Miscellaneous Topics

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Abstract

This chapter contains topics which do not fit naturally under any umbrella, but which I feel might be of some relevance. Basically TGD inspired comments to the work of the people not terribly relevant to quantum TGD itself are in question. For few years ago Witten's approach to 3-D quantum gravitation raised a considerable interest and this inspired the comparison of this approach with quantum TGD in which light-like 3-surfaces are in a key role. Few years later the entropic gravity of Verlinde stimulated a lot of fuss in blogs and it is interesting to point out how the formal thermodynamical structure (or actually its "square root") emerges in the fundamental formulation of TGD. Lisi's E_8 theory was a further blog favorite and some comments about its failures and possible manners to cure them are discussed. It is also shown how E_8 can be seed as being replaced with the Kac-Moody algebra associated standard model symmetry group in TGD framework.

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1 Introduction

As the title tells, this chapter contains topics which do not fit naturally under any umbrella, but which I feel might be of some relevance. Basically TGD inspired comments to the work of the people not terribly relevant to quantum TGD itself are in question.

For few years ago Witten's approach to 3-D quantum gravitation raised a considerable interest and this inspired the comparison of this approach with quantum TGD in which light-like 3-surfaces are in a key role.

Few years later the entropic gravity of Verlinde stimulated a lot of fuss in blogs and it is interesting to point out how the formal thermodynamical structure (or actually its "square root") emerges in the fundamental formulation of TGD. T-duality relating the physics in long and short length scales to each other is one of the basic dualities of string theory, and a natural question is whether it could have a counterpart in TGD. The proposal leads to a rather unexpected suggestion that biology might have counterpart at the level of particle physics. There are indeed several strong indications that in TGD Universe biology might not be at all something totally separate from the physics in CP_2 scale.

Lisi's E_8 theory was a further blog favorite and some comments about its failures and possible manners to cure them are discussed. It is also shown ho how E_8 can be seed as being replaced with the Kac-Moody algebra associated standard model symmetry group in TGD framework

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [?]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [?]. The topics relevant to this chapter are given by the following list.

- Emergent ideas and notions [?]
- Weak form of electric-magnetic duality [?]
- $\mathrm{M}^8 - Hduality$ [?]
- Hierarchy of Planck constants [?]
- Hyperfinite factors and TGD [?]
- Zero Energy Ontology (ZEO) [?]

2 Light-Like 3-Surfaces As Vacuum Solutions Of 3-D Vacuum Einstein Equations And Witten's Approach To Quantum Gravitation

There is an interesting relationship to the recent yet unpublished work of Witten related to 3-D quantum blackholes [B11], which allows to get additional perspective.

1. The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [B11, B6] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [B9]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that gravitational radiation is not possible. Hence AdS_3 theory in Witten's sense might define this conformal field theory.

2.1 Similarities With TGD

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory. Note however that in the recent formulation of Quantum TGD Kähler action and corresponding instanton density $J \wedge J$ define real and imaginary parts of complexified Kähler action. The imaginary part of the complexified Kähler function does not contribute to the WCW metric but gives first principle description of anyons and purely topological degrees of freedom.

2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.

3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD, gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

2.2 Differences From TGD

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

2. Partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [K2, K3]. Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [K3]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [K5].

6. To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   (a) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   (b) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   (c) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D light-like surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about S-matrix implied by the zero energy ontology [K2].

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices [K2]: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrödinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure [K2]. Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because S-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D light-like surfaces as sub-manifolds (analogs of black hole horizons and light-like boundaries). Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the Kähler-Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [K10].

5. The counterpart of the ordinary unitary S-matrix in mathematical sense is between zero energy states. I call it U-matrix [K2, K11]. It is quite possible and also natural that M-matrices would serve as building blocks of U-matrix so that also U-matrix would be experimentally measurable. This expectation seems to be true as the explicit construction of U-matrices demonstrates [K11]. It is crucial for understanding consciousness via *moment of consciousness as quantum jump* identification.

3 Entropic Gravity And TGD

Eric Verlinde has posted an interesting eprint titled *On the Origin of Gravity and the Laws of Newton* to arXiv.org [B8] . What Linde heuristically derives is Newton's $F = ma$ and gravitational force $F = GMm/R^2$ from thermodynamical considerations plus something else which I try to clarify (at least to myself!) in the following.

3.1 Verlinde's Argument For $F = Ma$

The idea is to deduce Newton's $F = ma$ and gravitational force from thermodynamics by assuming that space-time emerges in some sense. There are however various assumptions involved which more or less impy that both special and general relativity has been fed in besides quantum theory and thermodynamics.

1. Time translation invariance is required in order to have the notions of conserved energy and thermodynamics. This assumption requires not only require time but also symmetry with respect to time translations. This is quite a powerful assumption and time translation symmetry not hold true in General Relativity- this was actually the basic motivation for quantum TGD.

2. Holography is assumed. Information stored on surfaces, or screens and discretization is assumed. Again this means in practice the assumption of space-time since otherwise the notion of holography does not make sense. One could of course say that one considers the situation in the already emerged region of space-time but this idea does not look very convincing to me.

   Comment: In TGD framework holography is an essential piece of theory: light-like 3-surfaces code for the physics and space-time sheets are analogous to Bohr orbits fixed by the light-like 3-surfaces defining the generalized Feynman diagrams.

3. The first law of thermodynamics in the form

$$dE = TdS - Fdx \ .$$

Here F denotes generalized force and x some coordinate variable. In usual thermodynamics pressure P would appear in the role of F and volume V in the role of x. Also chemical potential and particle number form a similar pair. If energy is conserved for the motion one has

$$Fdx = TdS \ .$$

This equation is basic thermodynamics and is used to deduce Newton's equations.

After this some quantum tricks -a rather standard game with Uncertainty Principle and quantization when nothing concrete is available- are needed to obtain F=ma which as such does not involve $\hbar$ nor Boltzmann constant k_B. What is needed are thermal expression for acceleration and force and identifying these one obtains F=ma.

1. The condition $\Delta S = 2\pi k_B$ states that entropy is quantized with a unit of 2π appearing as a unit. log(2) would be more natural unit if bit is the unit of information.

2. The identification $\Delta x = \hbar/mc$ involves Uncertainty principle for momentum and position. The presence of light velocity c in the formula means that Minkowski space and Special Relativity creeps in. At this stage I would not speak about emergence of space-time anymore.

   This gives

$$F = T\frac{\Delta S}{\Delta x} = T\frac{2\pi mck_B}{\hbar} \ .$$

   F has been exressed in terms of thermal parameters and mass.

3. Next one feeds in something from General Relativity to obtain expression for acceleration in terms of thermal parameters. Unruh effect means that in an accelerted motion system measures temperate proportional to acceleration:

$$k_BT = \frac{\hbar a}{2\pi} \ .$$

   This quantum effect is known as Unruh effect. This temperature is extremely low for accelerations encountered in everyday life - something like 10^{-16} K for free fall near Earth's surface.

