopping codon it corresponds. The magnetic body of protein is a good candidate.

   Genetic code has several slightly differing variants. Could the 2 additional exotic AAs Pyl and Sec correspond in some situations to the additional AAs?

3. Essential for the bio-harmony as a fusion of harmonies is that one can select from each orbit single face as a representative of the AA it codes - kind of gauge choice is in question - and that the orbits corresponding to different AAs can be chosen to be disjoint. Otherwise codons belonging to the orbits of different Hamilton cycles can code for the same AA if the AA can be chosen to be in intersection. If not, the same codon can code for 2 different AAs - this can indeed occur in reality [L8]!

   The condition that orbits of different cycles do not interesect seems quite stringent but has not been proven. But what if it is actually broken? Indeed, in the case of icosahedral harmony with Z_1 symmetry tetrahedron and icosahedron could have common a doubled face the breaking of this condition would geometrically explain why ile belongs to both icosahedral and tetrahedral orbit.

   Ile is the problem also in the case if icosa-dodecahedral harmony. Dodecahedral singlet codes for ile as also icosahedral doublet. Could one talk about doubling of ile face so that it corresponds to a pair of triangle and pentagon (in 1-1 correspondence with triangle as chord).

4. The two copies of the dodecahedral code should correspond to 5 doublets and 2 singlets each. One expects that together they give rise to 10+2 +10+2 =24 faces. Do they? Mirror symmetry and rotation by π act as symmetries of the cycle so that neither can map the two cycles to each other. Dodecahedral (equivalently icosahedral) rotations give rise to new equivalent cycles. The action on pentagons corresponds to the action on vertices of icosahedron so that it easy to understand what happens.

   Each symmetry corresponds to a rotation around some axis and has opposite icosahedral vertices at this axis as fixed points. Hence any two cycles obtained in this manner have 2 common pentagons. This means reduction $24 \rightarrow 22$ unless one interprets the situation in terms doubled faces? Could the disappearing doublet correspond to stop-doublet? What about the remaining stop of the vertebrate code pairing with trp? Why does second singlet correspond to empty AA and not something else such as exotic AA.

5. There is also further problem. Suppose that an intersection of orbits takes place at single triangle. Suppose that one cannot choose this triangle to be "AA" triangle for both orbits. In this case it is not clear to which AA the codon codes. This kind of phenomenon actually takes place in some cases and is known as homonymy [L8]. It is associated with the deviations of the code from the vertebrate code and involves exotic AAs Pyl and Sec. Codons can serve as a stop codon or code for an exotic AA.

Clearly, the notion of bio-harmony involves many unclear aspects but my strong feeling is that there is very beautiful mathematics involved.

3 Hachimoji code and realizations of genetic code suggested by TGD inspired quantum biology

The discovery of Hachimoji code relates interestingly to two realizations of the genetic code inspired by TGD based quantum biology.

1. The first realization is dark genetic code with codons realized as 64 3-proton states made of dark protons having non-standard value $h_{eff} = n \times h_0$ of Planck constant [L5]. The hierarchy of Planck constants is predicted by adelic physics providing physical correlates for correlation in terms of p-adic physics [K3]. Dark genetic code would be fundamental in TGD and bio-chemical realization would be kind of shadow or mimicry of it and not even complete in some cases. One cannot talk about letter decomposition for dark proton triplets since the 3-proton states are entangled.

2. Second realization relies on the notion bio-harmony [L1, L9]: the realization of the genetic in terms of 3-chords of bio-harmony emerged as a by-product from a model of harmony.

3.1 Does dark realization of genetic code allow hachimoji code?

Could one realize hachimoji codons as dark codons?

1. If the proposed dark proton triplets [L5, L4] is the only fundamental realization of genetic codons, the real information storage capacity should not increase but the correspondence between dark codons and chemically realized codons would not be 1-to-1 but 1-to-8. Furthermore, the transcription of dark DNA to ordinary one would not be possible in 1-to-1 manner so that hachimoji code could not have evolved.

2. One can however imagine of having also neutrons rather than only protons in the dark nuclear string. If one can have both dark protons and neutrons, one could obtain effectively 8 letters. Also the number of dark RNA codons and perhaps also of ordinary AAs would increase - presumably by factor 8. Since the dark nucleons would be located along magnetic flux tube, Fermi statistics, which does not allow protons to have the same position, would not affect the situation and one would indeed obtain just the factor 8.

   There is however an objection. Dark proton sequences would be generated by the formation of exclusion zones in Pollack effect [L2] [L2], and would be stable against transformation to those containing neutrons since the energy needed to transform proton to neutron is about MeV and huge in the scale of biochemistry.

