Particle Massivation in TGD Universe

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Abstract

This chapter represents the most recent (2014) view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters. In this chapter my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

1. Physical states as representations of super-symplectic and Super Kac-Moody algebras

The basic constraint is that the super-conformal algebra involved must have five tensor factors. The precise identification of the Kac-Moody type algebra has however turned out to be a difficult task. The recent view is as follows. Electroweak algebra $U(2)_{ew} = SU(2)_L \times U(1)$ and symplectic isometries of light-cone boundary ($SU(2)_{rot} \times SU(3)_c$) give 2+2 factors and full supersymplectic algebra involving only covariantly constant right-handed neutrino mode would give 1 factor. This algebra could be associated with the 2-D surfaces X^2 defined by the intersections of light-like 3-surfaces with $\delta M^4_\pm \times CP_2$. These 2-surfaces have interpretation as partons.

For conformal algebra there are several candidates. For symplectic algebra radial light-like coordinate of light-cone boundary replaces complex coordinate. Light-cone boundary $S^2 \times R_+$ allows extended conformal symmetries which can be interpreted as conformal transformations of S^2 depending parametrically on the light-like coordinate of R_+. There is infinite-D subgroup of conformal isometries with S^2 dependent radial scaling compensating for the conformal scaling in S^2. Kähler-Dirac equation allows ordinary conformal symmetry very probably liftable to imbedding space. The light-like orbits of partonic 2-surface are expected to allow super-conformal symmetries presumably assignable to quantum criticality and hierarchy of Planck constants. How these conformal symmetries integrate to what is expected to be 4-D analog of 2-D conformal symmetries remains to be understood.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [**?**] The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [**?**]

2. Particle massivation

Particle massivation can be regarded as a generation of thermal mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The obvious objection is that Poincare invariance is lost. One could argue that one calculates just the vacuum expectation of conformal weight so that this is not case. If this is not assumed, one would have in positive energy ontology superposition of ordinary quantum states with different four-momenta and breaking of Poincare invariance since eigenstates of four-momentum are not in question. In Zero Energy Ontology this is not the case since all states have vanishing net quantum numbers and one has superposition of time evolutions with well-defined four-momenta. Lorentz invariance with respect to the either boundary of CD is achieved but there is small breaking of Poincare invariance characterized by the inverse of p-adic prime p characterizing the particle. For electron one has $1/p = 1/M_{127} \sim 10^{-38}$.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a

generator of some Lie-algebra (Virasoro generator L_0!)). What basically matters is the number of tensor factors involved and five is the favored number.

2. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

3. A natural identification of the non-integer contribution to the mass squared is as stringy contribution to the vacuum conformal weight (strings are now "weak strings"). TGD predicts Higgs particle and Higgs is necessary to give longitudinal polarizations for gauge bosons. The notion of Higgs vacuum expectation is replaced by a formal analog of Higgs vacuum expectation giving a space-time correlate for the stringy mass formula in case of fundamental fermions. Also gauge bosons usually regarded as exactly massless particles would naturally receive a small mass from p-adic thermodynamics. The theoretetical motivation for a small mass would be exact Yangian symmetry which broken at the QFT limit of the theory using GRT limit of many-sheeted space-time.

4. Hadron massivation requires the understanding of the CKM mixing of quarks reducing to different topological mixing of U and D type quarks. Number theoretic vision suggests that the mixing matrices are rational or algebraic and this together with other constraints gives strong constraints on both mixing and masses of the mixed quarks.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

1 Introduction

This chapter represents the most recent view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters [K1, K8, K12, K9, K10] of [K21]. In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the Kähler-Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected.

Since 2010 a further progress took place. These steps of progress relate closely to ZEO, bosonic emergence, the discovery of the weak form of electric-magnetic duality, the realization of the importance of twistors in TGD, and the discovery that the well-definedness of em charge forces the modes of Kähler-Dirac operator to 2-D surfaces - string world sheets and possibly also partonic

2-surfaces. This allows to assign to elementary particle closed string with pieces at two parallel space-time sheets and accompanying a Kähler magnetic flux tube carrying monopole flux.

Twistor approach and the understanding of the solutions of Kähler-Dirac Dirac operator served as a midwife in the process giving rise to the birth of the idea that all fundamental fermions are massless and that both ordinary elementary particles and string like objects emerge from them. Even more, one can interpret virtual particles as being composed of these massless on mass shell particles assignable to wormhole throats. Four-momentum conservation poses extremely powerful constraints on loop integrals but does not make them manifestly finite as believed first. String picture is necessary for getting rid of logarithmic divergences.

The weak form of electric-magnetic duality led to the realization that elementary particles correspond to bound states of two wormhole throats with opposite Kähler magnetic charges with second throat carrying weak isospin compensating that of the fermion state at second wormhole throat. Both fermions and bosons correspond to wormhole contacts: in the case of fermions topological condensation generates the second wormhole throat. This means that altogether four wormhole throats are involved with both fermions, gauge bosons, and gravitons (for gravitons this is unavoidable in any case). For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM. Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

One cannot avoid the question about the relation between p-adic mass calculations and Higgs mechanism. Higgs is predicted but does the analog of Higgs vacuum expectation emerge as the existence of QFT limit would suggest? Boundary conditions for Kähler-Dirac action with measurement interaction term for four-momentum lead to what looks like an as algebraic variant of masssless Dirac equation in Minkowski space coupled to the analog of Higgs vacuum expectation value restricted at fermionic strings. This equation does not however provide an analog of Higgs mechanism but a space-time correlate for the stringy mass formula coming from the vanishing of the scaling generator L_0 of superconformal algbra. It could also give a first principle explanation for the necessarily tachyonic ground state with half integer conformal weight.

