

# Shnoll effect decade later

M. Pitkänen

Email: matpitka6@gmail.com.  
<http://tgdtheory.com/>.

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## Abstract

Shnoll and collaborators have discovered strange repeating patterns of random fluctuations of physical observables such as the number  $n$  of nuclear decays in a given time interval. Periodically occurring peaks for the distribution of the number  $N(n)$  of measurements producing  $n$  events in a series of measurements as a function of  $n$  is observed instead of a single peak. The positions of the peaks are not random and the patterns depend on position and time varying periodically in time scales possibly assignable to Earth-Sun and Earth-Moon gravitational interaction.

These observations suggest a modification of the expected probability distributions but it is very difficult to imagine any physical mechanism in the standard physics framework. Rather, a universal deformation of predicted probability distributions could be in question requiring something analogous to the transition from classical physics to quantum physics.

TGD gives hints about the nature of the modification.

1. TGD inspired quantum measurement theory proposes a description of the notion of finite measurement resolution in terms of inclusions of so called hyper-finite factors of type  $\text{II}_1$  (HFFs) and closely related quantum groups parameterized by quantum phase  $q_m = \exp(i\pi/m)$ . Canonical identification mapping p-adic integers to their real counterparts is central element of TGD. For  $m = p$  one can consider also the quantum variant of p-adic integer  $n$  mapped to  $n_R$  by canonical identification. There are 2 candidates for quantum-p-adics depending on whether the binary digits are interpreted as quantum integers as such or mapped to a product of quantum counterparts of their prime factors.
2. Adelic physics provides a possible unification of real number based physics as physics of sensory experience and various p-adics physics as physics of cognition and predicts a hierarchy of Planck constants  $h_{eff} = nh_0$  and suggests the identification of preferred p-adic prime  $p$  as a ramified prime of extension of rationals associated with the adèle.

p-Adicization or perhaps even quantum-padicization could explain the findings of Shnoll.

1. The universality of the modified distribution  $P(n)$  would reduce to the interpretation of the integer  $n$  in the distribution  $P(n|\lambda)$  of counts as a p-adic integer or its counterpart mapped by canonical identification to a real number  $n_R$  appearing as argument of  $P(n|\lambda)$ . Same can be applied to  $n!$ . The fractality implied by the quantum criticality of TGD Universe suggests that  $P(n)$  should be approximately scaling invariant under  $n \rightarrow p^k n$ .
2. TGD can be regarded formally as complex square root of thermodynamics, which suggest the representation  $P(n) = |\Psi(n)|^2$ , where  $\Psi(n)$  would be wave function in the space of counts expressible as product of classical part and “quantum factor”. One could have wave functions in the space of counts  $n$  expressible as superpositions of “plane waves”  $q_m^{kn}$ , with  $k$  playing the role of momentum.

A more concrete model relies on wave function proportional to  $(kn)_{q_p} \propto q_m^{kn} + q_m^{-kn}$  - analog to a superposition of plane waves with momenta  $k$  propagating to opposite directions in the space of counts reduced effectively to a box  $0 \leq n < p = m$  representing modulo  $p$  counter. One would have effectively wave functions in finite field  $G_p$ . The symmetries of quantum factor would correspond to a multiplication or shift of  $k$  by element  $r$  of  $F_p$ .

Various additional rational-valued parameters characterizing the probability distribution can be mapped to (possibly quantum-) p-adics mapped to reals by canonical identification. The parameters taking care of the converge such as the parameter  $\lambda$  in Poisson distribution must be mapped to a power of  $p$  in p-adic context.

The model can be applied to explain the findings of Shnoll.

1. The model makes rather detailed predictions about the periodically occurring positions of the peaks of  $P(n)$  as function of  $p$  based on number theoretical considerations and in principle allows to determine these parameters for given distribution. There is  $p$ -periodicity due to the fact that the lowest pinary digit of  $n_R$  gives first approximation to  $n_R$ .
2. The slow variation of the p-adic prime  $p$  and integer  $m = p$  characterizing quantum integers could explain the slow variation of the distributions with position and time. The periodic variations occurring with both solar and sidereal periods could be understood in two manners.  
 The value of  $p$  could be characterized by the sum  $a_{net}$  of gravitational accelerations assignable to Earth-Sun and Earth-Moon systems and could vary. If the value of  $p$  is outcome of state function process, it is not determined by deterministic dynamics but should have a distribution. If this distribution is peaked around one particular value, one can understand the findings of Shnoll.
3. An alternative explanation would be based on slow dependence of quantum factor of  $\Psi(n)$  on gravitational parameters and on time. For instance, the momentum  $k$  defining the standing wave in the space of counts modulo  $p$  could change so that the peaks of the diffraction pattern would be permuted.

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## 1 Introduction

As I wrote the first version of this chapter about Shnoll effect for about decade ago [L1], I did not yet have the recent vision about adelic physics [L2, L3] as a unification of real physics and various

p-adics and real number based physics to describe the correlates of both sensory experience and cognition.

The recent view is that the hierarchy of extensions of rational numbers induces a hierarchy of extensions of p-adic number fields in turn defining adèle. This hierarchy gives rise to dark matter hierarchy labelled by a hierarchy of Planck constants and also evolutionary hierarchy. The hierarchy of Planck constants  $h_{eff} = n \times h_0$  is an essential element of quantum TGD and adelic physics suggests the identification of  $n$  as the dimension of extension of rationals.  $n$  could be seen as a kind of IQ for the system.

What is also new is the proposal that preferred p-adic primes labelling physical systems could correspond to so called ramified primes, call them  $p$ , of extension of rationals for which the expression of the rational p-adic prime as product of primes of extension contains less factors than that the dimension  $n$  of extension so that some primes of extension appear as higher powers. This is analog for criticality as the appearance of multiple roots of a polynomial so that the derivative vanishes at the root besides the polynomial itself.

Before continuing it is good to make some confessions. Already in the earlier approach [L1] I considered two options for explaining the Shnoll effect: either in terms of p-adic fractality or in terms of quantum phase  $q$  of both. I however too hastily concluded that the p-adic option fails and choose the quantum phase option.

In the following both options are seen as parts of the story relying on a principle: the approximate scaling invariance of probability distribution  $P(n)$  for fluctuations under scalings by powers of p-adic prime  $p$  implying that  $P(n)$  is approximately identical for the divisions for which the interval  $\Delta$  defining division differs by a power of  $p$ .

Second new idea is the lift of  $P(n)$  to wave function  $\Psi(n)$  in the space of counts. For quantum phase  $q_m$ ,  $m = p$ ,  $\Psi_m$  would have quantum factor proportional to a wave function in finite field  $F_p$ , and the notion of counting modulo  $p$  suggests that the wave function corresponds to particle in box - standing wave - giving rise to  $P(n)$  representing diffraction pattern.

## 1.1 Basic facts about Shnoll effect

Usually one is not interested in detailed patterns of the fluctuations of physical variables, and assumes that possible deviations from the predicted spectrum are due to the random character of the phenomena studied. Shnoll and his collaborators have however studied during last four decades the patterns associated with random fluctuations and have discovered a strange effect described in detail in [E1], [E1, E4, E5, E2, E6, E3]. The examples of [E1], [E1] give the reader a clear picture about what is involved.

1. Some examples studied by Shnoll and collaborators are fluctuations of chemical and nuclear decay rates, of particle velocity in external electric field, of discharge time delay in a neon lamp RC oscillator, of relaxation time of water protons using the spin echo technique, of amplitude of concentration fluctuations in the Belousov-Zhabotinsky reaction. Shnoll effect appears also in financial time series [E7] which gives additional support for its universality. Often the measurement reduces to a measurement of a number of events in a given time interval  $\tau$ . More generally, it is plausible that in all measurement situations one divides the value range of the studied observable to intervals of fixed length and counts the number of events in each interval to get a histogram representing the distribution  $N(n)$ , where  $n$  is the number of events in a given interval and  $N(n)$  is the number of intervals with  $n$  events. These histograms allow to estimate the probability distribution  $P(n)$ , which can be compared with theoretical predictions for the spectrum of fluctuations of  $n$ . Typical theoretical expectations for the fluctuation spectrum are characterized by Gaussian and Poisson distributions.
2. Contrary to the expectations, the histograms describing the distribution of  $N(n)$  has a distribution having several maxima and minima (see the figures in the article of Shnoll and collaborators (see <http://tinyurl.com/6kehe9b>). Typically -say for Poisson distribution - one expects single peak. As the duration of the measurement period increases, this structure becomes gets more pronounced: standard intuition would suggest just the opposite to take place. The peaks also tend to be located periodically. According to [E1] [E1] the smoothed out distribution is consistent with the expected distribution in the case that it can be predicted reliably.

