What Gravitons Are and Could One Detect Them in TGD Universe?

M. Pitkänen,

May 27, 2024
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

Contents

Contents

Abstract

What gravitons are in the TGD framework? This question has teased me for decades. It is easy to understand gravitation at the classical level in the TGD framework but the identification of gravitons has been far from obvious. Second question is whether the new physics provided by TGD could make the detection of gravitons possible?

The generalized Kähler structure for $M^4 \subset M^4 \times CP_2$ leads together with holography=generalized holomorphy hypothesis to the question whether the spinor connection of M^4 could have interpretation as gauge potentials with spin taking the role of the gauge charge. However, the induced M^4 spinor connection has a vanishing vielbein curvature. The M^4 Kähler gauge potential remains a candidate for giving rise to graviton-like state: the additional spin would come from the rotation of the monopole flux tube. Also the second fundamental form H^k has M^4 part M^k behaves like a spin 1 object.

One can assign the fundamental vertices with the splitting of closed string-like flux tubes representing elementary particles. The vertices would correspond to the defects of ordinary 4-D smooth structure making possible a theory allowing a creation of fermion pairs.

The Kähler part of the M^4 spinor connection could contribute to electroweak $U(1)$ gauge potential or define a new gauge force. It could also give rise to graviton-like states as monopole flux tubes containing fermion pairs with rotational angular momentum $L = 1$. There are however several objections against this idea.

H^k, generalizing the Higgs field, contains M^4 part M^k and CP_2 part S^k. S^k behaves like Higgs field and the M^4 part looks like a plausible candidate for graviton. H^k is concentrated at the singularities of space-time as a minimal surface and vanishes elsewhere and is identifiable as a generalized acceleration. The bosonic field equations imply that the vertex generalizes Newton's "F=ma" and gives rise to the TGD counterpart of Einstein's equations. The interpretation of graviton in terms of a generalized acceleration would conform with the Equivalence Principle.

The second question considered in this article is whether gravitons can be detected in the TGD Universe. It turns out dark protons at the monopole flux tube condensates give rise to a mild optimism in this respect.

1 Introduction

Consider first the general view of elementary particles and vertices in the TGD framework.

1. All elementary particles are identified as closed monopole flux tubes having parts at two Minkowskian space-time sheets connected by Euclidean wormhole contacts and carrying fermion lines at the light-like orbits of partonic 2-surfaces defining the interfaces between Minkowskian space-time sheets and an Euclidean wormhole contact. Fermionic lines correspond to the boundaries of string world sheets connecting different partonic orbits.

2. Only fermions are fundamental particles and bosons are bound states of fermions and antifermions. The basic objection is that fermion pair creation is impossible and fermion and antifermions are separately conserved. The solution of the problem is that the vertices for boson emission are analogous to their counterparts of quantum field theories in external classical fields and reduced to a creation of a fermion pair [L12].

The original TGD based proposal was that fermion spin gives the spin of graviton so that it would correspond to a closed monopole flux tube carrying a pair of fermion-antifermion pairs. One can also consider the proposal that graviton corresponds to a spin $S = 1$ boson associated with a rotating closed monopole flux tube and having rotational angular momentum $L = 1$ so that a $J = 2$ state would be the outcome.

There are several candidates for the spin 1 particle serving as a building block of graviton.

1. M^4 spinor connection has a vielbein part which can be however eliminated in linear M^4 coordinates. M^4 Kähler structure, or Hamilton-Jacobi structure [L13] as I call it, forced by the twistor lift of TGD [L3, L4], gives a $U(1)$ contribution to the M^4 spinor connection, which couples to fermion number. It could contribute to the electroweak $U(1)$ gauge potential or define an independent gauge field at QFT limit.

2. The trace of the second fundamental form as a generalization of the Higgs field has an M^4 part which has spin 1 and is the most plausible candidate for graviton.

The fundamental vertices involve the splitting of closed string-like flux tubes representing elementary particles.

1. The Equivalence Principle at quantum level would state that the minimal surface property fails only at the singularities and that vertices for fermion pair creation and gauge boson and graviton emission correspond to singularities. One could say that acceleration is associated only with the singularities and the motion at quantum level for a particle as 3-surfaces is a generalization of Brownian motion. Fermion pair creation as a turning of fermion backwards in time would correspond to this kind of singularity.

2. The proposal is that singularities are at most 2-D delta function type singularities assignable to the 3-D light-like partonic orbits at which the 4-metric is effectively 2-dimensional if holography= generalized holomorphy hypothesis holds true. This means that space-time surfaces are roots of two generalized holomorphic functions of H coordinates, one of which is light-like coordinate and the other three are complex coordinates. The singularities are analogous to poles so that the induced metric has also diagonal components in the generalized complex space-time coordinates consisting of light-like coordinate and complex coordinate identifiable as a subset of the generalized complex coordinates for H.

3. Singularities involve singular points at which a fermion pair is created (fermion turns backwards in time) and could be seen as intersections of representatives of 2-D homology equivalence classes identifiable as partonic 2-surfaces and involving points at which a closed monopole flux tube splits to two.

4. Exotic smooth structures [A2, A3, A1], which emerge first in dimension four and mean a grave difficulty for general relativity, can be seen as the standard smooth structure with defects and the conjecture is that the vertices as singularities correspond to these defects [L7]. The intersection form of the 4-manifold detects the exotic smooth structures and has a central role in 4-topology. The vertices would correspond to the defects of ordinary 4-D smooth structure making possible a theory allowing a creation of fermion pairs and I ended up with a proposal for the construction of gauge boson emission vertices [L12]. Space-time dimension would be the only possible space-time dimension.

There would be two kinds of vertices assignable to the singularities.

1. A term proportional to the trace H^k of the second fundamental form has delta function behavior the singularities. Its CP_2 part S^k behaves like Higgs whereas the M^4 part M^k is analogous to spin 1 particle. Bosonic field equations express H^k having interpretation as acceleration in terms of various generalized forces associated with gauge interactions. This equation generalizes Newton's "F=ma" and defines a counterpart for Einstein's equations. Graviton as a generalized acceleration at singularities codes for a very strong form of the Equivalence Principle and also the vision that the theory is free outside the singularities. Indeed, minimal surface equations correspond to massless field equations and minimal surface to a generalization of light-like geodesics.

2. Also the induced spinor connection develops a delta function behavior at the singularity since the diagonal components of the induced metric in generalized complex coordinates are non-vanishing and develop delta function singularity.

The second question considered in this article is whether gravitons can be detected in the TGD Universe. It turns out dark protons at the monopole flux tube condensates give rise to a mild optimism in this respect.

2 Brief summary of some basic ideas of TGD

In this section some background of classical and quantum TGD is described and also the question what gravitons are is considered.

2.1 Recent view of classical TGD

Before continuing, it is good to summarize the basic view about classical TGD as it is now.

1. In the TGD framework, one can understand classical gravitation in terms of the induced geometry of the space-time surface $X^4 \subset H = M^4 \times CP_2$. The gravitational constant G should be determined by the square of the CP_2 radius $R \sim 10^4 l_P$, $l_P^2 = G\hbar$. If one accepts the hierarchy of Planck constants $h_{eff} = nh_0$ predicted by the number theoretical vision about TGD [L17], the effective radius of CP_2, which is about 10^4 Planck lengths, would be apart from a numerical scale factor near unity $R_{eff}^2 = (\hbar_{eff}/h_0)l_P^2$.

2. Embeddability to H and the holography forced by the general coordinate invariance, implying that space-time surfaces are analogs of Bohr orbits, poses extremely strong constraints on the space-time surfaces so that they cannot directly correspond to the Einsteinian space-time.

   The QFT limit of TGD is obtained by replacing the many-sheeted space-time surface with a single metrically deformed region of M^4 such that gauge potentials are sums of the induced gauge potentials for the space-time sheets. Same applies to the deviations of the induced metric from the M^4 metric. This picture applies in long length scales in which Einsteinian view of space-time works [L11, L12, L15].

3. Holography is realized as a generalized holomorphy [L17, L19]. The twistor lift of TGD [L3, L4, L13] leads to the proposal that M^4 has a generalized Kähler structure, which combines ordinary complex structure and hypercomplex structure to its 4-D generalization so that H also allow generalized complex structure with 1 hypercomplex (light-like) coordinate and 1 complex coordinate for M^4 and two complex coordinates for CP_2. I have christened this generalization of the complex structure as Hamilton-Jacobi structure [L13]. A good guess is that there is a moduli space of Hamilton-Jacobi structures and in the first guess locally equal to a Cartesian product of the moduli space of ordinary complex structures and its hyper-complex analog.

   The generalized complex structure corresponds to the slicings of M^4 and X^4 by complex partonic 2-surfaces and hypercomplex string world sheets which are transversal or possibly even orthogonal locally.

2.1.1 About the bosonic action

The action associated with the singularities involves singular terms coming from the action defining the space-time surfaces as 4-D Bohr orbits. One could also assign a lower-dimensional action with the singularities as independent action. For instance, Chern-Simons-Kähler (C-S-K) action can emerge from the Kähler or appear as an independent action. Fermion lines can involve a 1-D Dirac action coupled to induced gauge potentials and string world sheets can involve separate action. It is not clear whether these all actions could emerge from a 4-D action at the singularities.

One can consider several options assuming that the exotic smooth structure can be regarded as an ordinary smooth structure with defects identifiable as singularities. One can consider several options depending on what the role of the instanton terms associated with the Kähler form is. Instanton terms gives $C-S-K$ boundary terms with an apparent violation of $U(1)$ gauge invariance (which actually corresponds to symplectic invariance as an approximate symmetry broken by gravitation). Therefore Kähler function can contain this term as a real contribution. Here only the most plausible option is considered.

Consider the bosonic action in more detail.

1. The exponential of the modified Dirac action is assumed to be imaginary and analogous to the phase defined by the action in QFTs. Unlike in gauge theories, the vacuum functional is a real exponent of the Kähler function identified as a purely classical bosonic action. This is possible since bosons are not primary quantum fields but expressible as bound states of fundamental fermions and antifermions.

2. Instanton term gives rise to a $C-S-K$ term associated with 3-D singularities. For the representations of Kac-Moody algebras, the coefficient of Chern-Simons action is $k/4\pi$ and allows an interpretation as quantization of α_K as $\alpha_K = 1/k$.

3. Volume action giving rise to cosmological constant is in a very special role in that it represents both the classical dynamics of particles as 3-D surfaces as analogs of geodesic lines, the classical geometrized dynamics of massless fields, and generalizes the Laplace equations of complex analysis.

4. The real exponential $exp(K)$ of the real Kähler function defined by $S_K + S_V$ would be visible in the WCW vacuum functional and bring in an additional dependence on the α_K and cosmological constant Λ, whose number theoretic evolution would fix the evolution of the other coupling strengths. Note that the induced spinor connection corresponds in gauge theories to gauge potentials for which the gauge coupling is absorbed as a multiplicative factor.

