Levy flight and TGD

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Abstract

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1 Introduction

I encountered an interesting Science Daily article (see) about the finding that in high energy heavy ion collisions in which quark-gluon plasma is created. What is studied is the distribution for the distances of hadrons travelled between two collisions during the rescattering phase during which hadrons collide, fuse and long-lived resonances decay. In [C1] this distribution is studied for pions. It is found that the distribution does not obey Gaussian distribution as in Brownian motion but has a long tail obeying a power law.

This distribution is associated with Levy flight (see) for which the distribution parameter corresponds to a Levy walk for which the second moment of the distribution is finite. Levy walk is the analog of Brownian motion for the distances travelled between subsequent collisions characterized by the distribution for the time durations for the periods during which the motion occurs with a constant velocity. The tail obeys a power law, which suggests a scaling symmetry and fractality associated with criticality.

In the sequel, Levy flight is discussed from the point of view of TGD. Zero energy ontology, forced by general coordinate invariance, predicts a slight failure of classical determinism. By holography, the space-time surfaces are analogs of orbits of particles as 3-surfaces replacing point-like particles. The non-determinism implies that the orbits define an analog of Brownian motion. The situation should be also quantum critical in order to explain the scaling and tail. In the QCD framework the quantum criticality could be due to the transition to a phase interpreted as quark-gluon plasma.

TGD suggests a different interpretation: the transition would correspond to a transition from the ordinary hadron physics characterized by Mersenne prime $M_{127} = 2^{127} - 1$ to M_{89} hadron physics with mass scale which is by a factor 512 higher than for the ordinary hadron physics [K1, K2]. Quantum criticality would however imply that the value of effective Planck constant h_{eff} increases by the same factor so that the hadronic length scale remains the same.

2 TGD view about the origin or random walks

Zero energy ontology (ZEO) replaces standard ontology of quantum theory and solves the basic problem of quantum measurement theory. ZEO is forced by the slight non-determinism of classical field equations of TGD which are exactly solvable by holography= holomorphy hypothesis. This non-determinism could explain Brownian motions and also Levy flight as a signature of quantum criticality.

2.1 Could random motion reflect the structure of space-time at the fundamental level

In TGD point-like particles are replaced by 3-surfaces and their orbits define space-time surfaces in $H = M^4 \times CP_2$ as analogs of slightly non-deterministic Bohr orbits satisfying holography forced by the realization of general coordinate invariance without path integral [L5, L6]. Holography= holomorphy principle implies that the solution of field equations is a minimal surface irrespective of the action principle as long as it is general coordinate invariant and constructed in terms of the geometry induced from H. One can wonder whether Brownian motion and Levy flight could at the fundamental level relate to the slight failure of the classical determinism in the TGD framework.

Space-time surfaces as solutions of field equations [L3] are determined as a root $(f_1, f_2) = (0, 0)$ of two analytic functions of one hypercomplex coordinate and 3 complex coordinates of H defining what I call Hamilton-Jacobi coordinates [L2]. Space-time as a minimal surface is expected to be slightly non-deterministic. The non-determinism would be located at 3-D surfaces analogous to the 1-D frames spanning 2-D soap film where the same occurs. At the frames the space-time surface can branch in several ways.

This could give rise to an analog of Brownian motion and Levy flight with point-like particles replaced with a 3-surface and the orbits with the preferred extremals as a minimal surface defining the analog of Bohr orbits.

2.2 How to describe classical non-determinism elegantly?

There are strong constraints to be satisfied at the loci of non-determinism since the field equations code for conservation laws of the isometry charges of H and these must be satisfied. This kind of conditions must be satisfied also for 2-D minimal surfaces at frames and they pose very strong conditions for what can happen at the frames. A natural guess is that the space-time surface decomposes to regions charracterized by different function pairs (f_1, f_2) having the loci of non-determinism as interfaces.

In TGD number theoretic vision is complementary to the geometric vision of physics. It involves p-adic physics for various values of p-adic prime characterizing the p-adic numbers field in question. Also the extensions of p-adic numbers, induced by extensions E of rationals, are allowed. In the holography= holomorphy vision, the extension of rationals would characterize the Taylor coefficients of analytic functions f_1, f_2 of the Hamilton-Jacobi coordinates of H. This is not enough to give p-adic primes without additional assumptions.

For ordinary polynomials of a complex argument the product for the difference of the roots defines what is known as discriminant. For rational coefficients it decomposes to a product of powers of so called ramified primes which define special p-adic primes and the proposal is that these primes define the p-adic primes characterizing the system. These p-adic primes near powers of 2 were found to characterize elementary particles in p-adic mass calculations that I performed around 1995 [L1].

2.3 How could the roots of polynomials of complex coordinates emerge?

One should somehow introduce naturally the roots of 2-D complex polynomials in order to get p-adic primes as ramified primes.

1. The basic observation is that the analytic maps $g: C^2 \rightarrow C^2$ allow to generate new solutions from the existing ones by the map $f = (f_1, f_2) \rightarrow g(f) = (g_1(f_1, f_2), g_2(f_1, f_2))$.

2. As a special case, one has $g = (P, Id)$, P a polynomial, giving rise to map $(f_1, f_2) \rightarrow (P(f_1), f_2)$. There are physical motivations for considering this restriction: the proposal is that $f_2 = 0$ represents a very long length scale constraint on the physics [L3], defining a slowly varying cosmological constant. The "world of classical worlds" (WCW) as the space of these 4-surfaces inside causal diamond (CD) would decompose to sub-WCWs inside with f_2 is fixed. One can calculate the roots for the polynomial P and assign to it ramified primes. The roots $(g_1(f_1), f_2) = (0, 0)$ give 4-surfaces representing roots of r of P as 4-surfaces $f_1 = r$.

