

Could a TGD analog of Weinstein's proposal help to define the QFT limit of TGD?

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Matti Pitkänen

orcid:0000-0002-8051-4364.

email: matpitka6@gmail.com,

url: http://tgdtheory.com/public_html/,

address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

Eric Weinstein has proposed "Geometric Unity", which is a proposal for a unification of the standard model and gravitation based on the notion of 14-D manifold $U(14)$, which according to my understanding is the bundle of metrics of X reducing locally to a product space-time and 10-D internal space which could consist of 4×4 symmetric matrices. Weinstein wants to endow $U(14)$ with some additional structure and explain gauge symmetries in terms of the fiber of $U(14)$ consisting of symmetric 4×4 matrices. Group $SO(10)$ acts as the 10-bein group of this space in the Euclidean case and the proposal is that it acts as a gauge group.

The first problem is that if the 10-bein group defines the gauge group, the gauge group for a Minkowskian signature of X is non-compact variant of $SO(10)$, which is the group of isometries for the space of M_{10} with Euclidean signature. In gauge theories non-compactness of the gauge group implies the loss of unitarity. Weinstein admits that his proposal works only in the Euclidean case.

Second problem is posed by the general coordinate invariance. General coordinate transformations do not induce a mere gauge transformation of the matrix of M_{10} as they should. This could mean severe difficulties in the realization of the general coordinate invariance.

In the TGD framework, one of the challenges is the more precise definition of the QFT limit of TGD. In this article I will consider a variant of Weinstein's theory obtained by replacing $H = M^4 \times CP_2$ with $M^4 \times S^n$ as a possible manner to approach the problem. For $n = 9$ and $n = 10$ one obtains $SO(n + 1)$ as maximal isometry group and holonomy group. It turns out that one can obtain standard model symmetries but the predicted number of fermion families turns out to be wrong. In TGD fermion families have a topological explanation. M can be replaced by a sphere S^n , and $n = 10$ gives 4 generations and $n = 8$ and $n = 9$ 2 generations. For larger values of n the number generations increases exponentially. Whether the QFT model could serve as a phenomenological description of the family replication phenomenon remains open.

In this article, I will consider a variant of Weinstein's theory obtained by replacing $H = M^4 \times CP_2$ with $M^4 \times S^n$. For $n = 9$ and $n = 10$ one obtains $SO(n + 1)$ as maximal isometry group and holonomy group. It turns out that one can obtain standard model symmetries but the predicted number of fermion families turns out to be wrong. In TGD fermion families have a topological explanation. M can be replaced by a sphere S^n , and $n = 10$ gives 4 generations and $n = 8$ and $n = 9$ 2 generations. For larger values of n the number generations increases exponentially. Whether the QFT model could serve as a phenomenological description of the family replication phenomenon remains open.

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1 Introduction

Eric Weinstein (see this) has proposed "Geometric Unity", which is a proposal for a unification of the standard model and gravitation based on the notion of 14-D manifold $U(14)$, which according to my understanding is the bundle of metrics of X reducing locally to a product space-time and 10-D internal space which could consist of 4×4 symmetric matrices. Weinstein wants to give $U(14)$ some additional structure and explain gauge symmetries in terms of the fiber of $U(14)$ consisting of symmetric 4×4 matrices. Weinstein has not published any article about his work but there is a draft of an article in web published almost two years ago (see this).

1.1 How do I understand the "Geometric Unity" of Weinstein?

Weinstein calls his approach "Geometric Unity".

1. The basic notion of Weinstein is observance, which means an immersion of space-time X to the infinite-D space $U(14)$ with bundle structure decomposing locally to X^4 and 10-D fiber consisting of symmetric tensors defining metric at a given point. A metric in X corresponds to a section of $U(14)$. This is just like in general relativity. One can assign to X fibers which would correspond to those associated with the fields of gauge theory. Gauge potentials and gauge fields correspond to sections in these fibers.