3. There are also other strange features involved with the effect. The anomalous distribution for the number  $n$  of events per fixed time interval (or more general value interval of measured observable) seems to be universal as the experiments carried out with biological, chemical, and nuclear physics systems demonstrate. The distribution seems also to be same at laboratories located far away from each other. The comparison of consecutive histograms shows that the histogram shape is likely to be similar to the shape of its nearest temporal neighbors. The shapes of histograms tend to recur with periods of 24 hours, 27 days, or 365 days. The regular time variation of consecutive histograms, the similarity of histograms for simultaneous independent processes of different nature and occurring in different geographical positions, and the above mentioned periods, suggest a common reason for the phenomenon possibility related to gravitational interactions in Sun-Earth and Earth-Moon system.

In the case that the observable is number  $n$  of events per given time interval, theoretical considerations predict a distribution characterized by some parameters. For instance, for Poisson distribution the probabilities  $P(n)$  are given by the expression

$$P(n|\lambda) = \exp(-\lambda) \frac{\lambda^n}{n!} . \quad (1.1)$$

The mean value of  $n$  is  $\lambda > 0$  and also variance equals to  $\lambda$ . The replacement of distribution with a many-peaked one means that the probabilities  $P(n|\lambda)$  are modified so that several maxima and minima result. This can occur of course by the randomness of the events but for large enough samples the effect should disappear.

The universality and position independence of the patterns suggest that the modification changes slowly as a function of geographic position and time. The interpretation of the periodicities as periods assignable to gravitational interactions in Sun-Earth system is highly suggestive. It is however very difficult to imagine any concrete physical models for the effect since distributions look the same even for processes of different nature. It would seem that the very notion of probability somehow differs from the ordinary probability based on real numbers and that this deformation of the notion of probability concept somehow relates to gravitation.

## 1.2 Quantum group inspired model for Shnoll effect

Usually quantum groups are assigned with exotic phenomena in Planck length scale. In TGD they are assignable to a finite measurement resolution [K6]. TGD inspired quantum measurement theory describes finite measurement resolution in terms of inclusions of hyper-finite factors of type  $\text{II}_1$  (HFFs) and quantum groups related closely to the inclusions and appear also in the models of topological quantum computation [B1] based on topological quantum field theories [A4].

Consider first the original version of the proposed model. If I would rewrite it now correcting also the small errors, the summary would be as follows. This slightly revised model can be included as such to the new model.

1. The possibility that direct p-adic variants of real distribution functions such as Poisson distribution might allow to understand the findings was discussed also in the original version. The erratic conclusion was that this cannot be the case. In fact, for  $\lambda = 1/p^k$  the sum of probabilities  $P(n)$  without normalization factor is finite, and the approximate scaling symmetry  $P(n) \simeq P(p^r n)$  emerges for  $k = 1$ . p-Adicity predicts approximate  $p$ -periodicity corresponding to the periodic variation of  $n_R$  with the lowest binary digit of  $n$ .
2. It was argued that one should replace the integer  $n!$  in  $P(n)$  with quantum integer [A3]  $(n!)_{q_m}$ ,  $q = \exp(i\pi/m)$ , identified as the product of quantum integers  $r_{q_m} = (q^r - q^{-r})/(q - q^{-1})$ ,  $r < n$ .

This however leads to problems since  $r_{q_m}$  can be negative. The problem can be circumvented by interpreting  $n!$  as p-adic number and expanding it in powers of  $p$  with binary digits  $x_k < p$ . For  $m = p$  the replacement of  $x_k$  with quantum integer yields positive binary digits.

The resulting quantum variant of p-adic integer can be mapped to its real counterpart by a generalization of canonical identification  $x = \sum x_n p^n \rightarrow \sum x_n p^{-n}$ . Whatever the detailed

definition, quantum integers are non-zero and positive. The quantum replacement  $r \rightarrow r_{q_m}$  of the integers appearing in rational parameters in  $P(n|\lambda_i)$  might therefore make sense. It however does not make sense in the exponents like  $\lambda^n$  and  $\lambda = p^k$ ,  $k > 1, 2, \dots$ , is forced by convergence condition.

3. I proposed also another modification of quantum integers  $x_{q_m}$ ,  $x < p = m - 1$  appearing in as binary digits by decomposing  $x$  into a product of primes  $s < p$  and replacing  $s$  with quantum primes  $s_{q_p}$  so that also the notion of quantum prime would make sense: one might talk about quantum arithmetics [K4, K3]. This is possible but is not necessary.

### 1.3 Adelic model for Shnoll effect

At the first re-reading the original model looked rather tricky, and this led to a revised model feeding in the adelic wisdom [L2, L3]. One implication hierarchy of Planck constants  $h_{eff}/h_0 = n$  with  $n$  identified as the dimension of Galois extension.

One also ends up to the proposal that preferred p-adic primes  $p$  correspond to so called ramified primes of the extension of rationals inducing the extensions of p-adic number fields defining the adèle. This kind of prime would naturally define a small- $p$  p-adicity associated with Shnoll effect, which would thus serve as a direct signature of adelic physics.

1. The first observation in conflict with the original belief is that one can actually define purely p-adic variant of the Poisson distribution  $P(n|\lambda)$  by replacing  $1/n!$  with its image  $(n!)_R$  under canonical identification. For instance, for Poisson distribution one must have  $\lambda = p^{-k}$ ,  $k = 1, 2, \dots$  for both real and p-adic distributions to make sense. The sum of the probabilities  $P(n)$  is finite. Poisson distribution with trivial quantum part is determined uniquely.
2. One can also consider quantization  $P(n) = |\Psi(n)|^2$ , suggested by the vision about quantum TGD as complex square root of thermodynamics and hierarchy of Planck constants making possible macroscopic quantum coherence in arbitrarily long scales. The complexity of  $\Psi(n)$  could genuine quantum interpretation. Quantum factor of  $\Psi(n)$  allows interpretation as a wave function in finite field  $F_p$  representing the space of counts modulo  $p$ . The existence of quantum p-adics requires  $m = p$ . Scaling by  $p$  is not a symmetry but multiplication by  $0 < k < p$  and shift by  $0 \leq k < p$  act as symmetries analogous to rotations and translations acting on waves functions in Euclidian 3-space.
3. The objections against Shnoll effect lead to an additional condition - or should one say principle - stating that the  $P(n)$  is approximately invariant under scalings  $n \rightarrow p^k n$ . This could be seen as a manifestation of p-adic fractality in turn reflecting quantum criticality of TGD Universe.
4. Taking  $n$  as the observable simplifies p-adicization crucially since the highly non-unique p-adicization of classical observables is avoided. One could speak of quantum measurement in the space of counts  $n$  defining universal observables. In quantum measurements the results are typically expressed as numbers of counts in given bin so that this kind of p-adicization is physically natural. The division of measurement interval would define an ensemble and  $n$  would be measured in each interval. State function reduction would localize  $\Psi(n)$  to  $n$  in each interval.

This picture leads to an alternative and simpler view about Shnoll effect. The scaling invariance is an essential additional condition now.

1. The factorials  $n!$  appearing in  $P(n) = (d^n f/dx^n)/n!$  identified as coefficients of Taylor series of its generating function developed in binary expansion for  $p = m$ . In this expansion one must invert powers of  $p$  in  $(n!)_R$  and could also replace the coefficients of powers of  $p$  with quantum integers or replace even primes in their prime composition with quantum primes. For given norm  $(n!)_R$  has period  $p$  approximately.
2. The  $n$ :th derivative  $X(n) = d^n f/dx^n$  appearing as coefficient of  $1/n!$  is replaced with  $X(n)_R/X(n)_p$  giving approximate periodicity and scaling invariance  $n \rightarrow pn$ .

3. Quantum phase is associated with the ansatz stating  $P(n) = |\Psi(n)|^2$ . In the “diffractive” situation quantum counterpart corresponds to  $|(kn)_{q_m}|^2$ ,  $0 < k < p - 1$ . This gives rise to periodicity with period  $m = p$ .

The universal modifications of the probability distributions  $P(n|\lambda_i)$  considered predict patterns analogous to the ones observed by Shnoll. The p-adic prime  $p = m$  characterizes the deformation of the probability distribution and implies approximate  $p$ -periodicity, which could explain the periodically occurring peaks of the histograms for  $N(n)$  as function of  $n$ .

One can imagine several explanations for the dependence of the time series distribution  $P(n)$  on the direction of the momentum of alpha particle [E2, E6] and on the dependence of  $P(n)$  on time.

