About the relationship of electroweak and strong interactions in the TGD Universe: part I

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Abstract

The purpose of this three-part article is to develop a more detailed view of the relationship between electroweak and strong interactions at the level of the embedding space H. In the first part of the article the general vision is described. In the second part a concrete model for the Weinberg angle is developed, leading to a vision about the relationship of weak and strong interactions in the TGD framework. In the third part the masses of W and Z^0 and their ratio are estimated, and questions related to the relationship of weak and strong interactions are discussed.

General view. Quantum TGD involves three levels corresponding to embedding space $H = M^4 \times CP_2$, space-time X^4 and "world of classical worlds" (WCW). Holography principle at X^4 level implies the absence of path integral and related divergence problems. Holography = holomorphy (H-H) principle allows an exact solution of field equations for volume action. For Kähler action this is true if the Kähler current is light-like. Also Lagrangian minimal surfaces are allowed as candidates for preferred extremals.

Bosons are not primary fields and correspond at H and WCW levels to bound states of fermions. At H and X^4 level bosons correspond to induced spinor connections and apparently only electroweak gauge bosons are present. These couplings however involve implicitly vector boson couplings identified as $U(1)$ counterparts of gluons by electric-magnetic duality. The challenge is to identify the gluon propagators at the level of X^4.

Fermions at the H level are identified as modes of H spinor fields forming a hierarchy of color multiplets. This predicts a hierarchy of standard model physics. At the X^4 level fermions correspond to induced spinor fields.

Construction of scattering amplitudes. Interactions are contact interactions with contacts defined by the 2-D string world sheets as intersections of space-time surfaces, including self intersections. Fermion lines correspond to intersections of string world sheets and partonic orbits.

Zero energy ontology (ZEO) and the associated mild classical non-determinism are fundamental. The loci of classical non-determinism as intersections of 3-D edges of space-time with 3-D partonic orbits define partonic 2-surfaces as geometric vertices. Feynman graphs are replaced with analogs of Brownian motion for 3-surfaces and associated fermion lines. Fermion propagators are free propagators in H. Bosonic propagators correspond to WCW correlation functions.

QCD serves as a role model. The notions of fragmentation and hadronization are generalized. H level corresponds to observed hadrons and leptons. At X^4 level quark gluon plasma is replaced by induced spinor fields for quarks and leptons. ZEO gives rise to statistical aspects appearing also in QCD.

Standard model and TGD. The Higgs mechanism is a key element of the standard model. p-Adic thermodynamics replaces it at the level of WCW. At the X^4 level massivation is described in terms of warping distinguishing between GRT and TGD.

Kähler-Chern-Simons action (KCS) and corresponding modified Dirac action define the dynamics for the partonic orbits. M^4 possesses an analog of Kähler structure and contributes to this action an additive part.

Color confinement involves two steps. First color triplets/singlets are formed from quark/leptonic color multiplets and then color triplets form hadrons. The color information is coded by WCW wave functions for the position of the fermion line and its edge at the partonic 2-surface. The 3 members of the color triplet are associated with the 3 coordinate patches of CP_2. A generalization of the Weinberg mixing is proposed. The assumption that the emission of photons having wrong helicity with respect to the direction of fermion momentum – lightlike in 8-D sense – vanishes, relates the Weinberg angle and the strong interaction coupling strength α_s to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing.

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1 Introduction

The purpose of this work is to develop a more detailed view of the relationship between electroweak and strong interactions at the level of the embedding space H. The initiative came from Martin Ebner who wanted to find clearcut predictions related to the standard model, in particular Weinberg angle. This led to a project involving Martin and two AI agents, who did extremely valuable work identifying the numerous errors in my paper and pencil calculations and also in the earlier view about Weinberg angle estimated in my thesis [K1] dating back to 1982. The project led to a rather dramatic increase in the understanding of the relationship between electroweak and strong interactions in the TGD framework.

1.1 Steps of progress during recent years

Several big steps of progress in TGD relevant for this goal have occurred during the last few years.

1. The emergence of holography = holomorphy (H-H) principle means that the dynamics of space-time surfaces reduces to algebraic equations [**?**]or volume action and also for Kähler action if Kähler current is assumed to be light-like. The original conjecture that his occurs for general action was too strong [L22]. The generalized holomorphy means that two holomorphic functions depending on one hypercomplex coordinate and 3 complex coordinates define Minkowskian regions of the space-time surfaces as their roots. A key role is played by the generalization of the notion of complex structure to Hamilton-Jacobi structure [L9]. The ansatz allows also to describe Euclidian regions (CP_2 type extremals).

   Also the Dirac equations at the level of $H = M^4 \times CP_2$ [L19] and at the level of space-time surfaces [L18] and also of 3-D light-like partonic orbits are solved by holomorphic ansatz.

2. The spectacular prediction following from the fact that both quarks and leptons move in color partial waves is that an infinite fractal hierarchy of scaled variants of standard model physics should exist [L20, L21, L24, L26].

3. The crucial difference between GRT and TGD is the possibility of warped space-time surfaces for which CP_2 projection is geodesic circle with angle coordinate as function $\Phi = \omega t$ of M^4 time coordinate t. Warping was one of the first discoveries of TGD and the proposal was that they could explain the arrival of several light and neutrino pulses from SN1987A. Warped space-time surfaces are flat but the light-velocity is reduced. In the rest frame, the induced metric is given by $(c_{\#}^2 = 1 - R^2\omega^2, -1, -1, -1)$.

   The notion of warping generalizes to the level of Hamilton-Jacobi structure so that it makes sense for a general surface. Together with the notion of gravitational Planck constant, whose

huge values imply quantum coherence in macroscopic and even in astrophysical scales, warping leads to a model for Allais anomaly and fluctuations of Newton's constant [L23].

A highly attractive idea is that phenomena like reflection and refraction occur at the boundaries of space-time regions with different values of $c_{\#}$ so that these notions would generalize to all particles. H-H principle predicts a slight violation of classical determinism. Space-time surfaces are predicted to be minimal surfaces analogous to soap films spanned by frames, which correspond to 3-D surfaces at which the derivatives of some H coordinates are discontinuous so that $c_{\#}$ is discontinuous. This kind of surfaces would naturally define loci of classical non-determinism at which "small" state function reductions (SSFR) giving rise to cognition would take place.

Warping leads also to the understanding of fermion massivation. The assumption that fermion lines are light-like geodesics of both H and X^4 located at the light-like 3-D parton orbits X^3 implies that fermions are massless as classical particles in H and X^4 but massive as particles in M^4.

The space-time surfaces (or at least string world sheets associated with fermion lines, which appear in the definition of scattering amplitudes) are warped with warping characterized by the parameter ω. A geometric description of fermion massivation suggests itself: the M^4 mass of fermion would correspond to $m = \hbar_{eff}\omega$ [L20]. It turns out that the TGD view of the Higgs mechanism implies that this is not quite correct but the proportionality to $\hbar_{eff}\omega$ is true.

4. Also a geometric view of interactions has emerged. The Bohr orbit would define an 8-D analog of Brownian motion. The 3-D edges at which $c_{\#}$ changes and at which a failure of classical determinism, minimal surface property and smoothness occurs, would be crucial for the definition of scattering amplitudes. The fermionic interaction vertices would be associated with the 2-D partonic surfaces X^2 identified as intersections of the edges Y^3 and partonic orbits X^3.

   The interaction between space-time surfaces is identified as contact interaction. If H-H principle is true, the intersection loci of 4-surfaces (including self-intersections) are 2-D string world sheets whereas the intersections of the latter with partonic orbits are 1-D light-like curves identifiable as fermion lines. The fermion vertices are intersections of fermion lines and partonic 2-surfaces.

5. QCD has emerged as a role model for TGD. The key idea is that incoming many fermion states with bosons described as color singlet bound states of fermions and antifermions represent incoming and outgoing states as generalization of hadrons. These states are defined in terms of second quantized free spinors of H. The propagators at space-time level correspond to the propagators for H spinor fields.

   The notion of quark-gluon plasma is replaced with the many-fermion states defined at space-time surfaces and defined in terms of induced spinor fields obeying modified Dirac equation. Gauge bosons do not appear as primary gauge fields and boson propagators emerge as correlation functions at the level of the "world of classical worlds" (WCW). The functional integral over WCW consisting of slightly non-deterministic "Bohr orbits of 3-surfaces" replaces path integral so that there are no problems with infinities.

6. The basic problem is how to understand fermion pair creation when the fermion fields at the level of H are free. The key observation is that just in the 4-D case exotic smooth structures which are ordinary smooth structures outside 3-D defects are possible [A3, A4, A1]: for the developing TGD interpretation see [L12, L6, L16]. In the annihilation vertex a fermion line would turn backwards in time at the edge of space-time. Pair creation would be reflection in time direction.

1.2 Questions to be answered

There are several questions to be answered.

1. Are SSFRs really state function reductions involving a loss of coherence? A competing idea is that the associated discontinuities give rise to a natural discretization of path integral. TGD inspired theory of consciousness requires the first interpretation. Also QCD as a strange mixture of kinetic theory (parton distribution functions and fragmentation functions) and QCD scattering amplitudes supports this interpretation.

   There are also deep questions related to the notions of BSFRs ("big" SFRs) and SSFRs, in particular to what really occurs in the sequences of SSFRs followed by BSFRs changing of the arrow of geometric time. These questions are discussed in [L27].

2. Bosons appear only in the initial and final states. Boson propagators are not needed at the space-time level. One should understand the WCW counterparts of boson propagators as correlation function between fermion vertices as edges. Here the geometrodynamics of partonic orbits and the dynamics of modified Dirac equation determined by the Kähler-Chern-Simons (K-C-S) action enters the game.

1.3 Application to electroweak and strong interactions

Concrete applications are the best way to make progress and in the second part of the article a model for Weinberg angle is developed. This leads to a nice overall vision consistent with basic standard model understanding of electroweak coupling constants and providing insights to the relationship between weak and strong interactions.

1. The components of the Higgs field can be seen as CP_2 parts of the electroweak gauge potentials. For W and Z^0 the space-time projections of CP_2 parts would give rise to the longitudinal polarization required by the massivation. For photons the space-time projections of fermion lines would be light-like geodesics of M^4 (no warping) and the projection of CP_2 gauge potential would be pure gauge and allow photons to have only two polarizations. The Higgs field as a dynamical degree of freedom would be the CP_2 part of the gauge potential coupling to the vector isospin.

2. p-Adic mass calculations predict the fermion masses and estimates for the boson masses. No potential model is needed to generate Higgs field vacuum expectation value. Fermion masses are predicted rather than fed into the model as fermion-Higgs couplings. Higgs vacuum expectation is however generated.

3. The warping of space-time surfaces distinguishing between GRT and TGD provides a space-time level description for the fermion massivation. Warping means that fermion lines X at light-like partonic orbits X^3 are light-like geodesics of both H and X^3 but massive as geodesics of M^4. The fermion mass is proportional to the frequency parameter ω associated with X via the correspondence $m \propto \hbar_{eff}\omega$.

The collaboration with Martin Ebner stimulated the writing of this article and his generous help made it possible to identify numerous mistakes in the earlier calculations and led to a dramatic increase in the understanding of the theory. The problems were not only related to my poor computational skills. It eventually became clear that the physical picture of TGD related to the standard model was not quite correct.

In the TGD framework, the geometric interaction vertices are partonic 2-surfaces identified as intersections of 3-D parton orbits and 3-D edges of space-time surface. Fermion lines are identified as boundaries of string world sheets as self-intersections of space-time surfaces. They intersect partonic 2-surfaces at a discrete set of points. This description makes it possible to express the basic coupling parameters of the standard model in terms of the wave function for the position of the intersection of the fermion line at the partonic 2-surface proposed to be analogous to the magnetic harmonic oscillator states.

TGD explains the coupling pattern of electroweak interactions but gluon couplings do not appear in the embedding space and space-time variants of the Dirac action. The fact that the holonomy group of CP_2 is identifiable as a subgroup of the color group acting as isometries of CP_2 motivates the conjecture that color interactions and electroweak interactions are parts of the same pattern. The pleasant surprise was that the induced gauge couplings included not only electroweak

gauge potentials but also a $U(1)$ gauge potential assignable to strong interactions and coupling to strong isospin so that the old-fashioned hadron physics makes a come back in the space-time description of strong interactions whereas the embedding space description relies on the notion of color partial waves. The generalization of the Weinberg mixing is proposed. The assumption that the emission of photons having wrong helicity with respect to the direction of fermion momentum – lightlike in 8-D sense – vanishes, relates the Weinberg angle and the strong interaction coupling strength α_s to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing.

This view would give at the space-time level a concrete realization for the idea that strong *resp.* weak interactions represent orbital angular momentum-like *resp.* spin-like aspects of color interactions and that strong interactions relate deeply to electroweak interactions as the old-fashioned hadron physics assumed. I have speculated for decades about a kind of duality between QCD description and low energy hadronic description in terms of strong isospin and it seems that this finding makes possible a precise quantitative realization of this vision.

TGD also predicts the M^4 counterpart of Kähler force as a new $U(1)$ interaction possibly related to gravitation. If this interaction is strong as compared to the gravitational force, it could relate to the hadronic strings with transversal M^4 degrees of freedom. Perhaps the more plausible option is that M^4 Kähler gauge potential generalizes the Newtonian gravitational potential.

2 A general view of the dynamics at the space-time level

Classical TGD is analogous to hydrodynamics in the sense that field equations for action principle defined by general coordinate action expressible in terms of the induced geometry, reduce to the conservation conditions for the Noether currents associated with the isometries of H. Holography = holomorphy principle implies that minimal surface equations are satisfied outside the edges of the space-time surface at which the derivatives of the embedding space coordinates are discontinuous.

1. Holography = holomorphy principle [L11, L17] allows the reduction of field equations to algebraic equations in Minkowskian regions of space-time surface X^4 and allows the solution of field equations also in the Euclidean regions emerging as blow-ups, using the language of algebraic geometry.

   Holography = holomorphy principle implies that the field equations are separately satisfied for the metric and Kähler contributions outside the singularities so that one has minimal surfaces outside singularities. This statement generalizes to any action which is general coordinate invariant and expressible in terms of induced geometry. The reason is that the contribution of the metric variation to the field equations involves a contraction of complex tensors of type (1,1) and (1,0)+(0,1). The variation with respect to the Kähler form gives a contraction of light-like current with a light-like vector and also vanishes. At edge singularities and partonic orbits holomorphy, smoothness and minimal surface property fails and these surfaces are analogous to the frames spanning soap films. The field equations reduce to continuity conditions at the singularities.

2. Boundary conditions stating the conservation of the isometry charges replace field equations at the edges. Edges are possible only if the action contains besides the volume term also additional parts, say Kähler action. Also boundaries can be considered. The cautious assumption has been that boundaries are not physically acceptable (see however [L7]). If boundary components with non-trivial homology charge can appear (boundary conditions might prevent their presence) their homology charges must sum up to zero.

3. The twistor lift of TGD [K10, K12, K13] [L4, L5] allows an alternative approach based on the identification of the 6-D analog of twistor space of X^4 as a sphere bundle over X^4 and identified as a 6-D surface in the 12-D Cartesian product of the twistor spaces $T(M^4)$ and $T(CP_2)$. Sphere bundle structure requires a dimensional reduction for the field equations and the 6-D Kähler action is reduced to a sum of 4-D Kähler action and volume term having interpretation in terms of a cosmological constant, which depends on p-adic length scale and approaches zero in long scales.

The nice feature of the twistor lift is that it fixes TGD completely. Only the twistor spaces of M^4 and CP_2 have Kähler structure [A2]. There are also arguments supporting the view that the twistor lift is not necessary and has a counterpart at the level of space-time surface. The twistor spheres of the twistor spaces of M^4 and CP_2 could correspond to the homologically trivial sphere around the tip of the line connecting the tips of CD as a hole in CD and of CP_2.

Field equations reduce to local conservation conditions for isometry currents. Besides isometry currents also their super counterparts associated with the modified Dirac action are conserved.