Is it possible to overcome this objection?

1. TGD inspired nuclear physics relies on nuclear string model [K2] for which unexpected correlations between nucleons (EMC effect) provide support. Nucleons would be connected bo nuclear string by color bonds having quark and antiquark at their ends. Bonds could be color neutral and color confinement would make the bonds stable.

   The bonds connecting nucleons to nuclear string would have u/d type quark and antiquark at their ends and could have total charges +, -1, and 0. This would predict new exotic states of nuclei with binding energy differences of order keV (small scale compared to MeV scale of nuclear binding energies). There is evidence for keV energy scale.

In fact, several scaled variants of dark nuclear physics are predicted [L5], and the nuclear binding energy scale would behave like $1/L$, where L is the size scale of dark nuclei identifiable as p-adic length scale in TGD framework. Even dark nuclear binding energy scale of order metabolic energy quantum of order .5 eV can be considered.

2. Same would apply to dark nuclei formed from dark protons. The bonds connecting dark protons to nuclear string could also have total charge +1,-1, and there could exist two states with charge 0. Only 3 spin states analogous to those of (neural) ρ_0 meson are accepted in the original model whereas neural pion-like state is not allowed. Now the states analogous to both ρ_0 and ρ_{-1} are accepted. One can denote the bond as $B(q)$, $q = 0, +1, -1$.

   The pair $p + B(-1)$ would behave like neutron effectively. The pair $p + B(1)$ would have charge +2 and could be be unstable due to repulsion whereas neutron like state could be stable by attraction. This could give rise to an effective doubling of letters.

   Remark: A possible objection is that the neutral ρ meson like color bond is expected to have energy higher than neutral pion by spin-spin splitting as in the case of ordinary mesons. A good argument for throwing out the pion-like bond is needed.

3.2 Is the realization of hachimoji code in terms of bio-harmony possible?

What about the realization of hachimoji code as a bio-harmony [L1, L9]? Bio-harmony makes it possible to transfer the genetic information at the level of dark variants of basic bio-molecules (also RNA, AAs, and tRNA) in terms of 3-chords of dark photons coupling via frequency resonance. The coupling to ordinary variants of DNA would take place via energy resonance and involve the transformation of dark photon to ordinary photon or vice versa coupling. Music expresses and induces emotions and the music of dark photons would provide fundamental expression of emotions realized at the bio-molecular level [L7].

3.2.1 Can one scale the number chords of bio-harmony by factor 8 by using icosa-dodecahedral bio-harmony?

The number 64 of 3-chords defining the bio-harmony should be scaled up by 8. As far as chords are considered, each note appearing in the chord should be doubled.

1. There are two variants for bio-harmony. 12-note scale is represented as a Hamiltonian cycle defined as a closed path (by octave equivalence) going through all vertices of as tesselation of sphere or torus and not intersecting itself. Both icosahedron and tetrahedron can be regarded as tesselations of sphere by triangles.

   The first realization [L1, L9] involves fusion of 3 Hamiltonian cycles at icosahedron defining 20 chord harmony H_{20} each and 1 cycle at tetrahedron defining 4-chord harmony H_4. This gives 20+20+20+4=64 3-chords defining the codons.

   Second realization [L6] is a fusion of 2 20-chord harmonies H_{20} defined by Hamiltonian cycles at icosahedron and 24-chord harmony H_{24} by cycle associated with torus tesselation. The fusion of two icosahedral cycles gives 20+20 3-chords and torus cycle gives 24 chords: 64 altogether. There are large number of Hamiltonian cycles and their fusions would correspond to different emotional states.

2. Can one imagine any modification of the model giving rise to 8-fold increase of the basic chords? One can consider doubling of the 4 basic frequencies to 8. For instance, splitting of each frequency could correspond to the doubling of the code letters. One can also imagine that each triplet of dark nucleons (dark neutron would be now dark proton+the bond with varying charge) corresponds to its own cyclotron frequency triplet so that 8-fold increase of 3-chords would become possible.

3. Could one have a geometric interpretation for the 8-fold increase of 3-chords realized as faces of Platonic solid or toric triangular tesselation. Could summand in the sum of 3 icosahedral harmonies and one tetrahedral harmony (of 2 icosahedral and toric harmonies) be replaced with an analog of tesselation having 8-fold number of triangles? The splitting of each triangle to 8 smaller equilateral triangles so that the 12-note scale would have now $8 \times 12 = 96$ notes, is not possible since the side of the smaller triangle should be $2^{-3/2}$ times smaller than that of the original triangle: inverse integer scaling would be required.

