# About TGD counterparts of classical field configurations in Maxwell's theory

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**Abstract**

Classical physics is an exact part of TGD so that the study of extremals of dimensionally reduces 6-D Kähler action can provide a lot of intuition about quantum TGD and see how quantum-classical correspondence is realized. In the following the goal is to develop further understanding about TGD counterparts of the simplest field configurations in Maxwell's theory.

In this article $CP_2$ type extremals will be considered from the point of view of quantum criticality and the view about string world sheets, their lightlike boundaries as carriers of fermion number, and the ends as point like particles as singularities acting as sources for minimal surfaces satisfying non-linear generalization of d'Alembert equation.

I will also discuss the delicacies associated with $M^4$ Kähler structure and its connection with what I call Hamilton-Jacobi structure and with $M^8$ approach based on classical number fields. I will argue that the breaking of CP symmetry associated with $M^4$ Kähler structure is small without any additional assumptions: this is in contrast with the earlier view.

The difference between TGD and Maxwell's theory and consider the TGD counterparts of simple em field configurations will be also discussed. Topological field quantization provides a geometric view about formation of atoms as bound states based on flux tubes as correlates for binding, and allows to identify space-time correlates for second quantization. These considerations force to take seriously the possibility that preferred extremals besides being minimal surfaces also possess generalized holomorphy reducing field equations to purely algebraic conditions and that minimal surfaces without this property are not preferred extremals. If so, at microscopic level only $CP_2$ type extremals, massless extremals, and string like objects and their deformations would exist as preferred extremals and serve as building bricks for the counterparts of Maxwellian field configurations and the counterparts of Maxwellian field configurations such as Coulomb potential would emerge only at the QFT limit.

# 1 Introduction

Classical physics is an exact part of TGD so that the study of extremals of dimensionally reduces 6-D Kähler action can provide a lot of intuition about quantum TGD and see how quantum-classical correspondence is realized. In the following the goal is to develop further understanding about TGD counterparts of the simplest field configurations in Maxwell's theory.

## 1.1 About differences between Maxwell's ED and TGD

TGD differs from Maxwell's theory in several important aspects.

1. The TGD counterparts of classical electroweak gauge potentials are induced from component of spinor connection of $CP_2$. Classical color gauge potentials corresponds to the projections of Killing vector fields of color isometries.

2. Also $M^4$ has Kähler potential, which is induced to space-time surface and gives rise to an additional $U(1)$ force. The couplings of $M^4$ gauge potential to quarks and leptons are of same sign whereas the couplings of $CP_2$ Kähler potential to B and L are of opposite sign so

that the contributions to 6-D Kähler action reduce to separate terms without interference term.

Coupling to induced $M^4$ Kähler potential implies CP breaking. This could explain the small CP breaking in hadronic systems and also matter antimatter asymmetry in which there are opposite matter-antimatter asymmetries inside cosmic strings and their exteriors respectively. A priori it is however not obvious that the CP breaking is small.

3. General coordinate invariance implies that there are only 4 local field like degrees of freedom so that for extremals with 4-D $M^4$ projection corresponding to GRT space-time both metric, electroweak and color gauge potentials can be expressed in terms four $CP_2$ coordinates and their gradients. Preferred extremal property realized as minimal surface condition means that field equations are satisfied separately for the 4-D Kähler and volume action reduces the degrees of freedom further.

   If the $CP_2$ part of Kähler form is non-vanishing, minimal surface conditions can be guaranteed by a generalization of holomorphy realizing quantum criticality (satisfied by known extremals). One can say that there is no dependence on coupling parameters. If $CP_2$ part of Kähler form vanishes identically, the minimal surface condition need not be guaranteed by holomorphy. It is not at all clear whether quantum criticality and preferred extremal property allow this kind of extremals.

4. Supersymplectic symmetries act as isometries of "world of classical worlds" (WCW). In a well-defined sense supersymplectic symmetry generalizes 2-D conformal invariance to 4-D context. The key observation here is that light-like 3-surfaces are metrically 2-D and therefore allow extended conformal invariance.

   Preferred extremal property realizing quantum criticality boils down to a condition that sub-algebra of SSA and its commutator with SSA annihilate physical states and that corresponding Noether charges vanish. These conditions could be equivalent with minimal surface property. This implies that the set of possible field patterns is extremely restricted and one might talk about "archetypal" field patterns analogous to partial waves or plane waves in Maxwell's theory.

5. Linear superposition of the archetypal field patterns is not possible. TGD however implies the notion of many-sheeted space-time and each sheet can carry its own field pattern. A test particle which is space-time surface itself touches all these sheets and experiences the sum of the effects caused by fields at various sheets. Effects are superposed rather than fields and this is enough. This means weakening of the superposition principle of Maxwell's theory and the linear superposition of fields at same space-time sheet is replaced with set theoretic union of space-time sheets carrying the field patterns whose effects superpose.

