Gödel, Lawvere, and TGD

February 24, 2025

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Abstract

The tweets of Curt Jaimungal inspired an attempt to understand Gödel's incompleteness theorem and related constructions from the TGD point of view. The basic idea is that the laws of physics, as they are formulated in the TGD framework, can be regarded as analogs for the axioms of a formal system.

1. Space-time surface which by holography= holomorphy vision is analogous to a Bohr orbit of particles represented as a 3-surface is analogous to a theorem. The slight classiocal non-determinism implies that there are several Bohr orbits associated with the same 3-surface at the passive boundary of causal diamond remaining un-affected in the sequence of small state function reductions (TGD counterpart of the Zeno effect).

   The holographic data would be in the role of the assumptions of a theorem, which need not to be proved and reduced to axioms, and the Bohr orbits would correspond to theorems deducible from these assumptions.

2. The adelization of physics means that real space-time surfaces obtained using extension of E of rationals are extended to adelic space-time surfaces. The p-adic analogs of the space-time surface would be correlates for cognition and cognitive representations correspond to the intersections of the real space-time surface and its p-adic variants with points having Hamilton-Jacobi coordinates in E.

3. Concerning Gödel, the most important question is how self references as a metamathematical notion is realized: how space-time surfaces can represent analogs of statements about space-time surfaces. In this framework, meta level could correspond to the maps $g: C^2 \rightarrow C^2$ mapping the function pairs $f = (f_1, f_2) : H \rightarrow C^2$ defining space-time surfaces as their roots to the composites $g \circ f$. g should act trivially at the passive boundary (PB) of CD. One can construct hierarchies of these composites.

4. Second realization would be in terms of the hierarchy of infinite primes analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory for an extension E of rationals. Also the Fock basis of WCW spinor fields relates to WCW like the set of statements about statements to the set of statements.

The TGD counterpart for the action of an object of a formal system acting as a morphism on another object is the action of the space-time surface X^4 to another surface Y^4 and vice versa. This correspond to the interaction of the Bohr orbits of X^3 and Y^3 involving a temporary formation of a connected 3-surface as an intermediate state (monopole flux tubes could connect the 3- surfaces).This action is highly unique and fixed apart from the weak classical non-determinism. The interaction would be analogous to sensory perception. The time reversal of this sensory perception involving two BSFRs changing the arrow of time would correspond to motor action. In this view, the infinite self reflection hierarchy is replaced with a finite SSFRs and self is a dynamical object.

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1 Introduction

The tweets of Curt Jaimungal (see this) inspired an attempt to understand Gödel's incompleteness theorem and related constructions from the TGD point of view.

It has remained somewhat unclear to me how the notion of conscious self is defined in theories based on pure mathematics. I however understand that the conscious system is identified as an object in a category X and the view of self about itself would be a set of morphisms of f_x of $X \rightarrow X$ as structure-preserving descriptions, morphisms, which would give information about x to the other selves y as objects of X. One can define X^Y as an object having as objects the morphisms $Y \rightarrow X$. X^Y would correspond to X as seen by object Y.

This associates to every object $x \in X$ morphism $f_x \in X^X$ of the category X into itself. One could say that X embedded in X^X and f_x corresponds to models of x for other selves of $y \in X$. Under conditions formulated by Lawvere [A1], any morphism f in X^X has a fixed point y_f. In particular, for f_x one can find y_x such that $f_x(y_x) = y_x$ is satisfied. In some cases this might be the case. Under the assumptions of Lawvere, one has $y_x = x$ and this might be the case always. These kinds of objects x are very special and one can wonder what its interpretation is.

In particular, Gödel's sentence is a fixed point for a sentence f_x, which associates to a sentence y a sentence $f_x(y)$ stating that y is not provable in the formal system considered. It turns out that $f_x(x) = x$ is true. Therefore x is not provable but is true. Could this mean that this kind of object is self-conscious and has a self model?

On the other hand, self-reflection, which means that one becomes aware of the content of one's own consciousness at least partially, can be claimed to create descriptions of itself and fixed point property suggests an infinite number of levels or possibly limit cycles: for Julia sets only non-trivial limit cycles are present. Infinite regression however means a paradox. On the other hand one can argue that self-representation is trivial for a fixed point.

What is the situation in TGD? In the following the idea about physics laws, identified in the TGD frameworks as the dynamics of space-time surfaces, is discussed in detail from the perspective of the metamathematics or metaphysics.

1.1 The laws of physics as analogs for the axioms of a formal system

The basic idea is that the laws of physics, as they are formulated in the TGD framework [L10, L11], can be regarded as analogs for the axioms of a formal system.

1. Space-time surface, which by holography= holomorphy vision is analogous to a Bohr orbit of particles represented as a 3-surface is analogous to a theorem. The slight classical non-determinism forces zero energy ontology (ZEO) [L2]: instead of 3 surfaces the analogs of Bohr orbits for a 3-surfaces at the the passive boundary (PB) of the causal diamond (CD) are fundamental objects. By the slight classical non-determinism, there are several Bohr orbits associated with the same 3-surface X^3 at the PB remaining un-affected in the sequence of "small" state function reductions (SSFRs). This is the TGD counterpart of the Zeno effect. The sequences of SSFRs defines conscious entity, self.

2. The adelization of physics means that real space-time surfaces obtained using extension of E of rationals are extended to adelic space-time surfaces. The p-adic analogs of the space-time surface would be correlates for cognition and cognitive representations correspond to the intersections of the real space-time surface and its p-adic variants with points having Hamilton-Jacobi coordinates in E [L4].

3. Concerning Gödel, the most important question is how self reference as a metamathematical notion is realized: how space-time surfaces can represent analogs of statements about space-time surfaces. In the TGD framework holography= holomorphy vision provides an exact solution of the classical field equations in terms of purely algebraic conditions. Space-time surfaces correspond to the roots function pairs (f_1, f_2), where f_i are analytic functions of the Hamilton Jacobi coordinates of $H = M^4 \times CP_2$ consisting of one hypercomplex and 3 complex coordinates.

   The meta level could correspond to the maps $g = (g_1, g_2) : C^2 \rightarrow C^2$, where g_i are also analytic functions or Hamilton-Jacobi coordinates, mapping the function pairs $f = (f_1, f_2) : H \rightarrow C^2$ and giving new, number theoretically more complex, solutions. The space-time surfaces obtained in this way correspond to the roots of the composites $g \circ f = (g_1(f_1, f_2), g_2(f_1, f_2))$.

   g should act trivially at the PB of CD in order to leave X^3 invariant. One can construct hierarchies of composites of maps g having an interpretation as hierarchies of metalevels. Iteration using the same g repeatedly would be a special case and give rise to the generalization of Mandelbrot fractals and Julia sets.

