Homology of "world of classical worlds" in relation to Floer homology and quantum homology

M. Pitkänen,

February 14, 2022
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Contents

Abstract

One of the mathematical challenges of TGD is the construction of the homology of "world of classical worlds" (WCW). The generalization of Floer homology looks rather obvious in the zero ontology (ZEO) based view about quantum TGD. ZEO, the notion of preferred extremal (PE), and the intuitive connection between the failure of strict non-determinism and criticality are essential elements. The homology group is defined in terms of the free group formed by preferred extremals $PE(X^3,Y^3)$ for which X^3 is a stable maximum of Kähler function K associated with the passive boundary of CD and Y^3 associated with the passive boundary is a more general critical point.

The identification of PEs as minimal surfaces with lower-dimensional singularities as loci of instabilities implying non-determinism allows to assign to the set $PE(X^3,Y_i^3)$ numbers $n(X^3,Y_i^3 \to Y_j^3)$ as the number of instabilities of singularities leading from Y_i^3 to Y_j^3 and define the analog of criticality index (number of negative eigenvalues of Hessian of function at critical point) as number $n(X^3,Y_i^3) = \sum_j n(X^3,Y_i^3 \to Y_j^3)$. The differential d defining WCW homology is defined in terms of $n(X^3,Y_i^3 \to Y_j^3)$ for pairs Y_i^3, Y_j^3 such that $n(X^3,Y_j^3) - n(X^3,Y_i^3) = 1$ is satisfied.

1 Introduction

One of the mathematical challenges of TGD is the construction of the homology of "world of classical worlds" (WCW). With my rather limited mathematical skills, I had regarded this challenge as a mission impossible. The popular article in Quanta Magazine with title "Mathematicians transcend the geometric theory of motion" (see `https://cutt.ly/vO4eb5V` however stimulated the attempts to think whether it might be possible to say something interesting about WWC homology.

The article told about a generalization of Floer homology by Abouzaid and Blumberg [A1] (`https://cutt.ly/ZPeOTSc`) published as 400 page article with the title "Arnold Conjecture and Morava K-theory". This theory transcends my mathematical skills but the article stimulated the idea WCW homology might be obtained by an appropriate generalization of the basic ideas of Floer homology (`https://cutt.ly/VO4dSPD`).

The construction of WCW homology as a generalization of Floer homology looks rather straightforward in the zero ontology (ZEO) based view about quantum TGD. The notions of ZEO and causal diamond (CD) [L1] [K4], the notion of preferred extremal (PE) [L5] [K2], and the intuitive connection between the failure of strict non-determinism and criticality pose strong conditions on the possible generalization of Floer homology.

WCW homology group could be defined in terms of the free group formed by preferred extremals $PE(X^3,Y^3)$ for which X^3 is a *stable* maximum of Kähler function K associated with the *passive* boundary of CD and Y^3 associated with the *active* boundary of CD is a more general critical point.

The stability of X^3 conforms with the TGD view about state function reductions (SFRs) [L1]. The sequence of "small" SFRs (SSFRs) at the active boundary of CD as a locus of Y^3 increases the size of CD and gradually leads to a PE connecting X^3 with stable 3-surface Y^3. Eventually "big" SFR (BSFR) occurs and changes the arrow of time and the roles of the boundaries of the CD changes. The sequence of SSFRs is analogous to a decay of unstable state to a stable final state.

The identification of PEs as minimal surfaces with lower-dimensional singularities as loci of instabilities implying non-determinism allows to assign to the set $PE(X^3,Y_i^3)$ numbers $n(X^3,Y_i^3 \to Y_j^3)$ as the number of instabilities of singularities leading from Y_i^3 to Y_j^3 and define the analog of criticality index (number of negative eigenvalues of Hessian of function at critical point) as number $n(X^3,Y_i^3) = \sum_j n(X^3,Y_i^3 \to Y_j^3)$. The differential d defining WCW homology is defined in terms of $n(X^3,Y_i^3 \to Y_j^3)$ for pairs Y_i^3, Y_j^3 such that $n(X^3,Y_j^3) - n(X^3,Y_i^3) = 1$ is satisfied. What is nice is that WCW homology would have direct relevance for the understanding of quantum criticality.

The proposal for the WCW homology also involves a generalization of the notion of quantum connectivity crucial for the definition of Gromow-Witten invariants. Two surfaces (say branes) can be said to intersect if there is a string world sheet connecting them generalizes. In ZEO quantum connectivity translates to the existence of a preferred extremal (PE), which by the weak form of holography is almost unique, such that it connects the 3-surfaces at the opposite boundaries of causal diamond (CD).