   This observation is also essential in the construction of QFT limit of TGD. The gauge potentials in standard model and gravitational field in general relativity are superpositions of those associated with space-time sheets idealized with slightly curved piece of Minkowski space $M^4$.

6. An important implication is that each system has field identity - field body or magnetic body (MB). In Maxwell's theory superposition of fields coming from different sources leads to a loss of information since one does not anymore now which part of field came from particular source. In TGD this information loss does not happen and this is essential for TGD inspired quantum biology.

   **Remark**: An interesting algebraic analog is the notion of co-algebra. Co-product is analogous to reversal of product AB= C in the sense that it assigns to C and a linear combination of products $\sum A_i \otimes B_i$ such that $A_iB_i = C$. Quantum groups and co-algebras are indeed important in TGD and it might be that there is a relationship. In TGD inspired quantum biology magnetic body plays a key role as an intentional agent receiving sensory data from biological body and using it as motor instrument.

7. I have already earlier considered a space-time correlate for second quantization in terms of sheets of covering for $h_{eff} = nh_0$. In [L2] it is proposed that $n$ factorizes as $n = n_1n_2$

such that $n_1$ ($n_2$) is the number sheets for space-time surface as covering of $CP_2$ ($M^4$). One could have quantum mechanical linear superposition of space-time sheets, each with a particular field pattern. This kind state would correspond to single particle state created by quantum field in QFT limit. For instance, one could have spherical harmonic for orientations of magnetic flux tube or electric flux tube.

One could also have superposition of configurations containing several space-time sheets simultaneously as analogs of many-boson states. Many-sheeted space-time would correspond to this kind many-boson states. Second quantization in quantum field theory (QFT) could be seen as an algebraic description of many-sheetedness having no obvious classical correlate in classical QFT.

8. Flux tubes should be somehow different for gravitational fields, em fields, and also weak and color gauge fields. The value of $n = n_1 n_2$ [L2] for gravitational flux tubes is very large by Nottale formula $\hbar_{eff} = \hbar_{gr} = GMm/v_0$. The value of $n_2$ for gravitational flux tubes is $n_2 \sim 10^7$ if one accepts the formula $G = R^2/n_2\hbar$. For em fields much smaller values of $n$ and therefore of $n_2$ are suggestive. There the value of $n$ measuring in adelic physics algebraic complexity and evolutionary level would distinguish between gravitational and em flux tubes.

   Large value of $n$ would mean quantum coherence in long scales. For gravitation this makes sense since screening is absent unlike for gauge interactions. Note that the large value of $h_{eff} = h_{gr}$ implies that $\alpha_{em} = e^2/4\pi\hbar_{eff}$ is extremely small for gravitational flux tubes so that they would indeed be gravitational in an excellent approximation.

   $n$ would be the dimension of extension of rationals involved and $n_2$ would be the number space-time sheets as covering of $M^4$. If this picture is correct, gravitation would correspond to much larger algebraic complexity and much larger value of Planck constant. This conforms with the intuition that gravitation plays essential role in the quantum physics of living matter.

   There are also other number theoretic characteristics such as ramified primes of the extension identifiable as preferred p-adic primes in turn characterizing elementary particle. Also flux tubes mediating weak and strong interactions should allow characterization in terms of number theoretic parameters. There are arguments that in atomic physics one has $h = 6h_0$. Since the quantum coherence scale of hadrons is smaller than atomic scale, one can ask whether one could have $h_{eff} < h$.

In this article $CP_2$ type extremals will be considered from the point of view of quantum criticality and the view about string world sheets, their lightlike boundaries as carriers of fermion number, and the ends as point like particles as singularities acting as sources for minimal surfaces satisfying non-linear generalization of d'Alembert equation.

I will also discuss the delicacies associated with $M^4$ Kähler structure and its connection with what I call Hamilton-Jacobi structure and with $M^8$ approach based on classical number fields. I will argue that the breaking of CP symmetry associated with $M^4$ Kähler structure is small without any additional assumptions: this is in contrast with the earlier view [K2].

The difference between TGD and Maxwell's theory and consider the TGD counterparts of simple em field configurations will be also discussed. Topological field quantization provides a geometric view about formation of atoms as bound states based on flux tubes as correlates for binding, and allows to identify space-time correlates for second quantization. These considerations force to take seriously the possibility that preferred extremals besides being minimal surfaces also possess generalized holomorphy reducing field equations to purely algebraic conditions and that minimal surfaces without this property are not preferred extremals. If so, at microscopic level only $CP_2$ type extremals, massless extremals, and string like objects and their deformations would exist as preferred extremals and serve as building bricks for the counterparts of Maxwellian field configurations and the counterparts of Maxwellian field configurations such as Coulomb potential would emerge only at the QFT limit.

