About honeycombs of hyperbolic 3-space and their relation to the genetic code

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Abstract

$M^8 - H$ duality and the realization of holography in M^8 strongly suggests the importance of tessellations of H^3 (analogous to lattices of E^3) in the TGD based physics. These tessellations form a scale hierarchy and can thus appear in all scales. The hierarchy of effective Planck constants labelling dark matter as phases of ordinary matter indeed predicts quantum coherence in arbitrarily long scales and gravitational quantum coherence corresponds to the largest scales of quantum coherence among basic interactions.

There are 5 Platonic tessellations known as honeycombs: the 4 regular honeycombs correspond to cubic, icosahedral, and 2 dodecahedral honeycombs and a quasiregular icosa-tetrahedral honeycomb having tetrahedra, octahedra and icosahedra as cells. The icosa-tetrahedral honeycomb might define a universal realization of the genetic code as an induced structure so that the genetic code would be much more than a biochemical accident. These 5 Platonic honeycombs could occur also in astrophysical scales as gravitational tessellations. The recent discovery of gravitational hum might have an explanation as gravitational diffraction in this kind of a honeycomb.

In this chapter the properties of hyperbolic honeycombs are considered in detail and also a detailed view about the realization of DNA double strand in terms of the icosa-tetrahedral honeycomb is considered. The emerging model is surprisingly quantitative. Also a connection with the notion of memetic code and the realization of memetic codons in terms of 21 DNA codons are suggested by the model.

1 Introduction

$M^8 - H$ duality and the realization of holography in M^8 strongly suggests the importance of tessellations of H^3 (analogous to lattices of E^3) in the TGD based physics. These tessellations form a scale hierarchy and can thus appear in all scales. The hierarchy of effective Planck constants labelling dark matter as phases of ordinary matter indeed predicts quantum coherence in arbitrarily long scales and gravitational quantum coherence corresponds to the largest scales of quantum coherence among basic interactions.

The 4 regular honeycombs correspond to cubic, icosahedral, and 2 dodecahedral tessellations. The quasiregular icosa-tetrahedral honeycomb has tetrahedra, octahedra and icosahedra as cells having triangular faces as cells. These honeycombs serve as candidates for physically interesting tessellations. These 5 honeycombs are unique in that they involve only Platonic solids. I have proposed that the icosa-tetrahedral tessellation might define a universal realization of the genetic code as an induced structure so that the genetic code would be much more than a biochemical accident. The details of this realization are discussed in [L7, L3].

These 5 Platonic tessellations (or honeycombs, I will used these terms interchangeably in the sequel) could occur also in astrophysical scales as gravitational tessellations. The recent discovery of gravitational hum might have an explanation as gravitational diffraction in this kind of a tessellation. The unexpectedly large intensity of hum could be due to the concentration of the radiation intensity in discrete directions and due the fact that in diffraction the amplitude of the scattered field is proportional to the square N^2 of the number N of scatterers rather than N.

Icosa-tetrahedral tessellation relates to the TGD based view of the genetic code. The TGD inspired view of genetic code has evolved during decades.

1. The first model of the genetic code was based on the so-called Combinatorial Hierarchy [K1] [L7] and predicted what I called memetic code realized as sequences of 21 DNA codons. Surprisingly, this model made a comeback as I prepared this article.

2. After several stray paths I ended up from a model of music harmony [K2, L1] [L6, L3] based on Hamiltonian cycles at the icosahedron to a model of genetic code also involving the tetrahedral Hamiltonian cycle.

   The basic observation was that the 12-note scale could correspond to a Hamiltonian cycle of icosahedron such that the steps of the cycle define a quint cycle. 12-note scale is obtained from the quint by octave equivalence. There are 3-types of icosahedral Hamiltonian cycles and each cycle defines 20 3-chords assignable to the triangular faces of the icosahedron and defines a musical harmony.

One obtains 20+20+20 chords for the 3 different harmonies with symmetry groups Z_6, Z_4 and Z_2. The orbits of these groups define sets of 3-chords. The surprising finding was that if these sets are identified as amino acids, the numbers of the chords are the same as the numbers of DNAs coding for a given amino acid. By adding a tetrahedral Hamiltonian cycle one obtains 64 3-chords. At the level of molecules the music would be "music of light". Since music expresses and generates emotions, the idea that emotions appear already at the molecular level was natural. Different combinations of 3 Hamiltonian cycles with symmetries Z_6, Z_4 and Z_2 would correspond to different moods at bio-molecular level (why just 3?)

The model made almost correct predictions for the numbers of mRNA codons coding for amino-acids. I have discussed a considerable number of its variants during years and even considered the replacement of icosahedron and tetrahedron with some other geometric object.

The basic problem was that gluing the tetrahedron and icosahedron together looked ugly and would have allowed only 63 codons. At that time I did not yet realize that an icosahedron and tetrahedron could be parts of a bigger structure.

3. Second model was based on the realization of codons as dark proton triplets assumed to reside at the monopole flux tubes parallel to DNA strands [L1, L3]. Dark proton triplets would neutralize the constant negative charge of -3 units per codon. The model suggested that it might be possible to understand the numbers of DNA, RNA, tRNA and amino acids in terms of entangled states of dark proton triplets representing codons. The model had also problems: in particular, one had to assume an additional binary degree of freedom to get the number DNA and mRNA codons correctly and the proposed identifications of this new degree of freedom did not look quite realistic.

4. Icosa-tetrahedral realization [L7] of the code in terms of icosatetrahedral honeycomb of H^3 was the next step in the evolution of ideas. It was made possible only by the dramatic development of understanding of TGD itself, in particular of its number theoretical aspects related to $M^8 - H$ duality [L4, L5].

   The tessellations of the hyperbolic 3-space H^3 represented as possibly complex mass shell in $M^4_c \subset M^8_c$ and as light-cone proper time = constant hyperboloids in $M^4 \subset M^4 \times CP_2$ are central in the the realization of holography in TGD. Icosa-tetrahedral honeycomb is a completely unique tessellation involving only Platonic solids and all possible platonic solids, tetrahedron, icosahedron, and octahedron are present. Kind of a quantum Platonic holy trinity is in question.

   This led to a proposal of the genetic code in terms of icosa-tetrahedral honeycomb induced to the 3-surface by restriction. This realization could be assignable to the magnetic body of the system involving dark matter in the TGD sense. The realization would be universal and would not be restricted to mere biology. Counterparts of codons and genes can be realized also for higher-dimensional objects, say cell membrane and even brain.

   Icosa-tetrahedral realization led to a proposal that the realizations of the code in terms of dark photon triplets and in terms of dark proton triplets are closely related. I did not however really understand the properties of the icosa-tetrahedral honeycomb when I published the first article about it [L7].

   Sequences of N dark cyclotron photon triplets as representations of genes consisting of N dark proton triplets would make possible communications between dark genes by 3N-resonance. Genes would serve as addresses, much like in LISP, and the message would be coded by the modulation of the frequency scale. The details of this picture that were not discussed at that time create problems that are solved by the model based on icosahedral honeycomb.

In this article the properties of hyperbolic honeycombs are considered in detail and also a detailed view about the realization of DNA double strand in terms of the icosa-tetrahedral tessellation is considered. The emerging model is surprisingly quantitative and suggests a lot of new understanding about the dark realization of genetic code. Also a connection with the notion of memetic code [K1] [L2] and the realization of memetic codons in terms of 21 DNA codons are suggested by the model.