2 Some background

In this section some background, including Morse theory, Floer homology, its generalization by Abouzaid and Blumberg, and the basic ideas of TGD proposal, is discussed.

2.1 The basic ideas of Morse theory

Torus as a 2-D example helps to understand the idea of homology and Morse theory. Homologically non-trivial surfaces are surfaces without boundary but are not boundaries themselves. Entire torus represents the element of H^2, the 2 homologically non-trivial circles, and points indeed have vanishing boundaries without being boundaries. The basic homological operation d represents the operation of forming a boundary: the boundary of a boundary is empty and this corresponds to $d^2 = 0$. d reduces degree of homology by one unit: $H_n \rightarrow H_{n-1}$.

How to understand the homology of torus? Morse theory based on the notion of Morse function provides the tool.

1. Consider the embedding of torus to 3-space. The height-coordinate h defines a Morse function at torus and one can assign to it $h = constant$ level surfaces. It has 4 critical points: h_0, h_1, h_2, h_3 at which the topology of level suraace changes.

   h has maximum h_3 at the top of torus and minimum h_0 at the bottom of the torus. h_3 corresponds to the entire torus, element of homology group H_2 and h_0 to a point as element of H_0.

   h has saddle points h_1, h_2 at the top and bottom of the "hole" of the torus. The level surfaces $h = h_1$ and $h = h_2$ correspond to two touching circles: the topology of the intersection changes from a contractible circle to a union of oppositely oriented incontractible small circles representing elements of the homology group H_1. That they have opposite orientations states conservation of homology charge in the topological reaction in which the level circle splits to two: $0 = 1 - 1$.

   Outside the critical points the topology of the $h = constant$ level surface is a circle or two disjoint circles. The critical points of h clearly code part of the homology of torus. What however remains missing is the homology group element, which corresponds to the large circle around the torus. This element of H_1 would be obtained if the height function h were a horizontal coordinate.

2. One can deform the torus and also add handles to it to get 2-D topologies with a higher genus. Morse function also helps to understand the homology of higher-dimensional spaces for which visual intuition fails.

   This situation is finite-D and too simple to apply in the case of the space of orbits of a Hamiltonian system. Now the point of torus is replaced with a loop as a single orbit in phase space. The loop space is infinite-dimensional and the Morse theory does not generalize as such. In Floer homology one studies even the homology of infinite-dimensional spaces.

Homology involves also the d operation. d can be indeed visualized in terms of dynamics of a gradient flow. Assume that torus is in the gravitational potential of Earth proportional to h. Gravitation defines a downwards directed gradient force. One can speak of critical directions as directions in which the particle forced to stay at the torus can fall downwards when subjected to an infinitesimal push.

1. At the top $h = h_3$ of the torus there are 2 critical directions: either along a small or large incontractible circle of torus. This number corresponds to the dimension $d = 2$ of torus as the element of the homology group H_2. At the bottom $h = h_0$ there are 0 critical directions and one has a point as an element of H_0. At the saddle points h_1, h_2 there is 1 critical direction and it corresponds to a nontrivial circle as an element of H_1. The number n of critical directions corresponds to the dimension for elements of the homology group H_n.

2. The particle at the top h_3 has 2 critical directions (criticality 2), and can fall to the saddle point h_2, having criticality 1, by moving along the small homologically non-trivial circle.

Criticality decreases by 1 unit so that one has a map $H_2 \to H_1$. The particle can also move along the large circle to the bottom, in which case criticality decreases by 2 units.

The particle at critical point h_2 moves to h_1 along a circle homologous to the large circle without a change in criticality and the particle at h_1 moves to h_0 also the small circle: the criticality changes by 1 unit so that one has a map $H_1 \to H_0$.

Therefore the elements of the homology group correspond to critical points for the gradient flow defined by the gravitational field and the effect of the map d can be represented dynamically as a motion in the gravitational field reducing the criticality by one unit.

The representability of homology elements as critical points of Morse function and the representation of d-operation in terms of gradient dynamics is extremely useful in higher dimensional spaces, where geometric intuition does not help much. In Floer homology this dynamics is applied as a tool.

2.2 The basic ideas of Floer homology

Consider first the motivations and ideas of Floer homology (`https://cutt.ly/lO6EMp6`). The original goal was to prove Arnold's conjecture. One considers a symplectic manifold with symplectic form ω. Arnold conjectured that the number of fixed points of a Hamiltonian symplectomorphism generated by an exponentiation of a Hamiltonian H, is bounded below by the number of critical points of a smooth function on M.