Here there is an analogy with TGD, where space-time X^4 is not fixed as in Weinstein's model but is a surface of $H = M^4 \times CP_2$. This makes space-time dynamical although H is non-dynamical and determines the symmetries of the theory. The space of space-time metrics in X^4 reduces to a much smaller space of metrics induced from the metric of $M^4 \times CP_2$.

2. Weinstein states that the curvature tensor of the Einstein's theory is incompatible with gauge theory. Riemannian connection and gauge connection transform in general coordinate transformations in a different way. I am unable to see why the induction of the Riemann connection from the metric would be somehow wrong. As a matter of fact, the transformation property of gauge potentials in a gauge transformation is identical with that of Riemann connection but for gauge potentials the general coordinate transformation of space-time is replaced with general coordinate transformation in the fiber reducing to local gauge rotation.

If I have understood correctly, the argument of Weinstein about asymmetry relies on the following observation using the old-fashioned language that I am used to.

- (a) One can project the one-forms $X_k dh^k$ of U to X giving form $X_\mu dx^\mu$, $X_\mu = \partial_\mu h^k A_k$. One-form could correspond to gauge potential. One cannot however project the contravariant vectors $X^k \partial_k$ in the same way since the Jacobian matrix $\partial_\mu h^k$ is not invertible when the dimension of U is higher than that of X . In the same way, one can induce the covariant metric g_{kl} of U as $g_{\mu\nu} = g_{kl} \partial_\mu h^k \partial_\nu h^l$ but not the contravariant form g^{kl} of the metric. This would be the asymmetry, which Weinstein sees as a problem.

- (b) Note however that one can lift $g^{\mu\nu}$ as the inverse of $g_{\mu\nu}$ to U as: $\hat{g}^{kl} = g^{\mu\nu} \partial_\mu h^k \partial_\nu h^l$. This allows local lifts of contravariant tensors of X to U . \hat{g}^{kl} is not identical with g^{kl} and is defined only in $X \subset U$.

3. Weinstein wants to assign particle physics symmetries with the 10-D fiber space M_{10} of 4×4 matrices defining the components of the metric for a given section and for given coordinate choice in X . Therefore the gauge symmetries would follow from the metric geometry of space-time. One would replace the 10-D fiber of the bundle of 4-D metrics with a bundle having M_{10} as a base space.

How to achieve this? One could assign mere gauge connection of some gauge group to M_{10} . Any gauge group seems to be possible. The situation would be the same as in ordinary gauge theories.

One can however introduce a metric in M_{10} and identify the connection as a Riemann connection in M_{10} . The gauge group would be the 10-bein group which is $SO(10)$ for the Euclidean signature of X .

The inner product in the linear space of matrices is obtained by increasing the indices of the matrix M_{ij} to give M^{ij} . The inner product would be $M^{ij} N_{ij}$. A contravariant flat metric in the linear space of 10×10 matrices would be $g^{ij,kl} = \eta^{ij} \eta^{kl}$ and the signature for the Minkowski metric $\eta = (1, -1, -1, -1)$ appears in it so that also M_{10} has non-Euclidian signature (it is easy to see the sign of the components $g^{ij,kl}$).

If M_{10} allows a metric, one can also define a spinor structure in M_{10} and one obtains spinor space with dimension $2^5 = 32$ to which fermions could be assigned. This suggests that one can assign to the 4-D X spinors isospin-like indices by forming a tensor product with M_{10} spinors. The properties of $SO(10)$ could explain fermion families.