4. Second realization would be in terms of the hierarchy of infinite primes [K4] analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory for an extension E of rationals and starting from a theory with single particle boson and fermions states labelled by ordinary primes. Here one can replace ordinary primes with the prime of an algebraic extension E of rationals. This gives a second hierarchy. Also the Fock basis of WCW spinor fields relates to WCW like the set of statements about statements to the set of statements.

1.2 How space-time surfaces could act on space-time surfaces as morphisms

Could one, by assuming holography= holomorphy principle, construct a representation for the action of space-time X^4 surface on other space-time surfaces Y^4 as morphisms in the sense that at least holomorphy is respected. In what sense this kind of action could leave a system associated with X^4 fixed? Can the entire X^4 remain fixed or does only the 3-D end X^3 of X^3 at the PB remain fixed? In ZEO this is indeed true in the sequence of SSFRs made possible by the slight failure of the classical determinism.

What the action of X^4 on Y^4 could be?

1. The action of X^4 on Y^4 would be a morphism respecting holomorphy if X^4 on Y^4 have a common Hamilton-Jacobi structure [L4]. This requirement is extremely strong and cannot be satisfied for a generic pair of disjoint surfaces X^4 and Y^4. The interpretation would be that this morphism defines a kind of perception of Y^4 about X^4, a representation of X^4 by Y^4.

   A naive proposal for the action of X^4 on Y^4 assumes a fixed point action for $Y^4 = X^4$. The self-perception of X^4 would be trivial. Non-triviality of self-representation since is in conflict with the fixed point property: this can be seen as the basic weakness of the proposal that conscious experience could be described using a formal system involving only the symbolic description but no semantics level.

2. The classical non-determinism of TGD comes to rescue here. It makes possible conscious memory and memory recall [L7] [K3] and the slightly non-deterministic space-time surface X^4 as an analog of Bohr orbit can represent geometrically the data making possible conscious memories about the sequence of SSFRs. The memory seats correspond to loci of non-determinism analogous to the frames spanning 2-D soap films. In the approach based on algebraic geometry, the non-determinism might be forced by the condition that space-time surfaces have non self-intersections. Second possibility is that space-time surfaces consist of regions, which correspond to different choices of (f_1, f_2) glued together along 3-D surfaces.

3. Purely classical self-representation would be replaced at the quantum level by a quantum superposition of the Bohr orbits for a given X^3. A sequence of "small" state function reductions (SSFRs) in which the superposition of Bohr orbits having the same end at the PB is replaced with a new one. SSFRs leave the 3-surfaces X^3 appearing as ends of the space-time surface at the PB invariant. The sequence of SSFRs giving rise to conscious entity self, would give rise to conscious self-representation.

4. The fixed point property for X^4 making the self-representation trivial would be weakened to a fixed point property for X^3, and more generally of 3-D holographic data.

1.3 How zero energy states identified as selves could act on each other as morphisms?

How the superposition $\Psi(X^3)$ of Bohr orbits associated with X^3 can act as a morphism on $\Psi(Y^3)$? The physical interpretation would be that $\Psi(X^3)$ and $\Psi(Y^3)$ interact: $\Psi(X^3)$ "perceives" $\Psi(Y^3)$ and vice versa and sensory representations are formed. This sensory representation is also analogous with the quantum counterpart of the learning process of language models producing associations and association sequences as analogs of sensory perceptions [L12].

1. These "sensory" representations must originate from a self-representation. This requires a geometric and topological interaction X^4 and Y^4 as a temporary fusion of X^4 and Y^4 to form a connected 4-surface Z^4. This would serve as a universal model for sensory perception. In the TGD inspired quantum biology, a temporary connection by monopole flux tubes serves as a model for this interaction. If the flux tubes serve as prerequisites and correlates for entanglement, entanglement could also be generated.

2. The holomorphy for Z^4 requires that X^4 on Y^4 have a common Hamilton-Jacobi structure during the fusion but not necessarily before and after the fusion. Therefore the defining analytic function pairs (f_1, f_2) [L6] can be different before and after the fusion and during the fusion and also for X^4 and Y^4 after and before the fusion. This might be an essential element of classical non-determinism. Continuity requirement poses very strong conditions on the function pairs involved. The representations produced in the interaction would be highly

unique. As already mentioned, also the absence of self-intersections could force classical non-determinism.

The outcome of the temporary fusion would give rise to a representation of the action of X^4 on Y^4 and vice versa. The representation would be a morphism in the sense that outcomes are holomorphic surfaces and the ends of X^4 and Y^4 at the PB of CD remain unaffected.

3. The fixed point property for Z^4 making the self-representation trivial would be replaced with the fixed point property for Z^3 and therefore also X^3 and Y^3.

4. The time reversed variant of sensory perception has an interpretation as motor action between them and would involve a pair of BSFRs induced by a subsystem of Z^4. Now the end of Z^4 at the PB of CD would be changed. X^4 would affect Y^4 in a non-deterministic way. The construction of the representation of X^4 on Y^4 would reduce to a construction of a self-representation for Z^4.

This view is inspired by the TGD view in which self is identified as a sequence of non-deterministic SSFRs and is thus not "provable" and has also free will. The holographic data would be in the role of the assumptions of a theorem, which need not to be proved and reduce to axioms, and the Bohr orbits would correspond to theorems deducible from these assumptions. In the interaction of X^3 and Y^3 a larger self Z^3 would be created and would involve quantum entanglement. In this view, the infinite self reflection hierarchy is replaced with a finite sequence of SSFRs providing new reflective levels and self is a dynamical object.

2 Gödel and Lawvere

In this section the incompleteness theorems of Gödel and the fixed point theorem of Lawvere are briefly summarized.

2.1 Gödel's incompleteness theorems

Gödel's incompleteness theorems [A2] (see this) apply to the arithmetics of natural numbers. They can be generalized and apply for instance to category theory. It is essential that there exist morphisms from $X \to X^X$ allowing fixed points.

2.1.1 Some background

There are two incompleteness theorems:
enumerate

There are always true statements which cannot be proven.

The system cannot demonstrate its own consistency.

Incompleteness theorems imply that Hilbert's program fails. There exists non-provable truths in any internally consistent and sufficiently strong axiomatic system of arithmetics. For complete number systems such as complex numbers there exists an infinite effective recursively enumerable axiomatization allowing to prove all truths but the theorems are about properties of complex numbers rather than integers and one cannot avoid incompleteness theorem for natural numbers.