# 2 $CP_2$ type extremals as ultimate sources for fields and singularities

$CP_2$ type extremals have Euclidian signature of induced metric and therefore represent the most radical deviation from Maxwell's ED, gauge theories, and GRT. $CP_2$ type extremal with light-like geodesic as $M^4$ projection represents a model for wormhole contact. The light-like orbit of partonic 2-surface correspond to boundary between wormhole contact and Minkowskian region and is associated with both throats of wormhole contact. The throats of wormhole contact can carry part of a boundary of string world sheet connecting the partonic orbits associated with different particles. These light-like lines can carry fermion number and would correspond to lines of TGD counterparts of twistor diagrams.

These world lines would correspond to singularities for the minimal surface equations analogous to sources of massless vector fields carrying charge [L1, L3]. These singularities would serve as ultimate sources of classical em fields. Various currents would consist of wormhole throat pairs representing elementary particle and carrying charges at the partonic orbits. Two-sheetedness is essential and could be interpreted in terms of a double covering formed by space-time sheet glued along their common boundary. This necessary since space-time sheet has a finite size being not continuable beyond certain minimal size as preferred extremal since some of the real coordinates would become complex.

## 2.1 Quantum criticality for $CP_2$ type extremals

TGD predicts a hierarchy of quantum criticalities. The increase in criticality means that some space-time sheets for space-time surface regarded as a covering with sheets related by Galois group of extension of rationals degenerate to single sheet. The action of Galois group would reduce to that for its subgroup.

This is analogous to the degeneration of some roots of polynomial to single root and in $M^8$ representation space-time sheets are indeed quite concretely roots of octonionic polymomial defined by vanishing of real or imaginary part in the decomposition $o = q_1 + iq_2$ of octonion to a sum quaternionic real and imaginary parts.

The hierarchy of criticalities is closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n = n_1n_2$ , where $n_1$ corresponds to number of sheets as covering over $CP_2$ and $n_2$ as covering over $M^4$. One can also consider special cases in which $M^4$ projection has dimension $D < 4$. The proposal is that $n$ corresponds to the dimension of Galois group for extension of rationals defining the level of dark matter hierarchy. If $n$ is prime, one has either $n_1 = 1$ or $n_2 = 1$.

It seems that the range of $n_2$ is rather limited since the expression for Newton's constant as $G = R^2/n_2\hbar$ varies in rather narrow range. If the covering has symmetries assignable to some discrete subgroup of $SU(3)$ acting as isometries of $CP_2$ this could be understood. The increase of criticality could mean that $n_1$ or $n_2$ or both are reduced.

What is the position of $CP_2$ type extremals in the hierarchies of Planck constants and quantum criticalities?

1. Consider first $n_2$. $CP_2$ type extremal have 1-D geodesic line as $M^4$ projection. The light-like geodesic as 1-D structure could be interpreted as covering for which two geodesic lines along the orbits of opposite throats of wormhole contact form a kind of time loop. In this case one would have $n_2 = 2$ and one could have $n = 2p$, $p$ prime.

   In this sense $CP_2$ type extremal or at least its core would be maximally critical. Deformations replacing the light-like geodesic as projection with higher-D region of $M^4$ presumably reduce criticality and one has $n_2 > 2$ is obtained. Whether this is possible inside wormhole contact is not clear. One can imagine that as one approaches partonic 2-surface, the criticality and degeneration increase in $CP_2$ degrees of freedom step by step and reach maximum in its core. This would be like realization of Thom's catastrophe involving parts with various degrees of criticalities.

   At the flux tubes mediating gravitational interaction $n_2 \sim 10^7$ would hold true in the exterior of associated $CP_2$ type extremals. This would suggests that $CP_2$ type extremals have maximal criticality in $M^4$ degrees of freedom and $M^4$ covering reduces to 2-fold covering for wormhole contacts.

2. What about criticality as $n_1$-fold covering of $CP_2$. This covering corresponds to a situation in which $CP_2$ coordinates as field in $M^4$ have given values of $CP_2$ coordinates $n_1$ times. A lattice like structure formed by $n_1$ wormhole contacts is suggestive. $n_1$ can be arbitrary large in principle and the gravitational Planck constant $h_{gr}/h_0 = n_1 n_2$ would correspond to this situation. Singularities would now correspond to a degeneration of some wormhole contacts to single wormhole contact and could have interpretation in terms of fusion of particles to single particle. One might perhaps interpret elementary particle reaction vertices as catastrophes.

