# TGD View about Coupling Constant Evolution

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**Abstract**

New results related to the TGD view about coupling constant evolution are discussed. The results emerge from the discussion of the recent claim of Atyiah that fine structure constant could be understood purely mathematically. The new view allows to understand the recently introduced TGD based construction of scattering amplitudes based on the analog of micro-canonical ensemble as a cognitive representation for the much more complex construction of full scattering amplitudes using real numbers rather than p-adic number fields. This construction utilizes number theoretic discretization of space-time surface inducing that of "world of classical worlds" (WCW) and makes possible adelization of quantum TGD.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years.

Could number theoretical constraints fix the evolution? Adelization suffers from serious number theoretical problem due to the fact that the action exponentials do not in general exist p-adically for given adele. The solution of the problem turned out to be trivial. The exponentials disappear from the scattering amplitudes! Contrary to the first beliefs, adelization does not therefore seem to determine coupling constant evolution.

TGD view about cosmological constant turned out to be the solution of the problem. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for $\alpha_K$, which can be studied numerically.

The approach is also related to the view about coupling constant evolution based on the inclusions of hyper-finite factors of type $II_1$, and it is proposed that Galois group replaces discrete subgroup of $SU(2)$ leaving invariant the algebras of observables of the factors appearing in the inclusion.

# Contents

# 1 Introduction

Atyiah has recently proposed besides a proof of Riemann Hypothesis also an argument claiming to derive the value of the structure constant (see `http://tinyurl.com/y8xw8cey`). The mathematically elegant arguments of Atyiah involve a lot of refined mathematics including notions of Todd exponential and hyper-finite factors of type II (HFFs) assignable naturally to quaternions. The idea that $1/\alpha$ could result by coupling constant evolution from $\pi$ looks however rather weird for a physicist.

What makes this interesting from TGD point of view is that in TGD framework coupling constant evolution can be interpreted in terms of inclusions of HFFs with included factor defining measurement resolution [K6, K1]. An alternative interpretation is in terms of hierarchy of extensions of rationals with coupling parameters determined by quantum criticality as algebraic numbers in the extension [L4, L5].

In the following I will explain what I understood about Atyiah's approach. My critics includes the arguments represented also in the blogs of Lubos Motl (see http://tinyurl.com/ycq8fhsy) and Sean Carroll (see http://tinyurl.com/y87f8psg). I will also relate Atyiah's approach to TGD view about coupling evolution. The hasty reader can skip this part although for me it served as an inspiration forcing to think more precisely TGD vision.

There are two TGD based formulations of scattering amplitudes.

1. The first formulation is at the level of infinite-D "world of classical worlds" (WCW) [K8] uses tools like functional integral. The huge super-symplectic symmetries generalizing conformal symmetries raise hopes that this formulation exists mathematically and that it might even allow practical calculations some day. TGD would be an analog of integrable QFT.

2. Second - surprisingly simple - formulation [L9] is based on the analog of micro-canonical ensemble in thermodynamics (quantum TGD can be seen as complex square root of thermodynamics). It relates very closely to TGD analogs of twistorialization and twistor amplitudes [K11, K13].

   During writing I realized that this formulation can be regarded as a generalization of cognitive representations of space-time surfaces based on algebraic discretization making sense for all extensions of rationals to the level of scattering amplitudes. In the adelization the key question is whether it is necessary to define the p-adic counterparts of action exponentials. The number theoretical constraints seem hopelessly strong. One solution would be that the action exponentials for allow space-time surfaces equal to one. This option fails. The solution of the problem is however trivial. Kähler function can have only single minimum for given values of zero modes and the action exponentials cancel from scattering amplitudes completely in this case. This formulation allows a continuation to p-adic sectors and adelization [L4, L5]. Note that no conditions on $\alpha_K$ are obtained contrary to the first beliefs.

One can also understand the relationship of the two formulations in terms of $M^8-H$ duality. This view allows also to answer to a longstanding question concerning the interpretation of the surprisingly successful p-adic mass calculations [K7]: as anticipated, p-adic mass calculations are carried out for a cognitive representation rather than for real world particles and the huge simplification explains their success for preferred p-adic prime characterizing particle as so called ramified prime for the extension of rationals defining the adeles.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years. TGD view about cosmological constant turned out to be the solution of the problem.

1. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

   In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for $\alpha_K$, which can be studied numerically.

2. I consider also the relationship to a second TGD based formulation of coupling constant evolution in terms of inclusion hierarchies of hyper-finite factors of type $\mathrm{II}_1$ (HFFs) [K6, K1]. I suggest that this hierarchy is generalized so that the finite subgroups of $SU(2)$ are replaced with Galois groups associated with the extensions of rationals. An inclusion of HFFs in which Galois group would act trivially on the elements of the HFFs appearing in the inclusion: kind of Galois confinement would be in question.

Ramified primes are conjecture to correspond to the preferred p-adic primes characterizing particles. Ramified primes are special in the sense that their expression as a product of primes $P_i$ of extension contains higher than first powers and the number $P_i$ is smaller than the maximal number $n$ defined by the dimension of the extension. It is not quite clear why ramified primes appear as preferred p-adic primes and in the following Dedekind zeta functions and what I call ramified zeta functions inspired by the interpretation of zeta function as analog of partition function are used in attempt to understand why ramified primes could be physically special.

The intuitive feeling is that quantum criticality is what makes ramified primes so special. In $O(p) = 0$ approximation the irreducible polynomial defining the extension of rationals indeed reduces to a polynomial in finite field $F_p$ and has multiple roots for ramified prime, and one can deduce a concrete geometric interpretation for ramification as quantum criticality using $M^8 - H$ duality.

# 2 Criticism of Atyiah's approach

The basic idea of Atyiah is that $\pi$ and the inverse of the fine structure constant $1/\alpha = 137.035999....$ are related by coupling constant evolution - that is renormalization - which is a basic operation in quantum field theory and has physical interpretation. For a physicist it is easy to invent objections.

1. In quantum field theory fine structure constant and all coupling strengths obey a continuous evolution as function of mass scale or length scale and one should predict the entire evolution rather than say its value at electron length scale. In TGD framework the coupling constant evolution becomes discrete and would basically labelled by the hierarchy of extensions of rationals.

2. $\pi$ is purely geometric constant - kind of Platonic transcendental having very special role in the mathematical world order - whereas fine structure constant is a dynamical coupling parameter. Atyiah does not have any proposal for why these constants would be related in this manner. Also no explanation for what it would mean that the circumference of unit circle would grow from $2\pi$ to $2/\alpha$ is given.

   **Remark**: In TGD actually the coverings labelled by the value $h_{eff}/n_0 = n$ identified as the order of Galois group of extension of rationals defining given level of the hierarchy of evolutionary levels (entanglement coefficients would belong to this extension as also S-matrix elements). The full angle using $M^4$ rotation angle as coordinate increases effectively to $n \times 2\pi$ for the covering spaces of extensions introducing $n$:th root of unity. In TGD would however have $n$ instead of $1/(\alpha\pi)$.

3. That $1/\alpha \sim 137$ should have interpretation as renormalized value of angle $\pi$ looks rather weird to me. The normalization would be very large and it is extremely difficult to see why $1/\pi$ have a role of fine structure constant say at high energy limit if one accepts coupling constant evolution and identifies $1/\alpha$ as the value of $1/\alpha$ at zero momentum transfer.

In fact, Atyiah proposes a discrete evolution of $\pi$ to $1/\alpha$ defined by approximations of HFF as a finite-D algebra. Forgetting $\pi$ as the starting point of the evolution, this idea looks beautiful. At first the idea that all numbers suffer a renormalization evolution, looks really cute. Coupling constant evolution is however not a sequence of approximations but represents a genuine dependence of coupling constants on length scale.

**Remark**: In TGD framework I propose something different.The length scale evolution of coupling constants would correspond to a hierarchy of inclusions of HFFs rather than a sequence of finite-D approximations approaching HFF. The included factor would represent measurement resolution. Roughly, the transformations of states by operations defined in included factor would leave state invariant in the measurement resolution defined by the included factor. Different values of coupling constant would correspond to different measurement resolutions.

1. Atyiah mentions as one of his inspirers the definition of $2\pi$ via a limiting procedure identifying it as the length of the boundary of $n$-polygon inside unit circle. Amusingly, I have proposed similar definition of $2\pi$ in p-adic context, where the introduction of $\pi$ would give rise to infinite extension.

Atyiah generalizes this definition to the area of quaternionic sphere so that the limiting procedure involves two integers. For sphere tesselations as analogs of lattices allow only Platonic solids. For torus one could have infinite hierarchy of tesselations [L8] allowing to define the area of torus in this manner. The value of $n$ defined by the extension of rationals containing root of unity $exp(i2\pi/n)$ such that $n$ is maximal. The largest $n$ for the roots of unity appearing in the extension of p-adics would determine the approximation of $2\pi$ used.

2. Atyiah suggests a concrete realization for the coupling constant evolution of numbers, not only coupling constants. The evolution would correspond to a sequence of approximation to HFF converging to HFF. One can of course define this kind of evolution but to physicist it looks like a formal game only.

3. HFF is interpreted as an infinite tensor product of $2 \times 2$ complex Clifford algebras $M_2(C)$, which can be also interpreted as complexified quaternions. One defines the trace by requiring that the trace of infinite tensor product of unit matrices equals to 1. The usual definition of schoolbooks would given infinite power of 2, which diverges. The inner product is the product of the usual inner products for the factors of the tensor product labelled by $n$ but divided by power $2^{-n_{max}}$ to guarantee that the trace of the identity matrix is unity as product of traces for factors otherwise equal to $2^n$. In fact, fermionic Fock algebra familiar to physicist is HFF although in hidden manner.

   **Remark**: The appearance of quaternions is attractive from TGD point of view since in $M^8 - H$ duality the dynamics at the level of $M^8$ is determined by associativity of either tangent or normal space of 4-surface in $M^8$ and associativity is equivalent with quaternionicity [L3]. The hierarchy of HFFs is also basic piece of quantum TGD and realizable in terms of quaternions.

4. Atyiah tells there is an algebra isomorphism from complex numbers $C$ to the subset of commuting matrices in HFF. One can define the map to $C$ as either eigenvalue of the matrix and obtains to isomorphisms: $t_+$ and $t_-$. One can define the renormalization map $C \to C$ in terms of the inverse of $t_- \circ t_+^{-1}$ or its inverse. This would assign to a complex numbers $z$ its normalized value.

   HFFs allow an excellent approximation by finite number of tensor factors and one can perform an approximation taking only finite number of tensor factors and at the limit of infinite number of factors get the desired normalization map. The approximation would be $t_-(n) \circ t_+(n)$. I must confess that I did not really understand the details of this argument.

   In any case, to me this does not quite correspond to what I understand with renormalization flow. Rather this is analogous to a sequence of approximations defining scattering amplitude as approximation containing only contributions up to power $g^n$. I would argue than one must consider the infinite sequence of inclusions of HFFs instead of a sequence of approximations defining HFF.

   In this manner one would the renormalization map would be $t_-(n+1) \circ t_+^{-1}(n)$, where $n$ now labels the hierarchy of HFFs in the inclusion hierarchy. $t_\pm n$ is now the exact map from commuting sub-algebra to complex numbers.

   There is however a rather close formal resemblance since simple inclusions correspond to inclusions of the sub-algebra with one $M_C^2$ factor replaced with mere identity matrix.

5. The proposal of Atyiah is that this renormalization of numbers is mediated by so called Todd exponentiation used in the construction of the characteristic classes. This map would be defined in terms of generating function $G(x) = x/(1 - exp{-}x)$ applied to $x = \pi$. If I understood anything about the explanation, this map is extended to infinite number of tensor factors defining the HFF and the outcome would be that $x = \pi$ for single tensor factor would be replaced with $1/\alpha$. Why Todd exponentiation? Atyiah also argues that one has $T(\pi)/\pi = T(\gamma)/\gamma$, where $\gamma$ is Euler's constant. My mathematical education is so limited that I could not follow these arguments.

6. Atyiah also claims that the approximation $1/\alpha = 137$ assumed by Eddington to be exact has actually deeper meaning. There are several formulas in this approximation such as $1/\alpha =$

$2^0+2^3+2^7=1+8+128$. If I understood correctly, Atyiah tells that the numbers 1, 8, and 128 appear in the Bott periodicity theorem as dimensions of subsequent stable homotopy groups. My own favorite formula is in terms of Mersenne primes: $1/\alpha = M_2+M_3+M_7 = 3+7+127$. The next Mersenne prime would be $M_{127}$ and corresponds to the p-adic length scale of electron.

**Remark**: A fascinating numerological fact is that $p \simeq 2^k$, $k \simeq 137$, corresponds to the p-adic length scale near to Bohr radius: kind of cosmic joke one might say. Fine structure constant indeed emerged from atomic physics!

It would be of course marvellous if the renormalization would not depend on physics at all but here physicist protests.

1. The coupling constant evolutions for the coupling strengths of various interactions are different and depend also on masses of the particles involved. One might however hope that this kind of evolution might make sense for fundamental coupling constants of the theory. In TGD Kähler coupling strength $1/\alpha_K$ would be such parameter.

2. The quantum criticality of TGD Universe suggests that Atyiah's claim is true in a weaker sense. Quantum criticality is however a dynamical notion. I have actually proposed a model for the evolution of $1/\alpha_K$ based on the complex zeros of Riemann Zeta [L1] and also a generalization to other coupling strengths assuming that the argument of zeta is replaced with its Möbius transform.

   Very strong consistency conditions should be met. Preferred primes would be primes near prime power of 2 and ramified primes of extension, and also the zero of zeta in question should belong to the extension in question. I am of course the first to admit that this model is motivated more by mathematical aesthetics than concrete physical calculations.

3. The idea about renormalization evolution in this manner could - actually should - generalize. One can consider a maximal set of commuting set of observables in terms of tensor product of HFFs and define for them map to diagonal $n \times n$ matrices with complex eigenvalues. One would have infinite sum over the eigenvalues of diagonal matrices over factors: just as one has for many particle state in QFT containing contribution from all tensor factors which are now however ordered by the label $n$. The length scale evolution of these observables could be defined by the above formula for inclusion. Fine structure constant basically reduces to charge as eigenvalue of charge operator so that this could make sense.

   The beauty of this view would be that renormalization could be completely universal. In TGD framework quantum criticality (QC) indeed strongly suggests this universality in some sense. The hierarchy of extensions of rationals would define the discrete coupling constant evolution.

# 3 About coupling constant evolution in TGD framework

It is often forgotten that fine structure constant depends on length scale. When Eddington was working with the problem, it was not yet known that fine structure constant is running coupling constant. For continuous coupling constant evolution there is not much point to ponder why its value is what it is at say electron length scale. In TGD framework - adelic physics - coupling parameters however obey discrete length scale evolution deriving from the hierarchy of extensions of rationals. In this framework coupling constants are determined by quantum criticality implying that they do not run at all in the phase assignable to given extension of rational. They are analogous to critical temperature and determined in principle by number theory.

### 3.0.1 Two approaches to quantum TGD

There are two approaches to TGD: geometric and number theoretic. The "world of classical worlds" (WCW) is central notion of TGD as a geometrization of quantum physics rather than only classical physics.