   Using this expression for T in previous equation one obtains the desired $F = ma$, which would thus have a thermodynamical interpretation. At this stage I have even less motivations for talking about emergence of space-time. Essentially the basic conceptual framework of Special and General Relativities, of wave mechanics and of thermodynamics are introduced by the formulas containing the basic parameters involved.

3.2 Verlinde's Argument For $F = Gmm/R^2$

The next challenge is to derive gravitational force from thermodynamic consideration. Now holography with a very specially chosen screen is needed.

Comment: In TGD framework light-like 3-surfaces (or equivalently their space-like duals) represent the holographic screens and in principle there is a slicing of space-time surface by equivalent screens. Also Verlinde introduces a slicing of space-time surfaces by holographic screens identified as surfaces for which gravitational potential is constant. Also I have considered this kind of identification.

1. The number of bits for the information represented on the holographic screen is assumed to be proportional to area.

$$N = \frac{A}{G\hbar} \quad .$$

This means bringing in blackhole thermodynamics and general relativity since the notion of area requires geometry.

Comment: In TGD framework the counterpart for the finite number of bits is finite measurement resolution meaning that the 2-dimensional partonic surface is effectively replaced with a set of points carrying fermion or anti-fermion number or possibly purely bosonic symmetry generator. The orbits of these points define braid giving a connection with topological QFTs for knots, links and braids and also with topological quantum computation.

2. It is assumed that the area of horizon corresponds to the area $A = 4\pi R^2$ for the sphere with radius which R which is the distance between the masses. This means a very special choice of the holographic screen. Entropy obviously depends very sensitively on R.

Comment: In TGD framework the counterpart of the area would be the symplectic area of partonic 2-surfaces. This is invariant under symplectic transformations of light-cone boundary. These "partonic" 2-surfaces can have macroscopic size and the counterpart for blackhole horizon is one example of this kind of surface. Anyonic phases are second example of a phase assigned with a macroscopic partonic 2-surface.

3. Special relativity is brought in via the bomb formula

$$E = mc^2 \quad .$$

One introduces also other expression for the rest energy. Thermodynamics gives for non-relativistic thermal energy the expression

$$E = \frac{1}{2} N k_B T \quad .$$

This thermal energy is identified with the rest mass. This identification looks to me completely ad hoc and I think that kind of holographic duality is assumed to justify it. The interpretation is that the points/bits on the holographic screen behave as particles in thermodynamical equilibrium and represent the mass inside the spherical screen. What are these particles on the screen? Do they correspond to gravitational flux?

Comment: In TGD framework p-adic thermodynamics replaces Higgs mechanism and identify particle's mass squared as thermal conformal weight. In this sense inertia has thermal origin in TGD framework. Gravitational flux is mediated by flux tubes with gigantic value of gravitational Planck constant and the intersections of the flux tubes with sphere could be TGD counterparts for the points of the screen in TGD. These 2-D intersections of flux tubes should be in thermal equilibrium at Unruh temperature. The light-like 3-surfaces indeed contain the particles so that the matter at this surface represents the system. Since all light-like 3-surfaces in the slicing are equivalent means that one can choose the representation of the system rather freely.

4. Eliminating the rest energy E from these two formulas one obtains $NT = 2mc^2$ and using the expression for N in terms of area identified as that of a sphere with radius equal to the distance R between the two masses, one obtains the standard form for gravitational force.

It is difficult to say whether the outcome is something genuinely new or just something resulting unavoidably by feeding in basic formulas from general thermodynamics, special relativity, and general relativity and using holography principle in highly questionable and ad hoc manner.

3.3 In TGD Quantum Classical Correspondence Predicts That Thermodynamics Has Space-Time Correlates

From TGD point of view entropic gravity is a misconception. On basis of quantum classical correspondence - the basic guiding principle of quantum TGD - one expects that all quantal notions have space-time correlates. If thermodynamics is a genuine part of quantum theory, also temperature and entropy should have the space-time correlates and the analog of Verlinde's formula could exist. Even more, the generalization of this formula is expected to make sense for all interactions.

Zero energy ontology makes thermodynamics an integral part of quantum theory.

1. In zero energy ontology quantum states become zero energy states consisting of pairs of the positive and negative energy states with opposite conserved quantum numbers and interpreted in the usual ontology as physical events. These states are located at opposite light-like boundaries of causal diamond (CD) defined as the intersection of future and past directed light-cones. There is a fractal hierarchy of them. M-matrix generalizing S-matrix defines time-like entanglement coefficients between positive and negative energy states. M-matrix is essentially a "complex" square root of density matrix expressible as positive square root of diagonalized density matrix and unitary S-matrix. Thermodynamics reduces to quantum physics and should have correlate at the level of space-time geometry. The failure of the classical determinism in standard sense of the word makes this possible in quantum TGD (special properties of Kähler action (Maxwell action for induced Kahler form of CP_2) due to its vacuum degeneracy analogous to gauge degeneracy). Zero energy ontology allows also to speak about coherent states of bosons, say of Cooper pairs of fermions- without problems with conservation laws and the undeniable existence of these states supports zero energy ontology.

2. Quantum classical correspondence is very strong requirement. For instance, it requires also that electrons traveling via several routes in double slit experiment have classical correlates. They have. The light-like 3-surfaces describing electrons can branch and the induced spinor fields at them "branch" also and interfere again. Same branching occurs also for photons so that electrodynamics has hydrodynamical aspect too emphasize in recent empirical report about knotted light beams. This picture explains the findings of Afshar challenging the Copenhagen interpretation.

   These diagrams could be seen as generalizations of stringy diagrams but do not describe particle decays in TGD framework. In TGD framework stringy diagrams are replaced with a direct generalization of Feynman diagrams in which the ends of 3-D light-like lines meet along 2-D partonic surfaces at their ends. The mathematical description of vertices becomes much simpler since the 2-D manifolds describing vertices are not singular unlike the 1-D manifolds associated with string diagrams ("eyeglass" in fusion of closed strings).

3. If entropy has a space-time correlate then also first and second law should have such and Verlinde's argument that gravitational force attraction follows from first law assuming energy correlation might identify this correlate. This of course applies only to the classical gravitation. Also other classical forces should allow analogous interpretation as space-time correlates for something quantal.

3.4 The Simplest Identification Of Thermodynamical Correlates In TGD Framework

The first questions that pop up are following. Inertial mass emerges from p-adic thermodynamics as thermal conformal weight. Could the first law for p-adic thermodynamics, which allows to calculate particle masses in terms of thermal conformal weights, allow to deduce also other classical forces? One could think that by adding to the Hamiltonian defining partition function chemical potential terms characterizing charge conservation it might be possible to obtain also other forces.

In fact, the situation might be much simpler. The basic structure of quantum TGD allows a very natural thermodynamical interpretation.

1. The basic structure of quantum TGD suggests a thermodynamic interpretation. The basic observation is that the vacuum functional identified as the exponent of Kähler function is analogous to a square root of partition function and Kähler coupling strength is analogous to critical temperature. Kähler function identified as Kähler action for a preferred extremal appears in the role of Hamiltonian. Preferred extremal property realizes holography identifying space-time surface as analog of Bohr orbit. One can interpret the exponent of Kähler function as the density of states in the world of classical worlds so that Kähler function would be analogous to entropy density. Ensemble entropy is average of Kähler function involving functional integral over the world of classical worlds. This exponent is the counterpart for the quantity Ω appearing in Verlinde's basic formula.

