Introduction to "TGD and Condensed Matter"

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Contents

1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

   Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A7]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes

for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of electromagnetic fields are nonvanishing. The correlations functions for weak fields are nonvanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement

theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

   TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

   TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [B4, B2, B3]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral

is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B1]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle

(EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

1.4.1 World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions

1.4.2 Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.4.3 WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H

or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

1.4.4 The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.5 Construction of scattering amplitudes

1.5.1 Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!

1.5.2 Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

1.5.3 The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K12]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

1.6.1 The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

1.6.2 Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are

defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p = 3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

1.6.3 p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since M^8-H duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K11].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.6.4 Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

1.7.1 Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \to f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

1.7.2 Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \quad (1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

1.7.3 Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique

and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

1.7.4 What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

1.7.5 Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary $M^8 - H$ duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of $M^8 - H$ duality as something more fundamental than the ordinary $M^8 - H$ duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.8.1 Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K22].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

1.8.2 Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K19, K20, K18]) support the view that dark matter might be a key player in living matter.

1.8.3 Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

1.9.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K27]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [?].

1.9.2 Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [?]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

2 Bird's Eye of View about the Topics of "TGD and Condensed Matter"

This book tries to provide a view about the applications of TGD to condensed matter physics. Quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry based on the notion of "World of Classical Worlds" (WCW) and physics as a generalized number theory. WCW picture generalizes Einstein's geometrization program to a geometrization of the entire quantum physics. Number theoretic vision states that so-called adelic physics provides a dual view about physics.

$M^8 - H$ duality realizes these dual views in terms of space-time surfaces $X^4 \subset H$ and $X^4 \subset M^8$ mapped to each other by$M^8 - H$ duality. This duality turns out to be a generalization of momentum-position duality of wave mechanics. Also the duality of number theory and geometry suggested by Langlands correspondence pops up into mind.

2.1 The view about physics at the level of H

An important guiding principle in the development of TGD has been quantum classical correspondence (QCC), whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory (here TGD differs from quantum field theories (QFTs)).

1. 4-D General Coordinate Invariance (GCI) forces holography: the space-time surface associated with a given 3-surface is almost unique as an analog of Bohr orbit. X^4 is therefore a preferred extremal of an action principle. This realizes QCC at space-time level and leads to zero energy ontology (ZEO) generalizing the ontology of standard physics.
2. The new view about space-time as 4-surface $X^4 \subset H = M^4 \times CP_2$ is central for applications. One manner to formulate this is that X^4 is simultaneously minimal surface and extremal of Kähler action S_K analogous to Maxwell action. The twistor lift of TGD forces the presence of both S_K and the volume term in the action.
3. Especially important minimal surfaces are CP_2 type extremals representing building bricks of elementary particles, cosmic strings and magnetic flux tubes as their deformations so that their M^4 and CP_2 projections have dimension larger than 2, and so called massless extremals (MEs). Magnetic flux tubes appear as two variants depending on whether they carry monopole flux or not. Monopole flux tubes require no current to create the magnetic field and are not possible in Maxwellian theory. Both are in a crucial role also in condensed matter applications.
4. The new view about space-time differs dramatically from that of GRT. The space-time surface is topologically non-trivial in all scales and many-sheeted in the sense that CP_2 coordinates as function of M^4 coordinates and vice versa are many-valued. The space-time of GRT is obtained from the many-sheeted one in long length scale limit by replacing the sheets with a single region of M^4 and by deforming its metric. The gauge potentials are defined as sums of induced gauge potentials for sheets. The deviation of the metric is the sum of the deviations of the induced metric from the M^4 metric.

   The new physics related to the many-sheetedness is not describable in terms of the QFT approach.

5. The classical field equations reduce to conservation laws for the conserved charges defined by the isometries of H. Therefore they are essentially hydrodynamical and this together with QCC is essential for TGD inspired quantum hydrodynamics (QHD). The conjecture that the extremals allow generalized Beltrami property, which implies the existence of a global coordinate varying along the flow lines of flow. For instance, Beltrami property provides purely classical geometric correlates for supra flows and supracurrents. Global coordinates allow identification of order parameters having interpretation in terms of quantum coherence.

6. The requirement that modified Dirac operator at the level of space-time surface is in a well-defined sense a projection of the Dirac operator of H implies that for preferred extremals the isometry currents are proportional to projections if the corresponding Killing vectors with proportionality factor constant along the projections of their flow lines. This implies as generalization of the energy conservation along flow lines of hydrodynamical flow ($\rho v^2/2+p = constant$).

   This also leads to a braiding type representations for isometry flows of H in theirs of their projections to the space-time surface and it seems that quantum groups emerge from these representations. Physical intuition suggests that only the Cartan algebra corresponding to commuting observables allows this representation so that the selection of quantization axes would select also space-time surface as a higher level state function reduction.

   One also ends up to a generalization of Equivalence Principle stating that the charges assignable to "inertial" or "objective" representations of H isometries in WCW affecting space-time surfaces as analogs of particles are identical with the charges of "gravitational" or subjective representations which act inside space-time surfaces. This has also implications for M^8-H duality.

2.2 The view about physics at the level of M^8

Over the years, the number theoretical vision has evolved to what I call adelic physics [?, ?]. $M^8 - H$ duality as analog of momentum-position duality and of Complementarity Principle crystallizes number theoretical vision.

1. Complexified octonions M^8_c have interpretation as an analog of momentum space [?, ?]. There are 4-surfaces in both M^8 and H and they are related by $M^8 - H$ duality. 4-surfaces $X^4 \subset M^8$ have associative normal spaces and are are defined as "roots" of an octonionic polynomial defined as an algebraic continuation of a real polynomial P with rational coefficients.

   A real polynomial (its roots) therefore defines the entire 4-surface, which means holography taken to an extreme. This also motivates the preferred extremal property at the H side, where one has partial differential equations and variational principle instead of algebraic equations. The analog of Bohr orbit property is one manner to formulate the restrictions.

2. The notion of cognitive representation is motivated by adelic physics, and corresponds to a subset of points of $X^4 \subset M^8$ in M^8_c such that the coordinates of the points are in the extension of rationals defined by P. They define a unique discretization of the 4-surface. Cognitive representation contains common points of the real 4-surface and its p-adic variants and makes possible the number-theoretical universality of adelic physics. It is not completely clear whether the cognitive representations are needed only on the M^8 side or at both sides of the duality.