2 About honeycombs in hyperbolic 3-space

This section, written in 2023, represents some new understanding related to the tessellations of H^3 known as honeycombs.

2.1 Some preliminaries

Some preliminaries are needed in order to understand Wikipedia articles related to tessellations in general.

1. Schläfli symbol $\{p,r\}$ (`rb.gy/j36tg`) tells that the possibly existing Platonic solid $\{p,r\}$ has r p-polygons as faces meeting at each vertex. For instance, icosahedron $\{3,5\}$ has 5 triangles as faces meeting at each vertex.

   Schläfli symbol generalizes to higher dimensions. The analog of Platonic solid $\{p,r,q\}$ possibly in 4-dimensions and assignable to 3-sphere has q 3-faces which are Platonic solids $\{p,r\}$. This description is purely combinatorial and is recursive. For instance, one can start from 3-D dimensional Platonic solid $\{p,q\}$ with 3-D objects in dimension 4 by replacing p with p,r. One can also project this object to dimension 3. In this manner one obtains a projection of 4-cube (tesseract) $\{4,3,3\}$ for which 3 cubes $\{4,3\}$ meet at each vertex ($2^4=16$ of them) and which has 8 3-cubes as faces as a 3-D object.

   In the case of hyperbolic tessellations also strange looking Schläfli symbols $\{(p,q,r,s)\}$ are encountered: icosa-tetrahedral tessellation involving only Platonic solids has symbol $\{(3,3,5,3)\}$. My understanding is that this object corresponds to $\{3,3,5,3\}$ as an analogue of Platonic solid associate with 4-sphere in 5-D Euclidian space and that the fundamental region of this tessellation in H^3 is analogous to a 3-D projection of this object. At a given vertex 3 objects $\{3,3,5\}$ meet. For these objects 5 tetrahedrons meet at a given vertex.

2. Vertex figure is a further central notion. It represents a view of the fundamental region of tessellation from a given vertex. The vertices of the figure are connected to this vertex. It does not represent the entire fundamental region. For instance, for a cube (octahedron) it contains only the 3 (4) nearest vertices. For icosa-tetrahedral tessellation the vertex figure is icosidodecahedron (`rb.gy/3u4pq`). The interpretation of the vertex symbol of the hyperbolic icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) is a considerable challenge.

3. One cannot avoid Coxeter groups and Coxeter symbols (`rb.gy/48qhg`) in the context of tessellations. They code the structure of the symmetry group of say Platonic solid (tessellation of S^2). This symmetry group is generated by reflections with respect to some set of lines, usually going through origin. For regular polygons and Platonic solids is its discrete subgroup of rotation group.

   The Coxeter group is characterized by the number of reflection hyperplanes H_i and the reflections satisfying $r_i^2=1$. The products $r_{ij}=r_ir_j$ define cyclic subgroups of order c_{ij} satisfying $r_{ij}^{c_{ij}}=1$. Coxeter group is characterized by a diagram in which vertices are labelled by i. The orders of the cyclic subgroups satisfy $c_{ij}\geq 3$. For c_{ij} the generators r_i and r_{ij} commute. For $c_{ij}=2$ the vertices are not connected, for $c_{ij}=3$ there is a line and for $c_{ij}>3$ the number c_{ij} is assigned with the line. For instance, hyperbolic tessellations are characterized by 4 reflection hyperplanes.

   For instance, for p-polygon the Coxeter group has 2 generators and the cyclic group has order p. For Platonic solids the Coxter group has 3 generators and the orders of cyclic subgroups are 3, 4, or 5. For icosa-tetrahedral tessellation the order is 4.

2.2 The most interesting honeycombs in hyperbolic 3-space

H^3 allows an infinite number of tessellations. There are 9 types of honeycombs. This makes 76 uniform hyperbolic honeycombs involving only a single polyhedron (`hrb.gy/rs9h5`).

4 of these honeycomes are *regular*, which means that they have identical regular faces (Platonic solids) and the same numbers of faces around vertices. The following list gives the regular uniform honeycombs and their Schläfli symbols $\{p, q, r\}$ telling that each edge has around it regular polygon $\{p, q\}$ for which each vertex is surrounded by q faces with p vertices.

1. H1: 2 regular forms with Schläfli symbol {5,3,4} (dodecahedron) and {4,3,5} (cube).
2. H2: 1 regular form with Schläfli symbol {3,5,3}(icosahedron)
3. H5: 1 regular form with Schläfli symbol {5,3,5} (dodecahedron).

There is a large number of uniform honeycombs involving several cell types. There exists however a "multicellular" honeycomb, which is completely unique in the sense that for it all cells are Platonic solids. This icosa-tetrahedral (or more officially, tetrahedral-icosahedral) honeycomb has tetrahedrons, octahedrons, and icosahedrons as its cells. All faces are triangles. The icosa-tetrahedral honeycomb is of special interest since it might make possible the proposed icosa-tetrahedral realization of the genetic code (`rb.gy/h8xx0`).

From the Wikipedia article about icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) one learns the following.

1. The Schläfli symbol of icosa-tetrahedral honeycomb is $\{(3, 3, 5, 3)\}$. This combinatorial symbol allows several geometric representations. The inner brackets would refer to the interpretation as an analogue of the Platonic solid assignable to a 4-sphere of Euclidian 5-space. At each vertex 3 objects of type $\{3, 3, 5\}$ would meet. At the vertex of $\{(3, 3, 5)\}$ in turn 5 tetrahedrons meet.

2. Icosa-tetrahedral honeycomb involves tetrahedron $\{(3, 3\}$, octahedron $\{(3, 4)\}$, an icosahedron $\{(3, 5)\}$ as cells. That there are no other honeycombs involving several Platonic solids and only them as cells makes this particular honeycomb especially interesting. Octahedron with Schläfli symbol $\{3, 4\}$ can be also regarded as a rectified tetrahedron havig Schläfli symbol $r\{3, 3\}$.

3. The vertex figure of icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`), representing the vertices a lines connecting them is icosidodecahedron (`rb.gy/q5w62`), which is a "fusion" of icosahedron and dodecahedron having 30 vertices with 2 pentagons and 2 triangles meeting at each, and 60 identical edges, each separating a triangle from pentagon. From a given vertex VF=60 vertices connected to this vertex by an edge can be seen. In the case of cube, octahedron, and dodecahedron the total number of vertices in the polyhedron is 2(VF+1). It is true also now, one would have 122 vertices in the basic structural unit. The total number of vertices for the disjoint polyhedra is 6+4+12= 22 and since vertices are shared, the number of polyhedra in the basic unit must be rather large.

4. The numbers called "cells by location" could correspond to numbers 30, 20, and 12 for octahedrons, tetrahedrons and icosahedrons respectively inside the fundamental region of the tessellation defining the honeycomb. That the number of icosahedrons is smallest, looks natural. These numbers are quite large. The counts around each vertex are given by (3.3.3.3), (3.3.3), *resp.* (3.3.3.3) for octahedra, tetrahedra, *resp.* icosahedra and tell the numbers of vertices of the faces meeting at a given vertex.