For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM.

Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets. In fact, this contribution can be assigned to the additional conformal weight assignable to the stringy curve. The extension of this conformal algebra to Yangian brings in third integer characterizing the poly-locality of the Yangian generator (n-local generator acts on n partonic 2-surfaces simultaneously. Therefore three integers would characterize the generators of the full symmetry algebra as the very naive expectation on basis of 3-dimensionality of the fundamental objects would suggest. p-Adic mass calculations should be carried out for Yangian generalization of p-adic thermodynamics.

1.1 Physical States As Representations Of Super-Symplectic And Super Kac-Moody Algebras

Physical states belong to the representations of super-symplectic algebra and Super Kac-Moody algebran. The precise identification of the two algebras has been rather tedious task but the recent progress in the construction of WCW geometry and spinor structure led to a considerable progress in this respect [K22].

1. In the generic case the generators of both algebras receive information from 1-D ends of 2-D string world sheets at which the modes of induced spinor fields are localized by the condition that the modes are eigenstates of electromagnetic charge. Right-handed neutrino is an exception since it has no electroweak couplings. One must however require that right-handed neutrino does not mix with the left-handed one if the mode is de-localized at entire space-time sheet.

   Either the preferred extremal is such that Kähler-Dirac gamma matrices defined in terms of canonical momentum currents of Kähler action consist of only M^4 or CP_2 type flat space gammas so that there is no mixing with the left-handed neutrino. Or the CP_2 and M^4 parts of the Kähler Dirac operator annihilate the right-handed neutrino mode separately. One can of course have also modes which are mixtures of right- and left handed neutrinos but these are necessarily localized at string world sheets.

2. The definition of super generator involves integration of string curve at the boundary of causal diamond (CD) so that the generators are labelled by *two* conformal weights: that associated with the radial light-like coordinate and that assignable with the string curve. This strongly suggests that the algebra extends to a 4-D Yangian involving multi-local generators (locus means partonic surface now) assignable to various partonic surfaces at the boundaries of CD - as indeed suggested [K23].

3. As before, the symplectic algebra corresponds to a super-symplectic algebra assignable to symplectic transformations of $\delta M^4_\pm \times CP_2$. One can regard this algebra as a symplectic algebra of $S^2 \times CP_2$ localized with respect to the light-like radial coordinate r_M taking the role of complex variable z in conformal field theories. Super-generators are linear in the modes of right-handed neutrino. Covariantly constant mode and modes decoupling from left-handed neutrino define the most important modes.

4. Second algebra corresponds to the Super Kac-Moody algebra. The corresponding Lie algebra generates symplectic isometries of $\delta M^4_\pm \times CP_2$. Fermionic generators are linear in the modes of induced spinor field with non-vanishing electroweak quantum numbers: that is left-hand neutrinos, charged leptons, and quarks.

5. The overall important conclusion is that overall Super Virasoro algebra has five tensor factors corresponding to one tensor factor for super-symplectic algebra, and 4 tensor factors for Super Kac-Moody algebra $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ (CP_2 isometries, S^2 isometries, electroweak $SU(2)_{ew} \times U(1)$). This is essential for mass calculations.

What looks like the most plausible option relies on the generalization of a coset construction proposed already for years ago but badly mis-interpreted. The construction itself is strongly supported and perhaps even forced by the vision that WCW is union of homogenous or even symmetric spaces of form G/H [K22], where G is the isometry group of WCW and H its subgroup leaving invariant the chosen point of WCW (say the 3-surface corresponding to a maximum of Kähler function in Euclidian regions and stationary point of the Morse function defined by Kähler action for Minkowskian space-time regions). It seems clear that only the Super Virasoro associated with G can involve four-momentum so that the original idea that there are two identical four-momenta identifiable as gravitational and inertial four-momenta must be given up. This boils dow to the following picture.

1. Assume a generalization of the coset construction so that the differences of G and H super-conformal generators O_n annihilate the physical states: $(O_n(G) - O_n(H))|phys\rangle = 0$.

2. In zero energy ontology (ZEO) p-adic thermodynamics must be replaced with its square root so that one consideres genuine quantum states rather than thermodynamical states. Hence the system is quantum coherent. In the simplest situation this implies only that thermodynamical weights are replaced by their square roots possibly multiplied by square roots irrelevant for the mass squared expectation value.

3. Construct first ground states with negative conformal weight annihilated by G and H generators G_n, L_n, $n < 0$. Apply to these states generators of tensor factors of Super Viraroso

algebras to obtain states with vanishing G and H conformal weights. After this construct thermal states as superpositions of states obtained by applying H generators and corresponding G generators G_n,L_n, $n > 0$. Assume that these states are annihilated by G and H generators G_n, L_n,$n > 0$ and by the differences of *all* G and H generators.

4. Super-symplectic algebra represents a completely new element and in the case of hadrons the non-perturbative contribution to the mass spectrum is easiest to understand in terms of super-symplectic thermal excitations contributing roughly 70 per cent to the p-adic thermal mass of the hadron.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the already generalized super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [K23]. The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [K23].

1.2 Particle Massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. The identification of elementary particles developed in three steps.