3. The interpretation of the points of M^8 as momenta leads to the proposal that the points of cognitive representation are algebraic integers. Contrary to the naive expectations, all algebraic numbers of extension belong to the cognitive representation for the roots of octonionic polynomials [?, ?]. A natural restriction is that 4-momenta correspond to algebraic integers. A further restriction is that the "active" points of the cognitive representation are occupied by quarks (in TGD leptons can correspond to bound states of quarks [?, ?]). Therefore the 4-surface in H is analogous to the Fermi ball containing discrete quark momenta as an analog of cognitive representation. Condensed matter physics relies strongly on the use of momentum space so that M^8 picture is central for the applications in condensed matter.

4. Galois group of the polynomial defining $X^4 \subset M^8$ acts as symmetries for the roots of the polynomials. The order of Galois group of P is identified as effective Planck constant

$h_{eff}/h_0 = n$, where the ordinary Planck constant is a multiple of h_0. n is in general not the same as the dimension of extension defined by the degree of P. The original identification was as the dimension of extension counting the number of roots. One must keep an open mind here.

The identification of as h_{eff} as the order of the Galois group (rather than dimension of extension) finds support from the following. The order gives the number of regions at the orbit of Galois group (in the case that the isotropy group of the point is trivial!) and at the level of H, the action is sum of identical contributions over these regions so that Planck constant would be nh_0.

The phases of matter with different values of h_{eff} behave like dark matter relative to each other (this does not of course imply that galactic dark matter and energy correspond to $h_{eff} > h$ phases). Various quantum scales, typically proportional to h_{eff}, can be arbitrarily long and quantum coherence is possible in all scales and assigned with the magnetic body (MB). Galois confinement [?] generalizes the notion of periodic boundary conditions and could serve as a universal mechanism for the formation of bound states. One implication is that the total momentum of a bound state formed by quarks has total momentum, which is a rational integer. This serves as an extremely powerful constraint.

The evolutionary hierarchies formed by the extensions of rationals defined by functional composition of polynomials are characterized by root inheritance if the condition $P(0) = 0$ is satisfied. This gives rise to an evolutionary hierarchy of bound states such that the new level contains all bound states of the previous level (conserved genes serve as an analogy). What is nice is that the states of Galois representations which are not Galois singlets can serve as composites of Galois singlets at the next level. For these reasons Galois confinement is expected to play a fundamental role in TGD and in the TGD based view about condensed matter.

5. Nottale introduced the notion of gravitational Planck constant [E1] $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, which allows us to understand the planetary planets as Bohr orbits. The form of $\hbar_{gr}$ is dictated by the Equivalence Principle. In the TGD framework, the hierarchy of Planck constants, the infinite range of gravitational interaction, and the absence of screening motivate this notion [K22, K15, K16, K2] .

   The gravitational Compton length for a particle with mass m is $\Lambda_{gr} = GM/v_0 = r_s c/2v_0$ and thus expressible in terms of Schwarzschild radius and has $r_s/2$ as lower bound. For Earth of order TGD view about living matter involves $\hbar_{gr}$ in an essential manner.

   Amazingly, there are indications that Λ_{gr} for Earth could play a key role in superconductivity [?], superfluidity, and in quantum analogies of hydrodynamical systems [?]. Also the role of Λ_{gr} for Sun is suggestive. Note that the original motivation for the large value of h_{eff} was the TGD based model for effects of em radiation at ELF frequencies on the vertebrate brain [K8, K19, K20].

2.3 $M^8 - H$ duality

The proposal is that the description of physics in terms of geometry and number theory are dual to each other. There are several observations motivating $M^8 - H$ duality.

1. There are four classical number fields: reals, complex numbers, quaternions, and octonions with dimensions $1, 2, 4, 8$. The dimension of the embedding space is $D(H) = 8$, the dimension of octonions. Spacetime surface has dimension $D(X^4) = 4$ of quaternions. String world sheet and partonic 2-surface have dimension $D(X^2) = 2$ of: complex numbers. The dimension $D(string) = 1$ of string is that of reals.
2. Isometry group of octonions is a subgroup of automorphism group G_2 of octonions containing $SU(3)$ as a subgroup. $CP_2 = SU(3)/U(2)$ parametrizes quaternionic 4-surfaces containing a fixed complex plane.

Could M^8 and $H = M^4 \times CP_2$ provide alternative dual descriptions of physics?

1. Actually a complexification $M^8_c \equiv E^8_c$ by adding an imaginary unit i commuting with octonion units is needed in order to obtain sub-spaces with real number theoretic norm squared. M^8_c

fails to be a field since $1/o$ does not exist if the complex valued octonionic norm squared $\sum o_i^2$ vanishes.

2. The four-surfaces $X^4 \subset M^8$ are identified as "real" parts of 8-D complexified 4-surfaces X_c^4 by requiring that $M^4 \subset M^8$ coordinates are either imaginary or real so that the number theoretic metric defined by octonionic norm is real. Note that the imaginary unit defining the complexification commutes with octonionic imaginary units and number theoretical norm squared is given by $\sum_i z_i^2$ which in the general case is complex.

3. The space H would provide a geometric description, classical physics based on Riemann metric, differential geometric structures and partial differential equations deduced from an action principle. M_c^8 would provide a number theoretic description: no partial differential equations, no Riemannian metric, no connections...

   M_c^8 has only the number theoretic norm squared and bilinear form, which are real only if M_c^8 coordinates are real or imaginary. This would define "physicality". One open question is whether all signatures for the number theoretic metric of X^4 should be allowed? Similar problem is encountered in the twistor Grassmannian approach.

4. The basic objection is that the number of algebraic surfaces is very small and they are extremely simple as compared to extremals of action principle. Second problem is that there are no coupling constants at the level of M^8 defined by action.

   Preferred extremal property realizes quantum criticality with universal dynamics with no dependence on coupling constants. This conforms with the disappearance of the coupling constants from the field equations for preferred extremals in H except at singularities, with the Bohr orbitology, holography and ZEO. $X^4 \subset H$ is analogous to a soap film spanned by frame representing singularities and implying a failure of complete universality.