2.1.2 Chern-Simons-Kähler action

The status of Chern-Simons-Kähler (C-S-K) action is not clear. It would result from the instanton term for the Kähler action. It if taken to be real as also the remaining part of the action it can contribute to Kähler function and the exponent of vacuum functional. If it is taken imaginary it does not contribute to the Kähler function its exponential and defines a complex phase of the vacuum functional.

1. At the first look, in the TGD framework Chern-Simons-Kähler action is the only possible action for 3-D light-like surfaces representing light-like orbits of partonic 2-surfaces appearing as interfaces of Euclidean and Minkowskian space-time regions or as boundaries of space-time surfaces. This first guess turns out to be too naive: the fact that minimal surface property fails at the singularities changes the situation.

   Irrespective of whether the instanton term is real or not, one can assume that the field equations expressing conservation of various charges are satisfied. If it is real, the boundary conditions allow the flow of interior Noether charges and complex charges to the boundary or interfaces. Otherwise there is no flow to the boundary.

   At these 3-surfaces 4-D induced metric degenerates to an effectively 3-dimensional metric. The twistor lift of TGD suggests that $C-S-K$ action involves contributions from both CP_2 and M^4 allowing a generalized Kähler structure [L13]. The M^4 contribution allows the assignment of non-vanishing Poincare charges to $C-S-K$ action.

2. By its topological nature, $C-S-K$ action does not involve the induced metric at all. The interior part of action makes itself visible in boundary conditions stating that quantum numbers do not flow out through boundaries and are conserved at light-like interfaces between regions of space-time surface with Euclidean and Minkowski signature [L8].

3. Modified Dirac action for the *entire action* would contain the fermionic counterpart of $C-S-K$ action, determined uniquely by consistency arguments predicting a far reaching generalization of superconformal symmetry and related Kac-Moody symmetry is used to describe all interactions at elementary particle level [K3] [L19]. If will be however found that the vertices at the singularities are defined by the volume part of the action but expressible in terms of the remaining parts of the action.

   Modified $C-S-K$ Dirac action involves couplings to the induced electroweak gauge potentials. The covariant derivatives contain the CP_2 spinor connection determined by the CP_2 metric. CP_2 scale appears as a counterpart of Planck length and could be equal to Planck length for the minimal value of effective Planck constant $h_{eff} = nh_0$. Also the M^4 part associated with the generalized Kähler structure is present if one accepts a twistor lift of TGD.

4. The light-like surface can also contain many-fermion states and I proposed for a long time ago that at the fundamental level FQHE (Fractional Quantum Hall Effect) type systems could correspond to the nanoscopic analogs of partonic 2-surfaces carrying a very large number of electrons [K2]. One possibility is that the partonic surface contains a very large number of handles behaving like particles but this is not the only possibility.

The couplings of this kind of systems to gauge bosons and gravitons would be described as in the case of elementary particles. One would have a sum over scattering amplitudes and quantum coherence would apply. 2-dimensionality would be essential and would raise FQHE type systems in a special role.

2.1.3 About the modified Dirac action

If the bosonic field equations are true at singularity and modified Dirac action is determined by the entire bosonic action, there would be no singularities and vertices since modified Dirac equation would be true everywhere. The solution of the problem is that the modified Dirac action involves only the volume term so that the modified gamma matrices are just induced gamma matrices, which indeed looks very natural.

This motivates the consideration of the proposal that only the induced gamma matrices $\Gamma^\alpha = g^{\alpha\beta}h^k_\beta\gamma_k$ (no contribution from L_K) corresponding to S_V appear in L_D and the bosonic action $S_B = S_K + S_V + S_I$, where the instant on term S_I, reducing to a Chern-Simons-Kähler term, is real, is defined by the twistor lift of TGD.

1. If the field equations are satisfied also at the singularities, the contributions from S_K+S_I and S_V cancel each other at the singularity in accordance with the idea that an exotic smooth structure is in question. Both S_K and S_I contributions would have an imaginary phase.

2. L_V, which involves cosmological constant Λ, would disappear from the scattering amplitudes by the field equations for L_B although it is implicitly present. The number theoretic evolution of the $S_K + S_I$ would make itself visible in the scattering vertices. Outside the singularities both terms vanish separately but at singularities this is not the case. Only lower-D singularities contribute to the scattering amplitudes.

   The number theoretical parameters of the bosonic action determined by the hierarchy of extensions of rationals would parametrize different exotic smooth structures and make themselves visible in the quantum dynamics in this way. S_I would contribute to classical charges and its M^4 part would contribute to the Poincare charges.

3. An objection against this proposal is that the divergence of the modified gamma matrices defined by the $S_K + S_I$ need not be well-defined. It should be proportional to a lower-dimensional delta function located at the singularity.

2.2 The failure of the minimal surface property at singularities

The failure of the minimal surface property at the singularities implied by generalized holomorphy implies an interesting and, as it seems, important delicacy. Since the contribution of the volume part of action gives a delta function like singularity also the contribution of other parts of the action give a delta function like singularity since otherwise the field equations would not be satisfied everywhere. This implies that the trace of the second fundamental form, call it H^k, at the singularity equals apart from the sign to the similar contribution of the parts of the action. This is nothing but the analog of Newton's equations with H^k representing acceleration for a particle as 3-surface.

What about modified Dirac action at singularity? If modified Dirac matrices are defined by the entire action, Dirac action vanishes and also the divergence of modified matrices vanishes everywhere. No vertices are obtained. However, if the modified gamma matrices are identified as induced gamma matrices, their divergence is proportional to H^k contracted with ordinary gamma matrices and is by field equations equal to the divergence associated with the remaining terms in the action. This vanishes everywhere except at the singularities. This is like Newton's equation. Could this give rise to fermionic vertices?

H^k has CP_2 part, which behaves group theoretically like a Higgs field. The M^4 part behaves like a spin 1 field. Gauge-gravitation duality of QFTs due to holography states that gauge theory at the boundary corresponds to gravitation in the interior. Does this mean that the spin one particles at the singularities acting as a source of H^k give rise to gravitation in the interior. Could gravitons correspond to monopole flux tubes, which carry spin 1 fermion-antifermion states at the wormhole throats? Could the rotation of the monopole flux tubes give rise to an additional spin so that graviton is obtained as $J = 2$ state.

2.2.1 About the singularities of the bosonic action

The Hamilton-Jacobi structure [L13] implies the existence of generalized complex coordinates $(u, v, w, \overline{w})$ of space-time as a subset of similar coordinates of H.

1. At the singularities generalizing the poles of analytic function the minimal surface property fails and the action is expected to give a singular delta function type contribution. This contribution should give rise to the scattering amplitudes. The failure of generalized holomorphy is analogous to that of 2-D holomorphy. The latter means that the pole of the action of Laplacian of the analytic function develops a delta function so that $\partial_z \partial_{\overline{z}} f$ is proportional to a delta function rather than vanishing. In a cut f in turn develops a discontinuity.

2. There are 3 kinds of pole-like singularities associated with light-like partonic orbits. The pole at $u = u_0$ gives a 3-D light-like partonic orbit. The pole at $z = z_0$ gives a string world sheet, whose boundary intersects the partonic orbit along the fermion line. The pole at $(u, v) = (u_0, v_0)$ gives a partonic 2-surface with a vertex interpretation. The pole with $(u, z) = (u_0, z_0)$ gives a fermionic line as the boundary of the string world s000heet. The pole $(u, v, z) = (u_0, v_0, z_0)$ gives a point-like singularity which could correspond to the turning of the light-like fermion line. At least turning back in the time direction is possible.

3. The cuts for partonic 2-surfaces would mean that the H complex coordinates as function of $(u, v, w, \overline{w})$ are discontinuous and this implies multiple covering property. These kinds of singularities could correspond to the non-minimal values of the effective Planck constant identified as a dimension of an extension of rationals. The roots of polynomials defining the space-time surfaces as holomorphic imbeddings indeed define multiple coverings. These cuts are possible both in CP_2 and M^4 degrees of freedom. In the CP_2 (M^4) case M^4 (CP_2) coordinates as functions of CP_2 (M^4) coordinates are many-valued.

4. In CP_2 case multiple coverings of M^4 analogous to those for Riemann surfaces would mean surfaces for which closed paths around the singularity turn the singularity several times. Anyons could correspond to this kind of singularities [K2]. In the M^4 case, the space-time surface could consist of a larger number or parallel monopole flux tubes behaving as a quantum coherent unit with a very large value of h_{eff}.

2.2.2 What singularities can correspond to vertices for fermion pair creation?

It is not clear whether all singularities have an interpretation in terms of exotic smooth structures. The physical criterion would be that the creation of a fermion pair takes place at the defect and that the minimal surface property fails. Fermions can correspond to induced spinor fields and fermion pairs could be created at surfaces of dimension $d < 4$.

1. For closed two-sheeted cosmic strings and monopole flux tubes, which split by reconnection, the interpretation makes sense and means a generalization of the basic vertex for closed strings. These objects can be 2-sheeted as elementary particles in which case the reconnection would occur in the direction of CP_2. If they are single sheeted, the reconnection would occur in the direction of M^4.

2. 3-D light-like light-partonic orbits appearing as interfaces between Euclidean and Minkowskian space-time regions and as boundaries of space-time surfaces are singularities [L8]. Boundary conditions state that the possible flows of conserved charges from the interior go to the partonic orbit so that the divergence of the Chern-Simons-Kähler canonical momentum current coming from instanton term equals to the sum of the normal components of the canonical currents associated with Kähler action and volume term.

    (a) Chern-Simons action at the light-like partonic orbit coming from the instanton term is well-defined and finite and field equations should not give rise to a singularity except possibly at partonic 2-surfaces, which have been identified as analogs of vertices at which the partonic 2-surface X^2 splits to two.

(b) At the light-like partonic orbit 4-metric has a vanishing determinant and is therefore effectively 2-D (the light-like components of $g_{uv} = g_{vu}$ of the 4-metric vanish). As a consequence, $\sqrt{g_4}$ vanishes like L^2 at the partonic orbit unless some coordinate gradients diverge.

The canonical momentum currents for the volume action are proportional to the contravariant induced metric appearing in the trace of the second fundamental form diverging like $1/L^2$ and to $\sqrt{g}$ so that they remain finite.

(c) Kähler action contains the contravariant metric twice and is proportional to $\sqrt{g_4}$. This can give rise to a divergence of type $1/L^2$ unless the boundary conditions make it finite. I have proposed that the electric-magnetic self-duality at the partonic orbit can transform the Kähler action to an instanton term giving Chern-Simons Kähler term. In this case, a separate instanton term would not be needed. In this case everything would be finite at the partonic orbit. Minimal surface property fails in a smooth manner.

The intuitive picture is that the contributions from the normal currents at the partonic orbit and the Chern-Simons term cancel each other and the partonic orbit cannot play a role of a vertex.

(d) The possible presence of $1/L^2$ divergence could however give rise to a 2-D defect and genuine vertex. If it is identified as a creation of a pair of partonic 2-surfaces, the interpretation in terms of a creation of a fermion pair is possible and could be assigned to the splitting of a monopole flux tube.

