An Overview About Quantum TGD: Part I

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Abstract

This chapter is the first one of the two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development.

The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" (WCW) identified as the infinite-dimensional WCW of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision in this chapter.

In this chapter the discussion is mostly concentrated on general ideas whereas the topics related to the construction of M-matrix are discussed on the second chapter. TGD relies heavily on geometric and number theoretical ideas gradually generalized during the years. The following summarizes the overall picture as it is now.

1. The basic vision is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property.
2. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants. Generalization of 2-D conformal symmetries generalized so that it applies light-like surfaces is conjectured to define quantum criticality mathematically: actually one has hierarchy of broken conformal symmetries defined by the hierarchy of sub-algebras of conformal algebra or associated algebra (say Kac-Moody type algebra).
3. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. WCW spinors define a von Neumann algebra known as hyperfinite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD.
4. p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.
5. The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. This leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. This leads to an explicit formula for the Dirac determinant. What is remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: they should correspond to preferred extremals of Kähler action.

   Thus hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality. The hierarchy of sub-algebras of conformal algebra consisting of generators for which conformal weight is multiple of integer n, would correspond to the value $h_{eff} = n \times h$ of effective Planck

constant. Conformal equivalence classes of space-time surfaces with same space-like ends at boundaries of CD would contain n surfaces.

6. HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory.

1 Introduction

This chapter is the first one of two chapters providing a summary about evolution of quantum TGD in nearly chronological order. By their nature these chapters are dynamical and I cannot guarantee internal consistency since the ideas discussed are those under most vigorous development.

The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the WCW or "world of the classical worlds" identified as the infinite-dimensional WCW of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis this vision.

1.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years.

1. The basic dynamical objects of TGD are 3-surfaces of 8-D imbedding space fixed uniquely by the symmetries of particle physics and the structure of standard model. 4-D general coordinate invariance allows to assume that these surfaces are light-like and the interpretation is as random light-like orbits of 2-dimensional partons. This picture leads immediately to an understanding of the fundamental super-conformal symmetries of the theory and realization that TGD can be seen as an almost topological quantum field theory.

2. The basic vision is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

3. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW Clifford algebra defines a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the Kähler-Dirac operator assigned to the light-like 3-surfaces.

4. The reduction of the WCW geometrization to second quantization of induced spinor fields at light-like 3-surface is crucial for the practical progress made in the geometrization. The Dirac determinant defined as the product of generalized eigenvalues of the Kähler-Dirac operator has identification as vacuum functional defined by Kähler function. By construction the generalized eigenvalues carry information about the preferred extremal of Kähler action, and their number for a given light-like 3-surface is finite so that finiteness of the theory is guaranteed and the notion of finite measurement resolution -forced originally by the properties of hyper-finite factors- emerges automatically.

5. p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

6. The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds.

7. HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

1.2 Ideas related to the construction of S-matrix

The construction of S-matrix has been the most difficult challenge of TGD and involves several ideas that have emerged during last years. It is not possible to represent explicit formulas yet but the general principles behind S-matrix, or rather its generalization to M-matrix, are reasonably well understood now.

1. Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory.

   One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2. The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Thus S-matrix is characterized by the measurement resolution analogous to length scale cutoff of quantum field theories. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree. The amazing conclusion interpreted in terms of asymptotic freedom is that at the never-reachable limit of infinite measurement resolution the S-matrix becomes trivial.

3. An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the

construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD: for instance, photons travelling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

4. Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory.

5. M^8-H duality or "number theoretical compactification" [K19] states that one can regard space-time surfaces X^4 either as associative (co-associative) surfaces in the space M^8 of hyper-octonions or as preferred extremals of Kähler action in $M^4\times CP_2$. Associativity means that the tangent space of X^4 at each point is some hyperquaternionic subspace $HQ=M^4$ of HO. Besides this a preferred plane $M^2\subset M^8$ identifiable as a plane of non-physical polarizations belongs to the tangent space at each point. This hypothesis provides a purely number theoretic interpretation of gauge conditions and implies a large number of "must-be-trues" of quantum TGD, and together with zero energy ontology leads to a precise view about the realization of zero energy states in terms of causal diamonds allowing to deduce p-adic length scale hypothesis and a general vision about coupling constant evolution in which time scales appear as power of 2 multiples of a basic length scale.

   One can ask whether this duality generalizes to H-H duality such that the image of associative (co-associative) surface in duality is associative (co-associative). If this were the case the dualities would make the space of space-time surfaces a category and one could iterate the duality to construct new preferred extremals of Kähler action.

   One important implication is a justification for the coset construction based on the lifting of Super Kac-Moody algebra (SKM) at a given light-like 3-surface to a sub-algebra of super-symplectic algebra (SC) lifted from $\delta M^{\pm}\times CP_2$ to algebra in H.

6. The outcome is a generalization of Feynman diagrammatics in which the lines of Feynman diagrams are replaced with 3-D light-like surfaces meeting at 2-D surfaces representing vertices. The contribution of a given Feynman diagram is calculated using the fusion rules of a generalized conformal field theory recursively rather than instead of the ordinary Feynman rules. A new element is symplectically invariant (invariant under symplectic/contact transformations of $\delta M^4_{\pm}\times CP_2$) factor of N-point function and thus expressible in terms of symplectic invariants constructed from the areas assignable to the geodesic triangles defined by the subsets of N points and satisfying fusion rules. Simple argument shows that this factor vanishes if any two arguments of N-point function are identical: this gives excellent hopes that infinities are avoided as general arguments indeed predict. The construction and classification of symplectic QFTs as analogs of conformal field theories becomes a basic mathematical challenge.

   The restriction of the arguments of N-point functions to a discrete set of points at partonic 2-surfaces and defining number theoretical braids is an essential ingredient of the approach making it possible the completion of the theory to real and various p-adic domains. These points correspond to the unique intersection of the hyper-quaternionic (and thus associative subset $M^4\subset M^8$ with the partonic 2-surfaces, where M^4 is now a fixed associative plane of M^8 which should not be confused with the varying associative plane assignable to each point of X^4.

   A structure resembling stringy perturbation theory involving fermionic propagators expressible as inverses of the super-generator G_0 is what one naively expects. The fact that G_0 must carry fermion number seems however to be a problem: the stringy propagator actually corresponds to $G{-}1p^k\gamma_k(G^{\dagger))^{-1}}$. There is thus no need for Majorana spinors leading to super string models and imbedding space dimension $D=8$ works.

1.3 Some general predictions of quantum TGD

TGD is consistent with the symmetries of the standard model by construction although there are definite deviations from the symmetries of standard model. TGD however predicts also a lot of new physics. Below just some examples of the predictions of TGD.

1. Fractal hierarchies meaning the existence of scaled variants of standard model physics is the basic prediction of quantum TGD. p-Adic length scale hypothesis predicts the possibility that elementary particles can have scaled variants with mass scales related by power of $\sqrt{2}$. Dark matter hierarchy predicts the existence of infinite number of scaled variants with same mass spectrum with quantum scales like Compton length scaling like $\hbar$.