Gödels incompleteness theorems apply at the syntactic level, not semantic. To understand the core of Gödel's theorem one must understand the difference semantic and syntax.

1. The language of mathematics involves only syntax. At the syntactic level there is no meaning yet. Formal systems, computer programs, in particular AI and LLMs, involve only the syntax. Classical deterministic physics can be seen as an analog of a formal system with classical time evolution as the analog of logical deduction and classical laws of physics as axioms. However, the local laws of physics depend on the action chosen and this does not conform with independence of the Boolean logic on the axiom system.

   Rather remarkably, in TGD the holography= holomorphy principle implies the same universal dynamics irrespective of action and predicting that space-time surfaces are minimal surfaces. 0

2. Semantic level involves meaning and this requires consciousness. The notion of apple serves a good illustration. Apple corresponds to a real world object, to the mental image created by its perception, and to its name which corresponds to syntactic aspect. The symbol "apple" corresponds to syntactic level. This level cannot catch the notion of truth in semantic sense.

Gödel proved that first order logic is semantically complete but not syntactically complete: there are sentences that cannot be proved or disproved in the axion system considered.

Some further notions related to the axiom system are needed.

1. Completeness means that any statement or its negation is provable to be true. Gödel proved that first order logic is semantically complete but not syntactically complete: there are sentences that cannot be proved or disproved in the axion system considered.

2. Consistency means that there is no statement such that both the statement and its negbation can be proven from axioms

3. Effective axiomatization means the existence of an algorithm, which can list the theorems following from a given axiomatization.

4. In mathematics, logic and computer science, a formal language is called recursively enumerable if it is a recursively enumerable subset in the set of all possible words over the alphabet of the language. This means that there exists a Turing machine which will enumerate all valid strings of the language and list them.

   So called true arithmetics is complete but does not have recursively enumerable set of axioms.

5. Algebraically closed fields with a given characteristic are complete, consistent, and have an infinite recursively enumerable set of axioms. If the characteristic is p, the multiplication of an element of the field by p gives zero. Finite fields have characteristic p and reals, complex numbers and p-adic numbers characteristic 0. The truths about integers cannot be however represented in these formal systems. The fields of complex algebraic numbers and real algebraic numbers are complete and consistent.

2.1.2 What does the first imcompleteness theorem state?

Intuitively, Gödel's first sentence states "I am not provable". This sentence is not about natural numbers but a meta level statement about statements about natural numbers. Gödel numbering, mapping the sentences of arithmetics in 1-1 and invertible way to to natural numbers, makes it possible to formulate metalevel statements as statements about natural numbers.

Gödel considers *sentence forms*, which are sentences about sentences. In a sufficiently general formal system for a given sentence form F there exists a statement p such that the Gödel numbers of p and $F(G(p))$ are identical. One can say that p defines a fixed point of the map of F. Other sentences than p are in general not fixed points for F. Gödel sentence is a special case and corresponds to a fixed point for a sentence form F_G stating that a given sentence is not provable.

Note that in the TGD framework, the fixed point property of p under F could mean that the action $(f_1, f_2) \rightarrow g \circ (f_1, f_2)$ of $g : C^2 \rightarrow C^2$ as the analog of the sentence form F reduces to a holomorphic general coordinate transformation of the space-time surface X^4 respecting its Hamilton-Jacobi structure so that X^4 does not change.

Some related theorems deserve to be listed:

1. Liar's paradox stating "I am false" is not equivalent with the Gödels first sentence". If it is false then it is true and vice versa.

2. The Gödel number of a false formula cannot be represented in arithmetics as shown by Tarski.

3. Tarski's undefinability theorem (see this) considers first-order arithmetic language and shows that the encoding by Gödel numbers cannot be done for semantic concepts such as truth: no sufficiently rich interpreted language can represent its own semantics.

4. Turing's theorem states that there are algorithms that do not halt.

Geometrically it is easy to understand the existence of unprovable truths if one looks the rules of first order logic in the axiomatic system adopted as rules for constructing paths in the space of statements. The can be true statements, which cannot be reached from axioms.

2.1.3 A rough sketch of the proof of the first completeness theorem

Gödel numbering (see this) is a basic notion. It assigns to each symbol of a formal language a natural number. All well-formed sentences can be transformed to sequences of the numbers associated with the symbols and one can assign a Gödel number to each sequence of this kind as a natural number. This maps the sentences as statements about numbers to numbers. The map is 1-1 so that the statement can be deduced from its Gödel number.

Both formulas and sequences of formulas representing proofs are encoded by Gödel numbers. Also statement forms making statements about Gödel numbers are possible. Statement forms are statements about statements so that one is now at a reflective level. Gödel numbering means that logical deductions can be encoded to maps of natural numbers to itself assigning to the assumptions the implication.

Gödel numbering makes it possible to encode statements about natural numbers and statements about provability of theorems about natural numbers. The Gödel number for the Gödel sentence is the same as that for the statement that the Gödel sentence is not provable. The sentence therefore states its own unprovability.

The proof of the first incompleteness theorem, as understood by a layman like me, goes roughly like the following.

1. Sentences are mapped to their Gödel numbers. This correspondence is 1-1 so that one can decode the sentence from the Gödel number. This requires that all symbols appearing in the sentences are coded to numbers x and the product of the numbers x is mapped to a product of powers p^x subsequent primes.

2. The statement "Bew" says that for a sentence with Gödel number y there exists a sentence with Gödel number x, which proves this sentence. More formally:

   Bew(y)= $\exists$ x(y is the Gödel number of a formula and x is the Gödel number of the proof of the formula encoded by y).

3. Statement forms F have as argument Gödel numbers of statements.

4. In a sufficiently strong axiomatics, for any *statement form* F there exists a statement p such that p is equivalent with $F(G(p))$:

   $$p \leftrightarrow F(G(p)) \ .$$

   This corresponds to the fixed point property. F defines a map from stements p to statement forms $F(G(p)$. In general $F(G(p))$ is not equivalent with p. In other words the Gödel numbers are not the same. F is the map and it always has a fixed point p implying that p is equivalent with F(G(p)).

5. Choose F to be the negation of $Bew(x)$. One obtains

   $$p \leftrightarrow \not{B}ew(G(p)) \ .$$

   p is therefore equivalent with the statement that there is no proof for p.

   There exists no x such that the Gödel number of x is the Gödel number of a formula proving y. The p is the formula and $y = G(p)$ is its Gödel number.