1. A general expression for the isometry currents [K2] for a given action is given in terms of canonical momentum currents Π_k^α, whose covariant divergence vanishes when field equations are satisfied: $D_\alpha \Pi_k^\alpha = 0$. This form gives rise to an equivalent form of field equations as local conservation of Noether currents associated with isometries of H. Let j_A^k be the Lie algebra generator of the symmetry

$$\begin{aligned} J_A^\alpha &= \Pi_k^\alpha j_A^k \ , \\ \Pi_k^\alpha &= \frac{\partial L}{\partial_{\partial_\alpha h^k}} \\ \partial_\alpha J_A^\alpha \sqrt{g} &= 0 \ . \end{aligned} \tag{2.1}$$

2. For the volume action density proportional to $\sqrt{g}$, the canonical momentum currents are apart from the multiplicative constant

$$\Pi^{\alpha,k} = g^{\alpha\beta}\partial_\alpha h^k \ . \tag{2.2}$$

The vanishing of the covariant divergence gives rise to minimal surface equations as analogs of massless field equations. The orbit of 3-surface is a generalization of the geodesic line as an orbit of a point-like particle to an orbit of a particle as 3-surface.

3. It seems now clear that both M^4 and CP_2 Kähler forms contribute to the induced Kähler form appearing in the Kähler action so that one has $J_{\alpha\beta} = J_{\alpha\beta}(M^4) + J_{\alpha\beta}(CP_2)$. Kähler action density is proportional to $J^{\alpha\beta}J_{\alpha\beta}\sqrt{g}$. The contribution of Kähler action to canonical momentum currents contains contributions coming from the variation of the action with respect to the induced metric and induced Kähler form.

$$\begin{aligned} \Pi_g^{\alpha,k} &= \frac{\partial L\sqrt{g}}{\partial_{g_{\alpha\beta}}}\partial_\beta h^k = T^{\alpha\beta}\partial_\beta h^k\sqrt{g} \ , \\ \Pi_J^{\alpha,k} &= \frac{\partial L}{\partial_{J_{\alpha\beta}}} J_l^k \partial_\beta h^l \sqrt{g} = 2J^{\alpha\beta}J_l^k\partial_\beta h^l\sqrt{g} \ , \end{aligned} \tag{2.3}$$

4. Kähler-Chern-Simons (K-C-S) action is the only possible action for light-like partonic orbits since the formal dependence on the singular metrically 2-D metric disappears and one has an analog of topological QFT. C-K-S action could emerge as a boundary term associated with the instanton term for Kähler action, which is total divergence. It is needed as a modified Dirac action associated with the partonic orbit and would give rise to modified Dirac action which can be restricted to fermion lines.

   The twistor lift of TGD strongly suggests that also M^4 has Kähler structure. I have christened it the Hamilton-Jacobi (H-J) structure [L9]. H-J structure involves an integrable distribution of orthogonal planes M^2 and E^2 of M^4 , which are hyper-complex and complex respectively. The coordinate lines associated with the hypercomplex coordinate u and its conjugate v have light-like tangent vectors. The notion of warping strongly suggests that the notion of warped H-J structure. The light-like geodesics of $H = M^4 \times CP_2$ and their duals would define coordinate lines at X^4 and the coordinates u and v for the warped H-J structure would vary along them. Even more general option would be that only the tangents of the generalized u and v coordinate lines are light-like.

The M^4 Kähler form and gauge potential are transversal and in a plane orthogonal to the local M^2 plane defined by Hamilton-Jacobi structure. This is the counterpart for the physical fact that massless gauge fields have no longitudinal polarization. The induced Kähler form giving U(1) part of the induced electroweak gauge field is the sum of contributions coming from M^4 and CP_2 degrees of freedom. The study of the fermion vertices as intersections of fermion lines demonstrates that M^4 hKähler structure is necessary for ordinary gauge interactions.

Field equations reduce to the conservation of isometry charges. The H-H principle involves generalized holomorphic maps $H \to C^2$. Holomorphic maps of H could be interpreted as general coordinate transformations but those of C^2 are genuine dynamical symmetries. Could the infinitesimal complex analytic deformations of generalized complex analytic functions f_1, f_2 of the hypercomplex coordinate of M^4 and 3 complex coordinates of H define conserved Noether charges? Holomorphic maps $g : C^2 \to C^2$ are dynamical symmetries acting via functional composition on maps f. Could maps g differing infinitesimally from identity mape define conserved Noether charges.

2.1 Classical field equations

The requirement that the action defined by the sum of Kähler action and volume term is stationary leads to the following field equations stating the conservation of isometry currents in the interior of the four-surface [K2].

There are two contributions to the field equations coming from the variations of the induced metric and Kähler form.

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &= j^\alpha J^k_l \partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu{}^\beta - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{2.4}$$

Here $T^{\alpha\beta}$ denotes the sum of the traceless canonical energy momentum tensor associated with the Kähler action and of the energy momentum tensor associated with the volume action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &= j^\alpha(J_\alpha{}^\beta h^k{}_\beta + J^k_l\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{2.5}$$

$H^k_{\alpha\beta}$ denotes the components of the second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

The field equations associated with the Chern-Simons term at light-like partonic orbits can be deduced from similar general formulas [K2]. Now the induced metric disappears from the field equations. Since Kähler gauge potential is not the primary dynamical variable, the field equations do not reduce to $J_{\alpha\beta} = 0$ as in the standard C-S theory. Also now the field equations can be transformed to local conservation laws for isometry currents and the presence of M^4 Kähler form gives a contribution to the M^4 isometry currents. It seems that the contribution to the momentum currents is transversal if the M^4 Kähler gauge potential is transversal.

2.1.1 Boundary conditions at partonic orbits and edges

Intersections of the edges and partonic orbit are define 2-D partonic surfaces as generalized vertices. If the condition that the string world sheet defines the contact interaction reduces the 2-D partonic vertex to a discrete set of points. Fermionic vertices as intersections with fermion of H defines which defines a null geodesics of H.

There are two cases to be considered.

1. $c_\#$ characterizing H-J structure changes at the edge. $c_\#$ characterizes fermion lines and H-J structures for fermionic surfaces. Edge could be a surface with a time-like normal but also space-like normals can be considered.

2. Partonic orbit is 3-D light-like surface. 4-metric degenerates to effectively 2-D metric. The technical challenge is to understand the boundary conditions.

The boundary conditions apply to both classical action and modified Dirac action. Conservation of isometry currents defines the boundary conditions.

1. Edges are not light-like 3-surfaces so that K-C-S action is not necessary. For partonic orbits would give modified Dirac equation as K-C-S Dirac for the fermion line as a special case. At the edge the normal component of the total isometry current is continuous:

$$\Delta T_A^n = 0 \ . \tag{2.6}$$

2. At the partonic orbit its equal to the K-C-S current flowing along the partonic orbit in the partonic case:

$$\Delta T_A^n = T_A^n(K - C - S) \ . \tag{2.7}$$

In Kähler action there are two contributions to the isometry currents coming from the variations of the induced metric and of Kähler form. Also the Kähler form of M^4 having only transversal component can be present.

2.1.2 Observations about boundary conditions

In the boundary conditions the divergence is replaced with difference of normal components at the two sides. Explicit expressions for the normal components of isometry currents are btained from the general formula for the isometry.

1. The boundary conditions at space-time edges pose a constraint on the value of cosmological constant Λ, which from twistor life is dynamical and determined by the p-adic length scale hypothesis and approaches zero at long scales. Kähler action density $J^{\mu\nu}J_{\mu\nu}$ and the scale Λ are of the same order of magnitude. In long scales $J_{\mu\nu}$ approaches zero. This solves the problem related to the huge value of Λ

2. For K-C-S case boundary conditions are very delicate. If the diverging quantities $g^{n_i\alpha}\sqrt{g_4}$ become proportional to 3-D delta function dominating at the partonic orbit over other terms in the boundary conditions and the boundary conditions are well-defined for the volume term of the action.

3. For Kähler action the vanishing of $g_{n_1\alpha}$ *resp.* $g_{n_2\alpha}$ and of $J_{n_1\alpha}$ *resp.* $J_{n_2\alpha}$ implies that also for Kähler action 3-D delta function gives the dominating term. 4-metric and Kähler form metrically 2-D. One would obtain square of 3-D deltafunction from the presence of two contravariant components of induced metric without the vanishing of $J_{n_1\alpha}$ and $J_{n_2\alpha}$. This follows by explicit study of the standard formulas for the components of contravariant 4-metric at the partonic orbit.

2.1.3 Model for the light-like partonic orbit

One should develope a model for the light-like partonic orbit and fermion line at it.

1. Could the partonic orbit X^3 be regarded as a homologically non-trivial geodesic 2-sphere CP_1 moving along a light-like geodesic of H. It would be sphere rotating in CP_2 with frequency ω. Non-vanishing K-C-S action (and of Kähler form) requires a homologically non-trivial geodesic sphere.

2. Fermion line would be light-like geodesic having projection to $M^4 \times S^1 \subset H = M^4 \times CP_2$, S^1 is a geodesic line. It is characterized by frequency ω: $m^0 = t$, $\Phi = \omega t$. Φ is the angle coordinate for the geodesic circle S^1. Any coordinate line of X2 along which t varies would be a light-like geodesic of and could serve as a fermion line.

   The induced metric at the fermion line would be $g_{tt} = 1 - R^2\omega^2 = c_\#^2$, where $c_\#$ is the reduced light velocity. Fermion behaves like a massless particle in H and in X^4. The M^4 component of 4-velocity is constant and equal to $v = R\omega$. Fermion behaves like a massive particle in M^4 with mass: the first guess for the mass is $m = \hbar_{eff}\omega$. However, the condition that the coupling of fermions to the induced gauge potential at the fermion line corresponds to the standard model coupling to Higgs expectation requires a weaker condition $m \propto \hbar_{eff}\omega$. This is one way to express M^4 massivation of fermions in TGD as a consequence of masslessness in H. Warping of the space-time surfaces distinguishing between GRT and TGD in turn justifies the masslessness in X^4 and the generalization of QCD approach to scattering amplitudes to TGD. I have discussed warping for Allais anomaly and fluctuations of gravitational constant in [L23] and its application to neutrinos in [L20].

3. Light-likeness of the partonic orbit implies that the covariant induced 4-metric is effectively 2-D and reduces to 2-metric for the partonic 2-surface X^2. Boundary conditions imply that this must be true also for the induced Kähler form. Metric and Kähler form are transversal: this is a geometric way to state gauge invariance means.

4. The induced metric cannot appear in the action for the light-like partonic orbit and K-C-S (Kähler-Chern-Simons) action is in question. The topological instanton term for Kähler action gives the K-C-S action as a boundary term but also the volume term of the action can contribute to it as a boundary term.

   The contravariant 4-metric and 3-metric are singular at the partonic orbit and must approach to something finite times a 3-D delta function if one wants to make sense of the boundary conditions. The difference of the isometry currents of Kähler action plus volume term at the two sides of partonic orbit is equal to the isometry current associated with K-C-S action.

K-C-S Dirac action can be defined as supersymmetric counterpart of K-C-S action.

1. The Dirac counterpart $\overline{\Psi}\Gamma^\mu D_\mu \Psi$ of the K-C-S action is defined for modified gamma matrices given by

$$\Gamma^\alpha = \frac{\partial L_{K-C-S}}{\partial_\alpha h^k}\Gamma^k \quad . \tag{2.8}$$

   The spinor modes at X^3 satisfy induced Dirac equation and it is also satisfied from fermion lines.

2. Fermion satisfies the modified Dirac equation associated with the K-C-S action restricted to the light-like fermion line at the light-like partonic orbit.

2.2 Fermionic dynamics

If the modified gamma matrices Γ^α are identified as contractions of the canonical momentum currents associated with the action density with the gamma matrices of H, the theory is supersymmetric in the sense that the contractions $\overline{u}_n\Gamma^\alpha\Psi$ with the modes of the modified Dirac equation define conserved super currents associated with the conserved fermion current $\overline{u}_n\Gamma^\alpha\Psi$.

Outside the 3-D edges the modified Dirac equation reduces to the field equations for the classical action when generalized holomorphy is assumed. The modes are holomorphic functions of the complex coordinate w and hypercomplex coordinate u of space-time surface X^4 (or their conjugates). u and w are associated with the Hamilton-Jacobi structure of X^4 induced by that of $M^4 \times CP_2$ and are annihilated by the conjugates of the modified gamma matrices. In string models spinor modes are constructed in the same way.

2.2.1 Dirac equations at the level of WCW, H and CD

Free massless Dirac equation in H. Masslessness at the level of H and massivation at the level of M^4 [L19, L18].

2.2.2 Modified Dirac action and modified Dirac equation

Induction procedure for fermions means the restriction of the second quantized free spinor fields of the embedding space to the space-time surface. The proposal discussed in [K14] is that it make sense to assume that the resricted spinor fields obey what I call modified Dirac equation reducing to induced Dirac equation when the action contains only the volume term. The Dirac equation is massless in 4-D sense.

1. The general expression for the modified gamma matrices is given by

$$\Gamma^\mu = \frac{\partial L}{\partial_{\partial_\mu h^k}}\Gamma_k \quad . \tag{2.9}$$

   Since the modified gamma matrices are deduced from the action, this implies an analog of supersymmetry. The modified Dirac equation requires classical field equations, which state the vanishing of the divergence $D_\alpha\Gamma^\alpha$ as a consistency condition guaranteeing hermiticity.

2. If the action contains only the volume term, one obtains induced gamma matrices and the Dirac equation is analogous to the ordinary massless Dirac equation. The conserved isometry currents have fermionic super counterparts. The presence of Kähler action gives additional contributions to the modified gamma matrices

3. This general formula applies also in the case of K-C-S action. For $L(K-C-S)$ the modified gamma matrices Γ^μ are well-defined since the singular induced metric does not appear in $L(K-C-S)$.

4. What about the restriction of the modified Dirac equation on the fermion line? At the light-like partonic orbit fermion line the Dirac equation for 3-D K-C-S action makes sense and is also necessary. This gives rise to coupling to the local Kähler form defining analog of monopole magnetic field in the case of parton orbit.

The modified Dirac equation can be solved by using the same ansatz as in the case of string theory. The modes are annihilated by the conjugate of complexified gamma matrices:

$$\Gamma^{\overline{w}}\Psi = 0 \quad , \quad \Gamma^v\Psi = 0 \quad . \tag{2.10}$$

Here v refers to the conjugate of hypercomplex coordinate u. Also the second half of Dirac operator annihilates Ψ

$$\Gamma^u D_u + \Gamma^w D_w\Psi = 0 \quad . \tag{2.11}$$

Analogous conditions hold true for the conjugates of the modes. H-H principle implies that the equations reduce to generalized holomorphy and are exactly solvable.

Concerning symmetries, some observations are in order.

1. The Noether currents associated with the isometries can be assigned to the modified Dirac equation by the standard formulas. Modified Dirac equation expresses the conservation of quark and lepton numbers: quarks and leptons correspond to the opposite H-chiralities of H-spinors. The modified Dirac equation implies the mixing of M^4 chiralities, which signals massivation.

2. It is possible to assign to the fermion current infinite number of super currents by replacing $\overline{\Psi}$ in $\overline{\Psi}\Gamma^\alpha\Psi$ with the solution $\overline{\Psi}_n$ of the modified Dirac equation. The conjugates of the super charges are obtained in an analogous way. Also the super counterparts of the isometry currents are obtained. Also the possibly conserved Noether currents assigned to the conformal symmetries modifying f_1 and f_2 would have conserved super counterparts.

There are some questions to be pondered.

1. The non-triviality of the vertex is due to the violation of separate conservation of fermion number and antifermion number which hold true in free theory. This is true at least for the standard smooth structure. Time orientation changes and gives rise to conservation. Fermion turns backwards in time and transforms to antifermion. What about the situation for the exotic smooth structure: could one say that fermion and antifermion numbers are separately conserved. Does the change of arrow of time together with exotic smooth structure replace the creation (annihilation-) operator for fermion with creation (annihilation-) operator for antifermion and vice versa.

   If the modified Dirac equation is in some sense true also at vertices, one cannot identify the propagator as the inverse of the modified Dirac operator since there would be no source term for the Dirac propagator. The problem can be solved by using the massless Dirac propagator in H or CD with endpoints restricted to X^4. M^4 Kähler structure (Hamilton-Jacobi structures) [L9] is also possible and modifies the notion of H masslessness [L18] since M^4 transversal degrees of freedom give a contribution to the mass squared besides CP_2. Also tachyonic states become possible and might relate to the color confinement.