1. Originally germions were identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian signature of the background space-time sheet. Gauge bosons and Higgs were identified as wormhole contacts with light-like throats carrying fermion and anti-fermion quantum numbers. Gravitons were identified as pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level. This picture generalizes to include super-symmetry. The fermionic oscillator operators associated with the partonic 2-surfaces act as generators of badly broken SUSY and right-handed neutrino gives to the not so badly broken $\mathcal{N} = 1$ SUSY consistent with empirical facts.

   Of course, "badly" is relative notion. It is quite possible that the mixing of right-handed neutrino with left-handed one becomes important only in CP_2 scale and causes massivation. Hence spartners might well have mass of order CP_2 mass scale. The question about the mass scale of right-handed neutrino remains open.

2. The next step was to realize that the topological condensation of fermion generates second wormhole throat which carries momentum and symplectic quantum numbers but no fermionic quantum numbers. This is also needed to the massivation by p-adic thermodynamics applied to the analogs of string like objects defined by wormhole throats with throats taking the role of string ends. p-Adic thermodynamics did not however allow a satisfactory understanding of the gauge bosons masses and it became clear that some additional contribution - maybe Higgsy or stringy contribution - dominates for weak gauge bosons. Gauge bosons should

also somehow obtain their longitudinal polarizations and here Higgs like particles indeed predicted by the basic picture suggests itself strongly.

3. A further step was the discovery of the weak form of electric-magnetic duality, which led to the realization that wormhole throats possess Kähler magnetic charge so that a wormole throat with opposite magnetic charge is needed to compensate this charge. This wormhole throat can also compensate the weak isospin of the second wormhole throat so that weak confinement and massivation results. In the case of quarks magnetic confinement might take place in hadronic rather than weak length scale. Second crucial observation was that gauge bosons are necessarily massive since the light-like momenta at two throats must correspond to opposite three-momenta so that no Higgs potential is needed. This leads to a picture in which gauge bosons eat the Higgs scalars and also photon, gluons, and gravitons develop small mass.

4. A further step was the realization that although the existence of Higgs is established, it need not contribute to neither fermion or gauge boson masses. CP_2 geometry does not even allow covariantly constant holomorphic vector field as a representation for the vacuum expectation value of Higgs. Elementary particles are string like objects and string tension can give additional contribution to the mass squared. This would explain the large masses of weak bosons as compared to the mass of photon predicted also to be non-vanishing in principle. Also a small contribution to fermion masses is expected.

   Higgs vacuum expectation would be replaced with the stringy contribution to the mass squared, which by perturbative argument should apart from normalization factor have the form $\Delta m^2 \propto g^2 T$, where g is the gauge coupling assignable to the weak boson, and T is the analog of hadronic string tension but in weak scale. This predicts correctly the ratio of W and Z boson masses in terms of Weinberg angle.

5. The conformal weight characterizing fermionic masses in p-adic thermodynamics can be assigned to the very short piece of string connecting the opposite throats of wormhole contact. The conformal weight associated with the long string connecting the throats of two wormhole contacts should give the dominant contribution to the masses of weak gauge bosons. Five tensor factors are needed in super-conformal algebra and super-symplectic and super-Kac Moody contributions assignable to symplectic isometries give five factors.

   One can assign conformal weights to both the light-like radial coordinate r_M of $\delta M^4_\pm$ and string. A third integer-valued quantum number comes from the extension of the extended super-conformal algebra to multi-local Yangian algebra. Yangian extension should take place for quark wormhole contacts inside hadrons and give non-perturbative multi-local contributions to hadron masses and might explain most of hadronic mass since quark contribution is very small. That three integers classify states conforms with the very naive first guess inspired by 3-dimensionality of the basic objects.

   The details of the picture are however still fuzzy. Are the light-like radial and stringy conformal weights really independent quantum numbers as it seems? These conformal weights however must be additive in the expression for mass squared to get five tensor factors. Could one identify stringy coordinate with the light-like radial coordinate r_M in Minkowskian space-time regions to explain the additivity? The dominating contribution to the vacuum conformal weight must be negative and half-integer valued. What is the origin of this tachyonic contribution?

The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces.

1. Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Chern-Simons action and its Dirac counterpart result as boundary terms of Kähler action and its Dirac counterpart for preferred extremals. This requires that $j \cdot A$ contribution to Kähler action vanishes for preferred extremals plus weak form of electric-magnetic duality.

   The addition of 3-D measurement interaction term - essentially Dirac action associated with 3-D light-like orbits of partonic 2-surfaces implies that Chern-Simons Dirac operator plus

Lagrangian multiplier term realizing the weak form of electric magnetic duality acts like massless M^4 Dirac operator assignable to the four-momentum propagating along the line of generalized Feynman diagram. This simplifies enormously the definition of the Dirac propagator needed in twistor Grassmannian approach [K23].

2. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is besides Loretnz invariance suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

   Mass squared is identified as the p-adic thermal expectation value of mass squared operator m^2 appearing as M^4 contribution in the scaling generator $L_0(G)$ in the superposition of states with vanishing total conformal weight but with varying mass squared eigenvalues associated with the difference $L_0(G) - L_0(H)$ annihilating the physical state. This definition does not break Lorentz invariance in zero energy ontology. The states appearing in the superposition of different states with vanishing total conformal weight give different contribution to the p-adic thermodynamical expectation defining mass squared and the ability to physically observe this as massivation might be perhaps interpreted as breaking of conformal invariance.

3. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

4. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses via a generation of Higgs vacuum expectation having possibly interpretation in terms of a coherent state. Before the detailed model for elementary particles in terms of pairs of wormhole contacts at the ends of flux tubes the picture about the situation was as follows. From the beginning it was clear that is that ground state conformal weight must be negative. Then it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to stringy contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. The possible Higgs vacuum expectation makes sense only at QFT limit perhaps allowing to describe the Yangian aspects, and would be naturally proportional to Δh so that the coupling to Higgs would only apparently cause gauge boson massivation.

5. A natural identification of the non-integer contribution to the conformal weight is as stringy contribution to the vacuum conformal weight. In twistor approach the generalized eigenvalues of Chern-Simons Dirac operator for external particles indeed correspond to light-like momenta and when the three-momenta are opposite this gives rise to non-vanishing mass. Higgs is necessary to give longitudinal polarizations for weak gauge bosons.

An important question concerns the justification of p-adic thermodynamics.

1. The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. The physical justification for p-adic thermodynamics is effective p-adic topology characterizing the 3-surface: this is the case if real variant of light-like 3-surface has large number of common algebraic points with its p-adic counterpart obeying same algebraic equations but in different number field. In fact, there is a theorem stating that for rational surfaces the number of rational points is finite and rational (more generally algebraic points) would naturally define the notion of number theoretic braid essential for the realization of number theoretic universality.

2. The most natural option is that the descriptions in terms of both real and p-adic thermodynamics make sense and are consistent. This option indeed makes if the number of generalized eigen modes of Kähler-Dirac operator is finite. This is indeed the case if one accepts periodic

boundary conditions for the Chern-Simons Dirac operator. In fact, the solutions are localized at the strands of braids. This makes sense because the theory has hydrodynamic interpretation. This reduces $\mathcal{N} = \infty$ to finite SUSY and realizes finite measurement resolution as an inherent property of dynamics.

The finite number of fermionic oscillator operators implies an effective cutoff in the number conformal weights so that conformal algebras reduce to finite-dimensional algebras. The first guess would be that integer label for oscillator operators becomes a number in finite field for some prime. This means that one can calculate mass squared also by using real thermodynamics but the consistency with p-adic thermodynamics gives extremely strong number theoretical constraints on mass scale. This consistency condition allows also to solve the problem how to map a negative ground state conformal weight to its p-adic counterpart. Negative conformal weight is divided into a negative half odd integer part plus positive part Δh, and negative part corresponds as such to p-adic integer whereas positive part is mapped to p-adic number by canonical identification.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

1.3 What Next?

The successes of p-adic mass calculations are basically due to the power of super-conformal symmetries and of number theory. One cannot deny that the description of the gauge boson and hadron massivation involves phenomenological elements. There are however excellent hopes that it might be possible some day to calculate everything from first principles. The non-local Yangian symmetry generalizing the super-conformal algebras suggests itself strongly as a fundamental symmetry of quantum TGD. The generalized of the Yangian symmetry replaces points with partonic 2-surfaces being multi-local with respect to them, and leads to general formulas for multi-local operators representing four-momenta and other conserved charges of composite states.

In TGD framework even elementary particles involve two wormhole contacts having each two wormhole throats identified as the fundamental partonic entities. Therefore Yangian approach would naturally define the first principle approach to the understanding of masses of elementary particles and their bound states (say hadrons). The power of this extended symmetry might be enough to deduce universal mass formulas. One of the future challenges would therefore be the mathematical and physical understanding of Yangian symmetry. This would however require the contributions of professional mathematicians.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L2].

2 Identification Of Elementary Particles

2.1 Partons As Wormhole Throats And Particles As Bound States Of Wormhole Contacts

The assumption that partonic 2-surfaces correspond to representations of Super Virasoro algebra has been an unchallenged assumption of the p-adic mass calculations for a long time although one might argue that these objects do not possess stringy characteristics, in particular they do not possess two ends. The progress in the understanding of the Kähler-Dirac equation and the introduction of the weak form of electric magnetic duality [K20] however forces to modify the picture about the origin of the string mass spectrum.

1. The weak form of electric-magnetic duality, the basic facts about Kähler-Dirac equation and the proposed twistorialization of quantum TGD [K23] force to conclude that both strings and bosons and their super-counterparts emerge from massless fermions moving collinearly at partonic two-surfaces. Stringy mass spectrum is consistent with this only if p-adic thermodynamics describes wormhole contacts as analogs of stringy objects having quantum numbers at the throats playing the role of string ends. For instance, the three-momenta of massless wormhole throats could be in opposite direction so that wormhole contact would become massive. The fundamental string like objects would therefore correspond to the wormhole contacts with size scale of order CP_2 length. Already these objects must have a correct correlation between color and electroweak quantum numbers. The colored super-generators taking care that anomalous color is compensated can be assigned with purely bosonic quanta associated with the wormhole throats which carry no fermion number.