The goal is to generalize Morse theory.

1. Morse theory involves the height function h in a finite-D manifold M and the critical points of h correspond to elements of homology groups H_n. The number n of negative eigenvalues of Hessian of f at critical points defines the index of criticality f and one can associate with the critical point an element of the homology group H_n. $n = 0$ corresponds to maximum of f. Note that in infinite-D case, Morse theory need not work since n can be arbitrarily large and if the convention for criticality is changed so that $n = 0$ corresponds to minimum, a different theory is obtained.

2. In Morse homology, the n-simplices of the simplicial homology are replaced by critical points with criticality index n and the homology groups are replaced with the Abelian group defined by the critical points and graded by the criticality index n. The gradient flow lines connecting critical points with $\Delta n = 1$ allow to define an analog of the exterior derivative d: it is defined by the number of flow lines connecting critical points with $\Delta n = 1$.

2.3 Floer homology

The motivation for the symplectic Floer homology is the conjecture by Arnold related to the Hamiltonian systems. These systems are defined in phase space, whose points are pairs of position and momentum of the particle. This notion is extremely general in classical physics.

1. One considers compact symplectic manifolds M and symplectic action $S = \oint p_i dq_i$ and its critical points, which are loops. Note that symplectic action has interpretation as an area. The general case $S = \oint (p_i dq_i/dt - H)dt$ is not considered in the Floer homology.

   Remark: A more general question is whether there exist closed orbits, kind of islands of order, in the middle of oceans of chaos consisting of non-closed chaotic orbits. This is indeed the case: there is a fractal structure formed by islands of order in oceans of disorder. Hamiltonian chaos differs from dissipative chaos in that the fractal has the same dimension as the symplectic manifold since symplectic transformations preserve area and high 2n-dimensional volumes.

2. Arnold's conjecture was that the number of critical points of a given criticality index of a symplectomorphism has as an upper bound the number of critical points for a generic function. The inspiration behind the Floer homology is the intuition that a generalization of Morse theory to the loop space $L(M)$ allows us to understand the homology. The conjecture is that the closed orbits serve as minimal area representatives for the homology of $L(M)$. These closed orbits would be critical points of S defining the area closed by the curve.

The goal is to understand the homology of a finite-dimensional compact symplectic manifold M and Floer homology provides the needed tool. Floer homology for the infinite-D loop space $L(M)$ serves as a tool to achieve this goal and the proof of Arnold's conjecture follows as an outcome.

In symplectic Floer homology, one is interested in closed loops as orbits of a symplectic flow in a compact symplectic space M. One wants to identify them as critical points of an analog of Morse function in the loop space $L(M)$.

1. In the symplectic Floer homology, M is a finite-D symplectic manifold and one deduces information about it from the homology of loop space $L(M)$ by generalizing Morse homology to the homology of $L(M)$.

2. The counterpart of the Morse function is unique and defined by the symplectic action functional $S = \oint p_i dq_i$ in $L(M)$. Note that S depends only on M. S defines the counterpart of free action with a vanishing Hamiltonian H. For a general Hamiltonian one would have $S = \oint (p_i dq_i/dt - H)dt$. Note that closed orbits are possible if M is compact. For a generic H the dynamic becomes chaotic.

   Closed loops for free flows define the analogs of critical points of Morse function. For instance, for 2-torus the closed orbits correspond to loops with winding numbers n_1, n_2.

3. One must identify the counterpart for the gradient flow lines connecting the critical points with $\Delta n = 1$ in order to define d. Here one considers a deformation of the system by a time dependent Hamiltonian H and hopes that the predictions do not depend on the choices of H. This gives to orbits of the closed loops in the loop space giving rise to cylinders in M.

   These cylinders define pseudoholomorphic curves and define the counterparts of the gradient flows connecting critical points as closed loops in X. The differential d for the Floer homology is defined in terms of the numbers of these curves between critical points with the property that the criticality index increases by one unit.

4. The basic result is a proof for the the Arnold conjecture and roughly states that for the ranks of homology groups of M are smaller than the Floer homology groups defined by arbitrary Hamilton.

Floer homology has a rich variety of applications discussed in the Wikipedia article (`https://cutt.ly/lO6EMp6`). One application relates to the Lagrangian manifolds of a symplectic manifold. Now the chain complex is generated by the intersection points of Lagrangian manifolds intersecting transversely.