4. The simplest manner to get 8-fold scaling for the number of chords is some kind of fusion of 8 octaves of bio-harmony. By octave equivalence the 8-letter code would bring new information at the level of bio-harmony perceived in an improved resolution only. New information would require that the fused scales differ slightly. A natural interpretation for the fusion would be as formation of a discrete bundle structure in which 8-fold increase of notes of the scale corresponds to 8-point fiber.

   The fusion of fundamental harmonies with 20, 4 or 24 3-chords is used in the proposed models of bio-harmonies. The geometric interpretation of the fusion is not quite clear. For a fusion of 3 icosahedral code one could imagine a discrete bundle structure in which 3 copies of note as points of icosahedron form a 3-point fiber. The addition of tetrahedron could be seen as a union of icosahedron and tetrahedron with gluing along common face. This does not however fit with the bundle interpretation.

   Same applies to the union of 2 icosahedral codes with $(V, F) = (12, 20)$ and 1 toric code with $(V, F) = (20, 24)$. One could ask whether the latter option could allow interpretation as singular bundle structure such that in the fiber space two tori collapse to spheres. This would correspond to a disappearance of 4 faces so that one has 20 faces instead of 24. This does not look like an attractive option.

5. Could one find a realization of the code consistent with the bundle interpretation? Could one have 64 codons by using fusion of 2 icosahedral and 2 dodecahedral codes (forget for a moment that the faces of dodecahedron are pentagons!)? Dodecahedron has 20 vertices (maybe 20-note scale might relate to micro-intervals used in Eastern music) and 12 faces. The fusion would give 20+20+12+12= 64 chords. Dodecahedral harmony is unique since there is only single Hamilton's cycle.

   One would have only single topology and the interpretation as fiber space with 2 points in the fiber would make sense if the dodecahedral tesselation is constructed as a dual of icosahedral one with new vertices as centers of icosahedral triangles. Music, even the music of light realized as triplets of dark photons with frequencies equal to those of the chords of bioharmony, expresses emotions and this leads to the suggestion that emotions are expressed even at the level of bio-molecules [**?**] Therefore I cannot avoid the temptation to ask whether the uniqueness of the dodecahedral harmony could relate to the eastern notion of empty mind empty of any emotions and thoughts.

6. For this realization of bio-harmony the fusion of 8 bio-harmonies could be seen as a transition to a higher hierarchy level considering structures made of structures and would produce the required number 96 of notes. These bio-harmonies would have slightly different 12-note scales. Octave equivalence would suggests that 12-note scale is effectively replaced with $8 \times 12 = 96$ note scale. The interpretation in terms of fiber space structure with 2×8 points in the fiber would make sense.

3.2.2 The problem of Pythagoras a motivation for the fusion 8 copies of bio-harmonies

Could one imagine any justification for the fusion of 8 copies bio-harmonies possibly with slightly differing scales? A problem that teased already Pythagoras suggests this kind of justification!

1. The basic problem of Pythagorean scale based on rational frequencies realized as quints of the basic frequency modulo octave equivalence is that octave equivalence is not quite exact. The octave projections by scaling by a power of 2 of the scale in higher octaves to the lowest octave do not quite quite co-incide with the 12-note scale assigned with it: the reason is that

no power of $x = 3/2$ can exactly co-incide with power of 2 so that $x^{12} = 2^7$ is true only with 1 per cent accuracy.

Pythagoras who firmly believed that Nature relies on the arithmetics of rationals was even ready to believe that Nature is imperfect! In TGD framework one could say that only the cognition based on rationals is imperfect (also cognition using algebraic numbers is predicted to be possible and evolution would mean increase of the complexity for the extension of rationals). Tempered scale would require the powers of algebraic number $x = 2^{1/2}$ to belong to the extension.

The problem is that Pythagorean scale seems however to have a deeper meaning (people with absolute ear love it) [L3]. Could some number of octaves - say 8 - give a more precise mathematical model of music experience in the case of people having absolute ear? Could it be that people with absolute ear have a better pitch resolution and are able to distinguish between notes of 96-note scale?

Remark : The realization of 12-note scale using irrational frequency ratios coming as $2^{1/12}$-powers of the fundamental frequency does not have problem with octave equivalence.