2. The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms.

   Measurement interaction terms would code the values of quantum charges to the space-time geometry. One can even consider the possibility of realizing quantum classical correspondence in the strong sense that the classical correlation functions for appropriate observables at space-time level are equal to their quantum counterparts and thus same for all space-time surfaces in the superposition. Here one could restrict the consideration to correlation functions with arguments restricted to 3-surfaces defined as union of space-like 3-surfaces at the ends of space-time surface and possibly also parton orbits.

   Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

   In Verlinde's formula there is exponential factor $exp(-E/T - Fx)$ analogous to this kind of Lagrange multiplier term. In TGD conserved charges multiplied by chemical potentials defining generalized forces appear in the exponent.

3. If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the Kähler-Dirac gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the Kähler-Dirac gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^{\alpha}_{Q_i} D_\alpha)\Psi = 0 \ ,$$

   where Q_i refers to i: th conserved charge.

4. This gives an analog of thermodynamics in the world of classical worlds (WCW) for fixed values of quantum numbers of the positive energy part of state. For zero energy states one however has also additional thermodynamics - or rather, its square root. This thermodynamics is for the conserved quantum numbers whose averages are fixed. For general zero energy states one has sum over state pairs labelled by momenta and various other quantum numbers labelling the positive energy part of the state. The coefficients of the conserved quantities of the measurement interaction term linear in conserved quantum numbers define the analogs of temperature and various chemical potentials. The field equations defined by Kähler function and chemical potential terms have thermodynamical interpretation and give

coupling to conserved charges and also to their thermal averages. What is important is that temperature and various chemical potentials assigned to positive and negative energy parts of the state allow a complete geometrization in a general coordinate invariant manner and allow explicit expressions in terms of functions expressible in terms of the induced geometry.

To sum up, TGD suggests two thermodynamical interpretations. p-Adic thermodynamics is for mass squared assuming conformal invariance and also the basic formulation of quantum TGD allows thermodynamical interpretation. Actually, thermodynamics in both cases can be replaced with its square root in ZEO. The thermodynamical structure of quantum TGD has of course served as a guiding principle for two decades. In particular, quantum criticality as the counterpart of thermal criticality has been extremely useful guide line and led to a breakthrough in the understanding of the Kähler-Dirac equation during the last year. Also p-adic thermodynamics has been in the scene for more than 15 years and makes TGD a theory able to make precise quantitative predictions.

Some conclusions drawn from Verlinde's argument is that gravitation should be entropic interaction, that gravitons do not exist, and that string models and theories introducing higher-dimensional space-time are a failure. TGD view is different. Only a generalization of string model allowing to realize space-time as surface is needed and this requires fixed 8-D imbedding space. Gravitons also exist and only classical gravitation as well as other classical interactions code for thermodynamical information by quantum classical correspondence. In any case, it is encouraging that also colleagues might be finally beginning to get on the right track although the path from Verlinde's arguments to quantum TGD as it is now will be desperately long and tortuous if colleagues continually refuse to receive the helping hand.

4 What Could Be The Counterpart Of T-Duality In TGD Framework?

Stephen Crowley sent me a book of Michel Lapidus [A4] about zeros of Riemann zeta and also about his own ideas in this respect. The book has been written in avery lucid manner and looks very interesting. The big idea is that the T-duality of string models could correspond to the functional equation for Riemann zeta relating the values of zeta at different sides of the critical line. T-duality [A1] is formulated for strings in space $M^d \times S^1$ or its generalization replacing S^1 with higher-dimensional torus and generalized to fractal strings. Duality states that the transformation $R \rightarrow 1/R$ with suitable unit for R defined by string tension is a duality: the physics for these different values of R is same. Intuitively this is due to the fact that the contributions of the string modes representing n-fold winding and those representing vibrations labelled by integer n are transformed to each other in the transformation $R \rightarrow 1/R$.

Lapidus is a mathematician and mathematicians often do not care too much about the physical meaning of the numbers. For a physicist like me it is extremely painful to type the equation $R \rightarrow 1/R$ without explicitly explaining that it should actually read as $R \rightarrow R_0^2/R$, where R_0 is length unit, which must represent fundamental length scale remaining invariant under the duality transformation. Only after this physicist could reluctantly put $R_0 = 1$ but still would feel himself guilty of unforgivable sloppiness. $R_0 = 1$ simplifies the formulas but one must not forget that there are three scales involved rather than only two. The question inspired by this nitpicking is how the physics in the length scales R_1 and R relates to the physics in length scale R. Are dualities - or perhaps holography like relations in question- so that T-duality would follow from these dualities?

4.1 Could One Replace Winding Number With Magnetic Charge And T-Duality With Canonical Identification?

How could one generalize T-duality to TGD framework? One should identify the counterpart of the winding number, the three fundamental scales, and say something about the duality transformation itself.

1. In TGD Universe partonic 2-surfaces are the basic object. Partonic 2-surface is not strings and the only reasonable generalization for winding number is as Kähler magnetic charge representing the analog of winding of the partonic 2-surface around magnetically charged

2-sphere of CP_2. Magnetic charge tells how many times partonic 2-surface wraps around the homologically non-trivial geodesic sphere with unit magnetic charge. If the generalization of T-duality holds true, one would expect that the contributions of the oscillations and windings of the partonic two-surface to ground state energy must be transformable to each other by the counterpart of the transformation $R \rightarrow R_0^2/R$ - or something akin to that. Also less concrete and more general interpretations are possible, and below the most plausible interpretation will be considered.

2. The duality $R \rightarrow R_0^2/R = R_1$ gives R_0 as a geometric mean $R_0 = \sqrt{RR_1}$ of the scales R and R_1. What are these three length scales in TGD Universe? The obvious candidate for R is CP_2 size scale. p- Adic mass calculations [K6] imply that the primary p-adic length scale $L_{p,1} = \sqrt{p}R$ is of order of Compton length of the elementary particle characterized by the p-adic prime p. The secondary p-adic length scale $L_{p,2} = pR$ in turn defines the size scale of causal diamond (CD) assignable to the magnetic body of the elementary particle characterized by prime p. For instance, for electron this scale corresponds to.1 seconds, a fundamental biological time scale.

   One indeed has $L_{p,1} = \sqrt{L_{p,2}R}$, and CP_2 scale and CD length scale are dual to each other if T-duality holds true. Therefore the duality would relate physics at CP_2 scale - counterpart of Planck length in TGD framework - and in biological scales and would have direct relevance to quantum biology. One has an infinite hierarchy of p-adic length scales and each of them would give rise to one particular instance of the T-duality. Adeles [K12] would provide appropriate formulation of T-duality in TGD framework. The corresponding mass scales would be $\hbar/R, \hbar/\sqrt{p}R$ and $\hbar/pR$. The third scale corresponds to a scale, which for electron corresponds to the 10 Hz frequency in the case of photons. The duality would suggest that the physics associated with the frequencies in EEG scale related to the communications from the biological body to magnetic body is dual to the physics in CP_2 scale.