5. In M^8, the dynamics determined by an action principle is replaced with the condition that the *normal* space of X^4 in M^8 is associative/quaternionic. The distribution of normal spaces is always integrable to a 4-surface.

   One cannot exclude the possibility that the normal space is complex 2-space, this would give a 6-D surface [**?**, **?**]. Also this kind of surfaces are obtained and even 7-D with a real normal space. They are interpreted as analogs of branes and are in central role in TGD inspired biology.

   Could the twistor space of the space-time surface at the level of H have this kind of 6-surface as M^8 counterpart? Could $M^8 - H$ duality relate these spaces in 16-D M_c^8 to the twistor spaces of the space-time surface as 6-surfaces in 12-D $T(M^4) \times T(CP_2)$?

6. Symmetries in M^8 number theoretic: octonionic automorphism group G_2 which is complexified and contains $SO(1,3)$. G_2 contains $SU(3)$ as M^8 counterpart of color $SU(3)$ in H. Contains also $SO(3)$ as automorphisms of quaternionic subspaces. Could this group appear as an (approximate) dynamical gauge group?

   $M^8 = M^4 \times E^4$ as $SO(4)$ as a subgroup. It is not an automorphism group of octonions but leaves the octonion norm squared invariant. Could it be analogous to the holonomy group $U(2)$ of CP_2, which is not an isometry group and indeed is a spontaneously broken symmetry.

   A connection with hadron physics is highly suggestive. $SO(4) = SU(2)_L \times SU(2)_R$ acts as the symmetry group of skyrmions identified as maps from a ball of M^4 to the sphere $S^3 \subset E^4$. Could hadron physics $\leftrightarrow$ quark physics duality correspond to $M^8 - H$ duality. The radius of S^3 is proton mass: this would suggest that M^8 has an interpretation as an analog of momentum space.

   One implication of $M^8 - H$ duality is that the image of a generic point of $X^4 \subset M^8$ is a single point of CP_2. There is complete localization in color degrees of freedom in Einsteinian regions of space-time having 4-D M^4 projection and one cannot even speak of color. This would solve in a trivial manner the problem of color confinement.

   CP_2 type extremals however correspond to singularities for which a single point of line has ill-defined normal space and normal spaces correspond to a 3-D surface in CP_2. In this case, one can assign representations of color group $SU(3)$ to the image of the line which is essentially CP_2. Color therefore makes sense inside the Euclidean wormhole contacts.

   For the string-like entities, the singularity at a given point of string world sheets corresponds to a 2-D surface of CP_2, which is a complex manifold or Lagrangian manifold. For the two geodesic spheres the color group reduces to $U(2)$ or $SO(3)$, and one can speak about spontaneous breaking of the color symmetry. Also S^1 singularity is possible for objects with 3-D M^4 projection. In this case the color symmetry reduces to $U(1)$.

7. What is the interpretation of M^8? Massless Dirac equation in M^8 for the octonionic spinors must be algebraic. This would be analogous to the momentum space Dirac equation. Solutions would be discrete points having interpretation as quark momenta! Quarks pick up discrete points of $X^4 \subset M^8$.

   States turn out to be massive in the M^4 sense: this solves the basic problem of 4-D twistor approach (it works y for massless states only). Fermi ball is replaced with a region of a mass shell (hyperbolic space H^3).

   $M^8 - H$ duality would generalize the momentum-position duality of the wave mechanics. QFT does not generalize this duality since momenta and position are not anymore operators.

2.4 Organization of "TGD and Condensed Matter"

1. The first part of the book contains some material related to the earlier work on this topic.

   Two old chapters about possible implications of TGD for condensed matter physics written for at least about 15 years ago at least and updated only slightly. The phases of CP_2 complex coordinates could define phases of order parameters of macroscopic quantum phases so that the deviations of induced gauge field concept from the standard one could have direct experimental implications visible for instance in the properties of living matter and even in hydrodynamics. For instance, Z^0 magnetic gauge field could make itself visible in hydrodynamics and also Z^0 magnetic vortices could be involved with super-fluidity.

2. The second part of the book is devoted to the most recent (2021-) work related to TGD view about condensed matter.

3 Sources

The eight online books about TGD [K29, K28, K21, K14, K5, K13, K7, K24] and nine online books about TGD inspired theory of consciousness and quantum biology [K26, K4, K17, K3, K6, K9, K10, K23, K25] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

4 The contents of the book

4.1 PART I: SOME EARLIER WORK (BEFORE 2021) RELATED TO CONDENSED MATTER

4.1.1 Hydrodynamics and CP_2 Geometry

The chapter is one of the earliest attempts to apply TGD to macroscopic physics and must be taken as such. The chapter begins with a brief summary of the basic notions related to many-sheeted space-time. A generalization of hydrodynamics to a p-adic hierarchy of hydrodynamics is considered and a mechanism of energy transfer between condensate levels is identified. It is suggested that TGD based generalization of Hawking-Bekenstein law holds even in macroscopic length scales and that hydrodynamical vortices behave in some aspects like elementary particles.

TGD leads to a formulation of a general theory of phase transitions: the new element is the presence of several condensate levels.

It has much later become clear that the vision about elementary particles Euclidian space-time regions defining lines of generalized Feynman diagrams generalizes to macroscopic scales and that every macroscopic body should accompany such space-time sheet and thus in some aspects behave like elementary particle.

A topological model for the generation of the hydrodynamical turbulence is proposed. The basic idea is that hydrodynamical turbulence can be regarded as a spontaneous Kähler magnetization leading to the increase the value of Kähler function and therefore of the probability of the configuration. Kähler magnetization is achieved through the formation of a vortex cascade via the decay of the mother vortex by the emission of smaller daughter vortices. Vortices with various values of the fractal quantum number and with sizes related by a discrete scaling transformation appear in the cascade. The decay of the vortices takes place via the so called phase slippage process.

An encouraging result is the prediction for the size distribution of the vortices: the prediction is practically identical with that obtained from the model of Heisenberg but on rather different physical grounds. The model is rather insensitive to the p-adic scaling of vortices in the transition as long as it is smaller than $\lambda = 2^{-5}$. The model is also consistent with the assumption that the decay of a vortex to smaller vortices corresponds to a phase transition from a given level of dark matter hierarchy to a lower level so that the value of $\hbar$ is reduced by a factor $\lambda = v_0/n \simeq 2^{-11}/n$, $n = 1, 2, ...$ so that Compton length scales as well as sizes of vortices are reduced by this factor.