1. The change of ramified prime  $p$  induced by the change of the extension of rationals would affect the periods. An interesting question is whether the effects understood in terms of the effect of the measurement apparatus on many-sheeted space-time topology and geometry on  $p$ . Can one speak about measurement of  $p$  and of extension of rationals?
2. The extension of rationals (and thus  $p$ ) need not change. The “quantum factor” of  $\Psi$  in  $P(n) = |\Psi(n)|^2$  has part depending on  $q_p$ . The dependence on  $q_p$  could change without change in  $p$  so that the extension of rationals need not change. One could speak about measurement of an observable related to the quantum factor of  $\Psi$ . A more concrete model relies on wave function proportional to  $(kn)_{q_p} \propto q_m^{kn} + q_m^{-kn}$  - analog to a superposition of plane waves with momenta  $k$  propagating to opposite directions in the space of counts and producing in  $P(n)$  diffraction pattern proportional to  $(qn)_{q_p}^2$ . Change of momentum  $k$  by scaling or shift induced by variation of the gravitational parameters or time evolution could be in question.

The p-adic primes  $p$  in question are rather small, not much larger than 100 and the periods of  $P(n)$  provide a stringent test for the proposal. If  $p$  corresponds to ramified prime as adelic physics suggests, it can be indeed small.

To sum up, I cannot avoid the thought that fluctuations regarded usually as a mere nuisance could be actually a treasure trove of new physics. While we have been busily building bigger and bigger particle accelerators, the truth would have been staring directly at our face and even winking eye to us.

## 2 Adelic view about Shnoll effect

In the sequel the adelic model for Shnoll effect is developed. The earlier model - with errors corrected - can be seen as a variant of this model.

### 2.1 General form for the deformation of $P(n|\lambda)$

Could Shnoll effect be a direct manifestation of adelic physics [L2, L3]? In TGD framework adelic physics is motivated as physics of cognition and sensory experience, and this could explain why Shnoll effect is associated even with financial time series. Instead of starting to make ad hoc guesses, consider first what kind of constraints adelic physics could pose on the deformation.

1. The basic idea is that since the effect is universal, the form of the probability distribution  $P(n|\lambda_i)$  should be modified in a universal manner, which depends on the experimental situation only very weakly.
2. Adelic physics suggests that the deformation of probability distributions  $P(n|\lambda_i)$  could depend on small p-adic prime  $p$  identifiable as ramified prime and on integer  $m$  defining defining quantum phase  $q_m = \exp(i2\pi/m)$  and giving rise to effective angle resolution in terms of allowed phases as roots of unity.

The first guess is that  $m$  could give rise to the periodic occurrence of the maxima and minima in the deformed distribution due to the  $m$ -periodicity of  $n_{q_m}$ . p-Adic prime  $p$  would define finite length scale resolution: it turns that also the map of factorials  $n!$  interpreted as p-adic

numbers by canonical identification to their counterparts gives an approximately  $p$ -periodic modulation of  $P(n)$ .

3. According to the standard definition quantum integers are real and given by  $n_q = (q^n - q^{-n})/(q - q^{-1})$ . The problem is that  $n_{q_m}$  vanishes if  $n$  is divisible by  $m$  so that one cannot replace the factorials appearing in Poisson distribution (say) with their quantum counterparts. The solution of the problem is the interpretation of  $n$  as  $p$ -adic integer and the replacement of binary coefficients with quantum integers  $n_{q_p}$  ( $m = p$ ), which are positive. One could also decompose them into a product of prime factors and replace them with their quantum counterparts  $n_{q_p}$ .

In the power  $\lambda^{-n}$  one could consider the replacement of  $n$  with  $n_{q_p}$  but this does not work in the  $p$ -adic case because  $p^{-n_q}$  in general does not belong to a finite extension of  $p$ -adics used. In the  $p$ -adic case  $\lambda = 1/p^k$  turns out to be the only possible option. For  $k = 1$  one obtains approximate scaling invariance  $n \rightarrow pn$ .

4. The hierarchy of Planck constants makes possible quantum coherence in all scales. This inspires the idea that the probabilities  $P(n)$  are moduli squared for a complex probability amplitude  $\Psi(n)$ :  $P(n) = |\Psi(n)|^2$ .  $\Psi_n$  could have having “quantum factor”  $\Psi_q$  containing a phase depending on  $n$ .

The simplest option is that quantum factor is has phase  $U(n) = q_m^n$  or its power. This does not give any effect visible in  $P(n)$ . A more general options is a quantum factor  $\Psi_q = \sum c_k q_m^k$ . In this case one obtains interference effects in the modulus squared. Complex quantum integer  $n_q = (q^n - q^{-n})/(1 - q)$  as a multiplicative factor would give rise to a diffractive factor  $\sin^2(n\pi/m)/\sin^2(\pi/m)$  in  $P(n)$ .

Speaking about amplitude for fluctuations and quantum diffraction in an ensemble defined by a division of the range of observable or a division of time interval to smaller intervals is of course quite a generalization of quantum mechanical thinking but it is interesting to look whether this could lead to sensible predictions. One could however whether the slow variations of the fluctuation patterns could correspond to different outcomes for quantum measurements measuring  $p$ -adic prime  $P$  and  $m$ .

Usually objections are the best manner to proceed and now the objections leads to an approximate scaling invariance  $n \rightarrow p$  of  $P(n)$  suggested also by the  $p$ -adic fractality implied by the quantum criticality of TGD Universe.

1. The first objection is that the findings of Shnoll are special in the sense that the replacement of observable with its diffeomorph cannot preserve the character of distribution  $P(n)$ . One can however claim that in practice the choice of observables is highly unique from physical constraints. Only simple scalings of the observable can be considered in same cases.
2. Second objection is that replacing the interval  $\Delta O$  for observable with  $\Delta O/m$  cannot leave the general shape of the distribution invariant. The naïve guess is that one has  $P(n) \rightarrow P(nm)$  for large enough values of  $n$ . This condition in a suitably restricted form can be posed as a constraint. The natural assumption is that the condition holds true only for the  $p$ -adic scalings  $m = p^k$ .

This condition can be used as a constraint on  $P(n)$ .  $P(n)$  would depend on  $n$  only through functions of  $n$  invariant under  $p$ -adic scalings  $n \rightarrow pn$ . An example about scaling invariant is provided by the function  $x_R = n_R/n_p$ , where  $n_p$  is the  $p$ -adic norm of  $n$  and  $n_R$  is obtained by canonical identification  $n = \sum n_k p^k \rightarrow n_k p^k \rightarrow n_R = \sum n_k p^{\rightarrow} n_k p^{-k}$ .

Any function of  $x_R$  is invariant under  $p$ -adic scalings and one can construct analogs for Gaussian, Poisson distribution, etc... by replacing  $n$  with  $x_R$ . Periodicity with period  $p$  is obtained if one replaces the  $p$ -adic unit  $nn_p$  with  $nn_p \bmod p$ . Since the higher binary digits do not affect strongly the behavior of  $x_R$ , approximate  $p$ -periodicity is obtain in any case.

3. The factorial  $n!$  appears in probability distributions having Taylor series as a generating function. A little calculation below using Legendre’s theorem shows that apart from an approximately periodic multiplicative function of  $n$  with period  $p$  one has  $p^{-n}/(n!)_R =$

$p^{-s(n)}$ , where  $s(n)$  is the sum of the binary digits of  $n$  in the expansion in powers of  $p$ .  $s(n)$  is invariant under scalings  $n \rightarrow p^k n$ . Therefore the deformed Poisson distribution is unique from the approximate scaling invariance and given apart from normalization by  $P(n) = p^{-n}/(n!)_R = p^{-s(n)}$ .

4. What about more general scalings  $n \rightarrow kn$ ? Under the scalings  $n \rightarrow kn$  for  $k$  not divisible by  $p$ , the norm of  $n$  is invariant. The rough scaling behavior of  $P(n)$  is however un-affected. The lowest binary digit is replaced in  $n \rightarrow n + 1$  with  $n + k \bmod p$  so that approximate  $p$ -periodicity is still present.

Consider next quantum phases.

1. The dependence on the quantum phase  $q_m = \exp(i\pi/m)$ ,  $m = p$ , cannot be invariant under  $p$ -adic scalings. The reason is that scaling by  $p$  takes all powers of  $q_p$  to unit and is thus not a bijection.

One can however consider different kind of symmetries. The integers  $n \bmod p$  form finite field  $G_p$  in which multiplication and sum define transformations analogous to rotations and translations acting naturally as symmetries in the space of probability amplitudes defined in the space of counts  $n \bmod p$  - modulo arithmetics means finite phase resolution for  $n$  represented as a phase. The wave functions can be interpreted as elements of finite-field algebra analogous to group algebra consisting of probability amplitudes in group.

One can interpret the plane waves  $q_p^{kn}$  as analogs of plane waves with momentum  $k$ . Multiplication and translations by  $r$  would correspond naturally to symmetries analogous to rotations and translations in Euclidian space.