   Note that one cannot exclude alternative variants of T-duality. In particular, Planck scale and CP_2 length scale as candidates R_1 and R could be considered.

3. What is the interpretation of these three length scales? CP_2 length scale corresponds naturally to the size scale of wormhole contacts. They are Euclidian regions of space-time surface and represent lines of generalized Feynman graphs. Both general arguments and the construction of elementary bosons forces [K7] to assign to these regions braid strands playing a role of Euclidian strings. Parallel translation along the strands is essential in the construction of fermionic bilinears as invariant under general coordinate transformations and gauge transformations [K7]. The ends of these strands carry fermion and anti-fermion numbers. The counterpart of string tension involved appearing in stringy mass formula implied by super-conformal invariance is indeed determined by R and p- adic thermodynamics [K6] leads to a detailed and successful calculations for elementary particle masses using only p-adic thermodynamics, super-conformal invariance, and p-adic length scale hypothesis as basic assumptions.

4. The wormhole throats carrying fermion number are Kähler magnetic monopoles and the wormhole must be accompanied by a second wormhole throat carrying opposite magnetic charge and also a neutrino pair neutralizing the weak isospin so that weak massivation takes place. The end of the flux tube containing the neutrino pair is virtually non-existent at low energies. The length scale for this string must correspond to Compton length for elementary particle given essentially by primary p-adic length scale $L_{p,1}$. The more restrictive assumption that this length scale corresponds to the Compton length of weak bosons looks un-necessarily restrictive and looks also un-natural.

5. The excitations with mass scale $\hbar/pR$ would correspond to excitations assignable to entire CD, maybe assignable to the flux tubes of the magnetic bodies of elementary particles defining also string like objects but in macroscopic scales. For electron the scale is of order of the circumference of Earth. This dynamics would naturally correspond to the dynamics in Minkowskian space-time regions. The dynamics at intermediate length scale would be intermediate between the Euclidian and Minkowskian dynamics and reduce to that for light-like orbits of partonic 2-surfaces with metric intermediate between Minkowskian and Euclidian.

6. A natural interpretation for T-duality in this sense is in terms of strong form of holography. The interior dynamics at length scale R *resp.* pR assigned to Euclidian *resp.* Minkowskian regions of space-time surface corresponds by holography to the dynamics of light-like orbits of partonic 2-surfaces identified as wormhole throats. Therefore the dynamics in Euclidian and Minkowskian regions are dual to each other. Therefore T-duality in TGD sense would follow from the possibility of having both Euclidian and Minkowskian holography. Strong form of holography in turn reduces to strong form of General Coordinate Invariance, which has turned out to be extremely powerful principle in TGD framework.

4.2 Is The Physics Of Life Dual To The Physics In CP_2 Scale?

The duality of life with elementary particle physics at CP_2 length scale - the TGD counterpart of Planck scale - looks rather far-fetched idea. There is however already earlier support for this idea.

1. p-Adic physics is physics of cognition, and one can say that living systems are in the algebraic intersection of real and p-adic worlds: the intersection of cognition and matter. Canonical identification maps p-adic physics to real physics. This map takes p-adic integers which are small in p-adic sense to larger integers in real sense and thus maps long real scales to short real scales. Clearly this map is highly analogous to the T-duality. p-Adic length scales are indeed explicitly related with the above identification of the T-duality so that canonical identification might be involved with T-duality.

   If this interpretation is correct, cognitive p-adic representations in long real length scales would give representations for the physics in short length scales. EEG range of frequencies allowing communication to the magnetic bodies is absolutely essential for brain function. CDs would correspond to the real physics scale associated with the cognitive representations. These cognitive representations are indeed exactly what our science is building so that T-duality would make also scientist as a part of the big vision!

2. The model for dark nucleons as three quark states led to one of the greatest surprises of my professional life [K8, K4]. Under rather general conditions the three quark states for nucleon are in one-one correspondence with the DNA, RNA, tRNA codons, and amino-acids for vertebrate genetic code and there is natural physical correspondence between DNA triplets and amino-acids. This suggests that genetic code is realized at the level of hadrons and that living matter is a kind of emulation for it, or that living matter is representation for matter at hadron level. This leads to rather far reaching speculations about biological evolution - not as random process - but a process analogous R&D applied in industry [K4]. New genes would be continually tested at the level of dark matter and the modifications of genome could be carried out if there is a transcription process transforming dark DNA to ordinary DNA.

3. The secondary p-adic mass scale of electron corresponds to the 10 Hz frequency, which defines a fundamental biorhythm. Also to current quark masses, which are actually not so well-known but are in MeV range, one can assign biologically interesting time scales in millisecond range. This suggests that all elementary particles induce physics in macroscopic time scales via their CD: s containing their magnetic bodies.

The unavoidable and completely crazy looking question raised by T-duality is whether there is intelligent life in the Euclidian realm below the CP_2 length scale - inside the lines of generalized Feynman graphs. This kind of possibility cannot be avoided if one takes holography absolutely seriously. In purely mathematical sense TGD suggests even stronger form of holography based on the notion of infinite primes [K9]. In this holography the number theoretic anatomy of given space-time point is infinitely complex and evolves. The notion of quantum mathematics replacing numbers by Hilbert spaces representing ordinary arithmetics in terms of direct sum and tensor product suggest the same [K12]. Space-time point would be in this picture its own infinitely complex Universe - the Platonia.

4.3 Could One Get Expression For Kähler Coupling Strength From Restricted Form Of Modular Invariance?

The contributions to the exponent of the vacuum functional, which is proportional to Kähler action for preferred extremal, are real *resp.* imaginary in Euclidian *resp.* Minkowskian regions. Under rather general assumptions (weak form of electric-magnetic duality defining boundary conditions at wormhole throats plus additional intuitively plausible assumption) these contributions are proportional to the same Chern-Simons term but with possibly different constant of proportionality [K2].

These terms sum up to a Chern-Simons term with a coefficient analogous to the complex inverse gauge coupling

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_K^2} \quad .$$

The real part would correspond to Kähler function coming from Euclidian regions defining the lines of generalized Feynman diagrams and imaginary part to Minkowskian regions. There are could arguments suggesting that With the conventions that I have used $\theta/2\pi$ is counterpart for $1/\alpha_K$ and there are good arguments that it corresponds to finte structure constant in electron length scale. Furthermore, T-duality would suggest that τ is proportional to $1+i$ so that one would have

$$\frac{\theta}{2\pi} = \frac{4\pi}{g_K^2} \quad .$$

This condition would fit nicely with the fact that Chern-Simons contributions from Minkowskian and Euclidian regions are identical. If this equation holds true the modular transformations must reduce to those leaving this relationship invariant and can only permute the complex and real parts and thus leave τ invariant. One could also interpret this value of τ as physically especially interesting representation and assign to all values of τ related by modular transformation an isotropy group leaving it fixed. All other physically equivalent values would be obtained as SL(2, Z) orbit of this value.

