Galois groups and genetic code

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Abstract

This article was inspired by the inverse problem of Galois theory. Galois groups are realized as number theoretic symmetry groups realized physically in TGD a symmetries of space-time surfaces. Galois confinement as an analog of color confinement is proposed in TGD inspired quantum biology .

Galois groups, in particular simple Galois groups, play a fundamental role in the TGD view of cognition. The TGD based model of the genetic code involves in an essential manner the groups A_5 (icosahedron), which is the smallest non-abelian simple group, and A_4 (tetrahedron). The identification of these groups as Galois groups leads to a more precise view about genetic code. The question why the genetic code is a fusion of 3 icosahedral codes and of only a single tetrahedral code remained however poorly understood.

The identification of the symmetry groups of the I, O, and T as Galois groups makes it possible to answer this question. Icosa-tetrahedral tesselation of 3-D hyperbolic space H^3, playing centrl role in TGD, can be replaced with its 3-fold covering replacing $I/O/T$ with the corresponding symmetry group acting as a Galois group. T has only only a single Hamiltonian cycle and its 3-fold covering behaves effectively as a single cycle. Octahedral codons can be regarded as icosahedral and tetrahedral codons so they do not contribute to the code.

Contents

1 Introduction

Galois groups, in particular simple Galois groups acting on cognitive representations consisting of points, whose coordinates in a number theoretically preferred coordinate system of octonions belong to EQ, play a fundamental role in the TGD view of cognition [L3]. The TGD based model of genetic code [L1, L2] involves in an essential manner the groups A_5 (icosahedron (I)), which is the smallest non-abelian simple group, and A_4 (tetrahedron (T)). Genetic code has as building bricks Hamiltonian cycles of I and T. Genetic code relates to information and therefore to cognition so that the interpretation of these symmetry groups as Galois groups is suggestive.

The most recent step of progress was the realization that genetic code can be represented in terms of icosa-tetrahedral tesselation of a hyperbolic 3-space H^3 [L4] and that the notion of genetic code generalizes dramatically. Also octahedron (O) is involved with the tesselation but plays a completely passive role. The question why the genetic code is a fusion of 3 icosahedral codes and of only a single tetrahedral code remained however poorly understood.

The progress in the understanding of the role of Galois groups inspired by a summary of inverse Galois problem [A2] (`https://cutt.ly/jmjpyDS`) in TGD framework [?] made it possible to answer this question. The proposal is that the symmetry groups of the I, O, and T can be identified as Galois groups.

Icosa-tetrahedral tesselation can be replaced with its 3-fold covering replacing $I/O/T$ with the corresponding symmetry group acting as a Galois group. Octahedral codons can be regarded as icosahedral and tetrahedral codons so they do not contribute to the code. T has only only a single Hamiltonian cycle and its 3-fold covering behaves as a single cycle. T has only only a single Hamiltonian cycle and its 3-fold covering behaves effectively as a single cycle.

2 Could the symmetries of icosa-tetrahedral realization of the genetic code correspond to Galois symmetries?

Abelian groups Z_p, p prime, are simple and the alternating group A_5 with order 60 is the smallest non-Abelian simple group. All groups A_n, $n \geq 5$ are simple and have $n!/2$ elements. A_5 corresponds to the icosahedral group isomorphic with the symmetry group of the dodecahedron.

The TGD based model of genetic code [L1, L2, L4] involves in an essential manner the groups A_5 (icosahedron) and A_4 (tetrahedron). Simple groups play a fundamental role in the TGD view of cognition. Could this mean that genetic code represents the lowest level of an infinite cognitive hierarchy?

2.1 The TGD inspired model model of genetic code, cognition, and Galois groups

TGD based model of bioharmony [L1, L2, L4] provides a model of genetic code as a fusion of 3 icosahedral Hamiltonian cycles and the unique tetrahedral Hamiltonian cycle (what "fusion" precisely means is far from clear and I have considered several options).

Icosahedral Hamiltonian cycles is a non-self-intersecting path at icosahedron connecting nearest points if icosahedron going through all 12 points of the icosahedron. It is interpreted as a representation of a 12-note scale with a scaling by quint assigned to a given step along the cycle. For a given Hamiltonian cycle, the allowed 3-chords of icosahedral harmony are identified as chords defined by the triangular faces of the icosahedron.