Wormhole contacts can be regarded as $CP_2$ type extremals having two holes corresponding to the 3-D orbits of wormhole contacts. Mathematician would probably speak of a blow up. $CP_2$ type extremals is glued to surrounding Minkowskian space-time sheets at the 3-D boundaries of these holes. At the orbit of partonic 2-surface the induced 4-metric degenerates to 3-D metric and 4-D tangent space becomes metrically 3-D. Light-likeness of the $M^4$ projection would correspond to this. For $CP_2$ type extremal 3 space-like $M^4$ directions of Minkowskian region would transmute to $CP_2$ directions at the light-like geodesic and time direction would become light-like. This is like graph of function for which tangent becomes vertical. For deformations of $CP_2$ type extremals this process could take place in several steps, one dimension in given step. This process could take place inside $CP_2$ or outside it depending on which order the transmutation of dimensions takes place.

# 3 Delicacies associated with $M^4$ Kähler structure

Twistor lift forces to assume that also $M^4$ possesses the analog of Kähler form, and Minkowskian signature does not prevent this [K2]. $M^4$ Kähler structure breaks CP symmetry and provides a very attractive manner to break CP symmetry and explain generation of matter antimatter symmetry and CP breaking in hadron physics. The CP breaking is very small characterized by a dimensionless number of order $10^{-9}$ identifiable as photon/baryon ratio. Can one understand the smallness of CP breaking in TGD framework?

## 3.1 Hamilton-Jacobi structure

Hamilton-Jacobi structure [K1] can be seen as a generalization of complex structure and involves a local but integrable selection of subspaces of various dimension for the tangent space of $M^4$. Integrability means that the selected subspaces are tangent spaces of a sub-manifold of $M^4$. $M^8-H$ duality allows to interpret this selection as being induced by a global selection of a hierarchy of real, complex, and quaternionic subspaces associated with octonionic structure mapped to $M^4$ in such a manner that this global selection becomes local at the level of $H$.

1. The 4-D analog of conformal invariance is due to very special conformal properties of light-like 3-surfaces and light-cone boundary of $M^4$. This raises hopes about construction of general solution families by utilizing the generalized form of conformal invariance. Massless extremals (MEs) in fact define extremely general solution family of this kind and involve light-like direction vector $k$ and polarization vector $\epsilon$ orthogonal to it defining decomposition $M^4 = M^2 \times E^2$. I have proposed that this decomposition generalizes to local but integrable decomposition so that the distributions for $M^2$ and $E^2$ integrate to string world sheets and partonic 2-surfaces.

2. One can have decomposition $M^4 = M^2 \times E^2$ such that one has Minkowskian analog of conformal symmetry in $M^2$. This decomposition is defined by the vectors $k$ and $\epsilon$. An unproven conjecture is that these vectors can depend on point and the proposed Hamilton-Jacobi structure would mean a *local* decomposition of tangent space of $M^4$ , which is integrable meaning that local $M^2$s integrate to string world sheet in $M^4$ and local $E^2$s integrate to closed 2-surface as special case corresponds to partonic 2-surface. Generalizing the terminology, one could talk about family of partonic surfaces. These decompositions could define families of exremals.

   An integrable decomposition of $M^4$ to string world sheets and partonic 2-surfaces would characterize the preferred extremals with 4-D $M^4$ projection. Integrable distribution would mean assignment of partonic 2-surface to each point of string world sheet and vice versa.

3. $M^4$ Kähler form defines unique decomposition $M^2 \times E^2$. This is however not consistent Lorentz invariance. To cure this problem one must allow moduli space for $M^4$ Kähler forms such that one can assign to each Hamilton-Jacobi structure $M^4$ Kähler form defining the corresponding integrable surfaces in terms of light-like vector and polarization vector whose directions depend on point of $M^4$.

   This looks strange since the very idea is that the imbedding space if unique. However, this local decomposition could be secondary being associated only with $H = M^4 \times CP_2$ and emerge in $M^8 - H$ duality mapping of space-time surfaces $X^4 \subset M^8$ to surfaces in $M^4 \times CP_2$. There is a moduli space for octonion structures in $M^8$ defined as a choice of preferred time axis $M^1$ (rest system), preferred $M^2$ defining hypercomplex place and preferred direction (light-like vector), and quaternionic plane $M^2 \times E^2$ (also polarization direction is included). Lorentz boosts mixing the real and imaginary octonion coordinates and changing the direction of time axis give rise to octonion structures not equivalent with the original one.

   Thus the choice $M^1 \subset M^2 \subset M^4 = M^2 \times E^2 \subset M^8$ is involved with the definition of octonion structure and quaternionion structure. The image of this decomposition under $M^8 - H$ duality mapping quaternionic tangent space of $X^4 \subset M^8$ containing $M^1$ and $M^2$ as sub-spaces would be such that the image of $M^1 \subset M^2 \subset M^2 \times E^2$ depends on point of $M^4 \subset H$ in integrable manner so that Hamilton-Jacobi structure in $H$ is obtained.