5. What looks intriguing is that the numbers 30, 20, and 12 for octahedrons (O), tetrahedrons (T) and icosahedrons (I) correspond to the numbers of vertices, faces, and edges for I. As if the fundamental region would be obtained by taking an icosahedron and replacing its 30 vertices with O, its 20 faces with T and its 12 edges with I, that is by using the rules *vertex* $\rightarrow$ *octahedron*; *edge* $\rightarrow$ *I, face* $\rightarrow$ *T*. These 3-D objects would be fitted together along their triangular faces.

   Do the statements about the geometry and homology of I translate to the statements about the geometry and homology of the fundamental region? This would mean the following replacements:

   (a) "2 faces meet at edge" $\rightarrow$ "2 T:s share face with an I".

(b) "5 faces meet at vertex" $\rightarrow$ "5 T:s share face with an O".

(c) "Edge has 2 vertices as ends" $\rightarrow$ "I shares a face with 2 different O:s".

(d) "Face has 3 vertices $\rightarrow$ "T shares a face with 3 different O:s".

(e) "Face has three edges" $\rightarrow$ "T has a common face with 3 I:s".

2.3 An attempt to understand the hyperbolic honeycombs

The following general observations might help to gain some understanding of the honeycombs.

The tessellations of E^3 and H^3 are in many respects analogous to Platonic solids as 2-D objects. The non-compactness implies that there is an infinite number of cells for tessellations. It is important to notice that the radial coordinate r for H^3 corresponds very closely to the hyperbolic angle and its values are quantized for the vertices of tessellation just like the values of spherical coordinates are quantized for Platonic solids. The tessellations for E^3 are scale covariant. For a fixed radius of H^3 characterized by Lorentz invariance cosmic time this is not the case. One can however scale the value of a. What distinguishes between regular tessellations in E^3 and H^3 is that the metric of H^3 is non-flat and has negative curvature. H^3 is homogeneous space meaning that all points are metrically equivalent (this is the counterpart of cosmological principle in cosmology). Since both spaces have rotations as symmetries, this does not affect basic Platonic solids as 2-D structures assignable with 2-sphere if the edges are identified as geodesic lines of S^2. Quite generally, isometries characterize the tessellations, whose fundamental region corresponds to coset space of H^3/Γ by a discrete group of the Lorentz group acting as isometries of H^3. The modifications induced by the replacement $E^3 \rightarrow H^3$ relate to the 3-D aspects of the tessellation. This is because the metric is non-flat in the radial direction. The negative curvature implies that the geodesic lines diverge. One can use a counterpart of the standard spherical coordinates and in these coordinates the solid angles assignable to the vertices of Platonic solid are smaller than in E^3. Also the hyperbolic planes H^2 emerging from edges of the tessellation of H^3 diverge in normal direction the angles involved are smaller.

It is useful to start from the description of the Platonic solids. They are characterized combinatorially by integers and geometrically by various kinds of angles. Denote by p the number of vertices/edges of the face and by q the number of faces meeting at vertex.

1. Important constraints come from the topology and combinatorics. Basic equations for the numbers V ,E, and F for the number of vertices, edges and faces are purely topological equations $VE + F = 2$, and the equation $pF = 2E = qV$. Manipulation of these equations gives $1/r + 1/p = 1/2 + 1/E$ implying $1/r + 1/p > 1/2$. Since p and q must be at least 3, the only possibilities for $\{p, q\}$ are $\{3, 3\}$, $\{4, 3\}$, $\{3, 4\}$, $\{5, 3\}$, and $\{3, 5\}$.

2. The angular positions of the vertices at S^2 are basic angle variables. In H^3 hyperbolic angle assignable to the radial coordinate is an additional variable of this kind analogous to the position of the unit cell in the E^3 tessellation. The cosmological interpretation is in terms of redshift.

3. There is the Euclidian angle ϕ associated with the vertex of the face given by π/p. Here there is no difference between E^3 and H^3.

4. The angle deficit δ associated with the faces meeting at a given vertex due to the fact that the faces are not in plane in which case the total angle would be 2π. δ is largest for tetrahedron with 3 faces meeting at vertex and therefore with the sharpest vertex and smallest for icosahedron with 5 triangles meeting at vertex. This notion is essentially 3-dimensional, being defined using radial geodesics, so that the δ is not the same in H^3. In H^3 δ is expected to be larger than in E^3.

5. There is also the dihedral angle θ associated with the faces as planes of E^3 meeting at the edges of the Platonic solid. θ is smallest for a tetrahedron with 4 edges and largest for a dodecahedron with 20 edges so that the dodecahedron is not far from the flat plane and this angle is not far from π. The H^3 counterpart of θ is associated faces identified as hyperbolic planes H^2 and is therefore different.

6. There is also the vertex solid angle Ω associated with each vertex of the Platonic solid $\{p,q\}$ given by $\Omega = q\theta - (q-2)\pi$. For tessellations in E^3 the sum of these angles is 4π. In H^3 its Euclidian counterpart is larger than 4π.

7. The face solid angle is the solid angle associated with the face when seen from the center of the Platonic solid. The sum of the face solid angles is 4π. For Platonic solid with n vertices, one has $\Omega = 4\pi/n$. The divergence of the geodesics of H^3 implies that this angle is smaller in H^3: there is more volume in H^3 than in E^3.

E^3 allows only single regular tessellation having cube as a unit cell. H^3 allows cubic and icosahedral tessellations plus two tessellations having a dodecahedron as a unit cell. Why does E^3 not allow icosahedral and dodecahedral tessellations and how the curvature of H^3 makes them possible? Why is the purely Platonic tetra-icosahedral tessellation possible in H^3?

The first guess is that these tessellations are almost but not quite possible in E^3 by looking at the Euclidian constraints on various angles. In particular, the sum of dihedral angles θ between faces should be 2π in E^3, the sum of the vertex solid angles Ω at the vertex should be 4π. Note that the scaling of the radial coordinate r decreases the dihedral angles θ and solid angles Ω. This flexibility is expected to make possible so many tessellations and honeycombs in H^3. The larger the deviation of the almost allowed tessellation, the larger the size of the fundamental region for fixed a.

Consider now the constraints on the basic parameters of the Platonic solids (`rb.gy/1cuav`) in E^3 while keeping their H^3 counterparts in mind.

1. The values of didedral angle for tetrahedron, cube, octahedron, dodecahedron, and icosahedron are

$$[\theta(T),\theta(C),\theta(O),\theta(D),\theta(I)] \approx [70.3^\circ,\ 90^\circ, 109.47^\circ,\ 116.57^\circ, 138.19^\circ]\ .$$

   Note that $r=5$ tetrahedra meeting at a single edge in E^3 would almost fill the space around the edge. In E^3 $r=4$ cubes can meet at the edge. In H^3 r should be larger. This is indeed the case for the cubic honeycomb $\{4,3,5\}$ having $r=5$. For $r=3$ icosahedrons the sum dihedral angles exceeds 2π which conforms with the that $\{3,5,3\}$ defines an icosahedral tessellation in H^3. For the $r=4$ dodecahedra meeting at the edge the total dihedral angle is larger than 360°: $r=4$ is therefore a natural candidate in H^3. There are indeed regular dodecahedral honeycombs with Schläfli symbol $\{5,3,r\}$, $r=4$ and $r=5$. Therefore it seems that the intuitive picture is correct.