2.  $\Psi(n)$  would have “quantum factor” expressible as a wave function in the space of counts  $n$ . For plane wave  $q_p^{kn}$ , the plane wave would not be visible in  $P(n)$ . The superposition  $q_p^{kn} + q_p^{-kn}$  of two plane waves propagating in opposite directions in the space of counts modulo  $p$  is proportional to quantum integer  $(kn)_{q_p} = \sin^2(kn\pi/p)/\sin(\pi/p)$  defining the analog of diffraction pattern. One has the analog of standing wave in a box having  $n = 0$  and  $n = p$  as its boundaries.

This is really nice mathematics but is “quantum factor” really needed? Can one do using just the deformation of say Poisson distribution or its quantum analog obtained by replacing  $n!$  interpreted as  $p$ -adic integer with its quantum counterpart? Or is “quantum factor” all that is needed? Or does this depend on situation? The following is just a list about the questions, which pop into mind and reflect my confusion more than my understanding.

1. The “quantum factor” of  $\Psi(n)$  - to be distinguished from “classical factor” depending on  $n$  without any analysis to binary digits interpreted as  $p$ -adic or quantum  $p$ -adic integers - can be regarded as a wave function in finite field  $F_p$  for the lowest binary digit of  $n$ .  $n_{q_p}^2$  gives the probability for the count  $n \bmod p$ . The modulo  $p$  condition for the binary digit of  $n$  can be interpreted as particle in box condition  $0 < n \bmod p < p$  so that states correspond to standing waves propagating in the space of counts and representable as sums of plane waves with wave vector  $0 \leq k < p$  propagating in opposite directions. This implies that quantum part of  $P(n)$  is universal and give by  $n_{q_p}^2 = \sin^2(kn\pi/p)/\sin^2(\pi/p)$ . Diffractive pattern results. Also  $p$ -periodicity is obtained from modulo  $p$  arithmetics. Approximate scale invariance  $n \rightarrow p^k n$  is not obtained. This could explain Shnoll effect.

“Quantum factor” alone is non-realistic since the probabilities for large values of  $n$  must be small. Should one interpret the “classical factor” of  $\Psi$  as a wave function for the remaining binary digits defining  $n_{rem}$ ? This would give the needed decrease for large values of  $n$ :  $p^{-s(k)}$  for Poisson distribution. Now approximate scaling  $p$ -adic scaling invariance would be true as also  $p$ -periodicity in the lowest binary digit.

3. Does it make sense to talk about separate wave functions for the lowest binary digits as wave functions for  $n_{rem} = n \bmod p^k - m \bmod p^{k-1}$  so that one would have product  $P(n) \propto \prod_k (n_k)_{q_p}^2$  of single digit wave functions? Physical intuition tells that the lowest digits are



the most important ones and cannot be independent. Could one consider lowest  $k$  binary digits as single entity with  $m = p^k$  and generalize quantum group picture by using quantum integers  $n_{q_{p^k}}$  with  $p^k$ -periodicity?

- Note that for  $p > n$  one would have only single binary digit and  $P(n) \propto n_{q_p}^2 = \sin^2(n\pi/p)/\sin^2(\pi/p)$ ? For  $n > p$  but near to it  $P(n)$  one would have single maximum: this would be nearer to what one expects with Gaussian intuition. Could one think of the analog of p-adic length scale hierarchy so that the increase of  $p$  would lead from quantum description to a classical description? Do ramified primes of extension determine this kind of hierarchy? The number of binary digits in “quantum factor” would correlate with the measurement resolution for  $n$ .

## 2.2 Deformation of Poisson distribution as an example

Consider next the p-adic modification of  $P(n)$  based on canonical identification, which I gave up in the original approach since I erratically concluded that the sum of probabilities without normalization fails to converge.

- Adelic physics suggests that prime  $p$  and quite generally, all preferred p-adic primes, could correspond to ramified primes for the extension of rationals defining the adèle. Ramified prime divides discriminant  $D(P)$  of the irreducible polynomial  $P$  (monic polynomial with rational coefficients) defining the extension (see <http://tinyurl.com/oyumsnk>).

Discriminant  $D(P)$  of polynomial whose roots give rise to extension of rationals, is essentially the resultant  $Res(P, P')$  for  $P$  and its derivative  $P'$  defined as the determinant of so called Sylvester polynomial (see <http://tinyurl.com/p67rdgb>).  $D(P)$  is proportional to the product of differences  $r_i - r_j$ ,  $i \neq j$  the roots of  $p$  and vanishes if there are two identical roots.

**Remark:** For second order polynomials  $P(x) = x^2 + bx + c$  one has  $D = b^2 - 4c$ .

Ramified primes divide  $D$ . Since the matrix defining  $Res(P, P')$  is a polynomial of coefficients of  $p$  of order  $2n - 1$ , the size of ramified primes is bounded and their number is finite. The larger coefficients  $P(x)$  has, the larger the value of ramified prime can be. Small discriminant means small ramified primes so that polynomials having nearly degenerate roots have also small ramifying primes. Galois ramification is of special interest: for them all primes of extension in the decomposition of  $p$  appear as same power. For instance, the polynomial  $P(x) = x^2 + p$  has discriminant  $D = -4p$  so that primes 2 and  $p$  are ramified primes.

- One can consider a p-adic modification of  $n!$  by expanding  $n! \equiv x$  as series  $x \sum x_n p^n$  in powers of the ramified prime and mapping the result to a real number by canonical identification  $\sum x_n \rightarrow \sum x_n p^{-n}$ . The outcome is approximately periodic for large  $n$  since the lowest binary digit gives dominating contribution and is periodic with period  $p$ . There would be two approximate periodicities for the peaks corresponding to  $p$ .

**Remark:** Canonical identification is applied in p-adic mass calculations [K1] [L5] and at the level of scattering amplitudes it would map Lorenz invariants appearing in the scattering amplitudes expressible in terms of rational functions with coefficients which are rational (or in an extension of rationals) to their real counterparts.

- Also the powers of  $\lambda$  should make sense p-adically, and the replacement of  $\lambda$  by a power of  $p$  indeed makes sense p-adically. In the case of Poisson distribution this would predict

$$P(n) \propto \frac{p^{-nk}}{(n!)_R}.$$

for  $\lambda_R = p^{-k}$ .

- $n!$  contains some power  $\nu_p(n!)$  of  $p$  given by Legendre’s formula (see <http://tinyurl.com/jdvwaph>):

$$\nu_p(n!) = \sum_{i=1}^{\infty} \left[ \frac{n}{p^i} \right] ,$$

where  $[x]$  denotes the value of floor function replacing  $xt$  with the largest integer smaller than  $x$ . The  $p$ -adic norm of the  $n!$  equals to  $p^{-\nu_p(n!)}$  so that the sum of probabilities converges for all values of  $k$  of one has  $\nu_p(n!) < p$ . It seems that this is the case quite generally. In fact, there is in the same sources also another formula for  $\nu_p(n!)$  making this manifest.

$$\nu_p(n!) = \frac{n - s_p(n)}{p - 1} .$$

Here  $s_p(n)$  is the sum of pinary digits of  $n$  expressed as power series of  $p$ . For  $k = 1$  the  $n$ :th term is  $p^{-s_p(n)}$  divided by the canonical images of a  $p$ -adic number with unit  $p$ -adic norm.

Since  $s_p(n)$  increases in step-wise manner, one obtains asymptotically a periodically modulated series with period  $p$  since the canonical image of  $n!$  approaches periodic function. The generalization to any  $P(n)$  expressible as  $n$ :th term in a Taylor series of some function serving as generating function for  $P(n)$  is obvious.

As already explained, one can modify this distribution by adding to  $\Psi(n) = \sqrt{P(n)}$  “quantum factor” as a wave function in the space of counts modulo  $p$  forming a finite field and particle in box analogy gives essentially  $(kn)_{q_p}$  as quantum factor characterized by momentum  $k$  leading to diffraction patter described by  $(kn)_{q_p}^2$ . This standing wave quantum factor could be universal and describe modulo  $p$  counter.

### 3 Explanation for some findings of Shnoll

One should be able to undertand both the many-peaked character of the distributions as well as their spatial and temporal variation involving correlations with the gravitational physics of Sun-Earth and Earth-Moon systems.

The really difficult problem is to understand how astrophysical objects could affect microscopic physics in even nuclear scales where physics is local.

In the sequel two kinds explanations inspired by number theory are considered. The first model is rather formal. The second model relies on the notion of gravitatonal Planck constant introduced by Nottale and explains the rate variations as being induced by the variation of the Planck constant.

#### 3.1 The temporal and spatial dependence of the distributions

One should also understand the variation of the shape of the distribution with time and its spatial variation.