4.1.2 Macroscopic Quantum Phenomena and CP_2 Geometry

Topological field quantization is applied to a unified description of three macroscopic quantum phases: super conductors, super fluids and quantum Hall phase. The basic observation is that the formation of connections identified as join along boundaries bonds makes possible the formation of macroscopic quantum system from topological field quanta having size of the order of the coherence length ξ for ordinary phase. The presence of the connections makes possible supra flow and the presence of two levels of the topological condensate explains the two-fluid picture of super fluids. In standard physics, the order parameter is constant in the ground state. In TGD context, the non-simply connected topology of the 3-surface makes possible ground states with a covariantly constant order parameter characterized by the integers telling the change of the order parameter along closed homotopically nontrivial loops. Later an alternative identification of connections as Kähler magnetic flux tubes carrying magnetic monopole flux has emerged but does not change the general vision.

The role of the ordinary magnetic field in super conductivity is proposed to be taken by the Z^0 magnetic field in super fluidity and the mathematical descriptions of super conductors and super fluids become practically identical. The generalization of the quantization condition for the magnetic flux to a condition involving also a velocity circulation, plays a central role in the description of both phases and suggests a new description of the rotating super fluid and some new effects. A classical explanation for the fractional Quantum Hall effect in terms of the topological field quanta is proposed. Quantum Hall phase is very similar to the supra phases: an essential role is played by the generalized quantization condition and the hydrodynamic description of the Hall electrons. The role of Z^0 magnetic field is suggested by large parity breaking effects in biology.

The results obtained support the view that in condensed matter systems topological field quanta with size of the order of $\xi \simeq 10^{-8} - 10^{-7}$ meters are of special importance. This new length scale is expected to have also applications to less exotic phenomena of the condensed matter physics (the description of the conductors and di-electrics and ferromagnetism) and in hydrodynamics (the failure of the hydrodynamic approximation takes place at this length scale). These field quanta of course, correspond to only one condensate level and many length scales are expected to be present.

4.1.3 Quantum Model for Bio-Superconductivity: I

The model for generalized EEG relates very closely to the general model of high T_c superconductivity. This motivates a separate discussion of the vision about bio-super-conductivity in TGD Universe.

1. *General mechanisms of bio-superconductivity*

The many-sheeted space-time concepts suggested a very general mechanism of superconductivity based on the "dropping" of charged particles from atomic space-time sheets to larger space-time sheets. The first guess was that larger space-time sheets are very dry, cool and silent so that the necessary conditions for the formation of high T_c macroscopic quantum phases are met. The criticism against "dropping" is that particle can topologically condense on several space-time sheets which therefore are not separate worlds: this is indeed assumed in the recent view about GRT and QFT limit of TGD. Dropping could therefore occur only at larger space-time sheet at the boundary of the smaller one. The expansion of the space-time sheet (flux tube) in p-adic phase transition liberates also zero point kinetic energy (cyclotron energy).

The possibility of large $\hbar$ quantum coherent phases makes the assumption about thermal isolation between space-time sheets un-necessary. The establishment of thermal equilibrium would rely on the phase transitions transforming ordinary to dark matter and vice versa. Biophotons could be produced from dark photons in this manner. The flow from a flux tube portion with larger value of h_{eff} to that with a smaller value liberates cyclotron energy.

A crucial element is quantum criticality predicting a new kind of superconductivity explaining the strange features of high T_c super-conductivity. This led to the proposal that there are two kinds of Cooper pairs, exotic Cooper pairs with spin $S = 1$ and counterparts of ordinary BCS type Cooper pairs with spin $S = 0$. Both correspond to a large value of Planck constant. Exotic Cooper pairs are quantum critical meaning that they can decay to ordinary electrons. Below temperature $T_{c_1} > T_c$ only exotic Cooper pairs with spin are present and their finite lifetime implies that superconductivity is broken to ordinary conductivity satisfying scaling laws characteristic for criticality. At T_c spinless BCS type Cooper pairs become stable and exotic Cooper pairs can decay to them and vice versa. An open question is whether the BCS type Cooper pairs can be present also in the interior of cell.

These two superconducting phases would compete in certain narrow interval around critical temperature for which body temperature of endotherms is a good candidate in the case of living matter. Also high T_c superfluidity of bosonic atoms dropped to space-time sheets of electronic Cooper pairs becomes possible besides ionic super conductivity. Even dark neutrino superconductivity can be considered below the weak length scale of scaled down weak bosons.

Magnetic flux tubes would be carriers of dark particles and magnetic fields would be crucial for super-conductivity. Two parallel flux tubes carrying magnetic fluxes in opposite directions is the simplest candidate for super-conducting system. This conforms with the observation that anti-ferromagnetism is somehow crucial for high temperature super-conductivity. The spin interaction energy is proportional to Planck constant and can be above thermal energy: if the hypothesis that dark cyclotron energy spectrum is universal is accepted, then the energies would be in bio-photon range and high temperature super-conductivity is obtained. If fluxes are parallel spin $S = 1$ Cooper pairs are stable. $L = 2$ states are in question since the members of the pair are at different flux tubes. These two kinds of Cooper pairs could correspond to BCS type and exotic Cooper pairs.

The fact that the critical magnetic fields can be very weak or large values of $\hbar$ is in accordance with the idea that various almost topological quantum numbers characterizing induced magnetic fields provide a storage mechanism of bio-information.

This mechanism is extremely general and in principle works for electrons, protons, ions, charged molecules and even exotic neutrinos and an entire zoo of high T_c bio-superconductors, super-fluids and Bose-Einstein condensates is predicted. Of course, there are restrictions due to the thermal stability it room temperature and it seems that only electron, neutrino, and proton Cooper pairs are possible at room temperature besides Bose-Einstein condensates of all bosonic ions and their exotic counterparts resulting when some nuclear color bonds become charged.

2. Hierarchies of preferred p-adic length scales and values of Planck constant

TGD inspired quantum biology and number theoretical considerations suggest preferred values for $r = \hbar/\hbar_0$. For the most general option the values of $\hbar$ are products and ratios of two integers n_a and n_b. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n_i)$, $i \in \{a, b\}$, in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of r.