Remark: In the sequel I will use the shorthands IH, OH, and TH for icosahedral, octahedral, and tetrahedral harmonies. Also the notation $I/O/T$ will be used for icosahedron/octahedron/tetrahedron unless there is a danger of confusing them with their symmetry groups with identical shorthand notations.

Galois groups are essential for cognition in the TGD framework. In particular, simple groups as primes for groups are also primes for cognition [L3]. Genes represent information and Galois groups are crucial for cognition in the TGD framework. Genes would correspond to sequences of 3-chords of bioharmony. This raises several questions.

Could genetic code relate to Galois group A_5 as the smallest simple non-abelian Galois group (and also to the fact that the only polynomials of order smaller than 5 are generically solvable)? Could genetic code correspond to the lowest level in a hierarchy of cognition and of analogs of genetic code?

The order $n = 60$ for A_5 suggests a fusion of 3 icosahedral codes to give 20+20+20 = 60 codons.

1. 3 Platonic solids, - icosahedron (I), tetrahedron (T) , and octahedron (O) - which have triangles as faces so that one can consider the possibility of constructing a lattice like structure by gluing these Platonic solids together along their faces. Hyperbolic space H^3 indeed allows isosa-tetrahedral tessellation, which also involves O:s. I have proposed that this allows a realization of genetic code and also of genes [L4]. The notion of gene generalizes so that genes can also be 2- or 3-D lattice-like structures.

2. A_5 has $A_3 = Z_3$ as a subgroup and I(cosahedron) corresponds to A_5/Z_3. I has several Hamiltonian cycles having as a symmetry group Z_6, Z_4 or Z_2. Z_2 can act either as rotations or reflections.

   Q: Could A_5 as a Galois group as 3-fold covering of I make it possible to understand why the fusion of just 3 icosahedral codes is possible?

3. Tetrahedral group T corresponds to the alternating group $A_4 = S_4/Z_2 = Z_4 \times Z_3$ with 12 elements and tetrahedron identification as A_4/Z_3. The tetrahedral Hamiltonian cycle (4-scale) is unique and has 4 3-chords. The 3-fold copy would correspond to A_4. Information about the unique Hamiltonian cycles of O and T can be found in [A1] (`https://cutt.ly/9mlMiV8`).

   Q: Could the factor that there is only one tetrahedral cycle explain why only a single tetrahedron contributes?

4. Octahedral group O has 24 elements and is the wreath product of Z_3 and Z_2^3 and has also the decomposition $O = S_2 \times S_4$. Octahedron can be identified as O/Z_3. Also octahedral Hamiltonian cycle representing 8-scale with 8 chords is unique.

   Q: Why don't octahedral codons contribute?

2.2 A model of the genetic code based on icosa-tetrahedral tessellation of hyperbolic 3-space

TGD leads to a proposal for a geometric representation of the genetic code in terms of icosa-tetrahedral tessellation of the hyperbolic 3-space H^3 (mass shell or light-cone proper time $a = constant$ hyperboloids of M^4) [L4]. Both I, O, and T having triangular faces appear in the tessellation. Recall that the corresponding harmonies are denoted by IH, OH and TH.

I do not completely understand the details of the icosa-tetrahedral tessellation. The following picture satisfies the constraints coming from the notion of harmony but I have not proven that it is correct. Here the help of a professional geometrician knowing about tessellations of H^3 would be needed.

1. The analog of the discrete translational symmetry for lattices can be assumed: all I:s , O:s and T:s are equivalent as far as common faces with neighboring Platonic solids are considered.

2. The term icosa-tetrahedral tessellation suggests that all octahedral faces are glued to tetrahedral and icosahedral faces so that octahedral chords reduce to either icosahedral or tetrahedral chords. OH would not be an independent harmony. This requires that the number of common faces between two O:s vanishes: $n_O^O = 0$.

3. T shares at least 1 face with a given I so that the number of tetrahedral chords is reduced to at most 3 for given T. 4 purely tetrahedral faces (not shared with I) are needed. I would have $n_{IT} \leq 4$ purely tetrahedral faces in such a way that the total number of purely tetrahedral 3-chords is 4.