1. The correlation of the fluctuation periods with astrophysical periods assignable to Earth-Sun system (diurnal period and period of Earth’s orbit) suggests that the gravitational interaction of the measurement apparatus with Sun is involved. Also the period 27.28 days which corresponds to sidereal period of Moon measured in the system defined by distant star. In [E1] , [E1] this period is somewhat confusingly referred to as synodic period of Sun with respect to Earth (recall that synodic period corresponds to a period for the appearance of third object (say Moon) in the same position relative to two other objects (say Earth and Moon)). Therefore also Moon-Earth gravitational force seems to be involved. Moon-Earth and Earth-Sun gravitational accelerations indeed have roughly the same order of magnitude.

That gravitational accelerations would determine the effect conforms with Equivalence Principle. The most natural dimensionless parameter characterizing the situation is  $|\Delta \mathbf{a}_{gr}|/\mathbf{a}_{gr}$  expressible in terms of  $\Delta R/R$  and  $\Delta r/r$ , where  $R$  *resp*  $r$  denotes the distance between Earth and Sun *resp.* Earth and Moon, and the ratio  $R/r$  and cosine for the angle  $\theta$  between the direction vectors for the positions of Moon and Sun from Earth. The observed palindrome

effect [E3] is consistent with the assumed dependence of the effect on the distances of Earth from Sun and Moon. Also the smallness of the effect as one approaches North Pole conforms with the fact that the variations of distances from Sun and Moon become small at this limit.

2. In 24 hour time scale it is enough to take into account only the Earth-Sun gravitational interaction. One could perform experiments at different positions at Earth's surface to see whether the variation of distributions correlates with the variation of the gravitational potential. The maximal amplitude of  $\Delta R/R$  is  $2R_E/R \simeq .04$  so that for  $\Delta p/p = k\Delta R/R$  one would have  $\Delta p/p = .04k$ . Already for  $p \sim 100$  the variation range would be rather small. For  $\Delta m/m$  one expects that analogous estimate holds true.
3. One observes in alpha decay rates periodicities which correspond to both sidereal and solar day [E2]. The periodicity with respect to solar day can be understood in terms of the periodic variation of Sun-Earth distance. The periodicity with respect to sidereal day would be due to the diurnal variation of the Earth-Moon distance. Similar doubling of periodicities are predicted in other relevant time scales.
4. In the case of alpha decay the effect reveals intricacies not explained by the simplest model [E2, E6]. In this case one studies random fluctuations for the numbers of alpha particles emitted in a fixed direction. Collimators are used to select the alpha particles in a given direction and this is important for what follows. Two especially interesting situations correspond to a detector which is located to North, East, or West from the sample. What is observed that the effect is different for East and West directions and there is a phase shift of 12 hours between East and West. In Northern direction the effect vanishes. Also other experiments reveal East-West asymmetry called local time effect by the authors [E4, E5].

The distribution for the counts of alpha particles in a given angle depend on time and the time dependence is sensitive to the direction angle of the alpha particle. This might be however only apparent since collimators are used to select alpha particles in given direction. The authors speak about anisotropy of space-time and Finsler geometry [A1] could be considered as a possible model. In this approach the geometry of space-time would be something totally independent of measurement apparatus.

One can identify a candidate for a scalar on which the magnitude of effect should depend.

1. At quantitative level the distribution for counts in a given direction can depend on angles defined by the vectors formed from relevant quantities. These include at least the tangential rotational velocity  $v = \omega \times r$  of the laboratory at the surface of Earth, the direction of the velocity  $v_\alpha$  of alpha particle with respect to sample actually reflecting the geometry of collimators, the net gravitational acceleration  $a_{net}$  caused by Earth, Sun, and Moon, and the direction of acceleration  $g$  in the Earth's gravitational field.
2. The first task is to construct from these vectors a scalar or a pseudo-scalar (if one is ready to allow large parity breaking effects), which vanishes for North-East direction, has opposite signs for East and West direction and has at least approximately a behavior consistent with the phase shift of 12 hours between East and West. The constraints are satisfied by the scalar

$$X = E \cdot a_{net} \quad , \quad E = \frac{(v \times g) \times v_\alpha}{|(v \times g) \times v_\alpha|} \quad . \quad (3.1)$$

Unit vector  $E$  changes sign in East-West permutation and also with a period of 12 hours meaning the change of the roles of East and West with this period in the approximation that the net acceleration vector is same at the opposite sides of Earth. The approximation makes sense if the change of sign induces much larger variation than the change of the Earth-Sun and Earth-Moon distances. If the parameters of the model are even functions of  $X$ , the predicted effect can be consistent with the experimental findings in the approximation that  $a_{net}$  is constant in 24 hour time scale.

This could explain the difference in the fluctuations associated with alpha particles emitted in East and West direction and the fact that there is no effect in North direction.  $v \times g$  points

to North and North direction for  $v_{alpha}$  has  $E = 0$  so that the magnitude of  $E$  proportional to the sine of the angle between North and  $v_\alpha$  should dictate the magnitude of the effect.

### 3.2 TGD based model for the Shnoll effect in alpha decay

In TGD framework the space-time is topologically non-trivial and dynamical in macroscopic scales and the presence of collimators making possible to select alpha particles in a given direction affect the geometry of many-sheeted space-time sheets describing the measurement apparatus and therefore the details of the interaction with the gravitational fields of Earth, Sun, and Moon. As a consequence, the value of  $p = m$  should reflect the geometry of the measurement apparatus and depend only apparently on the direction of  $v_\alpha$ . If this interpretation is correct, a selection of events from a sample without collimators should yield distributions without any dependence on the direction of  $v_\alpha$ .

The situation is sensitive to the value of  $p = m$  in the model described above. The changes should be such that the parameters of the smoothed out real probability distribution are not affected much. For instance, in the case of q-Poisson distribution the value of  $p = m$  should change in such a way that  $\langle n \rangle = \lambda$  is not unaffected much. The change of  $p$  would affect the positions of the peaks but small changes of  $p$  would not mean too dramatic changes. Periodic time dependence of these parameters would explain the findings of Shnoll. Gravitational interactions in Sun-Earth-Moon system and therefore the periodic variations of Sun-Earth and Earth-Moon distances is the first guess for the cause of the periodic variations.

In the case of alpha decay Shnoll effect is associated with temporal fluctuations in the number  $n$  of the measured events in time interval  $\Delta T$  characterized by  $P(n)$ .  $P(n)$  is reported to depend on the gravitational accelerations assignable to Earth-Moon and Earth-Sun systems. It is claimed that this dependence on gravitational parameters is quite general. In TGD framework this looks natural since gravitational flux tubes and gravitational Planck constant  $h_{gr}$  play a central role in TGD inspired biology. These accelerations have same order of magnitude.

There are two possible sources for the effect in the proposed model.

1. Classical option: the representation  $P(n) = |\Psi(n)|^2$  is not assumed. If one accepts the proposed scaling invariant ansatz, the only parameter affecting the p-adic part of the deformation determined by canonical identification is the value of  $p$ . Thus the change of  $p$  and presumably of also extension of rationals would be involved.  $p$ -periodicity is approximate.
2. Quantum option:  $P(n) = |\Psi(n)|^2$  is assumed. The existence of quantum p-adics requires  $p = m$  so that the space of counts modulo  $p$  is finite field  $G_p$ . The quantum factor of wave function  $\Psi(n)$  in the space of counts the most general quantum phase dependent combination  $\sum c_k (kn)_{q_p}$  in  $\Psi(m)$ , the parameters  $c_k$  appear as additional quantal parameters besides the parameters fixing the original distribution  $P(n)$  (Gaussian, Poisson,...). For quantum factor the  $p$ -periodicity in  $n$  is exact. Particle in box description for modulo  $p$  counter property suggests standing wave interpretation so that wave function would be  $|(kn)_{q_p}|^2$  with  $k$  having interpretation as an analog of momentum. The outcome would be diffraction pattern  $\sin^2(kn\pi/p)/\sin(\pi/p)$ . The value of  $k$  could be seen as analogous to  $G_p$ -valued momentum varying from measurement to measurement.

In this case the slow variations of  $P(n)$  could reflect slow change of  $c_k$  even when  $P$  remains unaffected. For instance, a scaling of complex quantum integer  $n_q$  to  $(kn)_{q_p}$ ,  $0 < k < p$  would induce permutation of the peaks of the diffraction pattern. The interpretation would be as finite field permutation induced by multiplication. Also finite field translation of  $n_q$  to  $(n+k)_{q_p}$ . This is the minimal option and suggests that genuine quantum effect is in question: the value of  $k$  could be seen as analogous to  $Z_m$ -valued momentum varying from measurement to measurement.

Suppose that the emitted alpha particle propagates along a magnetic flux tube. A natural question is whether the direction of  $a_{net}$  corresponds to a direction of flux tubes at which the gravitational effects of Sun and Moon sum up.