The counterpart of T-duality should somehow relate dynamics in Minkowskian and Euclidian regions and this raises the question whether it corresponds to $\tau \rightarrow i\tau$ and is represented by some duality transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad ,$$

where $(a, b; c, d)$ defines a unimodular matrix $(ad - bc = 1)$ with integer elements, that is in $SL(2, Z)$. The electric- magnetic duality $\tau \rightarrow -1/\tau$ [B7] and the shift $\tau \rightarrow \tau + 1$ are the generators of this group. It is not however quite clear whether they can be regarded as gauge symmetries in TGD framework. If they are gauge symmetries, then the critical values of Kähler coupling strength defined as fixed points of coupling constant evolution must form an orbit of $SL(2, Z)$. It could be also that modular symmetry is broken to a subgroup of $SL(2, Z)$ and this subgroup leaves τ invariant in the case of minimal symmetry.

1. $\tau \rightarrow i\tau$ would permute Euclidian and Minkowskian regions with each other and is therefore a candidate for the T-duality. This condition cannot be satisfied in generic case but one can ask whether for some special choices of τ these transformations could generate a non-trivial sub-group of modular transformations. This subgroup

   To see whether this is the case let us write explicitly the condition $\tau \rightarrow i\tau$:

   $$\frac{a\tau + b}{c\tau + d} = \frac{\theta}{2\pi} + i\frac{1}{\alpha_K} \quad , \quad \alpha_K = \frac{g_K^2}{4\pi} .$$

   The condition allows to solve τ as

   $$\tau = \frac{a - id}{2c}\left[1 + \epsilon_1\sqrt{1 + \frac{4ibc}{(d-ia)^2})}\right] \quad , \quad \epsilon_1 = \pm 1 \quad .$$

2. For

$$d = \epsilon a \ , \ \ \epsilon = \pm 1$$

implying $a^2 - bc = 1$, the solution simplifies since the argument of square root is real. One has

$$\tau = \frac{a}{2c}(1-\epsilon i)\left[1+\epsilon_1\sqrt{1-\epsilon\frac{\sqrt{a^2-1}}{a}}\right] \ .$$

The imaginary and real parts of τ are identical: this might allow an interpretation in terms of the fact that Chern-Simons terms from two regions are identical (normal derivatives are however discontinuous at wormhole throat). Certainly this is a rather strong prediction.

3. Does this mean that SL(2, C) is broken down to the 4-element isotropy group generated by this transformation? If so, a the condition just deduced could allow to deduce additional constraints on the value of Kähler coupling strength, which is in principle fixed by the criticality condition to have only finite number of values? By the earlier arguments - related to p-adic mass calculations and the heuristic formula for the gravitational constant - the value of Kähler coupling strength is in a good approximation equal to fine structure constant at electron length scale:

$$\alpha_K = \frac{g_K^2}{4\pi} \simeq \alpha \ , \ \ \frac{1}{\alpha} \simeq 137.035999084 \ .$$

4. One obtains the following estimate for $a/2c$ from the estimate for α_K by considering the imaginary part of τ:

$$\frac{a}{2c}\left[1+\epsilon_1\sqrt{1-\epsilon\frac{\sqrt{a^2-1}}{a}}\right] = \frac{1}{\alpha_K} \ .$$

At the limit $a \to \infty$ one has

$$\frac{a}{2c}\left[1+\epsilon_1\sqrt{1-\epsilon}\right] = \frac{1}{\alpha_K} \ .$$

The simplest option at this limit corresponds to $\epsilon = 1$ giving

$$\frac{a}{2c} \simeq 137.035999084 \ .$$

Note that $a/2c = 137$ is not allowed by determinant condition so that the deviation of α_K from $1/137$ is predicted. One must have $a > 137 \times 2c \geq 2 \times 137$. This implies

$$1+\epsilon_1\sqrt{1-\frac{\sqrt{a^2-1}}{a}} = 1 + .0026\epsilon_1 \ .$$

By expanding the square root in first order to Taylor series one obtains the condition

$$\frac{a}{2c}\left(1+\epsilon_1\frac{1}{2^{3/2}\times 137c}\right) \simeq \frac{1}{\alpha_K} \ .$$

For large enough values of a and c it is possible to have arbitrary good approximation to fine structure constant. Note that the integers a and c cannot have common factors since this together with determinant condition $a^2 - bc = 1$ would lead to contradiction.

5 E_8 Theory Of Garrett Lisi And TGD

Recently (towards end of the year 2007) there has been a lot of fuss about the E_8 theory proposed by Garrett Lisi [B10] in physics blogs, in media, and even New Scientist [B4] wrote about the topic. There are serious objections against Lisi's theory and it is interesting to find whether the theory could be modified so that it would survive the basic objections. Although it seems that Lisi's theory cannot be saved, one achieves further insights about HO-H ($M^8 - H$) duality. Number theoretical spontaneous compactification can be formulated in terms of the Kac-Moody algebra assignable to Poincare group and standard model gauge group having also rank 8. The representation can be constructed in standard manner using quantized M^8 coordinates at partonic 2-surfaces. Also E_8 representations are in principle possible and the question concerns their physical interpretation.

5.1 Objections Against Lisi's Theory

The basic claim of Lisi is that one can understand the particle spectrum of standard model in terms of the adjoint representation of a non-compact version E_8 group [B2]. There are several objections against E_8 gauge theory interpretation of Lisi.

1. Statistics does not allow to put fermions and bosons in the same gauge multiplet. Also the identification of graviton as a part of a gauge multiplet seems very strange if not wrong since there are no roots corresponding to a spin 2 two state.
2. Gauge couplings come out wrong for fermions and one must replace YM action with an ad hoc action.
3. Poincare invariance is a problem. There is no clear relationship with the space-time geometry so that the interpretation of spin as E_8 quantum numbers is not really justified.
4. Finite-dimensional representations of non-compact E_8 are non-unitary. Non-compact gauge groups are however not possible since one would need unitary infinite-dimensional representations which would change the physical interpretation completely. Note that also Lorentz group has only infinite-D unitary representations and only the extension to Poincare group allows to have fields transforming according to finite-D representations.
5. The prediction of three fermion families is nice but one can question the whole idea of putting particles with mass scales differing by a factor of order 10^{12} (top and neutrinos) into same multiplet. For some reason colleagues stubbornly continue to see fundamental gauge symmetries where there seems to be no such symmetry. Accepting the existence of a hierarchy of mass scales seems to be impossible for a theoretical physicist in main main stream although fractals have been here for decades.
6. Also some exotic particles not present in standard model are predicted: these carry weak hyper charge and color (6-plet representation) and are arranged in three families.

5.2 Three Attempts To Save Lisi's Theory

To my opinion, the shortcomings of E_8 theory as a gauge theory are fatal but the possibility to put gauge bosons and fermions of the standard model to E_8 multiplets is intriguing and motivates the question whether the model could be somehow saved by replacing gauge theory with a theory based on extended fundamental objects possessing conformal invariance.