1. WCW consists of 3-surfaces and by holography realized by assigning to these 3-surfaces unique 4-surfaces as preferred extremals. In zero energy ontology (ZEO) these 3-surfaces are pairs of 3-surfaces, whose members reside at opposite boundaries of causal diamond (CD) and are connected by preferred extremal analogous to Bohr orbit. The full quantum TGD would rely on real numbers and scattering amplitudes would correspond to zero energy states having as arguments these pairs of 3-surfaces. WCW integration would be involved with the definition of inner products.

2. The theory could be seen formally as a complex square root of thermodynamics with vacuum functional identified as exponent of Kähler function. Kähler geometry would allow to eliminate ill-defined Gaussian determinants and metric determinant of Kähler metric and they would simply disappear from scattering amplitudes. WCW is infinite-D space and one might argue that this kind of approach is hopeless. The point is however that the huge symmetries of WCW - super-symplectic invariance - give excellent hopes of really construction the scattering amplitudes: TGD would be integrable theory.

3. A natural interpretation would be that Kähler action as the analog of Hamiltonian defines the Kähler function of WCW and functional integral defined by it allows definition of full scattering amplitudes.

The number theoretic approach could be called adelic physics [L4, L5] providing also the physics of cognition.

1. At space-time level p-adicization as description of cognition requires discretization. Cognitive representations at space-time level consist of finite set of space-time points with preferred coordinates $M^8$ in extension of rationals inducing the extensions of p-adic number fields. These representations would realize the notion of finite measurement resolution. p-Adicization and adelization for given extension of rationals are possible only in this manner since these points can be interpreted as both real and p-adic numbers.

2. What about cognitive representations at the level of WCW? The discrete set of space-time points would replace the space-time surface with a finite discrete set of points serving also as its WCW coordinates and define the analog of discretization of WCW using polynomials in $M^8$ fixed by their values at these points [L3]. If the space-time surface is represented by a polynomial, this representation is all that is needed to code for the space-time surface since one can deduce the coefficients of a polynomial from its values at finite set of points. Now the coefficients belong to extension of rationals. If polynomials are replaced by analytic functions, polynomials provide approximation defining the cognitive representation.

While writing this I realized that what I have micro-canonical ensemble [L9] as kind of complex square root of its counterpart in thermodynamics can serve as a cognitive representation of scattering amplitudes. Cognitive representations of space-time surfaces would thus give also cognitive representations of WCW and micro-canonical ensemble would realize cognitive representations for the scattering amplitudes. Cognitive representations define only a hierarhcy of approximations. The exact description would involve the full WCW, its Kähler geometry, and vacuum functional as exponent of Kähler function.

The idea of micro-canonical ensemble as a subset of space-time surfaces with the same vanishing action would select a sub-set of surfaces with the same values of coupling parameters so that the fixing the coupling parameters together with preferred extremal property selects the subset with same value of action. There are two options to consider.

1. The real part of the action vanishes and imaginary part is multiple of $2\pi$ so that the action exponential is equal to unity. For the twistor lift this actually implies the vanishing of the entire action since volume term and Kähler term have the same phase (that of $1/\alpha_K$). The role of coupling parameters would be analogous to the role of temperature and applied pressure. In principle this condition is mathematically possible. The electric part of Kähler action in Minkowskian regions has sign opposite to magnetic part and volume term (actually magnetic $S^2$ part of 6-D Kähler action) so that these two contributions could cancel. The problem is that Kähler function would be constant and therefore also the Kähler metric.

2. I have also proposed [L9] that the analog of micro-canonical ensemble makes sense meaning that all space-time surfaces contributing to the scattering amplitude have the same action. As a consequence, the action exponential and the usual normalization factor would cancel each other and one would obtain just a sum over space-time surfaces with same action: otherwise action exponential would not appear in the scattering amplitudes - this is the case also in perturbative QFTs. This is crucial for the p-adicization and adelization since these exponential factors belong to the extension of rationals only under very strong additional conditions.

   This option has analog also at the level of WCW since Kähler function should have for give values of zero modes only single minimum so that localization in zero modes would mean that the action exponential cancels in the normalization of the amplitudes. It seems that this option is the only possible one.

   Note that the cancellation of the metric determinant and Gaussian determinant possible for Kähler metric with the exponent of Kähler function serving as vacuum functional reduces the perturbative integrations around the minima of Kähler action to a sum over exponents, and if only single minimum contributes for given values of the zero modes, the sum contains only single term.

## 3.1 Number theoretic vision about coupling constant evolution

Let us return to the question about the coupling constant evolution.

1. Each extension of rationals corresponds to particular values of coupling parameters determined by the extension so that it indeed makes sense to ponder what the spectrum of values for say fine structure constant is. In standard QFT this does not make sense.

2. Coupling constant evolution as a function of momentum or length scales reduces to p-adic coupling constant evolution in TGD as function of p-adic prime. Particles are characterized by preferred p-adic primes - for instance, electron corresponds to $M_{127} = 2^{127} - 1$ - the largest Mersenne prime which does not correspond to super-astronomical Compton length - and the natural identification is as so called ramified primes of extension.

Why the interpretation of p-adic primes as ramified primes?

1. As one increases length scale resolution particle decomposes to more elementary particles.

2. Particles correspond in TGD to preferred p-adic primes. This suggests that when a prime (ideal) of given extension is looked at improved precision determined by an extension of the orignal extension it decomposes into a product of primes. This indeed happens.

   The number of primes of the larger extension appearing in the decomposition to product equals to the dimension of extension as extension of the original extension. All these primes appear and only once in the generic case. Ramified primes of ordinary extension are however odd-balls. Some primes of extension are missing and some appear as higher powers than 1 in their decomposition.

3. Ramified primes are analogous to critical systems. Polynomial with a multiple root - now prime of extension appearing as higher power - corresponds to a critical system. TGD is quantum critical so that one expects that ramified primes are preferred physically and indeed correspond to quantum critical systems.

4. Only the momenta belonging to the extension of rationals are considered and one can identify them as real-valued or p-adic valued momenta. Coupling constants do not depend on the values of the momenta for given extension of rationals and are thus analogous to critical temperature.

   This involves interesting not totally resolved technical question inspired by p-adic mass calculations for which the p-adic mass squared value is mapped to its real value by canonical identification $S \sum x_n p^n \to \sum x_n p^{-n}$. The correspondence is continuous and can be applied to Lorentz invariants appearing in scattering amplitudes [K3].

Could this correspondence be applied also to momenta rather than only mass squared values and Lorentz invariants? $M^8 - H$ correspondence [L3] selects fixed Poincare frame as moduli space for octonionic structures and at $M^8$ level this could make sense.

## 3.2 Cosmological constant and twistor lift of Kähler action

Cosmological constant $\Lambda$ is one of the biggest problems of modern physics. Surprisingly, $\Lambda$ turned out to provide the first convincing solution to the problem of understanding coupling constant evolution in TGD framework. In QFTs the independence of scattering amplitudes on UV cutoff length scale gives rise to renormalization group (RG) equations. In TGD there is however no natural cutoff length scale since the theory is finite. Cosmological constant should however evolve as a function of p-adic length scales and cosmological constant itself could give rise to the length scale serving in the role of cutoff length scale. Combined with the view about cosmological constant provided by twistor lift of TGD this leads to explicit RG equations for $\alpha_K$ and scattering amplitudes.

Cosmological constant has two meanings.

1. Einstein proposed non-vanishing value of $\Lambda$ in Einstein action as a volume term at his time in order to get what could be regarded as a static Universe. It turned out that Universe expanded and Einstein concluded that this proposal was the greatest blunder of his life. For two decades ago it was observed that the expansion of the Universe acclerates and the cosmological constant emerged again. $\Lambda$ must be extremely small and have correct sign in order to give accelerating rather decelerating expansion in Robertson-Walker cooordinate. Here one must however notice that the time slicing used by Einstein was different and fr this slicing the Universe looked static.

2. $\Lambda$ can be however understood in an alternative sense as characterizing the dynamics in the matter sector. $\Lambda$ could characterize the vacuum energy density of some scalar field, call it quintessense, proportional to 3- volume in quintessence scenario. This $\Lambda$ would have sign opposite to that in the first scenario since it would appear at opposite side of Einstein's equations.

### 3.2.1 Cosmological constant in string models and in TGD

It has turned out that $\Lambda$ could be the final nail to the coffin of superstring theory.

1. The most natural prediction of M-theory and superstring models is $\Lambda$ in Einsteinian sense but with wrong sign and huge value: for instance, in AdS/CFT correspondence this would be the case. There has been however a complex argument suggesting that one could have a cosmological constant with a correct sign and even small enough size.

   This option however predicts landscape and a loss of predictivity, which has led to a total turn of the philosophical coat: the original joy about discovering the unique theory of everything has changed to that for the discovery that there are no laws of physics. Cynic would say that this is a lottery win for theoreticians since theory building reduces to mere artistic activity.

2. Now however Cumrun Vafa - one of the leading superstring theorists - has proposed that the landscape actually does not exist at all [B4] (see `http://tinyurl.com/ycz7wvng`). $\Lambda$ would have wrong sign in Einsteinian sense but the hope is that quintessence scenario might save the day. $\Lambda$ should also decrease with time, which as such is not a catastrophe in quintessence scenario.

3. Theorist D. Wrase et al has in turn published an article [B2] (see `http://tinyurl.com/ychrhuxk`) claiming that also the Vafa's quintessential scenario fails. It would not be consistent with Higgs Higgs mechanism. The conclusion suggesting itself is that according to the no-laws-of-physics vision something catastrophic has happened: string theory has made a prediction! Even worse, it is wrong.

   **Remark**: In TGD framework Higgs is present as a particle but p-adic thermodynamics rather than Higgs mechanism describes at least fermion massivation. The couplings of Higgs to fermions are naturally proportional their masses and fermionic part of Higgs mechanism is seen only as a manner to reproduce the masses at QFT limit.

4. This has led to a new kind of string war: now inside superstring hegemony and dividing it into two camps. Optimistic outsider dares to hope that this leads to a kind of auto-biopsy and the gloomy period of superstring hegemony in theoretical physics lasted now for 34 years would be finally over.

String era need not be over even now! One could propose that both variants of $\Lambda$ are present, are large, and compensate each other almost totally! First I took this as a mere nasty joke but I realized that I cannot exclude something analogous to this in TGD. It turned that this is not possible. I had made a delicate error. I thought that the energy of the dimensionally reduced 6-D Kähler action can be deduced from the resulting 4-D action containing volume term giving the negative contribution rather than dimensionally reducing the 6-D expression in which the volume term corresponds to 6-D magnetic energy and is positive! A lesson in non-commutativity!

The picture in which $\Lambda$ in Einsteinian sense parametrizes the total action as dimensionally reduced 6-D twistor lift of Kähler action could be indeed interpreted formally as sum of genuine cosmological term identified as volume action. This picture has additional bonus: it leads to the understanding of coupling constant evolution giving rise to discrete coupling constant evolution as sub-evolution in adelic physics. This picture is summarized below.

### 3.2.2 The picture emerging from the twistor lift of TGD

Consider first the picture emerging from the twistor lift of TGD.

1. Twistor lift of TGD leads via the analog of dimensional reduction necessary for the induction of 8-D generalization of twistor structure in $M^4 \times CP_2$ to a 4-D action determining space-time surfaces as its preferred extremals. Space-time surface as a preferred extremal defines a unique section of the induced twistor bundle. The dimensionally reduced Kähler action is sum of two terms. Kähler action proportional to the inverse of Kähler coupling strength and volume term proportional to the cosmological constant $\Lambda$.

   **Remark:** The sign of the volume action is negative as the analog of the magnetic part of Maxwell action and *opposite* to the sign of the area action in string models.

   Kähler and volume actions should have opposite signs. At $M^4$ limit Kähler action is proportional to $E^2 - B^2$ In Minkowskian regions and to $-E^2 - B^2$ in Euclidian regions.

2. Twistor lift forces the introduction of also $M^4$ Kähler form so that the twistor lift of Kähler action contains $M^4$ contribution and gives in dimensional reduction rise to $M^4$ contributions to 4-D Kähler action and volume term.

   It is of crucial importance that the Cartesian decomposition $H = M^4 \times CP_2$ allows the scale of $M^4$ contribution to 6-D Kähler action to be different from $CP_2$ contribution. The size of $M^4$ contribution as compared to $CP_2$ contribution must be very small from the smallness of CP breaking [L7] [K13].

   For canonically imbedded $M^4$ the action density vanishes. For string like objects the electric part of this action dominates and corresponding contribution to 4-D Kähler action of flux tube extremals is positive unlike the standard contribution so that an almost cancellation of the action is in principle possible.

3. What about energy? One must consider both Minkowskian and Euclidian space-time regions and be very careful with the signs. Assume that Minkowskian and Eucidian regions have *same time orientation*.

   (a) Since a dimensionally reduced 6-D Kähler action is in question, the sign of energy density is positive Minkowskian space-time regions and of form $(E^2 + B^2)/2$. Volume energy density proportional to $\Lambda$ is positive.

(b) In Euclidian regions the sign of $g^{00}$ is negative and energy density is of form $(E^2 - B^2)/2$ and is negative when magnetic field dominates. For string like objects the $M^4$ contribution to Kähler action however gives a contribution in which the electric part of Kähler action dominates so that $M^4$ and $CP_2$ contributions to energy have opposite signs.

(c) 4-D volume energy corresponds to the magnetic energy for twistor sphere $S^2$ and is therefore positive. For some time I thought that the sign must be negative. My blunder was that I erratically deduced the volume contribution to the energy from 4-D dimensionally reduced action, which is sum of Kähler action and volume term rather than deducing it for 6-D Kähler action and then dimensionally reducing the outcome. A good example about consequences of non-commutativity!

The identification of the observed value of cosmological constant is not straightforward and I have considered several options without making explicit their differences even to myself. For Einsteinian option cosmological constant could correspond to the coefficient $\Lambda$ of the volume term in analogy with Einstein's action. For what I call quintessence option cosmological constant $\Lambda_{eff}$ would approximately parameterize the total action density or energy density.

1. Cosmological constant - irrespective of whether it is identified as $\Lambda$ or $\Lambda_{eff}$ - is extremely small in the recent cosmology. The natural looking assumption would be that as a coupling parameter $\Lambda$ or $\Lambda_{eff}$ depends on p-adic length scale like $1/L_p^2$ and therefore decreases in average sense as $1/a^2$, where $a$ is cosmic time identified as light-cone proper time assignable to either tip of CD. This suggests the following rough vision.

   The increase of the thickness of magnetic flux tubes carrying monopole flux liberates energy and this energy can make possible increase of the volume so that one obtains cosmic expansion. The expansion of flux tubes stops as the string tension achieves minimum and the further increase of the volume would increase string tension. For the cosmological constant in cosmological scales the maximum radius of flux tube is about 1 mm, which is biological length scale. Further expansion becomes possible if a phase transition increasing the p-adic length scale and reducing the value of cosmological constant is reduced. This phase transition liberates volume energy and leads to an accelerated expansion. The space-time surface would expand by jerks in stepwise manner. This process would replace continuous cosmic expansion of GRT. One application is TGD variant of Expanding Earth model explaining Cambrian Explosion, which is really weird event [K2].