**Classical option:** The parameters  $\lambda_i$  - such as the parameter defining Poisson distribution - determining  $P(n|\lambda_i)$  could depend on  $X$  but only through  $P(X)$  in the model obeying the scaling invariance  $n \rightarrow pn$ .

The dependence could be through  $p = p(X)$  would affect the approximate  $p$ -periodicity. A purely  $p$ -adic deformation would require that the ramified prime  $p$  depends on  $X$  so that gravitational effects modify decay rates directly via the relative direction of the alpha particle flux tubes and various kinds of gravitational flux tubes. The extension of rationals assignable to the flux tubes along which alpha particles propagate would depend on  $X$ . Alpha particle interactions with gravitational flux tubes via wormhole contacts and this should determine the value of  $p$ .

**Quantum option:** If  $P$  and extension are not affected, only the diffractive quantum degrees of freedom remain under consideration. This would provide the minimal model. In quantum sector the quantum part of the distribution could depend on  $X$ , say by a scaling of the momentum  $k \rightarrow r(X)k$  or shift  $k \rightarrow k + r(X)$  modifying the diffraction pattern. Since the change is slow, shift looks more plausible option having interpretation as a discrete version of slow classical dynamics. Now the peaks of diffraction pattern would be permuted.

1. The dependence of the scaling factor  $r(X)$  on the parameter  $X$  appearing in  $n \rightarrow r(X)n$  in quantum factor would be given by piecewise constant function and discontinuities would occur at certain values of  $X$ .  $X$  is of form

$$X = vv_{\alpha}ga_{net} \times \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) .$$

Here  $\theta_3$  is the angle between  $v$  and  $g$ ,  $\theta_2$  is the angle between  $v \times g$  and  $v_{\alpha}$  and  $\theta_1$  is the angle between  $E$  and  $a_{net}$ .  $X$  has dimension  $(v^2/T^2)^2$ .

2. If only the relative directions matter, one could try to assign the integer  $r(X)$  to the quantity

$$X_0 = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) .$$

$X_0$  does not depend on the absolute values of various parameters such as  $v_{\alpha}$ , which is presumably the observable measured.

3. Some unit should be chosen for the trigonometric functions involved.  $\cos(2\pi/m)$  resp.  $\sin(2\pi/m)$  is the first  $P$  guess for the unit of  $\cos(\theta_1)$  resp.  $\sin(\theta_i)$ ,  $i = 2, 3$  so that  $n(X_0)$  would be given by

$$n(X_0) = \left[ \frac{\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)}{\cos(2\pi/m)\sin^2(2\pi/m)} \right] .$$

To sum up, the minimal model for the Shnoll effect would be based on the modification of diffractive part by scaling  $n \rightarrow r(X)n$  so that diffraction peaks are permuted but also the change of  $p = m$  can be consider.

### 3.3 What can one say about values about the period of $P(n)$ ?

The basic prediction is that the presence of an approximate period  $p = m$  identified as ramified prime for extension of rationals. In one of the experiments (see <http://tinyurl.com/6kehe9b> **Fig.1** of [E1], [E1] ) the histogram for  $N(n)$  has peaks, which seem to occur periodically with a separation  $\Delta n$  of about 100 units. If these periods correspond to  $P$ , its value must be smaller than 100. The nearest primes are  $p = 89, 97, 101, 113$ . In **Fig. 2** of same reference one has also periodicity and  $p$  must be near 10. Hence there are good hopes that the proposed model might be able to explain the findings.

There is an intriguing empirical finding possibly related to the value of  $p$  and to the dependence on gravitational fields bringing in basic periods of solar system.

1. The fluctuations appear with temporal periods of 24 hours, 27 days and 365 days. Quite recently I learned about 160 minute period which appears in astrophysics in very wide range starting from solar oscillations and ending to the physics of quasars [L4]. TGD inspired interpretation relies on the observation that  $Fe^{2+}$  cyclotron frequency in endogenous magnetic field  $B_{end} = .2$  Gauss playing key role in TGD inspired quantum biology is 10 Hz and in the interstellar magnetic field with average value of 2 nTesla this frequency corresponds to a period of 160 minutes. Cosmic alpha rhythm could be in question!

2. What is intriguing that 24 hours corresponds to  $3^2$ -multiple and 27 days to  $3^5$ -multiple of this period! Does this mean that  $p = 3$ -adicity is involved with Shnoll effect  $p = 3$  would be a ramified prime of the extension in question. 3-adicity is also observed to be characterize big steps in evolution besides 2-adicity [K2].

### 3.4 Are these models too formal?

The proposed rather formal models in principle have the potential of explaining the findings of Shnoll but detailed numerical work is required to find whether the model works also at the level of details.

1. Taking  $n$  as the observable simplifies p-adicization crucially since the p-adicization of classical observables is avoided. One could speak of quantum measurement in the space of counts  $n$  defining a universal observable. In quantum measurements the results are typically expressed as numbers of counts in given bin so that this kind of p-adicization is physically natural.

The division of measurement interval would define an ensemble and  $n$  would be measured in each interval. State function reduction would localize  $\Psi_n$  to  $n$  in each interval.

2. The universality of the modified distributions would reduce to the interpretation of the integer  $n$  in the distribution  $P(n|\lambda)$  as a p-adic integer mapped by canonical identification to a real number  $n_R$  appearing as argument of  $P(n|\lambda)$ . Same can be applied to  $n!$ .

It is essential that the sum of probabilities without normalization factor converges and that the distribution is approximately scaling invariant under  $n \rightarrow p^k n$ . Approximate scaling invariance can be interpreted in terms of fractality implied by the quantum criticality of TGD Universe.

3. One can consider also the quantum variant of p-adic integer  $n$  mapped to  $n_R$  by canonical identification. The parameter  $m$  defining quantum group and being possible for possible “quantum factor” in the complex square root of  $P(n)$  having interpretation as wave function satisfies  $m = p$  from the condition that the canonical images of quantum p-adics are positive. There are 2 candidates for quantum-padics depending on whether the binary digits are interpreted as quantum integers as such or mapped to a product of quantum primes.

Various additional rational-valued parameters characterizing the probability distribution can be mapped to (possibly quantum-) p-adics mapped to reals by canonical identification. The parameters taking care of the converge such as the parameter  $\lambda$  in Poisson distribution must be mapped to a power of  $p$  in p-adic context.

4. The small value of p-adic prime  $p$  involved can be understood in TGD framework in terms of adelic physics suggesting that preferred primes are so called ramified primes of extension of rationals. For given irreducible polynomial determining the extension one can calculate the ramified primes from the discriminant of the polynomial.

Model can be applied to the results of Shnoll.

1. The model makes rather detailed predictions about the periodically occurring positions of the peaks of  $P(n)$  as function of  $p$  based on number theoretical considerations and in principle allows to determine these parameters for given distribution.
2. The value of  $p$  could be characterized by the sum  $a_{net}$  of gravitational accelerations assignable to Earth-Sun and Earth-Moon systems and could vary. If the value of  $p$  is outcome of state function process, it is not determined by deterministic dynamics but should have a distribution. If this distribution is peaked around one particular value, one can understand the findings of Shnoll.

The periodic variations occurring with both solar and sidereal periods could be understood in two ways. The slow variation of  $p = m$  could explain the slow variation of the distributions with position and time. An alternative explanation would be based on slow dependence of quantum factor of  $\Psi(n)$  on gravitational parameters and on time. For instance, one could have wave functions proportional  $(kn)_{q_m}$ ,  $k = 1, \dots, p$ , so that the change of  $k$  would permute the diffraction peaks.

3. Various effects such as the dependence of the probability distributions on the direction of alpha particles selected using collimators and 12 hour phase shift between the directions associated with East and West direction could be perhaps understood as direct evidence for the effects of measurement apparatus on the many-sheeted space-time affecting either the value of  $p$  or the “quantum factor”: say the dependence of  $k$  on the momentum parameter  $X$  defined earlier.

However, one has good reasons to argue that these models are very formal and the connection with the empirical findings is rather loose.

### 3.5 Shnoll effect and quantum gravitation

Thanks for Ed Oberg for sending an email in which he mentioned Shnoll effect. I have discussed the Shnoll effect from the TGD point of view in [?]. Now I must say that the number theoretical ideas involved with explanation look a little bit too formal an approach when one wants to understand the splitting of the distribution for the number of counts per time interval in say alpha decays. A more direct connection with physics would be badly needed. Therefore I decided to take a fresh look on Shnoll effect with inspiration coming from the increase of the understanding of quantum gravitation in the TGD framework [L6, L7].