2. The values of the vertex solid angle Ω for cube, dodecahedron, and icosahedron are given by the formula $\Omega = q\theta - (q-2)\pi$ giving

$$[\Omega(C),\Omega(D),\Omega(I)] \approx [1.57080, 2.96174, 2.63455].$$

   The sum of these angles should be 4π for a tessellation in E^3. In E^3 This is true only for 8 cubes per vertex ($\Omega = \pi/2$) so that the cubic honeycomb is the only Platonic honeycomb in E^3. The minimal number of cubes per vertex is 9 in H^3. It is convenient to write the values of the vertex solid angles for D and I as

$$[\Omega(D),\Omega(I)] = [0.108174,\ 0.209651] \times 4\pi\ .$$

   The number of D:s *resp.* I:s must be at least 10 *resp.* 5 for dodecahedral *resp.* icosahedral honeycombs in H^3.

3. The basic geometric scales of the Platonic solids are circumradius R, surface area A and volume V. The circumradius is given by $R = (a/2)\tan(\pi/q)\tan(\theta/2)$, where a denotes the edge length. The surface area A of the Platonic solid $\{p,q\}$ equals the area of face multiplied by the number F of faces: $A = (a/2)^2 Fpcot(\pi/p)$. The volume V of the Platonic time is F times the volume of the pyramid whose height is the length a of the face: that is $V = FaA/3$.

   Choosing $a/2$ as the length unit, the circumradii R, total face areas A an the volumes V of the Platonic solids are given by

$$[R(T), R(C), R(O), R(D), R(I)] = [\sqrt{3}/2, \sqrt{3}, \sqrt{2}, \sqrt{3}\phi, \sqrt{3-\phi}\phi] \ ,$$

$$[A(T), A(C), A(O), A(D), A(I)] = [4\sqrt{3}, 24, 2\sqrt{3}, 12\sqrt{25+10\sqrt{5}}, 20\sqrt{3}] \ ,$$

   and

$$\begin{aligned}&[V(T), V(C), V(O), V(D), V(I)] \simeq [\sqrt{8}/3, 8, \ \sqrt{128}/3, \ 20\phi^3/(3-\phi), 20\phi^2/3]\\ &\simeq [.942809, 8, 3.771236, 61.304952, 17.453560] \ .\end{aligned}$$

What can one say about icosa-tetrahedral tessellation?

1. Consider first the dihedral angles θ. The values of dihedral angles associated T, O, and I in H^3 are reduced from that in E^3 so that their sum in E^2 scene must be larger than 2π. Therefore at least one of these cells must appear twice in H^3. It could be T but also O can be considered. For $2T+O+I$ and $T+2O+I$ the sum would be $388.26°$ *resp.* $427.43°$ in E^3. $2T+O+I$ *resp.* $T+2O+I$could correspond to 4 cells ordered cyclically as ITOT resp. IOTO.

2. The values of the vertex solid angle Ω for tetrahedron, octahedron, and icosahedron are given by $[\Omega(T), \Omega(O), \Omega(I)] = [0.043870, 0.108174, 0.209651]4\pi$ If the numbers of T, O and I are $[n(T), n(O), n(I)]$, one must have $[n(T)\Omega(T), +n(O)\Omega(O) + n(I)\Omega(I) > 4\pi$ in H^3.

   If the number of the cells for the fundamental domain are really $[N(T), N(O), N(I)] = [30, 20, 12]$, the first guess is that $[n(T), n(O), n(I)] \propto [N(T), N(O), N(I)$ is approximately true. For $[n(T), n(O), n(I)] = [2, 3, 1]n(I)$, one obtains $\Omega = n(T)\Omega(T) + n(O)\Omega(O) + n(I)\Omega(I) = n(I) \times .629 \times 4\pi$. This would suggest $n(I) = 2$ giving $[n(T), n(O), n(I)] = [4, 6, 2]$

3 New results about the relation of the icosa-tetrahedral tessellation to the dark genetic code

How could the icosa-tetrahedral tessellation relate to the proposed dark realizations of the genetic code [L6, L7]?

3.1 About the problems of the earlier view of the dark realizations of the genetic code

Consider first the problems of the earlier views of the realization of the dark genetic codes in terms of dark proton triplets at monopole flux tubes parallel to the ordinary DNA and to the realization in terms of dark photon triplets.

1. The TGD based inspired model of the dark photon genetic code [K2] [L3, L6] assumes that the dark realization of genetic code involves 3 icosahedral Hamiltonian cycles giving rise to 20+20+20 dark DNA codons and the unique tetrahedral Hamiltonian cycle giving the remaining 4 codons.

   The obvious problem of icosa-tetrahedral picture is that one must assume that icosahedron and tetrahedron are disjoint. If they have a common face, the number of faces reduces to 63 and one DNA codon is missing. This raises the question whether icosahedron and tetrahedron could be disjoint pieces of a larger structure.

2. Icosahedron and tetrahedron should have a physical realization: what could it be? How the Hamiltonian cycles are realized physically? The cycles are defined only modulo the isometry group I of icosahedron having 60 elements and Z_n $n = 6, 4$ or 2 leaves the cycle and the orbits of this group (amino-acids) invariant. The Hamiltonian cycle has $\#(I/Z_n)$ isometric copies (the numbers of copies are 10, 15, and 32). Does this have a physical significance? How are the 12 frequencies associated with the edges of the cycle realized physically? What is the physical interpretation of octave equivalence: does it have something to do with 2-adicity?

3. In the dark proton realization a given codon would correspond to a selected triangular face of I or T carrying dark protons at the vertices of this face. The original view was that dark 3-proton states would correspond to 64 codons. The problem was that one obtains only 8 states for dark proton triplets from spin and antisymmetrization in spin degrees of freedom would not allow any states unless the spatial wave function is totally antisymmetric and spins are in the same direction.

   In the original proposal also neutrons were assumed so that the codon corresponds to a sequence of 3 nucleons with both spins. 3 nucleons would give rise to 64 states as required. Dark protons can also be effectively neutrons as far as charge is considered. This might be possible if the bonds connecting the dark protons can be both neutral and negatively charged. Weak interactions are as strong as electromagnetic interactions in a given biological scale (such as DNA scale) if the dark Compton length proportional to h_{eff} is larger than this scale and the weak transitions change the dark protons to effective dark neutrons.

   This option leads to a problem with the fact that DNA nucleotides have negative unit charge. One should have protons to neutralize this charge and stabilize DNA. Also variants of the proposal in which there are flux tube connections between dark protons having 2 different neutral states analogous to neutral pion and neutral ρ meson.

   The simplest proposal, which is consistent with the idea that genetic codons correspond to cyclotron transitions of dark proton triplets assignable to the triangular faces of an icosahedron or tetrahedron is as follows. Besides 2 spin states, dark protons can also have 2 states with spin ± 1 corresponding to the analog of rotation in the discrete space defined by the vertices of the triangle. This would give $2^3 \times 2^3 = 64$ states.

The realizations of the genetic code in terms of dark photon triplets and dark proton triplets should correspond to each other. This requires that dark proton triplet realization should naturally correspond to the icosa-tetrahedral realization.