2. Second modification comes from the necessity to assume weak confinement in the sense that each wormhole throat carrying fermionic numbers is accompanied by a second wormhole throat carrying neutrino pair cancelling the net weak isospin so that only electromagnetic charge remains unscreened. This screening must take place in weak length scale so that ordinary elementar particles are predicted to be string like objects. This string tension has however nothing to do with the fundamental string tension responsible for the mass spectrum. This picture is forced also by the fact that fermionic wormhole throats necessarily carry Kähler magnetic charge [K20] so that in the case of leptons the second wormhole throat must carry a compensating Kähler magnetic charge. In the case of quarks one can consider the possibility that magnetic charges are not neutralized completely in weak scale and that the compensation occurs in QCD length scale so that Kähler magnetic confinement would accompany color confinement. This means color magnetic confinement since classical color gauge fields are proportional to induced Kähler field.

These modifications do not seem to appreciably affect the results of calculations, which depend only on the number of tensor factors in super Virasoro representation, they are not taken explicitly into account in the calculations. The predictions of the general theory are consistent with the earliest mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. What is new is the possibility to describe the massivation of gauge bosons by including the contribution from the string tension of weak string like objects: weak boson masses have indeed been the trouble makers and have forced to conclude that Higgs expectation might be needed unless some other mechanism contributes to the conformal vacuum weight of the ground state.

2.2 Family Replication Phenomenon Topologically

One of the basic ideas of TGD approach has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of ZEO this picture changed somewhat. It is the wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian, which correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components could allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface. The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and it is the moduli of these partonic 2-surfaces which appear in the elementary particle vacuum functionals naturally.

The first modification of the original simple picture comes from the identification of physical particles as bound states of pairs of wormhole contacts and from the assumption that for generalized Feynman diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats- also those appearing in internal lines- and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. This means that bosons and their super-partners correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. Free fermions and their superpartners could correspond to CP_2 type vacuum extremals with single wormhole throat. It turns however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as (g, g) type wormhole contacts.

2.2.1 *Do free fermions correspond to single wormhole throat or (g, g) wormhole?*

The original interpretation of genus-generation correspondence was that free fermions correspond to wormhole throats characterized by genus. The idea of $SU(3)$ as a dynamical symmetry suggested that gauge bosons correspond to octet and singlet representations of $SU(3)$. The further idea that all lines of generalized Feynman diagrams are massless poses a strong additional constraint and it is not clear whether this proposal as such survives.

1. Twistorial program assumes that fundamental objects are massless wormhole throats carrying collinearly moving many-fermion states and also bosonic excitations generated by super-symplectic algebra. In the following consideration only purely bosonic and single fermion throats are considered since they are the basic building blocks of physical particles. The reason is that propagators for high excitations behave like p^{-n}, n the number of fermions associated with the wormhole throat. Therefore single throat allows only spins 0,1/2,1 as elementary particles in the usual sense of the word.

2. The identification of massive fermions (as opposed to free massless fermions) as wormhole contacts follows if one requires that fundamental building blocks are massless since at least two massless throats are required to have a massive state. Therefore the conformal excitations with CP_2 mass scale should be assignable to wormhole contacts also in the case of fermions. As already noticed this is not the end of the story: weak strings are required by the weak form of electric-magnetic duality.

3. If free fermions corresponding to single wormhole throat, topological condensation is an essential element of the formation of stringy states. The topological condensation of fermions by topological sum (fermionic CP_2 type vacuum extremal touches another space-time sheet) suggest $(g, 0)$ wormhole contact. Note however that the identification of wormhole throat is as 3-surface at which the signature of the induced metric changes so that this conclusion might be wrong. One can indeed consider also the possibility of (g, g) pairs as an outcome of topological conensation. This is suggested also by the idea that wormhole throats are analogous to string like objects and only this option turns out to be consistent with the BFF vertex based on the requirement of dynamical $SU(3)$ symmetry to be discussed later. The structure of reaction vertices makes it possible to interpret (g, g) pairs as $SU(3)$ triplet. If bosons are obtained as fusion of fermionic and anti-fermionic throats (touching of corresponding CP_2 type vacuum extremals) they correspond naturally to (g_1, g_2) pairs.

4. p-Adic mass calculations distinguish between fermions and bosons and the identification of fermions and bosons should be consistent with this difference. The maximal p-adic temperature $T = 1$ for fermions could relate to the weakness of the interaction of the fermionic wormhole throat with the wormhole throat resulting in topological condensation. This wormhole throat would however carry momentum and 3-momentum would in general be non-parallel to that of the fermion, most naturally in the opposite direction.

p-Adic mass calculations suggest strongly that for bosons p-adic temperature $T = 1/n$, $n > 1$, so that thermodynamical contribution to the mass squared is negligible. The low p-adic temperature could be due to the strong interaction between fermionic and anti-fermionic wormhole throat leading to the "freezing" of the conformal degrees of freedom related to the relative motion of wormhole throats.

5. The weak form of electric-magnetic duality forces second wormhole throat with opposite magnetic charge and the light-like momenta could sum up to massive momentum. In this case string tension corresponds to electroweak length scale. Therefore p-adic thermodynamics must be assigned to wormhole contacts and these appear as basic units connected by Kähler magnetic flux tube pairs at the two space-time sheets involved. Weak stringy degrees of freedom are however expected to give additional contribution to the mass, perhaps by modifying the ground state conformal weight.

2.2.2 *Dynamical $SU(3)$ fixes the identification of fermions and bosons and fundamental interaction vertices*

For 3 light fermion families $SU(3)$ suggests itself as a dynamical symmetry with fermions in fundamental $N = 3$-dimensional representation and $N \times N = 9$ bosons in the adjoint representation and singlet representation. The known gauge bosons have same couplings to fermionic families so that they must correspond to the singlet representation. The first challenge is to understand whether it is possible to have dynamical $SU(3)$ at the level of fundamental reaction vertices.