   One can however raise a serious objection: since the volume term is part of 6-D Kähler action, the length scale evolution of $\Lambda$ should be dictated by that for $1/\alpha_K$ and be very slow: therefore cosmological constant identified as Einsteinian $\Lambda$ seems to be excluded.

2. It however turns that it possible to have a large number of imbedding of the twistor sphere into the product of twistor spheres of $M^4$ and $CP_2$ defining dimensional reductions. This set is parameterized by rotations sphere. The $S^2$ part of 6-D Kähler action determining $\Lambda$ can be arbitrarily small. This mechanism is discussed in detail in [L11, L12] and leads also to the understanding of coupling constant evolution. The cutoff scale in QFT description of coupling constant evolution is replaced with the length scale defined by cosmological constant.

### 3.2.3 Second manner to increase 3-volume

Besides the increase of 3-volume of $M^4$ projection, there is also a second manner to increase volume energy: many-sheetedness. The phase transition reducing the value of $\Lambda$ could in fact force many-sheetedness.

1. In TGD the volume energy associated with $\Lambda$ is analogous to the surface energy in superconductors of type I. The thin 3-surfaces in superconductors could have similar 3-surface analogs in TGD since their volume is proportional to surface area - note that TGD Universe can be said to be quantum critical.

   This is not the only possibility. The sheets of many-sheeted space-time having overlapping $M^4$ projections provide second mechanism. The emergence of many-sheetedness could also be caused by the increase of $n = h_{eff}/h_0$ as a number of sheets of Galois covering.

2. Could the 3-volume increase during deterministic classical time evolution? If the minimal surface property assumed for the preferred extremals as a realization of quantum criticality is true everywhere, the conservation of volume energy prevents the increase of the volume. Minimal surface property is however assumed to fail at discrete set of points due to the transfer of conserved charged between Kähler and volume degrees of freedom. Could this make possible the increase of volume during classical time evolution so that volume and Kähler energy could increase?

   **Remark:** While writing this for the first time, I did not yet realize that if the action contains also parts associated with string world sheets and their light-like boundaries as $M^8-H$ duality suggests, then the transfer of conserved quantities between space-time interior and string world sheets and string world sheets and their boundaries is possible, and implies the failure of the minimal surface property at these surfaces. One can however formulated precisely the proposed option and it implies that also string world sheets are quantum critical and therefore minimal surfaces: the question whether this occurs everywhere or only for the portions of string world sheets near the boundaries of causal diamonds remains open [L16].

3. ZEO allows the increase of average 3-volume by quantum jumps. There is no reason why each "big" state function reduction changing the roles of the light-like boundaries of CD could not decrease the average volume energy of space-time surface for the time evolutions in the superposition. This can occur in all scales, and could be achieved also by the increase of $h_{eff}/h_0 = n$.

4. The geometry of CD suggests strongly an analogy with Big Bang followed by Big Crunch. The increase of the volume as increase of the volume of $M^4$ projection does not however seem to be consistent with Big Crunch. One must be very cautious here. The point is that the size of CD itself increases during the sequence of small state function reductions leaving the members of state pairs at passive boundary of CD unaffected. The size of 3-surface at the active boundary of CD therefore increases as also its 3-volume.

   The increase of the volume during the Big Crunch period could be also due to the emergence of the many-sheetedness, in particular due to the increase of the value of $n$ for space-time sheets for sub-CDs. In this case, this period could be seen as a transition to quantum criticality accompanied by an emergence of complexity.

### 3.2.4 Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that $\Lambda$ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [K10].

This would solve the problems due to the huge value of $\Lambda$ predicted in GRT approach: the smoothed out behavior of $\Lambda$ would be $\Lambda \propto 1/a^2$, $a$ light-cone proper time defining cosmic time, and the recent value of $\Lambda$ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - $\Lambda$ would be large.

A simple solution of the problem would be the p-adic length scale evolution of $\Lambda$ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [K2]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of $\Lambda$ [L10]. Is there any cure to this problem?

1. The magnetic energy decreases with the area $S$ of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like $S$. The sum of these has minimum for certain radius of flux tube determined by the value of $\Lambda$. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV$^4$ (see `http://tinyurl.com/k4bwlzu`) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can $\Lambda$ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of $M^4$ and $CP_2$ give the same contribution to the induced Kähler form at twistor sphere of $X^4$, this term has maximal possible value!

   The original discussions in [K11, K10] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and $\Lambda$ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of $M^4$ and $CP_2$ if $CP_2$ size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for $\Lambda$ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres $S^2$ of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of $T_{CP_2}$ having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates of say $S^2(M^4)$ as coordinates and imbedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of $CP_2$. The identification map is unique apart from $SO(3)$ rotation $R$ of either twistor sphere possibly combined with reflection $P$. Could one consider the possibility that $R$ is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common $S^2$ coordinates, one has $J_{ind} = J + RP(J)$, where $R$ denotes a rotation and $P$ denotes reflection. Without reflection one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

   **Remark:** It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

   One can choose the rotation to act on $(y, z)$-plane as $(y, z) \rightarrow (cy + sz, -sz + cy)$, where $s$ and $c$ denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of $s$. Reflection $P$ can be chosen to correspond to $z \rightarrow -z$. Using coordinates $(u = cos(\Theta), \Phi)$ for $S^2(M^4)$ and $(v, \Psi)$ for $S^2(CP_2)$ and by writing the reflection followed by rotation explicitly in coordinates $(x, y, z)$ one finds $v = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in $s$ one has $v = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form $J^{ind}$ is sum of unrotated part $J(M^4) = du \wedge d\Phi$ and $J(CP_2) = dv \wedge d\Psi$. $J(CP_2)$ equals to the determinant $\partial(v, \Psi)/\partial(u, \Phi)$. A suitable spectrum for $s$ could reproduce the proposal $\Lambda \propto 2^{-k}$ for $\Lambda$. The $S^2$ part of 6-D Kähler action equals to $(J_{\theta\phi}^{ind})^2/\sqrt{g_2}$ and in the lowest order proportional to $s^2$. For small values of $s$ the integral of Kähler action for $S^2$ over $S^2$ is proportional to $s^2$.

One can write the $S^2$ part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of $F$ on $s$ and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2, k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with $k$ replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from $k$.

4. The sum $J^{ind} = J + RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of $J$.

5. The imbeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \to 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1, 1)$ and $(-1, -1)$ whereas metric and energy momentum tensor have only components of type $(1, -1)$ and $(-1, 1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant $\Lambda$ as function of $S^2$ coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of $\Lambda$ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

   One would have family of solutions of field equations but particular value of $\Lambda$ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations $R$ combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K4] obtained at this limit would not be lost.

## 3.3 Does p-adic coupling constant evolution reduce to that for cosmological constant?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

### 3.3.1 Basic notions and ideas

Consider first the basic notions and ideas.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

   Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of co-incide. The degeneration of roots of polynomials is a concrete realization for this.

Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L9].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L1]. These proposals are however highly ad hoc.

### 3.3.2 Could the area of twistor sphere replace cutoff length?

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the $S^2$ part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of $S^2$ possibly

combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to $s$. The variation with respect to $s$ would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

   One obtains explicit RG equation for $\alpha_K$ and $\Lambda$ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{(S_K(X^4)/Vol(X^4)) + S(S^2)} \frac{dlog(S(S^2))}{ds} \quad . \tag{3.1}$$

   It should be noticed that the choices of the parameter $s$ in the evolution equation is arbitrary so that the identification $s = sin(\epsilon)$ is not necessary. Note that one must use Kähler action per volume.

   The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

   In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes, and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of $\alpha_K$? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L10].

   The critical values of coupling constant evolution would correspond to the critical values of $S$ and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J^2_{u\Phi}$ and the numerator $1/\sqrt{det(g)}$ increase with $\epsilon$. If the rate for the variation of these quantities with $s$ vary it is possible to have a situation in which the one has

$$\frac{dlog(J^2_{u\Phi})}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{3.2}$$

5. One can make highly non-trivial conclusions about the evolution at general level. For the extremals with vanishing action and for which $\alpha_K$ is critical (vanishing derivate), also the second derivative of $d^2S(S^2)/ds^2 =$ holds true at the critical point. The QFT analogs of these points are points at which beta function develops higher order zero. The tip of cusp catastrophe is second analogy.

   The points at which that the action has minimum are also interesting. For magnetic flux tubes for which one has $S_K(X^4) \propto 1/S$ and $S_{vol} \propto S$ in good approximation, one has $S_K(X^4) =$

$S_{vol}$ at minimum (say for the flux tubes with radius about 1 mm for the cosmological constant in cosmological scales). One can write

$$\frac{dlog(\alpha_K)}{ds} = -\frac{1}{2}\frac{dlog(S(S^2))}{ds} \quad , \tag{3.3}$$

and solve the equation explicitly:

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \quad . \tag{3.4}$$

A more general situation would correspond to a model with $x \neq 1/2$: the deviation from $x = 1/2$ could be interpreted as anomalous dimension. This allows to deduce numerically a formula for the value spectrum of $\alpha_{K,0}/\alpha_K$ apart from the initial values.

6. One can solve the equation also for fixed value of $S(X^4)/Vol(X^4)$ to get

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \quad . \tag{3.5}$$

$$\frac{\alpha_K}{\alpha_{K,0}} = \frac{S_K(X^4)/Vol(X^4)) + S(S^2)}{S_K(X^4)/Vol(X^4))} \quad . \tag{3.6}$$

At the limit $S(S^2) =\rightarrow 0$ one obtains $\alpha_K \rightarrow \alpha_{K,0}$.

7. One should demonstrate that the critical values of $s$ are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter $s$ and the dependence should be such that the continuation to the p-adic sectors is possible.

   A naive guess is that the values of $s$ are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

8. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L4] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have $M^8$ coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different $X^4$ so that the singular points depend on the parameter $s$, which induces dependence of scattering amplitudes on $s$. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

### 3.3.3 Could the critical values of $\alpha_K$ correspond to the zeros of Riemann Zeta?

Number theoretical intuitions strongly suggests that the critical values of $1/\alpha_K$ could somehow correspond to zeros of Riemann Zeta. Riemann zeta is indeed known to be involved with critical systems.

The naivest ad hoc hypothesis is that the values of $1/\alpha_K$ are actually proportional to the non-trivial zeros $s = 1/2 + iy$ of zeta [L1]. A hypothesis more in line with QFT thinking is that they correspond to the imaginary parts of the roots of zeta. In TGD framework however complex values of $\alpha_K$ are possible and highly suggestive. In any case, one can test the hypothesis that the values of $1/\alpha_K$ are proportional to the zeros of $\zeta$ at critical line. Problems indeed emerge.

1. The complexity of the zeros and the non-constancy of their phase implies that the RG equation can hold only for the imaginary part of $s = 1/2 + it$ and therefore only for the imaginary part of the action. This suggests that $1/\alpha_K$ is proportional to $y$. If $1/\alpha_K$ is complex, RG equation implies that its phase RG invariant since the real and imaginary parts would obey the same RG equation.

2. The second - and much deeper - problem is that one has no reason for why $dlog(\alpha_K)/ds$ should vanish at zeros: one should have $dy/ds = 0$ at zeros but since one can choose instead of parameter $s$ any coordinate as evolution parameter, one can choose $s = y$ so that one has $dy/ds = 1$ and criticality condition cannot hold true. Hence it seems that this proposal is unrealistic although it worked qualitatively at numerical level.

It seems that it is better to proceed in a playful spirit by asking whether one could realize quantum criticality in terms of of the property of being zero of zeta.

1. The very fact that zero of zeta is in question should somehow guarantee quantum criticality. Zeros of $\zeta$ define the critical points of the complex analytic function defined by the integral

$$X(s_0, s) = \int_{C_{s_0 \to s}} \zeta(s) ds \ , \tag{3.7}$$

   where $C_{s_0 \to s}$is any curve connecting zeros of $\zeta$, $a$ is complex valued constant. Here $s$ does not refer to $s = sin(\epsilon)$ introduced above but to complex coordinate $s$ of Riemann sphere.

   By analyticity the integral does not depend on the curve $C$ connecting the initial and final points and the derivative $dS_c/ds = \zeta(s)$ vanishes at the endpoints if they correspond to zeros of $\zeta$ so that would have criticality. The value of the integral for a closed contour containing the pole $s = 1$ of $\zeta$ is non-vanishing so that the integral has two values depending on which side of the pole $C$ goes.

2. The first guess is that one can define $S_c$ as complex analytic function $F(X)$ having interpretation as analytic continuation of the $S^2$ part of action identified as $Re(S_c)$:

$$\begin{array}{ll} S_c(S^2) = F(X(s, s_0)) \ , & X(s, s_0) = \int_{C_{s_0 \to s}} \zeta(s) ds \ , \\ S(S^2) = Re(S_c) = Re(F(X)) \ , & \\ \zeta(s) = 0 \ , & Re(s_0) = 1/2 \ . \end{array} \tag{3.8}$$

   $S_c(S^2) = F(X)$ would be a complexified version of the Kähler action for $S^2$. $s_0$ must be at critical line but it is not quite clear whether one should require $\zeta(s_0) = 0$.

   The real valued function $S(S^2)$ would be thus extended to an analytic function $S_c = F(X)$ such that the $S(S^2) = Re(S_c)$ would depend only on the end points of the integration path $C_{s_0 \to s}$. This is geometrically natural. Different integration paths at Riemann sphere would correspond to paths in the moduli space $SO(3)$, whose action defines paths in $S^2$ and are indeed allowed as most general deformations. Therefore the twistor sphere could be identified Riemann sphere at which Riemann zeta is defined. The critical line and real axis would correspond to particular one parameter sub-groups of $SO(3)$ or to more general one parameter subgroups.

   One would have

$$\frac{\alpha_{K,0}}{\alpha_K} = (\frac{S_c}{S_0})^{1/2} \quad . \tag{3.9}$$

The imaginary part of $1/\alpha_K$ (and in some sense also of the action $S_c(S^2)$) would determined by analyticity somewhat like the real parts of the scattering amplitudes are determined by the discontinuities of their imaginary parts.

3. What constraints can one pose on $F$? $F$ must be such that the value range for $F(X)$ is in the value range of $S(S^2)$. The lower limit for $S(S^2)$ is $S(S^2) = 0$ corresponding to $J_{u\Phi} \to 0$.

   The upper limit corresponds to the maximum of $S(S^2)$. If the one Kähler forms of $M^4$ and $S^2$ have same sign, the maximum is $2 \times A$, where $A = 4\pi$ is the area of unit sphere. This is however not the physical case.

   If the Kähler forms of $M^4$ and $S^2$ have opposite signs or if one has $RP$ option, the maximum, call it $S_{max}$, is smaller. Symmetry considerations strongly suggest that the upper limit corresponds to a rotation of $2\pi$ in say $(y, z)$ plane ($s = sin(\epsilon) = 1$ using the previous notation).