1. In TGD framework H-HO duality allows to consider Super-Kac Moody algebra with rank 8 with Cartan algebra assigned with the quantized coordinates of partonic 2-surface in 8-D Minkowski space M^8 (identifiable as hyper-octonions HO). The standard construction for the representations of simply laced Kac-Moody algebras allows quite a number of possibilities concerning the choice of Kac-Moody algebra and the non-compact E_8 would be the maximal choice.
2. The first attempt to rescue the situation would be the identification of the weird spin 1/2 bosons in terms of supersymmetry involving addition of right-handed neutrino to the state giving it spin 1. This options does not seem to work.

3. The construction of representations of non-simply laced Kac-Moody algebras (performed by Goddard and Olive at eighties [A2]) leads naturally to the introduction of fermionic fields for algebras of type B, C, and F: I do not know whether the construction has been made for G_2. E_6, E_7, and E_8 are however simply laced Lie groups with single root length 2 so that one does not obtain fermions in this manner.

4. The third resuscitation attempt is based on fractional statistics. Since the partonic 2-surfaces are 2-dimensional and because one has a hierarchy of Planck constants, one can have also fractional statistics. Spin 1/2 gauge bosons could perhaps be interpreted as anyonic gauge bosons meaning that particle exchange as permutation is replaced with braiding homotopy. If so, E_8 would not describe standard model particles and the possibility of states transforming according to its representations would reflect the ability of TGD to emulate any gauge or Kac-Moody symmetry.

The standard construction for simply laced Kac-Moody algebras might be generalized considerably to allow also more general algebras and fractionization of spin and other quantum numbers would suggest fractionization of roots. In stringy picture the symmetry group would be reduced considerably since longitudinal degrees of freedom (time and one spatial direction) are non-physical. This would suggest a symmetry breaking to $SO(1,1)\times E_6$ representations with ground states created by tachyonic Lie allebra generators and carrying mass squared 2 in suitable units. In TGD framework the tachyonic conformal weight can be compensated by super-symplectic conformal weight so that massless states getting their masses via Higgs mechanism and p-adic thermodynamics would be obtained.

5.3 Could Super-Symmetry Rescue The Situation?

E_8 is unique among Lie algebras in that its adjoint rather than fundamental representation has the smallest dimension. One can decompose the 240 roots of E_8 to 112 roots for which two components of SO(7, 1) root vector are ± 1 and to 128 vectors for which all components are ± 1/2 such that the sum of components is even. The latter roots Lisi assigns to fermionic states. This is not consistent with spin and statistics although SO(3, 1) spin is half-integer in M^8 picture.

The first idea which comes in mind is that these states correspond to super-partners of the ordinary fermions. In TGD framework they might be obtained by just adding covariantly constant right-handed neutrino or antineutrino state to a given particle state. The simplest option is that fermionic super-partners are complex scalar fields and sbosons are spin 1/2 fermions. It however seems that the super-conformal symmetries associated with the right-handed neutrino are strictly local in the sense that global super-generators vanish. This would mean that super-conformal super-symmetries change the color and angular momentum quantum numbers of states. This is a pity if indeed true since super-symmetry could be broken by different p-adic mass scale for super partners so that no explicit breaking would be needed.

5.4 Could Kac Moody Variant Of E_8 Make Sense In TGD?

One can leave gauge theory framework and consider stringy picture and its generalization in TGD framework obtained by replacing string orbits with 3-D light-like surfaces allowing a generalization of conformal symmetries.

H-HO duality is one of the speculative aspects of TGD. The duality states that one can either regard imbedding space as $H = M^4 \times CP_2$ or as 8-D Minkowski space M^8 identifiable as the space HO of hyper-octonions which is a subspace of complexified octonions. Spontaneous compactification for M^8 described as a phenomenon occurring at the level of Kac-Moody algebra would relate HO-picture to H-picture which is definitely the fundamental picture. For instance, standard model symmetries have purely number theoretic meaning in the resulting picture.

The question is whether the non-compact E_8 could be replaced with the corresponding Kac Moody algebra and act as a stringy symmetry. Note that this would be by no means anything new. The Kac-Moody analogs of E_{10} and E_{11} algebras appear in M-theory speculations. Very little is known about these algebras. Already $E < sub > n < /sub >$, $n > 8$ is infinite-dimensional as an analog of Lie algebra. The following argument shows that E_8 representations do not work in TGD context unless one allows anyonic statistics.

1. In TGD framework space-time dimension is D=8. The speculative hypothesis of HO-H duality inspired by string model dualities states that the descriptions based on the two choices of imbedding space are dual. One can start from 8-D Cartan algebra defined by quantized M^8 coordinates regarded as fields at string orbit just as in string model. A natural constraint is that the symmetries act as isometries or holonomies of the effectively compactified M^8. The article "The Octonions" [A3] of John Baez discusses exceptional Lie groups and shows that compact form of E_8 appears as isometry group of 16-dimensional octo-octonionic projective plane $E_8/(Spin(16)/Z_2)$: the analog of CP_2 for complexified octonions. There is no 8-D space allowing E_8 as an isometry group. Only SO(1, 7) can be realized as the maximal Lorentz group with 8-D translational invariance.

2. In HO picture some Kac Moody algebra with rank 8 acting on quantized M^8 coordinates defining stringy fields is natural. The charged generators of this algebra are constructible using the standard recipe involving operators creating coherent states and their conjugates obtained as operator counterparts of plane waves with momenta replaced by roots of the simply laced algebra in question and by normal ordering.

3. Poincare group has 4-D maximal Cartan algebra and this means that only 4 Euclidian dimensions remain. Lorentz generators can be constructed in standard manner in terms of Kac-Moody generators as Noether currents.

4. The natural Kac-Moody counterpart for spontaneous compactification to CP_2 would be that these dimensions give rise to the generators of electro-weak gauge group identifiable as a product of isometry and holonomy groups of CP_2 in the dual H-picture based on $M^4 \times CP_2$. Note that in this picture electro-weak symmetries would act geometrically in E^4 whereas in CP_2 picture they would act only as holonomies.

Could one weaken the assumption that Kac-Moody generators act as symmetries and that spin-statistics relation would be satisfied?

1. The hierarchy of Planck constants relying on the generalization of the notion of imbedding space breaks Poincare symmetry to Lorentz symmetry for a given sector of the world of classical worlds for which one considers light-like 3-surfaces inside future and past directed light cones. Translational invariance is obtained from the wave function for the position of the tip of the light cone in M^4. In this kind of situation one could consider even E_8 symmetry as a dynamical symmetry.

2. The hierarchy of Planck constants involves a hierarchy of groups and fractional statistics at the partonic 2-surface with rotations interpreted as braiding homotopies. The fractionization of spin allows anyonic statistics and could allow bosons with anyonic half-odd integer spin. Also more general fractional spins are possible so that one can consider also more general algebras than Kac-Moody algebras by allowing roots to have more general values. Quantum versions of Kac-Moody algebras would be in question. This picture would be consistent with the view that TGD can emulate any gauge algebra with 8-D Cartan algebra and Kac-Moody algebra dynamically. This vision was originally inspired by the study of the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿. Even higher dimensional Kac-Moody algebras are predicted to be possible.