2. Could it be that only the metric part is present in gamma matrices in the interior. Both options seem to be possible by holomorphy predicting minimal surfaces always. Does this have any implications if the construction of the scattering amplitudes involve only the partonic orbits and partonic surfaces as intersections of 3-D eges and partonic orbits.

3 A general view of construction of scattering amplitudes

3.1 General vision

3.1.1 QCD as a role model

Consider first the general view of the construction of the scattering amplitudes.

1. The hadron phase is generalized to fermionic physics at the level of H. Massless Dirac equation in H for free fermions gives the spectrum of modes for quark and lepton spinors. At the level of H bosons are bound states of leptons and quarks. Only fermions appear as propagating particles. The basic prediction is an infinite hierarchy of scaled variants of standard model physics [L20, L24].

2. Generalization of quark-gluon phase means fermionic physics at the level of the space-time surface X^4. Induced-/modified Dirac equation is satisfied. Only fermions propagate and the propagators are H propagators with end points restricted to the space-time surface. The propagation occurs along partonic orbits and the fact that fermion lines are light-like geodesics of H implies that the fermions are almost massless in M^4 and massless in H and X^4. Bosonic correlation functions in WCW replace ordinary boson propagators and gauge bosons as fundamental particles disappear.

S-matrix factorizes to the product of a unitary matrix corresponding to the generalization of hadronization and to a product of S-matrices generalizing the dynamics of quark gluon plasma. Each period between two SSFRs connecting two loci of classical non-determinism corresponds to a unitary time evolution followed by SSFR. The presence of the sequence of SSFRs would conform with the fact that in QCD kinetic equations reflecting the presence of decoherence are an essential part of description. At space-time level only induced spinors appear as quantum fields obtained from the free spinor fields of H.

Hadronic reaction as a pair of BSFRs in which the arrow of geometric time temporarily changes. Interpretation could be as a quantum tunnelling. The period between BSFRs is a sequence of SSFRs involving state function reduction. This period corresponds to a reversed arrow of time during which CD increases and the states at its passive boundary are unaffected. This conforms with the view of perturbative QCD involving both kinetic description in terms of parton densities f and quark-hadron fragmentation functions D. The alternative proposal would be that the sequences of SSFRs are replaced with the analog of Feynman description so that no SFR would actually occur and there would be a product of unitary matrices. This alternative does not conform with the QCD view and with the TGD based view of consciousness.

3.1.2 Interactions as contact interactions

The assumption that interactions are contact interactions and are described in terms of data provided by string sheets identified as intersections of space-time surfaces representing "Bohr orbits" of 3-D particles implies that he scattering amplitudes can be formulated using the data provided by fermion fields associated with the string world sheets. This would be string model type description.

A stronger assuption is that fermion fields can be restricted on fermion lines defined by the intersections of string world sheets with 3-D light-like partonic orbits. This reduces the fermionic vertices to points as in QFTs. The vertex is the intersection of fermion line with 3-D edge singularity. One could justify this assumption by hypercomplex analyticy: imbedding space coordinates and induced fermion fields depend only on the second hypercomplex coordinate which varies along fermion lines.

3.1.3 Quantum criticality and generalized conformal invariance

The "spin-glass degeneracy of the maxima of the Kähler function" emerged at the non-physical (action reduces pure-Kähler-action) limit. This degeneracy corresponds to the vanishing of the induced Kähler form since the Kähler gauge potential is pure gauge. The U(1) gauge transformations correspond to symplectic transformations as dynamical symmetries respecting vacuum extremal property but affecting the induced metric. This degeneracy is dramatically reduced but does not disappear when volume term is added. It corresponds to the weak classical non-determinism of the minimal surfaces: the loci of non-determinism and of local the failure of the minimal surface property are analogous to the frames of soap films. This non-determinism would correspond to lona range quantum fluctuations at quantum criticality.

Already in 2-D conformal field theories, quantum criticality is implied by conformal invariance. In TGD holography = holomorphy principle extends the conformal invariance and gives rise to quantum criticality which fixes the spectrum of Kähler coupling strength as an analog of critical temperature.

3.2 ZEO and the role of classical nondeterminism

Classical non-determinism predicted by H-H principle could be the cornerstone of both classical and quantum dynamics.

3.2.1 Classical non-determinism, warping, and dynamics as generalized Brownian motion

Classical non-determinism suggests the following picture.

1. The moments of time involving classical non-determinism should correspond to 3-D singularities at which the derivatives of H coordinates are discontinuous and minimal surface property fails. The field equations state the conservation of isometry currents and would be satisfied in the sense that the normal components of isometry currents are cntinuous.

   One has an edge of space-time and holomorphy and smoothness fail at the edge. The proposal is that an exotic smooth structure replaces the ordinary smooth structure having these 3-D surfaces as defects [L14, L6, L20]. Milnor found that the 7-D sphere allows 7 exotic smooth structures. There exists an infinite number of exotic smooth structures in space-time dimension $D = 4$ where they first emerge [A3, A4, A1]. They are a problem of GRT and

cosmic censorship is required to get rid of them. The time evolution looks like Brownian walk just like path integral at the limit $\Delta T \to 0$. However, instead of analog of path integral one would have a sequence of SSFRs implying decoherence.

2. The phenomenon of warping distinguishes between GRT and TGD [L23, L20]. Space-time surfaces with flat metric but reduced light velocity given by $c_{\#} = \sqrt{g_{tt}}$ are possible. This suggests a generalization of the notion of Hamilton-Jacobi structure [L9] such that M^4 and CP_2 degrees become correlated. Fermion lines and the associated space-time surfaces would be characterized by $c_{\#}$.

3.2.2 Two alternative views of what could happen in quantum scattering event?

One can consider two alternative options.

1. It would be nice to interpret the sequence of these edge discontinuities as an analog of a similar sequence for Feynman path integral for which the time difference ΔT between the discontinuities goes to zero. This would require giving up the idea SSFR is a genuine state function reduction giving rise to decoherence. This would mean giving up the basic idea of TGD inspired theory of conciousness.

2. On the other hand, I propose that QCD serves as a role model for the TGD view of scattering events and thus involves incoming and outgoing particles as states defined in terms of fermionic oscillator operators for the free fermion fields in H. Fermion fields in H satisfy massless Dirac equation in 8-D sense and are massive.

   Quark gluon phase would be replaced with phases involving only induced fermion fields satisfying the massless Dirac equation. It seems that an excellent approximation is to consider fermions localized to fermion lines as massless light-like geodesics in H restricted to light-like geodesics in X^4 but with reduced light velocity $c_{\#}$. In M^4 the geodesics are massive and the mass satisfies $m \propto \hbar_{eff}\omega$. The coefficient corresponds to the coupling of the fermion to the constant induced gauge potential at the fermion line. The standard model counterpart is the coupling of the fermion to the Higgs expectation.

3. The formation of quark gluon phase would be a unitary transformation from H phase for fermions ton X^4 phase and hadronization would be a unitary transformation in an opposite direction.

4. The question is whether also the SSFRs actually just unitary transformations or whether they are genuine SFRs as TGD inspired consciousness requires?

   QCD is a strange looking mixture of QFT and kinetic theory. Scattering amplitudes are calculated by QFT but one has parton densities f and quark-hadron fragmentation functions D, which belong to kinetic theory involving quantum statistics and therefore state function reductions.

   If QCD view is correct, SSFRs are indeed genuine SFRs involving a loss of coherence and only the periods between two SSFRs would be unitary evolutions.

3.3 Propagators

Consider first the identification of propagators.

3.3.1 Fermion propagators

Fermion lines identified as intersections of string world sheets and parton orbits. Light-like geodesic both at the level of H and X^4. Massivation at the level of M^4.

The fermionic propagators are identified as H-propagators with the end points restricted to the edges of fermion lines. No problems in the calculation of the fermion propagator since there is no need to invert the modified Dirac operator. The propagators for the modes of H spinor fields are

massive in the M^4 sense but for light-like M^4 separations they behave like massless propagators. For the H spinor modes which generate mass (Higgs mechanism), light-likeness would be in 8-D sense and in 4-D sense for X^4 .

One should demonstrate that 8-D light-like propagators behave like massive M^4 propagators with a small mass (in the CP_2 mass scale) for 8-D light-like separations. Massless modified Dirac operator would provide holomorphic spinor modes representing wave functions for the position of the fermion vertex at the partonic 2-surface.

3.3.2 Bosonic propagators as WCW correlation functions

The translational degrees of freedom for space-time surfaces as Bohr orbits gives rise to momentum degrees of freedom after Fourier transform. Poincare invariance (exact at the limit of infinitely large CD) makes natural the dependence on s,u,t. Generalization of the conformal invariance makes natural the assumption that the analog of bosonic propagator as a correlation function in WCW degrees of freedom has the behavior predicted by conformally invariant theories.

Masslessness have several representations: minimal surfaces are analogous to non-linear massless fields, fundamental fermions are massless in $H = M^4 \times CP_2$ but massive in M^4 . Induces spinor fields are massless in X^4. The latest finding is that warped space-time surface realized masslessness in H and X^4 but represent massivation at the level of M^4. The particle mass is directly coded to the properties of a light-like geodesics of H representing the fermion line [L20].

When only the cm degrees of freedom for partonic orbits are left, point-like limit of QFT should emerge and give massless bosonic propagators for say photons and gravitons. For instance, super conformal invariance applied in p-adic mass calculations states that only transversal degrees of freedom are physical. In holography-holomorphy principle longitudinal degrees of freedom correspond to hypercomplex degrees of freedom.

3.3.3 Vertices as intersections of partonic orbits and edges of space-time surface

In ZEO one assigns to the opposite boundaries of CD opposite arrows of geometric time and opposite time orientations. The natural question is how this relates to the matter and antimatter. The proposal is that these two vacuum states for fermion correspond at least formally to Dirac seas for positive *resp.* negative energy particle.

The description of fermion pair creation as turning of a fermion backwards in time forces to look the situation again. The interpretation would be as time reflection in which the sign of the energy changes. This guarantees energy conservation and at the opposite boundary there must be for intance boson with the energy equal the sum of energies of fermion and antifermion.

The fermion line beginning from the positive energy boundary of CD is time reflected and returns back with positive energy and opposite time arrow . If the time reflection of creation operator transforms if to creation operator for antifermion, the standard view is obtained.

It seems that one must take events as basic notion rather than states. H-H principle indeed implies this at classical level. Bohr orbits are the basic notion in pair creation the Bohr orbit turns backward in time after time reflection. Also refraction is possible and would naturally occur at 3-surfaces where the value of $c_{\#}$ changes this could be interpreted as scattering from classical fields. The generalized Brownian motion would involve both refraction and reflection at the intersection of partonic orbits and singular 3-surfaces.

Annihilation of fermions and antifermions in cosmology, could be due to quantum transitions transforming pairs of fermion lines as boundaries of string world sheets identified as intersections of space-time surfaces (including self-intersections) connecting the opposite boundaries of CD to V-shaped lines connecting the points of the same boundary. Refraction would be replaced with reflection.

Could this new view provide new insights to the matter antimatter asymmetry? As in the standard cosmology, CP breaking implying generation of regions with slight excess of fermions over antifermions or vice versa is necessary to prevent complete annihilation of fermions and antifermions.

3.3.4 Is it possible to obtain the gauge theory vertices directly from the modified Dirac action?

A key question is whether the vertices are obtained directly from the modified Dirac action, in fact from the modified Dirac action as Kähler-Chern-Simons (K-C-S) action. The modified Dirac equation is satisfied at the partonic orbit. The question is what happens at the partonic 2-surface as intersections of the partonic orbit and 3-D edge of the space-time surface at which the derivatives of the embedding space coordinates are discontinuous and minimal surface property, holomorphy and smoothness fail?

The conceptual problem is that the divergence is that fermion current is conserved and fermion number must be conserved. However, in the annihilation-/creation vertices the fermion line turns backwards in time. One can say that fermion number is conserved if the event is interpreted as annihilation of a fermion-antifermion pair. The proposal is that exotic smooth structure [A3, A4, A1] identified as ordinary smooth structure having the edge as defect allows to formulate the following intuitive view at a rigorous mathematical level [L14, L6, L20]. In the annihilation vertex as a defect of exotic smooth structure, the positive energy fermion would transform to negative energy antifermion travelling with the opposite arrow of geometric time.

1. The 4-D modified Dirac action can be written as a covariant divergence reducing to the ordinary divergence. If the Dirac equation is true everywhere, it vanishes and the theory becomes trivial in fermionic degrees of freedom. The divergence representation is possible also for the K-C-S Dirac action. The action is proportional $\sqrt{g_3}$ is present but is eliminated since the 3-D permutation symbol is proportional to $\sqrt{g_3}$. Is the divergence at the vertex proportional to delta function and gives rise to the vertex. A non-vanishing divergence would define the current as a source for the bosonic particles.

2. The divergence contains several terms. $\partial_\mu \Gamma^\mu$, which givest the trace of the second fundamental form and diverges at the edge and is identifiable as a generalized acceleration. This gives rise to analogs of vector interactions in M^4 and cP_2 degrees of freedom. That acceleration is in question, strongly suggests the Equivalence Principle and gravitation. Could they have identification in terms of graviton and Higgs exchange? But how could the spin 2 property of gravitons as fermionic bound states emerge? Is an entirely new interaction in question? And what about the Kähler form associated with M^4: it should contribute additional term to the U(1) force.

3. Besides this there are the terms $(\partial_\mu \overline{\Psi})\Gamma^\mu \Psi$ and $\overline{\Psi}\Gamma^\mu \partial_\mu \Psi$ which, by using 3-D(!) modified Dirac at partonic orbit, can be transformed to $(\partial_\mu \overline{\Psi})A_\mu \Gamma^\mu \Psi$ and $\overline{\Psi}\Gamma^\mu A_\mu \Psi$. These terms should give rise to gauge interaction vertices. They are however are finite 2-D deltafunction would be required.

   Could the edge discontinuity for annihilation-creation vertex means that Ψ and $\overline{\Psi}$ are initially zero and appear our of nowehere so that these two terms are proportional to delta function at the partonic 2-surface. Does the same argument apply also to the vertices analogous to refraction of fermion in time direction.

3.3.5 Is the proposed picture consistent with the standard view of fermions

In TGD bosons would consist of fermion antifermion pairs. Do the members of pairs have opposite arrows of time? If boson is a pair of collinear massless on-mass-shell fermion and its antifermion in M^4, its spin vanishes. If the momenta are opposite, the spin equals 1 but the state is massive. What about massless spin 1 particles like photon?

This problem disappears when one notices that chiral symmetry is raised to the level of H and becomes a separate conservation of quark and lepton numbers. The signature for the massivation of fermion in M^4 is that fermion is a mixture of both M^4 helicities. The light-like 8-momenta of fermions at fermion lines would be parallel and the M^4 projection of the total 8-momentum would be time-like. Even photons could have tiny mass as p-adic thermodynamics strongly suggests. Warping would realize this picture at space-time level.

In TGD inspired theory of consciousness, the proposed mechanism of memory involves signalling backwards in time and time reflection. This time reflection could also correspond to a time reflection

for fermions, say neutrinos and also time reflection for bosons as fermion antifermion pairs. The reflected light provides information about the surface from which it is reflected.

3.4 Number theoretic view of coupling constant evolution

Correlation functions in WCW define fermion-boson coupling constants and their evolution reflecting the algebraic structure of the space-time surface if H-H holds true.

In the TGD framework, Weinberg angle and other coupling than Kähler coupling can in principle be computed when Kähler coupling is given. Also the spectrum of the Kähler coupling would be fixed by quantum criticality as an analog of critical temperature.

1. Holography = holomorphy principle predicts that outside the singularities minimal surface equations are true irrespective of the action principle as long as it is general coordinate invariant and depends only on the induced geometry. The coefficients of polynomials or even more general functions (f_1, f_2) belonging to extension of rationals would quite generally give rise to theoretical coupling constant evolution in terms of number. The polynomials f_i form also hierarchies hierarchy with respect to the degree.