Also $CP_2$ allows the analog of Hamilton-Jacobi structure as a local decomposition integrating to a family of geodesic spheres $S^2_I$ as analog of partonic 2-surfaces with complex structure and having at each point as a fiber different $S^2_I$ - these spheres necessary intersect at single point. This decomposition could correspond to the 4-D complex structure of $CP_2$ and complex coordinates of $CP_2$ would serve as coordinates for the two geodesic spheres.

Could one imagine decompositions in which fiber is 2-D Lagrangian manifold - say $S^2_{II}$ - with vanishing induced Kähler form and not possessing induced complex structure? $S^2_{II}$ does not have complex structure as induced complex structure and is therefore analogous to $M^2$. $S^2_{II}$ coordinates would be functions of string world sheet coordinates (in special as analytic in hypercomplex sense and describing wave propagating with light-velocity). $S^2_I$ coordinates would be analytic functions of complex coordinates of partonic 2-surface.

## 3.2 CP breaking and $M^4$ Kähler structure

The CP breaking induce by $M^4$ Kähler structure should be small. Is this automatically true or must one make some assumptions to achieve this.

Could one guarantee this by brute force by assuming $M^4$ and $CP_2$ parts of Kähler action to have different normalizations. The proposal for the length scale evolution of cosmological constant however relies on almost cancellation $M^4$ induced Kähler forms of $M^4$ and $CP_2$ parts due to the fact that the induced forms differ from each other by a rotation of the twistor sphere $S^2$. The $S^2$ part $M^4 \times S^2$ Kähler for can have opposite with respect to $T(CP_2) = SU(3)/U(1) \times U(1)$ Kähler so that for trivial rotation the forms cancel completely. If the normalizations of Kähler actions differ this cannot happen at the level of 4-D Kähler action.

To make progress, it is useful to look at the situation more concretely.

1. Kähler action is dimensionless. The square of Kähler form is metric so that $J_{kl}J^{kl}$ is dimensionless. One must include to the 4-D Kähler action a dimensional factor $1/L^4$ to make it dimensionless. The natural choice for $L$ is as the radius $R$ of $CP_2$ geodesic sphere to radius of twistor spheres for $M^4$ and $CP_2$. Note however that there is numerical constant involved and if it is changed there must be a compensating change of Kähler coupling strength. Therefore $M^4$ contribution to action is proportional to the volume of $M^4$ region using $R^4$ as unit. This contribution is very large for macroscopic regions of $M^4$ unless self-duality of $M^4$ Kähler form would not cause cancellation ($E^2 - B^2 = 0$).

2. What about energy density? The naive expectation based on Maxwell's theory is that the energy density assignable to $M^4$ Kähler form is by self-duality proportional to $E^2 + B^2 = 2E^2$ and non-vanishing. By naive order of magnitude estimate using Maxwellian formula for the energy of this kind extremal is proportional to $Vol_3/R^4$ and very large. Does this exclude

these extremals or should one assume that they have very small volume? For macroscopic lengths of one should assume extremely thin MEs with thickness smaller than $R$. Could one have 2-fold covering formed by gluing to copies of very thin MEs together along their boundaries. This does not look feasible.

Luckily, the Maxwellian intuition fails in TGD framework. The Noether currents associated in presence of $M^4$ Kähler action involve also a term coming from the variation of the induced $M^4$ Kähler form. This term guarantees that canonical momentum currents as $H$-vector fields are orthogonal to the space-time surface. In the case of $CP_2$ type extremals this causes the cancellation of the canonical momentum currents associated with Kähler action and corresponding contributions to conserved charges. The complete symmetry between $M^4$ and $CP_2$ and also physical intuition demanding that canonically imbedded $M^4$ os vacuum require that cancellation takes place also for $M^4$ part so that only the term corresponding to cosmological constant remains.

## 3.3 $M^4$ Kähler form and CP breaking for various kinds of extremals

I have considered already earlier the proposal that CP breaking is due to $M^4$ Kähler form [K2]. CP breaking is very small and the proposal inspired by the Cartesian product structure of the imbedding space and its twistor bundle and also by the similar decomposition of $T(M^4) = M^4 \times S^2$ was that the coefficient of $M^4$ part of Kähler action can be chosen to be much smaller than the coefficient of $CP_2$ part. The proposed mechanism giving rise to p-adic length scale evolution of cosmological constant however requires that the coefficients of are identical. Luckily, the CP breaking term is automatically very small as the following arguments based on the examination of various kinds of extremals demonstrate.