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes

$M_{G,k} = (1+i)k-1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four p-adic length scales sith $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG. This hypothesis - to be referred to as Mersenne hypothesis - replaces the earlier rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

3. Fractal hierarchy of magnetic flux sheets and the hierarchy of genomes

The notion of magnetic body is central in the TGD inspired theory of living matter. Every system possesses magnetic body and there are strong reasons to believe that the magnetic body associated with human body is of order Earth size and that there could be an entire hierarchy of these bodies with even much larger sizes. Therefore the question arises what one can assume about these magnetic bodies. The quantization of magnetic flux suggests an answer to this question.

1. The quantization condition for magnetic flux reads in the most general form as $\oint (p-eA)\cdot dl = n\hbar$. If supra currents flowing at the boundaries of the flux tube are absent one obtains $e\int B \cdot dS = n\hbar$, which requires that the scaling of the Planck constant scales up the flux tube thickness by r^2 and scaling of B by $1/r$. If one assumes that the radii of flux tubes do not depend on the value of r, magnetic flux is compensated by the contribution of the supra current flowing around the flux tube: $\oint (p-eA)\cdot dl = 0$. The supra currents would be present inside living organism but in the faraway region where flux quanta from organism fuse together, the quantization conditions $e\int B \cdot dS = n\hbar$ would be satisfied.
2. From the point of view of EEG especially interesting are the flux sheets which have thickness $L(151) = 10$ nm (the thickness of cell membrane) carrying magnetic field having strength of endogenous magnetic field. In absence of supra currents these flux sheets have very large total transversal length proportional to r^2. The condition that the values of cycloctron energies are above thermal energy implies that the value of r is of order 2^{k_d}, $k_d = 44$. Strongly folded flux sheets of this thickness might be associated with living matter and connect their DNAs to single coherent structure. One can of course assume the presence of supra currents but outside the organism the flux sheet should fuse to form very long flux sheets.
3. Suppose that the magnetic flux flows in head to tail direction so that the magnetic flux arrives to the human body through a layer of cortical neurons. Assume that the flux sheets traverse through the uppermost layer of neurons and also lower layers and that DNA of each neuronal nuclei define a transversal sections organized along flux sheet like text lines of a book page. The total length of DNA in single human cell is about one meter. It seems that single organism cannot provide the needed total length of DNA if DNA dominates the contribution. This if of course not at all necessarily since supra currents are possible and outside the organism the flux sheets can fuse together. This implies however correlations between genomes of different cells and even different organisms.

These observations inspire the notion of super- and hyper genes. As a matter fact, entire hierarchy of genomes is predicted. Super genes consist of genes in different cell nuclei arranged to threads along magnetic flux sheets like text lines on the page of book whereas hyper genes traverse through genomes of different organisms. Super and hyper genes provide an enormous representative capacity and together with the dark matter hierarchy allows to resolve the paradox created by the observation that human genome does not differ appreciably in size from that of wheat.

4.1.4 Quantum Model for Bio-Superconductivity: II

The models for EEG and its variants and for nerve pulse rely on a general model of high T_c superconductivity. The general vision behind model of cell membrane as super-conductor inspired by the identification of dark matter in terms of hierarchy of Planck constants and the notion of magnetic body was considered in the previous chapter. In this chapter the vision is tested by applying it to various anomalous findings about the behavior of the cell membrane.

The topics discussed are following.

1. There are several findings challenging the standard thermodynamical view about cell membrane. TGD suggests a model in which various transmembrane proteins (receptors, channels, pumps) act as Josephson junction between superconductors assignable to the interior and exterior of cell membrane.

   The most feasible model for cell membrane and charge transfer found hitherto relies on Pollack's observations about fourth gel like phase of water. The model for the findings leads to a generalization of the cell membrane as Josephson junction obtained by adding to Josephson energy the difference of the cyclotron energies of dark ion at two sides of the cell membrane. Cyclotron energy difference replaces chemical potential difference in the generalization of the thermodynamical model inspired by Zero Energy Ontology, and replacing thermodynamical distributions with their quantal "square roots". Charge transfer would be induced by a phase transition changing the value of Planck constant at either or both sides of the membrane. This would induce the change of the equilibrium concentrations of ions and also charge transfer.

2. Water memory, chiral selection of biomolecules, burning of water by radiowaves represent further intriguing effects whose understanding seems to require new physics. Dark matter identified in term of hierarchy of Planck constants and the notion of magnetic body define an attractive candidate in this respect. Scaled up variants of weak physics defined by the hierarchy of Planck constants and p-adic length scale hierarchy could explain chiral selection.

3. Hafedh Abdelmelek and collaborators have found evidence for effective super-conductivity in the sciatic nerves of both endotherms (rabbit) and poikilotherms (frog). The TGD based explanation would be in terms of dark supra currents.

4. DC currents of Becker have been known for a long time. An attractive interpretation is as supra currents. The basic prediction is that the resistance should not depend on he length of the conduction pathway. One can also construct a quantum model for the current.

5. TGD inspires two views about cell membrane which need not be contradictory. For the first model cell is far from vacuum extremal, for the second model nearly vacuum extremal. There are several constraints on the model coming from the TGD based identification of bio-photons, the new view about metabolism. It seem that that the first model might be enough when generalized along lines inspired by Pollack's findings about the fourth phase of water.

Physicists M. Tajmar and C. J. Matos and their collaborators working in ESA (European Satellite Agency) have made an amazing claim of having detected strong gravimagnetism with gravimagnetic field having a magnitude which is about 20 orders of magnitude higher than predicted by General Relativity.

Tajmar et al have proposed the gravimagnetic effect as an explanation of an anomaly related to the superconductors. The measured value of the mass of the Cooper pair is slightly larger than the sum of masses whereas theory predicts that it should be smaller. The explanation would be that actual Thomson field is larger than it should be because of gravimagnetic contribution to quantization rule used to deduce the value of Thomson field. The required value of gravimagnetic Thomson field is however 28 orders of magnitude larger than General Relativity suggests. TGD inspired proposal is based on the notion of gravitational Planck constant assignable to the flux tubes connecting to massive objects. It turns out that the TGD estimate for the Thomson field has correct order of magnitude. The identification $h_{eff} = h_{gr}$ at particle physics and atomic length scales emerges naturally.