Diagonal argument is essential. Cantor's diagonal argument proves that reals are not a countable set mappable to integers. The idea is to assume this kind of listing is defined by the sequences of binary digits for reals. One changes the diagonal bits to their opposites and obtains a new element which does not belong to the list.

Diagonal argument relates to the existence of a fixed point if the axiomatic system is strong enough. The existence of the Gödel number for the proof of the statement would require extension of the axiom system by making the Gödel sentence an axiom.

2.2 The fixed point theorem of Lawvere

Some layman comments about the fixed point theorem of Lawvere [A1] (see are in order since Gödel sentence is a fixed point mapping the natural number defined by sentence to its Gödel number.

Lawvere's theorem generalizes functions to morphisms of categories and states that, for any Cartesian closed category $\mathbf{C}$ and given an object B in it, if there is a weakly point-surjective morphism f from some object A to the exponential object B^A, then every endomorphism $g : B \to B$ has a fixed point. That is, there exists a morphism $b : 1 \to B$ (, where 1 is a terminal object in $\mathbf{C}$) such that $g \circ b = b$.

The fixed point theorem of Lawvere has an impressive list of implications.

- Cantor's theorem
- Cantor's diagonal argument
- Diagonal lemma
- Russell's paradox
- Gödel's first incompleteness theorem
- Tarski's undefinability theorem
- Turing's proof
- Löb's paradox
- Roger's fixed-point theorem
- Rice's theorem

3 TGD perspective

TGD provides a rather concrete geometric analogy for the natural numbers as a formal system. The laws of physics would define the ultimate axiomatic system. Besides the geometric view of physics, TGD also allows a number theoretic view in which real space-time surfaces are generalized to adelic space-time surfaces. The holography= holomorphy vision suggests a concrete realization of meta hierarchies such that the independence of the logic on axioms corresponds to the independence of the space-time surfaces on the action if it is general coordinate invariant and constructible using the induced geometry. Infinite primes suggest a second realization.

3.1 How quantum TGD could define a physical analog of a formal system?

In TGD, arithmetic and more general formal systems are replaced by their living quantum versions and entire hierarchies of them emerge.

1. It is good to summarize first what TGD is. Space-time surfaces, Bohr orbits, are minimal surfaces identified as roots $(f_1, f_2) = (0, 0)$ of holomorphic functions of Hamilton-Jacobi coordinates of H [L6]. These 4-surfaces are expected to be slightly non-deterministic since already 2-dimensional minimal surfaces are slightly non-deterministic: there are several minimal surfaces spanned by a given frame. Now the frames are replaced by 3-surfaces defining the holographic data. In the TGD inspired theory of consciousness they serve as seats of potentially conscious memories [L7] [K3].

   Spinor structure as square root of Kähler structure would define logic as a square root of geometry. In the fermionic sector, the free second quantized fermion fields of H satisfying Dirac equation in H can be induced to the space-time surfaces and satisfy the modified Dirac equation at the space-time surface. Hamilton-Jacobi coordinates of H allow to identify a hypercomplex and a complex coordinate for the space-time surface and the modified Dirac equation can be solved algebraically just as in string models.

At the level of the "world of classical worlds" (WCW), the fermionic oscillator operators can be used to define the Gamma matrices of WCW associated with its spinor structure. WCW spinors define many-fermion states and their Fock basis defines a Boolean algebra and a kind of quantum realization of logic and the fermionic dynamics could express at the quantum level what the implication $A \rightarrow B$ means in zero energy ontology, where A and B would correspond to many-fermion states at the passive and active boundaries of CD. By the failure of the exact classical determinism, the WCW spinor field could be seen as a not totally deterministic logical deduction leading from premises A to some conclusion B.

The laws of physics would involve an infinite number of conservation laws implied by the symmetries of WCW. Holographic= holomorphy principle would mean an infinite group of symmetries generalizing conformal symmetries. Also generalization of the symplectic group assignal the the product of light-cone boundary and CP_2 could correspond to symmetries [L5]. In fact, the field equations reduce conservation laws of Noether currents and their super counterparts.

2. In general, the laws of classical physics would replace an infinite number of axioms needed to prove all possible truths or at least some of them: their number depends on the axioms of the formal system. For natural numbers, only a subset can be proven to be true. The laws of classical physics would basically state that holography= holomorphy principle is satisfied so that space-time surfaces within a given CD are roots $(f_1, f_2) = (0, 0)$, where f_1 and f_2 and analytic functions of hypercomplex coordinate of M^4 and three complex coordinates of $H = M^4 \times CP_2$.

3. A space-time surface satisfying holography= holography principle could be interpreted as a proof of a theorem. The coefficients of f_1 and f_2 can in an extension E of rationals. They could be polynomials and even analytic functions. Hierarchies of formal systems would be obtained. If the coefficients are complex numbers, an analogy of complex numbers would be obtained. For each extension of rationals and polynomial pair (P_1, P_2), one obtains discretization of space-time surfaces in terms of points with coordinates whose values are algebraic integers of E or even subset of algebraic rationals. Also a discretization of WCW is obtained automatically since the Taylor coefficienst belong to E. A discretization of WCW is obtained by restricting the Taylor coefficients to algebraic integers.

4. These space-time surfaces, analogous to Bohr orbits, provide analogues of theorems. The 3-D loci as loci of the slight non-determinism associated with the four-surfaces/Bohr orbits as minimal surfaces produce an analog for a proof of the theorems.

In a consistent system, a sentence and its negation cannot be true simultaneously. An interesting question is what the negation of the sentence is geometrically and in the fermion sector.

1. For fermions this would mean that fermionic creation operators are replaced by annihilation operators and vice versa. Therefore the fermionic sector statement and its negation cannot be simultaneously true. Is this enough to make the system consistent at quantum level?

2. Zero energy ontology (ZEO) suggests that theorems and their negations correspond to the two possible arrows of geometric time. The failure of CP symmetry implies a failure of time reversal T as a symmetry and this might imply that either the theorem or its negation realized as a reversed time evolution, and defined by (f_1, f_2) and its time reversed counterpart respectively, is true, in other words allows a realization as a space-time surface. What is amusing is that the violation of time reversal symmetry, which indeed happens, would relate directly to the consistency of the axiomatic system defined by physics!

3. What could the time reversal mean concretely? CP could correspond naturally to complex conjugation in CP_2 and 3-D reflection of CD with respect to its center point. CPT should act as an identity but realized as an antiunitary transformation replacing fermionic oscillator operators with their conjugates: could the antiunitarity guarantee consistency of the system?