In accordance with the QFT picture, I have considered the possibility that the 2-D vertex could correspond to a branching of a partonic orbit. In the recent picture it would be accompanied by a creation of a fermion pair. The stringy view however suggests that pair creation occurs in the creation of partonic orbits in the splitting of monopole flux tubes. The stringy view is more attractive.

3. I have also proposed that 1-D singularities identifiable as boundaries of string world sheets and identifiable as fermion lines at the partonic orbits are important. The creation of a pair of fermion lines would give rise to the analogs of gauge theory vertices as 0-D singularities. It is however far from clear whether the stringy singularities are actually present and whether they could correspond to exotic smooth structures. One can imagine two options.

   (a) There are no string world sheets. Monopole flux tubes can be regarded as deformations of cosmic strings. Instead of strings several monopole flux tubes can emerge from a wormhole contact. For the minimal option, monopole flux tubes, CP_2 type extremals, and massless extremals as counterparts of radiation fields are the basic extremals and the splitting of monopole flux tubes gives rise to vertices as defects of the ordinary smooth structure.

   (b) String world sheets appear as singularities of the monopole flux tubes or even more general 4-surfaces and are analogous to wormhole contacts as blow-ups in which a point of X^4 explodes to CP_2 type extremal. I have indeed proposed that a blow-up at which the points of the string world sheet as surface $X^2 \subset X^4$ are replaced with a homologically non-trivial 2-surface $Y^2 \subset CP_2$ takes place. Y^2 could connect two parallel space-time sheets. Could these singularities correspond to defects of exotic smooth structures such that the ends of the string carry fermion number? The vertex for the creation of a pair of fermion and antifermion lines would correspond to a diffeo defect. Note that also these defects could reduce to a splitting of a monopole flux tube so that TGD would generalize the stringy picture.

2.3 How to handle the interfaces between Minkowskian and Euclidean regions of space-time?

The understanding of how vertices can emerge as singularities requires the treatment of the dynamics at the interfaces X^3 between Minkowskian and Euclidean regions X^3 of the space-time surface identified as light-like partonic orbits. This is a difficult technical problem. The vision is that by holomorphy as a realization of generalized holography, the 4-metric at X^3 degenerates to 2-D effective Euclidean metric apart from 2-D sub-singularities X^2 at which the generalized holomorphy fails.

One must treat both the bosonic and fermionic situations.

1. The densities of both volume action and Kähler action vanish at X^3 whereas C-S-K action density is non-vanishing. The canonical momentum densities appearing also in gamma matrices involving the singular contravariant metric have a has 2-D delta function singularity field equations are satisfied due to the generalized holomorphy except at X^2.

2. For the modified Dirac action, the modified gamma matrices have 2-D delta function singularity at X^3 but their divergences vanish by the holomorphy outside X^2f or each part of the action separately.

   The simplest way to make sense of the modified Dirac equation at X^3 is to assume that the covariant derivatives of the induced spinor field with respect to the light-like coordinates u and v vanish at X^3 except at X^2.

   At X^2 one would obtain effectively a 3-D delta function source analogous to a pole of an analytic function and giving rise to a vertex. The analogy with the 2-D electrostatics where point charges correspond to poles of analytic functions is obvious.

There are two options for the treatment of the interface dynamics.

1. The interface X^3 is regarded as an independent dynamic unit and carrier of charges. The earlier approaches rely on this assumption. By the light-likeness of X^3, C-S-K action is the only possible option and is indeed non-vanishing at X^3. The problem with $U(1)$ gauge invariance disappears if C-S-K action is identified as a total divergence emerging from the instanton term for Kähler action.

   One can assign to the instanton term a corresponding contribution to the modified Dirac action at X^3. It however seems that the instanton term associated with the 4-D modified Dirac action does not reduce to a total divergence allowing to localize it a X^3.

   In this approach, conservation laws require that the normal components of the canonical momentum currents from the Minkowskian and Euclidean sides add up to the divergence of the canonical momentum currents associated with the C-S-K action.

2. Since the interface is not a genuine boundary, one can argue that one should treat the situation as 4-dimensional. The interface would not be a carrier of charges. This approach is adopted in this article. In the bosonic degrees of freedom, the C-S-K term is present also for this option and could determine the bosonic dynamics of the boundary apart from a 2-D sub-singularities coming from the violation of the minimal surface property and of the generalized holomorphy. At vertices involving fermion pair creation this violation would occur.

In the 4-dimensional treatment there are no analogs of the boundary conditions at the interface.

1. It is essential that the 3-D light-like orbit X^3 is a 2-sided surface between Minkowskian and Euclidean domains. The variation of the C-S-K term emerging from a total divergence could determine the dynamics of the interface except possibly at the singularities X^3, where the interior contributions from the 2 sides give rise to a 2-D delta function term.

2. The contravariant metric diverges at X^3 since by holomorphy one has $g_{uv} = 0$ at X^3 outside X^2. For the same reason the divergences of canonical momentum currents vanish outside X^2.

   One can however consider stronger condition $J_{uv} = 0$ at X^3 outside X^2, which could guarantee that the contribution of the Kähler action remains finite. The contribution from Kähler action to field equations could be even reduced to the divergence of the instanton term at X^3 by what I have called electric-magnetic duality proposed years ago [K1]. At X^3, the dynamics would be effectively reduced to 2-D Euclidean degrees of freedom outside X^2. Everything would be finite as far as modified gamma matrices associated with the Kähler action outside X^2 are considered.

3. Since the metric at X^3 is effectively 2-D, the induced gamma matrices are proportional to 2-D delta function. If $J_{uv} = 0$ condition (, which is not necessary) is true, the contribution of the volume term to the modified gamma matrices dominates over the finite contribution of the Kähler action outside X^2.

   In order to obtain the counterpart of Einstein's equations the metric must be effectively 2-D also at X^2 so that $det(g_4) = 0$ is true although holomorphy fails. It seems that one must assume induced, rather than modified, gamma matrices (effectively reducing to the induced ones at X^3 outside X^2) since for the latter option the gravitational vertex would vanish by the field equations. The induced gamma matrices anticommute to the induced metric unlike the modified gamma matrices so that they are arguably a natural choice.

   What raises worries is that the vanishing of the covariant divergence of modified gammas, which guarantees hermiticity of the modified Dirac operator, is not true for the induced gamma matrices. The situation is therefore very delicate and I cannot claim that I understand it sufficiently. It seems that the edge of the partonic orbit due to the turning of the fermion line and involving hypercomplex conjugation is essential.

4. For the modified Dirac equation to make sense, the vanishing of the covariant derivatives with respect to light-like coordinates seems necessary. One would have $D_u\Psi = 0$ and $D_v\Psi = 0$ in X^3 except at the 2-D singularities X^2, where the induced metric would have non-vanishing diagonal components g_{uu} and g_{vv}. This would give rise to the gauge boson vertices involving emission of fermion-antifermion pairs.

5. By the generalized holomorphy, the second fundamental form H^k vanishes outside X^2. At X^2, H^k is proportional to a 2-D delta function and also the Kähler contribution can be of comparable size This should give the TGD counterpart of Einstein's equations and Newtonian equations of motion and to the graviton vertex.

   The orientations of the tangent spaces at the two sides are different. The induced metric at the Minkowskian side would become 4-D. At the Euclidean side it could be Euclidean and even metrically 2-D.

The overview about symmetry breaking through the generation of 2-D singularities is suggestive. Masslessness and holomorphy are violated via the generation of the 8-D analog of Higgs expectation at the vertices. The use of the induced gamma matrices violates supersymmetry guaranteed by the use of the modified gamma matrices but only at the vertices.

The basic objection is that the use of the induced gammas in the modified Dirac equation seems necessary although the non-vanishing of H^k seems to violate the hermiticity at the vertices. Can the turning around of the fermion line and the exotic smooth structure allow to get rid of this problem?

2.4 About the QFT limit of TGD

Just for fun, one can also look at the situation from the point of view of Einstein-Yang-Mills type theory, which should emerge as the QFT limit of TGD at which space-time surface can be assumed to have a 4-D M^4 projection so that the modelling of the many-sheeted space-time surfaces as slightly curved M^4 should make sense. The gauge potentials would correspond to the sums of the induced gauge potentials for various space-time sheets. Same would apply to the deviation of the metric from the M^4 metric.

One expects that in the case of effectively 2-D systems, the light-like partonic orbits cannot be completely eliminated even at this limit and FQHE systems could represent systems of this kind. In elementary particle length scales they could be replaced by point-like particles but in the case of multi-electrons states at nanoscopic parton surfaces this does not work.

What happens to the curvature scalar at the limit when the induced 4-metric becomes effectively 3-D?

1. The induced covariant 4-metric becomes degenerate at the partonic orbit and the contravariant metric has some divergent components. $\sqrt{-g_4}$ vanishes at this limit like $1/L$, $L \to 0$.

2. The curvature tensor $R^{\alpha}_{\beta\gamma\delta}$ has dimension zero and could remain finite. Ricci tensor $R^{\alpha\beta}$ and Einstein tensor $G^{\alpha\beta}$ could diverge like $1/L^4$. Curvature scalar could diverge like $1/L^2$. If Einstein's equations hold true, the energy momentum tensor is proportional to the Einstein tensor and could diverge like $1/L^4$. Multiplied with $\sqrt{-g}$ it would diverge like $1/L^3$. This suggests that the limit gives the analog of Chern-Simons-Kähler action or its QFT analog as a delta function like singularity. The modified Dirac action should also have a counterpart, which could be finite since it has vanishing dimension.

3 About the identification of gravitons in the TGD Universe

TGD leads to the identification of all elementary particles in terms of closed monopole flux tubes associated with pairs of two parallel space-time sheets. The Euclidean wormhole contacts at the "ends" of the flux tube correspond to light-like orbits of partonic 2-surfaces and would carry point-like fermions serving as building bricks of all elementary particles. In the case of particles with spin smaller than 2, either wormhole contact can carry the spin and electroweak quantum numbers and second wormhole contact possibly carries a neutrino pair neutralizing the weak isospin so that one has a weak analog of confinement. There are also closed half-monopole flux tubes having boundaries [L20] and both these and monopole flux tubes with closed cross section could be important in superconductivity [L2].

3.1 Graviton as a fermion antifermion pair?

The proposal has been that graviton spin reduces to fermionic spin so that both wormhole contacts should carry a fermion pair with spin 1. These kinds of states might well exist but in this picture it is difficult to understand how the expected value of the gravitational constant is coded to the structure of the state formed by the two spin 1 fermion pairs. The second problem is that it is not obvious how the Equivalence Principle could be realized at the level of gravitons in this picture.

The assumption of two fermion pairs is not necessary if the monopole flux tube rotates. One could have fermion and antifermion at the wormhole contacts defining the ends of this string-like object. Angular momentum $L = 0$ would give bosons with spin 0 and 1. $L = 1$ would allow bosons spin 2, 1, and 0 and $L = 2$ would allow bosons with spin $3, 2, 1$. At least formally, this picture would conform with the holography and the idea that gauge theory at boundaries corresponds to gravitation in the interior.