2. The 8×12-note would be obtained as follows. One performs first 12-fold iteration to get 12-note scale. The 12th iterate is very near to the basic note by octave equivalence. After that one repeats 12-iteration 7 times so that each note in the original 12-note scale is mapped to 8 notes. These notes must be within interval corresponding to half-note (say E-F), which corresponds to the scaling $r = 2^{1/12}$ in good approximation. This gives the condition $(x^{12\times 8}/2^8)^8 < r$ giving the condition $x < 2 \times r^{1/(8\times 12)} = 2 \times 2^{1/96} \simeq 2.0145$ satisfied for $x_8 = 27/17$.

3. The construction of bio-harmony was based on the assumption that the subsequent vertices along Hamiltonian cycle (neighboring points of tesselation) are related by the scaling of frequency by $x_7 = 3/2$ (Hamiltonian cycle would correspond to quint cycle especially familiar for jazz musicians) and projecting to the basic octave. 12 scalings of this kind give slightly more than 7 octaves ($(3/2)^{12} \simeq 129.746..$ rather than $2^7 = 128$): there relative error is about 1 per cent. $x_7 = 3/2$ would suggest 7 rather than 8 copies of the basic bio-harmony.

   Quint rule is consistent with 8-fold repetition of the basic 12-iteration but one can imagine also alternative rules for generating the notes of the scale using powers of some number x reduced to basic octave. Could a simple choice for $x = x_8$ give $x_8^{12} = 2^8$ in a better approximation than $x_7 = 3/2$ gives $x_7^{12} = 2^7$? The replacement $x_7 = 3/2 \rightarrow x_8 = (3/2) \times y$, where y is rational approximation for $2^{1/12}$, gives a natural guess for x_8. For $y = 18/17$ giving $x_8 = 27/17$ (to be compared with $x_7 = 27/18$ one obtains $x_8^{12}/2^8 = 1.006...$, so that the error is .6 per cent whereas for $x_7 = 3/2$ the corresponding error is around 1 per cent. Note that $p = 17$ is Fermat prime of form $F_n = 2^{2^n} + 1$ near to power of 2. Primes near power of two are in fundamental role in TGD.

4. It will be found that the recent proposals for bio-harmony have drawbacks, and that a more elegant identification of bio-harmony as a fusion of icosahedral and dodecahedral harmonies leads to a replacement of powers of quints $(C - G)$ with powers of slightly larger interval $(C - G\sharp)$ and a generalization of Hamiltonian cycle by introducing the analogy of gauge symmetry.

3.2.3 Details of the icosa-dodecahedral harmony

Consider now the details of the icosa-dodecahedral harmony.

1. Dodecahedral harmony involves $n_{20} = 20$ notes. The generalization of the quint cycle means that the frequencies in the basic octave are obtained from the base frequency as scalings by octave equivalence: $f/f_0 = x_{20}^k/2^{r(k)}$, $k = 0, 1, .., 19$ with $r(k)$ fixed by the condition that $1 \leq f/f_0 \leq 2$. x_{20} is a rational number determined by the condition that x_{20}^{20} is as near as possible to power $2^{k_{20}}$, where k_{20} can have several values.

   $k_{20} = 12$ gives $x_{20} = 127/40$ as optimal choice. $x_{20}^{20}/2^{12} = 1.0007$, so that the error is very small. What puts bells ringing that Mersenne prime 127 appears in the numerator of x_{20}: it appears also in the model of genetic code based on Combinatorial Hierarchy [K1].

2. One can argue that the values of x should be such that 20-note scale shares the notes of 12-note scale under octave equivalence. This requires that x_{12} and x_{20} differ by a power of 2. For $n_{12} = 12, k_{12} = 8$ $x_{12} = 127/80 = x_{20}/2$ gives $x_{12}^{12}/2^8 = 1.0007$, which is an excellent accuracy. Note that x_{12} is not very far from quint $x = 3/2$. 20-note scale shares under octave equivalence the notes of 12-note scale in the sense that one has $x_{12}^r 2^{-r} = x_{20}^r$.

   8 icosahedral octaves emerges a as a prediction of icosa-dodecahedral codes and this is the number of octaves required by hachimoji DNA. Presumably there is a connection between these two identical numbers.

3. To get some idea about dodecahedral harmony one can use the fact that $x_{12} = 1.5875..$ is near to $2^{8/12} = 1.5874...$, which corresponds to the interval $C - G\sharp$ rather than quint $C - G$. For case b) the notes of 4 pentagons containing 4-edge would can be transposed to $CG\sharp ECG\sharp$ so that the notes begin to repeat themselves approximately and one would indeed obtain only 3-chords modulo octave equivalence! If the notes of 3-chord correspond to same power of x, all 3-chords would be of the same type: the melancholic 3-chord with which so many finnish tangos end! Since the repetition is not exact the notes of dodecahedral scale cover the entire octave. The basic $CEG\sharp$ chord transponated by the powers of x_{20} covers entire octave.