The hypercomplex conjugation is realized as the permutation $(u, v) \rightarrow (v, u)$ of the light-like coordinates (u, v) associated with the Hamilton-Jacobi structure of M^4 and defining hypercomplex coordinate and its conjugate [L4].

For the special Hamilton-Jacobi structure $(u = t-z, v = t+z)$ and $w = x+iy$, time reversal $T : t \to -t$ would correspond to $(u,v) \to (-v,-u)$ and would be hypercomplex conjugation apart from the sign. P would correspond to $z \to -z$ and $w = x+iy \to -w$ and would give $(u,v) \to (v,u)$. PT would give $(u,v) \to (-u,-v)$. Does CPT act geometrically as an identity? In other words, can this transformation conjugating also CP_2 complex coordinates leave the space-time surface unaffected.

3.1.1 Holography= holomorphy vision

ZEO and holography= holomorphy vision provide some insights.

1. The most general option is that the 3-D loci X^3 of non-determinism divide space-time surfaces to regions such that the function pairs (f_1, f_2) can differ for different regions but continuity conditions are true at the 3-D interfaces X^3 of the regions. The simplest picture is that these regions correspond to a slicing of H by space-like surfaces ordered with respect to time coordinate.
2. Sentences as theorems would correspond to sequences of X^3 as loci of non-determinism defining an analogs of steps of a computer program assignable to the clicks of the computer clock [L9, L8, L12]. If this is the case, the surfaces X^3 or the regions associated with them could be analogous to the natural numbers assigned with basic logical primitives $a \to b$.

3.1.2 The formulation at the level of WCW and its adelic counterpart

One could formulate the analogs of statements as space-time surfaces in terms of the coefficients of analytic functions (f_1, f_2), in particular polynomials (P_1, P_2), with Taylor coefficients in an extension E of rationals defined inside CD. A good reason for this is that these indeed define space-time surfaces although it is not clear whether a given space-time surface as analog of Bohr orbit corresponds to a single pair of polynomials. It would be enough to consider only the situation for extensions E of rationals: a kind of reductionism would be achieved.

An attractive number theoretic idea is that the classical non-determinism means that the E-valued coefficients of the polynomials (P_1, P_2) are pseudo constants depending on the coordinates of H, perhaps only the light-cone proper time of the passive half-cone of CD so that in p-adic context everything would be continuous and smooth and in real contex only continuous. This would define the space-time surface as an adele having real part as a correlate of sensory experience and p-adic parts as correlate of cognition. This would mean the weakening of the assumption that the space-time surface decomposes to regions characterized by different (f_1, f_2): now the dependence of the coefficients on a finite number of pinary digits would distinguish different regions from each other.

This would mean a considerable weakening of the assumption that the space-time surface decomposes to regions characterized by different (f_1, f_2): now the the dependence of the coefficients on finite number of the pinary digits would distinguish separate regions from each other and would give better hopes of satisfying the continuity conditions at the interfaces of these regions.

3.1.3 Space-time surfaces as theorems

For a given coefficient field E of polynomials, a space-time surface which is realizable for a large enough CD could serve as the analog of a true sentence.

1. For a given CD, it may well be the case that for a given 3-surface X^3, defining the holographic data, there is no 4-surface $X^4(X^3)$ satisfying the condition $(f_1, f_2) = (0,0)$ also at the PB of the CD. However, for a larger CD this might be the case. The nature of the allowed polynomials or even analytic functions, in particular the degree of polynomial, also matters.

   The ramified primes correspond in p-adic mass calculations geometrically to p-adic primes defining p-adic length scales. If the p-adic length scale scale is larger than the size of the CD, one expects that all theorems assignable to polynomials as Bohr orbits are not realizable. The self having the CD as a perceptive field, which gradually increases in the sequences of SSFRs, should be able to live long enough for the size of the CD to become large enough.

2. This gives rise to a hierarchy of living analogs of formal systems labelled by the algebraic extensions of rationals, by polynomials (even analytic functions), and the size scale of the CD. The effective axiomatics would gradually become stronger as the dimension of the algebraic extension, degree of the polynomials, and the size of the CD grows. If the polynomials (P_1, P_2) (or even analytic functions (f_1, f_2)) can change at the loci of non-determinism, the increase of E and increase of the algebraic complexity of polynomials would indeed make the effective axiomatics evolving.

3. As the extension E of rationals and the size of CD increases, the limit of algebraic numbers is approached and the system could become complete in the sense that any 3-surface at either boundary of CD can allow space-time surfaces as roots. For the entire WCW allowing $f = (f_1, f_2)$, which are arbitrary analytic functions of H coordinates, one obtains analog of complex numbers and coefficients of polynomials are now complex numbers and it may happen that all the sentences or their negations can be proven to be true.

4. The development/evolution of the system would mean an increase of algebraic complexity and could be seen as an addition of axioms making it possible to prove new statements realized as Bohr orbits. The identification of the coefficient field E as rationals would be the lowest level, after that would emerge algebraic extensions of rational numbers and a hierarchy of polynomials and even analytic functions with Taylor coefficients in arbitrary extension E can be considered.

   For a given formal system, the superposition associated with the PB of CD would correspond to a version of some selected axioms, the assumptions of the quantum theorem. The non-deterministic time evolutions would correspond to the proof of theorems that follow from various quantum axioms.

   The classical laws of physics would define the set of axioms. The finite size of the CD allows the realization of only a subset of theorems. Liveliness would mean that new axioms would be added to the used, always finite, effective set of axioms. For example, the growth of CD in SSFR would allow polynomials with which the p-adic length scales determined by the ramified prime, which are bounded by the size of CD, would grow. The new allowed polynomials would give realizations to new theorems.

3.1.4 Number theoretic statements in terms of the adelic space-time surfaces

Adeles are formed by the real space-time surface and its various p-adic counterparts [L1], which are in rough sense Cartesian products of p-adic number fields or more generally, their extensions. Arithmetics relates to cognition and the p-adic factors of the space-time adele would serve as correlates for cognition.

1. In the TGD framework space-time surfaces are analogs of numbers since the function pairs (f_1, f_2) form a function field when either of them is fixed: in TGD there are physical arguments for why the fixing of f_2 (say) could define a very large sub-WCW: $f_2 = 0$ could define the analog of cosmological constant as slowly varying parameter [L6] and determine a map relating the twistor spheres of M^4 of CP_2 to each other. They are also analogous to theorems. Gödel numbers could encode the discretization of the space-time surface in the extension of E of rationals.