1. The codons identified as dark proton triplets assignable to one of the 20 triangular faces of icosahedron and tetrahedron have in quantum situations a wave function in the discrete space of the faces, which is in general delocalized. Could these wave functions in the set of faces give rise to states in 1-1 correspondence with the icosahedral and tetrahedral codons? There would be 20 wave functions for an icosahedron and 4 wave functions for a tetrahedron. The number of icosahedral states must be tripled to 60 corresponding to the 3 basic types of icosahedral Hamiltonian cycles with symmetries Z_n, $n = 6, 4, 2$.

   The 3 dark protons also have spin degrees of freedom. The dark proton triplet in the ground state(s) would be naturally spontaneously magnetized so that all spins are in the same direction. Also the states in which some dark protons are excited are allowed by Fermi statistics and are needed since these excitations could correspond to the spatial wave functions in face degrees of freedom.

2. Dark photon triplets are needed for communications. The vision is that they correspond to the representation of codons as frequency triplets represented by the realization of icosahedral and tetrahedral Hamiltonian cycles as frequency triplets. The assumption has been that the 3 frequencies of dark 3-photon are associated with the cyclotron (or Larmor transitions if only spin is dynamical) of dark protons of a dark proton triplet.

   Dark photon communications between identical codons would take place by 3-resonance. The de-excitation of the first codon would lead to the excitation of an identical codon: one would have a kind of flip-flop. Also dark genes as sequences of N dark codons could act as a single quantum coherent unit and 3-N resonances between identical dark genes would become possible. The mechanism is very similar to that used in the computer language LISP. The modulation of the frequency scale by modulating the thickness of the monopole flux tubes would make possible coding of the signal and it would be transformed to a sequence of resonance pulses at the receiving end.

Dark photon triplet states could correspond to wave functions in the space of icosahedral and tetrahedral faces.

3. Cyclotron transitions would be needed in order to generate dark photon triplets. This would require excitations of the dark protons of the spontaneously magnetized ground state(s). If only spin matters, the cyclotron transitions reduce to Larmor transitions. The correspondence with the icosahedral Hamiltonian cycles in terms of dark photon triplets would suggest that these excitations correspond to icosahedral genetic codons as wave functions in the set of faces. The cyclotron transition would provide the energy needed to excite the wave function in the set of faces. 64 transitions would be needed. It is important to notice that cyclotron transitions rather than cyclotron states of dark protons would correspond to codons of icosa-tetrahedral representation represented as wave functions in the set of facs.

   There are however only 8 states per face if only Larmor transitions are allowed. This is much less than the number 20 20+20+4=64 for icosahedral and tetrahedral Hamiltonian cycles. An additional two-valued degree of freedom is needed. The simplest possibility is the assignment to each dark proton an analog of angular momentum eigenstate with spin ± 1 corresponding to a discrete rotation around the triangle. This would give $8 \times 8 = 64$ states per face. Could the excitations of these states correspond to 20+20+20 icosahedral states plus 4 tetrahedral states?

4. Hitherto the considerations have been implicitly classical in that a localization in the set of faces has been assumed. Quantum theory allows us to give up this assumption. Icosahedral realization suggests that dark proton triplet has a icosahedral wave function delocalized to the set of 20 faces with symmetry fixed by the Hamiltonian cycle to Z_n, $n = 6, 4$ or 2, and that the excitation of the dark proton triplet in the face degrees of freedom provides the energy changing the wave function in the set of faces. The same would apply to the tetrahedron with symmetry Z_4 allowing 4 wave functions.

   The orbital and angular momentum degrees of freedom would be coupled. The transition from the ground state for dark proton triplet would excite wave function in the set of faces. This could imply the desired correspondence between the dark proton representations and dark photon realizations of the code.

5. There is a further problem. Spontaneously magnetized states of 3 dark protons would define ground states of codons. The ground state proton triplet cannot have lower energy states and cannot emit dark photon triplets and are therefore "mute" and unable to communicate, presumably necessary for processes like transcription and translation. Note that ground states are however not deaf.

The proposed general view is attractive but the details remain to be understood and problems solved. Here the notion of icosa-tetrahedral tessellation could help. The proposal of [L7] was that the icosa-tetrahedral honeycomb at the light-cone proper time $a = constant$ surfaces identifiable as hyperbolic 3-space H^3 allows to realize the dark genetic code.

The icosa-tetrahedral honeycomb is the unique honeycomb, which involves only Platonic solids. This inspires the question whether genetic code could be universal and realized in all scales by induction, which means that the tessellation of H^3 induces tessellation of 3-surface $X^3 \subset H^3$ by restriction. Also the induction to $H^3(a)$ projection of X^4 makes sense.

The TGD view of holography indeed predicts the special role of hyperbolic 3-spaces. The space-time surfaces in $H = M^4 \times CP_2$ are analogs of Bohr orbits, which go through $H^3(a_n) \subset M^4 \subset H$, where a_n corresponds to a root of the polynomial with integer coefficients determining to a higher degree a given region of the space-time surface by $M^8 - H$ duality [L4, L5].

In the sequel the detailed realization of the genetic code in terms of the icosahedral honeycomb will be discussed with an emphasis on the problems noticed above.

3.2 The realization of the code in terms of icosa-tetrahedral tessellation

The fundamental region of the icosa-tetrahedral tessellation contains 30 octahedrons, 20 tetrahedrons, and 12 icosahedrons and the cautiously proposed interpretation is that the cells meeting at each *edge* of the tessellation have either the cyclic structure TOTI or OTOI, and each vertex involve 3 O:s, 2 T:s and 1 I. Could one interpret this in terms of the dark icosahedral realization of the genetic code?

3.2.1 Ideas related to the detailed realization of the genetic code

The detailed realization of the dark genetic code is far from completely understood and one might hope that icosa-tetrahedral realization could bring in the constraints allowing us to fill in the details. It is useful to proceed by considering basic requirements on the realization of the dark code.

1. There are 3 O:s per single I in vertex if 10 instead of 12 icosahedral cells are included. The reasons for this become clear from the proposed relation between DNA double strand and fundamental cell of icosahedral honeycomb. What could the role of O:S be?

   Imagine that it is possible to arrange the polyhedrons for a given I to cycles as -I-O-T-O-T-O-: here cyclicity is assumed. The two tetrahedrons and I would be disjoint. This would solve the problem due to the common face of T and I (only 63 DNA codons) but give 60+4+4 faces and 68 dark DNA codons. There is however the problem posed by the mute codons. Could the presence of mute DNA codons reduce the number of DNA codons from 68 to 64. This would imply that their transcription allows only 64 dark mRNA codons. Could mute mRNA codons reduce the effective number of mRNA codons to 61 for the standard code (stop codons would be mute)? What about its variants with a smaller number of stop codons?

2. Bioharmony involves 3 icosahedral Hamiltonian cycles. All the combinations of the 3 -cycles with symmetries Z_6, Z_4 and Z_2 predict the same code. These bioharmonies are interpreted as correlates for emotional states appearing already at the basic bio-molecular level. The motivation comes from the fact that the icosa-tetrahedral harmony emerges as a geometric model for the music harmony and music indeed both creates and expresses emotions.

   Could icosahedral honeycomb allow us to understand the realization of these 3 icosahedral Hamiltonian cycles in terms of cyclotron frequency triplets? One must have closed magnetic monopole loops in order to have cyclotron transitions. Could these loops form triangles of form I-T-O. This would be 6 different triangles and 3 different positions of I for given T. This kind of loop would be assigned with each vertex of the face. Could the magnetic field strengths depend on the loop and for a given T give rise cyclotron frequency triplets characterizing a given icosahedral Hamiltonian cycle.