4. For 8 pentagons of type a) one would obtain 3-chord transposable to $CG\sharp E$ and 2-chord transposable to $CG\sharp$.

5. Should one allow also for the icosahedral harmonies only chords for which the notes belong to the cycle and triangle? This would allow 3-chords for triangles containing two edges of the cycle: these chords would be of type CGD involving two quints. Triangles containing single edge would correspond to 2-chords with separation by quint. The triangles containing no edges would correspond to notes. The choice of the note would not be unique. The model of icosahedral harmony indeed predicts this kind of 3-chords. For instance, dissonant chords involving 3 subsequent notes are possible [L1] and more natural interpretation would be as possible notes of melody.

3.2.4 Is gauging of sphere needed to make icosa-dodecahedral harmony non-trivial?

There is also a second objection. If the notes of the chord correspond to same power of $x_{20} = 127/40$, only the notes $C, EC, G\sharp$ would appear in the 3-chords the approximation that $x^{20}/2^{12} = 1$ as is obvious from the fact that one $x_{20} \simeq 2^{4/12}$. Both icosahedral and dodecahedral harmonies based on $x_{20} = 127/40$ would be trivial. As noticed, one obtains the 20 transposes of this chord but having only chords with same structure looks still trivial.

1. One could solve the problem by allowing combinations of notes of 3-chord with different values of k in x_{20}^k (or x_{12}^k). The division of octave to 20 (12) notes guarantees that the chords obtained in this manner allow to realize very rich repertoire of harmonies. Essentially $20^3 = 8000$ chords become possible. What looks like a weakness of Pythagorean view about music based on rationals would become a strength.

   The analogy with the non-uniqueness of gauge choice in gauge theories is obvious. Gauge transformations changing the value of k in local manner give new Hamiltonian cycles from given cycle. Mathematically this solution looks elegant since one can also choose $x_{20} = 127/40 = 2x_{12}$. This also gives 8 octaves for icosahedral harmony as hachimoji code requires.

2. Although the proposed solution is mathematically elegant, it is interesting to look also for the case $x_{12} = 3/2$. The first problem is that x_{12}^{20} deviates 20 per cent from base note, and would correspond to $E\flat$ rather than C. What is however nice is that the notes for a pentagon containing 4-edge would correspond to C, G, D, A, E, H. From these one can select major chords CEG, GHD, and minor chords ACE, EGH. One could obtain the basic harmonies from the dodecahedral part by allowing all possible choices.

   Could one assume a slightly modified quint scale and different scales for icosahedron and dodecahedron? Icosahedral and dodecahedral scales are roughly consistent if k_{20} corresponds to an integer multiple of k_{12}. For $k_{12} = 7$ and $k_{20} = 2k_{12} = 14$ one has $x_{12} = 3/2$ and $x_{20} = 13/8$. One has $x_{20}^2 0/2^{14} = 1.006$. One has $x_{20}/x_{12} = 13/12 = 1.08..$ to be compared with $2^{1/12} = 1.059...$ The difference is more than half-note that x_{20} corresponds roughly to $C - G\sharp$ interval as for $x_{20} = 127/40$ as above. Therefore this option does not look attractive.

3.3 Summarizing

Some concluding remarks are in order.

1. Hachimoji DNA turned out to be extremely inspiring discovery also from TGD point of view and led to a more refined vision about bio-harmony with elegant mathematical interpretation.

2. If the above arguments make sense, one cannot avoid the question whether the fact that some people have absolute ear mean that genetic code with 8-fold number of codons is realized at the level of dark codons and bio-harmony? Chemical realization would have been probably discovered.

3. 8×12-note scale would allow discretized glissandos and also discretized blue notes appearing in popular music. Purely electronic production of this kind of music using computer programs is possible using Garage Band or some other similar program, and it would be interesting to test how the discretized glissando is heard.

   One can imagine also instruments producing this kind of music. A hybrid of piano and violin comes first in mind. The keys of piano would be replaced by keys sensitive to touch - the technology used in smartphones would allow to realize this. The 8 1/16 notes associated with a given ordinary half-note would correspond in an increasing order to linearly ordered regions along the key, and one could change the note or chord by shifting the fingers along the key. The strength of touch could code for the volume. The chords of the harmony do not consist of arbitrary notes of the 8×12 note scale but are obtained by transposing the chords of the basic bio-harmony. This would help enormously in playing since one can shift all fingers along the keys defining the chord.

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