3. The maps g play a key role in the TGD based view about the physical analog of metamathematics and Gödel's incompleteness theorem [L7]. Space-time surfaces as almost deterministic systems would be analogous to theorems with the assumptions of the theorem represented by holographic data at the so called passive boundary of CD remaining unaffected in the sequence of small state function reductions (SSFRs) defining the analog of Zeno effect.

   The axiom system would be represented by the variational principle, which also gives minimal surfaces as its solutions by holography= holomorphy correspondence therefore one can say that classical physics defines the analog of logic, which cannot depend on axiomatics. The map g would mean a transition to a meta level and the surfaces obtained in this way would represent theorems about theorems. One would obtain hierarchies of composite maps representing statements about statements about....

   If this picture is correct, the p-adic primes as ramified primes would be associated with a metalevel accompanying already elementary particles and represented by g. Hierarchies of these metalevels are predicted the iterations of g gives rise to the analogs of Mandelbrot fractals and Julia sets and for the approach to chaos as increase of complexity.

2.4 Does the p-adic non-determinism of differential equations correspond to the classical non-determinism of Bohr orbits?

p-Adic differential equations differ from their real counterparts in that the integration constants are pseudo constants having vanishing p-adic derivatives. They depend on a finite number of pinary digits. This generalizes also to partial differential equations. Suppose that the space-time surfaces can be characterized by a p-adic prime p, or its analog for algebraic extension of rationals, identified as a ramified prime of $f = P$.

1. Could the classical non-determinism of field equations correspond to the p-adic nondeterminism for some ramified prime p in the sense that the coefficients of (f_1, f_2) are p-adic pseudo constants for p? Could the real coefficients and their p-adic counterparts be related by a canonical identification $x_p = \sum x_n p^n \rightarrow x_R = \sum x_n p^{-n}$ or be identical? Canonical identification would guarantee a continuous correspondence: otherwise one would obtain huge fluctuations.

2. Could the Brownian motion and Levy flight reflect the underlying p-adic non-determinism at the fundamental level at which particles are replaced by 3-surfaces which in turn are replaced by the analogs of Bohr orbits which are slightly non-deterministic.

3. Could the parameters characterizing the TGD analog of Brownian motion and Levy flight be p-adic pseudo constants depending on finite number of pinary digits for the Hamilton-Jacobi coordinates of H. Could they change their values at points with Hamilton-Jacobi coordinates with a finite number of pinary digits: this would conform with the idea of discretization as a representation for a finite measurement resolution.

One can consider two options for modelling ordinary random walks from this perspective.

1. The above picture suggests that one considers random walks as a smooth dynamical evolution at the p-adic level and that the replacement of the initial values or other parameters characterizing the orbits with p-adic pseudo constants as functions of time gives rise to discontinuities of say velocity in the random walk. The p-adic scale should make itself visible in statistical sense as a natural scale associated with the motion.

   This seems to require that first the real configuration space is mapped to its p-adic counterpart by the inverse of canonical identification. After the smooth p-adic orbit, say free motion or motion in gravitational field, is mapped to its real counterpart by canonical identification. One can require the orbit x_+vt is continuous (x_0 is constant) but v is pseudo constant. The orbit would be a zigzag curve having also the characteristic p-adic fractality.

2. Probability distributions suggests an alternative, perhaps more elegant approach. Also the p-adic calculations based on p-adic thermodynamics define this kind of approach. The key idea would be that everything is smooth at the p-adic level and canonical identification brings in fractallity. The probability distributions of durations $T_n = t_n - t_{n-1}$ and velocities v is a way to statistically characterize the random walk. Brownian walk and Levy flight serve as basic examples.

   Restrict first the consideration to the durations T_R of motion with a constant velocity. The inverse I^{-1} of the canonical identification would map the real durations T_R to p-adic durations $T_p = I^{-1}T_R$. The p-Adic distribution function $P_p(T_p)$ would be a smooth function in accordance with the idea that the fractality at the real side derives from p-adic smoothness via canonical identification. On the real side, canonical identification would give the real distribution as $P_R(T_R) = (P_p(T_p))_R$ by I. Normalization would be required to get probability interpretation.

   One would have $T_R \to T_p \to P_p(T_p) \to (P_p(T_p))_R = P_R(T_R)$ One would obtain a hierarchy of distributions labelled by the parameters of P_p and by p. A similar map $v_R \to v_p P_p(v_p) \to (P_p(v_p))_R = P_R(v_R)$ would give a fractal velocity distribution.

2.5 The role of quantum criticality

Classical non-determinism alone need not produce the Levy flight or Levy walk. In the particle physics experiments involving high energy ion collisions the total path lengths for particles during the period involving final state interactions are considered. It is indeed found that this distribution has a long tail obeying a scaling law. Since scaling laws mean fractality and are associated with the criticality, the natural question is whether quantum criticality could be involved.

The TGD based view of the generation of quark-gluon plasma suggests an interpretation in terms of quantum criticality against transition from the ordinary hadron physics characterized by the Mersenne prime M_{107} to M_{89} hadron physics characterized by a mass scales which is 512 times higher. By quantum criticality this hadron physics would however correspond to a large value of h_{eff}. For $h_{eff} = 512 = L(107)/L(89)$ the Compton length of M_{89} and M_{107} mesons a would be equal [K1, K2]. The transition to ordinary hadron physics could take place by p-adic cooling gradually, which would mean that p-adic prime corresponding to M_{89} is gradually reduced to M_{107} and the p-adic hadron mass scale reduced and Compton length would be preserved. I have considered the possibility M_{89} hadron physics and p-adic cooling as a possible explanation of the solar wind and energy production at the surface of the Sun [L4].

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