4. There is however a serious problem. If the 10-bein group defines the gauge group, the gauge group for a Minkowskian signature of X is non-compact variant of $SO(10)$, which is the group of isometries for the space of M_{10} with Euclidean signature. In gauge theories non-compactness of the gauge group implies the loss of unitarity. Weinstein admits that his proposal works only in the Euclidean case.
5. A further problem is posed by the general coordinate invariance. A general coordinate transformation of X does not only take the matrix defining the the metric at a point of X to its image point, as it would do for the fields of the YM theory, but would also rotate it by the matrix defined by the Jacobian of the transformation from right and left since a tensor is in question. This rotation is not a mere local $SO(10)$ rotation in the tangent space as the very idea of general coordinate invariance would require. This could mean severe difficulties in the realization of the general coordinate invariance.
6. Riemannian connection is determined by a metric. Weinstein however states that metric is determined by a connection. Does this mean that one can assign to a general connection a metric? This is certainly not the case. The Riemannian connection is symmetric as a metric connection unlike general connection. By looking at the case of Riemann connection one finds already in a 2-D situation that one ends up with integrability conditions as consistency conditions for which it is difficult to imagine an algorithmic solution.

Weinstein presumably means that the selection of a section in $U(14)$ means selection of horizontal space at every point of X . By definition of $U(14)$ this selects a metric and also the associated Riemann connection so that the statement contains nothing new. "Selection of a section in $U(14)$ defines a metric in X " would be a more precise statement.

7. Weinstein talks about topological spinors instead of metric spinors. One can assign to the metric of X^4 spinor structure if certain topological conditions are true. The spinor structure need not be unique. This is one of the problems of general relativity and TGD solves it by inducing the spinor structure from that of $H = M^4 \times CP_2$. If I understand correctly, the topological spinors of X would be possible without the metric given by the identification as a section of $U(14)$. I do not see any reason for this.

In TGD, the notion of the induced spinor structure solves these problems and is also essential for understanding standard model symmetries in terms of generalized spin structure of CP_2 .

The dream of Weinstein is that Geometric Unity could provide a description of gravitation and gauge fields not plagued by the standard problems. The dimension of the space of symmetric matrices is 10 and the rotation group $SO(10)$ is its maximal isometry group having dimension 45. As I see it, the first fatal problem is the signature of the Minkowski signature of the metric of X implying that the gauge group is a noncompact variant of $SO(10)$. One should somehow modify Weinstein's proposal to overcome this problem. Second fatal problem is that the matrices of M do not behave like YM fields in general coordinate transformations, that is, they do not suffer a mere $SO(10)$ gauge transformation.

1.2 Could TGD analog of Weinstein's proposal help to define the standard model and define the QFT limit of TGD?

In the TGD framework, one of the challenges is the more precise definition of the QFT limit of TGD. Intuitively, the QFT limit of TGD in long scales is defined by replacing the many-sheeted space-time with a slightly curved region of M^4 . One assumes that the M^4 projection of the space-time sheets is 4-dimensional in the region of M^4 considered: one could talk about Einsteinian space-time. Various gauge fields in a region of M^4 are defined as sums of induced gauge fields assignable to various space-time sheets. For a large number of space-time sheets the extreme simplicity of the induced gauge fields is lost (the 4 coordinates and their gradients define the gauge field and there is also holography) and Einstein-Yang Mills action provides a reasonable approximation for the dynamics. The Equivalence Principle could be seen as a remnant of the Poincare invariance.

The information about the topology of the space-time surface is lost and poses crucial limitations: in biological systems the topology would become especially significant but would be important even in astrophysical and cosmological scales. Cosmic strings and related monopole flux tubes would represent an example about the failure of the Einsteinian space concept.

One must have a phenomenological description of various topological aspects of TGD relevant to elementary particle physics in the framework of QFT. In particular, the topological explanation of family replication phenomena should be transformed to a QFT theoretic description. I have proposed [K1] [L1] that one can assign to the 3 fermion families identified as 3 lowest genera for the partonic 2-surface a combinatorial $SU(3)_g$ symmetry. The emission of charged $SU(3)_g$ bosons changes the genus of the partonic 2-surface of fermion and provides at least a partial explanation of CKM mixing and its leptonic counterpart.