A further application is associated with Yang- Mills theory. The action is the Chern-Simons action defining a topological quantum field theory. Its critical points are topologically non-trivial gauge connections with a trivial curvature form. Topological non-triviality means that the group defined by the parallel translations along closed curves is non-trivial. The counterpart of the gradient flow is defined by Yang-Mills action and the flow lines correspond to instantons approach at the ends of the counterpart of mapping cylinder trivial connections.

2.4 The generalization of Floer homology by Abouzaid and Blumberg

The work of mathematicians Abouzaid and Blumberg [A1] (`https://cutt.ly/ZPeOTSc`), which represents the generalization of Floer homology which, using popular terms, allows to "count holes" in the infinite-D space of loops.

The abstract of the article of Abouzaid and Blumberg is following.

> We prove that the rank of the cohomology of a closed symplectic manifold with coefficients in a field of characteristic p is smaller than the number of periodic orbits of any non-degenerate Hamiltonian flow.
> Following Floer, the proof relies on constructing a homology group associated to each such flow, and comparing it with the homology of the ambient symplectic manifold. The proof does not proceed by constructing a version of Floer's complex with characteristic p coefficients, but uses instead the canonical (stable) complex orientations of moduli spaces of Floer trajectories to construct a version of Floer homology with coefficients in Morava's K-theories, and can thus be seen as an implementation of Cohen, Jones, and Segal's vision for a Floer homotopy theory. The key feature of Morava K-theory that allows the construction to be carried out is the fact that the corresponding homology and cohomology groups of classifying spaces of finite groups satisfy Poincar duality.

I try to express what I understand as a physicist about this highly technical summary.

1. The main emphasis is in the homology of finite-D symplectic manifolds and the homology of the infinite-D loop space is only a tool to obtain this information.

2. The generalization of Arnold's conjecture is expressed in the first paragraph. For closed symplectic manifolds the cohomology groups of a closed symplectic manifold have rank smaller than the number of periodic orbits of *any* non-degenerate Hamiltonian flow.

   Therefore Hamiltonian flows give information about the cohomology and by Poincare duality also about homology of the symplectic manifold.

3. The coefficients of homology can be chosen in very many manners: rationals, integers, finite fields, p-adic number fields. Integers are however the natural ones in the situation in which one counts concrete objects. The homology has coefficients in finite field F_p, integers modulo prime p: for instance, the numbers of flow lines of gradient flow connecting the critical points of symplectic action are counted modulo p.

4. Time dependent Hamiltonians enter into the picture as perturbations of the symplectic action. One replaces the free symplectic action $S = \oint p_i dq_i/dt$ in loop space with $S = \oint (p_i dq_i/dt - H)dt$ playing a role analogous to that of Morse function. This is like adding an interaction term to free action. It is essential that the symplectic space is compact so that closed orbits as critical points of S are possible.

2.5 Gromow-Witten invariants

The proposed TGD based generalization of the notion of "being connected" by a flow line of gradient flow resonates with the definition of Gromow-Witten (G-W) invariant. G-W invariant emerges in enumerative geometry, which is essentially counting of particular kinds of points of enumerative geometry which is a branch of algebraic geometry.

G-W invariants (`http://tinyurl.com/y9b5vbcw`) are rational number valued topological invariants useful in algebraic and symplectic geometry. These quantum invariants give information about these geometries not provided by classical invariants. Despite being rational numbers in the general case G-W invariants in some sense give the number of string world sheets connecting given branes.

The definition of G-W invariant involves a non-locality, which is completely analogous to the non-locality in the proposed definition of WCW homology. In TGD, the string world sheet as connector of branes is replaced with PE as a connector of the boundaries of opposite boundaries of CD taking the role of brane.

Here is the definition of G-W invariants with some TGD induced coloring taken from [K3, K1].

1. One considers a collection of n surfaces ("branes") with even dimensions in some symplectic manifold X of dimension $D = 2k$ (say Kähler manifold) and pseudo-holomorphic curves ("string world sheets") X^2, which have the property that they connect these n surfaces in the sense that they intersect the "branes" in the marked points x_i, $i = 1,..,n$.

"Connect" does not reduce to intersection in a topologically stable sense since connecting is possible also for branes with dimension smaller than $D-2$. One allows all surfaces X^2 that intersect the n surfaces at marked points if they are pseudo-holomorphic even if the basic dimension rule is not satisfied. In the 4-dimensional case this does not seem to have implications since the partonic 2-surfaces automatically satisfy the dimension rule. The n branes intersect or touch in a quantum sense: there is no concrete intersection but intersection with the mediation of "string world sheet".