3. One can criticize the assumption that there is only a single codon per single I and T. I:s could in principle carry several codons. This however gives a restriction that the codons inside given I and T are different and restricts the representative power of the code if it involves more than 2 strands. This restriction is however automatically satisfied for the base-paired codon and anticodon in the DNA double strand!

3.2.2 Dark photon realization of the icosahedral part of the code

Consider first the realization of the icosahedral part of the code in terms of dark photons.

1. The 3 icosahedral Hamiltonian cycles have symmetries. The 20 codons with Z_6 symmetry correspond to 3 6-plets and 1 doublet of Z_6 and for unbroken symmetry the codons inside these multiplets code for the same amino acid. This means 3+1= 4 amino acids. Z_4 symmetry has 5 4-plets and in absence of symmetry breaking this corresponds to 5 amino-acids. Z_2 symmetry as 10 2-plets, and also this symmetry is also almost exact and corresponds to the almost exact symmetry with respect to the third letter of the codon analogous to isospin symmetry.

2. Icosahedral part of the icosa-tetrahedral realization involves 3 icosahedral Hamiltonian cycles characterized by different symmetries. For Z_6 symmetry, there are 6+6+6+2=20 codons codons. These sets of codons can be regarded as orbits of Z_6 and correspond to amino-acids. This if the Z_6 symmetry is not broken. This means 3+1 amino acids in absence of symmetry breaking.

   Z_4 symmetry has 5 4-plets and in absence of symmetry breaking this corresponds to 5 amino-acids coded by 4 codons each. Z_2 symmetry has 10 2-plets and this symmetry is also almost exact. This symmetry corresponds to the almost exact symmetry with respect to the third letter of the codon.

3. Dark photon codons are represented as cyclotron frequency triplets of dark photons created in 3-cyclotron transitions for dark proton triplets involving simultaneous emission of 3 dark photons made possible by quantum coherence. In the case of genes with N codons one has 3N-cyclotron transition and 3N dark proton-state represents a gene as a quantum coherent unit.

3.2.3 Dark proton realization of the icosahedral part of the code

Consider next the dark proton realization of the icosahedral part of the code.

1. The basic problem of the dark proton realization of the code is that there are only 8 dark proton spin states. If one assumes that each dark proton can have spin ± 1 this problem the number of dark proton states is 4 and one obtains 64 states.

   If one allows the states with vanishing spin so that one would have 3 orbital states per dark proton, the number of cyclotron transitions per dark proton is 4. Since lowest energy states are mute and transitions define codons, this could be the correct identification.

2. Icosa-tetrahedral realization should give 20+20+20+4 =64 dark proton triplets assignable to the faces of I and T. Suppose that the cells can be thought of as forming a cycle O-I-O-T-O-T with O and T ends connected. The two T:s have no common faces with O and without additional conditions give rise to 4+4 additional codons giving 68 codons. How can one reduce the number of dark DNA codons to 64?

3. Dark proton codons have a ground state, or possibly several of them, which by definition cannot decay to lower energy states by emission of dark photon cyclotron triplet. Ground state codon is mute since it cannot produce dark photon triplets as 3-chords.

   The natural first guess is that the ground states correspond to the 6 combinations 3 icosahedral Hamiltonian cycles and 2 tetrahedral cycles assignable to $2\times$ T. The 3 stop codons are transcribed but not translated so that the interpretation of 3 DNA stop codons as icosahedral ground state dark codons unable to send 3-photon signals is not correct. For mRNA this interpretation could make sense if the mRNA images of DNA stop codons represent ground state codons.

4. Cyclotron excitations of ground state codons are induced by dark photon triplets. Conversely cyclotron de-excitatons generate dark proton triplets except for the ground state codons with minimum total energy. Suppose that there are 6 ground state codons as combinations of 3 dark codon ground states assignable to the 3 icosahedral Hamiltonian cycles and 2 dark proton ground states assignable to tetrahedral cycles of the two T:s. This would give 8 mute states. The total number of dark DNA codons is 60+8=68. Note that the mute states are not deaf: they can receive messages.

   One would obtain only 60 DNA codons, which can be transcribed to mRNA codons if the transcription involving dark photon codons. How could one get 64 as an effective number of DNA codons?

   One can imagine transitions between otherwise mute codons, which generate dark photon triplets coupling to mRNA associated with DNA. Let A, B and C the ground state codons with minimal total dark cyclotron energies in an increasing order for the 3 icosahedral Hamiltonian cycles. If for a given T (two options) the cyclotron transitions are possible only between codons C and B and B and A one obtains 2 DNA-mRNA pairings for both T:s. One would have 60+2+2=64 mRNAs pairing with DNA and effectively 64 DNA codons.

Note that the transcription produces only 64 dark mRNA codons from 68 dark DNA codons.

For 64 mRNA codons it could happen that there are no transitions between the 3 icosahedral codons for both choices of T so that there are 6 mute mRNA codons. If there are transitions $C \rightarrow B$ and $B \rightarrow A$, the number of mute icosahedral codons is 4. If there are no transitions between tetrahedral ground state codons, one has effectively 60 mRNA codons since the translation stops due to the absene of dark 3-photon signals to tRNA. If there is a transition between the 2 ground state nRNA codons associated with the two T:s, one obtains 61 effective mRNA codons of the standard realization of the code. The transitions between tetrahedral codons can increase the effective number of mRNA codons.

5. What about tRNA appearing as a pair of amino-acid and single RNA codon. Could the RNA of tRNA and amino-acids correspond to the unique icosahedral honeycomb of H^3 and to icosahedral Hamiltonian cycles so that the number of dark codons in absence of tetrahedral degeneracy would reduce to 32, which is the minimal number of ordinary tRNA codons, which is increased by the non-uniqueness of the ordinary tRNA itself? Note that mute tRNA codons are not deaf: they can receive messages but cannot send them. Obviously, tRNA and amino-acids would correspond to the lowest evolutionary level.

The tentative conclusion would be that in the TGD framework DNA-mRNA transcription is not 1-to-1: information is lost and could say that RNA represents a lower level of evolutionary hierarchy. This would conform with the RNA world vision. The numbers of dark proton DNA and mRNA codons are 68 and 64 respectively. The unavoidable existence of mute codons gives effective DNA codon number 64 as the number of mRNA codons. 3 icosahedral codons can be mute and one obtains 3 stop codons unable to communicate with tRNA. The number of mute codons can also be smaller.

The dark DNA and RNA codons are dynamical and are not fixed to be the same as ordinary codons. This is required only during the communications with ordinary DNA possibly taking place by dark photons transforming to ordinary photons and inducing resonant transitions of ordinary DNA and other basic biomolecules. This strongly suggests that dark DNA and RNA act as kinds of R&D laboratories making it possible to test variants of the genes. Actually their ground states would correspond to 3 icosahedral representations and 2 tetrahedral representations and would correspond to aminoacids via transcription and translation.

Needless to say, this picture is highly speculative and one can probably imagine variants for it. The basic idea is however clear: icosa-tetrahedral tessellation could explain the details of the standard genetic code and its modifications.

3.2.4 Realization of the flux tube structures associated with dark codons

The following represents an attempt to make the above picture more concrete.