2. In number theoretic vision, real numbers serve as correlates of sensory experience and p-adic number fields as correlates of cognition. They combine to adeles as analogs of Cartesian products of reals and various p-adic number fields. One must use the extensions of the p-adic number fields induced by E and E induces also the extension of the rational adele. Space-time surfaces can be generalized to adelic surfaces and since space-time surfaces are regarded as generalized numbers, space-time surfaces as theorems correspond to numbers in a generalized sense.

3. The discretizations of the space-time surfaces by points, which have Hamilton-Jacobi coordinates of H belonging to the extension E, are natural. This poses strong restrictions. For instance, the surface $x^n + y^n = z^n$, $n \geq 3$ has only the point $(0, 0, 0)$ as a set rational points and this result generalizes to extensions E of rationals although it is not so strong.

The basic number theoretic question could be as follows. Does a given point of the space-time surface as a solution of the $(f_1, f_2) = (0, 0)$ belong to the extension E of rationals defining the coefficients of E? Could adelization allow an answer to this question?

1. The polynomial equations with rational coefficients have a rational solution if they have solution in all p-adic numbers fields for which the solution is finite also in real sense. The equations for p-adic numbers are much simpler due to the modular arithmetics. This is applied in the proof of the Fermat's theorem which can be formulated also in for space-time surfaces: the equations $\xi_1^n + \xi_2^n = w^n$ make sense for space-time surfaces and the space-time surface would be no rational points for $n \geq 3$.

2. This statement seems to hold true for the extensions E of rationals. The root of $(f_1, f_2) = 0$ would exists as a point in E discretization of H if it exists in all E-extended p-adic variants H such that the Hamilton-Jacobi coordinates are finite as algebraic numbers.

3. These conditions hold true for every point of E-discretization separately that one entire set of number theoretical statements for each point of space-time surface is obtained. The solutions can be said to be in the intersection of real and various p-adic numbers and define a cognitive representation of the real space-time surface.

3.2 Self reference: could space-time surfaces represent statements about space-time surfaces as analogs of sentences?

It is good to start by making questions and try to understand what one tries to understand. The basic goal of a meta-mathematician is to reduce statements about the sentences of arithmetics as sentences about natural numbers.

How the notion of self reference could be formulated for space-time surfaces? If the space-time surfaces correspond to numbers, the theorems about space-time surfaces are space-time surfaces as analogs of Bohr orbits satisfying the classical field equations. The metalevel statements about these theorems should be also realized as space-time surfaces. What could this mean?

In arithmetics, the list of the sentences about natural numbers is based on Gödel numbers and statements would be statements about Gödel numbers. This is essentially self reference. In the case of TGD,the notion of Gödel numbers does not look useful.

3.2.1 Hierarchies of extensions of rationals defined by analytic maps $G : C^2 \rightarrow C^2$ as meta hierarchies?

What is new in the TGD framework is that one has a geometric description of the sentences. Could the meta hierarchy correspond to the space-time surfaces which are associated with hierarchies of extensions of rationals and Galois group defined by the analytic maps $g : C^2 \rightarrow C^2$ acting as $g(f_1, f_)) = (g_1(f_1, f_2), g_2(f_1, f_2)$? These kinds of hierarchies give rise to generalizations of Mandelbrot fractals and Julia sets.

1. Could the maps g realize the meta level statements about a surface determined by $(f_1, f_2) = (0, 0)$ as the surfaces $(g_1(f_1, f_2), g_2(f_1, f_2) = (0, 0)$ and be realized also as space-time surface when one extends the analog of axiomatization. What Gödel's incompleteness theorems would mean in this interpretation? Could the analog of the first Gödel sentence mean that the 3-surfaces at the PB of the CD cannot remain invariant for this kind statement?

2. Is it possible to find any analytic functions g, whose action is trivial at the PB of CD? In a complex plane this is not possible since one should have an analytic function vanishing outside the boundary of the region considered.

   Does the light-likeness of PB make this possible vanishing of g in the space-like exterior of the PB under some additional conditions? Note that PB corresponds formally to a single point, moment of big bang, in the cosmological context and the induced metric at the passive light-cone boundary vanishes. Could the Hamilton-Jacobi structure for the CD code for this. The ratio w/a of complex coordinate would correspond to spherical complex coordinates. The powers w^n would approach zero at PB for $n > 1$.

3.2.2 Meta hierarchies and the hierarchy of infinite primes?

What comes to mind in the TGD framework, is the hierarchy of infinite primes [K4, K1, K2] having an interpretation as a hierarchy of statements about statements about.... statements. At the level the statements correspond to ordinary primes. The hierarchy has several interpretations.

1. It can be regarded as a repeated second quantization of a supersymmetric arithmetic quantum field theory in which the single particle states at a given level correspond to many particle states of the lower level. What is remarkable is that the analogs of bound states are included in the spectrum.

2. The interpretation in terms of a hierarchy of polynomials with an increasing number of variables makes sense.

3. Geometrically this hierarchy is analogous to the hierarchy of many-sheeted space-time with sheets containing smaller sheets containing... and at the level of the theory of consciousness and self-hierarchy. Here one can also consider the possibility that many-fermion states with an odd number of fermions at a given level give rise to single fermion states at the next level.

4. This also brings in mind the hierarchies of algebraic extensions of rationals in particular hierarchies defined by functional compositions of polynomials and even analytic functions. The compositions of the polynomials define hierarchies of increasingly complex polynomials with increasing size of algebraic extensions and Galois groups.

5. These kinds of hierarchies can be defined for any extension E of rationals as extensions of extensions of represented as functional compositions of polynomials.

6. The meta hierarchies of infinite primes are also structurally similar to the hierarchies assignable to extensions E formed by the space-time surfaces represented as roots $g \circ (f_1, f_2) = (0, 0)$, where g is analytic map $g : C^2 \to C^2$? If f is restricted to C so that f_1 remains fixed, this would give rise to hierarchies of extensions of extensions of rationals and corresponding hierarchies of Galois groups having lower level Galois as a normal subgroup. These hierarchies could be relevant for the understanding of mathematical cognition [L3].

4 Gödel and Lawvere

In this section the incompleteness theorems of Gödel and the fixed point theorem of Lawvere are briefly summarized.

4.1 Gödel's incompleteness theorems

Gödel's incompleteness theorems (see this) apply to arithmetics of natural numbers. They can be generalized and apply for instance to category theory. It is essential that there exist morphisms from $X \to X^X$ allowing fixed points.

4.1.1 Some background

There are two incompleteness theorems:
enumerate

There are always true statements which cannot be proven.

The system cannot demonstrate its own consistency.