2. Pseudo-holomorphy means that the Jacobian df of the imbedding map $f: X^2 \rightarrow X$ commutes with the symplectic structures j *resp.* J of X^2 *resp.* X: i.e. one has $df(jT) = Jdf(T)$ for any tangent vector T at given point of X^2. For $X^2 = X = C$ this gives Cauchy-Riemann conditions.

   In the symplectic case X^2 is characterized topologically by its genus g and homology class A as the surface of X. In algebraic geometry context the degree d of the polynomial defining X^2 replaces A. In TGD X^2 corresponds to a string world sheet having also a boundary. X^2 has also n marked points $x_1, ..., x_n$ corresponding to intersections with the n surfaces.

3. G-W invariant $GW^{X,A}_{g,n}$ gives the number of pseudo-holomorphic 2-surfaces X^2 connecting n given surfaces in X - each at single marked point. In TGD these surfaces would be partonic 2-surfaces and marked points would be carriers of sparticles.

3 About the generalization of Floer homology in the TGD framework

A generalization of homotopy and homology groups could help to understand WCW topology. One of the intuitive visions behind TGD has indeed been that, despite the explicit appearance of metric, TGD in a certain sense is a topological quantum theory. A mathematical motivation for this intuition comes from the fact that minimal surfaces provide representations for homological equivalence classes. Floer homology suggests concrete ideas, which might help to understand the homology of WCW.

3.1 Key ideas behind WCW homology

The encounter with Floer homology inspired the question whether one could say something interesting about WCW homology by an appropriate generalization of the concepts involved with it.

3.1.1 Preferred extremals (PEs) as counterparts of critical points

PEs are an obvious candidate for the counterparts of critical points. ZEO however implies some important delicacies crucial for WCW homology.

1. In the TGD Universe, space-time is a 4-surface in $H = M^4 \times CP_2$, in a loose sense an orbit of 3-surface. General Coordinate Invariance (GCI) requires that the dynamics associates to a given 3-surface a highly unique 4-surface at which the 4-D general coordinate transformations act. This 4-surface is a PE of the action principle determing space-time surfaces in H and analogous to Bohr orbit. GCI gives Bohr orbitology as an exact part of quantum theory and also holography.

   These PEs as 4-surfaces are analogous to the closed orbits in Hamiltonian systems about which Arnold speculated. In the TGD Universe, only these PEs would be realized and would make TGD an integrable theory. The theorem of Abouzaid and Blumberg allows to prove Arnold's conjecture in homologies based on cyclic groups Z_p. Maybe it could also have use also in the TGD framework.

2. WCW generalizes the loop space considered in Floer's approach. Very loosely, loop or string is replaced by a 3-D surface, which by holography induced is more or less equivalent with

4-surface. In TGD just these minimal representatives for homology as counterparts of closed orbits would matter.

3. Symplectic structure and Hamiltonian are central notions also in TGD. Symplectic (or rather, contact) transformations assignable to the product $\delta M^4_+ \times CP_2$ of the light-cone boundary and CP_2 act as the isometries of the infinite-D "world of classical worlds" (WCW) consisting of these PEs, or more or less equivalently, corresponding 3-surfaces. Hamiltonian flows as 1-parameter subgroups of isometries of WCW are symplectic flows in WCW with symplectic structure and also Kaehler structure.

4. The space-time surfaces are 4-D minimal surfaces in H with singularities analogous to frames of soap films. Minimal surfaces are known to define representatives for homological equivalence classes of surfaces. This has inspired the conjecture that TGD could be seen as a topological/homological quantum theory in the sense that space-time surfaces served as unique representatives or their homological classes.

5. There is also a completely new element involved. TGD can be seen also as number theoretic quantum theory. $M^8 - H$ duality can be seen as a duality of a geometric vision in which space-times are 4-surfaces in H an of a number theoretic vision in which one consideres 4-surfaces in octonionic complexified M^8 determined by polynomials with dynamics reducing to the condition that the normal space of 4-surface is associative (quaternionic). M^8 is analogous to momentum space so that a generalization of momentum-position duality of wave mechanics is in question.

3.1.2 The first sketch for WCW homology

A suitable generalization of Floer's theory might allow us to define WCW homology.

1. The PEs would correspond to the critical points of an analog of Morse function in the infinite-D context. In TGD the Kähler function K defining the Kahler geometry of WCW is the unique candidate for the analog of Morse function.