The M^4 part of the trace of the second fundamental form and M^4 Kähler gauge potential are the natural candidates for defining graviton-like state in this way.

1. The graviton candidate in question should couple in the same way to all fermions and therefore to fermion number. The objection is that, unlike gravitons, gauge bosons couple with opposite signs to fermions and antifermions. Could the rotation of the monopole flux tube cure the problem?

2. A further condition is that graviton couplings are proportional to four-momenta. If graviton corresponds to the ordinary spin 1 gauge boson in the proposed way, the rotational motion should give rise to this dependence.

3. One should also understand the value of gravitational constant. The M^4 part of the second fundamental form is the most natural candidate and has a delta function singularity at the singularities of space-time surfaces as minimal surfaces. At these surfaces also the generalized holomorphy fails.

3.2 About the physical interpretation of the trace of the second fundamental form

The trace of the second fundamental form, which can be regarded as an analog and a generalization of the Higgs field of the standard model, has a delta function like singularity at the singularities of the space-time surface at which the minimal surface property fails.

1. The divergence of $g^{\mu\nu}\partial_\nu$ vertex as the trace of the second fundamental form $H^k \equiv D_\alpha h^k\beta$ defined by the covariant derivatives of coordinate gradients, appears in the vertex. The second fundamental form H^k is orthogonal to the space-time surface and can be written as

$$H^k \equiv g^{\mu\nu} D_\nu \partial_\mu h^k = P^k_l H^l_0 \ , \quad P^k_l = h^k_l - g^{\mu\nu} h^k_\mu h_{lr} h^r_\nu \ ,$$
$$H^k_0 = g^{\alpha\beta}(\partial_\alpha + B^k_\alpha)(g^{\alpha\beta} h^k_\beta \ , \quad B^k_\alpha = B^k_{lm} h^m_\alpha \ . \tag{3.1}$$

P^k_l projects H^k_0 to the normal space of the space-time surface. H^k_0 is covariant derivative of h^k_α and $B^k_\alpha = B^k_{lm} h^m_\alpha$ is the projection of the Riemann connection of H to the space-time surface.

2. In linear Minkowski coordinates for M^4, the induced M^4 vielbein connection vanishes: $B^k_\alpha = 0$ and the direct M^4 contribution to the M^4 part of the trace of the second fundamental form vanishes $M^k_0 = 0$. However, the presence of the CP_2 contribution coming from the orthonormal projection implies that M^k is non-vanishing and proportional to the radius squared of CP_2. This is expected to give rise to a vertex that is proportional to H^k, whose CP_2 part, call it S^k, is analogous to the Higgs field of the standard model.

   This field is vanishing in the interior by the minimal surface property in analogy with the generalized Equivalence Principle. M^4 part, call it M^k, with spin 1 property would also vanish in the interior and could contribute to the graviton vertex if the corresponding particle corresponds to a rotating monopole flux tube with $L = 1$. The quantum variant of the Equivalence Principle would be extremely strong.

   H^k is a generalization of acceleration from 1-D case to 4-D situation so that the interaction vertices are lower-dimensional regions of the space-time surface which experience acceleration. The regions outside the singularities represent massless fields geometrically. At the singularities the Higgs-like field is non-vanishing so that there is mass present. The analog of Higgs vacuum expectation is non-vanishing only at the defects.

3.2.1 Could the trace of the second fundamental form be enough?

Gauge-gravitation duality forces to ask whether generalized Higgs H^k as the trace of the second fundamental define a universal vertex? This would be extremely nice but does not look plausible. The proposal that the electroweak gauge potentials and M^4 Kähler gauge potential defined gauge theory vertices [L12] looks more plausible.

1. The trace of the second fundamental form H^k is expressible in terms of the divergence of the remaining contribution to the action is present giving the TGD counterpart of sum over gauge currents. Gravitational-gauge theory duality would suggest that this is enough.

   The analogous contribution in gauge theory is proportional to the divergence of canonical momentum current and therefore gauge current and would indeed make sense. Now gauge potentials as dynamical variables are replaced with H coordinates and the contribution from Kähler action involves a term proportional Kähler gauge current j^α contracted with a quantity proportional to $J_\alpha^{\ \beta}(h^k_\beta - J^k_{\ l} h^l_\beta)$. This term can be non-vanishing if j^α is non-vanishing. For massless extremels j^α is light-like. Also the contribution from energy momentum tensor expressible as $T^{\alpha\beta} H^k_{\alpha\beta}$ is present in the vertex.

2. It is however no need to express the trace of the second fundamental form in this way. Furthermore, the fermionic couplings at singularity coming from Kähler action involve only Kähler form. At the vertices there would be no couplings to electroweak charges and therefore no parity violation. The electroweak couplings are present in the modified Dirac equation, which in turn induced from the massless Dirac equation at the level of H involving electroweak gauge couplings gauge potentials. Second quantized fermions defining fermion propagators obey electroweak dynamics. Is the appearance of electroweak interactions at the level of H enough? If this picture is correct, generalized Higgs could be indeed called the God particle.

3. One could replace Kähler action in CP_2 degrees of freedom with electroweak action based on spinor connection to get the couplings to electroweak gauge currents without changing the situation. Now the counterparts of the bosonic electro-weak currents would appear in the vertices as they appear in YM theory. But again the couplings could be expresses solely in terms of the generalized acceleration and there would be independence on variational principle in this sense. The vacuum functional as exponent of Kähler function of WCW could however depend on the action.

4. The original intuitive expectation [L12] was that the vertex should contain a singular contribution proportional to the induced spinor connection, present also if the volume term defines the modified Dirac action, and giving rise to the analogs of gauge theory vertices is expected on the basis of physical intuition [L12]. In CP_2 degrees this would give rise to electroweak vertices.

   In M^4 degrees of freedom, the vielbein part of the spinor connection can be eliminated in linear M^4 coordinates but the M^4 Kähler form gives rise to a $U(1)$ gauge potential. Does this contribution give an additional contribution to the electroweak $U(1)$ gauge potential, a new $U(1)$ force, or a graviton-like state in the same way as M^k. The first option looks at first implausible since it is not clear how the R^2 proportionality of the coupling could emerge. Note however that the vertices correspond to partonic 2-surfaces. I have considered the possibility that the M^4 Kähler form could give rise to a CP violation.

   It however seems that it is not possible to obtain this coupling at the singularity.

A little technical remark is in order. The modified Dirac action must be dimensionless so that the scaling dimension of the induced spinors should be $d = -3/2$ and therefore the same as the scaling dimension of M^4 spinors. This looks natural since CP_2 is compact. The volume term included in the definition of the induced gamma matrices must be normalized by $1/L_p^4$. L_p is a p-adic length scale and is roughly of order of a biological scale $L_p \sim 10^{-4}$ meters if the scale dependent cosmological constant Λ corresponds to the inverse squared for the horizon radius. One has $1/L_p^4 = 3\Lambda/8\pi G$. This guarantees the expected rather slow coupling constant evolution induced by that of α_K diverging in short scales.

3.2.2 Could the trace of the second fundamental form unify Higgs and graviton?

The CP_2 part of the second fundamental form has quantum numbers of the Higgs field of the standard model. The M^4 part in turn contains a term proportional to the square of CP_2 radius identifiable as Planck length for the minimal value of $h_{eff} = nh_0$. The effective value CP_2 radius scales $\sqrt{h_{eff}/h_0}$.

One can start from the following tentative physics inspired picture.

1. The vertex which corresponds to a $d < 3$-dimensional surfaces as a defect of the standard smooth structure and as singularity of the space-time as minimal surface. At it the trace of the second fundamental form, vanishing elsewhere, has a delta function like singularity. The M^4 part of the trace is spin 1 object and a natural guess is that it corresponds to gravitaton.

2. In linear M^4 coordinates, only the contribution from CP_2 degrees of freedom to it is non-vanishing and proportional to CP_2 radius squared having interpretation in terms of Newton's constant. Could a rotating flux tube having angular momentum $L = 1$ and carrying M^4 Higgs with spin 1 at the wormhole throat give rise to gravitons as a fermion-antifermion pair and having $J = 2$?

3. The field equations for S_B are satisfied at the vertex so that the trace of the second fundamental form can be expressed in terms of divergences of the canonical momentum currents associated with the Kähler and instanton parts of S_B. This is essentially generalization of Newton's equation and the TGD counterpart of geodesic equations of motions in general relavity for particles as 3-surfaces and experience accelerations concentrated at the lower-dimensional vertices.

Note that also the covariant divergences for the canonical momentum currents of a more general action than volume term, having group theoretical properties of Higgs field in CP_2 degrees of freedom, could appear in the vertex and would be slashed between induced spinor and its conjugate.

3.2.3 Could M^4 Kähler form give rise to graviton like state?

Also a second candidate for graviton-lilke state must be considered. The vector bosons defined by the M^4 part of H^k and M^4 Kähler form could give rise to graviton-like state if the monopole flux tube corresponds to $L = 1$ state.

1. The induced M^4 spinor connection contains a $U(1)$ (Kähler) part and vielbein part. The Kähler part cannot be eliminated by a general coordinate transformation or gauge symmetry so that it defines a candidate for graviton. Note that instead of a genuine gauge symmetry one has an M^4 analog of symplectic symmetry [L13].

   Should the M^4 Kähler gauge potential be counted as a contribution to the $U(1)$ gauge potential of electroweak interactions? Or could it give rise to an independent degree of freedom at the QFT limit and give rise to graviton?

2. Can the induced M^4 Kähler gauge potential produce a realistic quantum theory of gravitation? The strongest objection is that in the QFT framework spin 1 fields give repulsion/attraction between changes of same/different sign. Whether the $L = 1$ rotational state could change the situation is not clear to me. Gravitational-gauge theory duality suggests this but one must be cautious: spin 2 particle coupling with opposite signs to matter and antimatter could be in question. On the other hand, the sign of the coupling of the trace of the second fundamental form to fermions and antifermions is the same that this looks a more promising option.

   In the QFT picture, M^4 Kähler gauge potential would correspond to a spin 1 particle. However, gravitons could correspond to closed monopole flux tubes with $L = 1$ angular momentum associated with the rotation of the flux tube and one would obtain a connection with the string picture. This might relate to the ability to approximate classical gravitation with a Maxwellian gauge theory using Newtonian gravitational potential as a counterpart of electric potential.

3. Can one understand the smallness of the gravitational constant for the Kähler option? The M^4 Kähler form gives rise to a spin one particle and that the rotation of the flux tube could give rise to a massless graviton-like state. The vertex term $\Gamma^\mu A_\mu$, where A_μ denotes M^4 spinor connection, contains a contribution from the CP_2 gamma matrices proportional to CP_2 radius squared and would naturally correspond to graviton coupling. There is however also a large contribution to the induced metric from the matrices of M^4. This contribution would come from 2-D partonic surfaces as singularities of partonic orbits and the M^4 contribution the metric should be of the same order of magnitude as CP_2 metric. This conforms with the intuitive picture about partonic 2-surfaces.