2. The proposal motivated by the notion of generalized Langlands correspondence between geometric and number theoretic visions is that the values coupling parameters depend on number theoretical invariants. The extension E of rationals appearing in map $f = (f_1, f_2)$ (polynomial hierarchy is the simplest guess) defining space-time surface as their root; the generalized Galois group acting as a flow a permuting the roots of (f_1, f_2) as space-time regions, and the ordinary Galois group tentatively assigned with partonic 2-surfaces.

3. The positions and shape of the singular surfaces depend on coupling parameters. The reason is that the boundary conditions guaranteeing the conservation of isometry charges at partonic orbits and at the edges of the space-time pose constraints on the coupling parameters. For instance, for mere volume action edges and partonic orbits are impossible. The contributions from volume action and, say, Kähler action must compensate each other in the boundary conditions.

   If the conditions defining edges and partonic orbits are algebraic equations, their intersection as a partonic 2-surface defining the generalized vertex satisfies algebraic equations. H-H principle implies that the addition of a third function f_3 having the same dependence of hypercomplex coordinate as f_i and vanishing at the partonic 2-surface selects a finite set of points as its roots. These points are in general roots of an algebraic equation and the roots of algebraic equations for algebraic functions with coefficients in extension of rationals quite generally allow the analog of the Galois group.

4. What about the coefficient of the volume term interpreted as cosmological constant Λ? Kähler action as 6-D action for the twistor lift of TGD gives as extremals 6-D surfaces with induced twistor structure meaing CP_1 bundle structure. The twistor space of H is the product of twistor spaces of M^4 and CP_2. Only these two spaces have a twistor space with Kähler structure. 6-D Kähler action is dimensionally reduced to 4-D Kähler action and volume term coming from the CP_1 fiber of X^6.

   The coefficient Λ of volume term is dynamically generated and depends on p-adic length scale approaching zero in long scales. There is no explicit breaking of conformal symmetry present if Λ is put in by hand.

3.5 Three kinds of Mersenne primes and TGD

In the TGD framework, there are 3 especially interesting real or complex primes and corresponding algebraic extensions: ordinary , Gaussian, and Eisensteinian Mersenne primes. What makes them interesting is that for the corresponding finite field arithmetics is very simple. For this reason they could be regarded as "cognitively" simple in the TGD view of cognition. Also more general Mersenne primes can be defined in terms of primitive roots of unity but prime decomposition need not be unique for the corresponding integers so that they are not "cognitively simple".

3.5.1 Basic mathematical facts

Ordinary Mersenne primes have the form $M_n = 2^n - 1$, n prime, and have norm equal to M_n.

Gaussian Mersenne primes are of the form $(1+i)^n - 1$. Both n and the algebraic norm, equal to $2^n - 2^{(n+1)/2} + 1$ for $n \equiv 1, 7 \pmod 8$, $2^n + 2^{(n+1)/2} + 1$ for $n \equiv 3, 5 \pmod 8$ and 5 for $n = 2$, are primes.

By replacing the base 2 of the algebraic extension with the base elements $(1-\omega)$ or $(1-\omega^2)$, having an absolute norm of 3, one can define Eisensteinian Mersenne prime as $E_n = (1-\omega)^n - 1$. Also now both n and the algebraic norm $N(E_n)$ must be a standard rational prime.

The rational norm of $N(E_n)$ is for $n \equiv 1, 11 \pmod{12}$ given by $3^n - 3^{(n+1)/2} + 1$ and for $n \equiv 5, 7 \pmod{12}$ $3^n + 3^{(n+1)/2} + 1$. The norms $n = 2$, $n = 5$ and $n = 7$ are given by 7, 271 and 2269. The sequence of exponents of Eisenstein primes goes as $n = 2, 5, 7, 11, 17, 19, 79, 163, 193, ...$

The exponent of a given Mersenne prime can belong to several of these three sets simultaneously. The same integer n can also define both ordinary or Gaussian and Eisensteinian prime.

1. Ordinary Mersenne primes satisfy $M_n = 3 \pmod 4$, and ordinary Mersenne primes are also primes for Gaussian integers but not Gaussian Mersenne primes. In particular, $n = 127$ for the electron satisfies the condition.

2. The lists for the exponents of ordinary (O), Gaussian (G) and Eisensteinian (E) Mersenne primes are

$$\begin{aligned} &\text{O:} \quad \{2,3,5,7,13,17,19,31,61,89,107,127,521,...\} \ , \\ &\text{G:} \quad \{2,3,5,7,11,19,29,47,73,79,113,151,157,163,167,239,241,283,353,...\} \ , \\ &\text{E:} \quad \{2,5,7,11,17,19,79,163,193,239,317,353...\} \ . \end{aligned} \tag{3.1}$$

3. There are exponents defining several kinds of primes. Using shorthand notation Ordinary-O, Gaussian-G and Eisensteinian-E, the cases are O-G, O-E , G-E and O-E-G. The lists of common exponents are

$$\begin{aligned} &\text{O-G:} \quad \{2,3,5,7,19\} \\ &\text{O-E:} \quad \{2,5,7,17,19\} \ , \\ &\text{G-E:} \quad \{2,5,7,11,19,79,163,239,353\} \ , \\ &\text{O-G-E:} \quad \{2,5,7,19\} \ . \end{aligned} \tag{3.2}$$

It is possible to define analogs of Mersenne primes for arbitrary cyclotomic fields as $(1 - exp(i2\pi/m))^n - 1$, where $exp(i2\pi/m)$ is a primitive root of unity. m need not be a prime but n must be prime.

Cyclotomic fields $Q(\zeta_n)$, $\zeta_n = exp(i2\pi/n)$ allow a unique factorization in 30 cases, which are $1, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19, 20, 21, 24, 25, 27, 28, 32, 33, 35, 36, 40, 44, 45, 48, 60, 84$. $n = 2, 4$ and 3 gives the standard, Gaussian and Eisensteinian integers. The additional condition that the degree of the field is at most 2 is satisfied for n in the set $1, 2, 3, 4, 6$. This selects these 3 fields, Q and the only quadratic cyclotomic fields $Q(i)$ and $Q(\omega)$, all allowing a unique decomposition to prime factors. They are "cognitively" preferred.

What makes ordinary, Gaussian *resp.* Eisensteinian primes so interesting in the TGD framework are that they are near powers of a number x, which for these 3 cases is 2, $1+i$ *resp.* $1 - exp(i2\pi/3)$ and has norm 2, 2 *resp.* 3. One can ask whether Eisensteinian primes could define important p-adic length scales. Eisensteinian Mersenne primes define a length scale hierarchy coming as powers of $\sqrt{3}$ containing a sub-hierarchy coming as powers of 3.

3.5.2 Possible particle physics implications

For Gaussian Mersenne prime $M_{G,n}$ with $n = 79$, the Gaussian p-adic length scale is $L(79) = 2^{(79-151)/2} L(151)$, $L(151) \simeq 10$ nm. The Eisensteinian p-adic length scale for $k = 79$ is $L_E(79) = (3/2)^{79/2} L(79) \simeq 1.314 \times 10^{-12}$ m, to be compared with the p-adic length scale $L(127) = 2.44 \times 10^{-12}$ m. The electron could therefore have a companion with mass nearly twice the electron mass. An interesting question is whether this might relate to the leptopion hypothesis [K11].

3.5.3 Astrophysical quantum coherence and quantum biology

One can also consider the possible quantum biological implications of Eisensteinian Mersenne primes. The new element is the possibility of quantum coherence in astrophysical scales.

In particular, quantum gravitation is expected to be in a central role. Quantum gravitation plays a key role in the TGD inspired view of quantum biology already now. Gravitational Planck constant defines gravitational Compton length Λ_{gr}, which has no dependence on the small mass m. For the Earth it is $\sim .5$ cm and for the Sun nearly one half of the radius of the Earth [L13] and appears in the physics of the Sun.

1. Concerning Eisensteinian Mersenne primes, it is interesting to notice that there is evidence for evolutionary time scales coming as powers of 3 [I1, I2].

2. $L(k = 163) \simeq .64$ μm is one of the miracle Gaussian Mersenne primes associated with $k = 151, 157, 163, 167$ defining biologically important length scales. The corresponding Eisensteinian p-adic length scale is $L_E(163) = 3^{(163-79)/2} L_E(79) = 3^{42} L_E(79) \simeq 1.438 \times 10^8$ m, which should be compared with the solar radius $R(Sun) = 6.96 \times 10^8$ m. $L_E(163)$ is 21 percent of solar radius. The thickness of the convection zone is about 30 percent. The ratio $L_E(163)/L(163)$ is $L_E(163)/L(163) = 2.25 \times 10^{14}$.

3. The proposed role of astrophysical quantum coherence scales in quantum biology suggests that there could be a resonant interaction between length scales $L_E(163)$ and $L(163)$, which corresponds to the third level for the hierarchically coiled DNA. Note that the largest miraculous Gaussian Mersenne with $k = 167$ corresponds to the size of the cell nucleus. In this interaction between widely different length scales ordinary biophotons [K3, K4] would transform to dark Eisensteinian photons with the same energy or vice versa. The origin and realization of this kind of interaction between widely different scales is one of the basic problems of biology.

   Gravitational resonance would transform ordinary photons to gravitationally dark Eisensteinian photons with the same energy or vice versa. The gravitational dark photons could also decay to bunches of ordinary Eisensteinian photons in an energy conserving way. The gravitational resonance does *not* require $k = k_E$ since the wavelength of a dark Eisensteinian photon does not carry information about its origin.

   The most general situation would involve ordinary p-adic length scale $L(k)$ and Eisensteinian length scale $L_E(k_E)$, $k_E \in \{163, 193\}$. $\hbar_{gr}/\hbar$ would scale the wavelength $\lambda \leq L(k)$ to dark wave length $\lambda_E(k) = \hbar_{gr}/\hbar\lambda \leq L_E(k_E)$. Both $\lambda(k)$ *resp.* $\lambda_E(k_E)$ would be subharmonics $\lambda = L(k)/n$ *resp.* $\lambda_E(k_E) = L_E(k_E)/n_E$ of the fundamental $L(k)$ *resp.* $L_E(k_E)$.

   The resonance condition is $(\hbar_{gr}/\hbar)L(k)/n \simeq L_E(k_E)/n_E$, that is $n_E/n \simeq L_E(k_E)/[(\hbar_{gr}/\hbar)L(k)]$. For $k = k_E$ and $n = n_E = 1$ it reduces to $\hbar_{gr}/\hbar \simeq L_E(k_E)/L(k_E)$. Since any k not too far from k_E is possible, with $L(k)$ in half octaves, and so are several wavelength subharmonics and a resonance width, the condition can be satisfied with reasonably small n and n_E for many values of $\hbar_{gr}/\hbar$: the point is a general mechanism of communication between long and short scales, not a prediction of a unique resonance frequency, and a particular resonance found in this way illustrates the mechanism rather than tests it.

4. Could the $L(163) - L_E(163)$ resonance be made possible by the gravitational Planck constant $r(Earth, p) = \hbar_{gr}(Earth, p)/\hbar = r_s(E)/(2\beta_0 L_p)$ of Nottale [E1] for the Earth-proton system? The velocity parameter β_0 can be identified as the reduced light-velocity $c_{\#} \leq 1$ due to warping and $\beta_0 \simeq 1$ seems to work for the Earth. From the Schwarzschild radius of the Earth $r_s(E) \simeq .01$ m and from the reduced proton Compton length $L_p = \hbar/m_p$, one obtains $r(Earth, p) = \hbar_{gr}(E, p)/\hbar = .238 \times 10^{14}$.

   $\hbar_{gr}(M, m)$ defines gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = r_s(M)/2\beta_0$ as basic length scale which, in accordance with the Equivalence Principle, does not depend on m. One can however ask whether the scaling of p-adic length scales by $\hbar_{gr}/\hbar$ could give physically relevant length scales. The ratio $L_E(163)/L(163)$ is $(3/2)^{163/2} \simeq 2.25 \times 10^{14}$, 9.45 times $r(Earth, p)$, which gives the gravitationally dark scale from $L(163)$.

An optimist would ask whether the discrepancy factor 9.45 could be interpreted in terms of a power 3^2. $\lambda_{k=163}$ would be 9:th subharmonic.

There is no need to be so optimistic. The scaling $k \to k+6 = 169$ scaling L(163) by factor 8 to $L(169)$ is allowed. The magnetic length of the Earth's magnetic field $B_E \simeq .5$ Gauss is 3.6 μ m and slightly longer than $L(167) = 2.5$ μ m. The endogenous magnetic field $B_{end} \simeq 2B_E/5$ [J1] is identifiable as monopole magnetic field carrying gravitational flux of the Earth, and plays a key role in the TGD inspired biology. The magnetic length is $l_B = 5.7$ μ m and not far from $L(169) = 5$ μm. Resonance would be obtained for $n_E/n \simeq 9.45/8 \simeq 1.18 \simeq 6/5$.

5. The scale $\hbar_{gr}(Earth, e)/\hbar$ for Earth-electron pair is by the ratio m_e/m_p shorter and the gravitationally dark scale obtained from $L(163)$ would be about 8.3 km. According to Google, 10 km is an important scale in planetary science. Nottale introduces the scale 10 km in his scale relativity involving fractal transitions. In planetary formation models, the system splits to structures smaller than 10 km unless there is sufficient mass to form larger structures. The oceanic crust has thickness in the range 5-10 km. Troposphere has thickness roughly 10 to 11 km. In computational Earth science, 10 km represents the holy grail below which computation becomes very difficult.

6. Note that the value of $\hbar_{int}/\hbar$ can be estimated also from the condition that DNA cyclotron frequency $f_c = 1$ Hz in the endogenous magnetic field $B_{end} = .2$ Gauss [J1] corresponds to frequency 10^{14} Hz of proton. The corresponding energy .41 eV is in the near IR and of the order of the metabolic energy currency .5 eV. For $r_E = (3/2)^{163/2} = 2.25 \times 10^{14}$ the energy would be .92 eV below the visible range beginning at 1.59 eV.

What about the relation of the gravitational Planck constant of the Sun-proton pair to the scale $L_E(193)$?

1. The solar Schwarzschild radius is $r_s(Sun) = 3$ km satisfying $r_s(Sun) \simeq 3 \times 10^5 r_s(Earth)$. Nottale's model for the inner planets of the planetary system [E1] gives the estimate $\beta_0 \simeq 2^{-11}$ for the velocity parameter for the Sun.

   This gives $r(Sun, p)/r(Earth, p) = 6 \times 10^8$ implying $r(Sun, p) \simeq 1.43 \times 10^{22}$. This is by a factor 6.4×10^7 larger than $L_E(163)/L(163) \simeq 2.25 \times 10^{14}$. The scaled up $L(163)$ would be 0.92×10^{16} m, which corresponds .97 ly, which might be regarded as a biologically natural scale in Earth-Sun system.

   For the Sun-electron pair the value of gravitational Planck constant is reduced by the mass ratio $m_e/m_p = 1/1836 \simeq 5.446 \times 10^{-4}$, which is 13 percent larger than the velocity parameter $\beta_0(Sun) = 4.8 \times 10^{-4}$ introduced by Nottale [E1] differing from 2^{-11} by factor .99. This gives $r(Sun, e) \simeq 0.779 \times 10^{19}$. The scaled up $L(163)$ would be about 33.3 AU: 30.1 AU is the orbital distance of the outermost planet Neptune used as a definition for the size of the planetary system.

2. Could the gravitational Planck constant for the Sun-proton pair and the Eisensteinian scale $L_E(193)$ relate to each other? The first thing to notice is that $k = 193$ defines neither ordinary nor Gaussian Mersenne prime but the p-adic length scale hypothesis allows also more general p-adic length scales, in particular those for which k is prime. Furthermore, the gravitational quantum coherence pairing of p-adic length scales does not require $k = k_E$.

   For $k = 193$, ordinary p-adic length scale $L(193) = 2^{(193-151)/2} \times L(151) = 2^{21} \times L(151)$ about 2.1 cm and nearly twice the Schwarzschild radius of the Earth. The corresponding Eisensteinian p-adic length scale $L_E(193) = 3^{(193-163)/2} L_E(163) = 3^{15} L_E(163) \simeq 2.06 \times 10^{15}$ m, which corresponds to .22 ly, would define the resonance wavelength of $k = 193$ Eisensteinian photons. Google informs that this scale has astrophysical relevance. Stellar close approaches correspond to minimal distance of .22 ly. This scale is also associated with wide binary separations.