2. TGD predicts that standard model fermions and gauge bosons differ topologically in a profound manner. Free fermions correspond to light-like wormhole throats associated with topologically condensed CP_2 type extremals whereas gauge bosons correspond to fermion-anti-fermion states associated with the throats of wormhole contacts connecting two space-time sheets with opposite time orientation. The implication is that Higgs vacuum expectation value cannot contribute to fermion mass: this conforms with the results of p-adic mass calculations. TGD predicts also so called super-symplectic quanta and these give dominating contribution to most hadron masses. These degrees of freedom correspond to those of hadronic string and should not reduce to QCD.

3. The most fascinating applications of zero energy ontology are to quantum biology and TGD inspired theory of consciousness. Basic new element are negative energy photons making possible communications to the direction of geometric past. Here also dark matter hierarchy is involved in an essential manner.

4. In cosmology the mere imbeddability required for Robertson-Walker cosmology implies that critical and over-critical cosmologies are almost unique and characterized by their finite duration. The cosmological expansion is accelerating for them and there is no need to assume cosmological constant. Macroscopic quantum coherence of dark matter in astrophysical scales is a dramatic prediction and allows also to assign periods of accelerating expansion to quantum phase transition changing the value of gravitational Planck constant. The dark matter parts of astrophysical systems are predicted to be quantum systems.

5. The notion of hyper-finite factors suggesting the representation of finite measurement resolution as gauge symmetry suggests that the physics of TGD Universe is universal in the sense that it is possible to engineer a system able to mimic the physics of any consistent gauge theory. Kind of analog of Turing machine would be in question.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L5]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L6].

2 Physics as geometry of WCW spinor fields

The construction of the geometry of WCW ("world of classical worlds" or simply WCW) has proceeded rather slowly. The experimentation with various ideas has however led to the identification of the basic constraints on WCW geometry. The most recent vision is described in [K31].

The basic philosophical motivation for the hypothesis that quantum physics could reduce to the construction of WCW Kähler metric and spinor structure, is that infinite-dimensional Kähler geometric existence could be unique not only in the sense that the geometry of the space of 3-surfaces could be unique but that also the dimension of the space-time is fixed to $D = 4$ by this requirement and $M^4_+ \times CP_2$ is the only possible choice of imbedding space. This optimistic vision derives from the work of Dan Freed with loops spaces demonstrating that they possess unique Kähler geometry and from the fact that in $D > 1$ case the existence of Riemann connection, finiteness of Ricci tensor, and general coordinate invariance poses even stronger constraints.

2.1 Constraints on WCW geometry

The detailed considerations of the constraints on WCW geometry suggests that it should possess at least the following properties.

1. Metric should be Kähler metric. This property is necessary if one wants to geometrize the oscillator algebra used in the construction of the physical states and to obtain a well defined divergence free functional integration in the configuration space.

2. Metric should allow Riemann connection, which, together with the Kähler property, very probably implies the existence of an infinite dimensional isometry group as the construction of Kähler geometry for the loop spaces demonstrates [A10].

3. The so called symmetric spaces classified by Cartan [A12] are Cartesian products of the coset spaces G/H with maximal isometry group G. Symmetric spaces possess G invariant metric and curvature tensor is constant so that all points of the symmetric space are metrically equivalent. Symmetric space structure means that the Lie-algebra of G decomposes as

$$g = h \oplus t \ ,$$
$$[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ ,$$

   where g and h denote the Lie-algebras of G and H respectively and t denotes the complement of h in g. The existence of the $g = t + h$ decomposition poses an extremely strong constraint on the symmetry group G.

   In the infinite-dimensional context symmetric space property would mean a drastic calculational simplification. The most one can hope is that WCW is expressible as a union $\cup_i (G/H)_i$ of symmetric spaces. Reduction to a union of G/H is the best one can hope since 3-surface of Planck size cannot be metrically equivalent with a 3-surface having the size of galaxy! The coordinates labelling the symmetric spaces in this union do not appear as differentials in the line element of WCW and are thus zero modes. They correspond to non-quantum fluctuating degrees of freedom in a well defined sense and are identifiable as classical variables of quantum measurement theory.

4. Metric should be Diff^4 (not only $Diff^3$!) invariant and degenerate and the definition of the metric should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 to act on. This requirement is absolutely crucial for all developments.

5. Divergence cancellation requirement for the functional integral over WCW requires that the metric is Ricci flat and thus satisfies vacuum Einstein equations.

2.2 WCW as a union of symmetric spaces

In the finite-dimensional context, globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. Good guess is that same holds true in the infinite-dimensional context. The task is to identify the infinite-dimensional groups G and H. Only quite recently, more than seven years after the discovery of the candidate for Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from Diff^4 invariance and Diff^4 degeneracy.

The crux of the matter is Diff^4 : all 3-surfaces on the orbit of 3-surface X^3 must be physically equivalent so that one can effectively replace all 3-surfaces Z^3 on the orbit of X^3 with a suitably chosen surface Y^3 on the orbit of X^3. The Lorentz and Diff^4 invariant choice of Y^3 is as the intersection of the 4-surface with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light-cone: effectively the imbedding space can be replaced with the product $\delta M^4_+ \times CP_2$ as far as vibrational degrees of freedom are considered. More precisely: WCW has a fiber structure: the 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ correspond to the base space and the 3-surfaces on the orbit of given Y^3 and diffeomorphic with Y^3 correspond to the fiber and are separated by a zero distance from each other in WCW metric.

These observations lead to the identification of the isometry group as some subgroup G of the group of the diffeomorphisms of $\delta H = \delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of the 3-surfaces in δH. Therefore one can identify the WCW as the union of the coset spaces G/H, where H corresponds to the subgroup of G acting as diffeomorphisms for a given X^3. H depends on the topology of X^3 and since G does not change the topology of the 3-surface, each 3-topology defines a separate orbit of G. Therefore, the union involves the sum over all topologies of X^3 plus possibly other "zero modes".

The task is to identify correctly G as a sub-algebra of the diffeomorphisms of δH. The only possibility seems to be that the symplectic transformations of δH generated by the function algebra of δH act as isometries of WCW. The symplectic transformations act nontrivially also in δM^4_+ since δM^4_+ allows Kähler structure and thus also symplectic structure.

2.2.1 The magic properties of the light like 3-surfaces

In case of the Kähler metric, G- and H Lie-algebras must allow a complexification so that the isometries can act as holomorphic transformations. The unique feature of the δM^4_+, realized already seven years ago, is its metric degeneracy: the boundary of the light-cone is metrically 2-dimensional sphere although it is topologically 3-dimensional! This implies that light-cone boundary allows an infinite-dimensional group of conformal symmetries generated by an algebra, which is a generalization of the ordinary Virasoro algebra! There is actually also an infinite-dimensional group of isometries (!) isomorphic with the group of the conformal transformations! Even more, in case of δH the groups of the conformal symmetries and isometries are local with respect to CP_2. Furthermore, light-cone boundary allows infinite dimensional group of symplectic transformations as the symmetries of the symplectic structure automatically associated with the Kähler structure. Therefore 4-dimensional Minkowski space is in a unique position in TGD approach. δM^4_+ allows also complexification and Kähler structure unlike the boundaries of the higher-dimensional light-cones so that it becomes possible to define a complexification in the tangent space of the WCW, too.