1. For $CP_2$ type extremals with light-like $M^4$ geodesics as $M^4$ projection the induced $M^4$ Kähler form vanishes so that there is no CP breaking. For small deformations $CP_2$ type extremals thickening the $M^4$ projection the induced $M^4$ Kähler form is non-vanishing. An attractive hypothesis is that the small CP breaking parameter quantifies the order of magnitude of the induced $M^4$ Kähler form. This picture could allow to understand CP breaking of hadrons.

2. Canonically imbedded $M^4$ is a minimal surface. A small breaking of CP symmetry is generated in small deformations of $M^4$. In particular, for massless extremals (MEs) having 4-D $M^4$ projection the action associated with $M^4$ part of Kähler action vanishes at the $M^4$ limit when the local polarization vector characterizing ME approaches zero. The small CP breaking is characterized by the size of the polarization vector $\epsilon$ giving a contribution to the induced metric. This conforms with the perturbative CP breaking.

3. String like objects of type $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where $X^2$ is minimal surface and $Y^2$ is 2-surface in $CP_2$. The $M^4$ projection contains only electric part but no magnetic part. The $M^4$ part of action is proportional to the volume $Y^2$ and therefore very small. This in turn guarantees smallness of CP breaking effects.

   (a) If $Y^2$ is homologically non-trivial (magnetic flux tube carries monopole flux), $CP_2$ part of action is large since action density is proportional $1/\sqrt{det(g_2)}$ for $Y^2$ and therefore large. The thickening of the flux tube however reduces the value of the action by flux conservation as discussed already earlier.

   $M^4$ and $CP_2$ contributions to the actions are of opposite sign but $M^4$ contribution os however very small as compared to $CP_2$ contribution. One can look the situation in $M^2 \times S^2$ coordinates. The transverse deformation would correspond to the dependence of $E^2$ coordinates on $S^2$ coordinates. The induced Kähler form would give a contribution to the $S^2$ part of induced Kähler form whose size would characterize CP breaking.

   (b) $Y^2$ can be also homologically trivial. In particular, for $Y^2 = S^2_{II}$ the $CP_2$ contribution to the total Kähler action vanishes and only the small $M^4$ contribution proportional to the area of $Y^2$ remains.

# 4 About TGD counterparts for the simplest classical field patterns

What could be the TGD counterparts of typical configurations of classical fields? Since minimal surface equation is a nonlinear generalization of massless field equations, one can hope that the simplest solutions of Maxwell's equations have TGD analogs. The strong non-linearity poses a strong constraint, which can be solved if the extremal allows generalization of holomorphic structure so that field equations are trivially true since they involve in complex coordinates a contraction of tensors of type (1,1) with tensors of type (2,0) or (0,2). It is not clear whether minimal surface property reducing to holomorphy is equivalent with preferred extremal property.

Can one have the basic field patterns such as multipoles as structures with 4-D $M^4$ projection or could it be that flux tube picture based on spherical harmonics for the orientation of flux tube is all that one can have? Same question can be made for radiation fields having MEs as archetypal representatives in TGD framework. What about the possible consistency problems produced by $M^4$ Kähler form breaking Lorentz invariance?

I have considered these questions already earlier. The following approach is just making questions and guesses possibly helping to develop general ideas about the correspondence.

1. In QFT approach one expresses fields as superpositions of partial waves, which are indeed very simple field patterns and the coefficients in the superposition become oscillator operators. What could be the analogs of partial waves in TGD? Simultaneous extremals of Kähler action and volume strongly suggest themselves as carriers of field archetypes but the non-linearity of field equations does not support the idea that partial waves could be realized at classical level as extremals with 4-D $M^4$ projection. A more plausible option is that they correspond to spherical harmonics for the orientation of flux tube carrying say electric flux. Could the flux tubes of various kinds serve as building of all classical fields?

2. String-like objects $X^2 \times Y^2 \subset M^4 \times CP_2$, where string world sheet $X^2$ is minimal surface and $Y^2$ is sub-manifold of $CP_2$ and their deformations in $M^4$ degrees of freedom transversal to $X^2$ and depending on the coordinates $Y^2$ are certainly good candidates for archetypal field configurations.

   $Y^2$ can be homologically trivial and could correspond to Lagrangian sub-manifold. $Y^2$ can also carry homology charge $n$ identifiable as Kähler magnetic charge and correspond to complex sub-manifold of $CP_2$ with complex structure induced from that of $CP_2$.

The simplest option corresponds to geodesic sphere $Y^2 = S^2$. There are two geodesic spheres in $CP_2$ and they correspond to simplest string like objects.

1. $S^2_I$ has Kähler magnetic charge of one unit and the cosmic and its deformations carry monopole flux. These field configurations are not possible in Maxwell's electrodynamics and the proposal is that they appear in all length scales. The model for the formation of galaxies solving also the problem of galactic dark matter relies on long cosmic strings. They are proposed to appear also in biology.