   The resonance condition requires $\hbar_{int}/\hbar = L_E(193)/L(193) = (3/2)^{193/2} \simeq 9.8 \times 10^{16}$. The scaling of $L(193)$ associated with the $\hbar_{gr}(Sun, p)/\hbar = 1.43 \times 10^{22}$ is about 1.5×10^5 larger. Note that the gravitational scaling gives a scale of 3.0×10^{20} m, that is 31,700 ly. The

distance of Earth from the galactic center is about 26,000 ly and Galactic disk to the outer edge of Milky Way 50,000 ly.

The scaling associated with $r(Sun,e) = \hbar_{gr}(Sun,e)/\hbar = .779 \times 10^{19}$ is by factor 79 larger than $L_E(193)/L(193)$. In this case the gravitational scaling of $L(193)$ gives the scale 17 ly. Google informs that the local interstellar cloud of hydrogen and helium has radius of ~ 15 ly.

The pairing of $k = 193$ with $k_E = 193$ is not the only option: the scaling down of $L(193)$ by a factor 64, not far from 79, to $L(181) \simeq 3 \times 10^{-4}$ m ($k = 181$ is prime) would give $r(Sun,e)L(181)/L_E(193) = 79/64 \simeq 1.234$, and the mechanism already discussed could make resonance possible for wavelength subharmonics.

3. Could the scalings of $2r_s(E) \simeq L(193)$ by $r(Earth,e) = \hbar_{gr}(Earth,e)/\hbar$ and $r(Earth,p) = \hbar_{gr}(Earth,p)/\hbar$ have an astrophysical interpretation?

   For the proton the gravitationally dark scale $L_{gr}(E,p,193)$ is 4.76×10^{11} m and equals to 3.18 AU (the distance of the Earth from the Sun).

   For the electron the gravitational dark scale $L_{gr}(E,e,193) = 2.62 \times 10^8$ m is 38 per cent of the solar radius $R(Sun) = 6.96 \times 10^8$ m. The thickness of the convection zone of the Sun is about 30 percent of the solar radius. The size of the magnetotail of the Earth varying in the range $[3.8, 64] \times 10^8$ m and $L_{gr}(E,e,193)$ is 70 percent of the lower end of the range.

One should take a critical attitude for above identifications. Astrophysics involves enormous number of scales and it is probably easy to find interpretation for any scale just by going to Google. What is clear that the scalings induced by the gravitational Planck constant cannot give the ratio of Eisensteinian and ordinary p-adic length scales.

$k = 239$ corresponds to both Gaussian and Eisensteinian scales. The secondary p-adic length scale $L(2,127) = \sqrt{M_{127}}L(127)$ corresponds in a good approximation to 10 Hz frequency defining a biorhythm appearing as alpha resonance frequency in EEG. The primary Gaussian p-adic scale $L_G(239) = 2^{-15/2}L(2,127) \simeq L(2,127)/181$ corresponds to 1.7 kHz.

3.5.4 Could $L(163) - L_E(163)$ resonance be important for consciousness?

Astrophysical quantum coherence is central in TGD inspired theory of consciousness and one can wonder whether $L(163) - L_E(163)$ resonance could be important for consciousness?

1. In music quint cycle with frequencies coming as powers of 3/2 is important and is central also for the model for bioharmony [L2, L8]. One can ask whether Eisensteinian and ordinary or Gaussian Mersennes with the same exponent n could be of special interest from the point of view of consciousness theory. Here $n \in \{2,5,7,19\}$ are of special interest.

   For Mersenne primes M_n the quint cycle could be defined in terms of the associated finite field. Since the powers $1/2^k$ correspond in finite field to powers 2^{n-k}, the scalings by powers 3/2 could correspond to finite field integer scalings by powers of $(3 \times 2^{n-1})^k$. This would bring a cutoff to the number of representable octaves.

   This cutoff might relate to the problem, which bothered Pythagoras: the powers $(3/2)^k$ of frequency give frequencies, which can be reduced to the basic octave by octave equivalence. For ordinary integers, the quint cycle can give an integer number of octaves only approximately. In finite fields the cycle closes and for p-adics it closes modulo $O(p)$. As Martin Ebner noticed, for $n = 5, 7$ and 19 it is 6, 18 and 27594 quints, so that twelve quints make whole octaves for $n = 5$ but not for 7 or 19. In any case, if music experience relies on finite fields, the problem of Pythagoras would disappear.

   Powers of 2 have Gaussian and Eisensteinian complex analogs and one can wonder whether they allow an interpretation in terms of conscious experience.

2. The ordinary, Gaussian *resp.* Eisensteinian Mersenne primes are also interesting from the perspective of the TGD inspired view of cognition. In a finite field associated with these Mersenne primes, the inverses x^{-k} for powers of x are given by x^{n-k} so that the finite field arithmetics and its p-adic counterpart become very simple unlike in the general case. These 3 types of number fields are cognitively simple, one might say.

4 Can TGD reproduce the basic aspects of the standard model view?

Consider now the question whether TGD is able to reproduce the basic aspects of the standard model view.

4.1 A vision about the TGD counterpart of Higgs mechanism

The development of a detailed view of the TGD counterpart of Higgs mechanism involves several challenges and the attempt to understand Weinberg angle gave strong hints about how to proceed.

1. The detailed structure of vertices must be understood. Both M^4 and CP_2 contributions to the modified gamma matrices for K-C-S action are needed. B_α and $J_{\alpha\beta}$ must be sums over M^4 and CP_2 contributions also for L(K-C-S). I have earlier concluded that twistor lift and the notion of (possibly warped) Hamilton-Jacobi structure imply it.

   Without both contributions, one does not get the counterpart of $\gamma^k A_k$ vertices, where M^4 gamma matrices γ^k appear. This is so because the modified gamma matrices $\Gamma^\alpha(K-C-S)$ determined by $L(K-C-S)$ contain only the CP_2 parts of M^4 contribution is not included.

   The explicit formulas for the vertex factor, which involves the transversal Kähler form of M^4, are needed. Note that the interior part of the Kähler action would also contain the M^4 part. Does this predict new effects possibly related to gravitation or does this only affect the standard $U(1)$ force? The most plausible interpretation is that the M^4 Kähler form gives rise to Newtonian gravitational potential.

2. The induced and modified gamma matrices contain both M^4 and CP_2 contributions. Also for $L(K-C-S)$ this allows decomposition of the vertex $\Gamma^\alpha A_\alpha$ into M^4 and CP_2 parts.

   Group theoretically, the CP_2 part of the gauge boson behaves like the corresponding component of Higgs. Higgs could be seen as CP_2 part of the gauge boson with polarization in CP_2 direction! The only difference is that Higgs couples the right and left M^4 chiralities of fermion and antifermion. The mixing of M^4 chiralities for a fixed H-chirality makes also possible the coupling of Higgs to photon as a bound state of collinear fermion and antifermion with opposite M^4 chiralities allowing helicity ± 1.

   The presence of both M^4 chiralities in fermion modes follows from the 8-D character of the massless Dirac equation and from the conservation of H-chirality guaranteeing separate conservation of B and L. This mixes both M^4 chiralities and means massivation.

   This conforms with the old intuition that Higgs components are the CP_2 counterparts of weak bosons.

3. The basic view about gauge bosons eating Higgs field components translates to the statement that after massivation W and Z have the usual M^4 and the CP_2 part identifiable as a gauge boson with CP_2 spin 1. For massive bosons it becomes the longitudinal scalar component.

   In the standard model γ remains massless and does not eat the corresponding Higgs component. In TGD, the non-eaten Higgs component can couple also to photons since a photon with spin 1 is necessarily a pair of parallel fermion and antifermion with opposite M^4 chiralities. This is possible because the massless Dirac equation in H fermions and modified Dirac equation imply that fermions are superpositions of opposite M^4 chiralities. At the space-time level warping gives rise to the mass parameter associated with the light-like geodesic of H with S^1 projection.

4. Standard model predicts that Higgs coupling to fermion is proportional to the mass of fermion and that the mass is proportional to the Higgs expectation. One must choose the associated fermion coupling appropriately to reproduce the fermion mass so that the model is a fit, not a predictive theory.

   (a) In p-adic thermodynamics the mass comes from thermodynamics and there is no Higgs potential. Also now Higgs develops a vacuum expectation since ¡H¿ as expectation of the

CP_2 counterpart of $\langle \Gamma^w \gamma_w \rangle$ can no longer vanish. The reason is that the vertex factors appearing in $\langle H \rangle$ and $\langle \Gamma^z \gamma_z \rangle$ coming from the M^4 and CP_2 part of the Kähler form are different. The generalization of the formula applied to get Weinberg angle from $\langle \Gamma^w \gamma_w \rangle$, when applied to the CP_2 part of γ, picks up the Higgs vacuum expectation $\langle H \rangle = h$ as the part of photon proportional to vector isospin.

(b) The mass-proportionality of the fermion coupling to Higgs follows in TGD from the induced/modified Dirac equation. The solution of Dirac equation is of the form $\Psi = D\Psi_0$, $D = \Gamma^\alpha D_\alpha$.

Both Γ^α and D_α contains M^4 and CP_2 parts. CP_2 part transforms M^4 chiralities to each other and couples right- and left-handed M^4 spinors and has interpretation in terms of Higgs-fermion coupling. Warping implies that the 1-D Dirac operator is massless at the fermion lines (light-like geoesics in H) as particles in H and X^4 but massive in M^4.

(c) Fermion mass satisfies $m \propto \hbar_{eff}\omega$, where the induced metric at fermion lines $\Phi = \omega t$ is $g_{tt} = 1 - R^2\omega^2 = c_\#^2$. Proportionality involves gauge potential, which is constant at the fermion line and corresponds to Higgs expectation value in the standard model. The appearance of warped $D(X^4)$ thus implies that Higgs-fermion vertex is of form

$$m\Psi_R \Gamma^t(CP_2)\Psi_L \ . \tag{4.1}$$

The desired mass proportionality of Higgs coupling to the fermion mass follows as a consequence.

5. The mass ratio m_W/m_Z is predicted by the standard model to be $m_W/m_Z = cos(\theta_W)$. How to understand this in the TGD framework? In p-adic thermodynamics this should follow from group theory alone.

(a) W and Z are superpositions of fermion pairs with coefficients proportional to the gauge couplings g_{mn} of the boson to the fermion pair $F_m\overline{F_n}$.

(b) The mass squared is additive as a conformal weight (note the connection with string picture) [K7]. The mass squared is proportional to

$$\sum_{mn} g_{mn}^2(m_m^2 + m_n^2) \ .$$

Group theory determines the coupling parameters g_{mn} and involves the Weinberg angle. This model should predict the m_W/m_Z correctly. If so, this would give a connection between p-adic thermodynamics as WCW level description and $\langle \Gamma^w \gamma_w \rangle$and $\langle Higgs \rangle = h$ conditions related to the QFT type description at the space-time level. The Higgs couplings do not depend on the charge but only on the vector isospin, so that the Higgs would correspond to the part of $\Gamma^\alpha A_\alpha$ involving CP_2 gamma matrices and coupling to vector isospin. The p-adically calculated fermion masses should in turn be proportional to the warping frequencies ω at the space-time level.

4.1.1 Stringy view of electroweak bosons and mesons

TGD unification of electroweak and strong interactions in terms of CP_2 geometry has evolved as responses to objections.

1. Electroweak bosons and mesons are proposed to emerge as bound states of fermion and antifermion assignable to string-like objects identifiable as closed two-sheeted monopole flux tubes associated with pairs of space-time sheets. Mesons would have fermion and antifermion at the wormhole throats of *opposite* wormhole contacts and would be very much like ordinary mesons modelled in terms of open strings.

   Electroweak bosons, Higgs like particles, and also gluons, would consist of a fermion and antifermion at the opposite throats of a *single* wormhole contact. Pairs of left- right-handed

neutrinos at the other wormhole contact would screen the weak isospin I_L^3 so that the Higgs mechanism could be understood as weak screening.

The absence of the stringy interactions between fermion and antifermion of the gauge boson and Higgs-like particle mediated by string would explain why electroweak bosons do not form Regge trajectories assignable to hadronic states.

2. The most obvious objection is that massless bound states of a massless fermion and antifermion with helicity ± 1 are impossible in M^4: the helicities for fermion and antifermion must be opposite and the state would be an M^4 scalar. In TGD, 8-D chiral symmetry makes possible the 8-D notion of light-likeness (masslessness).

   The H-chirality as the product of M^4 and CP_2 chiralities is fixed and quarks and leptons correspond to opposite H-chiralities. H spinor modes are therefore superpositions of tensor products of M^4 and CP_2 spinors having a fixed H-chirality defined as a product of M^4 and CP_2 chiralities. M^4 chirality is not fixed anymore and has interpretation as a signature of massivation.

3. This makes possible spin 1 bound states of fermion and antifermion with opposite M^4 chiralities. Also the photon and other massless particles would be states of this kind. p-Adic thermodynamics suggests that also photons can have a small mass due to the p-adic temperature $T = 1/n < 1$ associated with the fermion and antifermion. I have proposed that the fermion and antifermion for weak bosons have $T = 1/2$. The second option is that the p-adic mass scale for bound state fermions is reduced by a power of $\sqrt{2}$.

   For $p = M_{89}$ and $T = 1$ the mass scale of weak bosons would be roughly by a factor $\sqrt{2}$ too large for $T = 1$. p-Adic length scale $L(k)$ with $k = 90$ is suggestive. There are two options. Either the p-adic length scale of the bound state fermions increases by a factor $\sqrt{2}$ or the reduction of p-adic temperature to $T = 1/2$ produces an effective p-adic mass scale $k = 90$. In the latter case however additional assumptions are required in order to avoid practically vanishing mass. This suggests that the genuine reduction of the p-adic mass scale, reducing the mass squared and conformal weight, is a general aspect of being a bound state.

4. 8-D light-likeness implies that there are only spin 1 bosons in the 8-D sense having polarization in M^4 or CP_2 degrees of freedom. Gauge bosons would correspond to spin 1 in M^4 and Higgs like states to spin 1 in CP_2. Isospin doublet nature of Higgs would correspond to 8-D masslessness in CP_2 sector. Only two polarization directions are possible for massless states. The TGD counterpart of the Higgs mechanism would transform the neutral CP_2 helicity of Higgs doublet as a fermion pair to the third spin component of a massive weak boson.

   With respect to fermionic isospin, weak bosons couple left-handedly and Higgs vectorially. As far as charge is considered, Higgs behaves like the counterpart of the W boson. Could also Z^0 *resp.* the proposed G_s have Higgs-like counterparts? Note that the vector part of the axial part V_{03} of the induced spinor connection has the same form as Z^0 and could correspond to vectorially coupling Higgs.

4.1.2 What is the TGD counterpart for the transition to the unitary gauge?

In standard model, the surviving Higgs component in the electromagnetic direction is left in the transition to unitary gauge transferring the disappearing Higgs components to longitudinal components to W and Z and rotating the Higgs to a fixed direction invariant under electromagnetic gauge rotations. This component cannot be gauge transformed away.

In TGD, the Higgs is replaced by the CP_2 gauge potentials or rather their contractions with CP_2 matrices so that the analog of unitary gauge would mean that the charged part of the induced CP_2 gauge potential vanishes at the partonic 2-surface. Note that the charged part of the induced gauge field need not vanish unless the CP_2 projection is a homologically non-trivial geodesic sphere. Concerning the construction of vertices this would be enough.

Quite generally, for the TGD counterpart of unitary gauge the classical W boson fields would by definition vanish in the ground state at partonic 2-surfaces and the fluctuations of W would give rise to the counterpart of quantum field realized as fermion antifermion pairs. This condition conforms

with the intuitive view that the classical fields do not induce a mixing of different electromagnetic charge states for spinors.

4.1.3 What does one mean with the monopole flux tubes?

How to identify the monopole flux fluxes? Monopole flux tubes are basic solutions of field equations of TGD and appear in all scales. Holography = holomorphy vision implies that they appear for practically any general coordinate invariant action principle dictated by the induced geometry only.