1. The selection of 1 O from 3 O:s could mean a selection of an icosahedral Hamiltonian cycle with symmetry group Z_6, Z_4, or Z_2. This gives for icosahedral realization 20+20+20 =60 icosahedral codons. Tetrahedral Hamiltonian cycles associated with the two T:s should give the remaining 4 codons. One can however imagine several ways for how this could occur.

2. The selection of O should correspond to a choice of the icosahedral cycle. What does this mean geometrically? To each dark proton of the codon, one must assign a closed monopole flux tube. The strength of the magnetic field of the flux tube fixes the cyclotron frequency scale for each flux tube. The 20 dark-photon chords defining a given icosahedral bioharmony differ for different choices of O and T. The frequencies are fixed if the Hamiltonian cycle corresponds to a quint cycle such that the frequencies associated with the neighboring vertices of the Hamiltonian cycle differ by a scaling 3/2. This requires that the magnetic field strengths along the cycle differ by scaling 3/2.

3. How to concretely realize the correlation of the bioharmony with the choice of O and T for a given I? Suppose that for a given I, the closed flux tube connects I and the selected O and

T. There would be a closed I-O-T flux tube for each vertex of the face defining the codon. This kind of flux tube would define an analog of a string of a musical instrument.

These closed flux tubes would be hyperbolic analogies of closed circuits formed by Euclidian nearest neighbour lattice bonds. If makes sense to assign to each I a cycle O-I-O-T-O-T, with O and T at ends being connected, the cycle I-O-T would go through the either T, and this implies that tetrahedral codons correspond to the other face of T. One would obtain 64 dark proton codons with 3 mute dark proton codons identifiable as stop codons. In the transcription the signal as a dark photon triplet would not reach the dark RNA codon and the transcription would stop. Could this mean that dark RNA codon attaches first to dark DNA codon and the transcription of DNA to ordinary RNA occurs after that in the usual way.

4. The proposed transitions between ground state codons for icosahedral Hamiltonian cycles modify the cycle geometrically since the O in cycle I-O-T changes. If the transitions for given T are only of C→ B and $B \rightarrow A$ with energies in increasing order, one can imagine that the O is replaced by a neighboring O in the transition in the O-I-O-T-O-T.

Several questions remain to be answered.

1. The symmetry breaking for the icosahedral codons with Z_n, $m = 6, 4, 2$ should be understood. This symmetry breaking can be assumed to occur at the level of dark mRNA and modify the frequency triplets from those for completely symmetric mRNA codons. The replacement $T \rightarrow U$ might relate to the symmetry breaking.

   UUG, CUG, and the very common AUG appear as start codons. They correspond to symmetry breaking for 6-plet (Z_6) coding for leu and 4-plet (Z_4) coding for ile. All symmetry breakings occur for start codons UUG, CUG, and for codons UAA and UAG and UGA and UGG closely related to stop codons.

2. Can one understand the reduction of the number of mRNA stop codons to 2 or 1 occurring for some variants of the code? In these situations, the stop codon of mRNA can code for an exotic amino acid pyrrolysine and selenocysteine. Could the transition between stop codon of dark mRNA icosahedral Hamiltonian cycle to a stop codon of another Hamiltonian cycle take place such that the dark photon triplet generated couples to tRNA involving the exotic amino acid. Situation would be almost like in the case of DNA where only two ground state codons stop the transcription.

3. What can one say about the strength of the magnetic fields assignable with the monopole flux tubes? Nanometer length scale 1 nm, naturally assignable to the DNA double strand, corresponds from the formula $l_B = 26nm/\sqrt{B/Tesla}$ to 12.2 GHz. What is interesting is that the gravitational Compton frequency for Earth is 67 GHz and defines a lower bound for the gravitational quantum coherence time. If the strengths of the magnetic fields span 7 octaves, the thickness of the flux tube would vary by a factor 10 in the range about .1 nm - 1 nm.

4. Note that the 12-note scale can be realized using powers $(3/2)^k$, $k = 1, ..., 12$, of the fundamental and by using octave equivalence to reduce the note to the basic octave. Since the monopole flux is quantized, the realization of the scale requires variation of flux tube thickness inducing variation of magnetic field strength and therefore of that cyclotron frequency scale.

   There is nothing cherished in the rational quint cycle as the basis of the 12-note scale. For instance, the well-tempered scale actually replaces the Pythagorean scale with an algebraic scale coming in powers of $2^{1/12}$.

3.3 Description of the entire DNA double strand in terms of icosatetrahedral tessellation

The most ambitious model would describe the entire DNA double strand and relate the model bioharmony to the properties of the icosa-tetrahedral tessellation. There are however many questions remaining.

1. Single DNA and RNA strand would correspond to a "half realization" for which the T and I cells would contain only single codon. The splitting of DNA could have a geometric interpretation as an effective replication of the induced tessellation to two tessellations to RNA type tessellations.

2. There are 20 amino-acids and an icosahedron involves 20 faces. Is this a mere accident? Could icosahedral honeycomb describe amino-acid sequences geometrically. tRNA appears as a single unit. tRNA-amino-acid pairing would involve pairing of two icosahedral tessellations as also the pairing of RNA and tRNA in the translation. tRNA would naturally correspond to a single cell of icosahedral tessellation. This would also explain why the number of tRNA molecules is considerably smaller than RNA codons.

3. Does RNA correspond to icosahedral or icosa-tetrahedral tessellation? Tetrahedral Hamiltonian cycles are needed, in particular the dark proton triplets associated with the tetrahedral faces. Therefore icosa-tetrahedral tessellation is the natural option also for RNA.

4. It is thought that DNA and RNA nucleotides float freely in the cellular water and DNA and RNA codons are built from them in replication/transcription. This is probably the case at the biochemical level, whose dynamics is controlled by dark level (I have however considered the possibility that freely floating nucleotides could actually form loosely bound codons).

   At the ark level both replication and transcription would involve replication of the induced icosa-tetrahedral tessellation: a similar process occurs for clay crystals, and is suggested to be a precursor of DNA replication. This process is a holistic quantum process occurring in a single quantum jump. This would explain the incredible accuracy of these processes, which is extremely difficult to understand in the chemical approach.

   The replication would determine the outcome, be it a pair of DNA double strands or of DNA and RNA. After this the chemical processes leading to the formation of chemical codons from nucleotides and their pairing with dark codons of the induced icosa-tetrahedral tessellation would take place.

DNA has a helical structure. Helical tessellations are known to exist (`rb.gy/5ova6`). If icosa-tetrahedral tessellation is induced, the helical structure would most naturally reflect the dynamics of the corresponding space-time surface. This suggests that only a sequence of I:s is selected from the set of 12 I:s in a given fundamental region of the icosa-tetrahedral tessellation.

To see whether this hypothesis can make sense one must use geometrical facts about DNA double helix, which has A-, B-, and Z forms `rb.gy/4kcrm`).

1. B-form is believed to dominate in cells. From the table of the Wikipedia article one learns that for the B-form the rise per base pair (bp) is 3.32 $\AA$, that full turn corresponds to 10.5 bps, and that the pitch of the helix per turn 33.2 $\AA$, which corresponds to 10 bps per turn. The pitch/turn should be equal to $10.5 \times 3.32 = 34.52$ $\AA$. There is obviously a mistake in the table.