   The space-time surfaces for which the exponent $exp(-K)$ of the Kähler function is stationary (so that the vacuum functional is maximum) would define PEs. Also other space-time surfaces could be allowed and it seems that the continuity of WCW requires this. However the maxima or perhaps extrema would provide an excellent approximation and number theoretic vision would give an explicit realization for this approximation.

   It is however important to notice that the K for, in general non-unique, preferred external $PE(X^3, Y^3)$ can be maximum for X^3 and a more general critical point for Y^3. This option conforms with the ZEO view about SFRs in which the passive boundary of CD is stable and a sequence of SSFRs takes place at the active boundary and increases its size. The homology would be assigned to the criticality of the active boundary of CD.

   This would require a varying CD size, which should therefore be determined by PE and appear as a parameter in PE. By $M^8 - H$ duality the boundary of CD corresponds to the image of a mass shell H^3 in M^3. Perhaps this property at the active end of PE codes for the size scale of the CD. The size scale of CD, not necessarily the size, should correspond to the p-adic length scale L_p determined by the largest ramified prime of the polynomial coding for PE. Does this mean that L_p remains the same during the entire sequence of SSFRs or can it increase? The size could increase by factor $\sqrt{p}$ with change ibn L_p and for large p-adic primes such as $M_{127} = 2^{127} - 1$ this would mean very large scaling.

   Remark: Since WCW Kähler geometry has an infinite number of zero modes, which do not appear in the line element as coordinate differentials but only as parameters of the metric tensor, one expects an infinite number of maxima.

2. The PEs would correspond by $M^8 - H$ duality to roots of polynomials P in the complexified octonionic M^8 so that a connection with number theory emerges. $M^8 - H$ duality strongly strongly suggests that $exp(-K)$ is equal to the image of the discriminant D of P under canonical identification $I : \sum x_n p^n \to \sum x_n p^{-n}$ mapping p-adic numbers to reals. The prime p would correspond to the largest ramified prime dividing D [L7, L8].

3. The number theoretic vision could apply only to the critical points of $exp(-K)$ with respect to both ends of PE and give rise to what I call a hierarchy of p-adic physics as correlates of cognition. Everything would be discrete and one could speak of a generalization of computationalism allowing also the hierarchy of extensions of rationals instead of only rationals as in Turing's approach. The real-number based physics would also include the non-maxima via a perturbation theory involving a functional integral around the maxima. Here Kähler geometry allows to get rid of ill-defined metric and Gaussian determinants.

3.1.3 G-W invariants and ZEO

Enumerative geometry is also a central element of adelic physics.

1. M^8-H duality involves the notion of cognitive representations consisting of special points of 4-surface, in particular, points of 3-D mass shell $H^3 \subset M^4_c \subset M^8_c$. The "active" points containing quark are identified as quark momenta. A generalization of momentum-position duality is in question.

2. The points of the cognitive representation, having interpretation as four-momenta [L2, L3, L7, L8], are identified are algebraic integers in the extensions defined by the real polynomial P with rational coefficients continued to a polynomial of a complexified octonion. P defines mass shells as its roots with $m^2 = r_n$ defining the spectrum of virtual mass squared values for quarks. The finite number of mass shells guarantees the absence of divergences due to momentum space integrations.

3. By the symmetries of H^3, the number of points in cognitive representations is especially high at the mass shells. Physical states correspond to Galois singlets (Galois confinement implying conformal confinement) for which the sum of quark momenta is an ordinary integer as one uses as unit the p-adic mass scale defined by the largest ramified prime associated with P.

4. The mass shells associated with a given polynomial P are connected by a 4-surface X^4 as a deformation of M^4_c, which defines M^8-H duality by assigning to $X^4 \subset M^8$ space-time surface in $H = M^4 \times CP_2$. This surface is a minimal surface with singularities analogous to frames of a soap film. M^8-H duality maps the points of cognitive representation to $X^4 \subset H$ [L6].

The TGD view about WCW homology could perhaps be regarded as a generalization of the quantum connectedness behind G-W invariants. The role of the string world-sheet as a quantum connector is taken by PE so that there is no need to introduce gradient dynamics separately. The quantum connection between X^3_1 and X^3_2 at the boundary A of CD exists if $X^3_1 = CPT(Y^3_1)$ is true for a PE having X^3_1 and Y^3_1 as ends. $\Delta n = \pm 1$ translates to an appearance or dis-appearance of minimal number of critical directions. The attribute "quantum" is well-deserved since the classical non-determinism serves as a space-time correlate for quantum jumps at WCW level [L6, L4, L7, L8].

3.2 A more concrete proposal for WCW homology as a generalization of the Floer homology

Consider first the notion of "world of classical worlds" (WCW).