The space of the vector fields on $\delta H = \delta M^4_+ \times CP_2$ inherits the complex structure of the light-cone boundary and CP_2. The complexification can be induced from the complex conjugation for the functions depending on the radial coordinate of the light-cone boundary playing the same role as the time coordinate associated with string space-time sheet. In M^4_+ degrees of freedom complexification works only provided that the radial vector fields posses zero norm as WCW vector fields (they have also zero norm as vector fields).

The effective two-dimensionality of the light-cone boundary allows also to circumvent the no-go theorems associated with the higher-dimensional Abelian extensions. First, in the dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite dimensional Abelian group rather than central extensions by the group $U(1)$. In the present case the extension is a symplectic extension analogous to the extension defined by the Poisson bracket $\{p, q\} = 1$ rather than the standard central extension but is indeed 1- dimensional and well defined provided that the configuration space metric is Kähler. Secondly, $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [A18]. The point is that light-cone boundary is metrically and conformally 2-sphere and therefore the gauge algebra is effectively the algebra associated with the 2-sphere and, as a consequence, also WCW metric is Kähler.

There is counter argument against complexifixation. The Kähler structure of the light-cone boundary is not unique: various complex structures are parameterized by $SO(3,1)/SO(3)$ (Lobatchewski space). The definition of the Kähler function as absolute minimum of Kähler action however makes it possible to assign unique space-time surface $X^4(Y^3)$ to any Y^3 on the light-cone boundary and the requirement that the group $SO(3)$ specifying the Kähler structure is isotropy group of the classical four-momentum associated with $X^4(Y^3)$, fixes the complex structure uniquely as a function of Y^3. Thus it seems that Kähler action is necessary ingredient of the group theoretical approach.

2.2.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X_l^3 -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X_l^3)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

2. One could also say that light-like 3-surfaces X_l^3 and the space-like 3-surfaces X^3 in the intersections of $X^4(X_l^3) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M_\pm^4 \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M_\pm^4 \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X_l^3 \times \delta M_\pm^4 \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD: s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

2.2.3 Symmetric space property reduces to conformal and symplectic invariance

The idea about symmetric space is extremely beautiful but it millenium had to change before I was ripe to identify the precise form of the Cartan decomposition. The solution of the puzzle turned out to be amazingly simple.

The algebra is a direct sum $g = g_1 \oplus g_2$ such that g_1 has $h = n$ as conformal weights and g_2 has more general conformal weights. This motivates the guess that the ground state conformal weights are given by $h = i/2 + y$. It is actually possible to regard the imaginary part of h as a pseudo conformal weight, which can be eliminated by a natural choice of the light-like radial coordinate of δM_+^4. Conformal invariance suggests integer spectrum for y whereas Riemann hypothesis favors zeros of Riemann Zeta.

The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of WCW, which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories.

The light-cone conformal invariance differs in many respects from the conformal invariance of string theories. In particular, the finite-dimensional group defining Kac-Moody group is replaced by an infinite-dimensional symplectic group.

2.3 An educated guess for the Kähler function

The turning point in the attempts to construct WCW geometry was the realization that four-dimensional $Diff$ invariance (not only 3-dimensional $Diff$ invariance!) of General Relativity must have a counterpart in TGD. In order to realize this symmetry in the space of 3-surfaces, the definition of WCW metric should somehow associate to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$ for Diff^4 to act on. Physical considerations require that the metric should be, not only Diff^4 invariant, but also Diff^4 degenerate so that infinitesimal Diff^4 transformations should correspond to zero norm vector fields of WCW.

Since Kähler function determines Kähler geometry, the definition of the Kähler function should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3. The natural physical interpretation for this space-time surface is as the classical space-time associated with X^3 so that in TGD classical physics ($X^4(X^3)$) becomes a part of WCW geometry and of the quantum theory.

2.3.1 Kähler function as Kähler action for preferred extremal

One could try to construct WCW geometry by finding the metric for a single representative 3-surface at each orbit of G and extending it by left translations to the entire orbit of G. The metric for this representative should be $Diff^3$ invariant and somehow it should associate a unique space-time surface to the 3-surface in question. The original attempt was however more indirect and based on the realization that the construction of the Kähler geometry reduces to that of finding Kähler function $K(X^3)$ with the property that it associates a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 and possesses mathematically and physically acceptable properties. The guess for the Kähler function is the following one.

By Diff^4 invariance one can restrict the consideration on the set of 3-surfaces Y^3 on the "light-cone boundary" $\delta H = \delta M^4_+ \times CP_2$ since one can define the space-time surface associated with $X^3 \subset X^4(Y^3)$ to be $X^4(X^3) = X^4(Y^3)$ in case that the initial value problem for X^3 has $X^4(Y^3)$ as its solution. This implies $K(X^3) = K(Y^3)$.

The value of the Kähler function K for a given 3-surface Y^3 on light-cone boundary is obtained in the following manner.

1. Consider all possible 4-surfaces $X^4 \subset M^4_+ \times CP_2$ having Y^3 as its sub-manifold: $Y^3 \subset X^4$. If Y^3 has boundary then it belongs to the boundary of X^4: $\delta Y^3 \subset \delta X^4$.

2. Associate to each four surface Kähler action as the Maxwell action for the Abelian gauge field defined by the projection of the CP_2 Kähler form to the four-surface. For a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density whereas for an Euclidian signature the action density is always non-positive.

3. Define the value of the Kähler function K for Y^3 as the absolute minimum of the Kähler action S_K over all possible four-surfaces having Y^3 as its sub-manifold: $K(Y^3) = Min\{S_K(X^4)|X^4 \supset Y^3\}$.

This definition of the Kähler function has several physically appealing features.

1. Kähler geometry associates with each X^3 a unique four-surface, which will be interpreted as the classical space-time associated with X^3. This means that the so called classical space time (and physics!) in TGD approach is not defined via some approximation procedure (stationary phase approximation of the functional integral) but is an essential part of not only quantum theory, but also of WCW geometry, which in turn might be determined by a mere mathematical consistency! Since quantum states are superpositions over these classical space-times, it is clear that the observed classical space-time is some kind of effective, quantum average space-time, presumably defined as an absolute minimum for the effective action of the theory.

2. The space-time surface associated with a given 3-surface is analogous to a Bohr orbit of the old fashioned quantum theory. The point is that the initial value problem in question differs from the ordinary initial value problem in that although the values of the H coordinates h^k as functions $h^k(x)$ of X^3 coordinates can be chosen arbitrarily, the time derivatives $\partial_t h^k(x)$

at X^3 are uniquely fixed by the principle selecting preferred extremals as generalized Bohr orbits (absolute minimization or probably something more delicate such as criticality [K27], existence of quaternionic tangent space structure [K19], or Hamilton-Jacobi structure [K2]) unlike in the ordinary variational problems encountered in the classical physics. This implies something closely analogous to the quantization of the symplectic momenta so that the space-time surface can be regarded as a generalized Bohr orbit. The classical quantization of electric charge and mass are possible consequences of the Bohr orbit property.