One can imagine two options for what monopole charges could mean.

1. Can one assign monopole flux to fundamental fermion as wormhole throat or boundary component? The universality of the condition $\langle \Gamma^w \gamma_w \rangle$ would not be lost. However, in the simplest view lepton would correspond to a single partonic 2-surface, and must have a vanishing monopole charge. This leads to problems.

   In early days of TGD, I considered the idea that it is possible to associate with the valence quarks homology charges which sum up to 0. Fluxes 2,-1,-1 for the three valence quarks would be the simplest choice for baryons. This inspires the question whether color hypercharge Y with values 2/3, -1/3, -1/3 of quark could be proportional to the flux. Entire hierarchy of color partial waves defining a hierarchy of standard model physics is predicted in the recent formulation of TGD and the hypothesis would predict the color fluxes associated with the quarks and leptons in various color partial waves.

2. Or is the monopole charge a property of the wormhole contact at the either end of the monopole flux tube connecting two parallel space-time sheets? This option looks more plausible one. Fundamental fermions would be associated with closed wormhole monopole flux tube loops travelling from throat A_1 to A_2 along space-time sheet A. flowing from A_2 to B_2 through wormhole contact at sheet B, and along B from B_2 to B_1 and back to A_1 at A through wormhole contacts. The two wormhole contacts are needed since flux loops must be closed and they are needed to screen the weak isospin of both quarks and leptons as an alternative view of weak symmetry breaking. Note that in TGD the color hyper charge and isospin and their weak counterparts correspond to each other as fermion quantum numbers (CP_2 spin quantum numbers to be distinguished from color quantum numbers of color partial waves). Weak screening means color confinement for CP_2 spin degrees of freedom.

   The vanishing of homology charge would occur already at the level of quarks and leptons and the Weinberg angle could be the same in the quark and lepton sectors. For this option the simplest choice of homology charge is 1 unit and this would fix the model and imply independence on the details of the partonic 2-surface, which conforms with the TQFT aspect of K-C-S action. Note however that the condition $\langle \Gamma^w \gamma_w \rangle = 0$ depends very weakly on the details and even topology of the partonic 2-surface.

4.1.4 A warped variant of M^4 Kähler structure is required by the TGD counterpart of Higgs mechanism

Whether the Hamilton-Jacobi (H-J) structure [L9] as Kähler structure for M^4 is necessary has been a long standing open question. H-J structure involves a local integrable decomposition of tangent space of M^4 two longitudinal and transversal parts. Longitudinal tangent space as a local analog of M^2 has hypercomplex coordinates u and v whose coordinate curves have light-like tangent vectors. The Kähler form has only transversal part: this is the counterpart for the allowance of only transversal polarizations in gauge theories.

Warping [L23, L20] strongly suggests the possibility of warped H-J structure so that u and v can vary along light-like geodesics of H at fermion lines located at 3-D partonic orbits. Even more general coordinate lines with light-like tangential vectors can be considered. The warping is characterized by a reduced light velocity $c_\# = \sqrt{g_{tt}} = \sqrt{1 - R^2\omega^2}$, where the light-like geodesic having a geodesic circle as CP_2 is represented as $\Phi = \omega t$.

Higgs mechanism can be understood in this way geometrically. The restriction to the partonic orbit and fermion lines eliminates a degree of freedom. The frequency ω associated with the warped light-like geodesic of H suggests that fermion mass scale is proportional to $\hbar_{eff}\omega$ determined by

the spinor dynamics at the level of H. Note that ω is inversely proportional to $\hbar_{eff}$ so that the mass is the same for the dark variants of the fermion. For massless particles there is no warping: massless extremals [K2] represent a basic example of this.

Warping at partonic orbit corresponds to the development of the vacuum expectation value of Higgs and the absorption of longitudinal degrees of freedom to the Z^0 longitudinal degree of freedom. The hypercomplexity for the functions f_1 and f_2 appearing in H-H principle means that they depend only on the second hypercomplex coordinate, call it u. The complex coordinate of the partonic orbit corresponds to the degree of freedom absorbed to the longitudinal degrees of freedom of W bosons.

4.1.5 Warped M^4 Kähler structure makes possible gauge theory vertices

The following argument shows that M^4 Kähler structure and its warped counterpart are necessary also from the point of view of the construction of scattering amplitudes.

1. The counterpart of $\Gamma^\alpha A_\alpha$ as a counterpart of $\gamma^k \epsilon_k$ appearing in the gauge theory vertices must be obtained. M^4 gammas must appear in Γ^α.

2. $\Gamma^\alpha A_\alpha$ are associated with the points of the partonic orbit and their 2-D ends X^2 define generalized vertices. Ordinary fermin vertices correspond to the intersection of the fermion lines and X^2. The gamma matrices in $\Gamma^\alpha A_\alpha$ are determined by Kähler-Chern-Simons action L_{K-C-S}.

   L_{K-C-S} for CP_2 alone involves only CP_2 gamma matrices and one does not bet the desired M^4 gamma matrices at the vertices. The propagators for gauge bosons would be effectively correlation functions in CP_2. This does not make sense. Polarization vectors would be in the CP_2 directions. One would have only the analog of the Higgs field as spin 1 field in CP_2.

3. Also M^4 gamma matrices are needed. This requires the M^4 H-J structure and corresponding transversal M^4 Kähler form. An H-J structure with a transversal complex coordinate would give modified gamma containing both M^4 and CP_2 part and these would determine the direction of the local polarization vector. It would be transversal in accordance with gauge invariance.

4. The Kähler-Chern-Simons action L_{K-C-S} can be seen as a generalization of the action $S/\hbar = g \int A_\mu dx^\mu/hbar$ describing the interaction of point like charge with an Abelian gauge field $F = dA$. In the recent case g correspond to the U(1) coupling g_1 and is absorbed to A. $J = dA$ corresponds to F/g_1. This would gives

   $$L_{K-C-S} = \frac{1}{\hbar}\epsilon^{\alpha\beta\gamma} A_\alpha J_{\beta\gamma} \quad . \tag{4.2}$$

   One could also see L_{K-C-S} as a boundary term emerging from the Abelian instanton term $\frac{1}{2g_K^2}\int J^J \sqrt{g_4} dV$ as surface terms implying the replacement $L_{K-C-S} \to (1/2g_K^2) L_{K-C-S}$. The induced fields A and J appearing in L_{K-C-S} are sums of M^4 and CP_2 contributions.

5. The modified gammas are given as

   $$\Gamma^\alpha = \Pi_k^\alpha \Gamma^k \quad , \Pi_k^\alpha = \partial_{\partial_\alpha h^k} \quad . \tag{4.3}$$

   Here Π_k^α are canonical momentum densities, whose covariant divergences vanish by field equations: this gives rise to field equations as conservation laws for isometry charges.

   The partial derivatives with respect to M^4 complex coordinate are proportional to transversal M^4 gamma matrices and their contractions with induced gauge potentials define the counterparts of $\epsilon_k \gamma$. The CP_2 part gives polarization analogous to those for Higg bosons interpreted as spin 1 particles in CP_2. 3 of them would be absorbed to longitudinal degrees of freedom.

6. If $J(M^4)$ is transversal and longitudinal part of B_α as the sum of M^4 and CP_2 parts vanishes, the M^4 part of the modified M^4 gamma matrices and thus polarization can be only longitudinal.

   The simplest option would be that for M^4 the Kähler gauge potential B has only the transversal component. This would give

   $$B_u = B_u(CP_2) \ , \quad B_w = B_w(M^4) + B_w(CP_2) \ , \quad J_{w\overline{w}} = J_{w\overline{w}}(M^4) + J_{w\overline{w}}(CP_2). \tag{4.4}$$

   The CP_2 part of B_u must be non-trivial and is non-trivial for the light-like geodesics defining the fermion line at the partonic 2-surface. This would give to the modified gamma matrix M^4 contribution

   $$\Gamma^\alpha(M^4) = \epsilon^{u\alpha\beta} B_u(P_2) J(M^4)_{kl} \partial_\beta m^l \gamma_l \ . \tag{4.5}$$

   $B_u(CP_2)$ would be CP_2 Kähler gauge potential in the direction of u coordinate varying in the direction of the fermion line. The contraction $\Gamma^\alpha(M^4)A_\alpha$, where A_α is the complete gauge potential involving also non-Abelian contributions from CP_2, involves a gauge coupling term involving also the contraction of M^4 gamma matrices with A. The WCW correlation function for this quantity would define the counterpart of the boson propagator. The CP_2 contribution would give the analog of the Higgs propagator.

7. The explicit expression for the TGD counterpart of the ordinary M^4 contribution to the vertex $\Gamma^\alpha A_\alpha$ is

   $$\frac{1}{g_1^2} \epsilon^{u\alpha\beta} B_u(CP_2) J_{kl}(M^4) \partial_\beta m^l A_\alpha \ . \tag{4.6}$$

   B_α is the sum of M^4 and CP_2 Kähler gauge potentials and A_α includes also W and Z parts. There is also analogous vector potential contracted with the gamma CP_2 matrices associated with CP_2 degrees of freedom and would relate to Higgs absorbed to W and Z^0 to give them longitudinal degrees of freedom. The Higgs boson as a dynamical field corresponds to the CP_2 part of weak gauge potential coupling only to the vector isospin. The neutral part of the Higgs as a dynamical field is not the entire CP_2 part (projection to CP_2 directions) of the massless γ, whose couplings are proportional to the electromagnetic charge, but only the projection of the part coupling to the vector isospin. Since the modified gamma matrices involved with $\langle \Gamma^w \gamma_w \rangle$ and its counterpart $\langle H \rangle$ are different, $\langle H \rangle$ is non-vanishing. The condition $\langle \Gamma^z \gamma_z \rangle = 0$ fixes the vacuum expectation of Higgs.

4.2 What is the correct choice of Kähler-Chern-Simons action density $L(KCS)$?

The following questions are highly relevant for the proposal considered.

1. Are the integrals over CP_1 really what they are thought to be?

2. Is the Kähler-Chern-Simons action density $L(KCS)$ defined by the sum of Kähler forms of CP_2 and M^4 or is it a sum of $L(KCS)(M^4)+L(KCS)(CP_2)$? Is it possible to select between these options by requiring that the $\langle \Gamma^w \gamma_w \rangle = 0$ argument remains as it is?

It is important to check whether the integrals defining "$\langle 1/r \rangle$" and "$\langle r \rangle$" do not involve any additional factors depending on r. The overall normalization does not matter. The induced spinor connection of CP_2 in the vertex should not produce nasty surprises. But what about $L(KCS)$ action density?

1. There are two options for $L(KCS)$.

   Option a). Induced Kähler gauge potentials of M^4 and CP_2 sum up to give $B_K = B_K(M^4) + B_K(CP_2)$ and $L(KCS)$ is defined by B_K and J_K. The counterpart of the standard model $U(1)$ gauge field would contain an additional contribution. This would affect our estimate for $p = sin^2(\theta_W)$ from $\langle \Gamma^w \gamma_w \rangle = 0$ condition by adding an additional term. Therefore this option is not favoured. Note that $L(KCS)$ is proportional to $B_t J_{\xi\bar{\xi}}$.

   Option b). $B_K(M^4) + B_K(CP_2)$ are independent fields and one has $L(M^4, KCS) + L(M^4, KCS)$. The interpretation of $B_K(M^4)$ as an analog of Newtonian gravitational potential is attractive.

2. $J(M^4)$ plays a key role in making the theory non-trivial.

   (a) $J(M^4)$ has only the transversal part $J_{w\overline{w}}$. This reflects the physical fact that only transversal degrees of freedom are physical in gauge theories, in string models and also in TGD. This degeneracy reflects the Minkowskian signature of M^4.

   (b) The presence of $J(M^4)$ in $L(KCS)$ contributes to the modified gamma matrics parts involving M^4 gamma matrices: these appear n the M^4 part of condition for $\langle \Gamma^w \gamma_w \rangle = 0$. Otherwise one obtains only vertices for CP_2 parts of induced gauge bosons behaving like Higgs type scalar fields coupling left and right hand scales. $J(M^4)$ is necessary for a non-trivial theory.

   (c) What about $B(M^4)$? It certainly has a transversal part $(B_w, B_{\overline{w}})$ giving rise to $J_{w\overline{w}}$. $B(M^4)$ could also be a longitudinal part which is gradient so that it would not contribute to $J(M^4)$. The simplest option is $B_t(M^4) = constant$. This is necessary to make $L(KCS, M^4)$ non-vanishing. Since B_K is not a genuine gauge potential but defines Kähler form the gauge transformations correspond to generalized symplectic transformations affecting the space-time surface. Therefore also gradient has physical meaning and would have such content if the interpretation as the TGD counterepart of gravitational gauge potential is correct.

3. Options a) and b) differ with respect to their dependence on Kähler gauge potentials of M^4 and CP_2.

   (a) For option a) one as $B = B(M^4) + B(CP_2)$ and $J = J(M^4) + J(CP_2)$. For option b) and has $B = B(M^4)$ and $J = J(M^4)$ in $L(KCS)(M^4)$ and $B = B(CP_2)$ and $J = J(CP_2)$ in $L(KCS)(CP_2)$.

   (b) If $B(M^4)$ is transversal, $L(KCS)(M^4) \propto B_t(M^4)$ vanishes identically and one does not obtain gauge boson propagators for option b). Therefore only the option a) works for $B_t(M^4) = 0$ since $B_t(CP_2) \neq 0$ guarantees that $L(KCS)$ is nonvanishing. $J(M^4)$ is non-vanishing also in this case and contributes to $L(KCS)$.

4. Option a) has however problems.

   (a) Kähler gauge potential is the sum $B(M^4) + B(CP_2)$ and this modifies the $\langle \Gamma^w \gamma_w \rangle = 0$ argument involving the non-vanishing transversal part of $B(M^4)$.

   (b) $B_t(CP_2) = Be_t^3$ is proportional to $r^2/(1+r^2) = (F-1)/F$. One has $B_t = B_t(M^4) + B_t(CP_2)$ and for $B_t(M^4) = 0$ this would mean the replacements

   $$\langle (F-1)/F \rangle \to \langle ((F-1)/F)^2 \rangle \ ,$$
   $$\langle 1/F \rangle \to \langle (F-1)/F \rangle \ .$$

   This would reduce the contribution from small values of r to these expectations by factor r^2. The nice conformal invariance predicting $Z = 1/2$ for holomorphic sections would be probably lost.

5. These arguments suggest that option b) with $L(KCS) = L(KCS, M^4) + (KCS, CP_2)$ is the physical option. Suppose that $B(M^4)_t = constant$.

(a) The earlier argument $\langle \Gamma^w \gamma_w \rangle = 0$ remains unaffected and one avoids problems due to the addition of $B(M^4)$ to standard model $U(1)$ gauge potential. The interpretation of $B(M_4)_t$ as the analog of Newtonian gravitational potential is attractive.

(b) A connection with warping, distinguishing between GRT and TGD, is highly suggestive. In warping $g_{tt} = 1$ is replaced with $g_{tt} = c_{\#}^2 = 1 - R^2\omega^2$, where $c_{\#}$ is the reduced light velocity using unit $c = 1$. $1 - c_{\#}^2$ is analogous to constant gravitational potential $2\Phi_{gr}$. Could $B_t = constant$ at fermion lines correspond to the constant gravitational potential $2\Phi_{gr} = 1 - c_{\#}^2$. Intriguingly, this term relates directly to mass squared of the fermion.

4.3 Electric magnetic duality and two views of strong interactions

The outcome of the attempts to understand Weinberg angle to be discussed in the next section led to rather concrete ideas about the realization of the old idea about electric magnetic-duality [K2].

4.3.1 Electric-magnetic duality

Electric magnetic duality [K2] is one of the key ideas of TGD. It has several motivations.

1. The self-duality of CP_2 Kähler geometry suggests electric-magnetic duality. The electric description would be in terms of ordinary Coulomb charges, say color charges. The magnetic description would rely on magnetic monopoles. The proposed description of Weinberg mixing based on WCW wave functions for the locations of fermion vertex a partonic 2-surface identified as holomorphic monopole harmonics with monopole charge identified as homology charge gives support for the magnetic picture.