1. For instance, alpha decay rates are studied: overall decay rates or in fixed direction for alpha rays. Number of counts per time interval  $\tau$  varies. Poisson distribution becomes many-peaked.
2. Is there a dependence on the period  $\tau$ ? How many peaks? Are the numbers of peaks the same for various values of  $\tau$ ? The natural assumption is that there are several rates. If so, the number  $N$  at the maximum of the peak  $I$  is  $N = rate(I) \times \tau$ .
3. There are periodicities of the peak structure related to sidereal time and solar time. There are correlations with the dynamics of the Sun, Earth, and even galaxy. There is also a dependence on the direction of the alpha ray.
4. The splitting of the decay rates as the emergence of almost degenerate states of nuclei would be the simplest explanation. The astrophysical correlations suggest that this should be due to the gravitational effects.

The recent TGD view of quantum gravitation could provide a simple explanation.

1. A splitting of the state of the emitting nucleus to  $N$  states occurs such that the  $N$  states have different decay rates. Where does this degeneracy come from? Could the degenerate states be dark variants of the ordinary nucleus in the TGD sense and therefore have different values of  $h_{eff}$ . The gravitational Planck constants  $\hbar_{gr}$  for astrophysical objects are suggested by the observed astrophysical correlations.
2. Why would these almost degenerate states of nuclei have different alpha decay rates? These rates are determined by nuclear physics. In the TGD framework, the only variable parameter is effective Planck  $h_{eff}$  which affects the rates in higher order in perturbation expansion. Lowest order is not affected. In higher orders the effect is non-trivial and could be large for strong interactions.
3. The quantum gravitational effects characterized by  $\hbar_{gr}$  are expected to be the largest ones. Could the almost degenerate nuclei be attached to gravitational flux tubes of different astrophysical objects and have different effective/gravitational Planck constants? Sun, Earth, Moon, galaxy, and planet come first in mind.
4. The Shnoll effect depends on the directions and distances of the Sun and Moon and there is a periodic variation. The dependence on the sidereal time suggests that even galactic gravitational fields matter. Since the attachment of the local system to the gravitational flux tube is required, the probability for the presence of a gravitational flux tube connecting Earth and the gravitational source is what matters. The probability is proportional to the density of flux tubes per unit area and decreases with the distance between the Earth and the source. This could explain the periodic variations of the rates.

5. This model applies also to electromagnetic interactions and could explain the Shnoll effect in chemistry. The basic prediction is that the splitting of the Poisson distribution is qualitatively similar independent of the system studied.

It is perhaps fair to conclude that the explanation of the Shnoll effect in terms of gravitational quantum coherence in astrophysical scales is the more plausible option. The study of fluctuations could open a completely new field of research and a very abstract form of quantum theory. From TGD point of view this could mean theoretical and experimental work to deduce and test the predictions of adelic physics.

## 4 Appendix: p-Adic primes as ramified primes, quantum p-adics, and quantum primes

The following describes still rather speculative ideas about the physical role of number theory inspired by adelic physics.

### 4.1 Preferred p-adic primes as ramified primes?

As I wrote the first version of this chapter, I had not yet developed the vision about adelic physics. Adelic physics corresponds to a hierarchy of extensions of rationals inducing extensions of p-adic number fields and the proposal is that ramified primes of extension correspond to preferred p-adic primes.

1. Adelic physics suggests that prime  $p$  and quite generally, all preferred p-adic primes, could correspond to ramified primes for the extension of rationals defining the adèle. Ramified prime divides discriminant  $D(P)$  of the irreducible polynomial  $P$  (monic polynomial with rational coefficients) defining the extension (see <http://tinyurl.com/oyumsnk>).

Discriminant  $D(P)$  of polynomial whose roots give rise to extension of rationals, is essentially the resultant  $Res(P, P')$  for  $P$  and its derivative  $P'$  defined as the determinant of so called Sylvester polynomial (see <http://tinyurl.com/p67rdgb>).  $D(P)$  is proportional to the product of differences  $r_i - r_j$ ,  $i \neq j$  the roots of  $p$  and vanishes if there are two identical roots.

**Remark:** For second order polynomials  $P(x) = x^2 + bx + c$  one has  $D = b^2 - 4c$ .

2. Ramified primes divide  $D$ . Since the matrix defining  $Res(P, P')$  is a polynomial of coefficients of  $p$  of order  $2n - 1$ , the size of ramified primes is bounded and their number is finite. The larger coefficients  $P(x)$  has, the larger the value of ramified prime can be. Small discriminant means small ramified primes so that polynomials having nearly degenerate roots have also small ramifying primes. Galois ramification is of special interest: for them all primes of extension in the decomposition of  $p$  appear as same power. For instance, the polynomial  $P(x) = x^2 + p$  has discriminant  $D = -4p$  so that primes 2 and  $p$  are ramified primes.

**Remark:** All polynomials having pair of complex conjugate roots have  $p = 2$  as ramified prime.

3. What does ramification mean algebraically? The ring  $\mathcal{O}(K)/(p)$  of integers of the extension  $K$  modulo  $p = \pi_i^{e_i}$  can be written as product  $\prod_i \mathcal{O}/\pi_i^{e_i}$  (see <http://tinyurl.com/y6yskkas>). If  $p$  is ramified, one has  $e_i > 1$  for at least one  $i$ . Therefore there is at least one nilpotent element in  $\mathcal{O}(K)/(p)$ .

Could one interpret nilpotency quantum physically?

1. For Galois extensions one has  $e_i = e > 1$  for ramified primes.  $e$  divides the dimension of extension. For the quadratic extensions ramified primes have  $e = 2$ . Quadratic extensions are fundamental extensions - kind of conserved genes -, whose further extensions give rise to physically relevant extensions.

On the other hand, fermionic oscillator operators and Grassmann number used to describe fermions “classically” are nilpotent. Could they correspond to nilpotent elements of order



$e_i = e = 2$  in  $\mathcal{O}(K)/(p)$ ? Fermions are building bricks of all elementary particles in TGD. Could this number theoretic analogy for the fermionic statistics have a deeper meaning?

2. What about ramified primes with  $e_i = e > 2$ ? Could they correspond to para-statistics (see <http://tinyurl.com/y4mq6j22>) or braid statistics (see <http://tinyurl.com/psuq45j>)?

Both parabosonic and parafermionic fields of order  $n$  have the representation  $\Psi = \sum_{i=1}^n \Psi_i$ . For parafermion field one has  $\{\Psi_i(x), \Psi_i(y)\} = 0$  and  $[\Psi_i(x), \Psi_j(y)] = 0$ ,  $i \neq j$ , when  $x$  and  $y$  have space-like separation. For parabosons the roles of commutator and anti-commutator are changed.

The states containing  $N$  identical parafermions are described by a representation of symmetric group  $S_N$  with  $N$  rows with at most  $n$  columns (anti-symmetrization). For states containing  $N$  identical parabosons one has  $N$  columns and at most  $n$  rows. For parafermions the wave function is symmetric in horizontal direction but the modes are different so that Bose-Einstein condensation is not possible.

For parafermion of order  $n$  operator  $\sum_{i=1}^n \Psi_i$  one has  $(\sum_{i=1}^n \Psi_i)^n = \prod \Psi_1 \Psi_2 \dots \Psi_n$  and higher powers vanish so that one would have  $n + 1$ -nilpotency. Therefore the interpretation for the nilpotent elements of order  $e$  in  $\mathcal{O}(K)/(p)$  in terms of parafermion of order  $n = e - 1$  might make sense.

It seems impossible to build a nilpotent operator from parabosonic field  $\Psi = \sum_i \Psi_i$ : the reason is that the powers  $\Psi_i^n$  are non-vanishing for arbitrarily high values of  $n$ .

3. Braid statistics differs from para-statistics and is assigned with quantum groups. It would naturally correspond to quantum phase  $\exp(i\pi/p)$  assignable to the exchange of particles by braid operation regarded as a homotopy permuting braid strands. Could ramified prime  $p$  would correspond to braid statistics and the index  $e_i = e$  characterizing it to parafermion statistics of order  $e - 1$ ? This possibility cannot be excluded since this exotic physics would be associated in TGD framework to dark matter assigned to algebraic extensions of rationals whose dimension  $n$  equals to  $h_{eff}/h_0$ .

Why the primes, which do not split maximally in given extension would be physically special?

1. Do ramified primes possess exceptional evolutionary fitness and are ramified primes present for lower-dimensional extensions present also for higher-dimensional extensions? If higher extensions are formed as extensions of already existing extensions, this is the case. Hierarchy of polynomials of polynomials would to this kind of hierarchy with ramified primes of starting point polynomials analogous to conserved genes.
2. Quadratic extensions are the simplest ones and could serve as starting point extensions. Polynomials of form  $x^2 - c$  are the simplest among them. Discriminant is now  $D = -4c$ .
3. Why  $c = M_n = 2^n - 1$  allowing  $p = 2$  and Mersenne prime  $p = M_n$  as ramified primes would be favored? Extension of rationals defined by  $x = 2^n$  is non-trivial for odd  $n$  and is equivalent with extension containing  $\sqrt{2}$ .  $c = M_n = 2^n - 1$  as a small deformation of  $c = 2^n$  gives an extension having both 2 as  $M_n$  as ramified primes.