Incompleteness theorems imply that Hilbert's program fails. There exists non-provable truths in any internally consistent and sufficiently strong axiomatic system of arithmetics. For complete number systems such as complex numbers there exists an infinite effective recursively enumerable axiomatization allowing to prove all truths but the theorems are about properties of complex numbers rather than integers and one cannot avoid incompleteness theorem for natural numbers.

Gödels incompleteness theorems apply at the syntactic level, not semantic. To understand the core of Gödel's theorem one must understand the difference semantic and syntax.

1. The language of mathematics involves only syntax. At the syntactic level there is no meaning yet. Formal systems, computer programs, in particular AI and LLMs, involve only the syntax. Classical deterministic physics can be seen as an analog of a formal system with classical time evolution as the analog of logical deduction and classical laws of physics as axioms. However, the local laws of physics depend on the action chosen and this does not conform with independence of the Boolean logic on the axiom system.

   Rather remarkably, in TGD the holography= holomorphy principle implies the same universal dynamics irrespective of action and predicting that space-time surfaces are minimal surfaces. 0

2. Semantic level involves meaning and this requires consciousness. The notion of apple serves a good illustration. Apple corresponds to a real world object, to the mental image created by its perception, and to its name which corresponds to syntactic aspect. The symbol "apple" corresponds to syntactic level. This level cannot catch the notion of truth in semantic sense.

Gödel proved that first order logic is semantically complete but not syntactically complete: there are sentences that cannot be proved or disproved in the axion system considered.

Some further notions related to the axiom system are needed.

1. Completeness means that any statement or its negation is provable to be true. Gödel proved that first order logic is semantically complete but not syntactically complete: there are sentences that cannot be proved or disproved in the axion system considered.

2. Consistency means that there is no statement such that both the statement and its negbation can be proven from axioms

3. Effective axiomatization means the existence of an algorithm, which can list the theorems following from a given axiomatization.

4. In mathematics, logic and computer science, a formal language is called recursively enumerable if it is a recursively enumerable subset in the set of all possible words over the alphabet of the language. This means that there exists a Turing machine which will enumerate all valid strings of the language and list them.

   So called true arithmetics is complete but does not have recursively enumerable set of axioms.

5. Algebraically closed fields with a given characteristic are complete, consistent, and have an infinite recursively enumerable set of axioms. If the characteristic is p, the multiplication of an element of the field by p gives zero. Finite fields have characteristic p and reals, complex numbers and p-adic numbers characteristic 0. The truths about integers cannot be however represented in these formal systems. The fields of complex algebraic numbers and real algebraic numbers are complete and consistent.

4.1.2 What does the first imcompleteness theorem state?

Intuitively, Gödel's first sentence states "I am not provable". This sentence is not about natural numbers but a meta level statement about statements about natural numbers. Gödel numbering, mapping the sentences of arithmetics in 1-1 and invertible way to to natural numbers, makes it possible to formulate metalevel statements as statements about natural numbers.

Gödel considers *sentence forms*, which are sentences about sentences. In a sufficiently general formal system for a given sentence form F there exists a statement p such that the Gödel numbers of p and $F(G(p))$ are identical. One can say that p defines a fixed point of the map of F. Other sentences than p are in general not fixed points for F. Gödel sentence is a special case and corresponds to a fixed point for a sentence form F_G stating that a given sentence is not provable.

Note that in the TGD framework, the fixed point property of p under F could mean that the action $(f_1, f_2) \to g \circ (f_1, f_2)$ of $g : C^2 \to C^2$ as the analog of the sentence form F reduces to a holomorphic general coordinate transformation of the space-time surface X^4 respecting its Hamilton-Jacobi structure so that X^4 does not change.

Some related theorems deserve to be listed:

1. Liar's paradox stating "I am false" is not equivalent with the Gödels first sentence". If it is false then it is true and vice versa.

2. The Gödel number of a false formula cannot be represented in arithmetics as shown by Tarski.

3. Tarski's undefinability theorem (see this) considers first-order arithmetic language and shows that the encoding by Gödel numbers cannot be done for semantic concepts such as truth: no sufficiently rich interpreted language can represent its own semantics.

4. Turing's theorem states that there are algorithms that do not halt.

Geometrically it is easy to understand the existence of unprovable truths if one looks the rules of first order logic in the axiomatic system adopted as rules for constructing paths in the space of statements. The can be true statements, which cannot be reached from axioms.

4.1.3 A rough sketch of the proof of the first completeness theorem

Gödel numbering (see this) is a basic notion. It assigns to each symbol of a formal language a natural number. All well-formed sentences can be transformed to sequences of the numbers associated with the symbols and one can assign a Gödel number to each sequence of this kind as a natural number. This maps the sentences as statements about numbers to numbers. The map is 1-1 so that the statement can be deduced from its Gödel number.

Both formulas and sequences of formulas representing proofs are encoded by Gödel numbers. Also statement forms making statements about Gödel numbers are possible. Statement forms are statements about statements so that one is now at a reflective level. Gödel numbering means that logical deductions can be encoded to maps of natural numbers to itself assigning to the assumptions the implication.

Gödel numbering makes it possible to encode statements about natural numbers and statements about provability of theorems about natural numbers. The Gödel number for the Gödel sentence is the same as that for the statement that the Gödel sentence is not provable. The sentence therefore states its own unprovability.

The proof of the first incompleteness theorem, as understood by a layman like me, goes roughly like the following.

1. Sentences are mapped to their Gödel numbers. This correspondence is 1-1 so that one can decode the sentence from the Gödel number. This requires that all symbols appearing in the sentences are coded to numbers x and the product of the numbers x is mapped to a product of powers p^x subsequent primes.

2. The statement "Bew" says that for a sentence with Gödel number y there exists a sentence with Gödel number x, which proves this sentence. More formally:

   Bew(y)= $\exists$ x(y is the Gödel number of a formula and x is the Gödel number of the proof of the formula encoded by y).

3. Statement forms F have as argument Gödel numbers of statements.

4. In a sufficiently strong axiomatics, for any *statement form* F there exists a statement p such that p is equivalent with $F(G(p))$:

   $$p \leftrightarrow F(G(p)) \ .$$

   This corresponds to the fixed point property. F defines a map from stements p to statement forms $F(G(p)$. In general $F(G(p))$ is not equivalent with p. In other words the Gödel numbers are not the same. F is the map and it always has a fixed point p implying that p is equivalent with F(G(p)).

5. Choose F to be the negation of $Bew(x)$. One obtains

   $$p \leftrightarrow \not{B}ew(G(p)) \ .$$

p is therefore equivalent with the statement that there is no proof for p.