2. $S^2_{II}$ is homologically trivial so that magnetic flux over it vanishes although magnetic field is non-vanishing. Note that although the Kähler magnetic field is vanishing, the electromagnetic ordinary magnetic field is non-vanishing because em field is a combination of Kähler form and component of $CP_2$ curvature form with vanishing weak isospin. The total flux of ordinary magnetic field over $S^2_{II}$ vanishes whereas electric flux can be non-vanishing.

## 4.1 Coulomb fields

By the vanishing of magnetic flux flux tubes for $S^2_{II}$ cannot represent ordinary magnetic field. They can however serve as radial flux tubes carrying electromagnetic flux. Magnetic flux tubes indeed allow time dependent deformations for which the phase angles of $CP_2$ coordinates depend linearly of $M^4$ time coordinate. This would give rise to an archetypal flux tube representation of the electric field created by point charge. Also gravitational flux tubes should correspond to this kind flux tubes emanating radially from the source.

Charge quantization suggests that these flux tubes carry unit charge. In the case of charged elementary particle there would be only single flux tubes but there would be wave function for its orientation having no angular dependence. In principle, this wave function can any spherical harmonic.

Does the orientation angle dependence of flux distribution have any counterpart in Maxwell's theory. One would have the analog of $1/r$ Coulomb potential with the modulus squared of spherical harmonic $Y_{lm}$ modulating it. Could one consider the possibility that in atoms the spherical harmonics for excited states correspond to this kind of distribution for the electric flux coming from nucleus. The probability amplitude for electrons touching the flux tube would inherit this distribution.

For many particle system with large em charge there would be large number of radial flux tubes and the approximation of electric field with Coulomb field becomes natural. In the case of atoms this limit is achieved for large enough nuclear charges. This does not exclude the possibility of having space-time surfaces carrying Coulomb potential in Maxwellian sense: in this case however the field equations cannot solved by holomorphy and quantum criticality might exclude these configurations.

What about gravitation? The notion of gravitational Planck constant requires that Planck mass replaced in TGD framework by $CP_2$ mass defining the unit of gravitational flux - $h_{gr}0GMm/v_0$ cannot be smaller than $h_0$. What happens in systems possessing mass smaller than $CP_2$ mass? Are gravitational flux tubes absent. Is gravitational interaction absent in this kind of systems or is its description analogous to string model description meaning that $h_{gr} = h_0$ for masses smaller than $CP_2$ mass?

## 4.2 Magnetic fields

As such $S^2_{II}$ flux tubes cannot serve as counterparts of ordinary magnetic fields. The flux tubes have now boundary and the current at boundary creates the magnetic field inside the tube. This would mean cutting of a disk $D^2$ from $S^2_{II}$ so that the net magnetic flux becomes non-vanishing.

The assumption has been that genuine boundaries are not possible since conservation laws very probably prevent them (the normal components of canonical momentum currents should vanish at boundaries but this is not possible). This requires that this flux tube must be glued along the boundary of $D^2 \times D^1$ to surrounding space-time surface $X^4$, which has a similar hole. At the boundary of this hole the space-time surface must turn to the direction of $CP_2$ meaning that the dimension of $M^4$ projection is reduced to $D = 2$. Algebraic geometer would talk about blow-up.

Ordinary multipole magnetic field could correspond to spherical harmonic for the orientation of this kind flux tubes. They could also carry electric flux but the em charge could be fractionized. These flux tubes might relate to anyons carrying fractional em charge. Also the fractional charges of quarks could classically correspond to flux tubes mediating both color magnetic field and em flux. The spherical harmonic in question corresponds to that associated with electron in atoms.

## 4.3 Magnetic and electric fields associated with straight current wire

Magnetic and electric fields associated with straight current wire need not allow representation as archetypes since they are obviously macroscopic entities.

1. Is the magnetic field associated with straight current wire representable in terms of extremal with 4-D $M^4$ projection. The magnetic field lines rotate around the current and it is does not seem natural to model it the field in terms of flux tubes. Forget the presence of $M^4$ Kähler form. One can imbed this kind of magnetic field as a surface with 4-D $M^4$ projection and possessing cylindrical symmetry. Line current would correspond to a source of the magnetic field and could be realized as a flux tube carrying em current and topologically condensed to the space-time sheet in question.

   The imbedding however fails at certain critical radius and the assumption is that no boundaries are allowed by conservation laws. Should one glue the structure to the surrounding space-time surface at this radius. In Maxwell's theory one would have surface current in direction opposite to the source cancelling the magnetic field outside. Could this current have interpretation as a return current?