This is a highly non-trivial constraint. For instance, the vertices in which n wormhole throats with same (g_1, g_2) glued along the ends of lines are not consistent with this symmetry. The splitting of the fermionic worm-hole contacts before the proper vertices for throats might however allow the realization of dynamical $SU(3)$. The condition of $SU(3)$ symmetry combined with the requirement that virtual lines resulting also in the splitting of wormhole contacts are always massless, leads to the conclusion that massive fermions correspond to (g, g) type wormhole contacts transforming naturally like $SU(3)$ triplet. This picture conformsl with the identification of free fermions as throats but not with the naive expectation that their topological condensation gives rise to $(g, 0)$ wormhole contact.

The argument leading to these conclusions runs as follows.

1. The question is what basic reaction vertices are allowed by dynamical $SU(3)$ symmetry. FFB vertices are in principle all that is needed and they should obey the dynamical symmetry. The meeting of entire wormhole contacts along their ends is certainly not possible. The splitting of fermionic wormhole contacts before the vertices might be however consistent with $SU(3)$ symmetry. This would give two a pair of 3-vertices at which three wormhole lines meet along partonic 2-surfaces (rather than along 3-D wormhole contacts).

2. Note first that crossing gives all possible reaction vertices of this kind from $F(g_1)\overline{F}(g_2) \rightarrow B(g_1, g_2)$ annihilation vertex, which is relatively easy to visualize. In this reaction $F(g_1)$ and $\overline{F}(g_2)$ wormhole contacts split first. If one requires that all wormhole throats involved are massless, the two wormhole throats resulting in splitting and carrying no fermion number must carry light-like momentum so that they cannot just disappear. The ends of the wormhole throats of the boson must glued together with the end of the fermionic wormhole throat and its companion generated in the splitting of the wormhole. This means that fermionic wormhole first splits and the resulting throats meet at the partonic 2-surface.

   his requires that topologically condensed fermions correspond to (g, g) pairs rather than $(g, 0)$ pairs. The reaction mechanism allows the interpretation of (g, g) pairs as a triplet of dynamical $SU(3)$. The fundamental vertices would be just the splitting of wormhole contact and 3-vertices for throats since $SU(3)$ symmetry would exclude more complex reaction vertices such as n-boson vertices corresponding the gluing of n wormhole contact lines along their 3-dimensional ends. The couplings of singlet representation for bosons would have same coupling to all fermion families so that the basic experimental constraint would be satisfied.

3. Both fermions and bosons cannot correspond to octet and singlet of $SU(3)$. In this case reaction vertices should correspond algebraically to the multiplication of matrix elements

e_{ij}: $e_{ij}e_{kl} = \delta_{jk}e_{il}$ allowing for instance $F(g_1, g_2) + \overline{F}(g_2, g_3) \rightarrow B(g_1, g_3)$. Neither the fusion of entire wormhole contacts along their ends nor the splitting of wormhole throats before the fusion of partonic 2-surfaces allows this kind of vertices so that BFF vertex is the only possible one. Also the construction of QFT limit starting from bosonic emergence led to the formulation of perturbation theory in terms of Dirac action allowing only BFF vertex as fundamental vertex [K6] .

4. Weak electric-magnetic duality brings in an additional complication. $SU(3)$ symmetry poses also now strong constraints and it would seem that the reactions must involve copies of basic BFF vertices for the pairs of ends of weak strings. The string ends with the same Kähler magnetic charge should meet at the vertex and give rise to BFF vertices. For instance, $F\overline{F}B$ annihilation vertex would in this manner give rise to the analog of stringy diagram in which strings join along ends since two string ends disappear in the process.

If one accepts this picture the remaining question is why the number of genera is just three. Could this relate to the fact that $g \leq 2$ Riemann surfaces are always hyper-elliptic (have global Z_2 conformal symmetry) unlike $g > 2$ surfaces? Why the complete bosonic de-localization of the light families should be restricted inside the hyper-elliptic sector? Does the Z_2 conformal symmetry make these states light and make possible de-localization and dynamical $SU(3)$ symmetry? Could it be that for $g > 2$ elementary particle vacuum functionals vanish for hyper-elliptic surfaces? If this the case and if the time evolution for partonic 2-surfaces changing g commutes with Z_2 symmetry then the vacuum functionals localized to $g \leq 2$ surfaces do not disperse to $g > 2$ sectors.

2.2.3 *The notion of elementary particle vacuum functional*

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals, is made.

The basic assumptions underlying the construction are the following ones:

1. Elementary particle vacuum functionals depend on the geometric properties of the two-surface X^2 representing elementary particle.

2. Vacuum functionals possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting preferred extremal of the Kähler action as a generalized Bohr orbit [K7]) and determined in $Diff^3$ invariant manner.

3. ZEO allows to select uniquely the partonic two surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower boundary of $CD \times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.

5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.

6. Elementary particle vacuum functionals are stable against the decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g - 1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized Feynman diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K1] the construction of elementary particle vacuum functionals is described in more detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered.

2.3 Critizing the view about elementary particles

The concrete model for elementary particles has developed gradually during years and is by no means final. In the recent model elementary particle corresponds to a pair of wormhole contacts and monopole flux runs between the throats of of the two contacts at the two space-time sheets and through the contacts between space-time sheets.