   For $s \to s_0$ the value of $S_c$ approaches zero: this limit must correspond to $S(S^2) = 0$ and $J_{u\Phi} \to 0$. For $Im(s) \to \pm\infty$ along the critical line, the behavior of $Re(\zeta)$ (see http://tinyurl.com/y7b88gvg) strongly suggests that $|X| \to \infty$ . This requires that $F$ is an analytic function, which approaches to a finite value at the limit $|X| \to \infty$. Perhaps the simplest elementary function satisfying the saturation constraints is

$$F(X) = S_{max} tanh(-iX) \quad . \tag{3.10}$$

   One has $tanh(x + iy) \to \pm 1$ for $y \to \pm\infty$ implying $F(X) \to \pm S_{max}$ at these limits. More explicitly , one has $tanh(-i/2-y) = [-1+exp(-4y)-2exp(-2y)(cos(1)-1)]/[1+exp(-4y)-2exp(-2y)(cos(1)-1)]$. Since one has $tanh(-i/2 + 0) = 1 - 1/cos(1) < 0$ and $tanh(-i/2 + \infty) = 1$, one must have some finite value $y = y_0 > 0$ for which one has

$$tanh(-\frac{i}{2} + y_0) = 0 \quad . \tag{3.11}$$

   The smallest possible lower bound $s_0$ for the integral defining $X$ would naturally to $s_0 = 1/2 - iy_0$ and would be below the real axis.

4. The interpretation of $S(S^2)$ as a positive definite action requires that the sign of $S(S^2) = Re(F)$ for a given choice of $s_0 = 1/2 + iy_0$ and for a propertly sign of $y - y_0$ at critical line should remain positive. One should show that the sign of $S = a \int Re(\zeta)(s = 1/2 + it)dt$ is same for all zeros of $\zeta$. The graph representing the real and imaginary parts of Riemann zeta along critical line $s = 1/2 + it$ (see http://tinyurl.com/y7b88gvg) shows that both the real and imaginary part oscillate and increase in amplitude. For the first zeros real part stays in good approximation positive but the amplitude for the negative part increase be gradually. This suggests that $S$ identified as integral of real part oscillates but preserves its sign and gradually increases as required.

A priori there is no reason to exclude the trivial zeros of $\zeta$ at $s = -2n$, $n = 1, 2, ....$

1. The natural guess is that the function $F(X)$ is same as for the critical line. The integral defining $X$ would be along real axis and therefore real as also $1/\alpha_K$ provided the sign of $S_c$ is positive: for negative sign for $S_c$ not allowed by the geometric interpretation the square root would give imaginary unit. The graph of the Riemann Zeta at real axis (real) is given in MathWorld Wolfram (see http://tinyurl.com/55qjmj).

2. The functional equation

$$\zeta(1-s) = \zeta(s)\frac{\Gamma(s/2)}{\Gamma((1-s)/2)} \qquad (3.12)$$

allows to deduce information about the behavior of $\zeta$ at negative real axis. $\Gamma((1-s)/2)$ is negative along negative real axis (for $Re(s) \leq 1$ actually) and poles at $n+1/2$. Its negative maxima approach to zero for large negative values of $Re(s)$ (see `http://tinyurl.com/clxv4pz`) whereas $\zeta(s)$ approaches value one for large positive values of $s$ (see `http://tinyurl.com/y7b88gvg`). A cautious guess is that the sign of $\zeta(s)$ for $s \leq 1$ remains negative. If the integral defining the area is defined as integral contour directed from $s<0$ to a point $s_0$ near origin, $S_c$ has positive sign and has a geometric interpretation.

3. The formula for $1/\alpha_K$ would read as $\alpha_{K,0}/\alpha_K(s=-2n) = (S_c/S_0)^{1/2}$ so that $\alpha_K$ would remain real. This integration path could be interpreted as a rotation around z-axis leaving invariant the Kähler form $J$ of $S^2(X^4)$ and therefore also $S = Re(S_c)$. $Im(S_c) = 0$ indeed holds true. For the non-trivial zeros this is not the case and $S = Re(S_c)$ is not invariant.

4. One can wonder whether one could distinguish between Minkowskian and Euclidian and regions in the sense that in Minkowskian regions $1/\alpha_K$ correspond to the non-trivial zeros and in Euclidian regions to trivial zeros along negative real axis. The interpretation as different kind of phases might be appropriate.

What is nice that the hypothesis about equivalence of the geometry based and number theoretic approaches can be killed by just calculating the integral $S$ as function of parameter $s$. The identification of the parameter $s$ appearing in the RG equations is no unique. The identification of the Riemann sphere and twistor sphere could even allow identify the parameter $t$ as imaginary coordinate in complex coordinates in $SO(3)$ rotations around z-axis act as phase multiplication and in which metric has the standard form.

### 3.3.4 Some guesses to be shown to be wrong

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L4], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions $\Delta k$ should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio $c/s$ would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of $c/s$ would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of $dS/ds$. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

## 3.4 An alternative view about the coupling constant evolution in terms of cosmological constant

The above view about the evolution of cosmological constant relies crucially on the identification of $M^4 \times S^2$ as twistor space of $M^4$, and the assumption that the radii of twistor spheres $S^2(M^4$ and $S^2(CP_2)$ assignable to the twistor bundle of $CP_2$ are same.

One can however argue that the standard twistor space $CP_3$ of $M^4$ with Minkowskian signature (3,-3) is a more feasible candidate for the twistor space of $M^4$. Accepting this, one ends up to a modification of the above vision about coupling constant evolution [L22, L23]. The progress in understanding SUSY in TGD framework led also to a dramatic progress in the understanding of the coupling constant evolution [L20].

### 3.4.1 Getting critical about geometric twistor space of $M^4$

Let us first discuss the recent picture and how to modify it so that it is consistent with the hierarchy of CDs. The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of $M^4$ simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of $M^4$ and choices of $M^2$ as preferred complex sub-space of octonions with $S^2$ parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines $M^2$. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of $M^2$ and by the fact that it seems to work.

   **Remark:** $M^8 = M^4 \times E^4$ is complexified to $M^8_c$ by adding a commuting imaginary unit $i$ appearing in the extensions of rationals and ordinary $M^8$ represents its particular sub-space. Also in twistor approach one uses often complexified $M^4$.

2. The objection is that it is ordinary twistor space identifiable as $CP_3$ with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. $CP_3$ has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?

   Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of $M^8$ in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of $M^2$ reducrs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.

3. But what about the space $H$? The first guess is $H = M^4_{conf} \times CP_2$. According to [B1] (see `http://tinyurl.com/y35k5wwo`) one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying $E^3$ to $S^3$ and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$.

   The conformally compactified Minkowski space $M^4_{conf}$ should be analogous to base space of $CP_3$ regarded as bundle with fiber $S^2$. The problem is that one cannot imagine an analog of fiber bundle structure in $CP_3$ having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of $M^4_{conf}$ - call it $cd_{conf}$. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of $M^4_{conf}$. In the case of $CP_2$ one has 3 homologically trivial spheres defining

coordinate patches. This suggests that $cd_{conf}$ is simply $CP_2$ with second complex coordinate made hypercomplex. $M^4$ and $E^4$ differ only by the signature and so would do $cd_{conf}$ and $CP_2$.

The twistor spheres of $CP_3$ associated with points of $M^4$ intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. $CP_3$ has also bundle structure $CP_3 \rightarrow CP_2$. The $S^2$ fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of $S^2$ to each point of $CP_2$.

The $M^4$ points must belong to the interior of cd and this poses constraints on the distance of $M^4$ points from the tips of cd. One expects similar hierarchy of cds at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of $M^4$ and therefore of $M^4_{conf}$. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over $CP_2$: now one would have $CP_2$ bundle over $U(2)$. This is perhaps not an accident, and one can ask whether these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2)\times CP_2$. This would give to metric cross terms between $U(2)$ and $CP_2$.

6. The proposed identification can be tested by looking whether it generalizes. What the twistor space for entire $M^8$ would be? $cd = CD_4$ is replaced with $CD_8$ and the discussion of the preceding chapter demonstrated that the only possible identification of the twistor space is now is as the 12-D hyperbolic variant of $HP_3$ whereas $CD_{8,conf}$ would correspond to 8-D hyperbolic variant of $HP_2$ analogous to hyperbolic variant of $CP_2$.

The outcome of these considerations is surprising.

1. One would have $T(H) = CP_3 \times F$ and $H = CP_{2,H} \times CP_2$ where $CP_{2,H}$ has hyperbolic metric with metric signature $(1, -3)$ having $M^4$ as tangent space so that the earlier picture can be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle projection. Since $M^8 - H$ duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in $M^8$.

2. The hyperbolic variant Kähler form and also spinor connection of hyperbolic $CP_2$ brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to $M^4$ earlier, and suggested to explain the observed CP breaking effects and matter antimatter asymmetry for which there are two explanations [L20].

Some comments about the Minkowskian signature of the hyperbolic counterparts of $CP_3$ and $CP_2$ are in order.

1. Why the metric of $CP_3$ could not be Euclidian just as the metric of $F$? The basic objection is that propagation of fields is not possible in Euclidian signature and one completely loses the earlier picture provided by $M^4 \times CP_2$. The algebraic dynamics in $M^8$ picture can hardly replace it.

2. The map assigning to the point $M^4$ a point of $CP_3$ involves Minkowskian sigma matrices but it seems that the Minkowskian metric of $CP_3$ is not explicitly involved in the construction of scattering amplitudes. Note however that the antisymmetric bi-spinor metric for the spin 1/2 representation of Lorentz group and its conjugate bring in the signature. $U(2,2)$ as representation of conformal symmetries suggests $(2,2)$ signature for 8-D complex twistor space with 2+2 complex coordinates representing twistors.

The signature of $CP_3$ metric is not explicitly visible in the construction of twistor amplitudes but analytic continuations are carried out routinely. One has also complexified $M^4$ and $M^8$

and one could argue that the problems disappear. In the geometric situation the signatures of the subspaces differ dramatically. As already found, analytic continuation could allow to define the variants of twistor spaces elegantly by replacing a complex coordinate with a hyperbolic one.

**Remark**: For $E^4$ $CP_3$ is Euclidian and if one has $E^4_{conf} = U(2)$, one could think of replacing the Cartesian product of twistor spaces with $SU(3)$ group having $M^4_{conf} = U(2)$ as fiber and $CP_2$ as base. The metric of $SU(3)$ appearing as subgroup of quaternionic automorphisms leaving $M^4 \subset M^8$ invariant would decompose to a sum of $M^4_{conf}$ metric and $CP_2$ metric plus cross terms representing correlations between the metrics of $M^4_{conf}$ and $CP_2$. This is probably mere accident.

### 3.4.2 How the vision about coupling constant evolution would be modified?

The above described vision about coupling constant evolution in case of $T(M^4) = M^4 \times S^2$ would be modified since the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$. Now the signatures are opposite and Kähler forms differ by factor $i$ (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres).

1. $M^8 - H$ duality has two alternative forms with $H = CP_{2,h} \times CP_2$ or $H = M^4 \times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or $M^4$. Let us denote the twistor space $SU(3)/U(1) \times U(1)$ of $CP_2$ by $F$.

2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to $M^4$ by $M^8 - H$-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of $CP_2$ as a unit so that $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.

   Coupling constant evolution must correspond to a variation for the ratio of $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda \propto 1/r^2 \propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.

3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. $\Lambda$ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. $\Lambda$ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. $CP_2$ scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order $10^{-4}$ meters defined by the density of dark energy in the recent Universe and thus biological length scale.

4. The above model for the length scale evolution of coupling parameters would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only, and also now the zeros of Riemann zeta could naturally correspond to the quantum critical points.

### 3.4.3 TGD vision about SUSY and coupling constant evolution

TGD view about SUSY leads to radical modification and re-interpretation of SUSY [L22, L20], and to a dramatic progress in the understanding of coupling constant evolution.

Quarks would be the only fundamental fermion fields, and leptons would be spartners of quarks identified as local composites of 3 quarks. Imbedding space coordinates would have an expansion in terms of local super-monomials of quarks and antiquarks with vanishing baryon number and appearing as sums of monomial and its conjugate to guarantee hermiticity. Super-spinors would have similar expansion involving only odd quark numbers. This picture is forced by the requirement that propagators are consistent with the statistics of the spartner. Theta parameters would be replaced by creation and annihilation operators for quarks so that super-symmetrization would mean also second quantization. Number theoretic vision requires that only a finite number of Wick contractions of oscillator operators can vanish. These conditions have interpretation as conservation for the Noether currents of some symmetries.

This picture leads to a concrete view about S-matrix for the preferred extremals of a SUSY-variant of the basic action principle relying on the notion of super-variant of imbedding space and super-variant of the modified Dirac action. Coupling constant evolution discretizes and would reduce to an increase of the finite number of non-vanishing Wick contractions interpreted as radiative corrections as the dimension of the extension of rationals defining the adele increases. This evolution reflects directly the corresponding evolution at the level of $M^8$ in terms of octonionic polynomials determining the extension of rationals involved. Whether this view is consistent with the above general vision remains to be seen.

## 3.5 Generalized conformal symmetry, quantum criticality, catastrophe theory, and analogies with thermodynamics and gauge theories

The notion of quantum criticality allows two realizations: as stationarity of $S^2$ part of the twistor lift of Kähler action and in terms of zeros of zeta are key elements in the explicit proposal for discrete coupling constant evolution reducing to that for cosmological constant.

### 3.5.1 Quantum criticality from different perspectives

Quantum criticality is however much more general notion, and one must ask how this view relates to the earlier picture.

1. At the real number side continuous coupling constant evolution makes sense. What does this mean? Can one say that quantum criticality makes possible only adelic physics together with large $h_{eff}/h_0 = n$ as dimension for extension of rationals. This hierarchy is essential for life and cognition.

   Can one conclude that living systems correspond to quantum critical values of $S(S^2)$ and therefore $\alpha_K$ and in-animate systems correspond to other values of $\alpha_K$? But wouldn't his mean that one gives up the original vision that $\alpha_K$ is analogous to critical temperature. The whole point was that this would make physics unique?

   From mathematical view point also continuous $\alpha_K$ can make sense. $\alpha_K$ can be continuous if it corresponds to a higher-dimensional critical manifold at which two or more preferred extremals associated with the same parameter values co-incide - roots of polynomial $P(x, a, b)$ depending on parameters $a, b$ serves as the canonical example. The degree of quantum criticality would vary and there would be a hierarchy of critical systems characterized by the dimension of the critical manifold. One would have a full analog of statistical physics. For mathematician this is the only convincing interpretation.

   2-D cusp catastrophe serves as a basic example helping to generalize [A2]. Cusp corresponds to the roots of $dP_4/dx = 0$ of third order polynomial $P_4(x, a, b)$, where $(a, b)$ are control variables. The projection of region with 3 real roots to $(a, b)$-plane is bounded by critical lines forming a roughly V-shaped structure. $d^2P_4/dx^2$ vanishes at the edges of V, where two roots co-incide and $d^3P_4/dx^3$ vanishes at the tip of V, where 3 roots co-incide.