2. The second observation relates to the proposed identification of classical gluon fields as $G_{\alpha\beta} = H^A J_{\alpha\beta}$, where H^A are Hamiltonians of color isometries. The counterpart of color YM action density is proportional to $G^{\alpha\beta}G_{\alpha\beta} = H^A H_A J^{\alpha\beta} J_{\alpha\beta}$. $H^A H_A$ is invariant under color isometries and must be constant so that the action density is proportional to the Kähler action density.

   Does the description of color interactions at the space-time level reduce to that based on Kähler action? It might be however warned that the proposed identification of strong $U(1)$ force is not in terms of Kähler gauge potential but the vector part of V_{12} of spinor connection of CP_2.

3. The above argument works also in M^4 sector. The light-cone boundary δM_+^4 is metrically a 2-sphere and the Hamiltons of SO(3) isometries are analogous to color Hamiltonians so that the analogs of color gauge fields $G_{\alpha\beta}(M^4) = H^A J_{\alpha\beta}(M^4)$ make sense and also M^4 Kähler action two interpretations. Now $SO(3)$ and $U(1)$ descriptions would be dual and $U(1)$ description would rely on monopoles as homologically charged 2-surfaces associated with light-cone boundary with a missing tip defining a time-like slicing of the particle's CD.

4. The Hamiltonians defined in $SO(3)$ and CP_2 indeed occur in the construction of WCW geometry and are associated with the symplectic algebra of light-cone boundary acting as isometries of WCW. They form an analog of Kac-Moody algebra [K6, K9]and the Hamiltonians generating isometries are in a special role in that they do not act as mere gauge transformations as the higher Hamiltonians would naturally do.

4.3.2 Electric-magnetic duality and strong interactions

This vision leads to rather fascinating speculations and conforms with the general vision. Consider first the electric-magnetic duality already mentioned.

1. Electric-magnetic duality would suggest that the strong interaction coupling strengths for the descriptions based on $U(1)$ and color coupling strengths are related by the equation $\alpha_s(U(1)) = 1/\alpha_s(QCD)$. In the next section it will be found that this allows us to understand the relation between weak and strong interaction parameters and the picture is consistent with the empirical facts.

2. Electroweak quantum numbers assignable to CP_2 are analogous to fermionic spin quantum numbers and the electroweak group as holonomy group is a subgroup of color group $SU(3)$. This looks paradoxal first since color hyper charge and isospin are definitely not identical with their electroweak counterparts. The representations of color quantum numbers at space-time level as quantum numbers of monopole harmonics solves the apparent paradox and here the non-point-like nature of partons is essential.

   Note that electric-magnetic duality does not identify electric and magnetic fluxes. This would imply electroweak confinement and also electromagnetic charges are zero for physical states in some scale (in fact, they vanish at a certain, long enough scale, consider only atoms!). There is also electroweak confinement of weak isospin implying massivation and proposed to be realized in terms of pairs of left- and right-handed neutrinos.

3. There are two identification of spinor modes at embedding space level. The color partial waves are defined in $H = M^4 \times CP_2$ *resp.* $CD = cd \times CP_2$ as products color partial waves in CP_2 an plane waves in M^4 *resp.* partial waves in cd organized into irreps of Lorentz group for the light-cone boundary: in this case the line connecting the tips of CD is regarded as a hole behaving like magnetic monopole in cd and one obtains monopole harmonics.

Can the electric-magnetic duality map the spinor wave harmonics to monopole harmonics.

1. The hypothesis deserving to be tested is that color hyper-charge *resp.* color isospin corresponds to magnetic charge *resp.* vector isospin assigned to strong $U(1)$ gauge potential V_{12} maps the color quantum numbers of spinor modes at the embedding space level to those at the space-time level. The hierarchy of color representations for the H-spinors would be mapped to a hierarchy of homologically non-trivial partonic 2-surfaces and monopole harmonics at them.

2. Is it really possible to map color partial waves to monopole harmonics? The monopole harmonics could be associated with the position of the edge of the fermion line at a partonic 2-surface acting as a vertex in which case $(q, k) = (0, 0)$ is suggestive. They could be also associated with the position of the fermion line at the general partonic surface.

   There are two quantum numbers at both sides so that this conjecture is worth testing. In the case of $cd \subset M^4$ case, the two quantum numbers would relate to the representations of Lorentz group and should correspond to monopole harmonics or analogous structure, such as the exceptional representation of the Lorentz group (see the Appendix).

It turns out that this proposal is too ambitious.

4.3.3 The identification of the color quantum numbers of CP_2 spinors and of monopole harmonics fails

The detailed study of the hypothesis that the color quantum numbers of spinor modes correspond to the quantum numbers of monopole harmonics demonstrated that it cannot be true.

1. The strongest form of electric-magnetic duality is that all spinor modes are in one-one correspondence with monopole harmonics is extremely strong and does not seem possible. They correspond to groups $SU(3)$ and $SU(2) \times SU(2)$. The same dimension for Cartan algebra is not enough to give a natural 1-1 correspondence for the representations and their quantum numbers.

   In particular, the proportionality of color hyper-charge to the monopole magnetic charge q proportional to the homology charge is not realistic as it is easy to see already for the quark triplet. What goes wrong and what might cure the problem?

2. The hint comes from physics. The success of QCD and electroweak screening require the assumption that color singlet hadrons are constructed by first constructing color triplet quark states by using pairs of right and left-handed neutrinos, which move in triality zero color partial waves. Neutrino pairs also screen the weak isospin in the scale of string like monopole

flux tube so that the system has only Kähler charge and vectorial isospin I_V^3 identifiable as strong isospin. Same happens to the leptonic color multiplets.

This would predict that the hierarchy of hadron physics consists of almost copies of each other. Only the number of color triplets might depend on the level in the hierarchy.

The problem of realizing the electric-magnetic duality would reduce to the level of color triplet for quarks. This is an enormous simplification. Only the problem with the space-time representation of color charges of the color triplet remains.

The information about the color triplet of quarks should be represented at the space-time level in terms of WCW wave functions for the location of the fermion line and its edge at the partonic 2-surface.

1. A given WCW wave function at a partonic 2-surface should represent one particular state of the color triplet. The $(z/F^{1/2})$ as an analog of oscillator vacuum and covariantly holomorphic powers $(z/F)^n$ are the basic building bricks appearing in the holomorphic monopole harmonics. The factor $1/F = 1/(1+r^2)$ is eksponent $exp(-K)$ of the Kähler function $K = log(1+r^2)$ of CP_2 restricted to partonic 2-surface or, equivalently, to its CP_1 projection. The presence of F guarantees covariant holomorphy as the vanishing of $D_{\overline{z}}(z/F)^n$ with respect to the covariant derivative $D_{\overline{z}} = \partial_{\overline{z}} - B_z$, where one has $B_z = \partial_z K$ and $B_{\overline{z}} = -\partial_{\overline{z}} K$. Different signs are essential: otherwise the Kähler gauge potential would reduce to a gradient.

2. The minimal condition is that the color triplet corresponds to a triplet at the partonic orbit. Could the member of the triplet be restricted by induction procedure to the partonic orbit and partonic 2-surface. Could this color triplet define a model for WCW wave functions for the position of the fermion line?

   There is a problem. $\xi^1/(1+r^2)$ and $\xi^2/(1+r^2)$ are half-covariantly constant (covariantly holomorphic) in CP_2 but the third section $1/(1+r^2)$ representing quark with hypercharge 2 and vanishing isospin: only two modes are half-covariantly constant $\xi_i/(1+r^2)$. Even worse, the 2-dimensionality of the CP_1 allows only 1 independent holomorphic mode $z/F^{1/2}$ rather than 3! How to save the holomorphy?

3. Here the projective geometry of CP_2 comes to the rescue. CP_2 has 3 poles. The quark triplet could correspond to a partonic 2-surfaces inside the 3 coordinate patches of CP_2 to whose 3 poles 3 homologically non-trivial geodesic spheres can be assigned. The choices of the patch would code the color quantum numbers of the triplet!

   At a given geodesic sphere, the member of the color triplet would correspond to $z/F^{1/2}$, which is covariantly constant. The choice of the coordinate patch defining S^2 as a twistor sphere would geometrically code for the quark color quantum numbers $(Y, I_3) in \{(2/3, 0), (-1/3, \pm 1/2)\}$. z corresponds to one of the projective coordinates (z_1, z_2, z_3) of CP_2. Color confinement would mean that all 3 coordinate patches are populated by quarks, one quark for each pole. There would be no need to identify the color quantum numbers with those of monopole harmonics.

It would seem that only the section $z/F^{1/2}$ is needed in a given coordinate patch? What about the modes $(z/F^{1/2})(z/F)^n$ for $n > 0$. Are they needed at all or is just $n = 1$ enough? And what about fractional powers $1/n$ and m/n allowed by holomorphy? What could their presence mean physically?

1. Conformal invariance would suggest that higher powers z^n correspond to gauge degrees of freedom. Perhaps conformal invariance would be a manner to say that sections proportional to z^n are not needed.

2. What about fractional powers? Here the TGD based explanation [K8] of FQHE [D1] and charge fractionization enters the game here and obtains a connection with the monopole charge identifiable as homology charge. Identify z as a coordinate of $CP_1 \subset \delta M_+^4$ or of $CP_1 \subset CP_2$.

3. When z is replaced with $z^{1/n}$, $(z/F^{1/2})$ *resp.* z/F is replaced with $z^{1/n}/\sqrt{1+r^{2/n}}$ *resp.* $z^{1/n}/(1+r^{2/n})$. What happens? Taking $w = z^{1/n}$ as a new internal complex coordinate the section is proportional $w/(1+w^2)$. A 2π rotation for w leads to the original point but corresponds to n full 2π turns in CP_1 identified as the δM^4_+ or CP_2 projection. With respect to M^4 or CP_2 the spin and color isospin are fractional but with respect to the internal complex coordinates they have the normal values. One still has a holomorphy but there is a cut along the real axis since n 2π turns are needed to get to the original point.

4. Physically and geometrically the two situations are not identical. The many-sheeted partonic 2-surface is a singular covering of CP_1 and $z = 0$ is one of its branch points. This gives for the partonic 2-surface non-trivial homology in δM^4 and CP_2 so that it has a non-vanishing monopole flux. Homology equivalence class indeed corresponds to the degree n of the covering. The sheets of the covering touch at its fixed points of the covering map (see the Appendix) so that the genus of the surface is higher than $g = 0$.

   This gives a way to build algebraic geometric representations of higher quark generations with $g > 1$. Fractional powers of z would provide the way to build many-sheeted surfaces as coverings and with higher monopole flux. The states of the quark triplet can be described geometrically by the same section $z/F^{1/2}$ in the 3 coordinate patches of CP_2. The values of the parameter Z and Weinberg angle would be universal and this conforms with the calculations based on monopole harmonics for which magnetic charge corresponds to the homological charge.

5. What does the improved view mean for the model based on monopole harmonics? It seems that the only change is the restriction to holomorphic sections, already found to give universal predictions $Z = 1/2$ and $p = 1/4$. The condition for universality is $k = l/2 = q - l/2$. Since q is integer, the half-odd integer value of l and the condition $l \geq q$ requires $l_{min} = q + 1/2$. This does not allow the $q - l/2 = l/2$ condition and the prediction $Z = 1/2$ is lost. The universality requires that sections with half-odd integer l drop from consideration.

4.3.4 Electric-magnetic duality, color confinement, asymptotic freedom, and genus correspondnce

Consider now the relation to color confinement and asymptotic freedom.

1. The original naive proposal, made during the first years of TGD, was that the color hyper charge could be proportional to homological charge. If so, color confinement corresponds to homological confinement in the $U(1)$ picture. Higher color multiplets of H spinors have large color hyper-charges and would require large homology charges in conflict with the intuitive expectation that homology charges are small. It was already found that the correspondence between color multiplets and monopole harmonics cannot be realized. The confinement to color triplets preceding hadronic confinement cannot cure this problem.

   The identification of the color hyper charge as a homology charge would also predict a correlation between genus and color hyper charge. This does not make sense physically.

2. The UV type asymptotic freedom of QCD type description relates to IR type asymptotic freedom in the $U(1)$ type description. Standard QFT picture suggests that at $\alpha_s = 1$ a phase transition forcing a transition between the two descriptions would occur. Below this length scale $U(1)$ description would work and above it the QCD type description would work. The identification of this phase transition as a transition to the analog of quark gluon plasma phase from hadron dominated phase comes first in mind. The problem is that space-time picture does not allow description of color quantum numbers and classical gluons fields are absent.

   The problem due to the large $U(1)$ coupling strength however disappears since holography eliminates the path integral and perturbation theory. In the TGD framework, this phase transition would be between M_{107} and M_{89} hadron physics and would occur at the level of H and WCW.

Electric-magnetic duality forces to challenge the genus-generation correspondence and the view about color confinement.

1. The genus of the partonic surface correlates strongly with the polynomial bidegree of the polynomial defined in $CP_1 \times CP_1 \subset \delta M^4_+ \times CP_2$ (see Appendix). Higher genera require higher homology charges in holography = holomorphy vision in which Riemannian geometry becomes algebraic geometry. These genera are proposed to behave like many-handle states with a mass spectrum consisting of bands rather than discrete masses, just as in condensed matter physics. These states are not like ordinary elementary particles. What comes to mind are anyonic systems and FQHE.

2. Color confinement requires color singlet states. There are two ways to realize the confinement in the case of quarks.

   Option a) Hadrons are just color singlets without additional conditions.

   Option b): Hadrons are color singlets as composites of color triplets formed first. The success of QCD favours this option.

   For both a) and b) the number of hadrons increases with a given level but the number of hadrons is smaller for b). For b) the number of triplets can increase with the level. The increase of hadronic complexity is a testable prediction: for instance, does ordinary hadron physics provide evidence for additional triplets. Also the proposed genus-generation correspondence for which p-adic mass calculations provide support [K5] must be challenged: could whether fermion families correspond to the existence of several color triplets.

4.3.5 WCW correlation functions for bosons and super symplectic algebra

In the next section it will be found that the electric-magnetic duality suggesting $\alpha_s(U(1)) = 1/\alpha_s(QCD)$ allows to fix the parameters of $U(1)$ view of strong interactions and relates only TGD view to QCD description assumed to emerge at the QFT limit. This however provides only the $U(1)$ description of strong interactions. Can the $U(1)$ description be useful?

1. In the standard QFT the perturbation theory does not exist for $U(1)$ description for values of $\alpha_s(QCD)$ for which perturbative QCD would converge. The solution of the problem comes from holography. The path integral is absent and radiative corrections and coupling constant evolution is discrete and occurs by phase transitions increasing algebraic complexity.

2. Also $U(1)$ approach should provide a representation for the gluon propagator but there is no direct coupling between spinor modes and gluons. Fermionic quantum numbers correspond to those for monopole harmonics. How are the color quantum numbers of the emitted gluon at the vertex represented and how the gluon propagators are represented at WCW level?

3. The observation that $H_A J_{\alpha\beta}$ behaves like a classical gluon field suggests that gluon propagators correspond to WCW correlators for Kähler gauge potential B_K multiplied by the Hamiltonian H_A of color isometry at the vertex. Could gluon propagators reduce to WCW correlation functions for $H_A B_K$? By the preceding considerations also the correlators $H_A V_{12}$ are highly suggestive and can be extracted from the fermion vertex. Boson emission vertex must involve the matrix element of H_A between spinor modes multiplied by holomorphic ScMHs.

4. This suggests that gluons at vertices are represented by the Hamiltonians of color isometries. This generalizes to the Hamiltonians of the entire symplectic algebra of CP_2 and also to the symplectic algebra of light-cone boundary δM^4_+ reducing metrically to the sphere CP_1. Also the products $H_A(\delta M^4_+)H_A(CP_2)$ of $\delta M^4{}_+$ and CP_2 Hamiltonians must be allowed. If the symplectic invariance is analogous to conformal gauge invariance it implies that only the isometry generators are important at the QFT limit.

5. This speculative picture emerged from a bottom-up approach in an attempt to understand Weinberg angle. What is intriguing that this picture conforms with the original top-down vision that scattering matrix elements must reduce to WCW correlation functions for the

isometry generators of WCW with isometries identified as symplectic transformation of $\delta M^4 \times CP_2$ representable as Hamiltonians.