Since $SO(n)$ could allow a phenomenological description of a family replication, it is interesting to look for the TGD analog of Weinstein's proposal. Rather surprisingly, one can reproduce the TGD analog of Weinstein's view (as I understand it!), except that 4 or 2 fermion generations implied by holonomies are obtained. In TGD, fermion families have a topological explanation [?, L1]. M can be replaced by a sphere S^n , and $n = 10$ gives 4 generations and $n = 8$ and $n = 9$ 2 generations. For larger values of $n = 2k$ the number of generations given by 2^{k-3} increases exponentially. Whether the QFT model could serve as a phenomenological description of the family replication phenomenon remains an interesting question.

It should be also noticed that in the TGD framework the isometries of the internal space, which would be 10-D in Weinstein's model, are crucial and give color gauge fields whereas the holonomies give electroweak fields. In the color sector there is an analogy between TGD and Kaluza-Klein theories. Whether Weinstein considers the isometries for the space M of 4×4 symmetric matrices or assumes them, is not clear to me. I understand that a 10-bein connection would give rise to gauge potentials but I am not sure.

2 TGD variant of Weinstein's proposal

The TGD variant of Weinstein's proposal means only that one replaces CP_2 with space of S^n : $n = 10$ gives four generations and $n = 9$ gives 2 generations. Otherwise the basic picture of the TGD remains as such.

2.1 Basic ideas

Let us describe the basic ideas first.

1. Weinstein $U(14)$ bundle with projection X^4 is replaced by $X^4 \subset M^4 \times M$ and might be called observance in the terminology of Weinstein. The construction is in a way a special case of the $U(14)$ construction. Here M is the space of symmetric matrices, the determinant can be chosen as one because it produces compactness. Even now, space time is the surface. In TGD the counterpart of $U(14)$ or its generalization would have a metric and its spinor connection would define gauge fields by induction.
2. One obtains Poincare invariance and internal symmetries as isometries and holonomies of the metric of M corresponding to the generalized color and electroweak symmetries. The gauge fields are reduced to projections of the spinor connection of M just like in TGD. There are very few degrees of freedom. The field theory limit is obtained as in TGD.
3. M can be chosen as the space of symmetric 4×4 matrices with determinant 1. Therefore M is the sphere S^9 and $SO(10)$ can act on it as a maximal isometry and holonomy group.

2.2 Symmetry breaking at the level of geometry

What is needed is a symmetry breaking for $SO(10)$ that produces $SU(3)$ isometries and holonomies $U(2)$.

1. The great insight is that S^5 is a $U(1)$ bundle over CP_2 and the fiber space S^1 corresponds to the Kähler gauge potential whose different coupling to quarks and leptons produces color triplets for quarks and singlets for leptons as CP_2 partial waves.
2. The symmetry breaking proceeds step by step: S^9 into spheres S^8 , S^8 into spheres S^7 , S^7 into spheres S^6 and S^6 into spheres S^5 . After this one stops. A good reason is that the isometry group of S^4 is $SU(2) \times SO(3)$ and is not simple. Instead of color quantum numbers, one would have two isospins. Note that the isometry group of S^5 is $SO(6) = SU(4)$: symmetry breaking from $SU(4)$ to $SU(3) \times U(1)$ would take place.
3. A simplest example of the decomposition to lower-dimensional spheres is provided by S^2 . S^2 can be regarded as an S^1 corresponding to the $\theta = \text{constant}$ meridians. The equator is exceptional because it is a geodesic line. At the equator, the Riemann connection vanishes and the equator is in a dynamic equilibrium. More generally, one can expect that geodesic spheres are in a special position: they would have a hierarchy: S^8, S^7, S^6, S^5 .

One can also regard S^2 as a bundle-like structure consisting of geodesic circles going through the poles. An analog of this option seems to be realized for S^5 in the sense that it decomposes to a U^1 bundle over CP_2 .