One can also imagine glueing its copy to it along the boundary at critical radius. It would seem that the magnetic fields must have same direction at the boundary and therefore also in interior.

2. What about current ring? Separation of variables is essential for the simplest imbeddings implying a reduction of partial different equations to differential equation. There is rather small number of coordinates system in $E^3$ in which Laplacian allows separation of variables. The metric is diagonal in these coordinates. One example is toroidal coordinates assignable with a current ring having toroidal geometry. This would allow a construction of minimal surface solution in some finite volume. Minimal surface property would *not* reduce to complex analyticity for these extremals and they would be naturally associated $M^4 \times S^2_{II}$.

**Remark**: This kind of extremals are not holomorphic and could be excluded by quantum criticality and preferred extremal property. GRT space-time would be idealization making sense only at the QFT limit of TGD.

## 4.4 Time dependent fields

What about time dependent fields such as the field created by oscillating dipole and radiation fields? One can imagine quantal and classical option.

1. The simplest possibility is reduction to quantum description at single particle level. The dipole current corresponds to a wave function for the source particle system consisting of systems with opposite total charge.

   Spherical harmonics representing multipoles would induce wave function for the orientations of MEs (topological light ray) carrying radial wave. This is certainly the most natural options as far radiation field at large distances from sources is considered. One can also have second quantization in the proposed sense giving rise to multi-photon states and one can also define coherent states.

   One should also understand time dependent fields near sources having also non-radiative part. This requires a model for source such as oscillating dipole. The simplest possibility is that in the case of dipole there are charges of opposite sign with oscillating distance creating Coulomb fields represented in the proposed manner. It is however not obvious that preferred extremals of this kind exist.

2. One can consider also classical description. The model of elementary particle as consisting of two wormhole contacts, whose throats effectively serve as end of monopole flux tubes at the two sheets involved suggests a possible model. If the wormhole contacts carry opposite em charges realized in terms of fermion and antifermions an oscillating dipole could correspond to flux tube whose length oscillates. This means generation of radiation and for elementary particles this would suggest instability against decay. One can however consider excitation which decay to ground states - say for hadrons. For scaled up variants of this structure this would not mean instability although energy is lost and the system must end up to non-oscillating state.

   One possibility is that there are two charges at different space-time sheets connected by wormhole contacts and oscillating by their mutual interaction in harmonic oscillator state. Ground state would be stable and have not dipole moment.

## 4.5 Effectively 2-D systems

In classical electrodynamics effectively 2-D systems are very special in that they allow conformal invariance assignable to 2-D Laplacian.

1. Since minimal surface equation is generalization of massless d'Alambertian and since field equations are trivially true for analytic solutions, one can hope that the basic solutions of 4-D d'Alembertian generalize in TGD framework. This would conform with the universality of quantum criticality meaning that coupling parameters disappear from field equations.

Conformal invariance or its generalization would mean huge variety of field patterns. This suggests that effectively 2-D systems serve as basic building bricks of more complex field configurations. Flux tubes of various kinds would represent basic examples of this kind of surfaces. Also the magnetic end electric fields associated with straight current wire would serve as an example.

2. Are there preferred extremals analogous to the solutions of field equations of general relativity in faraway regions, where they become simple and might allow an analog in TGD framework? If our mathematical models reflect the preferred extremals as archetypal structures, this could be the case.

   Forget for a moment the technicalities related to $M^4$ Kähler form. One can construct a spherically symmetric ansatz in $M^4 \times S^2_{II}$ as a minimal surface for which $\Phi$ depends linearly on time $t$ and $u$ is function of $r$. The ansatz reduces to a highly non-linear differential equation for $u$. In this case hyper-complex analyticity is obviously not satisfied. This ansatz could give the analog of Schwartschild metric giving also the electric field of point charge appearing as source of the non-linear variant of d'Alembertian. It is however far from clear whether this kind extremals is allowed as preferred extremals.

   Under which conditions spherically symmetric ansatz is consistent with $M^4$ Kähler form? Obviously, the $M^4$ Kähler form must be spherically symmetric as also the Hamilton-Jacobi structure it. Suppose local Hamilton-Jacobi structures for which $M^2$s integrate to $t, r$ coordinate planes and $E^2$s integrate to $(\theta, \phi)$ sphere are allowed and that $M^4$ Kähler form defines this decomposition. In this case there are hopes that consistency conditions can be satisfied. Note however that $M^4$ Kähler form defines in this case orthogonal magnetic and electric monopole fields defining an analog of instanton. Can one really allow this or should one exclude the time line with $r = 0$?

   Similar $M^4$ Kähler structure can be associated with cylindrical coordinates and other separable coordinates system. $M^4$ Kähler structure would define Hamilton-Jacobi structure.

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