1. In TGD, point-like particles are replaced by 3-surfaces. Zero energy ontology (ZEO) is assumed, which means that space-time surfaces X^4 as "orbits" of 3-surfaces are inside causal diamonds. These 4-surfaces are PEs of the action principle. For the exact holography, 3-surface at either boundary of CD would determine X^4 uniquely but determinism is expected to be slightly violated so that there are several PEs associated with a given X^3 at either boundary of CD. The failure of strict determinism is analogous to the failure of determinism for soap films with frames.

   Let PE have X^3 *resp.* Y^3 as its ends at the opposite boundaries A *resp.* B of CD.

2. WCW is identified as the space of PEs. One could regard WCW also as covering a space such that for a given X^3 at (say) A, the fiber contains the PEs having X^3 as the first end. WCW has symplectic and even a Kähler structure and symplectic transformations at the light-like boundaries of CD are conjectured to define isometries of WCW but not symmetries of S_K.

3. Kähler function K, serving as the analog of symplectic action, defines Kähler form and symplectic structure. K corresponds to 4-D Kähler action S_K plus volume term for a PE. This action is obtained as a dimensional reduction of 6-D Kähler action for the 6-D surface X^6 in the 6+6-D twistor space of $T(M^4) \times T(CP_2)$. X^6 carries induced twistor structure and has X^4 as base space and S^2 as fiber.

3.2.1 WCW homology based on minimal surfaces with singularities

The challenge is to identify the counterpart of gradient flow as a counterpart of quantum connectivity. This should not bring anything new to the existing picture. The following proposal is perhaps the simplest one and conforms with the physical intuition.

1. Morse theory and Floer homology would suggest that one should consider the Hessian of Kähler function $K(PE(X^3)$ of WCW as functional of preferred extremal $PE(X^3, Y^3)$. One could calculate the numbers n_+ resp. n_- of positive and negative eigenvalues of Hessian and identify n_- as the criticality index and number of unstable directions.

2. There are several problems. The identification of the analog of gradient flow seems very difficult. However, by the weak holography due the failure of strict determinism, for a given X^3, there are several 3-surfaces Y_i^3 at the opposite boundary of CD defining PEs. The meaning of criticality is far from obvious since instability for a given time direction looks like stability in the opposite time direction. This is a potential problem since in ZEO [L1] [K4] both arrows of time are possible. There should be a clear distinction between the ends of a CD.

3. By the failure of the strict determinism, the basic objects in ZEO are pairs (X_i^3, Y_j^3) connected by $PE(X_i^3, Y_j^3)$ identifiable as critical points of K with respect to variations of at least one end. The physical picture suggests that criticality is possible for both ends and that a maximum for the passive boundary of CD and criticality for the opposite active boundary of CD (where quantum fluctuations due to "small" state function reductions (SSFRs) are located) is possible. The instabilities associated with criticality at active end would correspond to a definite time direction. It is however difficult to proceed without a more concrete picture.

4. WCW homology could also involve a generalization of the notion of quantum connectivity crucial for the definition of Gromow-Witten invariants. The idea is that two surfaces (say branes) can be said to intersect when there is a string world sheet connecting them, generalizes.

   In ZEO this translates to the existence of a preferred extremal (PE), which by the weak form of holography is almost unique, such that it connects the boundaries of causal diamond (CD), which plays the role of brane.

The identification of PEs as minimal surfaces [L6] allows us to make this picture more concrete and gives a direct connection to quantum criticality as it would be realized in terms of classical non-determinism. One would not count critical directions but critical transitions assignable to singularities of minimal surfaces.

1. PEs are identified as minimal surfaces with singularities analogous to the frames of soap film. At the singularities the minimal surface property fails and the Kähler action and volume term couple together in field equations so that conservation laws are satisfied.

2. The singular surfaces have dimension $d < 4$ and and can be regarded as loci of instability leading to non-determinism. By suitably perturbing the singularities, one can generate new

preferred extremals $PE(X^3, Y_j^3)$ from $PE(X^3, Y_i^3)$. The maximum property of K with respect to the variations of X^3 would suggest that one cannot replace X^3 with a new maximum in this way.

3. For each Y_i^3, one can count the number of deformations of the singularities leading to $PE(X^3, Y_j^3)$ and call this number $n(X^3, Y_i^3 \rightarrow Y_j^3)$ as an the analog for the number of gradient lines between given critical points in Floer homology.