4. The metric for S^9 can be constructed in the standard way by first decomposing S^9 into spheres S^8 and continuing recursively.

$$\begin{aligned}
 ds^2(S^9) &= d\theta_1^2 + \sin^2(\theta_1)ds^2(S^8) , \\
 ds^2(S^8) &= d\theta_2^2 + \sin^2(\theta_2)ds^2(S^7) , \\
 ds^2(S^7) &= d\theta_3^2 + \sin^2(\theta_3)ds^2(S^6) , \\
 ds^2(S^6) &= d\theta_4^2 + \sin^2(\theta_4)ds^2(S^5) ,
 \end{aligned} \tag{2.1}$$

S^5 is not decomposed into spheres S^4 but is interpreted as a $U(1)$ bundle over CP_2 .

$$ds^2(S^5) = d\phi^2 + ds^2(CP_2) . \tag{2.2}$$

The isometries and holonomies of CP_2 are obtained: these are the symmetries of the standard model TGD in the picture.

2.3 How does one obtain quarks and leptons?

1. Instead of M , S^9 , S^{10} , or even S^n can be considered and gives the symmetry group $SO(n+1)$, which has the representation that the fundamental or spinor representation has dimension $2^{\lfloor (n+1)/2 \rfloor}$.
2. The picture inspired by the physical picture and TGD framework is as follows. Spinor representation first breaks down into two chiralities (quarks and leptons), which involve $2^{\lfloor (n+1)/2 \rfloor - 1}$ states each such that these chiralities correlate with the chiralities of the M^4 spinors. For the quarks, the product of M^4 chiralities and M (S^n) chiralities is 1, and for leptons -1. A separate conservation of B and L is obtained. The splitting of S^n spinors into doublets identified as fermion generations also motivates the $SO(10)$ GUT.
3. Quarks and lepton spinors have $2^{\lfloor (n+1)/2 \rfloor - 2}$ spinor components and produce $2^{\lfloor (n+1)/2 \rfloor - 3}$ doublets, i.e. generations. $n = 9$ gives 4 generations for quarks and leptons instead of 3, which is empirically favoured. Both $n = 7$ and 8 give 2 generations. As n increases, the number of generations increases exponentially.

2.4 Riemann connection and the analog of Higgs mechanism

What can be said about the Riemannian connection and the analogy of the Higgs mechanism.

1. The Riemann connection on the sphere gives guidelines. The ground state corresponds to a hierarchy of geodesic spheres for which the gravitational forces disappear, i.e. the Riemannian connection vanishes.

The case of the sphere S^2 gives a simplified picture of the situation. The the Riemannian connection, dictating the spinor connection defining the gauge potentials, has the components $\Gamma_{\phi\phi}^\theta$ and $\Gamma_{\theta\phi}^\phi$. At the equator, they disappear: the gravitational force is zero and it is an equilibrium position. The angle θ would be analogous to the Higgs field and would produce a symmetry breaking: $\theta = \text{constant}$ the circle would not be a geodesic circle anywhere other than at the equator.

2. For the hierarchy of geodesic spheres, the same result is obtained and it is an equilibrium position with respect to the gravitation of S^n . The angles θ_i , $i = 1, 2, 3, 4$ are analogous to the components of the Higgs field. There are four of them, as in the standard model.

The hierarchy of geodesic spheres within geodesic spheres is a kind of 5-layered Russian doll. S^5 is split into CP_2 , i.e. S^5 is a $U(1)$ bundle over CP_2 (Kähler potential). All S^n points are also included, but those of geodesic spheres dominate. One obtains $SU(3)$ isometries and holonomies.

2.5 Spinor connection

The holonomy group of the spinor connection should contain a part corresponding to the electroweak $U(2)$ acting in the same way on all fermion generations. The spinor connection cannot reduce to a mere CP_2 spinor connection except for the geodesic spheres. The connection must have parts, which mix different spinorial tensor factors and therefore fermion generations and these parts could be seen as causing the CKM mixing and its leptonic counterpart.