2. A hierarchy of quantum criticalities has been actually assumed. The hierarchy of representations for super-symplectic algebra realizing 4-D analog of super-conformal symmetries allows an infinite hierarchy of representations for which infinite-D sub-algebra isomorphic to a full algebra and its commutator with the full algebra annihilate physical states. Also classical Noether charges vanish. What is new is that conformal weights are non-negative integers. The effective dimensions of these systems are finite - at least in the sense that one one has finite-D Lie algebra (or its quantum counterpart) or corresponding Kac-Moody algebra as symmetries. This realization of quantum criticality generalize the idea that conformal symmetry accompanies 2-D criticality.

This picture conforms also with the vision about hierarchy of hyper-finite-factors with included hyper-finite factor defining measurement resolution [K6]. Hyper-finiteness indeed means finite-dimensionality in excellent approximation.

### 3.5.2 TGD as catastrophe theory and quantum criticality as prerequisite for the Euclidian signature of WCW metric

It is good to look more precisely how the catastrophe theoretic setting generalizes to TGD.

1. The value of the twistor lift of Kähler action defining Kähler function very probably corresponds to a maximum of Kähler function since otherwise metric defined by the second derivatives could have non-Euclidian signature. One cannot however exclude the possibility that in complex WCW coordinates the (1,1) restriction of the matrix defined by the second derivatives of Kähler function could be positive definite also for other than minima.

   It would seem that one cannot accept several roots for given zero modes since one cannot have maximum of Kähler function for all of them. This would allow only the boundary of catastrophe region in which 2 or more roots co-incide. Positive definiteness of WCW metric would force quantum criticality.

   For given values of zero modes there would be single minimum and together with the cancellation of Gaussian and metric determinants this makes perturbation theory extremely simple since exponents of vacuum functional would cancel.

2. There is an infinite number of zero modes playing the role of control variables since the value of the induce Kähler form is symplectic invariant and there are also other symplectic invariants associated with the $M^4$ degrees of freedom (carrying also the analog of Kähler form for the twistor lift of TGD and giving rise to CP breaking). One would have catastrophe theory with infinite number of control variables so that the number of catastrophes would be infinite so that standard catastrophe theory does not as such apply.

3. Therefore TGD would not be only a personal professional catastrophe but a catastrophe in much deeper sense. WCW would be a catastrophe surface for the functional gradient of the action defining Kähler function. WCW would consists of regions in which given zero modes would correspond to several minima. The region of zero mode space at which some roots identifiable as space-time surfaces co-incide would be analogous to the V-shaped cusp catastrophe and its higher-D generalizations. The question is whether one allows the entire catastrophe surface or whether one demands quantum criticality in the sense that only the union of singular sets at which roots co-incide is included.

4. For WCW as catastrophe surface the analog of $V$ in the space of zero modes would correspond to a hierarchy of sub-WCWs consisting of preferred extremals satisfying the gauge conditions associated with a sub-algebra of supersymplectic algebra isomorphic to the full algebra. The sub-WCWs in the hierarchy of sub-WCWs within sub-WCWs would satisfy increasingly stronger gauge conditions and having decreasing dimension just like in the case of ordinary catastrophe. The lower the effective dimension, the higher the quantum criticality.

5. In ordinary catastrophe theory the effective number of behavior variables for given catastrophe can be reduced to some minimum number. In TGD framework this would correspond to the reduction of super-symplectic algebra to a finite-D Lie algebra or corresponding Kac-Moody (half-)algebra as modes of supersymplectic algebra with generators labelled by non-negative integer $n$ modulo given integer $m$ are eliminated as dynamical degrees of freedom by the gauge conditions: this would effectively leave only the modes smaller than $m$. The fractal hierarchy of these supersymplectic algebras would correspond to the decomposition of WCW as a catastrophe surface to pieces with varying dimension. The reduction of the effective dimension as two sheets of the catastrophe surface co-incide would mean transformation of some modes contributing to metric to zero modes.

#### 3.5.3 RG invariance implies physical analogy with thermodynamics and gauge theories

One can consider coupling constant evolution and RG invariance from a new perspective based on the minimal surface property.

1. The critical values of Kähler coupling strength would correspond to quantum criticality of the $S^2$ part $S(S^2)$ of 6-D dimensionally reduced Kähler action for fixed values of zero modes. The relative $S^2$ rotation would serve as behavior variable. For its critical values the dimension of the critical manifold would be reduced, and keeping zero modes fixed a critical value of $\alpha_K$ would be selected from 1-D continuum.

   Quantum criticality condition might be fundamental since it implies the constancy of the value of the twistor lift of Kähler action for the space-time surfaces contributing to the scattering amplitudes. This would be crucial for number theoretical vision since the continuation of exponential to p-adic sectors is not possible in the generic case. One should however develop stronger arguments to exclude the continuous evolution of Kähler coupling strength in $S^2$ degrees of freedom for the real sector of the theory.

2. The extremals of twistor lift contain dependence on the rotation parameter for $S^2$ and this must be taken into account in coupling constant evolution along curve of $S^2$ connecting zeros of zeta. This gives additional non-local term to the evolution equations coming from the dependence of the imbedding space coordinates of the preferred extremal on the evolution parameter. The derivative of Kähler action with respect to the evolution parameter is by chain rule proportional to the functional derivatives of action with respect to imbedding space coordinates, and vanish if 4-D Kähler action and volume term are *separately* stationary with respect to variations. Therefore minimal surface property as implied by holomorphy guaranteeing quantum criticality as universality of the dynamics would be crucial in simplifying the equations! It does not matter whether there is coupling between Kähler action and volume term.

Could one find interpretation for the miminal surface property which implies that field equations are separately satisfied for Kähler action and volume term?

1. Quantum TGD can be seen as a "complex" square root of thermodynamics. In thermodynamics one can define several thermodynamical functions. In particular, one can add to energy $E$ as term $pV$ to get enthalpy $H = E + pV$, which remains constant when entropy and pressures are kept constant. Could one do the same now?

   In TGD $V$ replaced with volume action and $p$ would be a coupling parameter analogous to pressure. The simplest replacement would give Kähler action as outcome. The replacement would allow RG invariance of the modified action only at critical points so that replacement would be possible only there. Furthermore, field equations must hold true separately for Kähler action and volume term everywhere.

2. It seems however that one must allow singular sets in which there is interaction between these terms. The coupling between Kähler action and volume term could be non-trivial at singular sub-manifolds, where a transfer of conserved quantities between the two degrees of freedom would take place. The transfer would be proportional to the divergence of the canonical momentum current $D_\alpha(g^{\alpha\beta}\partial_\beta h^k)$ assignable to the minimal surface and is conserved outside the singular sub-manifolds.

   Minimal surfaces provide a non-linear generalization of massless wave-equation for which the gradient of the field equals to conserved current. Therefore the interpretation could be that these singular manifolds are sources of the analogs of fields defined by $M^4$ and $CP_2$ coordinates.

   In electrodynamics these singular manifolds would represented by charged particles. The simplest interpretation would be in terms of point like charges so that one would have line singularity. The natural identification of world line singularities would be as boundaries of string world sheets at the 3-D light-like partonic orbits between Minkowskian and Euclidian regions having induced 4-metric degenerating to 3-D metric would be a natural identification. These world lines indeed appear in twistor diagrams. Also string world sheets must be assumed and they are are natural candidates for the singular manifolds serving as sources of charges in 4-D context. Induced spinor fields would serve as a representation for these sources. These strings would generalize the notion of point like particle. Particles as 3-surfaces would be connected by flux tubes to a tensor network and string world sheets would connected fermion lines at the partonic 2-surfaces to an analogous network. This would be new from the standard model perspective.

Singularities could also correspond to a discrete set of points having an interpretation as cognitive representation for the loci of initial and final states fermions at opposite boundaries of CD and at vertices represented topologically by partonic 2-surfaces at which partonic orbits meet. This interpretation makes sense if one interprets the imbedding space coordinates as analogs of propagators having delta singularities at these points. It is quite possible that also these contributions are present: one would have a hierarchy of delta function singularities associated with string worlds sheets, their boundaries and the ends of the boundaries at boundaries of CD, where string world sheet has edges.

3. There is also an interpretation of singularities suggested by the generalization of conformal invariance. String world sheets would be co-dimension 2 singularites analogous to poles of analytic function of two complex coordinates in 4-D complex space. String world sheets would be co-dimension 2 singularities analogous to poles at light-like 3-surfaces. The ends of the world lines could be analogous of poles of analytic function at partonic 2-surfaces.

   These singularities could provide to evolution equations what might be called matter contribution. This brings in mind evolution equations for $n$-point functions in QFT. The resolution of the overall singularity would decompose to 2-D, 1-D and 0-D parts just like cusp catastrophe. One can ask whether the singularities might allow interpretation as catastrophes.

4. The proposal for analogs of thermodynamical functions suggests the following physical picture. More general thermodynamical functions are possible only at critical points and only if the extremals are miminal surfaces. The singularities should correspond to physical particles, fermions. Suppose that one considers entire scattering amplitude involving action exponential plus possible analog of pV term plus the terms associated with the fermions assigned with the singularities. Suppose that the coupling constant evolution from 6-D Kähler action is calculated *without* including the contribution of the singularities.

   The derivative of $n$-particle amplitude with respect to the evolution parameter contains a term coming from the action exponential receiving contributions only from the singularities and a term coming from the operators at singularities. RG invariance of the scattering amplitude would require that the two terms sum up to zero. In the thermodynamical picture the presence of particles in many particle scattering amplitude would force to add the analog of pressure term to the Kähler function: it would be determined by the zero energy state.

   One can of course ask how general terms can be added by requiring minimal surface property. Minimal surface property reduces to holomorphy, and can be true also for other kinds of actions such as the TGD analogs of electroweak and color actions that I considered originally as possible action candidates.

   This would have interpretation as an analog for YM equations in gauge theories. Space-time singularities as local failure of minimal surface property would correspond to sources for $H$ coordinates as analogs of Maxwell's fields and sources currents would correspond to fermions currents.

## 3.6 TGD view about inclusions of HFFs as a manner to understand coupling constant evolution

The hierarchy of inclusions of HFFs is an alternative TGD inspired proposal for understanding coupling constant evolution and the intuitive expectation is that the inclusion hierarchies of extensions and their Galois groups contain the inclusion hierarchies of HFFs as special case. The included factor would define measurement resolution in well-defined sense. This notion can be formulated more precisely using Connes tensor product [A1, A3].

### 3.6.1 How Galois groups and finite subgroups of could $SU(2)$ relate

The hierarchy of finite groups associated with the inclusions of HFF corresponds to the finite subgroups of $SU(2)$. The set of these groups is very small as compared to the set of Galois groups - if I have understood correctly, any finite group can appear as Galois group. Should the hierarchy of inclusions of HFFs be replaced by much more general inclusion hierarchy? Is there a map projecting Galois groups to discrete subgroup of $SU(2)$?

By $M^8-H$ duality quaternions appear at $M^8$ level and since $SO(3)$ is the automorphism group of quaternions, the discrete subgroups of $SU(2)$ could appear naturally in TGD. In fact, the appearance of quaternions as a basic building brick of HFFs in the simplest construction would fit with this picture.

It would seem that the elements of the discrete subgroups of $SU(2)$ must be in the extension of rationals considered. The elements of finite discrete subgroups $G$ of $SU(2)$ are expressible in terms of rather small subset of extensions of rationals. Could the proper interpretation be that the hierarchy of extensions defines a hierarchy of discrete groups with elements in extension and the finite discrete subgroups in question are finite discrete subgroups of these groups. There would be correlation with the inclusion and extension. For instance, the fractal dimension of extension is an algebraic number defined in terms of root of unity so that the extension must contain this root of unity.

For icosahedron and dodecahedron the group action can be expressed using extension of rationals by $cos(\pi/n)$ and $sin(\pi/n)$ for $n=3,5$. For tetrahedron and cube one would have $n=2,3$. For tetrahedon, cube/octahedron and icosahedron basic geometric parameters are also expressible in terms of algebraic numbers in extension but in case of dodecahedron it is not clear for me whether the surface area proportional to $\sqrt{25+20\sqrt{5}}$ allows this (see `http://tinyurl.com/p4rwc7`).

It is very feasible that the finite sub-groups of also other Lie groups than $SU(2)$ are associated with the inclusions of HFFs or possibly more general algebras. In particular, finite discrete subgroups of color group $SU(3)$ should be important and extension of rationals should allow to represent these subgroups.

### 3.6.2 Once again about ADE correspondence

For a non-mathematician like me Mc-Kay correspondence is an inspiring and frustrating mystery (see `http://tinyurl.com/y8jzvogn`). What could be its physical interpretation?

Mac-Kay correspondence assigns to the extended Dynkin diagrams of ADE type characterizing Kac-Moody algebras finite subgroups of $SU(2)$, more precisely the McKay diagrams describing the tensor product decomposition rules for the fundamental representation of the finite subgroup of $SU(2)$. In the diagram irreps $\chi_i$ and $\chi_j$ are connected by $n_{ij}$ arrows if $\chi_j$ appears $n_{ij}$ times in the tensor product $V\otimes\chi_i$, where $V$ is but need not be fundamental representation.

One can assign also to inclusions of HFFs of index $d\geq 4$ with ADE type Dynkin diagrams. To inclusions with index $d<4$ one can assign subset of ADE type diagrams for Lie groups (rather than Kac-Moody groups) and they correspond to sub-groups of $SU(2)$. The correspondence generalizes to subgroups of other Lie groups.

1. As explained in [B3] , for $\mathcal{M}:\mathcal{N}<4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra $g$ with $h$ equal to the Coxeter number $h$ of the Lie algebra given in terms of its dimension and dimension $r$ of Cartan algebra $r$ as $h=(dim(g)-r)/r$. For $\mathcal{M}:\mathcal{N}<4$ ordinary Dynkin graphs of $D_{2n}$ and $E_6,E_8$ are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, $E_7$ and $D_{2n+1}$ are however not allowed. $E_6,E_7$, and $E_8$ correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M}:\mathcal{N}=4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A3] is following.

The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: $A_\infty$ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of $A_n$ for cyclic groups, of $D_n$ dihedral groups, and of $E_n$ with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

The extension is constructed by constructing first factor $R$ as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor $R_0$ consists elements of of $R$ of form $Id \otimes x$. $SU(2)$ preserves $R_0$ and for any subgroup $G$ of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of $R_0$ and $R$ under the action of $G$. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup $G$.

Physicist might try to interpret this by saying that one considers only sub-algebras $R_0^G$ and $R^G$ of observables invariant under $G$ and obtains extended Dynkin diagram of $G$ defining an ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under $R_0$ defining measurement resolution. Besides this the states are also invariant under finite group $G$? Could $R_0^G$ and $R^G$ correspond just to states which are also invariant under finite group $G$.

Could this kind of inclusions generalize so that Galois groups would replace $G$. If this is possible it would assign to each Galois group an inclusion of HFFs and give a precise number theoretic formulation for the notion of measurement accuracy.

2. At $M^8$-side of $M^8 - H$ duality the construction of space-time surfaces reduces to data at finite set of points of space-time surface since they are defined by an octonionic extension of a polynomial of real variable with coefficients in extension of rationals. Space-time surfaces would have quaternionic tangent space or normal space. The coordinates of quaternions are restricted to extension of rationals and the subgroup of automorphisms reduce to a subgroup for which matrix elements belong to an extension of rationals.