For  $c = M_n$  the number of ramified primes is smallest possible and equal to 2: why minimal number of ramified primes would give rise to a fittest extension? Why smallest number of fermionic p-adic mass scales assignable to the ramified primes would be the fittest option?

The p-adic length scale corresponding to  $M_n$  would be maximal and mass scale minimal. Could one think that other quadratic extension are unstable against transforming to Mersenne extensions with smallest p-adic mass scale?

## 4.2 p-Adic topology and canonical identification

p-Adic physics has become gradually a central part of quantum TGD [K5] and the notion of p-adic probability has already demonstrated its explanatory power in the understanding of elementary particles masses using p-adic thermodynamics [K1]. This encourages the attempt to understand

Shnoll effect in terms of an appropriate modification of probability concept based on p-adic numbers.

p-Adic topology [A2] is characterized by p-adic norm given by  $|x|_p = p^{-k}$  for  $x = p^k(x_0 + \sum_{k>0} x_k p^k)$ ,  $x_0 > 0$ . This notion of nearness differs radically from its real counterpart. For instance, numbers differing by a large power of  $p$  are p-adically near to each other. Therefore p-adic continuity means short range chaos and long range correlations in real sense. One might hope that p-adic notion of nearness allow the existence of p-adic variants of standard probability distributions characterized by rational valued parameters and transcendental numbers existing also p-adically such that these distributions can be mapped to their real counterparts by canonical identification mapping sum of probabilities to the sum of the images of the probabilities.

In the case of p-adic thermodynamics [K1] the map of real integers to p-adic integers and vice versa relies on canonical identification and its various generalizations and canonical identification is also now a natural starting point.

1. The basic formula for the canonical identification for given prime  $p$  characterizing p-adic number field  $Q_p$  is obtained by using for a real number  $x$  binary expansion  $x = \sum x_n p^{-n}$ ,  $x_n \in \{0, p-1\}$  analogous to decimal expansion. The map is very simple and given by

$$\sum_n x_n p^{-n} \rightarrow I(x) = \sum_n x_n p^n . \quad (4.1)$$

The map from reals to p-adics is two-valued in the case of real numbers since binary expansion itself is non-unique ( $p = (p-1) \sum_{k>0} p^{-k}$  as the analog of  $1 = .99999..$  for decimal expansion). The inverse of the canonical identification has exactly the same form. Canonical identification maps p-adic numbers to reals in a continuous manner and also the inverse map is continuous apart from the 2-valuedness eliminated if one introduces binary cutoff which is indeed natural when finite measurement resolution is assumed.

2. The first modification of canonical identification replaces binary expansion of real number in powers of  $p$  with expansion in powers of  $p^k$ :  $x = \sum x_n p^{-nk}$ ,  $x_n \in \{0, p^k - 1\}$  and reads as

$$\sum_n x_n p^{-nk} \rightarrow I_k(x) = \sum_n x_n p^{nk} . \quad (4.2)$$

3. A further variant applies to rational numbers. By using the unique representation  $q = r/s$  of given rational number as ratio of co-prime integers one has

$$I_k(q = \frac{r}{s}) = \frac{I_k(r)}{I_k(s)} . \quad (4.3)$$

### 4.3 Quantum integers

TGD based motivation for the notion of quantum integer comes from the fact that the so called hyper-finite factors of type II<sub>1</sub> (HFFs) play a key role in quantum TGD and allow to formulate the notion of finite measurement resolution in terms of inclusions of HFFs [K6] to which the quantum groups assignable to roots of unity are closely related. The findings of Shnoll would therefore relate to the delicacies of quantum measurement theory with finite measurement resolution.

In TGD framework one can consider modifications of the notion of quantum integer [A3]. One can ask what is the quantum counterpart of p-adic integer. One can also wonder whether prime decomposition of ordinary integers could generalize in some manner. Ordinary integers are positive and one can ask whether quantum integers should also have this property.

The quantum group is parameterize quantum phase

$$q = q_m = \exp(i\phi_m) , \quad \phi_m = \frac{\pi}{m} . \quad m \geq 3 \quad (4.4)$$

appear in TGD framework and the long standing intuitive expectation has been that there might exist a deep connection between p-adic length scale hypothesis and quantum phases defined by roots of unity defining algebraic extensions of p-adic numbers.

### 4.3.1 The standard definition of quantum integer has problems

The first thing to do is to see whether the standard notions of quantum integer and quantum factorial [A3] could allow to get rid of the problems.

The definition of quantum integers for  $q = q_m$  is given by

$$n_{q_m} = \frac{q_m^n - \bar{q}_m^n}{q_m - \bar{q}_m} = \frac{\sin(n\phi_m)}{\sin(\phi_m)} . \quad (4.5)$$

For  $n \ll m$  one has

$$n_{q_m} \simeq n . \quad (4.6)$$

These quantum integers are real. This property makes quantum integers a good candidate if one wants to generalize the notion of Poisson distribution and more generally, any probability distribution  $P(n|\lambda_i)$  parameterized by rationals. The rule would be very simple: replace all integers by their quantum counterparts:  $n \rightarrow n_q$ .

The proposal has however some problematic features.

1.  $n_q$  is negative for  $n \bmod 2m > m$  so that in the case of Poisson distribution modified by replacing  $n!$  by its quantum counterpart one would have negative probabilities in real context. In the p-adic context there is no well-defined notion of negative number so that one might avoid this difficulty if one can map p-adic probabilities to positive real probabilities. Quantum integers have unit norm p-adically so that p-adic Poisson distribution makes sense for  $N_p(\lambda) < 1$ .
2.  $n_{q_m}$  vanishes for  $n = m$  always. Therefore  $n_q!$  defined as a product of quantum integers smaller than  $n$  vanishes for all  $n > m$ . One way out is to restrict the values of  $n$  to satisfy  $n < m$ . This number theoretic cutoff would mean in the p-adic case that the sum of p-adic probabilities is finite without the condition  $N_p(\lambda) < 1$ .

### 4.3.2 Quantum p-adicity guarantees positivity of quantum integers

The elegant solution to the negativity problem comes from a simple observation. If one has  $m = p$ , the quantum integers  $n_{q_m} = (q^n - q^{-n})(q - q^{-1})$  ( $q_p = \exp(i\pi/p)$ ) are positive for  $n < p$ , vanish for  $n = p$  and become negative at  $n = p + 1$ . Scaling invariance  $n \rightarrow np$  is not obtained. One has however more general invariance. For  $m = p$ , the integers  $0 \leq k < p$  and therefore the phases  $q_m^k$  behave elements of finite field  $G_p$  and the scaling  $r \bmod p \neq 0$  for the quantum factor of  $\Psi$  acts as a permutation in the set formed by them. One has  $Z_p$  invariance. Also translations  $n \rightarrow n + r$  act as symmetries of  $G_p$ .

This suggests the interpretation of  $n$  as a p-adic integer so that one can write  $n = \sum n_k p^k$ . Assume  $m = p$ . The binary coefficients  $0 \leq n_k < p = m$  satisfy  $n_k < m$  so that their quantum counterparts are positive. One can regard them as numbers in algebraic extensions of p-adic numbers defined by the  $q_m$ . One can call these numbers quantum p-adics.

One can also map quantum p-adics to reals by using identification map as such. The same map is used also for algebraic extensions of p-adic numbers. There are however restrictions on  $p$  and  $m$ :  $m$  must be such that  $q_m$  does not allow representation as non-vanishing ordinary p-adic number. For  $p = m$  the condition is satisfied.

### 4.3.3 Should quantum integers allow a factorization to quantum primes

Physics as a generalized number theory vision [K5] suggests a way to circumvent above described problems.

1. Quantum integers defined in the standard manner do not respect the decomposition of integers to a product of factors- that is one does not have

$$(mn)_q = m_q n_q . \quad (4.7)$$

The preferred nature of the quantum phases associated with primes in TGD context however suggests that one should guarantee this property by hand by simply defining the quantum integer as a product of quantum integers associated with its prime factors:

$$n_q \equiv \prod (p_i)_q^{n_i} \text{ for } n = \prod p_i^{n_i} . \quad (4.8)$$

This would guarantee that the notion of primeness and related notions crucial for p-adic physics would make sense also for quantum integers. Note that this deformation would not be made for the exponents of integers for which sum is the natural operation.

2. This definition has problems. The quantum primes can have negative sign and if  $m$  is prime, quantum prime  $p_{q^m}$  vanishes. For  $m = p$  allowing the definition of quantum p-adics and their real counterparts, one can restrict prime decomposition to the primes appearing as factors of the binary digits  $k < p$  of quantum primes.

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