A vision about the fundamental role of quantum gravitation in living matter emerges. The earlier hypothesis that dark EEG photons decay to biophotons with energies in visible and ultraviolet range receives strong quantitative support. Also a mechanism for how magnetic bodies couple bio-chemistry emerges. The vision conforms with Penrose's intuitions about the role of quantum gravity in biology.

4.1.5 Quantum Hall effect and Hierarchy of Planck Constants

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

1. The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (Kähler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by delocalized electron at the boundary of the vortex. One can raise two questions.

   Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

   Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

2. Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge fractionalization and fractional braiding statistics. Here the vacuum degeneracy of Kähler action suggests the explanation. Vacuum degeneracy implies that the correspondence between the normal component of the canonical momentum current and normal derivatives of embedding space coordinates is 1- to-n. These kind of branchings result in multi-furcations induced by variations of the system parameters and the scaling of external magnetic field represents one such variation.

3. At the orbits of wormhole throats, which can have even macroscopic M^4 projections, one has $1 \rightarrow n_a$ correspondence and at the space-like ends of the space-time surface at light-like boundaries of causal diamond one has $1 \rightarrow n_b$ correspondence. This implies that at partonic 2-surfaces defined as the intersections of these two kinds of 3-surfaces one has $1 \rightarrow n_a \times n_b$ correspondence. This correspondence can be described by using a local singular n-fold covering of the embedding space. Unlike in the original approach, the covering space is only a convenient auxiliary tool rather than fundamental notion.

4. The fractionalization of charge can be understood as follows. A delocalization of electron charge to the n sheets of the multi-furcation takes place and single sheet is analogous to a sheet of Riemann surface of function $z^{1/n}$ and carries fractional charge $q = e/n$, $n = n_a n_b$. Fractionalization applies also to other quantum numbers. One can have also many-electron stats of these states with several delocalized electrons: in this case one obtains more general charge fractionalization: $q = \nu e$.

5. Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the braid rotation by $\Delta\Phi = 2\pi$, where Φ corresponds to M^4 angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate Φ of the n-branched covering $\Delta\Phi = 2/pi$ corresponds to $\Delta\Phi = n \times 2\pi$. If one identifies the sheets of multi-furcation and therefore uses Φ as angle coordinate, single valued angular momentum eigenstates become in general n-valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.

6. How to understand the exceptional values $\nu = 5/2, 7/2$ of the filling fraction? The non-abelian braid group representations can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible. It seems that the minimum number of braids is $n > 2$ from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives $n > 3$. The minimum value is $n = 4$ consistent with the fractional charge $e/4$.

   The model introduces Z_4 valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a Z_4 valued momentum characterizing the four delocalized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would corresponds to that of 4-fold covering space of embedding space serving as a convenient auxiliary tool. The more standard explanation is that $Z_4 = Z_2 \times Z_2$ such that Z_2:s correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. Electroweak gauge group defined non-abelian braid group in large h_{eff} phase weak length above atomic length scale so that weak bosons and even fermion behave as effectively massless particles below scaled up weak scale. TGD also predicts the possibility of dynamical gauge groups and maybe this kind of gauge group indeed emerges. Dynamical gauge groups emerge also for stacks of N branes and the n sheets of multifurcation are analogous to the N sheets in the stack for many-electron states.

4.2 PART II: THE MOST RECENT WORK RELATED TO CONDENSED MATTER PHYSICS

4.2.1 TGD Towards the End of 2021

This article tries to give a rough overall view about Topological Geometrodynamics (TGD) as it is towards the end of 2021. The two views about TGD and their relationship are discussed at the general level.

1. The first view generalizes Einstein's program for the geometrization of physics. Entire quantum physics is geometrized in terms of the notion of "world of classical worlds" (WCW), which by its infinite dimension has unique Kähler geometry.
2. Second vision reduces physics to number theory. Classical number fields (reals, complex numbers, quaternions, and octonions) are central as also p-adic number fields and extensions of rationals. The physics is classically coded by algebraic 4-surfaces in complexified M^8 having octonionic structure and "roots" of octonionic polynomials obtained as algebraic continuations of real polynomials with rational coefficients. M^8_c has an interpretation as an analog of momentum space.

The preparation of this summary led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.
2. The understanding of the details of the M^8-H duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. M^8-H duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H. Mass shell (its positive and negative energy parts) is mapped to a light-like boundary of CD with size $T = h_{eff}/m$, m the mass associated with momentum.
3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

Article includes also a section about neutrinos and TGD. The motivation is that the recent results related to neutrino mixing led to a dramatic progress in the understanding of the role of right-handed neutrino solving long-standing problems of quantum TGD.

4.2.2 TGD and Condensed Matter Physics

Condensed matter physics is under rapid evolution, one might even speak of revolution. New exotic states of matter are discovered and their theoretical understanding in the existing theoretical framework is highly challenging. The findings challenge the existing reductionistic framework and it is quite possible that new physics is required. This motivates the question whether the new physics provided by TGD could provide some understanding. The general view about condensed matter in TGD Universe is following.

1. TGD is analogous to hydrodynamics in the sense that field equations at the level of H reduce to conservation laws for isometry charges. The preferred extremal property meaning that space-time surfaces are simultaneous extremals of volume action and Kähler action allows interpretation in terms of induced gauge fields. The generalized Beltrami property implies the existence of an integrable flow serving as a correlate for quantum coherence. Conserved Beltrami flows currents correspond to gradient flows. At the QFT limit this simplicity would be lost.
2. The fields H, M, B and D, P, E needed in the applications of Maxwell's theory could emerge at the fundamental level in the TGD framework and reflect the deviation between Maxwellian and the TGD based view about gauge fields due to CP_2 topology.
3. The understanding of macroscopic quantum phases improves. The role of the magnetic body carrying dark matter is central. The understanding of the role of WCW degrees of freedom improves considerably in the case of Bose-Einstein condensates of bosonic particles such as polaritons. M^8 picture allows us to understand the notion of skyrmion. The formation of Cooper pairs and analogous states with higher energy would correspond to a formation of Galois singlets liberating energy used to increase h_{eff}. What is new is that energy feed makes possible supra-phases and their analogs above the critical temperature. TGD description of Cooper pairs is consistent with fermion number conservation and the notion of Bogoliubov quasiparticles does not seem necessary or relevant in TGD.
4. Fermi surface emerges as a fundamental notion at the level of M^8 but has a counterpart also at the level of H. Galois groups would be crucial for understanding braids, anyons and fractional Quantum Hall effect. Galois confinement suggests a universal mechanism for the formation of bound states. Space-time surface could be seen as a curved quasicrystal associated with the lattice of M^8 defined by algebraic integers in an extension of rationals. Also the TGD analogs of condensed matter Majorana fermions emerge.
5. TGD also provides new insights about topological physics and space-time topology provides a new element of topological physics.