3. Kähler function is Diff^4 invariant in the sense that the value of the Kähler function is same for all 3-surfaces belonging to the orbit of a given 3-surface. As a consequence, WCW metric is Diff^4 degenerate. The implications of the Diff^4 invariance have turned out to be decisive, not only for the geometrization of WCW, but also for the construction of the quantum theory. For instance, time like vibrational modes tangential to the 4-surface imply tachyonic mass spectrum unless they correspond to the zero modes of WCW metric. Diff^4 invariance however guarantees the required kind of degeneracy of the metric.

4. The non-determinism of Kähler action means that the complete reduction to the light-cone boundary is not possible. This means a mathematical challenge but is physically a highly desirable feature since otherwise time would be lost as it is lost in the canonically quantized general relativity.

The most general expectation is that WCW can be regarded as a union of coset spaces: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is determined as absolute minimum of Kähler action.

It will be found that in the case of $M^4_+ \times CP_2$ Kähler geometry, or strictly speaking contact Kähler geometry, characterized by a degenerate Kähler form (Diff^4 degeneracy and plus possible other degeneracies) seems possible. Although it seems that this construction must be generalized by allowing all light like 7-surfaces $X^3_l \times CP_2$, at least those for which X^3_l is boundary of light-cone inside M^4_+ or M^4, with the physical interpretation differing dramatically from the original one, the original construction discussed in the sequel involves the most essential aspects of the problem.

2.3.2 How to identify preferred extremals of Kähler action?

The first guess for preferred extremals of Kähler action defining the Bohr orbits was that they correspond to absolute minima of Kähler action. One can criticize this assumption, and I have proposed several identifications of preferred extremals [K2, K31] and some of them could be equivalent.

The number theoretical vision discussed in [K19] would suggest the separate minimization of magnitudes of positive and negative contributions to the Kähler action. It must be emphasized that this option need not conform nicely with number theoretical universality since in p-adic context absolute minimization does not make sense and should be replaced by some algebraic notion. The non non-vanishing determinant for Hessian of Kähler action would be such a purely algebraic condition characterizing absolute minimum and maximum but would not be able to distinguish between then. This notion is not consistent with the idea that quantum criticality has criticality of preferred extremals as space-time correlate [K27] since at criticality the Hessian is degenerate.

For this option Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant: note that they would be only inertial vacua and carry non-vanishing density gravitational energy. The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself.

The 3-surfaces for which CP_2 projection is at least 2-dimensional and not a Lagrange manifold would correspond to non-vacua since conservation laws do not leave any other option. The variational principle would favor equally magnetic and electric configurations whereas absolute minimization of action based on S_K would favor electric configurations. The positive and negative contributions would be minimized for 4-surfaces in relative homology class since the boundary of X^4 defined by the intersections with 7-D light-like causal determinants would be fixed. Without this constraint only vacuum bubbles would result.

The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable solution to the construction of Kähler function.

It should be noticed that the considerations of this chapter relate only to the extremals of Kähler action which need not be absolute minima nor more general preferred extremals discussed in [K19] although this is suggested by the high symmetries. The number theoretic approach based on the properties of quaternions and octonions discussed in the chapter [K19] leads to a proposal for the general solution of field equations based on the generalization of the notion of calibration [A11] providing absolute minima of volume to that of Kähler calibration. This approach will not be discussed in this chapter.

2.4 The construction of WCW geometry from symmetry principles

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

The gigantic size of the isometry group suggests that it might be possible to deduce very detailed information about the metric of the WCW by group theoretical arguments. This turns out to be the case. In order to have a Kähler structure, one must define a complexification of WCW. Also one should identify the Lie algebra of the isometry group and try to derive explicit form of the Kähler metric using this information. One can indeed construct the metric in this manner but a rigorous proof that the corresponding Kähler function is the one defined by Kähler action does not exist yet although both approaches predict the same general qualitative properties for the metric. The argument stating the equivalence of the two approaches reduces to the hypothesis stating electric-magnetic duality of the theory. For the Bohr orbit like preferred extremals of Kähler action magnetic WCW Hamiltonians derivable from group theoretical approach are essentially identical with electric WCW Hamiltonians derivable from Kähler action.

2.4.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of WCW consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as absolute minimum of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$

Kähler function would have the same value so that $Diff^4$ invariance and degeneracy would be the outcome.

This picture is however too simple.

1. The degeneracy of the absolute minima caused by the classical non-determinism of Kähler action however brings in additional delicacies, and it seems that the reduction to the light-cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving more general light like 7-surfaces $X^3_l \times CP_2$.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_+ \times CP_2$ manifest themselves as the properties of propagators and vertices, and that M^4 is favored over M^4_+. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of WCW s associated with various 7-D causal determinants. The minimum assumption is that all possible unions of future and past light-cone boundaries $\delta M^4_\pm \times CP_2 \subset M^4 \times CP_2$ label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^3_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers. One cannot exclude the possibility that even more general light like surfaces $X^3_l \times CP_2$ of M^4 are important as causal determinants.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 is unique among all its $Diff^4$ translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3 and also a preferred light like 7-surface $X^3_l \times CP_2$.

This is indeed possible. The possibility of negative values of Poincare energy(or equivalently inertial energy) inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action in the set of 4-surfaces going through X^3. These space-time sheets should also define uniquely the light like 7-surface $X^3_l \times CP_2$, most naturally as the "earliest" surface of this kind. Note that this means that it become possible to assign a unique value of geometric time to the space-time sheet.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

2.4.2 Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy

and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

The most natural option is that symplectic and Kac-Moody algebras together generate the isometry algebra and that the corresponding transformations leaving invariant the partonic 2-surfaces and their 4-D tangent space data act as gauge transformations and affect only zero modes.

2.4.3 Does the symmetric space property reduce to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{2.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

WCW geometry allows two super-conformal symmetries assignable the coset space decomposition G/H for a sector of WCW with fixed values of zero moes. One can assign to the tangent space algebras g resp. h of G resp. H analogous to Kac-Moody algebras super Virasoro algebras and construct super-conformal representation as a coset representation meaning that the differences of super Virasoro generators annihilate the physical states. This obviously generalizes Goddard-Olive-Kent construction Sugawara.

The original conjecture was that the four-momenta associated with the two representations are identical. The physical interpretation would be in terms of Equivalence Principle (EP). This need not to be the case and the four-momenta associated with H vanish naturally. Later a more feasible identification of quantal and classical variants of EP has emerged [K23].

The identification of the two algeras is not a mechanical task and has involved a lot of trial and error. The algebra g should be be spanned by the generators of super-symplectic algebra of light-cone boundary and by the Kac-Moody algebra acting on light-like orbits of partonic 2-surfaces. The sub-algebra h should be spanned by generators which vanish for a preferred point of WCW analogous to origin of $CP_2 = SU(3)/U(2)$. Now this point would correspond to maximum or minimum of Kähler function (no saddle points are allowed if the WCW metric has definite signature). In hindsight it is obvious that the generators of both symplectic and Kac-Moody algebras are needed to generate g and h: already the effective 2-dimensionality meaning that 4-D tangent space data of partonic surface matters requires this.