   If the subgroup is finite, only the subgroups appearing in ADE correspondence are possible and the extension must be such that it allows the representation of this group. Does this mean that the extension can is obtained from an extension allowing this representation? For $\mathcal{M} : \mathcal{N} = 4$ case this sub-group would leave the states invariant.

## 3.7 Entanglement and adelic physics

In the discussion about fine structure constant I asked about the role entanglement in coupling constant evolution. Although entanglement does not have direct relationship to coupling constant evolution, I will discuss entanglement from number theoretic point of view since it enlightens the motivations of adelic physics.

1. For given extension of rationals determining the values of coupling parameters by quantum criticality, the entanglement coefficients between positive and negative energy parts of zero energy states are in the extension of rationals. All entanglement coefficients satisfy this condition.

2. Self the counterpart of observer in the generalization of quantum measurement theory - as conscious entity [L6] corresponds to sequence of unitary evolutions followed by weak measurements. The rule for weak measurements is that only state function for which the eigenvalues of the density matrix is in the extension of rationals can occur. In general they are in a higher-D extension as roots of $N$:th order polynomials, $N$ the dimension of density matrix. Therefore state function reduction cannot occur in the generic case. State cannot decohere and entanglement is stable under weak measurements except in special situations when the eigenvalues of density matrix are in original extension.

3. The extension can change only in big state function reductions in which the arrow of clock time changes: this can be seen as an evolutionary step. From the point of view of consciousness theory big state function reduction means what might be called death and reincarnation of system in opposite time direction.

4. The number theoretical stabilization of entanglement at the passive boundary of CD makes possibility quantum computation in longer time scales than possible in standard quantum theory. $h_{eff}/h_0 = n$ equals to the dimension of extension of rationals and is therefore directly related to this.

This could have profound technological implications.

1. Ordinary quantum computation as single unitary step is replaced by a sequence of them followed by the analog of weak measurement.

2. ZEO allows also quantum computations in opposite time direction. This might allow shorten dramatically the duration of quantum computations from the perspective of the observed since most of the computation could be done with opposite arrow of clock time.

   The philosophy of adelic physics is discussed in article in book published by Springer [L5, L4] (see `http://tinyurl.com/ybzkfevz` and `http://tinyurl.com/ybqpkwg9`).

# 4 Trying to understand why ramified primes are so special physically

Ramified primes (see `http://tinyurl.com/m32nvcz` and `http://tinyurl.com/y6yskkas`) are special in the sense that their expression as a product of primes of extension contains higher than first powers and the number of primes of extension is smaller than the maximal number $n$ defined by the dimension of the extension. The proposed interpretation of ramified primes is as p-adic primes characterizing space-time sheets assignable to elementary particles and even more general systems.

In the following Dedekind zeta functions (see `http://tinyurl.com/y5grktvp`) as generalization of Riemann zeta [L10, L13] are studied to understand what makes them so special. Dedekind zeta function characterizes given extension of rationals and is defined by reducing the contribution from ramified reduced so that effectively powers of primes of extension are replaced with first powers.

If one uses the naive definition of zeta as analog of partition function and includes full powers $P_i^{e_i}$, the zeta function becomes a product of Dedekind zeta and a term consisting of a finite number of factors having poles at imaginary axis. This happens for zeta function and its fermionic analog having zeros along imaginary axis. The poles would naturally relate to Ramond and N-S boundary conditions of radial partial waves at light-like boundary of causal diamond CD. The additional factor could code for the physics associated with the ramified primes.

The intuitive feeling is that quantum criticality is what makes ramified primes so special. In $O(p) = 0$ approximation the irreducible polynomial defining the extension of rationals indeed reduces to a polynomial in finite field $F_p$ and has multiple roots for ramified prime, and one can deduce a concrete geometric interpretation for ramification as quantum criticality using $M^8 - H$ duality.

$M^8 - H$ duality central concept in following and discussed in [L3, L17, L14, L15] [L22]. Also the notion of cognitive representation as a set of points of space-time surface with preferred imbedding space coordinates belonging to the extension of rationals defining the adele [L4] is important and discussed in [L19, L18, L21].

## 4.1 Dedekind zeta function and ramified primes

One can take mathematics and physical intuition guided by each other as a guideline in the attempts to understand ramified primes.

1. Riemann zeta can be generalized to Dedekind zeta function $\zeta_K$ for any extension $K$ of rationals (see `http://tinyurl.com/y5grktvp`). $\zeta_K$ characterizes the extension - maybe also physically in TGD framework since zeta functions have formal interpretation as partition function. In the recent case the complexity is not a problem since complex square roots of partition functions would define the vacuum part of quantum state: one can say that quantum TGD is complex square root of thermodynamics.

   $\zeta_K$ satisfies the same formula as ordinary zeta expect that one considers algebraic integers in the extensions $K$ and sums over non-zero ideals $a$ - identifiable as integers in the case of rationals - with $n^{-s}$ replaced with $N(a)^{-s}$, where $N(a)$ denotes the norm of the non-zero ideal. The construction of $\zeta_K$ in the extension of rationals obtained by adding $i$ serves as an illustrative example (see `http://tinyurl.com/y563wcwv`). I am not a number theorists but the construction suggests a poor man's generalization strongly based on physical intuition.

2. The rules would be analogous to those used in the construction of partition function. $log(N(a))$ is analogous to energy and $s$ is analogous to inverse temperature so that one has Boltzmann weight $exp(-log(N(a)s)$ for each ideal. Since the formation of ideals defined by integers of extension is analogous to that for forming many particle states labelled by ordinary primes and decomposing to primes of extension, the partition function decomposes to a product over partition functions assignable to ordinary primes just like in the case of Riemann zeta. Let $K$ be an extension of rationals $Q$.

3. Each rational prime $p$ decomposes in the extension as $p = \prod_{i=1,...g} P_i^{e_i}$, where $n$ is the dimension of extension and $e_i$ is the ramification degree. Let $f_i$ be so called residue degree of $P_i$ defined as the dimension of $K$ mod $P_i$ interpreted as extension of rational integers $Z \ mod \ p$. Then one has $\sum_1^g e_i f_i = n$.

   **Remark**: For Galois extensions for which the order of Galois group equals to the dimension $n$ of the extension so that for given prime $p$ one has $e_i = e$ and $f_i = f$ and $efg = n$.

4. Rational (and also more general) primes can be divided into 3 classes with respect to this decomposition.

   For ramified primes dividing the discriminant $D$ associated with the polynomial ($D = b^2 - 4c$ for $P(x) = x^2 + bx + c$) one has $e_i > 1$ at least for one $i$ so that $f_i = 0$ is true at least for one index. A simple example is provided by rational primes (determined by roots of $P(x) = x^2 + 1$ with discriminant $-4$): in this case $p = 2$ corresponds to ramified prime since on has $(1 + i)(1 - i) = 2$ and $1 + i$ and $1 - i$ differ only by multiplication by unit $-i$.

5. Split primes have $n$ factors $P_i$ and thus have $(e_i = 1, f_i = 1, g = n)$ . They give a factor $(1 - p^{-s})^{-n}$. The physical analogy is $n$-fold degenerate state with original energy energy $nlog(p)$ split to states with energy $log(p)$.

   Inert primes are also primes of extension and there is no splitting and one has $(e_1 = e = 1, g = 1, f_1 = f = n)$. In this case one obtains factor $1/(1 - p^{-ns})$. The physical analogy is n-particle bound state with energy $nlog(p)$.

6. For ramified primes the situation is more delicate. Generalizing from the case of Gaussian primes $Q[i]$ (see `http://tinyurl.com/y563wcwv`) ramified primes $p_R$ would give rise to a factor

$$\prod_{i=1}^{g} \frac{1}{1 - p_R^{-f_i s}} \quad .$$

   $g$ is the number of *distinct* ideals $P_i$ in the decomposition of $p$ to the primes of extension.

   For Gaussian primes $p = 2$ has $g = 1$ since one can write $(2) = (1+i)(1-i) \equiv (1+i)^2$. This because $1 + i$ and $1 - i$ differ only by multiplication with unit $-i$ and thus define same ideal in $Q[i]$. Only the number $g$ of distinct factors $P_i$ in the decomposition of $p$ matters.

   One could understand this as follows. For the roots of polynomials ramification means that several roots co-incide so that the number of distinct roots is reduced. $e_i > 1$ is analogous to the number co-inciding roots so that number if distinct roots would be 1 instead of $e_i$. This would suggests $k_i = 1$ always. For ramified primes the number of factors $Z_p$ the number $\sum_{i=1}^{g} f_i k_i = n$ for un-ramified case would reduce from to $\sum_{i=1}^{g} f_i k_i = n_d$, which is the number of distinct roots.

7. Could the physical interpretation be that there are $g$ types of bound states with energies $f_i log(p)$ appearing with degeneracy $e_i = 1$ both in ramified and split case. This should relate to the fact that for ramified primes $p$ $L/p$ contains non-vanishing nilpotent element and is not counted. One can also say that the decomposition to primes of extension conserves energy: $\sum_{i=1,...,g} e_i f_i log(p) = n_d log(p)$.

   For instance, for Galois extensions ($e_i = e, f_i = f, g = n_d/ef$) for given $p$ the factor is $1/(1-p^{-es})^{fg}$: $efg = n_d$. If there is a ramification then all $P_i$ are ramified. Note that $e, f$ an $g$ are factors of $n_d$.

8. One can can extract the factor $1/(1-p^{-s})$ from each of the 3 contributions and organize these factors to give the ordinary Riemann zeta. The number of ramified primes is finite whereas the numbers of split primes and inert primes are infinite. One can therefore extract from ramified primes the finite product

$$\zeta^1_{R,K} = \prod_{p_R} (1-p_R^{-s}) \times \zeta^2_{R,K} \ , \quad \zeta^2_{R,K} = \prod_{p_R} [\prod_{i=1}^{g} \frac{1}{1-p^{-f_i s}}] \ .$$

   One can organize the remaining part involving infinite number of factors to a product of $\zeta$ and factors $(1-p^{-s})/(1-\prod p^{-s}))^n$ and $(1-p^{-s})/(1-p^{-ns})$ giving rise to zeta function -call it $\zeta_{si,K}$ - characterizing the extension. Note that $\zeta^2_{R,K}$ has interpretation as partition function and has pole of order $n_d$ at origin.

   One therefore can write the $\zeta_L$ as

$$\zeta_K = \zeta^1_{R,K} \times \zeta_{si,K} \times \zeta \ .$$

   where $\zeta_{si,K}$ is the contribution of split and inert primes multiplied by $(1-p^{-s})$

$\zeta_L$ has pole only at $s = 1$ and it carries in no obvious manner information about ramified primes. The naive guess for $\zeta_L$ would be that also ramified primes $p_R$ would give rise to a factor

$$\prod_{i=1}^{g} \frac{1}{(1-p_R^{-f_i s})^{e_i}} \ .$$

One could indeed argue that at the limit when $e_i$ prime ideals $P_i$ of extension co-incide, one should obtain this expression. The resulting $\zeta$ function would be product

$$\zeta_{naive,K} = \zeta_{R,K}\zeta_K \ , \qquad \zeta_{R,K} = \prod_{p_R} X(p_R)$$

$$X(p_R) = \prod_{i=1}^{g} \frac{1}{(1-p_R^{-f_i s})^{e_i-1}} \ .$$

Note that the parameters $e_i, f_i, g$ depend on $p_R$ and that for Galois extensions one has $e_i = d, f_i = f$ for given $p_R$. $\zeta_{R,L}$ would have poles at along imaginary axis at points $s = -n2\pi/log(p)$. Ramified primes would give rise to poles along imaginary axis. As far as the proposed physical interpretation of ramified primes is considered, this form looks more natural.

### 4.1.1 Fermionic counterparts of Dedekind zeta and ramified $\zeta$

One can look the situation also for the generalization of fermionic zeta as analog of fermionic partition function, which for rationals has the expression

$$\zeta^F(s) = \prod_p (1+p^{-s}) = \frac{\zeta(s)}{\zeta(2s)} \ .$$

Supersymmetry of supersymmetric arithmetic QFT suggest the product of fermionic and bosonic zetas. Also the supersymmetry of infinite primes for which first level of hierarchy corresponds to irreducible polynomials suggests this. On the other hand, the appearance of only fermions as fundamental particles in TGD forces to ask whether the ramified part of fermionic zeta might be fundamental.

1. By an argument similar to used for ordinary zeta based on interpretation as partition function, one obtains the decomposition of the fermionic counterpart of $\zeta_K^F$ Dirichlet zeta to a product $\zeta_K^F = \zeta_{R,K}^F \zeta_{si,K}^F \zeta^F$ of ramified fermionic zeta $\zeta_{R,K}^F$, $\zeta_{si,K}^F$, and ordinary fermionic zeta $\zeta^F$. The basic rule is simple: replace factors $1/(1-p^{-ks}$ appearing in $\zeta_K$ with $(1+p^{-ks})$ in $\zeta_K^F$ and extract $\zeta^F$ from the resulting expression. This gives

$$\zeta_{R,K}^{F,1} = \prod_{p_R}(1-p_R^{-s})\zeta_{R,K}^F \ , \quad \zeta_{R,K}^F = \prod_{p_R}[\prod_{i=1}^g (1+p_R^{-f_i s})] \ .$$

where $p_R$ is ramified prime dividing the discriminant. $\zeta_{R,K}^F$ is analogous to a fermionic partition function for a finite number of modes defined by ramified primes $p_R$ of extension.

2. Also now one can wonder whether one should define $\zeta_K^F$ as a product in which ramified primes give factor

$$\prod_{p_R}[\prod_{i=1}^g (1+p_R^{-f_i s})^{e_i}]$$

so that one would have

$$\zeta_{naive,K}^F = \zeta_{R,K}^F \zeta_K^F \ , \qquad \zeta_R^F = \prod_{p_R} Y(p_R) \ ,$$

$$Y(p_R) = \prod_{i=1}^g (1+p_R^{-f_i s})^{e_i - 1}$$

$\zeta_F(naive, K)$ would have zeros along imaginary axis serving as signature of the ramified primes.

### 4.1.2 About physical interpretation of $\zeta_{R,K}$ and $\zeta_{R,K}^F$

$\zeta_{R,K}$ and $\zeta_{R,K}^F$ are attractive from the view point of number theoretic vision and the idea that ramified primes are physically special. TGD Universe is quantum critical and in catastrophe theory the ramification for roots of polynomials is analogous to criticality. Maybe the ramification for p-adic primes makes them critical. $K/(p_R)$ has nilpotent elements, which brings in mind on mass shell massless particles.

1. $\zeta_{R,K}$ has poles at

$$s = i\frac{2n\pi}{log(p)f_i}$$

and $p_R^s = exp(in2\pi/f_i)$ is a root of unity, which conforms with the number theoretical vision. Only $P_i$ with $e_i > 1$ contribute.

2. $Z_{R,K}^F$ has zeros

$$s = i\frac{(2n+1)\pi}{log(p)f_i}$$

and $p_R^s = exp(i(2n+1)\pi/f_i)$ is a root of unity. Zeros are distinct from the poles of $Z_{R,K}$.