There exists no x such that the Gödel number of x is the Gödel number of a formula proving y. The p is the formula and $y = G(p)$ is its Gödel number.

Diagonal argument is essential. Cantor's diagonal argument proves that reals are not a countable set mappable to integers. The idea is to assume this kind of listing is defined by the sequences of binary digits for reals. One changes the diagonal bits to their opposites and obtains a new element which does not belong to the list.

Diagonal argument relates to the existence of a fixed point if the axiomatic system is strong enough. The existence of the Gödel number for the proof of the statement would require extension of the axiom system by making the Gödel sentence an axiom.

4.2 The fixed point theorem of Lawvere

Some comments about the fixed point theorem of Lawvere (see are in order since Gödel sentence is a fixed point mapping the natural number defined by sentence to its Gödel number.

Lawvere's theorem generalizes functions to morphisms of categories and states that, for any Cartesian closed category $\mathbf{C}$ and given an object B in it, if there is a weakly point-surjective morphism f from some object A to the exponential object B^A, then every endomorphism $g : B \rightarrow B$ has a fixed point. That is, there exists a morphism $b : 1 \rightarrow B$ (, where 1 is a terminal object in $\mathbf{C}$) such that $g \circ b = b$.

The fixed point theorem of Lawvere has an impressive list of implications.

- Cantor's theorem
- Cantor's diagonal argument
- Diagonal lemma
- Russell's paradox
- Gödel's first incompleteness theorem
- Tarski's undefinability theorem
- Turing's proof
- Löb's paradox
- Roger's fixed-point theorem
- Rice's theorem

5 TGD does not lead to the paradox with the notion of self

Self-awareness means the presence of a reflective level: being conscious about being conscious of something. This leads to an infinite self-reference hierarchy, which can be seen as a problem. This sequence could lead to a fixed point or limit cycle or a kind of strange attractor: in the case of iterations of analytic maps of the complex plane to itself, one obtains limit cycles but not fixed points. The second possibility would be that self-awareness corresponds to a fixed point from the beginning but is this really self-awareness? Note that Gödel sentence is a fixed point of a process producing sentences which are not provable in the formal system.

Clearly, the problem is that one assumes that self is something fixed rather than evolving. Zero energy ontology (ZEO) [L2] avoids this problem in the TGD framework. ZEO also solves the basic problem of quantum measurement theory.

In the ZEO based theory of consciousness, self is *not* a fixed entity but evolves by a sequence of "small" state function reductions (SSFRs).

1. The sequence of SSFRs is the ZEO counterpart for the sequence of repeated measurements of the same observables in standard quantum theory (Zeno effect). In fermionic degrees of freedom, the zero energy states can be seen in fermionic degrees of freedom as pairs of states at the boundaries. In geometric degrees of freedom one has a superpositions of the analogs of Bohr orbits.

2. At the PB of CD, the state is not changed (Zeno effect) in SSFRs. At the active boundary (AB) of CD, the state is changed. This gives rise to subjective time evolution and self. Also CD changes in size and this gives rise to a geometric time evolution as increase of the distance between the tips of CD correlating with the subjective time evolution. Each SSFR in the sequence of SSFRs gives rise to an updated self containing more information about the previous selves.

3. This information, self knowledge, is realized as memories represented geometrically by the Bohr orbit like space-time surfaces leading from fixed 3-surfaces at the PB to the changing 3-surfaces at the AB.

   These space-time surfaces are minimal surfaces. Their mild classical non-determinism (non-determinism occurs already for 2-D soap films) makes it possible for quantum states in ZEO to contain information about previous quantum states in the sequence of SSFRs as memories. Non-determinism also makes it possible to recall these memories. The memories in general change somewhat in the memory recall [L7].

REFERENCES

Mathematics

[A1] Lawvere FW. *Diagonal arguments and Cartesian closed categories*. Berlin: Springer, 1969.

[A2] Raatikainen P. Gödel's Incompleteness Theorems. Stanford Encyclopedia of Philosophy. 2013. Retrieved November 7,2022. Available at: https://plato.stanford.edu/entries/goedel-incompleteness/.

Books related to TGD

[K1] Pitkänen M. Infinite Primes and Consciousness. In *Mathematical Aspect of Consciousness. https://tgdtheory.fi/tgdhtml/mathconsc.html*. Available at: https://tgdtheory.fi/pdfpool/infpc.pdf, 2023.

[K2] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory: Part III. https://tgdtheory.fi/tgdhtml/Btgdnumber3.html*. Available at: https://tgdtheory.fi/pdfpool/infmotives.pdf, 2023.

[K3] Pitkänen M. Quantum Model of Memory. In *TGD Inspired Theory of Consciousness: Part I. https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/memoryc.pdf, 2023.

[K4] Pitkänen M. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionc.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Philosophy of Adelic Physics. Available at: https://tgdtheory.fi/public_html/articles/adelephysics.pdf., 2017.

[L2] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https://tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[L3] Pitkänen M. The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group. Available at: https://tgdtheory.fi/public_html/articles/SSFRGalois.pdf., 2020.

[L4] Pitkänen M. Holography and Hamilton-Jacobi Structure as 4-D generalization of 2-D complex structure. https://tgdtheory.fi/public_html/articles/HJ.pdf., 2023.

[L5] Pitkänen M. Symmetries and Geometry of the "World of Classical Worlds" . https://tgdtheory.fi/public_html/articles/wcwsymm.pdf., 2023.

[L6] Pitkänen M. About Langlands correspondence in the TGD framework. https://tgdtheory.fi/public_html/articles/Frenkel.pdf., 2024.

[L7] Pitkänen M. How subjective memories are realized in TGD inspired theory of consciousness? https://tgdtheory.fi/public_html/articles/memorytgd.pdf., 2024.

[L8] Pitkänen M. Quartz crystals as a life form and ordinary computers as an interface between quartz life and ordinary life? https://tgdtheory.fi/public_html/articles/QCs.pdf., 2024.

[L9] Pitkänen M. Space-time surfaces as numbers, Turing and Goödel, and mathematical consciousness . https://tgdtheory.fi/public_html/articles/compuTGD.pdf., 2024.

[L10] Pitkänen M. TGD as it is towards the end of 2024: part I. https://tgdtheory.fi/public_html/articles/TGD2024I.pdf., 2024.

[L11] Pitkänen M. TGD as it is towards the end of 2024: part II. https://tgdtheory.fi/public_html/articles/TGD2024II.pdf., 2024.

[L12] Pitkänen M. A hybrid of classical and quantum computer and quantum model for associative learning. https://tgdtheory.fi/public_html/articles/qcomplearn.pdf., 2025.