The maxima of Kähler function could correspond to this kind of points and could play also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories. It took quite a long time to realize that Kähler function must be identified as Kähler action for the Euclidian region of preferred extremal. Kähler action for Minkowskian regions gives imaginary contribution to the action exponential and has interpretation in terms of Morse function. This part of Kähler action can have and is expected to have saddle points and to define Hessian with signature which is not positive definite.

2.4.4 What effective 2-dimensionality and holography really mean?

Concerning the interpretation of Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with holography.

1. Holography suggests that light-like 3-surfaces with fixed ends give rise to same WCW metric and the deformations of these surfaces by Kac-Moody algebra correspond to zero modes just like the interior degrees of freedom for space-like 3-surface do. The same would be true for space-like 3-surfaces at the ends of space-time surface with respect to symplectic transformations.

2. The non-trivial action of Kac-Moody algebra in the interior of X_l^3 together with effective 2-dimensionality and holography would encourage the interpretation of Kac-Moody symmetries acting trivially at X^2 as gauge symmetries. Light-like 3-surfaces having fixed partonic 2-surfaces at their ends would be equivalent physically and effective 2-dimensionality and holography would be realized modulo gauge transformations. As a matter fact, the action on WCW metric would be a change of zero modes so that one could identify it as analog of conformal scaling. The action of symplectic transformations vanishing in the interior of space-like 3-surface at the end of space-time surface affects only zero modes.

3. Gauge symmetry property means that the Kähler metric of the WCW is same for all gauge equivalent choices of X_l^3 and Kac-Moody deformations correspond to zero modes. Kähler function could differ by a real part of a holomorphic function of configuration space coordinates representing now Kac-Moody transforms of X_l^3. If Dirac determinant gives the exponent of Kähler function, the eigenvalues of the Kähler-Dirac action can differ only by scalings with are products of holomorphic function of WCW coordinates and its conjugates labeling different Kac-Moody transforms of X_l^3. This condition makes sense if one restricts the consideration to the finite number of eigenvalues λ_k assigned to D_K. The introduction of instanton term transforming the eigenvalues to $\lambda_k + \sqrt{n}$ would not allow his scaling.

   Either one must assume more general spectrum of form $\lambda_k + \sqrt{n}x_k$ with λ_k and x_k scaling in identical manner or that $n = 0$ modes are enough to define Kähler function. The latter option might be correct since the preferred extremal realizes effective 2-dimensionality at space-time level and conformal excitations break it so that they should not contribute to Kähler function. Also number theoretic universality favors this option. One cannot however exclude the first option. It must be admitted that the situation is not completely understood.

2.5 Attempts to identify WCW Hamiltonians

I have made several attempts to identify WCW Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate is based on the formulation of quantum TGD using 3-D light-like surfaces identified as orbits of partons. The proposal is out-of-date but the most recent proposal is obtained by a very straightforward generalization from the proposal for magnetic Hamiltonians discussed below.

2.5.1 Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM_+^4 have zero norm, one ends up with an explicit identification of the symplectic structures of WCW. There is almost unique identification for the symplectic structure. WCW counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$Q_m(H_A, X^2) = Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ ,$$

$$Q_m^+(H_A, r_M) = Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ ,$$

$$J \equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q_m^+.

$$Q_m^{\alpha,\beta}(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q_m^+(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the WCW metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

2.5.2 Generalization

The generalization for definition WCW super-Hamiltonians defining WCW gamma matrices is discussed in detail in [K31] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the Kähler-Dirac action: in particular, about their localization at string worlds sheets (right handed neutrino could be an exception). Second quantized Noether charges in turn define representation of WCW Hamiltonians as operators.

The basic formulas generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Right handed neutrino spinor is replaced with any mode of the Kähler-Dirac operator localized at string world sheet in the case of Kac-Moody sub-algebra of super-symplectic algebra corresponding to symplectic isometries at light-cone boundary and CP_2. The original proposal involved only the contractions with covariantly constant right- handed neutrino spinor mode but now one can allow contractions with all spinor modes - both quark like and leptonic ones. One obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

The resulting super Hamiltonians define WCW gamma matrices. They are labelled by two conformal weights. The first one is the conformal weight associated with the light-like coordinate of $\delta M^4_\pm \times CP_2$. Second conformal weight is associated with the spinor mode and the coordinate along stringy curve and corresponds to the usual stringy conformal weight. The symplectic conformal weight can be more general - I have proposed its spectrum to be generated by the zeros of Riemann zeta. The total conformal weight of a physical state would be non-negative integer meaning conformal confinement. Symplectic conformal symmetry can be assumed to be broken: an entire hierarchy of breakings is obtained corresponding to hierarchies of sub-algebra of the symplectic algebra isomorphic with it quantum criticalities, Planck constants, and dark matter.

The presence of two conformal weights is in accordance with the idea that a generalization of conformal invariance to 4-D situation is in question. If Yangian extension of conformal symmetries is possible and would bring an additional integer n telling the degree of multilocality of Yangian generators defined as the number of partonic 2-surfaces at which the generator acts. For conformal algebra degree of multilocality equals to $n = 1$.

2.6 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states angular momentum (and possibly also of Lorentz boost), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to "positive" frequencies and which to "negative frequencies" and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{2.2}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{2.3}$$

   In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to use the "half Poisson bracket"

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{2.4}$$

Here the subscript + and − refer to complex isometry current and its complex conjugate in terms of which the "half Poisson bracket" can be expressed.

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

2.7 WCW spinor structure

Quantum TGD should be reducible to the classical spinor geometry of WCW. In particular, physical states should correspond to the modes of WCW spinor fields. The immediate consequence is that WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of WCW spinor structure there are some important clues.

1. The classical bosonic physics is coded into the definition of WCW metric; therefore the classical physics associated with the spinors of the imbedding space should be coded into the definition of WCW spinor structure. This means that the generalized massless Dirac equation for the induced spinor fields on $X^4(X^3)$ should be closely related to the definition of WCW gamma matrices.

2. Complex probability amplitudes (scalar fields) in the WCW correspond to the second quantized boson fields in X^4. Hence the spinor fields of WCW should correspond to the second quantized, free, induced spinor fields on X^4. The space of WCW spinors should be just the Fock space of the second quantized fermions on X^4!

3. Symplectic algebra might generalize to a super symplectic algebra and that super generators should be linearly related to the gamma matrices of WCW. If this indeed is the case then the construction of WCW spinor structure becomes a purely group theoretical problem.

The realization of these ideas is simple in principle. Perform a second quantization for the free induced spinor field in X^4. Express WCW gamma matrices and symplectic super generators as superpositions of the fermionic oscillator operators. This means that WCW gamma matrices are analogous to spin 3/2 fields and can be regarded as a superpartner of the gravitational field of WCW. Deduce the anti-commutation relations of the spinor fields from the requirement of super symplectic invariance. Generalize the flux representation for the WCW Hamiltonians to a spinorial flux representation for their super partners.

2.7.1 WCW gamma matrices as super algebra generators

The basic idea is that the space of WCW spinors must correspond to the Fock space for the second quantized induced spinor fields. In accordance with this the gamma matrices of the configuration space must be expressible as superpositions of the fermionic oscillator operators for the second quantized induced free spinor fields in X^4 so that they are analogous to spin 3/2 fields. The Dirac equation is fixed from the requirement of super symmetry and has same vacuum degeneracy as Kähler action. A further assumption is that the contractions of the gamma matrices with isometry currents correspond to super charges of the group of isometries of WCW so that the construction reduces to group theory.