3. The product $\zeta_{R,tot,K} = \zeta_{R,K}\zeta_{R,K}^F$ has the poles and zeros of $\zeta_{R,K}$ and $\zeta_{R,K}^F$. In particular, there is $n$:th order pole of $Z_{R,K}$ at $s=0$. The zeros of $z_{F,K}$ along imaginary axis at $p^{iy} = -1$ also survive in $\zeta_{R,tot,K}$.

   $\zeta_{R,K}^F$ has only zeros and since fundamental fermions are primary fields in TGD framework, one could argue that only it carries physical information. On the other hand, supersymmetric arithmetic QFT [K5] and the fact that TGD allows SUSY [L20] suggests that the product $\zeta_{R,K} \times Z_{R,K}^F$ is more interesting.

From TGD point of view the ramified zeta functions $\zeta_{R,K}$, $\zeta_{R,K}^F$ and their product $\zeta_{R,K} \times \zeta_{R,K}^F$ look interesting.

1. $\zeta_{R,K}$ behaves like $s^{-n_d}$, $n_d = \sum_1^g (e_i - 1)$ near the origin. Could $n_d$-fold pole at $s = 0$ be interpreted in terms of a massless state propagating along light-cone boundary of CD in radial direction? This would conform with the proposal that zeros of zeta correspond to complex radial conformal weights for super-symplecti algebra. That ramified primes correspond to massless particles would conform with the identification of ramified prime as p-adic primes labelling elementary particles since in ZEO their mass would result from p-adic thermodynamics from a mixing with very massive states [L15].

   Besides this there would be stringy spectrum of real conformal weights along negative real axis and those coming as non-trivial zeros and these could correspond to ordinary conformal weights.

2. The zeros of $\zeta_{R,K}^F$ along imaginary axis might have interpretation as eigenvalues of Hamiltonian in analogy with Hilbert-Polya hypothesis. Maybe also the poles of $\zeta_{R,K}$ could have similar interpretation. The real part of zero/pole would not produce troubles (on the other hand, for waves along light-cone boundary it can be however absorbed to the integration measure.

3. A possible physical interpretation of the imaginary conformal weights could be as conformal weights associated with radial waves assignable to the radial light-like coordinate $r$ of the light-cone boundary: $r$ indeed plays the role of complex coordinate in conformal symmetry in the case of super-symplectic algebra suggested to define the isometries of WCW. Poles and zero could correspond to radial modes satisfying periodic/anti-periodic boundary conditions.

   The radial conformal weights $s$ defined by the zeros of $\zeta_{R,K}^F$ would be number theoretically natural since one could pose boundary condition $p^{is(r/r_0)} = -1$ at $r = r_0$ requiring $p^{is} = -1$ at the corner of cd (maximum value of $r$ in $CD = cd \times CP_2$.

   For the poles of $\zeta_{R,K}$ the periodic boundary condition $p^{is(r/r_0)} = 1$ is natural. The two boundary conditions could relate to Ramond an N-S representations of super-conformal algebras (see `http://tinyurl.com/y49y2ouj`). With this interpretation $s = 0$ would correspond to a radial plane-wave constant along light-like radial direction and therefore light-like momentum propagating along the boundary of CD. Other modes would correspond to other massless modes propagating to the interior of CD.

4. I have earlier considered an analogous interpretation for a subset zeros of zeta satisfying similar condition. The idea was that for given prime $p$ as subset of $s = 1/2 + iy_i$ of non-trivial zeros $\zeta$ $p^s = p^{1/2+iy_i}$ is an algebraic number so that $p^{iy_i}$ would be a root of unity. Zeros would decompose to subsets labelled by primes $p$. Also for trivial zeros of $\zeta$ (and also poles) the same holds true as for the zeros and poles $\zeta_R$. This encourages the conjecture that the property is true also for L-functions.

The proposed picture suggests an assignment of "energy" $E = nlog(p)$ to each prime and separation of "ramified" energy $E_d = n_d log(p)$, $n_d = \sum_1^g f_i(e_i - 1)$, to each ramified prime. The interpretation as partition function suggests that that one has $g$ types of states of $f_i$ identical particles and energy $E_i = f_i log(p)$ and that this state is $e_i$-fold degenerate with energies $E_i = f_i log(p)$. For inert primes one would have $f_i = f = n$. For split primes one would have $e_i = 1, f_i = 1$. In case of ramified primes one can separate one of these states and include it to the Dedekind zeta.

### 4.1.3 Can one find a geometric correlate for the picture based on prime ideals?

If one could find a geometric space-time correlate for the decomposition of rational prime ideals to prime ideals of extensions, it might be also possible to understand why quantum criticality makes ramified primes so special physically and wha this means.

What could be correlate for $f_i$ fundamental fermions behaving like single unit and what degeneracy for $e_i > 1$ does mean? One can look the situation first at the level of number fields $Q$

and $K$ and corresponding Galois group $Gal(K/Q)$, finite fields $F = Q/p$ and $F_i = K/P_i$, and corresponding Galois group $Gal(F_i/F)$. Appendix summarizes the basic terminology.

1. Inertia degree $f_i$ is the number of elements of $F_i/F_p$ ($F_i = K/P_i$ is extension of finite field $F_p = Q/p$). The Galois group $Gal(F_i/F_p)$ is identifiable as factor group $D_i/I_i$, where the *decomposition group* $D_i$ is the subgroup of Galois group taking $P_i$ to itself and the *inertia group* $I_i$ leaving $P_i$ point-wise invariant. The orbit under $Gal(F_i/F_p)$ in $F_i/F_p$ would behave like single particle with energy $E_i = f_i log(p)$.

   For inert primes with $f_i = n$ inertia group would be maximal. For split primes the orbits of ideals would consist of $f_i = 1$ points only and isotropy group would be trivial.

2. Ramification for primes corresponds intuitively to that for polynomials meaning multiple roots as is clear also from the expression $p = \prod P_i^{e_i}$. In accordance with the intuition about quantum criticality, ramification means that the irreducible polynomial reduced to a reducible polynomial in finite field $Q/p$ has therefore a multiple roots with multiplicities $e_i$ (see Appendix). For Galois extensions one has ($e_i = e, f_i = f$) Criticality would be seen at the level of finite field $F_p = Q/p$ associated with ramified prime $p$.

The interpretation of roots of corresponding octonionic polynomials as $n$-sheeted covering space like structures encourages to ask whether the independent tensor factors labelled by $i$ suggested by the interpretation as a partition function could be assigned with the sheets of covering so that ramification with $e_i > 1$ would correspond to singular points of cognitive representation for which $e_i$ sheets co-incide in some sense, maybe in finite field approximation ($O(p) = 0$). Galois groups indeed act on the coordinates of point of cognitive representation belonging to the extension $K$. In general the action does not take the point to a point belonging to a cognitive representation but one can consider quantum superpositions of cognitive representations.

This suggests an interpretation in terms of space-time surfaces accompanied by cognitive representation under Galois group. Quantum states would be superpositions of preferred extremals at orbits of Galois group and for cognitive representations the situation would be discrete.

1. To build a concrete connection between geometric space-time picture and number theoretic picture, one should find geometric counterparts of integers, ideals, and prime ideals. The analogs of prime ideals should be associated with the discretizations of space-time surfaces/cognitive representations in $O(p) = 0$ or $O(P_i) = 0$ approximation. Could one include only points of cognitive representations differing from zero in $O(p) = 0$ approximation and form quantum states as quantum superpositions of these points of cognitive representation?

   in $O(p) = 0$ approximation and for ramified primes irreducible polynomials would have multiple roots so that $e_i$ sheets would co-incide at these points in $O(p) = 0$ approximation. Th conjecture that elementary particles correspond to this kind of singularities has been speculated already earlier with inspiration coming from quantum criticality.

2. In $M^8$ picture the octonionic polynomials obtained as continuation of polynomials with rational coefficients would be reduced to polynomials in finite field $F_p$. One can study corresponding discrete algebraic surfaces as discrete approximations of space-time surfaces.

3. One would like to have only single imbedding space coordinate since the probability that all imbedding space coordinates correspond to the same $P_i$ is small. $M^8 - H$ duality reduces the number of imbedding space coordinates characterizing partonic 2-surfaces containing vertices for fundamental fermions to single one identifiable as time coordinate.

   At the light-like boundary of 8-D CD in $M^8$ the vanishing condition for the real or imaginary part (quaternion) of octonionic polynomial $P(o)$ reduces to that for ordinary polynomial, and one obtains $n$ roots $r_n$, which correspond to the values of $M^4$ time $t = r_n$ defining 6-spheres as analogs of branes. Partonic 2-surfaces corresponde to intersections of 4-D roots of $P(o)$ at partonic 2-surfaces. Galois group of the polynomial naturally acts on $r_n$ labelling these partonic 2-surfaces by permuting them. One could form representations of Galois group using states identified as quantum superpositions of these partonic 2-surfaces corresponding to different values of $t = r_n$. Galois group leaves invariant the degenerate roots $t = r_n$.

4. The roots can be reduced to finite field $F_p$ or $K/P_i$. Ramification would take place in this approximation and mean that $e_i$ roots $t = r_n$ are identical in $O(p) = 0$ approximation. $e_i$ time values $t = r_n$ would nearly co-incide. This gives more concrete contents to the statement of TGD inspired theory of consciousness that these time values correspond to very special moments in the life of self. Since this is the situation only approximately, one can argue that one must indeed count each root separately so that partition function must be defined as product of the contribution form ramified primes an Dedekind zeta.

   The assignment of fundamental fermions to the points of cognitive representations at partonic 2-surfaces assignable to the intersections of 4-D roots and universal 6-D roots of octonionic polynomials (brane like entities) conforms with this picture.

5. The analogs of 6-branes would give rise to additional degrees of freedom meaning effectively discrete non-determinism. I have speculated with this determinism with inspiration coming from the original identification of bosonic action as Kähler action having huge 4-D spin glass degeneracy. Also the number theoretic vision suggest the possibility of interpreting preferred extremals as analogs of algebraic computations such that one can have several computations connecting given states [L2]. The degree $n$ of polynomial would determine the number of steps and the degeneracy would correspond to $n$-fold degeneracy due to the discrete analogs of plane waves in this set.

### 4.1.4 What extensions of rationals could be winners in the fight for survival?

It would seem that the fight for survival is between extensions of rationals rather than individual primes $p$. Intuition suggests that survivors tend to have maximal number of ramified primes. These number theoretical speciei can live in the same extension - to "co-operate".

Before starting one must clarify some basic facts about extensions of rationals.

1. Extension of rationals are defined by an irreducible polynomial with rational coefficients. The roots give $n$ algebraic numbers which can be used as a basis to generate the numbers of extension ast their rational linear combinations. Any number of extension can be expressed as a root of an irreducible polynomial. Physically it is is of interest, that in octonionic picture infinite number of octonionic polynomials gives rise to space-time surface corresponding to the same extension of rationals.

2. One can define the notion of integer for extension. A precise definition identifies the integers as ideals. Any integer of extension are defined as a root of a monic polynomials $P(x) = x^n + p_{n-1}x^{n-1}x^{n-1} + ... + p_0$ with integer coefficients. In octonionic monic polynomials are subset of octonionic polynomials and it is not clear whether these polynomials could be all that is needed.

3. By definition ramified primes divide the discriminant $D$ of the extension defined as the product $D = \prod_{i\neq j}(r_i - r_j)$ of differences of the roots of (irreducible) monic polynomial with integer coefficients defining the basis for the integers of extension. Discriminant has a geometric interpretation as volume squared for the fundamental domain of the lattice of integers of the extension so that at criticality this volume interpreted as p-adic number would become small for ramified primes an vanish in $O(p)$ approximation. The extension is defined by a polynomial with rational coefficients and integers of extension are defined by monic polynomials with roots in the extension: this is not of course true for all monic polynomials polynomial (see `http://tinyurl.com/k3ujjz7`).

4. The scaling of the $n-1$-tuple of coefficients $(p_{n-1}, ....., p_1)$ to $(ap_{n-1}, a^2p_{n-1}....., a^np_0)$ scales the roots by $a$: $x_n \rightarrow ax_n$. If $a$ is rational, the extension of rationals is not affected. In the case of monic polynomials this is true for integers $k$. This gives rational multiples of given root.

   One can decompose the parameter space for monic polynomials to subsets invariant under scalings by rational $k \neq 0$. Given subset can be labelled by a subset with vanishing coefficients $\{p_{i_k}\}$. One can get rid of this degeneracy by fixing the first non-vanishing $p_{n-k}$ to a non-vanishing value, say 1. When the first non-vanishing $p_k$ differs from $p_0$, integers label the polynomials giving rise to roots in the same extension. If only $p_0$ is non-vanishing, only the scaling by powers $k^n$ give rise to new polynomials and the number of polynomials giving rise to same extension is smaller than in other cases.

**Remark:** For octonionic polynomials the scaling symmetry changes the space-time surface so that for generic polynomials the number of space-time surfaces giving rise to fixed extension is larger than for the special kind polynomials.

Could one gain some understanding about ramified primes by starting from quantum criticality? The following argument is poor man's argument and I can only hope that my modest technical understanding of number theory does not spoil it.

1. The basic idea is that for ramified primes the minimal monic polynomial with integer coefficients defining the basis for the integers of extension has multiple roots in $O(p) = 0$ approximation, when $p$ is ramified prime dividing the discriminant of the monic polynomial. Multiple roots in $O(p) = 0$ approximation occur also for the irreducible polynomial defining the extension of rationals. This would correspond approximate quantum criticality in some p-adic sectors of adelic physics.

2. When 2 roots for an irreducible rational polynomial co-incide, the criticality is exact: this is true for polynomials of rationals, reals, and all p-adic number fields. One could use this property to construct polynomials with given primes as ramified primes. Assume that the extension allows an irreducible olynomial having decomposition into a product of monomials $= x - r_i$ associated with roots and two roots $r_1$ and $r_2$ are identical: $r_1 = r_2$ so that irreducibility is lost.

   The deformation of the degenerate roots of an irreducible polynomial giving rise to the extension of rationals in an analogous manner gives rise to a degeneracy in $O(p) = 0$ approximation. The degenerate root $r_1 = r_2$ can be scaled in such a manner that the deformation $r_2 = r_1(1+q))$, $q = m/n = O(p)$ is small also in real sense by selecting $n >> m$.

   If the polynomial with rational coefficients gives rise to degenerate roots, same must happen also for monic polynomials. Deform the monic polynomial by changing $(r_1, r_2 = r_1)$ to $(r_1, r_1(1+r))$, where integer $r$ has decomposition $r = \prod_i p_i^{k_i}$ to powers of prime. In $O(p) = 0$ approximation the roots $r_1$ and $r_2$ of the monic polynomial are still degenerate so that $p_i$ represent ramified primes.

   If the number of $p_i$ is large, one has high degree of ramification perhaps favored by p-adic evolution as increase of number theoretic co-operation. On the other hand, large p-adic primes are expected to correspond to high evolutionary level. Is there a competition between large ramified primes and number of ramified primes? Large $h_{eff}/h_0 = n$ in turn favors large dimension $n$ for extension.