3.3 Exotic differential structures in 4 dimensions, particle vertices, and the new view of gravitons

What remains to be understood are the counterparts of the basic vertices of the gauge theory and quantum gravity.

3.3.1 How to avoid separate conservation of fermion and antifermion numbers?

One can start from a long standing problem of TGD. The idea that all bosons are bound states of fundamental fermions is extremely nice but leads to a problem: it seems that the creation from fermion pairs from vacuum is not possible. The numbers of fermions and antifermions appearing as free particles and building bricks of bosons are separately conserved. The experimental fact is that fermion and antifermion numbers cannot be separately conserved.

1. The conservation law for the total fermion number can be expressed as a vanishing of the divergence of the fermion current $J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi\sqrt{g}$, where Γ^α are modified gamma matrices defined in terms of the action.

2. The vertex serves as a source for the modified Dirac equation since this equation cannot be satisfied at the vertex. This source must be also proportional to a delta function at singularity.

3. Intuitively, the pair creation vertex corresponds to a situation in which the fermion line turns back in time directions. This direction could also be some other direction, and in standard perturbation theory it could be even space-like, and should be normal to the singularity as a surface of the space-time surface. This raises the question whether one should identify the spinor fields emerging from the vertex as Ψ and its T- (CP-) conjugate proportional to $\overline{\Psi}$. The ordinary perturbation suggests that nothing special needs to be done.

3.3.2 Do exotic smooth structures make fermion pair creation possible?

The solution came from the discovery that 4-dimensional space-times are completely unique in the sense that they allow an infinite number of exotic smooth structures [L7]. Apart from a subset of measure zero they are reduced to ordinary differentiable structures. These subsets are physically analogous to defects and the simplest defects are point defects but one can also imagine 1-, 2-, and even 3-D defects. This finding means a serious difficulty for general relativity. Should some kind of cosmic censorship hypothesis deny their existence?

In the TGD framework, an attractive identification of the defects would be as singularities at which the minimal surface property for space-time surfaces as generalized complex surfaces fails. These singularities are analogs of poles and cuts in the complex analysis. In fact, hypercomplex poles are 1-D geodesic lines in M^2 and would correspond to light-like curves in the general case. Therefore 3-D light-like partonic orbits would be analogous to poles. String world sheet could serve as a counterpart for a hypercomplex cut.

The identification

defects of the ordinary smooth structure ↔ singularities at which the minimal surface property fails ↔ poles and other singularities where generalized holomorphy fails

looks highly attractive. The 3-D light-like orbits of partonic orbits, string world sheets, strings, and points at which light-like orbits of point-like fermions split, could correspond to these singularities identifiable as generalized vertices. It is not clear whether 3-D defects can be space-like 3-surfaces.

These structures could be essential for the definition of creation and annihilation vertices for fermion-antifermion pairs. The intuitive picture is that a fermion turns backwards in time in this kind of vertex.

1. In QFTs a standard approximation is to replace the gauge boson of the vertex with a classical gauge potential. In TGD there are no bosons as fundamental particles and this replacement is necessary. This would correspond to a turning of fermion lines at the orbit of a partonic 2-surface backwards in time which is somehow special. Could this point correspond to a defect of the ordinary differentiable structure which is actually exotic smooth structure?

2. There is also another problem. Modified Dirac action should give rise to all fundamental vertices. At the fermion line the modified Dirac equation is satisfied but it puts modified Dirac action to zero so that the action would be trivial in the gauge theory sense. At the singularity the modified Dirac equation could however fail and one would obtain a delta function like singularity giving the standard classical vertex for the creation of a fermion pair or a particle with spin smaller than 2 as a bound fermion pair. This picture generalizes also to higher-dimensional defects. Interesting quantum physics would be possible only in space-time dimension four!

3. Gauge-theory-gravitation duality suggests that gravitons correspond to fermion pairs annd to M^4 Kähler gauge potential assigned to rotating monopole flux tubes. Second option is that gravitons correspond to the M^4 part of the second fundamental form. These two views are equivalent since field equations are satisfied at the singularities although the minimal surface property fails.

   One can ask whether gravitons analogs of gauge bosons with gauge group $SO(1,3)$ or its compact subgroup as required by unitarity unless one allows infinite-dimensional representations of $SO(1,3)$, which in fact are naturally associated with the causal diamonds (CDs), which are basic objects in zero energy ontology (ZEO) [L14]: the Poincare invariance which is problematic at the level of CD would be realized in the moduli space of CDs. This option does not look plausible however.

The unitary representations of $SO(1,3)$ would however appear naturally in the Dirac equation in CD= $cd \times CP_2$. The mass spectrum (Equivalence Principle) would be the same as for Dirac equation in H since the only difference is that the separation of variables takes place only in different coordinates. Indeed, future light-cone can be regarded as a slicing by the hyperboloids H^3 parameterized by the light-cone proper time. Instead of momenta the SO(1,3) quantum numbers would appear in the graviton vertex.

4. If gravitons are pairs of fermion pairs, the vertex involves two separate vertices in an essential way. This is possible but does not look elegant since two separate gauge boson vertices would be needed. This would conform with the idea that gravitation is in some sense square of a gauge theory but does not look an attractive idea.

In this framework one could understand basic vertices as splitting of two-sheeted closed monopole flux tubes with Euclidean wormhole contacts at ends [L12]. The splitting of the flux tube as a generalization of reconnection for closed strings would produce two closed flux tubes. The simplest reconnection would involve creation of a fermion-antifermion pair such that the fermion and antifermion pair go to separate wormhole contacts. The defects of the smooth structure would correspond to situations in which the topology of 3-surface is between two topologies. The pinching of torus to produce two spheres represent the basic example of this.

In the case of a fermion with a neutrino-antineutrino (left- and right-handed neutrinos) pair at the second wormhole contact neutralizing the weak isospin of the fermion as a geometric object, the reconnection would produce a pair of monopole flux tubes. The first one would represent a fermion. The second one would represent a boson with fermion and antifermion at opposite wormhole contacts. If the string does not rotate, the boson has spin 1 or 0 corresponding to a gauge boson and Higgs type scalar or pseudoscalar. If the string rotates one obtains a boson with spin 2 or 1 for the simplest option if the M^4 spinor connection contributes.

One can of course worry about the triviality of the vielbein part of M^4 spinor connection. Maybe it gives only rise to a topological gravitation whereas the Kähler part would give rie to graviton. The failure of the standard smooth structure at the defect could however imply that the elimination of the vielbein spinor connection by a general coordinate transformation fails just at the defect which is 2-D and has complex CP_2 coordinates as more natural coordianates.

3.4 How could modified Dirac action determine the scattering amplitudes?

Holography=generalized holomorphy property means that minimal surface field equations are true outside singularities for any general coordinate invariant action constructible in terms of the induced geometry. However, the twistor lift of TGD suggests that 6-D Kähler action is the fundamental action. It reduces to 4-D Kähler action plus volume term in the dimensional reduction guaranteeing that the 6-surface can be regarded as a generalization of twistor space having space-time surface as a base-space and 2-sphere.

3.4.1 Are modified gamma matrices defined by the entire bosonic action or induced gamma matrices?

One can express the induced spinor field obtained as a restriction of the massless second quantized H spinor field to the space-time surface and it satisfies modified Dirac equation [L19].

Modified Dirac action L_D is defined for the induced spinor fields. One must distinguish between two cases. The modified Dirac action involves the modified gamma matrices defined in terms of the entire bosonic action whereas its variant involves induced gamma matrices determined by volume action alone (cosmological constant). The original proposal was that the first option is correct but it seems that the simpler option based on induced gamma matrices is what makes possible the construction of the scattering amplitudes.

1. The modified Dirac action fixed by the condition of hermiticity stating that the canonical momentum currents appearing in it have a vanishing divergence. If the modified gamma matrices Γ^α are defined by a bosonc action S_B defining the space-time surface itself, they are indeed divergenceless by field equations. This implies a generalization of conformal symmetry to the 4-D situation [L13] and the modes of the modified Dirac equation define super-symplectic and generalized conformal charges defining the gamma matrices of WCW [L19]. The problem is that the modified Dirac action vanishes identically by a modified Dirac equation and one cannot construct non-trivial scattering amplitudes since ordinary perturbation theory is not possible.

2. Generalized holomorphy implies that minimal surface equations hold true outside singularities. Modified gamma matrices could be therefore replaced by the *induced* gamma matrices and modified Dirac action would vanish everywhere except at the singularities where the action density has delta function like singularity.

   This conforms with the original, hard-to-justify, intuition that only the light-like partonic orbits and possibly other singularities contribute to the interaction vertices so that effectively fermions reside only at the singularities.

   Therefore there are two options: the modified gamma matrices are defined by the sum of L_B or by L_V defining the induced gamma matrices. The latter option looks more plausible since it gives the analog of Newton's equations at the vertices identified as singularities.

3. An attractive guiding physical idea is that the singularities are not actually singularities if exotic smooth structure is introduced. Field equations hold true but with S_B. The singularities would cancel for the exotic smooth structure. One would avoid problems with the conservation laws by using exotic smooth structure. What the precise meaning of this idea is, remains unclear and requires precise formulatikon of the exotic smooth structure.

4. At the short distance limit for which α_K is expected to diverge as a $U(1)$ coupling, the action reduces to S_V and the defects would be absent. Only closed cosmic strings and monopole flux tubes would be present but wormhole contacts and string world sheets identifiable as defects are absent: this would be the situation in the primordial cosmology [L18]. Only dark energy as classical energy of the cosmic strings and monopole flux tubes would be present and there would be no elementary particles and elementary particle scattering at this limit.

In [L12] the construction of vertices was discussed and the conclusion was that the vertices are generalizations of ordinary vertices in which induced gauge potentials replaced standard model electroweak gauge potentials. The key idea is that one can overcome the problem due to the fact that gauge fields are not primary fields in TGD is that classical gauge potentials give rise to a pair creation. The discussion was based on physical intuition and far from rigorous.

3.4.2 How to obtain vertices for gauge bosons?

What about the gauge boson vertices?

1. In the earlier article [L12] I ended up with the vision that the induced spinor connection of CP_2 must give the TGD counterparts of electroweak vertices but could not represent a precise argument for why this should be the case. The induced M^4 vielbein connection is trivial but M^4 Kähler gauge potential gives rise to $U(1)$ gauge potential. This could contribute and additional term to $U(1)$ part of electroweak gauge fields or define and new $U(1)$ field. One would have vertices and therefore concrete representations for electroweak gauge bosons, Higgs and graviton plus possibly for this new spin 1 boson.

2. Since the gauge potential terms *seem* (!) to remain finite at the singularity, one can argue that only the trace of second fundamental form appears in the vertex and gives rise to the sum of the terms associated with the remaining parts of S_B and can be interpreted as source currents as divergences of the bosonic canonical momentum currents slashed between fermion field and its conjugate much like in gauge theories. Classical gauge potentials would disappear altogether, which would conform with gauge invariance.