The super Kac Moody algebra was assigned originally with light like 3-D causal determinants but has a more natural identification as the Kac-Moody algebra of symplectic isometries. The corresponding gammma matrices (super Hamiltonians) are essentially inner products of the modes of induced spinor field with the second quantized spinor field and all modes of indued spinor fields with all possible charge states are allow. For the entire symplectic algebra only the inner products with right-handed neutrino spinors define the super-generators. This implies that super-generators are labelled by two conformal weights. The first conformal weight is associated with the imbedding space Hamiltonians and corresponds to the light-like radial coordinate of light-cone boundary. Second conformal weight labels the spinor modes localized at 2-D string world sheets. The super generators are integrals of the spinor modes localized at 1-D stringy curves so that one has formally a 3-D situation [K27, K31]. Holography implied by the strong form of general coordinate invariance however implies effective 2-dimensionality. Gamma matrices define the components of WCW metric as anti-commutators.

2.7.2 The Kähler-Dirac equation and gamma matrices

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the vacuum functional of the theory is identifiable as Dirac determinant. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the Kähler-Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique Kähler-Dirac action, which is internally consistent and super-symmetric. Space-time geometry must carry information about conserved quantum charges assignable to partonic 2-surfaces and it took considerable to to realize that this is achieved via a measurement interaction terms which are Lagrangian multiplier terms expressing that conserved classical charges are identical with their quantum counterparts in Cartan algebra for the space-time surfaces in quantum superposition representing the outcome of measurement. This makes sense if classical charges parametrize zero modes.

Second key idea [K27, K31] is that the well-definedness of em charge eigenvalue for spinor modes requires their localization to 2-D string world sheets. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet. Due to the presence of classical W boson fields this is possible only if localization takes plce at 2-D string world sheets and partonic 2-surfaces. Therefore string theory like structure emerges as part of TGD. The super Hamiltoanians defined in terms fluxes of Hammiltonians over partonic 2-surfaces are modified: a super-Hamiltonian at point of partonic 2-surface is replaced with an integral over stringy curve connecting points of two partonic 2-surfaces.

2.8 What about infinities?

The construction of a divergence free and unitary inner product for the WCW spinor fields is one of the major challenges. In the sequel constraints on the geometry of WCW posed by the finiteness of the inner product are analyzed.

2.8.1 Inner product from divergence cancellation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of WCW over the reduced WCW containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ("light-cone boundary") using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{2.5}$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light-cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by WCW integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of WCW integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the non-compact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [B10]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \ . \tag{2.6}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancellation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the WCW into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \rightarrow y,\beta) = \sum_{r,s} |S(r,\alpha \rightarrow s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since WCW metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in WCW integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems stating that semiclassical approximation is exact for certain systems (for example Duistermaat-Hecke theorem for integrable systems [A7]). Symmetric space property suggests that Kähler function might possess the properties guaranteeing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving WCW spinor fields would be

completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that WCW integrals are continuable to p-adic number fields requires this kind of reduction.

2.8.2 Divergence cancellation, Ricci flatness, and symmetric space and Hyper Kähler properties

In the case of the loop spaces left invariance implies that Ricci tensor is a multiple of the metric tensor so that Ricci scalar has an infinite value. Mathematical consistency (essentially the absence of the divergences in the integration over WCW) forces the geometry to be Ricci flat: in other words, vacuum Einstein's equations are satisfied. It can be shown that Hyper Kähler property guarantees Ricci flatness. The reason is that the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(\infty)$ generators instead of $U(\infty)$ generators as in case of loop spaces, so that the traces vanish.

Hyper Kähler property requires a quaternionic structure in the tangent space of WCW. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper-Kähler property means the possibility to perform complexification in S^2-fold manners. An interesting possibility raised by the notion of visionb is that hyper Kähler structure could be replaced with what might be called "hyper-hyper-Kähler structure" resulting when quaternionic tangent space is replaced with its hyper-quaternionic variant. This would conform with the Minkowski signature of the space-time surface. In this framework also hyper-octonionic structure might be considered. An interesting question not yet even touched, is whether the conjectured $M^8 - -M^4 \times CP_2$ duality is realized also at the level of the WCW of 3-surfaces.

Consider now the arguments in favor of Ricci flatness of the WCW.

1. The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property. In the following argument reader can well consider replacing the attribute "quaternionic" with "hyper-quaternionic".

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of WCW in vibrational modes is indeed multiple of four as required by Hyper

Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of WCW. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of WCW. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at $X^2_+ \times CP_2$ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

3. One can see the super-symplectic conformal weights as points in a particular complex plane of the quaternionic space and the choice of this plane corresponds to a selection of one WCW Kähler structure which are parameterized by S^2. The necessity to restrict the conformal weights to a complex plane brings in mind the commutativity constraint on simultaneously measurable quantum observables.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of WCW and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

3 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K4] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

3.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on Kähler-Dirac gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

3.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal

and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{3.1}$$

A more general form of this duality is suggested by the considerations of [K8] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{3.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{3.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

3.1.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1], [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (3.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (3.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (3.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (3.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

3.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K15] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{3.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{3.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

3.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{3.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K16]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K23]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

3.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

3.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

3.2.2 Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

3.2.3 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D1].

3.2.4 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K7]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K10]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K11].

3.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K2]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \quad . \tag{3.11}$$

The (1, 1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^{\alpha}\partial_{\alpha}\phi = -j^{\alpha}A_{\alpha} \ . \tag{3.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^{\alpha}/dt = j_K^{\alpha}$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_{\beta}^K \partial_{\gamma} j_{delta}^K \ = \ 0 \ . \tag{3.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B7] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K2]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^{\alpha}\partial_{\alpha}\phi = \partial_{\alpha}j^{\alpha}\phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^{\alpha}\phi$ and $j_I^{\alpha}\phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^{\alpha}\partial_{\alpha}\phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_{\alpha}(j^{\alpha}\phi) \ = \ 0 \ . \tag{3.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_{\phi}^e = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_{\phi}^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential

couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with 1-D Dirac action in induced metric at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

9. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

4 Von Neumann algebras and TGD

The work with TGD inspired model [K24] for topological quantum computation [K24] led to the realization that von Neumann algebras [A19, A25, A20, A6], in particular so called hyper-finite factors of type II_1 [A14], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [A17] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter.

4.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A14].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

4.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [K27] based on the notion of delta function, plus the emergence of s [A9], the possibility to formulate the notion of delta function rigorously in terms of distributions [A13, A23], and the emergence of path integral approach [A21] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A22, A26] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A16] relate closely to type II_1 factors. In topological quantum computation [K24] based on braid groups [A27] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B5] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B9, B4].

4.3 Factors of type II_1 and quantum TGD

For me personally the realization that TGD Universe is tailored for topological quantum computation [K24] led also to the realization that hyper-finite (ideal for numerical approximations) von Neumann algebras of type II_1 have a direct relevance for TGD.