How does the tensor product of 2-D spinor spaces defining the fundamental representation of $SO(10)$ decompose to a direct sum of quark doublets and lepton doublets? The construction recipe of spinors in S^9 (or S^n) provides answers to these questions.

1. The basic idea comes from the construction of the gamma matrices and spinors of 4-D space. 4-D spinors are tensor products of two copies of 2-D spinors. In the case of a flat space, 2-D gamma matrices can be represented as anticommuting Pauli spin matrices σ_i , $i = 1, 2$. 4-D flat space gamma matrices are of form $\sigma_i \otimes 1$ and $\sigma_0 \otimes \sigma_i$ for $i = 1, 2$. One can construct gamma matrices for curved 4-space as linear combinations of these. One gets from Euclidean to Minkowskian signature by multiplying space-like gamma matrices by imaginary unit.

One can do the same for 4-D gamma matrices to construct 8-D gamma matrices. Take flat 4-D space gamma matrices and define matrix γ_5 as their product $\gamma_5 = \gamma_1 \times \gamma_2 \times \gamma_3 \times \gamma_4$. In Euclidian case it has square $\gamma_5^2 = 1$. The 4+4 8-D gamma matrices are $\gamma_i \times 1$ and $\gamma_5 \times \gamma_i$.

2. An important delicacy is that the dimensions of spinor spaces for the spaces with dimension $2n$ and $2n + 1$ are same since the product of gamma matrices for $D = 2n$ becomes gamma matrix γ_{2n+1} in $2n + 1$ -D case. One has dimension 2 for S^2 and S^3 , dimension 4 for S^4 and S^5 , dimension 8 for S^6 and S^7 , and dimension 16 for S^8 and S^9 and dimension 32 for 10-D space. If one assumes S^9 allowing $SO(10)$ symmetries one obtains only 2 fermion generations. The space M has the symmetries of $SO(10)$ but is non-compact and having the value of the determinant of the matrix as an additional coordinate Λ . M would give 4 fermion generations. The non-compactness of M is not an attractive feature from the TGD perspective since space-time surfaces could be arbitrary large in internal degrees of freedom. One can replace M with S^{10} to achieve compactness. Now the maximal isometry group would be $SO(11)$.
3. One can proceed step by step from dimension 9 to 5 by 4 steps. The dimension of the spinor space increases by 2 units at steps θ_1, θ_3 , and S^5 as $U(1)$ bundle over CP_2 increases dimension by 4 to give 16-D dimensional spinor space. M or S^{10} gives a 32-D spinor space.

$SO(n)$ symmetry dictates the structure of the spinor connection. One can restrict the consideration to S^9 since the treatment of S^{10} and higher dimensional spheres proceeds along similar lines.

1. The spinor connection is dictated by the Riemann connection and determined by the exterior derivatives of vielbein vectors e^A which define the metric as $g_{kl} = \eta_{AB} e_k^A e_l^B$, where η denotes flat metric with a signature which is now Euclidean. The exterior derivatives $de^A = V_B^A \wedge e^B$ define the spinor connection $V_B^A = V_{BC}^A e^C$ as $V_{BC}^A e^C \wedge e^B$.
2. The metric of S^9 is diagonal with respect to the coordinates $\theta_1, \theta_2, \theta_3, \theta_4, \phi$. The metric of 10-D space M and S^{10} is diagonal whereas the CP_2 metric fails to be diagonal. The CP_2 spinor connection couples only to the 4 spinor indices of the S^5 spinors reducing to CP_2 spinors. This guarantees that the spinors of M or S^{10} *resp.* S^9 effectively decomposes to a direct sum of spinors of 4 *resp.* 2 families.
3. The diagonality of the metric of S^9 (of S^n) makes it easy to deduce the structure of vielbein connection. The inspection of the formula for the recursive construction of the S^9 metric shows that the S^5 contribution is proportional to the factor

$$S = \prod_{i=1}^4 \sin^2(\theta_i) . \quad (2.3)$$

The dependence of S on $\theta_i, i = 1, \dots, 4$ contributes to the Riemann connection components of type $\Gamma_{kl}^{\theta_i}$ and $\Gamma_{k\theta_i}^i$, where k, l refers to the coordinates of S^5 . Besides this there are components Γ_{lm}^k proportional to S and to S^5 connection acting on single fermion generation defining the electroweak gauge potentials.