In this article, the basic notions of condensed matter physics are discussed from the TGD point view, some concrete problems of condensed matter are considered, and some tests are proposed.

4.2.3 TGD and Quantum Hydrodynamics

The purpose of this article is to consider possible applications of Topological Geometrodynamics (TGD) to hydrodynamics. The basic question is what quantum hydrodynamics could mean in the TGD framework.

The mathematical structure of TGD is essentially that of hydrodynamics in the sense that field equations reduce to conservation laws for the charges associated with the isometries of $H = M^4 \times CP_2$.

Hydrodynamical turbulence represents one of the unsolved problems of physics and therefore as an excellent test bench for the TGD based vision. How turbulence is generated and how it decays? What is the role of vortices and their reconnections? These are the basic questions. The central notion of the TGD based model is that of a magnetic body (MB) carrying dark $h_{eff} = nh_0$ phases and controlling ordinary matter. Z^0 magnetic field is proportional to the circulation in the proposed model and electroweak symmetry restoration below scaled up weak Compton length is in an essential role. This picture is applied to several problems including also the problems related to the magnetic reconnection rate and to the survival of magnetic fields in even cosmic scales. Monopole flux tubes provide the solution here.

The hydrodynamic quantum analogs is a fascinating field and TGD picture is applied to this case. The basic prediction is that the Faraday wave length playing the role of Compton wavelength corresponds to the gravitational Compton length predicted by the generalization of the Nottale hypothesis. The value is very near to the minimal value predicted by TGD.

In the TGD framework it might be possible to understand viscosity in terms of dark angular momentum unit $\hbar_{eff}$. A proposal which allows us to understand the critical values of Reynolds numbers for the generation of turbulence in terms of the gravitational Compton lengths associated with Sun and Earth is made. Also this success supports the view that new quantum theory provided by TGD is needed in order to understand the generation of turbulence.

The universality of QHD according to TGD motivates the proposal for an application to hadron and nuclear physics. The general description of quantum tunnelling could be in terms of ZEO involving two BSFRs and therefore temporary time reversal at the MB of the system of colliding particles. Quantum hydrodynamics and large values of h_{eff} would be involved with this period. A model of "cold fusion" is one practical application.

4.2.4 TGD Inspired Model for Freezing in Nano Scales

Freezing is a phase transition, which challenges the existing view of condensed matter in nanoscales. In the TGD framework, quantum coherence is possible in all scales and gravitational quantum coherence should characterize hydrodynamics in astrophysical and even shorter scales. The hydrodynamics at the surface of the planet such as Earth the mass of the planet and even that of the Sun should characterize gravitational Planck constant h_{gr} assignable to gravitational flux tubes mediating gravitational interactions. In this framework, quantum criticality involving $h_{eff} = nh_0 > h$ phases of ordinary matter located at the magnetic body (MB) and possibly controlling ordinary matter, could be behind the criticality of also ordinary phase transitions.

In this article, a model inspired by the finding that the water-air boundary involves an ice-like layer. The proposal is that also at criticality for the freezing a similar layer exists and makes possible fluctuations of the size and shape of the ice blob. At criticality the change of the Gibbs free energy for water would be opposite that for ice and the Gibbs free energy liberated in the formation of ice layer would transform to the energy of surface tension at water-ice layer.

This leads to a geometric model for the freezing phase transition involving only the surface energy proportional to the area of the water-ice boundary and the constraint term fixing the volume of water. The partial differential equations for the boundary surface are derived and discussed.

If $\Delta P = 0$ at the critical for the two phases at the boundary layer, the boundary consists of portions, which are minimal surfaces analogous to soap films and conformal invariance characterizing 2-D critical systems is obtained. Clearly, 3-D criticality reduces to rather well-understood 2-D criticality. For $\Delta P \neq 0$, conformal invariance is lost and analogs of soap bubbles are obtained.

In the TGD framework, the generalization of the model to describe freezing as a dynamical time evolution of the solid-liquid boundary is suggestive. An interesting question is whether this boundary could be a light-like 3-surface in $M^4 \times CP_2$ and thus have a vanishing 3-volume. A huge extension of ordinary conformal symmetries would emerge.

4.2.5 Spin Glasses, Complexity, and TGD

Spin glasses represent an exotic phenomenon, which remains poorly understood in the standard theoretical framework of condensed matter physics. Actually, spin glasses provide a prototype of complex systems and methods used for spin glasses can be applied in widely different complex systems.

In this chapter a TGD inspired view about spin glasses is discussed.

1. TGD view about space-time leads to the notion of magnetic flux tubes and magnetic body. Besides spins also long closed magnetic flux tubes would contribute to magnetization. The basic support for this assumption is the observation that the sum of the NFC magnetization and the FC remanence is equal to the NFC magnetization. Magnetic field assignable to spin glass would correspond to a kind of flux tube spaghetti and the couplings J_{ij} between spins would relate to magnetic flux tubes connecting them.
2. Quantum TGD leads to the notion of "world of classical worlds" (WCW) and to the view about quantum theory as a "complex square root" of thermodynamics (of partition function). The probability distribution for $\{J_{ij}\}$ would correspond to ground state functional in the space of space-time surfaces analogous to Bohr orbits.
3. Spin glass is a prototype of a complex system. In the TGD framework, the complexity reduces to adelic physics fusing real physics with various p-adic physics serving as correlates of cognition. Space-time surfaces in $H = M^4 \times CP_2$ correspond to images of 4-surfaces $X^4 \subset M^8_c$ mapped to H by $M^8 - H$ duality. X^4 is identified as 4-surface having as holographic boundaries 3-D mass shells for which the mass squared values are roots of an octonionic polynomial P obtained as an algebraic continuation of a real polynomial with rational coefficients. The higher the degree of P, the larger the dimension of the extension of rationals induced by its roots, and the higher the complexity: this gives rise to an evolutionary hierarchy. The dimension of the extension is identifiable as an effective Planck constant so that high complexity involves a long quantum coherence scale.