The basic facts about hyper-finite von Neumann factors of type II_1 suggest a more concrete view about the general mathematical framework needed.

1. The effective 2-dimensionality of the construction of quantum states and WCW geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the generators of conformal algebras, the gamma matrices of WCW, and the modes of the induced spinor fields are labelled by discrete labels. Hence the tangent space of WCW is a separable Hilbert space and its Clifford algebra is a hyper-finite type II_1 factor. Super-symmetry requires that the bosonic algebra generated by WCW Hamiltonians and the Clifford algebra of WCW both correspond to hyper-finite type II_1 factors.

2. Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, WCW can be regarded as union of big-bang/big crunch type WCWs obtained as a union of light-cones parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4

are hyper-finite and of type III_1 in algebraic quantum field theory [B9]. The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

3. Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner to imbed interacting sub-systems to a universal II_1 factor which now naturally corresponds to the infinite Clifford algebra of the tangent space of WCW of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, the formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations might be described by Jones inclusion [A1, A8] characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and quantum group in case of $\mathcal{M} : \mathcal{N} < 4$ and conformal theory with $k = 1$ Kac Moody for $\mathcal{M} : \mathcal{N} = 4$ [B6] .

4. von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that dimensional regularization works only in $D = 4 - \epsilon$. Dimensional regularization with space-time dimension $D = 4 - \epsilon \rightarrow 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

4.4 Does quantum TGD emerge from local version of HFF?

There are reasons to hope that the entire quantum TGD emerges from a version of HFF made local with respect to $D \leq 8$ dimensional space H whose Clifford algebra $Cl(H)$ raised to an infinite tensor power defines the infinite-dimensional Clifford algebra. Bott periodicity meaning that Clifford algebras satisfy the periodicity $Cl(n + k8) \equiv Cl(n) \otimes Cl(8k)$is an essential notion here [K26, K6]. The points m of M^k can be mapped to elements $m^k \gamma_k$ of the finite-dimensional Clifford algebra $Cl(H)$ appearing as an additional tensor factor in the localized version of the algebra.

The requirement that the local version of HFF is not isomorphic with HFF itself is highly non-trivial. The only manner to achieve non-triviality is to multiply the algebra with a non-associative tensor factor representing the space of hyper-octonions M^8 identifiable as sub-space of complexified octonions with tangent space spanned by real unit and octonionic imaginary unit multiplied by commuting imaginary unit (for a good review about properties of octonions see [A5]).

Space-times could be regarded equivalently as surfaces in M^8 or in $M^4 \times CP_2$ and the dynamics would reduce to associativity (hyper-quaternionicity) or co-associativity condition. It is rather remarkable that CP_2 forced by the standard model symmetries has also a purely number theoretic interpretation as parameterizing hyper-quaternionic four-planes containing a preferred hyper-octonionic imaginary unit defining hyper-complex structure in M^8. Physically this choice corresponds to a choice of Cartan algebra of Poincare algebra for which the system is at rest so that a connection with quantum measurement theory is suggestive. Color group is identifiable as a subgroup of octonionic automorphism group G_2 respecting this choice.

4.5 Quantum measurement theory with finite measurement resolution

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ [A1, A15] of these algebras lead to quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ [K26, K6]. Quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ interpreted as $\mathcal{N}$-module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by $\mathcal{N}$-rays and the notions of unitarity, hermiticity, and eigenvalue generalize [K3, K6].

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole.

At the level of conscious experience, the entanglement below measurement resolution would give rise to a pool of shared and fused mental images giving rise to "stereo consciousness" (say stereovision) [K9] so that contents of consciousness would not be something completely private as usually believed. Also fuzzy logic emerges naturally since ordinary spinors are replaced by quantum spinors for which the discrete spectrum of the eigenvalues of the moduli of its spinor components can be interpreted as probabilities that corresponding belief is true is [E2] [K26].

4.6 Cognitive consciousness, quantum computations, and Jones inclusions

Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1, 0) so that quantum qubits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

4.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [J2] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J3]. The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i, j), where i, j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha, \beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha - \beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha - \beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J2] are not consistent with this prediction [J1] and this is identified as the anomaly.

2. The QM prediction $E(\alpha, \beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha - \beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J1].

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly 2) but not anomaly a). A "mundane" explanation for anomaly 1) can be imagined [K26].

5 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

5.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E3] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K17, K14] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K18]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K21].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E3] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra

currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K21], [L2].

5.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the Kähler-Dirac gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

5.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and multiplies YM action action by n_an_b which is equivalent with the $\hbar = n_an_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_an_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	n_an_b	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_an_b}$

5.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K5].

5.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

5.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

5.7.1 1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \to CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

5.7.2 Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi^n_k = \partial L_K/\partial h^k_n$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi^n_k = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π^0_k and π^n_k. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

   (b) This picture is not quite correct yet. The fixing of π^0_k and π^n_k should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π^0_k and π^n_k must be fixed at X^3 and X^3_l in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X^3_l as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π^k_0 would give n_b branches in CP_2 degrees of freedom and the conditions for π^n_k would split each of these branches to n_a branches.

   (c) The existence of these two kinds of conserved charges (possibly vanishing for π^n_k) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g^2_K$ factor of the action with $\hbar/g^2_K$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = n g_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One

can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

   The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

5.7.3 Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

5.8 Updated view about the hierarchy of Planck constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

5.8.1 Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K22].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K15] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E3] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

   It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the Kähler-Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

5.8.2 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the Kähler-Dirac gamma matrices [K27] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is “prepared” meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

5.8.3 Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \to k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible

coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

5.8.4 Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$: th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N - 1$ and the maximum orbital angular momentum would correspond the sum

$\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

5.8.5 What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the Kähler-Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

   $$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \ . \tag{5.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

   $$p^2 = \frac{n^2\hbar^2}{K^2 L^2} \ .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

   $$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} \ . \tag{5.2}$$

   For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 \quad . \tag{5.3}$$

   This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{5.4}$$

   This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

5.8.6 Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E1, E4] discussed in [K30] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

1. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

2. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

3. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K29, K28] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   (a) If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K5] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   (b) For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

   (c) Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to.62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.

   Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

   It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

   (d) The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is.2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

4. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached

elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

5. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38, .99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

6. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

5.8.7 Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K27]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level, to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K20] so that conformal dynamics represents conformal evolution) [K19].

6 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M^8_c using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M^8_c composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions

$q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M_c^8. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCWs associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

2. One can formulate associativity in M^8 (or M_c^8) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H$... by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the Kähler-Dirac gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by Kähler-Dirac gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be

considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

6.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M^8_c. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

3. Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \to S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \to M^4$ (this is modification

to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition generalizes to the case of M^8_c: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M^8_c$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a manner to build quaternionic space-time surfaces properly.

2. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

3. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K2]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

4. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K27] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

5. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

6. One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

6.2 Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K25]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H$....