The first criticism relates to twistor lift of TGD [L3]. In the case of Kähler action the wormhole contacts correspond to deformations for pieces of CP_2 type vacuum extremals for which the 1-D M^4 projection is light-like random curve. Twistor lift adds to Kähler action a volume term proportional to cosmological constant and forces the vacuum extremal to be a minimal surface carrying non-vanishing light-like momentum (this is of course very natural): one could call this surface CP_2 extremal. This implies that M^4 projection is light-like geodesic: this is physically rather natural.

Twistor lift leads to a loss of the proposed space-time correlate of massivation used also to justify p-adic thermodynamics: the average velocity for a light-like random curve is smaller than maximal signal velocity - this would be a clear classical signal for massivation. One could however conjecture that the M^4 projection for the light-like boundaries of string world sheets becomes light-like geodesic of $M^4 \times CP_2$ instead light-like geodesic of M^4 and that this serves as the correlate for the massivation in 4-D sense.

Second criticism is that I have not considered in detail what the monopole flux hypothesis really means at the level of detail. Since the monopole flux is due to the CP_2 topology, there must be a closed 2-surface which carries this flux. This implies that the flux tube cannot have boundaries at larger space-time surface but one has just the flux tube with closed cross section obtained as a deformation of a cosmic string like object $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a complex surface of CP_2 characterized by genus. Deformation would have 4-D M^4 projection instead of 2-D string world sheet.

Note: One can also consider objects for which the flux is not monopole flux: in this case one would have deformations of surfaces of type $X^2 \times S^2$, S^2 homologically trivial geodesic sphere: these are non-vacuum extremals for the twistor lift of Kähler action (volume term). The net magnetic flux would vanish - as a matter fact, the induced Kähler form would vanish identically for the simplest situation. These objects might serve as correlates for gravitons since the induced metric is the only field degree of freedom. One could also have non-vanishing fluxes for flux tubes with disk-like cross section.

If this is the case, the elementary particles would be much simpler than I have though hitherto.

1. Elementary particles would be simply closed flux tubes which look like very long flattened squares. Short sides with length of order CP_2 radius would be identifiable as pieces of deformed CP_2 type extremals having Euclidian signature of the induced metric. Long sides would be deformed cosmic strings with Minkowskian signature with apparent ends, which are light-like 3-surfaces at which the induced 4-metric is degenerate. Both Minkowskian and Euclidian regions of closed flux tubes would be accompanied by fermionic strings. These objects would topologically condense at larger space-time sheets with wormhole contacts that do not carry monopole flux: touching the larger space-time surface but not sticking to it.

2. One could understand why the genus for all wormhole throats must be the same for the simplest states as the TGD explanation of family replication phenomenon demands. Of course, the change of the topology along string like object cannot be excluded but very probably corresponds to an unstable higher mass excitation.

3. The basic particle reactions would include re-connections of closed string like objects and their reversals. The replication of 3-surfaces would remain a new element brought by TGD.

The basic processes at fermionic level would be reconnections of closed fermionic strings. The new element would be the presence of Euclidian regions allowing to talk about effective boundaries of strings as boundaries between the Minkowskian or Euclidian regions. This would simplify enormously the description of particle reactions by bringing in description topologically highly analogous to that provided by closed strings.

4. The original picture need not of course be wrong: it is only slightly more complex than the above proposal. One would have two space-time sheets connected by a pair of wormhole contacts between, which most of the magnetic flux would flow like in flux tube. The flux from the throat could emerges more or less spherically but eventually end up to the second wormhole throat. The sheets would be connected along their boundaries so that 3-space would be connected. The absence of boundary terms in the action implies this. The monopole fluxes would sum up to a vanishing flux at the boundary, where gluing of the sheets of the covering takes place.

There is a further question to be answered. Are the fermionic strings closed or not? Fermionic strings have certainly the Minkowskian portions ending at the light-like partonic orbits at Minkowskian-Euclidian boundaries. But do the fermionic strings have also Euclidian portions so that the signature of particle would be 2+2 kinks of a closed fermionic string? If strong for of holography is true in both Euclidian and Minkowskian regions, this is highly suggestive option.

If only Minkowskian portions are present, particles could be seen as pairs of open fermionic strings and the counterparts of open string vertices would be possible besides reconnection of closed strings. For this option one can also consider single fermionic open strings connecting wormhole contacts: now possible flux tube would not carry monopole flux.

2.4 Basic Facts About Riemann Surfaces

In the following some basic aspects about Riemann surfaces will be summarized. The basic topological concepts, in particular the concept of the mapping class group, are introduced, and the Teichmueller parameters are defined as conformal invariants of the Riemann surface, which in fact specify the conformal equivalence class of the Riemann surface completely.

2.4.1 *Mapping class group*

The first homology group $H_1(X^2)$ of a Riemann surface of genus g contains $2g$ generators [A2, A5, A4] : this is easy to understand geometrically since each handle contributes two homology generators. The so called canonical homology basis can be identified (see **Fig. 1**).

One can define the so called intersection $J(a,b)$ for two elements a and b of the homology group as the number of intersection points for the curves a and b counting the orientation. Since $J(a,b)$ depends on the homology classes of a and b only, it defines an antisymmetric quadratic form in $H_1(X^2)$. In the canonical homology basis the non-vanishing elements of the intersection matrix are:

$$J(a_i,b_j) \ = \ -J(b_j,a_i) = \delta_{i,j} \ . \tag{2.1}$$

J clearly defines symplectic structure in the homology group.