Could the interpretation be in terms of gravitation-gauge theory duality? Different interactions would make their existence manifest only in the structure of incoming and outgoing states. The couplings to the spinor connections of CP_2 and M^4 are present in the massless Dirac equation at the level of H determining the fermionic propagators. Is this enough to give scattering amplitudes with correct physical properties?

Could the entire electroweak physics be induced from that for free massless H spinor fields? To me this looks unrealistic option. Somehow one should obtain the the couplings to the induced gauge potentials.

To understand how one could obtain gauge boson vertices, one must study in detail what happens at the singularities.

1. The modified Dirac action contains $\overline{\Psi}\Gamma^\mu(\partial_\mu + A_\mu)\Psi\sqrt{g}$ term and its conjugate. Here the induced gamma matrix is proportional to $\Gamma^\mu = g^{\mu\nu}\partial_\nu h^k\Gamma_k$.

2. In order to obtain non-trivial gauge boson vertices, $\Gamma^\mu A_\mu$ should have a delta function singularity at the vertex which corresponds to a singular 2-surface at at the 3-D light-like orbit of partonic 2-surface forming an interface between Euclidean and Minkowskian space-time regions.

   At this 2-D singularity the trace of the second fundamental form develops a delta function singularity. Elsewhere it vanishes identically since it does not have common non-vanishing tensor components with the induced metric. The singularity must correspond to the breaking of the generalized analyticity. Embedding space coordinates are not anymore functions of generalized complex coordinates of say (u, w) of space-time surface but functions of also (v, w) at the singularity. Singularity would be like a pole of an analytic complex function.

3. The covariant 4-metric (not only the 3-metric) is degenerate and effectively 2-D at the partonic orbit. In light-like coordinates (u, v) for the Minkowskian space-time region the induced metric of form $ds^2 = 2g_{uv}dudv + 2g_{w\overline{w}}dwd\overline{w}$ becomes degenerate since the generalized complex coordinates for the imbedding space depend only on u or v but not both so that one has $g_{uv} = g_{vu} = 0$.

   The tangent space of the space-time surface is therefore metrically 2-dimensional and $\sqrt{g_4}$ vanishes. The quantity $g^{uv}\sqrt{g}$ behaves like $1/\sqrt{g_4}$ and has 2-D delta function singularity at the entire partonic orbit. At the partonic orbit the modified Dirac equation must be true outside the 2-D singularity but the delta function like divergence means that this is not trivially true. Generalized holomorphy makes Ψ should guarantee this outside the singularity meaning that at the singularity Ψ depends on both u and v rather on only u or v.

4. At the singularity, the second fundamental form is non-vanishing and g_{uu} and g_{vv} are non-vanishing which correspond to the failure of hypercomplex analyticity. If one assumes $\sqrt{g_4} = 0$ also now one obtains a delta function singularity for the trace. At the singularity Γ_μ is a combination of gammas of both Γ_U and Γ_V. This means that A_U and A_V appear at vertex and are multiplied by the required delta function. By the loss of hypercomplex analyticity, the Dirac equation at the vertex is not satisfied and this gives the non-vanishing $\Gamma^\mu A_\mu$ giving rise to the vertex.

5. What does the singular 2-surface look like? The fermion should turn backwards in time. This means that at the vertex the time coordinate T of M^4 for the partonic orbit must have extremum. Hence the derivatives of T with respect to the coordinates of the 2-surface must vanish: time stops. As a consequence, the derivatives of the coordinates of 2-surface with respect to T diverge. The two branches meeting at the singularity could correspond to hypercomplex analytic *resp.* anti-analytic functions meaning that one has $(U = u, h^k = h^k(u, ...))$ *resp.* $(V = v, h^k = h^k(v, ...))$.

6. A more general condition would be that the meeting fermion lines are also complex analytic *resp.* anti-analytic functions. Fermions and antifermions could correspond to analytic *resp.* ant-ianalytic surfaces, that is generalized complex conjugates of each other. Since a single connected space-time region must be either analytic or antianalytic, this might relate closely to the matter antimatter asymmetry.

3.4.3 How to obtain the vertices from the modified Dirac equation

How could one deduce the vertex from the modified Dirac action by using these assumptions.

1. Should one assign a $d < 4$-dimensional fermionic term to the singular surface? Could one add to the Dirac action a total divergence expressing the separate conservation laws for the total fermion number outside the vertices and only the conservation of their sum at the vertex? This divergence can be transformed to an $d < 4$-dimensional integral over the vertex for the flux of the normal component of the non-conserved part of the fermion current through the surface.

2. One can check what comes out from this guess. Let us forget the possible complications due to the reversal of the direction of time at the vertex possibly due to T reflection. The total divergence is given by $\partial_\mu J^\mu$, $J^\mu = \overline{\Psi}\Gamma^\mu\Psi\sqrt{g}$ for the surface representing the particle (it can have dimension smaller than 4 and even correspond to a fermion line).

   One can write the divergence as a covariant divergence and for the contributions involving bilinears of the creation operators for fermions and antifermions, one obtains a sum of two terms which should not vanish at the singularity whereas the remaining contribution vanishes everywhere. This gives

$$\begin{array}{l} \partial_\mu J^\mu = \sum_\pm \overline{\Psi}_\pm[D^\leftarrow + D^\rightarrow + D_\mu\Gamma^\mu]\Psi_\mp]\sqrt{g} \ , \\ D^\leftarrow = (-i\partial_\mu^\leftarrow + A_\mu)\Gamma^\mu \ , \\ D^\rightarrow = \Gamma^\mu(i\partial_\mu^\rightarrow + A_\mu) \ , \end{array} \tag{3.2}$$

   $D_\mu\Gamma^\mu$ corresponds to the generalized Higgs term and is singular at the vertex.

3. What one wants is the following. At the singularity for the $\Psi_\pm\Gamma^\alpha\Psi_\mp$ parts of the current, the contributions of ordinary derivatives, at least of the normal derivatives, cancel each other also at the singularity. The contributions from the gauge potential terms should be nonvanishing at the singularity and should give the gauge couplings so that one would have

$$\partial_\mu J^\mu = \sum_\pm \overline{\Psi}_\pm[A_\mu\Gamma^\mu + \Gamma^\mu A_\mu + D_\mu\Gamma^\mu]\Psi_\mp\sqrt{g} \ . \tag{3.3}$$

   Kähler gauge potential commutes with the modified gamma matrices so that one obtains what one wants.

4. The Higgs term is singular but the gauge potential term does not seem able to develop a singularity unless the induced gauge potential behaves like a gauge potential of a point charge. Something still goes wrong in the above proposal. The modified Dirac equation is true outside the singularity. Singularity however serves as a fermionic source of the Dirac field. This means that the terms $\overline{\Psi}_\pm D^\leftarrow$ and $D^\rightarrow\Psi_\mp$ have a delta function like behavior at the singularity just like field equations have a singularity. This of course conforms with the super-symmetry associated with the modified Dirac equation.

   To get correct couplings, the sum of the source terms should give $A_\mu\Gamma^\mu + \Gamma^\mu A_\mu$ times a $d-2$-dimensional delta function associated with the singularity. $\sqrt{g_d}$ must appear in the integration measure to get the dimensions correctly. The delta function would compensate for the reduction of the dimension in the integral appearing in the Dirac action.

   One could worry about the gauge invariance of the scattering amplitudes at the fundamental level. In TGD, the notion of gauge invariance for Kähler gauge potential could be only approximate and correspond to symplectic transformations which are not isometries of the imbedding spaces. Classical gravitation would cause it's breaking as a gauge symmetry.

3.5 More about the singularities

3.5.1 About the singularities of the bosonic action

The Hamilton-Jacobi structure [L13] implies the existence of generalized complex coordinates $(u, v, w, \overline{w})$ of space-time as a subset of similar coordinates of H.

1. At the singularities generalizing the poles of analytic function the minimal surface property fails and the action is expected to give a singular delta function type contribution. This contribution should give rise to the scattering amplitudes. The failure of generalized holomorphy is analogous to that of 2-D holomorphy. The latter means that the pole of the action of Laplacian of the analytic function develops a delta function so that $\partial_z\partial_{\overline{z}}f$ is proportional to a delta function rather than vanishing. In a cut f in turn develops a discontinuity.

2. There are 3 kinds of pole-like singularities associated with light-like partonic orbits. The pole at $u = u_0$ gives a 3-D light-like partonic orbit. The pole at $z = z_0$ gives a string world sheet, whose boundary intersects the partonic orbit along the fermion line. The pole at $(u, v) = (u_0, v_0)$ gives a partonic 2-surface with a vertex interpretation. The pole with $(u, z) = (u_0, z_0)$ gives a fermionic line as the boundary of the string world s000heet. The pole $(u, v, z) = (u_0, v_0, z_0)$ gives a point-like singularity which could correspond to the turning of the light-like fermion line. At least turning back in the time direction is possible.

3. The cuts for partonic 2-surfaces would mean that the H complex coordinates as function of $(u, v, w, \overline{w})$ are discontinuous and this implies multiple covering property. These kinds of singularities could correspond to the non-minimal values of the effective Planck constant identified as a dimension of an extension of rationals. The roots of polynomials defining the space-time surfaces as holomorphic imbeddings indeed define multiple coverings. These cuts are possible both in CP_2 and M^4 degrees of freedom. In the CP_2 (M^4) case M^4 (CP_2) coordinates as functions of CP_2 (M^4) coordinates are many-valued.

4. In CP_2 case multiple coverings of M^4 analogous to those for Riemann surfaces would mean surfaces for which closed paths around the singularity turn the singularity several times. Anyons could correspond to this kind of singularities [K2]. In the M^4 case, the space-time surface could consist of a larger number or parallel monopole flux tubes behaving as a quantum coherent unit with a very large value of h_{eff}.

3.5.2 What singularities can correspond to vertices for fermion pair creation?

It is not clear whether all singularities have an interpretation in terms of exotic smooth structures. The physical criterion would be that the creation of a fermion pair takes place at the defect and that the minimal surface property fails. Fermions can correspond to induced spinor fields and fermion pairs could be created at surfaces of dimension $d < 4$.

1. For closed two-sheeted cosmic strings and monopole flux tubes, which split by reconnection, the interpretation makes sense and means a generalization of the basic vertex for closed strings. These objects can be 2-sheeted as elementary particles in which case the reconnection would occur in the direction of CP_2. If they are single sheeted, the reconnection would occur in the direction of M^4.

2. 3-D light-like light-partonic orbits appearing as interfaces between Euclidean and Minkowskian space-time regions and as boundaries of space-time surfaces are singularities [L8]. Boundary conditions state that the possible flows of conserved charges from the interior go to the partonic orbit so that the divergence of the Chern-Simons-Kähler canonical momentum current coming from instanton term equals to the sum of the normal components of the canonical currents associated with Kähler action and volume term.