This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

6.3 Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the M^8-H duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

6.3.1 Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

6.3.2 Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

6.3.3 Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1+1+3+\overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1+kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

4. The square of Dirac operator gives potential term proportional to $r^2 = x^2+y^2+z^2+t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1+ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

   Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos

so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

6.3.4 What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

6.4 How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H...$ iteration generating new solutions from existing ones.

6.4.1 Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

6.4.2 Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a,b,c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{6.1}$$

   The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{6.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^By^Cz^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A5] (see Fig. 6.4.2) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

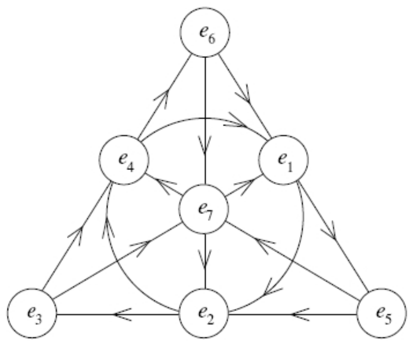

Figure 1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

6.5 Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A5] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2: s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

6.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

6.6.1 Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

   The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H.

3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

6.6.2 Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

6.6.3 Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

   This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

6.6.4 $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K13].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

6.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

7 Does Kähler-Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the Kähler-Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action. One cannot however get rid of Kähler action since the gamma matrices appearing in Kähler-Dirac action are defined in terms of canonical momentum densities of Kähler action. The most one can hope is that Dirac determinant reduces to the exponent of Kähler action for preferred extremals.

7.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. An infinite hierarchy of these currents is expected and they would define fermionic counterparts for zero modes. In number theoretic vision space-time surfaces are proposed to be identifiable as associative (co-associative) surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking [?]

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the Kähler-Dirac (Kähler-Dirac) equation. The requirement that there are deformations of the space-time surface -actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the Kähler-Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

3. The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [K27], [L4] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [K27], [L4]. If Kähler current defines Beltrami flow [B7] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

4. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The Kähler-Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

7.2 Quantum criticality and Kähler-Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the Kähler-Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

7.2.1 Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a far reaching generalization of conformal symmetries. The development of the understanding of conservation laws has been slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

1. The Kähler-Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the Kähler-Dirac action under this deformation vanishes. The vanishing of the first variation

for the Kähler-Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the Kähler-Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J_k^\alpha \Psi \ , \\ J_k^\alpha &= \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^k + \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h^l}\delta h^l \ . \end{aligned} \tag{7.1}$$

Here h_β^k denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. It is essential that the Kähler-Dirac equation holds true so that the Kähler-Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the Kähler-Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{7.2}$$

Here $1/D$ is the inverse of the Kähler-Dirac operator defining the counterpart of the fermionic propagator.

3. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{7.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the Kähler-Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [A3, A4]. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing Kähler-Dirac gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{7.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [A2].

4. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the Kähler-Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

5. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P\times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

   (a) Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [A24] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [A2]. Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

   (b) The understanding of the contributions to Kähler-Dirac action has been slow. It seems that what is needed is Chern-Simons Dirac action assigned to partonic orbits: this was the original proposal. The condition that the action of C-S-D operator reduces to that of massless M^4 Dirac operator. $Gamma^n\Psi = p^k\gamma_k\Psi$ would be space-time counterpart for the massless Dirac equation at the level of imbedding space. I have called this condition earlier generalized eigenvalue condition.

   The assumption that C-S-D is present strongly suggests that also Kähler action contains C-S term meaning that the C-S terms from Kähler action are cancelled at partonic orbits for preferred extremals. If C-S term is present also at space-like ends of space-time surface Kähler action and therefore also Kähler function vanishes identically. At the ends of space-time surface one would therefore have $\Gamma^n\Psi = 0$ if C-S-D term is not present. Hence this assumption seems unphysical. One would have massless Dirac propagator at the fermionic lines defined by the partonic boundaries of Kähler-Dirac equation and on-mass-shell condition at the space-like ends of the space-time surface.

   If this is correct interpretation then the fermionic lines identified as boundaries of string world sheets correspond to massless fermion propagators and the stringy propagators

$1/L_0$ could be associated with fermion fermion scattering at wormhole contacts (see fig. **??** in the appendix of this book). The generalized Feynman diagrammatics would be a combination of stringy and Feynman diagrammatics. External fermion lines would carry massless on-shell momenta and wormhole contacts could be seen as massive bound states of massless fermions falling into representations of super-conformal algebras assignable to wormhole contacts. This would allow stringy variant of twistor approach.

2. The action defined by four-volume gives a first glimpse about what one can expect. In this case Kähler-Dirac gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

3. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

4. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

5. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for Kähler-Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. The conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires additional conditions to be satisfied and the holomorphy of string world sheets (partonic 2-surfaces) and associated Kähler-Dirac gamma matrices makes this possible [K27].

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the Kähler-Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K6] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the

understanding of the relationship between criticality and hierarchy of Planck constants [K8], [L3]. Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

7.2.2 Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

The vanishing of second variations of preferred extremals - at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that WCW metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B3] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

7.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

7.3.1 Why Kähler-Dirac action?

1. Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K4, K19].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

2. Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial {h^k}_\alpha} L_K \ . \end{aligned} \tag{7.5}$$

Here T^α_k is canonical momentum current of Kähler action. If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{7.6}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{7.7}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{7.8}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T^\alpha_l \Gamma^l \ . \end{aligned} \tag{7.9}$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{7.10}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with Kähler-Dirac gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \tag{7.11}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

As a matter fact, any mode of Kähler-Dirac equation contracted with second quantized induced spinor field or its conjugate defines a conserved super charge. Also super-symplectic Noether charges and their super counterparts can be assigned to symplectic generators as Noether charges but they need not be conserved.

3. Does the Kähler-Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K19]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the Kähler-Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the Kähler-Dirac action so that there is no need to introduce this mixing by hand.

7.3.2 Overall view about Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence.

The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.

2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localiztion to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the ferminonic liness expressing 8-D light-likeness or 4-D light-likeness in effective metric.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.

   In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action

$$L_{ind} = \int \overline{\Psi} \Gamma^t_{ind} \partial_t \Psi \sqrt{g_1} dt$$

defined by the induced gamma matrix Γ^t_{ind} and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int \sqrt{g_1}dt$. It forces the boundary line to be a geodesic line. Dirac equation gives

$$\Gamma^t_{ind} D_t \Psi = ip^k(M^8)\gamma_k \Psi = 0 \ .$$

The square of the Dirac operator gives $(\Gamma^t_{ind})^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.

Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H. These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.

2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the imbedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.

   At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At imbedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n \Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

7.3.3 A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of

the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the Kähler-Dirac action? At least this question might make sense.

   (b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^\alpha &= \overline{\Psi} O \hat{\Gamma}^\alpha \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \tag{7.12}$$

   Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

   (c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

   (d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

   (e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

   (f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \to K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0}$= $kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

   The formulation of quantum TGD in terms of the Kähler-Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the Kähler-Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z)+\overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

7.3.4 How to calculate Dirac determinant?

If the modes of the Kähler-Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (Kähler-Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the 1-D Dirac operator associated with the induced metric at the light-like partonic orbits with vanishing metric determinant g_4.

The spectrum of light-like 8-momenta p^k is determined by the boundary conditions for 1-D Dirac operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of - now 8-D - mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of octonions in quaternionic sub-algebra defined by p^k. For

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

1. Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. By the generalizaton of AdS/CFT duality the action in question should be proportional to the area of the string world sheet in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices at string world sheet.

2. The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

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