   One can define the analog of criticality index $n(X^3, Y_i^3)$ as $n(X^3, Y_i^3) = \sum_j n(X^3, Y_i^3 \rightarrow Y_j^3)$ as the analog of n_- of the negative eigenvalues of Hessian. One defines an Abelian group as the complex formed by $PE(X_i^3, Y_j^3)$. $n(X^3, Y_i^3)$ defines the grading for $PE(X^3, Y_j^3)$ as an element of this complex.

   The differential d for WCW homology can be defined in the same way as in Floer homology. Assume $n(X^3, Y_j^3) - n(X^3, Y_i^3) = 1$. Define the action of d as $d(PE(X^3, Y^i)) = \sum_j n(X^3, Y_i^3 \rightarrow Y_j^3) PE(X^3, Y^j)$.

The non-determinism of 6-D Kähler and 4-D action would be essential as also the asymmetry between the active and passive boundaries of CD crucial for TGD based quantum measurement theory. Nondeterminism is also essential for the non-triviality of scattering amplitudes since quantum non-determinism in WCW degrees of freedom has classical non-determinism as a space-time correlate [L6]. If the determinism were exact the homology groups H_n would correspond directly to the groups C_n and one would have a Cartesian product of spaces with the homology group $H_n = C_n$. Interesting questions relate to the interpretation of PE pairs with $\Delta n \neq 1$.

3.2.2 Could CPT allow the concretization of quantum connectedness

The quantum connectedness in some sense identifies the 3-surfaces connected by $PE(X_1^3, Y_1^3)$ such that X_1^3 and Y_1^3 are at opposite boundaries of $CD = cd \times CP_2$. If one could assign to Y_1^3 at B a 3-surface X_2^3 at A, quantum connectedness would become more concrete. There is no compelling reason to effectively for this but can ask whether PE could allow to achieve this formally.

1. This formal connection is achieved if there is a discrete symmetry mapping the boundaries A and B of CD to each other. This symmetry must involve time reflection T with respect to the center point of cd. If one requires that the symmetry is an exact symmetry of quantum theory, CPT remains the only candidate. C would act as charge conjugation, realized as a complex conjugation in CP_2.

   CPT maps the boundaries of CDs to each other and therefore also the positive and negative energy parts of zero energy states. The 3-surfaces $(X_1^3\ X_2^3)$ at a given boundary of CD are quantum connected if one has $X_2^3 = CPT(Y_1^3)$ for a PE connecting X_1^3 and Y_1^3.

2. Critical points of K must be mapped to critical points so that CPT should act as a symmetry of the variational principle. If M^4 has Kähler structure the self-dual covariantly constant Kähler form of M^4, strongly suggested by the twistor lift of TGD, must be invariant under CPT and this is indeed the case. The Kähler gauge potential would be also fixed apart from the decomposition $M^4 = M^2 \times E^2$ defined by electric and magnetic parts of $J(M^4)$.

REFERENCES

Mathematics

[A1] Abouzaid A and Blumberg AJ. Arnold conjecture and morava k-theory, 2021. Available at: https://arxiv.org/abs/2103.01507.

Books related to TGD

[K1] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory.* Available at: https:/tgdtheory.fi/pdfpool/infmotives.pdf, 2011.

[K2] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I.* Available at: https:/tgdtheory.fi/pdfpool/prext.pdf, 2019.

[K3] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? In *TGD as a Generalized Number Theory: Part II.* Available at: https:/tgdtheory.fi/pdfpool/ratpoints1II.pdf, 2019.

[K4] Pitkänen M. Zero Energy Ontology. In *Towards M-Matrix: part I.* Available at: https:/tgdtheory.fi/pdfpool/ZEO.pdf, 2021.

Articles about TGD

[L1] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https:/tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[L2] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part I. Available at: https:/tgdtheory.fi/public_html/articles/M8H1.pdf., 2020.

[L3] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part II. Available at: https:/tgdtheory.fi/public_html/articles/M8H2.pdf., 2020.

[L4] Pitkänen M. The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group. Available at: https:/tgdtheory.fi/public_html/articles/SSFRGalois.pdf., 2020.

[L5] Pitkänen M. TGD as it is towards the end of 2021. https://tgdtheory.fi/public_html/articles/TGD2021.pdf., 2021.

[L6] Pitkänen M. What could 2-D minimal surfaces teach about TGD? https://tgdtheory.fi/public_html/articles/minimal.pdf., 2021.

[L7] Pitkänen M. About TGD counterparts of twistor amplitudes. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L8] Pitkänen M. About TGD counterparts of twistor amplitudes. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.