   (a) Chern-Simons action at the light-like partonic orbit coming from the instanton term is well-defined and finite and field equations should not give rise to a singularity except possibly at partonic 2-surfaces, which have been identified as analogs of vertices at which the partonic 2-surface X^2 splits to two.

(b) At the light-like partonic orbit 4-metric has a vanishing determinant and is therefore effectively 2-D (the light-like components of $g_{uv} = g_{vu}$ of the 4-metric vanish). As a consequence, $\sqrt{g_4}$ vanishes like L^2 at the partonic orbit unless some coordinate gradients diverge.

The canonical momentum currents for the volume action are proportional to the contravariant induced metric appearing in the trace of the second fundamental form diverging like $1/L^2$ and to $\sqrt{g}$ so that they remain finite.

(c) Kähler action contains the contravariant metric twice and is proportional to $\sqrt{g_4}$. This can give rise to a divergence of type $1/L^2$ unless the boundary conditions make it finite. I have proposed that the electric-magnetic self-duality at the partonic orbit can transform the Kähler action to an instanton term giving Chern-Simons Kähler term. In this case, a separate instanton term would not be needed. In this case everything would be finite at the partonic orbit. Minimal surface property fails in a smooth manner.

The intuitive picture is that the contributions from the normal currents at the partonic orbit and the Chern-Simons term cancel each other and the partonic orbit cannot play a role of a vertex.

(d) The possible presence of $1/L^2$ divergence could however give rise to a 2-D defect and genuine vertex. If it is identified as a creation of a pair of partonic 2-surfaces, the interpretation in terms of a creation of a fermion pair is possible and could be assigned to the splitting of a monopole flux tube.

In accordance with the QFT picture, I have considered the possibility that the 2-D vertex could correspond to a branching of a partonic orbit. In the recent picture it would be accompanied by a creation of a fermion pair. The stringy view however suggests that pair creation occurs in the creation of partonic orbits in the splitting of monopole flux tubes. The stringy view is more attractive.

3. I have also proposed that 1-D singularities identifiable as boundaries of string world sheets and identifiable as fermion lines at the partonic orbits are important. The creation of a pair of fermion lines would give rise to the analogs of gauge theory vertices as 0-D singularities. It is however far from clear whether the stringy singularities are actually present and whether they could correspond to exotic smooth structures. One can imagine two options.

   (a) There are no string world sheets. Monopole flux tubes can be regarded as deformations of cosmic strings. Instead of strings several monopole flux tubes can emerge from a wormhole contact. For the minimal option, monopole flux tubes, CP_2 type extremals, and massless extremals as counterparts of radiation fields are the basic extremals and the splitting of monopole flux tubes gives rise to vertices as defects of the ordinary smooth structure.

   (b) String world sheets appear as singularities of the monopole flux tubes or even more general 4-surfaces and are analogous to wormhole contacts as blow-ups in which a point of X^4 explodes to CP_2 type extremal. I have indeed proposed that a blow-up at which the points of the string world sheet as surface $X^2 \subset X^4$ are replaced with a homologically non-trivial 2-surface $Y^2 \subset CP_2$ takes place. Y^2 could connect two parallel space-time sheets. Could these singularities correspond to defects of exotic smooth structures such that the ends of the string carry fermion number? The vertex for the creation of a pair of fermion and antifermion lines would correspond to a diffeo defect. Note that also these defects could reduce to a splitting of a monopole flux tube so that TGD would generalize the stringy picture.

3.6 About symmetry between gravitational and gauge interactions

The beauty of the proposal is that it implies a symmetry between gravitational and gauge interactions. Higgs and graviton are parts of the second fundamental form. Color interactions couple to the isometry charges of CP_2 and gravitational interactions couple to the isometry charges of M^4. The extreme weakness of the gravitation can be understood as being due to the fact that by the flatness of M^4, the M^4 part of the second fundamental form involves only the CP_2 contribution.

One of the ideas related to gravitation is that gravitation could correspond to $SO(1,3)$ gauge theory. The problem is that the finite-D irreps of $SO(1,3)$ are non-unitary. In ZEO, the finite-D space of causal diamonds (CDs) forms [L14] the backbone of WCW and Poincare invariance and Poincare quantum numbers can be assigned with wave functions in this space. For CD, the infinite-D unitary representations of $SO(1,3)$ satisfying appropriate boundary conditions are a highly attractive identification for the counterparts of finite-D unitary representations associated with gauge multiplets.

One could replace the spinor fields of H with spinor fields restricted to CD with spinor fields for which M^4 parts sinor nodes as plane waves are replaced with spinor modes in CD labelled by spin and its hyperbolic counterpart assignable to Lorentz boosts with respect to either tip of CD. One could also express these modes as superpositions of the plane wave modes defined in the entire H. In TGD, gravitation could be seen as a $SO(1,3)$ gauge theory if it can be regarded as a gauge theory for the Poincare group.

Does color confinement have any counterpart at the level of M^4? The idea that physical states have vanishing four-momenta does not look attractive.

> The analog of color confinement would hold true for particles as unitary representations of $SO(1,3)$ in CD. One could say that $SO(1,3)$ appears as an internal isometry group of an observer's perceptive field represented by CD and Poincare group as an external symmetry group treating the observer as a physical object.By separation of variables the spinor harmonics in CD factorize phases depending on the mass of the particle determined by CP_2 and spinor harmonic of hyperbolic 3-space $H^3 = SO(1,3)/SO(3)$. $SO(1,3)$ allows an extremely rich set of representations in the hyperbolic space H^3 analogous to spherical harmonics. A given infinite discrete subgroup $\Gamma \subset SO(1,3)$ defines a fundamental domain of Γ as a double coset space $\Gamma \setminus SO(1,3)/SO(3)$. This fundamental domain is analogous to a lattice cell of condensed matter lattice defined by periodic boundary conditions. The graphics of Escher give an idea about these structures in the case of H^2. The products of wave functions defined in $\Gamma \subset SO(1,3)$ and of wave functions in Γ define a wave function basis analogous to the space states in condensed matter lattice. TGD allows gravitational quantum coherence in arbitrarily long scales and I have proposed that the tessellations of H^3 define the analogs of condensed matter lattices at the level of cosmology and astrophysics [L16]. The unitary representations of $SO(1,3)$ would be central for quantum gravitation at the level of gravitationally dark matter. They would closely relate to the unitary representations of the supersymplectic group of $\delta M^4_+ \times CP_2$ in M^4 degrees of freedom and define their continuations to the entire CD. There exists a completely unique tessellation known as icosa tetrahedral tessellation consisting of icosahedrons, tetrahedrons, and octahedrons glued along boundaries together. I have proposed that it gives rise to a universal realization of the genetic code of which biochemical realizations is only a particular example [L1, L9]. Also this supports a deep connection between biology and quantum gravitation emerging also in classical TGD [L6, L5]. Also electromagnetic long range classical fields are predicted to be involved with long length scale quantum coherence [L10].

The challenge is to understand the implications of this picture for $M^8 - H$ duality [L17]. The discretization of M^8 identified as octonions O with the Minkowskian norm defined by $Re(Im(o^2))$ is linear M^8 coordinates natural for octonions. The discretization obtained by the requirement that the coordinates of the points of M^8 (momenta) are algebraic integers in an algebraic extension of rationals would make sense also in p-adic number fields.

In the Robertson-Walker coordinates for the future light-cone M^4_+, sliced by H^3:s. In M^8, the analog of time coordinate is defined by mass and in H by the light-cone proper time. Hyperbolic angle and spherical angles defined the coordinates of H^3. The discretizations defined by the spaces $\Gamma \setminus SO(1,3)/SO(3)$ would define a discretization and one can define an infinite hierarchy of discretizations defined by the discrete subgroups of $SO(1,3)$ with matrix elements belonging to an extension of rationals. This number theoretically universal discretization defines a natural alternative for the linear discretization. Maybe the linear *resp.* non-linear discretization could be assigned to the moduli space of CDs *resp.* CD.

4 Could TGD allow the detection of gravitons?

Could an effectively 2-dimensional system make it possible to observe real gravitons and why should this be the case? Could the couplings to quantum coherent many-particle states such as electrons or protons at light-like 3-surfaces assignable to the fractional quantum Hall effect (FQHE) in the TGD description amplify the effects induced by gravitons? Also gravitational field bodies can be considered as an amplification mechanism.

One can consider two ways to detect gravitons. For the first option, one would have a condensed matter system with a large value of h_{eff} allowing large scale quantum coherence so that the rate for the Compton scattering of graviton would be high. The second option is based on the notion of a field body. One might hope that dark protons or even electrons associated with the gravitational magnetic body residing at the monopole flux tubes and half-monopole flux tubes make possible the detection of gravitons. Kind of gravitational quantum antenna would be in question.

The scattering amplitude of graviton from electrons or protons or even heavier particles would be a sum of a very large number of identical amplitudes when the wavelength of real graviton is much longer than the size of the partonic 2-surface representing the particle. One would obtain an analog of diffraction, somewhat like in the TGD based models of the recently discovered gravitational hum identified in the TGD framework as diffraction in astrophysical length scales [L16].

4.1 Is the detection of gravitons possible in FQHE type systems?

One might hope that the coupling of real gravitons to condensed matter gravitons and/or nanoscale quantum coherence could make the amplitude for the Compton scattering of graviton from a particle [B1] proportional to the square of the total number N of electrons.

One can consider fractional quantum Hall effect (FQHE) as an example. The stimulus for this came from a popular article (see this) telling about the work of Liang et al with title "Evidence for chiral graviton modes in fractional quantum Hall liquids" published in Nature [D1]. Chiral gravitons are not ordinary gravitons but this notion inspired the possible identification of gravitons as fermion-antifermion pairs at rotating monopole flux tubes discussed in this article.

1. FQHE occurs in 2-D electron gas and (see this) and the typical densities of electrons are of order $10^{11}/cm^2$. For an area cm^2 one would have $N^2 \sim 10^{22}$.

2. The differential cross section for gravitational Compton scattering from a particle with mass m [B1] is

$$\frac{d\sigma}{d\Omega} = \frac{G^2 m^2}{sin^4(\theta/2)}(cos^8(\theta/2) + sin^8(\theta/2)) \ .$$

   For the electron, the order of magnitude is $\sigma \sim 10^{-42} l_e^2$, where l_e is electron's Compton length, unless θ is very near to the forward direction. There is no hope of detecting gravitons in this way unless one has an analog of nearly forward scattering. Even in the presence of quantum coherence, giving for the rate of scattering events for N electron system a rate proportional to N^2 instead of N, the hopes for detection seem rather meager. If θ is of order 10^{-5}, the order of magnitude for the differential cross section is of the order of l_e^2.

4.2 What about dark protons at the monopole flux tubes and half-monopole flux tubes?

Proton mass is roughly 2000 times larger than electron mass so that the gravitons scattering from protons at the gravitational magnetic body of Earth is a promising idea to consider.