3. The condition that two roots of a polynomial co-incide means that both polynomial $P(x)$ and its derivative $dP/dx$ vanish at the roots. Polynomial $P(x) = x^n + p_{n-1}x^{n-1} + ..p_0$ is parameterized by the coefficients which are rationals (integers) for irreducible (monic) polynomials. $n-1$-tuple of coefficients $(p_{n-1}, ....., p_0)$ defines parameter space for the polynomials. The criticality condition holds true at integer points $n - 1 - D$ surface of this parameter space analogous to cognitive representation.

   The condition that critical points correspond to rational (integer) values of parameters gives an additional condition selecting from the boundary a discrete set of points allowing ramification. Therefore there are strong conditions on the occurrence of ramification and only very special monic polynomials are selected.

   This suggests octonionic polynomials with rational or even integer coefficients, define strongly critical surfaces, whose p-adic deformations define p-adically critical surfaces defining an extension with ramified primes $p$. The condition that the number of rational critical points is non-vanishing or even large could be one prerequisite for number theoretical fitness.

4. There is a connection to catastrophe theory, where criticality defines the boundary of the region of the parameter space in which discontinuous catastrophic change can take place as

replacement of roots of $P(x)$ with different root. Catastrophe theory involves polynomials $P(x)$ and their roots as well as criticality. Cusp catastrophe is the simplest non-trivial example of catastrophe surface with $P(x) = x^4/4 - ax - bx^2/2$: in the interior of V-shaped curve in $(a, b)$-plane there are 3 roots to $dP(x) = 0$, at the curve 2 solutions, and outside it 1 solution. Note that now the parameterization is different from that proposed above. The reason is that in catastrophe theory diffeo-invariance is the basic motivation whereas in $M^8$ there are highly unique octonionic preferred coordinates.

If p-adic length scale hypothesis holds true, primes near powers of 2, prime powers, in particular Mersenne primes should be ramified primes. Unfortunately, this picture does not allow to say anything about why ramified primes near power of 2 could be interesting. Could the appearance of ramified primes somehow relate to a mechanism in which $p = 2$ as a ramified prime would precede other primes in the evolution. $p = 2$ is indeed exceptional prime and also defines the smallest p-adic length scale.

For instance, could one have two roots $a$ and $a + 2^k$ near to each other 2-adically and could the deformation be small in the sense that it replaces $2^k$ with a product of primes near powers of 2: $2^k = \prod_i 2^{k_i} \to \prod_i p_i$, $p_i$ near $2^{k_i}$? For the irreducible polynomial defining the extension of rationals, the deforming could be defined by $a \to a + 2^k$ could be replaced by $a \to a + 2^k/N$ such that $2^k/N$ is small also in real sense.

## 4.2 Appendix: About the decomposition of primes of number field $K$ to primes of its extension $L/K$

The followings brief summary lists some of the basic terminology related to the decomposition of primes of number field $K$ in its extension.

1. A typical problem is the splitting of primes of $K$ to primes of the extension $L/K$ which has been already described. One would like to understand what happens for a given prime in terms of information about $K$. The splitting problem can be formulated also for the extensions of the local fields associated with $K$ induced by $L/K$.

2. Consider what happens to a prime ideal $p$ of $K$ in $L/K$. In general $p$ decomposes to product $p = \prod_{i=1}^{g} P_i^{e_i}$ of powers of prime ideals $P_i$ of $L$. For $e_i > 1$ ramification is said to occur. The finite field $K/p$ is naturally imbeddable to the finite field $L/P_j$ defining its extension. The degree of the residue field extension $(L/P_i)/(K/p)$ is denoted by $f_i$ and called inertia degree of $P_i$ over $p$. The degree of $L/K$ equals to $[L : K] = \sum e_i f_i$.

   If the extension is Galois extension (see `http://tinyurl.com/zu5ey96`), one has $e_i = e$ and $f_i = f$ giving $[L : K] = efg$. The subgroups of Galois group $Gal(L/K)$ known as decomposition group $D_i$ and inertia group $I_i$ are important. The Galois group of $F_i/F$ equals to $D_i/I_i$.

   For Galois extension the Galois group $Gal(L/K)$ leaving $p$ invariant acts transitively on the factors $P_i$ permuting them with each other. Decomposition group $D_i$ is defined as the subgroup of $Gal(L/K)$ taking $P_i$ to itself.

   The subgroup of $Gal(L/K)$ inducing identity isomorphism of $P_i$ is called inertia group $I_i$ and is independent of $i$. $I_i$ induces automorphism of $F_i = L/P_i$. $Gal(F_i/F)$ is isomorphic to $D_i/I_i$. The orders of $I_i$ and $D_i$ are $e$ and $ef$ respectively. The theory of Frobenius elements identifies the element of $Gal(F_i/F) = D_i/I_i$ as generator of cyclic group $Gal(F_i/F)$ for the finite field extension $F_i/F$. Frobenius element can be represented and defines a character.

3. Quadratic extensions $Q(\sqrt{n})$ are simplest Abelian extensions and serve as a good starting point (see `http://tinyurl.com/zofhmb8`) the discrimant $D = n$ for $p \ mod\ 4 = 1$ and $D = 4n$ otherwise characterizes splitting and ramification. Odd prime $p$ of the extension not dividing $D$ splits if and only if $D$ quadratic residue modulo $p$. $p$ ramifies if $D$ is divisible by $p$. Also the theorem by Kronecker and Weber stating that every Abelian extension is contained in cyclotomic extension of $Q$ is a helpful result (cyclotonic polynomials has as it roots all $n$ roots of unity for given $n$)

Even in quadratic extensions $L$ of $K$ the decomposition of ideal of $K$ to a product of those of extension need not be unique so that the notion of prime generalized to that of prime ideal becomes problematic. This requires a further generalization. One ends up with the notion of ideal class group (see `http://tinyurl.com/hasyllh`): two fractional ideals $I_1$ and $I_2$ of $L$ are equivalent if the are elements $a$ and $b$ such that $aI_1 = bI_2$. For instance, if given prime of $K$ has two non-equivalent decompositions $p = \pi_1\pi_2$ and $p = \pi_3\pi_4$ of prime ideal $p$ associated with $K$ to prime ideals associated with $L$, then $\pi_2$ and $\pi_3$ are equivalent in this sense with $a = \pi_1$ and $b = \pi_4$. The classes form a group $J_K$ with principal ideals defining the unit element with product defined in terms of the union of product of ideals in classes (some products can be identical). Factorization is non-unique if the factor $J_K/P_K$ - ideal class group - is non-trivial group. $Q(\sqrt{-5})$ gived a representative example about non-unique factorization: $2 \times 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$ (the norms are $4 \times 9$ and $6 \times 6$ for the two factorizations so that they cannot be equivalent.

This leads to class field theory (see `http://tinyurl.com/zdnw7j3` and `http://tinyurl.com/z3s4kjn`).

1. In class field theory one considers Abelian extensions with Abelian Galois group. The theory provides a one-to-one correspondence between finite abelian extensions of a fixed global field $K$ and appropriate classes of ideals of $K$ or open sub-groups of the idele class group of $K$. For example, the Hilbert class field, which is the maximal unramified abelian extension of $K$, corresponds to a very special class of ideals for $K$.

2. Class field theory introduces the adele formed by reals and p-adic number fields $Q_p$ or their extensions induced by algebraic extension of rationals. The motivation is that the very tough problem for global field $K$ (algebraic extension of rationals) defines much simpler problems for the local fields $Q_p$ and the information given by them allows to deduce information about $K$. This because the polynomials of order $n$ in $K$ reduce effectively to polynomials of order $n \ mod \ p^k$ in $Q_p$ if the coefficients of the polynomial are smaller than $p^k$. One reduces monic irreducible polynomial $f$ characterizing extension of $Q$ to a polynomial in finite field $F_p$. This allows to find the extension $Q_p$ induced by $f$.

   An irreducible polynomial in global field need not be irreducible in finite field and therefore can have multiple roots: this corresponds to a ramification. One identifies the primes $p$ for which complete splitting (splitting to first ordinary monomials) occurs as unramified primes.

3. Class field theory also includes a reciprocity homomorphism, which acts from the idele class group of a global field $K$, i.e. the quotient of the ideles by the multiplicative group of $K$, to the Galois group of the maximal abelian extension of $K$. Wikipedia article makes the statement "*Each open subgroup of the idele class group of K is the image with respect to the norm map from the corresponding class field extension down to K*". Unfortunately, the content of this statement is difficult to comprehend with physicist's background in number theory.

# 5 Appendix: Explicit formulas for the evolution of cosmological constant

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \ , \tag{5.1}$$

where $P$ is projection taking tensor quantity $T_{kl}$ in $S^2(M^4) \times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates $y^k$ for $S^2(M^4)$ or $S^(CP_2)$ and $x^\mu$ for$S^2$, $P$ is defined as

$$P : T_{kl} \to T_{\mu\nu} = T_{kl}\frac{\partial y^k}{\partial x^\mu}\frac{\partial y^l}{\partial x^\nu} \ . \tag{5.2}$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \ . \tag{5.3}$$

The expression for the coefficient $K$ of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu}J_{\mu\nu}\sqrt{det(g)} \ . \tag{5.4}$$

(Note that $J_{\mu\nu}$ refers to $S^2$ part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu}J_{\mu\nu})^2\frac{1}{\sqrt{det(g)}} \ . \tag{5.5}$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of $K$ over $S^2$:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J_{u\Phi}^2}{\sqrt{det(g)}} \ . \tag{5.6}$$

$(u,\Phi) \equiv (cos(\Theta,\Phi)$ are standard spherical coordinates of $S^2$) varying in the ranges $[-1,1]$ and $[0,2\pi]$.

This the quantity that one must estimate.

## 5.1 General form for the imbedding of twistor sphere

The imbedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4) \times S^2(CP_2)$ must be known. Dimensional reduction requires that the imbeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u,\Phi) \ ,$$

$$\left[u(S^2(CP_2)), \Phi(S^2(CP_2))\right] \equiv (v,\Psi) = RP(u,\Phi)$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere $S^2$). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates $u$ and $\Phi$ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.$(v,\Psi)$ are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection $P$ is absent from the above formula (RP $\rightarrow$ R).
2. $\epsilon = 1$ in which case $P$ is present. $P$ can be represented as reflection $(x,y,z) \rightarrow (x,y,-z)$ or equivalently $(u,\Phi) \rightarrow (-u,\Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle $\phi$ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

## 5.2 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g))$ using the induction formula expressing them as sums of projections of $M^4$ and $CP_2$ contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates $(u,\Phi)$ of $S^2(M^4)$ and to the coordinates $(v,\Psi)$ of $S^2(CP_2)$.

In coordinates $(u,\Phi)$ one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi)S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v,\Psi)}{\partial(u,\Phi)} \quad . \tag{5.7}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\frac{\partial(v,\Psi)}{\partial(u,\Phi)} = \frac{\partial v}{\partial u}\frac{\partial \Psi}{\partial \Phi} - \frac{\partial v}{\partial \Phi}\frac{\partial \Psi}{\partial u} \quad . \tag{5.8}$$

## 5.3 Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \quad .$$

where $P$ denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \quad .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \quad ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \quad .$$

This gives

$$\begin{aligned} & P[ds^2(S^2(CP_2))] \\ & = [(\tfrac{\partial v}{\partial u})^2 \tfrac{1}{1-v^2} + (1-v^2)(\tfrac{\partial \Psi}{\partial u})^2]du^2 \\ & +[(\tfrac{\partial v}{\partial \Phi})^2 \tfrac{1}{1-v^2} + (\tfrac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\ & +2[\tfrac{\partial v}{\partial u}\tfrac{\partial v}{\partial \Phi}\tfrac{1}{(1-v^2)} + \tfrac{\partial \Psi}{\partial u}\tfrac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \quad . \end{aligned}$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$\begin{aligned} g_{uu} &= \tfrac{1}{1-u^2} + (\tfrac{\partial v}{\partial u})^2 \tfrac{1}{1-v^2} + (1-v^2)(\tfrac{\partial \Psi}{\partial u})^2] \quad , \\ g_{\Phi\Phi} &= 1-u^2 + (\tfrac{\partial v}{\partial \Phi})^2 \tfrac{1}{1-v^2} + (\tfrac{\partial \Psi}{\partial \Phi})^2(1-v^2) \\ g_{u\Phi} = g_{\Phi u} &= \tfrac{\partial v}{\partial u}\tfrac{\partial v}{\partial \Phi}\tfrac{1}{(1-v^2)} + \tfrac{\partial \Psi}{\partial u}\tfrac{\partial \Psi}{\partial \Phi}(1-v^2) \quad . \end{aligned} \tag{5.9}$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \quad . \tag{5.10}$$

## 5.4 Coordinates $(v, \Psi)$ in terms of $(u, \Phi)$

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for $(v, \Psi)$ as functions of $(u, \Phi)$ and for partial derivations of $(v, \Psi)$ with respect to $(u, \Phi)$.

Let us restrict the consideration to the RP option.

1. P corresponds to $z \to -z$ and to

$$u \to -u \quad . \tag{5.11}$$

2. The rotation $R$ $(x, y, z) \to (x', y', z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \quad , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \quad . \tag{5.12}$$

   Here one has $(s, c) \equiv (sin(\epsilon), cos(\epsilon)$, where $\epsilon$ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

   From these formulas one can pick $v$ and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \quad . \tag{5.13}$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \quad . \tag{5.14}$$

## 5.5 Various partial derivatives

Various partial derivates are given by

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \quad , \\
\frac{\partial v}{\partial \Phi} &= -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \quad , \\
\frac{\partial \Psi}{\partial \Phi} &= (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \quad , \\
\frac{\partial \Psi}{\partial u} &= \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \quad , \\
X &= cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \quad .
\end{aligned} \tag{5.15}$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain $S^2$ coordinates as external parameters so that each point of $S^2$ corresponds to a slightly different space-time surface.

## 5.6 Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral $S$ over $S^2$ as function of the parameter $s = sin(\epsilon)$). One should also find the extrema of $S$ as function of $s$.

Especially interesting values are very small values of $s$ since for the cosmological constant becomes small. For small values of $s$ the integrand (see Eq. 5.6) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta)+O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of $S^2$. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on $s$ so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on $s$.

In the numerical calculation one must decompose the integral over $S^2$ to three parts.

1. There are parts coming from the small disks $D^2$ surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order $s$.

2. Besides this one has a contribution from $S^2$ with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \rightarrow \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J^2_{u\phi}$ vanishes like $s^2$ at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity $X$ in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \rightarrow \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of $us$.

Concerning practical calculation, the relevant formulas are given in Eqs. 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, and 5.15.

The calculation would allow to test/kill the key conjectures already discussed.

1. There indeed exist extrema satisfying $dS(S^2)/ds = 0$.

2. These extrema are in one-one correspondence with the zeros of zeta.

There are also much more specific conjctures to be killed.

1. These extrema correspond to $s = sin(\epsilon) = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis but since the choice of evolution parameter is to high extent free, the conjecture is perhaps too spesific.

2. For certain integer values of integer $k$ the integral $S(S^2)$ of Eq. 5.6 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where $x$ is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension $\Delta d$ would vanish.

   The condition $\Delta d = 0$ should be equivalent with the vanishing of the $dS/ds$. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

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