3. It must be emphasized that these considerations relate in TGD framework to Super-Kac Moody algebra only. The so called super-symplectic algebra is the second quintessential part of the story. In particular, color is not spin-like quantum number for quarks and quark color corresponds to color partial waves in the world of classical worlds or more concretely, to the rotational degrees of freedom in CP_2 analogous to ordinary rotational degrees of freedom of rigid body. Arbitrarily high color partial waves are possible and also leptons can move in triality zero color partial waves and there is a considerable experimental evidence for color octet excitations of electron and muon but put under the rug.

5.5 Can One Interpret Three Fermion Families In Terms Of E_8 In TGD Framework?

The prediction of three fermion generations by E_8 picture must be taken very seriously. In TGD three fermion generations correspond to three lowest genera $g = 0, 1, 2$ (handle number) for which all 2-surfaces have Z_2 as global conformal symmetry (hyper-ellipticity [K1, K6]). One can assign to the three genera a dynamical SU(3) symmetry. They are related by SU(3) triality which brings in mind the triality symmetry acting on fermion generations in E_8 model. SU(3) octet and singlet bosons correspond to pairs of light-like 3-surfaces defining the throats of a wormhole contact and since their genera can be different one has color singlet and octet bosons. Singlet corresponds to ordinary bosons. Color octet bosons must be heavy since they define neutral currents between fermion families.

The three E_8 anyonic boson families cannot represent family replication since these symmetries are not local conformal symmetries: it obviously does not make sense to assign a handle number to a given point of partonic 2-surface! Also bosonic octet would be missing in E_8 picture.

One could of course say that in E_8 picture based on fractional statistics, anyonic gauge bosons can mimic the dynamical symmetry associated with the family replication. This is in spirit with the idea that TGD Universe is able to emulate practically any gauge - or Kac-Moody symmetry and that TGD Universe is busily mimicking also itself.

To sum up, the rank 8 Kac-Moody algebra - emerging naturally if one takes HO-H duality seriously - corresponds very naturally to Kac-Moody representations in terms of free stringy fields for Poincare-, color-, and electro-weak symmetries. One can however consider the possibility of anyonic symmetries and the emergence of non-compact version of E_8 as a dynamical symmetry, and TGD suggests much more general dynamical symmetries if TGD Universe is able to act as the physics analog of the Universal Turing machine.

6 Could The Notion Of Hyperdeterminant Be Useful In TGD framework?

The vanishing of ordinary determinant tells that a group of linear equations possesses non-trivial solutions. Hyperdeterminant [B3] generalizes this notion to a situation in which one has homogenous multilinear equations. The notion has applications to the description of quantum entanglement and has stimulated interest in physics blogs [B1, B5]. Hyperdeterminant applies to hyper-matrices with n matrix indices defined for an n-fold tensor power of vector space - or more generally - for a tensor product of vector spaces with varying dimensions. Hyper determinant is an n-linear function of the arguments in the tensor factors with the property that all partial derivatives of the hyper determinant vanish at the point, which corresponds to a non-trivial solution of the equation. A simple example is potential function of n arguments linear in each argument.

6.1 About The Definition Of Hyperdeterminant

Hyperdeterminant was discovered by Cayley for a tensor power of 2-dimensional vector space V_2 (n-linear case for n-fold tensor power of 2-dimensional linear space) and he gave an explicit formula for the hyperdeterminant in this case. For $n = 3$ the definition is following.

$$A^1_{i_3j_3} = \frac{1}{2}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3j_3}A_{i_1i_2i_3}A_{j_1j_2j_3} \quad .$$

In more general case one must take tensor product of $k = 2$ hyper-matrices and perform the contractions of indices belonging to the two groups in by using n 2-D permutations symbols.

$$det(A) = \frac{1}{2^n}(\prod_{a=1}^{n}\epsilon^{i^a_kj^a_k})A_{i^a_1i^a_2\ldots i^a_n}A_{j^a_1j^a_2\ldots j^a_n} \quad .$$

The first guess is that the definition for V_k, $k > 2$ is essentially identical: one takes the tensor product of k hyper-matrices and performs the contractions using k-dimensional permutation symbols.

Under some conditions one can define hyperdeterminant also when one has a tensor product of linear spaces with different dimensions. The condition is that the largest vector space dimension in the product does not exceed the sum of other dimensions.

6.2 Could Hyperdeterminant Be Useful In The Description Of Criticality Of Kähler Action?

Why the notion of hyperdeterminant- or rather its infinite-dimensional generalization- might be interesting in TGD framework relates to the quantum criticality of TGD stating that TGD Universe involves a fractal hierarchy of criticalities: phase transitions inside phase transitions inside... At classical level the lowest order criticality means that the extremal of Kähler action possesses non-trivial second variations for which the action is not affected. The system is critical. In QFT context one speaks about zero modes. The vanishing of the so called Gaussian (of functional) determinant associated with second variations is the condition for the existence of critical deformations. In QFT context this situation corresponds to the presence of zero modes.

The simplest physical model for a critical system is cusp catastrophe defined by a potential function $V(x)$ which is fourth order polynomial. At the edges of cusp two extrema of potential function stable and unstable extrema co-incide and the rank of the matrix defined by the potential function vanishes. This means vanishing of its determinant. At the tip of the cusp the also the third derivative vanishes of potential function vanishes. This situation is however not describable in terms of hyperdeterminant since it is genuinely non-linear rather than only multilinear.

In a complete analogy, one can consider also the vanishing of n: th variations in TGD framework as higher order criticality so that the vanishing of hyperdeterminant might serve as a criterion for the higher order critical point and occurrence of phase transition.

1. The field equations are formally multilinear equations for variables which correspond to imbedding space coordinates at different space-time points. The generic form of the variational equations is

$$\int \frac{\delta^n S}{\delta h^{k_1}(x_1)\delta h^{k_2}(x_2)...\delta h^{k_n}(x_n)} \delta h^{k_2}(x_2)...\delta h^{k_n}(x_n) \prod_{i=2}^{n} d^4x_k = 0 \ .$$

   Here the partial derivatives are replaced with functional derivatives. On basis of the formula one has formally an n-linear situation. This is however an illusion in the generic case. For a local action the equations reduce to local partial differential equations involving higher order derivatives and field equations involve products of field variables and their various partial derivatives at single point so that one has a genuinely non-linear situation in absence of special symmetries.

2. If one has multi-linearity, the tensor product is formally an infinite tensor power of 8-D (or actually 4-D by General Coordinate Invariance) linear tangent spaces of H associated with the space-time points. A less formal representation is in terms of some discrete basis for the deformations allowing also linear ordering of the basis functions. One might hope in some basis vanishing diagonal terms in all orders and multilinearity.

3. When one uses discretization, the equations stating the vanishing of the second variation couple nearest neighbour points given as infinite-D matrix with non-vanishing elements at diagonal and in a band along diagonal. For higher variations one obtains similar matrix along a diagonal of infinite cube and the width of the band increases by two units as n increases by 1 unit. One might perhaps say that the range of long range correlations increases as n increases. The vanishing of the elements at the diagonal- not necessarily in this representation- is necessary in order to achieve multi-linear situation.