4. The intuitive guess is that in the spinor connection these non-diagonal components induce the mixing of fermion families. The vielbein components e^A for S^5 are proportional to \sqrt{S} and de^A gives to the vielbein connection components of type

$$(1/2)\partial_i(\log(S))e_k^A - \partial_k e_i^A = \partial_i(\log(S))e_k^A$$

appearing in $de^A = V_B^A \wedge e^B = V_{BC}^A e^C \wedge e^B$. This gives components V_{BC}^A for which the indices B and C correspond to S^5 and to the spinor tensor factors a θ_i appearing in S .

These sigma matrices associated appear in $V_{BC}^A \Sigma_{BC}$ and act on pairs of spinorial tensor factors, which correspond to S^5 and its tensor factor complement. The gauge boson associated

with this part of the spinor connection induces a mixing of fermion generations by bringing to the Dirac action bilinears involving different fermion generations. These mixing terms vanish if S^5 corresponds S^5 as gravitational equilibrium state in S^9 .

2.6 Could $SO(n)$ model provide a phenomenological model for the TGD view of family replication?

In the TGD framework the origin of the fermion families is topological and corresponds to the genus of partonic 2-surfaces [K1] [L1]. The three lowest genera are in a special role in that they allow a global Z_2 symmetry which could make possible a bound state of handles for the genus $g = 2$. For $g > 3$ handles would behave like pairs or single free handles and the many-handle state would have a continuum mass spectrum and would not correspond to an elementary fermion.

I have proposed [K1] [L1] that one can assign to the 3 fermion families identified as 3 lowest genera for the partonic 2-surface a combinatorial $SU(3)_g$ symmetry. The emission of charged $SU(3)_g$ bosons changes the genus of the partonic 2-surface of fermion and provides at least a partial explanation of CKM mixing and its leptonic counterpart.

The topological mixing of families means the disappearance or appearance of a handle. The nice feature of the $SO(n)$ model is that it predicts the mixing matrices of fermion generations. Could both the Higgs mechanism and the topological mixing assigned be described as a mixing of spinorial tensor factors phenomenologically using the $S(10)$ model? Topological mixing is indeed assigned with the mass matrix in QFTs and mass squared matrix in TGD.

One should describe the handle effectively as a tensor factor. Handles form a many-particle state in the classical sense whereas the spinorial tensor factors effectively form a many-particle state of isospin 1/2 objects. This raises several questions.

1. Could the spherical partonic topology corresponding to the lowest generation having no handles and acting as a vacuum state correspond to the missing generation of the S^9 model? Or could the bound state of two handles for $g = 2$ correspond to the disappearance of a single spinorial tensor factor in the $SO(10)$ model?
2. TGD predicts that generations with $g > 2$ have continuous mass spectrum. Could S^n , $n > 9$ describe phenomenologically states with handle number $g > 2$. What would be the counterpart of the continuity of the mass spectrum in the S^n model?
3. TGD also allows the existence of bosons which correspond to wormhole contacts for which throats have different genus. They could allow interpretation as $SU(3)_g$ charged bosons responsible for the mixing of fermion generations. Could these bosons correspond to non-diagonal $SO(n)$ bosons in the QFT model. Note that also direct time evolutions between different genera could contribute to the mixing?

There is empirical evidence for the decay of Higgs-like particles to $e\mu$ pairs and these Higgs like particles could correspond to the charged $SU(3)_g$ bosons [L1]. Also gauge bosons should have similar variants.

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