1. Could the dark protons at magnetic monopole flux tubes give rise to a quantum coherence amplifying the interactions with gravitons. Monopole flux tube condensates involve a very large number of parallel monopole flux tubes, which form a quantum coherent region. By quantum coherence, the dark proton system at the magnetic body would behave like a particle

with total protonic mass Nm_p. Even if the magnetic body involves particles with different masses, the Equivalence Principle coded to the gravitational Planck constant implies that the field body responds with its total mass.

Could the quantum coherent Compton scattering of graviton, or even better, of an analog of laser beam for gravitons, from the gravitational magnetic body of Earth lead to detectable recoil effects such as the transformation of dark protons to ordinary protons by the reverse Pollack effect? These effects are of course not observable using the existing technology which knows nothing about field bodies and dark matter in the TGD sense.

2. The number of monopole flux tubes corresponds to h_{eff}/h and this can be as large as 10^{14}. This would give a factor of order $N^2 \sim 10^{28}$ to the scattering cross section. In the case of dark protons, one would have a scaling factor $(m_p/m_e)^2 \sim 4 \times 10^6$. This would give a factor of order 10^{34} giving $\sigma \sim 10^{-8} l_e^2 \sim 10^{-2} l_p^2$. Could this make the detection possible?

3. Half-monopole flux tubes appear in the TGD based model for the transition to superconductivity as an intermediate, not yet superconducting, flux tubes carrying dark electrons but not their Cooper pairs [L20]: the pair of dark electron and corresponding hole at the level of ordinary matter replaces the notion of Bogoliubov quasiparticle as a superposition of electron and hole in such a way that the total fermion number is conserved. Half-monopole flux tubes have boundaries, which should be light-like and can be so as a static structure in the induced geometry [L8], which could carry dark protons.

   Note that also the light-like surfaces associated with the Quantum Hall systems would be naturally half-monopole flux tubes since electrons in these systems are known to form bound states with magnetic fluxes.

Appendix: GPT summary about exotic smooth structures

I do not count myself as a real mathematician. The GPT summary of exotic smooth structures kindly posted by Gary Ehlenberg however suggests deep connections between the TGD view of particle physics and 4-D gauge theories and theory of 4-manifolds.

The study of exotic R^4's has led to numerous significant mathematical developments, particularly in the fields of differential topology, gauge theory, and 4-manifold theory. Here are some key developments.

1. Donaldson's Theorems

   Simon Donaldson's groundbreaking work in the early 1980s revolutionized the study of smooth 4-manifolds. His theorems provided new invariants, known as Donaldson polynomials, which distinguish between different smooth structures on 4-manifolds.

   Donaldson's diagonalization theorem states that the intersection form of a smooth, simply connected 4-manifold must be diagonalizable over the integers, provided the manifold admits a smooth structure. This result was crucial in showing that some topological 4-manifolds cannot have a smooth structure.

   Donaldson's polynomial invariants help to classify and distinguish different smooth structures on 4-manifolds, particularly those with definite intersection forms.

2. Freedman's Classification of Topological 4-Manifolds

   Michael Freedman's work, which earned him a Fields Medal in 1986, provided a complete classification of simply connected topological 4-manifolds. His results showed that every such manifold is determined by its intersection form up to homeomorphism.

   h-Cobordism and the Disk Embedding Theorem: Freedman's proof of the h-cobordism theorem in dimension 4 and the disk embedding theorem were instrumental in his classification scheme.

3. Seiberg-Witten Theory

The development of Seiberg-Witten invariants provided a new set of tools for studying smooth structures on 4-manifolds, complementing and sometimes simplifying the methods introduced by Donaldson.

Seiberg-Witten Invariants are simpler to compute than Donaldson invariants and have been used to prove the existence of exotic smooth structures on 4-manifolds.

4. Gauge Theory and 4-Manifolds

   Gauge theory, particularly through the study of solutions to the Yang-Mills equations, has provided deep insights into the structure of 4-manifolds.

   The study of instantons (solutions to the self-dual Yang-Mills equations) has been crucial in understanding the differential topology of 4-manifolds. Instantons and their moduli spaces have been used to define Donaldson and Seiberg-Witten invariants.

5. Symplectic and Complex Geometry

   The interaction between symplectic and complex geometry with 4-manifold theory has led to new discoveries and techniques.

   Robert Gompf's work on constructing symplectic 4-manifolds has provided new examples of exotic smooth structures. His techniques often involve surgeries and handle decompositions that preserve symplectic structures.

   Symplectic Surgeries, that is techniques such as symplectic sum and Luttinger surgery have been used to construct new examples of 4-manifolds with exotic smooth structures.

6. Floer Homology

   Floer homology, originally developed in the context of 3-manifolds, has been extended to 4-manifolds and provides a powerful tool for studying their smooth structures.

   Instanton Floer Homology associates a homology group to a 3-manifold, which can be used to study the 4-manifolds that bound them. It has applications in understanding the exotic smooth structures on 4-manifolds.

7. Exotic Structures and Topological Quantum Field Theory (TQFT)

   The study of exotic R^4's has also influenced developments in TQFT, where the smooth structure of 4-manifolds plays a crucial role. TQFTs are sensitive to the smooth structures of the underlying manifolds, and exotic R^4's provide interesting examples for testing and developing these theories.

To sum up, the exploration of exotic R^4's has led to significant advances across various areas of mathematics, particularly in the understanding of smooth structures on 4-manifolds. Key developments include Donaldson and Seiberg-Witten invariants, Freedman's topological classification, advancements in gauge theory, symplectic and complex geometry, Floer homology, and topological quantum field theory. These contributions have profoundly deepened our understanding of the unique and complex nature of 4-dimensional manifolds.

The singularity X^2, where the minimal surface property and generalized complex structure fail should correspond to a defect of the ordinary smooth structure. This is the conjecture that I would like to understand better and here my limitations as a mathematician are the problem.

I can only ask questions inspired by the result that the intersection form $I(X^4)$ for 2-D homologically non-trivial surfaces of X^4 detects the defects of the ordinary smooth structure, which should correspond to surfaces X^2, i.e. vertices for a pair creation.

1. In homology, the defect should correspond to an intersection point of homologically non-trivial 2-surfaces identifiable as wormhole throats, which correspond to homologically non-trivial 2-surfaces of CP_2. This suggests that $X_1^4 \supset X_2^4$ containing the singularity/vertex differs from $I(X_2^4)$ when X_2^4 does not contain the vertex.

2. CP_2 has an intersection form corresponding to the homologically non-trivial 2-surfaces for which minimal intersection corresponds to a single point. The value of intersection form for 2 2-surfaces is essentially the product of integers characterizing their homology equivalence classes. If each wormhole contact contributes a single CP_2 summand to the total intersection form, there would be two summands per elementary particle as monopole flux tube.

3. 2-D singularity gives rise to a creation of an elementary particle and would therefore add two CP_2 summands to the intersection form. The creation of a fermion-antifermion pair has an interpretation in terms of a closed monopole flux tube. A closed monopole flux tube having wormhole contacts at its "ends" splits into two by reconnection.

REFERENCES

Mathematics

[A1] Etesi G. Exotica or the failure of the strong cosmic censorship in four dimensions, 2015. Available at: http://arxiv.org/abs/1503.04945v4.

[A2] Gompf RE. Three exotic R^4's and other anomalies. *J. Differential Geometry*, 18:317–328, 1983.Available at: http://projecteuclid.org/download/pdf_1/euclid.jdg/1214437666.

[A3] Asselman-Maluga T and Brans CH. World Scientific. https://docs.google.com/document/d/1Hp656gzbKzlrHtmWvTUtOA5zsYZ415R3HHIe2eI-tPO/edit., 2007.

Theoretical Physics

[B1] Rothman T and Boughn S. Can Gravitons Be Detected?, 2006. Available at: https://arxiv.org/pdf/gr-qc/0601043.pdf.

Condensed Matter Physics

[D1] Liang J et al. Evidence for chiral graviton modes in fractional quantum Hall liquids. *Nature*, 628:78–83, 2024. Available at: https://www.nature.com/articles/s41586-024-07201-w.

Books related to TGD

[K1] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/prext.pdf, 2023.

[K2] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter. https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K3] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L2] Pitkänen M. TGD and Condensed Matter. https://tgdtheory.fi/public_html/articles/TGDcondmatshort.pdf., 2021.

[L3] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L4] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L5] Pitkänen M. Comparison of Orch-OR hypothesis with the TGD point of view. https://tgdtheory.fi/public_html/articles/penrose.pdf., 2022.

[L6] Pitkänen M. How animals without brain can behave as if they had brain. https://tgdtheory.fi/public_html/articles/precns.pdf., 2022.

[L7] Pitkänen M. Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD. https://tgdtheory.fi/public_html/articles/finitefieldsTGD.pdf., 2022.

[L8] Pitkänen M. TGD inspired model for freezing in nano scales. https://tgdtheory.fi/public_html/articles/freezing.pdf., 2022.

[L9] Pitkänen M. About tessellations in hyperbolic 3-space and their relation to the genetic code . https://tgdtheory.fi/public_html/articles/tessellationH3.pdf., 2023.

[L10] Pitkänen M. About long range electromagnetic quantum coherence in TGD Universe. https://tgdtheory.fi/public_html/articles/hem.pdf., 2023.

[L11] Pitkänen M. About Platonization of Nuclear String Model and of Model of Atoms. https://tgdtheory.fi/public_html/articles/nuclatomplato.pdf., 2023.

[L12] Pitkänen M. About the Relationships Between Weak and Strong Interactions and Quantum Gravity in the TGD Universe . https://tgdtheory.fi/public_html/articles/SW.pdf., 2023.

[L13] Pitkänen M. Holography and Hamilton-Jacobi Structure as 4-D generalization of 2-D complex structure. https://tgdtheory.fi/public_html/articles/HJ.pdf., 2023.

[L14] Pitkänen M. New result about causal diamonds from the TGD view point of view. https://tgdtheory.fi/public_html/articles/CDconformal.pdf., 2023.

[L15] Pitkänen M. Symmetries and Geometry of the "World of Classical Worlds" . https://tgdtheory.fi/public_html/articles/wcwsymm.pdf., 2023.

[L16] Pitkänen M. The TGD view of the recently discovered gravitational hum as gravitational diffraction. https://tgdtheory.fi/public_html/articles/gravhum.pdf., 2023.

[L17] Pitkänen M. A fresh look at M^8H duality and Poincare invariance. https://tgdtheory.fi/public_html/articles/TGDcritics.pdf., 2024.

[L18] Pitkänen M. About the Recent TGD Based View Concerning Cosmology and Astrophysics. https://tgdtheory.fi/public_html/articles/3pieces.pdf., 2024.

[L19] Pitkänen M. Modified Dirac equation and the holography=holomorphy hypothesis. https://tgdtheory.fi/public_html/articles/modDir.pdf., 2024.

[L20] Pitkänen M. What happens in the transition to superconductivity? https://tgdtheory.fi/public_html/articles/halfmonopole.pdf., 2024.