Conformal invariance for the construction of WCW geometry strongly favors the proposal in which the data coding for gamma matrices of WCW as super-symplectic generators comes from partonic 2-surfaces and possibly their normal spaces. I have referred to this condition as a strong form of holography, which is indeed natural for 4-D conformal invariance as dynamical symmetries of classical field equations in holography = holomorphy vision.

4.3.6 Electric-magnetic duality and the integration of the levels of the hierarchy of standard model physics

TGD predicts a hierarchy of standard model physics.

1. TGD predicts an entire hierarchy of standard model physics, in particular of hadron physics, with levels characterized by the irreducible representations (irreps) associated with the multiplets of color partial wave for the spinor fields of CP_2 [L19, L18]. These hierarchies accompany both quarks and leptons and the levels are in 1-1 correspondence. U and D type quarks and charged leptons and neutrinos correspond to different color multiplets.

2. The proposal is that these, not necessarily copies, of standard model physics correspond to special p-adic length scales and Mersenne primes and their Gaussian counterparts are proposed as candidates for these special levels. One would have a hierarchy of physics labelled by Mersenne primes $M(n) = M_{k(n)} = 2^{k(n)} - 1$ and by their Gaussian counterparts.

   A feature making Mersenne primes "cognitively" very special is that both finite field and p-adic arithmetic are very simple in binary basis. The powers 2^{-k} for G_{M_n} for finite fields correspond to integers 2^{n-k} and $k = n - 1$ defines the binary cutoff. For $0 < k < n$ the positive powers $2^{k+n} = 2^k + 2^k M_n$ correspond to 2^k in the finite field and have the p-adic digits $2^k, 2^k$, and in p-adic arithmetic $2^{-k} = 2^{n-k}/(1 + M_n)$ has the digits 2^{n-k}, then $M_n - 2^{n-k}$ and $2^{n-k} - 1$ alternating: in binary a single bit, all n bits but one, and $n-k$ ones. Unlike for a general prime, finite field and p-adic arithmetics are rather easy. This applies also to Gaussian and Eisensteinian Mersenne primes.

3. For instance, ordinary hadron physics would correspond to Mersenne prime M_{107} and M_{89} hadron physics would correspond to a new hadron physics emerging at mass scale 512 times protons mass scale at LHC energies. The strange and unexpected features of quark-gluon plasma transition might be understood if it corresponds to a transition from M_{107} hadron physics to M_{89} hadron physics. Part of hadrons would leak to M_{89} sector.

Is it possible to integrate the hadron physics of the hierarchy to each other assuming the proposed duality?

1. The first thing to come in mind is that at the energy range between p-adic length scales L_p associated with $p = M(n)$ and $p = M(n + 1)$ the QCD type description is good for $M(n)$ hadron physics and $U(1)$ description works for $M(n + 1)$ hadron physics.

2. At intermediate p-adic length scale for which $\alpha_s(QCD) = \alpha_s(U(1)) = 1$, a phase transition between the two hadron physics becomes possible.

3. Are the QCD type hadron physics for $M(n)$ and $U(1)$ type hadron physics for $M(n + 1)$ e equivalent descriptions of the same hadron physics or genuinely different? For the proposed applications [L20, L21] and different color multiplet for the corresponding H SpMHs suggest that this is not the case.

A finite number of values $h_{eff} \leq h$ is given by the number 405 of factors of $(7!)^2$ is possible. A possible interpretation for the factors is as the number of space-time sheets for the space-time surface as a covering of M^4 and CP_2 such that both coverings have $7! = 5040$ sheets. Could this structure be assigned with partonic 2-surfaces assignable to fundamental fermions or does it characterize more complex space-time surfaces?

4.4 M^4 Kähler structure and gravitational interactions: electric-magnetic duality for gravitational interactions?

There are several questions related to the interpretation of M^4 Kähler structure.

1. Can gravitation be described at the level of CDs as M^4 Kähler force whereas the space-time description would be in terms of induced metric? What is the relation between Newton's gravitational potential and $B_K(M^4)$? Can the description of gravitation as M^4 Kähler force be consistent with the observation that the gravitational force is attractive between matter and antimatter and how could this relate to matter-antimatter asymmetry. Does electric-magnetic duality generalize to the description gravity in terms of M^4 Kähler force? How does the M^4 Kähler structure at level of CD relate to Hamilton-Jacobi structure and how warping modeifies H-J structure.

2. Does gravitational constant G appear as a coefficient of $J(M^4)$ and how does this relate to the notion of Hamilton-Jacobi structure? How does gravitational constant G relate to R^2?

In the following possible answers to these questions are discussed.

1. Can a fermion with mass $m \propto \hbar_{eff}\omega$, determined by warping giving $g_{tt} = 1 - R^2\omega^2 = c_\#^2$ at fermion line, be associated with a gravitational potential that corresponds to $B_K(M^4)$ in longitudinal degrees of freedom, which is thus a gradient. Could the simplest H-J structure correspond to $B_K(M^4) = Gm/r$, where r is the M^4 distance along the light ray at δM^4_+ emanating from the tip of δM^4_+ and m is fermion mass. Gravitational potential would therefore be a property of CD associated with the particle. This would provide a description of gravitation at the level of CD.

   Does the gravitational flux Gm defined by the gradient of B_K over a CP_1 of radius r and also to the Kähler magnetic monopole flux over the homologically non-trivial surface of the particle related to CD. Could this give realize an electric-magnetic duality in CD?

2. Fermion mass is determined by warping at the fermion line. $m \propto \hbar_{eff}\omega$. ω is determined by warping. $g_{tt} = 1 - R^2\omega^2$. The warping would be a property of CD and would mean the replacement of half-light-cones with the light whose boundary is defined by the light-like geodesics of H satisfying $m^0 = (t, r = c_\# t, \phi = \omega t)$ with M^4 projection satisfying $r_M = c_\# m^0$. The warped light-cones are balls $r_M \leq c_\# m^0$ rotating along a geodesic circle $s^2 \subset CP_2$.

3. The analog of Newtonian description in terms of M^4 Kähler form in CD involves a slicing of future/past light-cone by light-cone boundaries. The Einsteinian metric description is at the level of space-time surfaces. $B_K(M^4)$ is gradient in longitudinal degrees of freedom and could define Newtonian potential in these degrees of freedom. Since Kähler structure is involved, gauge invariance does not imply that gradient property makes it unphysical.

   The gradient of B_K defining gravitational form is the dual of a monopole magnetic field in transversal degrees of freedom. Gravitational flux of gravitational potential equal to gravi-magnetic monopole flux. This is the counterpart of self-duality for CP_2. This fixes the H-J structure in CD. This option might not be the only one.

   CD can be sliced by shifted light-cone boundaries. $\Phi(U(1), M^4) \equiv \Phi_{gr} = Gm/r \propto G\hbar_{eff}\omega/r$, where r is the M^4 distance from the tip of CD associated with the fermion, is the TGD counterpart of the gravitational potential defined at the edge of the light cone for this H-J.

4. Interactions would reduce to contact interaction between space-time surfaces associated with particles satisfying holography and identifiable as analogs of Bohr orbits if holography holds true. If holography = holomorphy vision holds they are 2-D string world sheets for the same H-J structures and points otherwise. They have fermion lines as intersections with partonic orbits. The value of the gravitational potential at the intersection points determines gravitational force in terms of gradient of $\Phi(U(1))$. The vision about monopole flux tubes as mediators of gravitational interactions playing a key role in TGD inspired biology and astrophysics is compatible with this picture.

One can imagine that the partonic 2-surfaces develop extrusions/tentacles of arbitrary radial size scale in δM^4_+, which give rise to contact interactions. The 3-D conformal invariance for δM^4_+ means that the radial light-like coordinate r_M of δM^4_+ is a gauge degree of freedom so that there is no energy cost involved with the creation of the tentacle as a quantum fluctuation. This would give a connection with the conformal invariance as it is realized in string models.

5. There is an objection against this picture. Why the induced $B_K(M^4)$ and $B_K(CP_2)$ do not superpose to give a single $U(1)$ force? Could the solution of the problem be that there are two conserved H chiralities corresponding to quarks and leptons and M^4 and CP_2 Kähler forces corresponding to these chiralities. The positive chirality ($\Gamma_9\Psi = \Psi$) would correspond to M^4 $U(1)$ with the same sign of $U(1)$ force for quarks and leptons. The negative chirality ($\Gamma_9\Psi = -\Psi$) would correspond to CP_2 $U(1)$ Kähler force in accordance with the charge assignments.

   The second objection is why the gravitational mass as a $U(1)$ charge for M^4 is not opposite for particles and antiparticles as experimental findings suggest. At this moment there is no obvious answer to this question. The description in terms of metric at space-time level distinguishes between positive and negative gravitational masses. Could the consistency with the metric description exclude negative gravitational masses and instablize antimatter? Or could the negative gravitational masses correspond to a reversed arrow of geometric time predicted by ZEO.

How does the gravitational constant relate to the CP_2 radius R satisfying $R^2/G \in [10^7 - 10^8]$?

1. R^2 is roughly $R^2 = 10^7 l_P^2$, where Planck length is given by $l_P^2 = G\hbar$. The relevant dimensions are $[G] = L^2/[\hbar]$ and $[m] = [\hbar]/L$. $\Phi_{gr} = Gm/r$ is dimensionless ($c = 1$) so that one has $[G] = [R^2]/[\hbar]$.

   The natural idea is that there is only a single fundamental length scale, Planck length, and the real CP_2 radius equals l_P. R would actually be a "dark" scale $R = \sqrt{\hbar_{eff}/\hbar_{min}}\, l_P$ evaluated for $\hbar_{eff} = \hbar$.

   The definition of the dark scale $R^2_{eff} \equiv R^2(\hbar_{eff})$ can be written as

$$R^2_{eff}/\hbar_{eff} = G\hbar/\hbar_{eff,min} \quad . \tag{4.7}$$

   This is an old proposal [L3] motivated by the plausible assumption that there is only a single fundamental scale and it is Planck length identifiable as CP_2 radius. This gives

$$\begin{aligned} R^2 \equiv R^2_{eff} &= l_P^2 \times \frac{\hbar_{eff}}{\hbar_{eff,min}} \quad , \\ R^2_{min} &= \frac{\hbar_{eff,min}}{\hbar} R^2_{eff}(\hbar) = l_P^2 \quad . \end{aligned} \tag{4.8}$$

   R^2_{min} would correspond to the real CP_2 radius equal to the Planck length $l_P = \sqrt{G\hbar}$. R^2 corresponds to $h_{eff} = \hbar = (7!)^2\hbar_{min}$.

   The integer quantization $\hbar_{eff}/\hbar_{min} = n$ together with the estimate $R^2/G \in [10^7 - 10^8]$ leads to the proposal $\hbar/\hbar_{min} = (7!)^2 = 25401600 \simeq 2.5402 \times 10^7$ with 7! having all integers smaller than 8 as factors. The prime factors of $7! = 5040$ are $2, 3, 5$ and 7. The real CP_2 size given by Planck length and effectively scales up by $\hbar_{eff}/\hbar_{min}$. A finite number of values $h_{eff} \leq h$ is given by the number 405 of factors of $(7!)^2$ is possible. A possible interpretation for the factors is as the number of space-time sheets for the space-time surface as a covering of M^4 and CP_2 such that both coverings have $7! = 5040$ sheets. Could this structure be assigned with partonic 2-surfaces assignable to fundamental fermions or does it characterize more complex space-time surfaces?

2. There is some evidence for the values of $h_{eff} < h$ coming from the TGD based model of [L1] of hydrinos claimed by Randell Mills [D2]. Randell Mills has written a book and numerous articles about the hydrino concept and many of them are published in respected journals. The simplest TGD based model for hydrino would be understood as an atom with $h_{eff} = h/2^k$, $k \in 1, ..., 8$ corresponding to the even factors 2,4,and 6 of 7! and $7!^2$. The company of Mills has a homepage containing besides commercial side also list of the abstracts with links to the corresponding articles related to the experimental aspects of the hydrino concept giving a brief summary about what is known about hydrinos (see this).

   The TGD based view of biological and geological evolution [L25] explaining also the formation of Moon assumes that after the formation of there was a phase transition increasing the value of $\hbar_{eff}$ from $\hbar/2$ to $\hbar$ and releasing a huge amount of atomic Coulomb energy.

4.5 Should also Lagrangian mimimal surfaces be allowed?

The above observations re-raise the question whether also the minimal surface solutions of field equations, which do not satisfy holography = holomorphy principle [L11, L17], should be allowed. I have considered this problem in [L15].

1. Lagrangian minimal surfaces having a vanishing induced Kähler form solve the field equations for the sum of Käction and volume term. These superminimal surfaces are an important class of minimal surfaces in C^2 and appear also as minimal 3-surfaces in Calabi-Yau spaces. Both holomorphic and supermininal surfaces minimize the volume in a given homology class so that one can speak of homological QFT at the classical level.

2. These kinds of surfaces are possible as string-like objects. M^4 projection defining a string world sheet would be a Lagrangian manifold for $J(M^4)$ having only the transversal part, and CP_2 projection would be a Lagrangian manifold for $J(CP_2)$. Both must be minimal surfaces. Homologically trivial geodesic sphere represents the simplest example in the case of CP_2. Kähler magnetic flux for them would vanish but R_{12} contribution in γ would give rise to a magnetic flux analogous to ordinary Maxwellian magnetic flux. This suggests that they should be regarded as physical solutions. An interesting question to ponder is whether W bosons could correspond to homologically trivial geodesic spheres or more general Lagrangian manifolds.

In [L10] I have given examples of homologically trivial and non-trivial geodesic spheres as

$$\begin{aligned} S_I^2 &: \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ , \\ S_{II}^2 &: \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ . \end{aligned} \tag{4.9}$$

Could the presence of both spheres allow new insights concerning the correlation between electroweak gauge potentials, masses and partonic orbits?

1. The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the trace of the second fundamental form is also easy to verify. S_I^2 is homologically trivial: in fact, the induced Kähler form vanishes identically for S_I^2. S_{II}^2 is a homologically nontrivial complex submanifold and the flux of the Kähler form gives its homology equivalence class. The string-like objects associated with these two kinds of geodesic spheres are minimal surfaces but holography= holomorphy principle is true only for S_{II}^2.

2. For S_{II}^2, e^1 and e^2 vanish (see the section 'Basic formulas of TGD for gauge potentials and fields in relation to standard model formulas' in part II) since ξ^1 and ξ^2 are functionally dependent so that σ_1 and σ_2 vanish. Hence W gauge potentials and - fields vanish. One can however worry about how to obtain non-trivial W propagators. For S_I^2, $e^1 = 2Im(\xi^1 d\bar{\xi}^1)/(r\sqrt{F})$ and $e^2 = 0$ so that W gauge potentials do not vanish and propagators are possible.

   The symplectic transformations of CP_2 act as gauge transformations on $B_K(CP_2)$ but since they also affect the space-time surface, they cannot be used to eliminate $B_K(CP_2)$ at S_I^2

even if $J(CP_2) = 0$ is true. The proposed interpretation is as approximate symmetries and as candidates for isometries at the level of WCW.

3. Should W (Z and γ) be assigned with S_I^2 (S_{II}^2), and more generally, with Lagrangian (holomorphic) minimal surfaces in CP_2? The non-holomorphy of Lagrangian surfaces as submanifolds (they have however an inherent complex structure as 2-manifolds) implies that for S_I^2 the inherent conformal invariance remains but the induced conformal invariance is lost. For S_{II}^2 these two conformal invariances are equivalent.

   How could this relate to the different masses of W and Z? The spheres S_I^2 and S_{II}^2 in general position intersect at a single point so that the parameters ω (warping) and the induced gauge potentials appearing in the formula for mass squared, can differ for Z and W. Hence Z^0 and W masses can differ.

4. For S_I^2, the modified gamma matrices appearing in the modified Dirac action would involve only the transversal gamma matrices of M^4 and the projection of CP_2 gamma matrices in the direction of the coordinate u of the light-like geodesic of H: this would give rise to the coupling between right- and left-handed chiralities of spinors.

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