2. The solution of the puzzle is that straight DNA in solution has 10.5 bps/turn and 10 bps/turn in solid state (`rb.gy/wqjbh`). If DNA double helix corresponds to solid state then 10 codons correspond to 3 full turns. Therefore my earlier assumption 10 bps/turn in the double helix is correct. 10 codons would correspond 3 full turns and to the length 99.6 $\AA \simeq 10$ nm, which in TGD framework corresponds to the p-adic length scale $L(151)$.

Double DNA strands cannot pair with all 12 I:s associated with the dark DNA. The length $L(151)$ should correspond to 10 I:s taking 80 per cent of the icosahedral volume. Is helical winding enough to achieve this?

1. The total volume of the fundamental region is $V = 20V(T) + 30V(O) + 12V(I) = 341.44$ using $2a$ as length unit. Using the estimate $V_{real} = L(151)^3 = 10^6 \ \AA^3$, one obtains $a = L(151)/2V^{1/3} \simeq 0.07 \times L(151)$. The volume fraction of single icosahedron would be $17.45/V \simeq .05$ and 10 I:s would take 1/2 of the volume.

2. The circumradius of single icosahedron would be $R = \sqrt{3-\phi}\phi a/2 \simeq .1 \times L(151) = 1$ nm. This conforms with the assumption that there are 10 codons per length $L(151)$! The diameter of the B-type DNA strand is 20 $\AA$ is also consistent with the value of the circumradius. Maybe the proposed picture works!

3. Notice that if an icosahedral cell corresponds to 2 tetrahedral cells and 3 tetrahedral cells, then 10 codons is the maximum for the realizable DNA codon.

What can one say about the straight form of DNA?

1. For 10.5 bps/turn for a straight DNA in solution, the smallest portion of strand, which corresponds to integer numbers of turns and of codons is 6 full turns. This corresponds to 63 bps and 21 codons.

2. With an inspiration coming from the notion of Combinatorial Hierarchy [A1, A2] defined in terms of Mersenne primes $M_n = 2^n - 1$ defined by the recursive formula $M(k) = M_{M(k-1)} = 2^{M(k-1)} - 1$, I proposed decades ago that ordinary genetic code could correspond to Mersenne prime $M_7 = 2^7 - 1$ [K1] [L2]. The basic idea is that a system with $2^7 - 1$ states corresponds to a Boolean logic with 7 bits but with one state missing: this state would correspond to empty set in the set theoretic realization or fermionic vacuum state in the realization as a basis for fermionic Fock states. Only 6 full bits can be realized and the number of realizable statements is 64, the number of genetic codons.

3. Memetic code corresponds to the Mersenne prime $M_{127} = M_{M_7} - 1 = 2^{127} - 1$. Now the number of codons would be $2^{126} = 2^{6\times 21}$ and is realizable as sequences of 21 DNA codons! Note that higher Mersenne numbers in the hierarchy were proposed by Hilbert to correspond to Mersenne primes but for obvious reasons this has not been proven.

4. Could 6 full turns of straight DNA define a memetic codon? During the transcription and replication, DNA double strand opens and becomes straight. Could memetic code be established during the transcription and replication periods? A further intriguing observation is that the cell membrane involves proteins consisting of 21 amino-acids.

3.4 Some questions

Many questions remain to be answered.

1. Hamiltonian cycles are fixed only modulo the 60-element isometry group I of icosahedron. Subgroups Z_n, $n = 6, 4$ or 2 as invariance groups of their orbits defining amino-acids coded by DNA codons assigned to them. Therefore the space I/Z_n corresponds to the space of orbits of Hamiltonian cycles having 10, 15, *resp.* 32 elements for $n = 6, 4$, *resp.* 2 . Suppose that the Hamiltonian cycles for various icosahedrons of the fundamental region proposed to be associated with the sequence of 10 DNA codons differ by a non-trivial isometry assignable to I/Z_n. Does this have physical implications or is it mere gauge degeneracy?

2. The wave functions defining quantal variants of the genetic codons can be assumed to be products of wave functions for the position of the face and 3-proton states assignable to a given face should form an orthonormal set. The face wave functions associated with tetrahedra are trivially orthogonal with those of second tetrahedron and icosahedron. For a fixed choice of the icosahedral or tetrahedral Hamiltonian cycle orthogonality can be realized for the wave functions associated with the position of the face.

   If the icosahedral face wave functions correspond to different Hamiltonian cycles then orthogonality of protonic states for a given face can guarantee the orthogonality. This is possible if the number of protonic states is larger than the number of icosahedral wave functions. This requires 20+20+20+20 protonic states so that four protonic 4 states are left if their number is 64.

3. Why Hamiltonian cycle and quint cycle? Without Hamiltonian cycles the number of frequencies defining 3-chords would be 30 and is reduced to 12 for Hamiltonian cycles. Hamiltonian cycles assigned to the genetic code define an additional symmetry as shifts along the cycle, which are represented as 3/2 scalings modulo octave equivalence. The quint cycle defines the 12 frequencies for a given magnetic field strength and the chords of different cycles consist of different combinations of frequencies.

   What does the Hamiltonian cycle as a 1-D closed path correspond physically?

The proposal that the fundamental region of the icosa-tetrahedral honeycomb could have interpretation as a kind of super-icosahedron raises several interesting questions.

1. Assume that the sequence of 10 DNA (2 codons missing) to the super-icosahedron having icosahedrons as 12 super-edges, tetrahedrons as 20 super-faces, and 30 octahedrons as super-vertices. Combinatorial equivalence suggests that one cdefine icosahedral Hamiltonian cycles as sequences of 12 icosahedrons serving as superedges. Could one define higher level icosahedral genetic codes in terms of icosahedral Hamiltonian cycles. The orthogonality of the face wave functions for the different Hamiltonian cycles would require the assignment of the analogs of dark proton triplets to the super-faces.

2. What could the notion of a super-Hamiltonian cycle as a sequence of 12 dark DNAs mean? The proposed interpretation is that the collection of tetrahedral and 3 icosahedral Hamilton's cycles defines a correlate of a mood, emotional state. It is difficult to say whether the mood is the same for all cells of the entire organism, for the genome of a single cell, for the genes, for the sequences of 10 DNAs, or for codons.

   Super-Hamiltonian cycle associated with the super-icosahedron would have as its edges icosahedrons with the associated 12 dark DNA codon. If the 12 icosahedrons can correspond to different Hamiltonian cycles, one would have a correlate for a sequence of moods. Hamiltonian cycle property allows only 60 sequences of this kind. Without this restriction one would have N^{12} mood sequences, where N is the number of Hamiltonian cycles.

3. One can of course ask whether super-octahedron and super-tetrahedron could make sense and whether they could combine to form a super-icosa-tetrahedron. Does one have any tessellation for which fundamental region would correspond to super-tetrahedron with tetrahedron as interior, 4 octahedrons as 4 super-vertices and 4 icosahedrons as super-edges. There is no mention of this kind of tessellation but it is known that hyperbolic tessellations constructible using the standard methods do exist.

   One could even ask whether there could exist a fractal hierarchy of these super-structures constructible from the super-Platonic solids of the previous level and whether it could be realized as a hierarchy associated with dark DNA. This would mean a hierarchy of increasingly refined emotions emerging as the length of genes and DNA increases.

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