   TGD Universe can be quantum critical in all scales, and the assumption that the spin glass transition is quantum critical, explains the temperature dependence of NFC magnetization in terms of long range large h_{eff} quantum fluctuations and quantum coherence at critical temperature.
4. Zero energy ontology predicts that there are two kinds of state function reductions (SFRs). "Small" SFR would be preceded by a unitary time evolution which is scaling and generated by the scaling generator L_0. This conforms with the fact that relaxation rates for magnetization obey power law rather than exponent law. "Big" SFRs would correspond to ordinary SFRs and would change the arrow of time. This could explain aging, rejuvenation and memory effects.
5. Adelic physics leads to a proposal that makes it possible to get rid of the replica trick by replacing thermodynamics with p-adic thermodynamics for the scaling operator L_0 representing energy. What makes p-adic thermodynamics so powerful is the extremely rapid converges of Z in powers of p-adic prime p.

4.2.6 Comparing Berry phase model of super-conductivity with TGD based model

Hiroyasu Koizumi has proposed a new theory of superconductivity (SC) based on the notion of Berry phase related with an effective magnetic field assignable to adiabatically evolving systems. The model shares similarities with the TGD inspired view about SC. The article also mentioned anomalies that were new to me. This motivated a fresh look in the TGD inspired model. The outcome was an integration of two separate ideas about supraphases.

1. Space-time surfaces as preferred extremals with CP_2 projection of dimension $D = 2$ or $D = 3$ would naturally correspond to 4-D generalizations of so called Beltrami flows, which are integrable flows defined by the flow lines of the induced Kähler field. The existence of a global coordinate z varying along flow lines requires the integrability of the flow. Classical dissipation is absent so that these surfaces are excellent candidates for the space-time correlates of supra flows. The exponential of z gives a phase factor associated with the complex order parameter of a coherent state of Cooper pairs as a counterpart of the Berry phase. Kähler magnetic monopole flux defines the TGD counterpart of "novel" magnetic field.
2. The identification of supra phases as dark matter as $h_{eff} > h$ phases at magnetic flux quanta (tubes and sheets) implies that Cooper pairs correspond to dark fermions associated with the members of flux tube pair, which actually combine to form a closed flux tube. Also single electrons can define supraflow.

3. The Cooper pairs must be created by bosonic oscillator operators constructed from fermionic oscillator operators by bosonization. This is possible only in 1+1-dimensional situations. Thanks to the Beltrami flow the situation is effectively 1+1-dimensional. Bosonization makes it possible to identify SU(2) Kac-Moody algebra, which has an interpretation in the TGD framework.

The assumption that Cooper pairs reside at the magnetic flux quanta solves the 4 problems of standard framework mentioned by Koizumi: high-Tc SCs have two transition temperatures; electron mass m_e instead of its effective mass m_e^* appears in Thomson moment; the reversible phase transition in an external magnetic field inducing a splitting of Cooper pairs does not involve dissipation; why the erratic calculation of the Josephson frequencies in standard model neglecting the chemical potentials gives a correct result?.

The formation of the Cooper pairs appears as a condition stabilizing the space-time sheets carrying dark matter and all preferred extremals could satisfy the conditions guaranteeing integrable flow and existence of a phase factor varying along flow lines. Could supra phases exist in all scales? Could the breaking of supra phases be only due to the finite size of the space-time sheets? Could even hydrodynamic flow involve super-fluidity of some kind - perhaps based on neutrino Cooper pairs as speculated earlier?

4.2.7 Can quasi-time crystal be created by a Fibonacci process?: TGD point of view

This article is a commentary of the work of Dumitrescu et al, which is based on a computer simulation of a quantum computer program realizing unitary evolution believed to make sense as a model for that of time quasi-lattice. We do not really understand what makes the time (quasi-)lattices: the needed physics is not understood.

Therefore the key question is how time (quasi-)lattices are possible in TGD Universe: here zero energy ontology (ZEO) provides a mechanism minimizing the dissipation: time reversal occurring in state function reductions gives rise to time reversed dissipation and dissipation in reversed time direction looking like automatic error correction. This relates to the long lifetime of entanglement, easy to achieve in the unitary evolution but more difficult in the dissipative real world.

The popular article at Phys.org talks somewhat misleadingly about 2-D time although the time values in discretization span 2-D algebraic extension of rationals. The effective N-dimensionality in the algebraic sense is a basic prediction of adelic physics, which involves cognitive representations as unique number theoretical discretization of space-time surface relying on the hierarchy of extensions of rationals. In the real physics sense one would have 1-D time but in algebraic sense N-dimensional time.

The claimed dynamical emergence of symmetries making possible symmetry protected short range entanglement for edge states of the ion array is not really understood and is therefore interesting from the TGD viewpoint. Same applies to the notion of topologically preserved long range entanglement: also here the new physics predicted by TGD can help.

The article mentions also the possibility of quantum coherent units of N qubits behaving like single multi-qubit. The notion of dark N-particles emerges naturally from the number theoretical view of TGD. The dark N-particle would be an analog of the color singlet hadron, and the color group would be replaced by the Galois group. The existence of these kinds of states would mean a revolution in quantum computation and there already exists evidence for N-photons.

4.2.8 How do AdS/CFT- and TGD based holographic dualities relate?

The article *"Traversable wormhole dynamics on a quantum processor"* by Jafferis et al published in Nature received a lot of media attention. My original reaction was due to frustration caused by the media hype. What was done was a quantum computer simulation of the so-called SYK (Sachdev-Ye-Kitaev) model proposing AdS/CFT duality for a particular condensed matter system. The attempts to understand what is involved soon led to a realization that since TGD predicts the analog of AdS/CFT holographic duality, the quantum computational aspects of the experiment should be understandable also using the holographic duality of TGD. This raises the question whether can one translate the notions of AdS holography to TGD holography. In particular, what could be the TGD counterparts for the notions of wormhole and negative energy shock waves needed to stabilize the wormhole. This article deals with these kinds of questions and leads to a rather detailed view of TGD based holography.

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