6.3 Could The Field Equations For Higher Variations Be Multilinear?

The question is whether for some highly symmetric actions- say Kähler action for preferred extremals- the notion of functional (or Gaussian) determinant could have a generalization to hyperdeterminant allowing to concisely express whether the solutions allow deformations for which the action is not affected.

1. In standard field theory framework this notion need not be of much use but in TGD framework, where Kähler action has infinite-dimensional vacuum degeneracy, the situation is quite different. Vacuum degeneracy means that every space-time surface with at most 2-D CP_2 projection which is so called Lagrangian manifold is vacuum extremal. Physically this correspond to Kähler gauge potential, which is pure gauge and implies spin glass degeneracy. This dynamical and local $U(1)$ symmetry of vacua is induced by symplectic transformations of CP_2 and has nothing to do with U(1) gauge invariance. For non-vacua it corresponds to isometries of "world of classical worlds". In particular, for M^4 imbedded in canonical manner to $M^4 \times CP_2$ fourth order variation is the first non-vanishing variation. The static mechanical analogy is potential function which is fourth order polynomial. Dynamical analogy is action for which both kinetic and potential terms are fourth order polynomials.

2. The vacuum degeneracy is responsible for much of new physics and mathematics related to TGD. Vacuum degeneracy and the consequent complete failure of canonical quantization and path integral approach forced the vision about physics as geometry of "World of Classical Worlds" (WCW) meaning a generalization of Einstein's geometrization of physics program. 4-D spin glass degeneracy is of the physical implications and among other things allows to have a failure of the standard form of classical determinism as a space-time correlate of quantum non-determinism. There are reasons to hope that also the hierarchy of Planck constants reduces to the 1-to-many correspondence between canonical momentum densities and time derivatives of imbedding space-coordinates. Quantum criticality and its classical counterparts is a further implication of the vacuum degeneracy and has provided a lot of insights to the world according to TGD. Therefore it would be nice if the generalization of the hyperdeterminant could provide new insights to quantum criticality.

6.4 Multilinearity, Integrability, And Cancellation Of Infinities

The multilinearity in the general sense would have a very interesting physical interpretation. One can consider the variations of both Kähler action and Kähler function defined as Kähler action for a preferred extremal.

1. Multilinearity would mean multi-linearization of field equations in some discrete basis for deformations- say the one defined by second variations. Dynamics would be only apparently non-linear. One might perhaps say that the theory is integrable- perhaps even in the usual sense. The basic idea behind quantum criticality is indeed the existence of infinite number of conserved currents assignable to the second variations hoped to give rise to an integrable theory. In fact, the possibility -or more or less the fact - that also higher variations can vanish for more restricted configurations would imply further conserved currents.

2. Second implication would be the vanishing of local divergences. These divergences result in QFT from purely local interaction terms with degree higher than two. Even mass insertion which is second order produces divergences. If diagonal terms are absent from Kähler function, also these divergences are absent in the functional integral. The main idea behind the notion of Kähler function is that it is a non-local functional of 3-surface although Kähler action is a local functional of space-time sheet serving as the analog of Bohr orbit through 3-surface. As one varies the 3-surface, one obtains a 3-surface (light-like wormhole throats with degenerate four-metric) which is also an extremal of Chern-Simons action satisfying weak form of electric magnetic duality.

3. The weak of electric magnetic duality together with the Beltrami property for conserved currents associated with isometries and for Kähler current and corresponding instanton current imply that the Coulomb term in Kähler action vanishes and it reduces to Chern-Simons term at 3-D light-like wormhole throats plus Lagrange multiplier term taking care that the weak electric magnetic duality is satisfied. This contributes a constraint force to field equations so that the theory does not reduce to topological QFT but to what could be called almost topological QFT.

4. Chern-Simons term is a local functional of 3-surface and one argue that the dangerous locality creeps in via the electric-magnetic duality after all. By using the so called Darboux coordinates (P_i, Q_i) for CP_2 Chern-Simons action reduces to a third order polynomial proportional to $\epsilon^{ijk}P_i dP_j dQ_k$ so that one indeed has multi-linearity rather than non-linearity. The Lagrangian multiplier term however breaks strict locality and also contributes to higher functional derivatives of Kähler function and is potentially dangerous. It contains information about the preferred extremals via the normal derivatives associated with the Kähler electric field in normal direction and it higher derivatives.

5. One has however good hopes about multi-linearity of higher variations Kähler function and of Kähler action for preferred extremals on basis of general arguments related to the symmetric space property of WCW . As a matter fact, effective two-dimensionality seems to guarantee genuine non-locality. Recall that effective two-dimensionality is implied by the strong form of General Coordinate Invariance stating that the basic geometric objects can be taken to be either light-like 3-surfaces or space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond. This implies that partonic 2-surfaces defining the intersections of these surfaces plus their 4-D tangent space-data code for physics. By effective 2-dimensionality Chern-Simons action is a non-local functional of data about partonic 2-surface and its tangent space. Hence the n: th variation of 3-surface and space-time surface reduces to a non-local functional of n: th variation of the partonic 2-surface and its tangent space data. This is just what genuine multi-linearity means.

6.5 Hyperdeterminant And Entanglement

A highly interesting application of hyperdeterminants is to the description of quantum entanglement-in particular to the entanglement of n qubits in quantum computation. For pure states the matrix describing entanglement between two systems has minimum rank for pure states and thus vanishing determinant. Hyper-matrix and hyperdeterminant emerge naturally when one speaks about entanglement between n quantum systems. The vanishing of hyperdeterminant means that the state is not maximally non-pure.

For the called hyper-finite factor defined by second quantized induced spinor fields one has very formally infinite tensor product of 8-D H-spinor space. By induced spinor equation the dimension effectively reduces to four. Similar formal $8 \rightarrow 4$ reduction occurs by General Coordinate Invariance for the n: th variations. Quantum classical correspondence states that many-fermion states have correlates at the level of space-time geometry. The very naive question inspired also by supersymmetry is whether the vanishing of n-particle hyperdeterminant for the fermionic entanglement has as a space-time correlate n: th order criticality. If so, one could say that the non-locality with all its beautiful consequences is forced by quantum classical correspondence!

6.6 Could Multilinear Higgs Potentials Be Interesting?

It seems that hyperdeterminant has quite limited applications to finite-dimensional case. The simplest situation corresponds to a potential function $V(x_1, .., x_n)$. In this case one obtains also partial derivatives up to n: th order for single variable and one has genuine non-linearity rather than multi-linearity. This spoils the possibility to apply the notion of hyperdeterminant to tell whether critical deformations are possible unless the potential function is multilinear function of its arguments. An interesting idea is that Higgs potential of this form. In this case the extrema allow scalings of the coordinates x_i. In 3-D case 3-linear function of 6 coordinates coming as doublets (x_i, y_i), $i = 1, 2, 3$ and characterized by a matrix $A_{i_1 i_2 i_3}$, where i_k is two-valued index, would provide an example of this kind of Higgs potential.

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