   The simplest possibility is that I shares a common face with 2 T:s. Each T shares 2 faces with O providing 2 purely tetrahedral 3-chords and shares the remaining 2 faces with distinct I:s. One would have $n_T^I = 2$, $n_T^O = 2$, $n_T^T = 0$.

   Since each I defines independently 20 chords, 2 I:s cannot have common faces. One would have $n_I^T = 2$, $n_I^I = 0$ and $n_I^O = 18$ to give $n_I^T + n_I^O + n_I^I = 2 + 18 + 0 = 20$.

4. What remains to be fixed are the numbers n_O^I and n_O^T satisfying $n_O^I + n_O^T = 8$. The conditions $n_O^T \geq 1$ and $n_O^I \geq 1$ must be satisfied since both T and I share faces with Os.

   Music comes to rescue here. The 8 3-chords of OH could define OH sub-harmony of IH. Analogously, the 4 3-chords of TH could define TH as a sub-harmony of OH.

   Could IH sharing 18 3-chords with OH contain 2 transposed copies of OH plus 2 chords of TH? IH cannot of course contain the entire TH as a sub-harmony.

   Could OH contain one copy of TH? This would give $n_O^I = n_O^T = 4$. Could the IH part of OH actually be TH as a sub-harmony of IH so that OH would reduce to 2 copies of TH?

To sum up, if the answers to the questions are positive, the incidence matrix n_i^j, $i, j \in \{I, T, O\}$, telling how many faces i shares with j would be given by

$$\begin{bmatrix} n_I^I & n_I^O & n_I^T \\ n_O^I & N_O^O & n_O^T \\ n_T^I & N_T^O & n_T^T \end{bmatrix} = \begin{bmatrix} 0 & 18 & 2 \\ 4 & 0 & 4 \\ 2 & 2 & 0 \end{bmatrix} . \tag{2.1}$$

2.3 3-fold cover of the icosa-tetrahedral tessellation

The proposed model does not yet explain the fusion of 3 icosahedral Hamiltomnian cycles. A 3-fold cover of the icosa-tetrahedral tessellation which replaces Platonic solids with their symmetry groups is highly suggestive. This raises a series of questions.

1. How could this representation relate to a possible interpretation in terms of the Galois groups $I = A_5$ and $O = S_2 \times S_4$ and $T = A_4$? Z_3 appears as a sub-group of all these groups and these Platonic solids are coset spaces I/Z_3, O/Z_3, and T/Z_3.

2. Could one lift the icosa-tetrahedral tessellation to a 3-sheeted structure formed by the geometric representations of the Galois groups of this structure acting as symmetry groups? Platonic solids would be replaced with their symmetry groups acting as Galois groups.

3. Could the 3 different icosahedral Hamiltonian cycles correspond to different space-time sheets - roughly CP_2 coordinates as 3-valued functions of M^4 coordinates whereas 20 regions representing icosahedral vertices would correspond to different loci of $E^3 \subset M^4$ just as one intuitively expects?

4. Same should apply to the tetrahedral and octahedral parts of the tessellation. But don't the 3 identical copies of the tetrahedral Hamiltonian cycle give 64+8=72 codons? How can one overcome this problem?

The following is a possible answer to these questions.

1. $h_{eff} = 60h_0$ corresponds to 60-sheeted space-time (here also $60k$-sheeted space-time is possible if 60-D extension of k-dimensional extension is in question). For T and O an analogous picture would apply. One could say that the projections of I and O and T are in M^4. At each sheet one would have icosa-tetrahedral tessellation.

2. I has 3 types of Hamiltonian cycles with symmetry groups Z_6, Z_4, and Z_2 and can give 3 different copies. However, only a single copy of tetrahedral harmony appears in the model: otherwise the number of codons would be larger than 64. Could the 3 identical Hamiltonian cycles for T and O effectively correspond to a single Hamiltonian cycle?

3. The fusion of Hamiltonian cycles is analogous to a formation of many-boson states. For T and O all Hamiltonian cycles would be identical: one would have only one Hamiltonian cycle effectively. The 3-chords associated with the 3 octahedral and tetrahedral cycles are identical so that only single tetrahedral harmony would be present.

To sum up, the lift of the icosa-tetrahedral complex to that defined by the respective Galois groups could explain why just 3 icosahedral Hamiltomian cycles and effectively only 1 tetrahedral cycle.

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