The dual to the canonical homology basis consists of the harmonic one-forms $\alpha_i, \beta_i, i = 1,..,g$ on X^2. These 1-forms satisfy the defining conditions

$$\begin{array}{ll} \int_{a_i} \alpha_j = \delta_{i,j} & \int_{b_i} \alpha_j = 0 \ , \\ \int_{a_i} \beta_j = 0 & \int_{b_i} \beta_j = \delta_{i,j} \ . \end{array} \tag{2.2}$$

The following identity helps to understand the basic properties of the Teichmueller parameters

$$\int_{X^2} \theta \wedge \eta \ = \ \sum_{i=1,..,g} [\int_{a_i} \theta \int_{b_i} \eta - \int_{b_i} \theta \int_{a_i} \eta] \ . \tag{2.3}$$

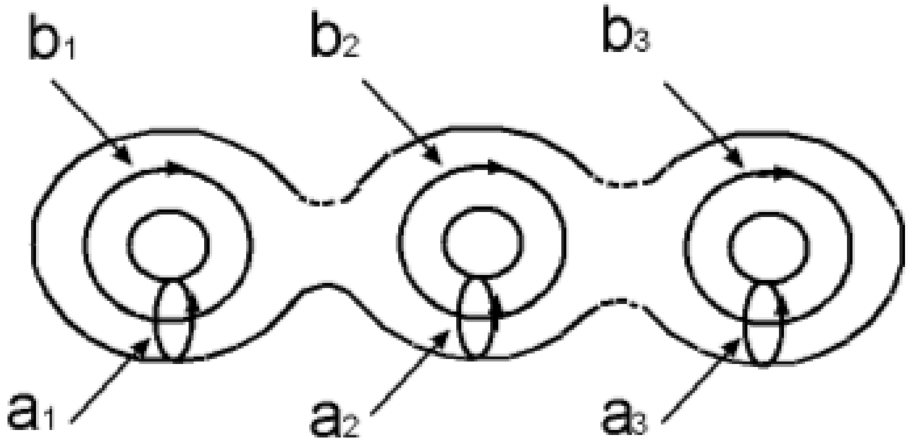

Figure 1: Definition of the canonical homology basis

The existence of topologically nontrivial diffeomorphisms, when X^2 has genus $g > 0$, plays an important role in the sequel. Denoting by $Diff$ the group of the diffeomorphisms of X^2 and by $Diff_0$ the normal subgroup of the diffeomorphisms homotopic to identity, one can define the mapping class group M as the coset group

$$M = Diff/Diff_0 \ . \tag{2.4}$$

The generators of M are so called Dehn twists along closed curves a of X^2. Dehn twist is defined by excising a small tubular neighborhood of a, twisting one boundary of the resulting tube by 2π and gluing the tube back into the surface: see **Fig. 2**.

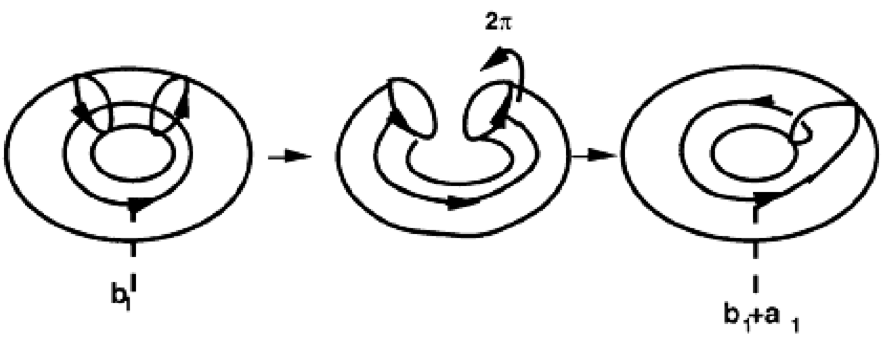

Figure 2: Definition of the Dehn twist

It can be shown that a minimal set of generators is defined by the following curves

$$a_1, b_1, a_1^{-1}a_2^{-1}, a_2, b_2, a_2^{-1}a_3^{-11}, ..., a_g, b_g \quad . \tag{2.5}$$

The action of these transformations in the homology group can be regarded as a symplectic linear transformation preserving the symplectic form defined by the intersection matrix. Therefore the matrix representing the action of $Diff$ on $H_1(X^2)$ is $2g \times 2g$ matrix M with integer entries leaving J invariant: $MJM^T = J$. Mapping class group is often referred also and denoted by $Sp(2g, Z)$. The matrix representing the action of M in the canonical homology basis decomposes into four $g \times g$ blocks A, B, C and D

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad , \tag{2.6}$$

where A and D operate in the subspaces spanned by the homology generators a_i and b_i respectively and C and D map these spaces to each other. The notation $D = [A, B; C, D]$ will be used in the sequel: in this notation the representation of the symplectic form J is $J = [0, 1; -1, 0]$.

2.4.2 *Teichmueller parameters*

The induced metric on the two-surface X^2 defines a unique complex structure. Locally the metric can always be written in the form

$$ds^2 = e^{2\phi} dz d\bar{z} \quad . \tag{2.7}$$

where z is local complex coordinate. When one covers X^2 by coordinate patches, where the line element has the above described form, the transition functions between coordinate patches are holomorphic and therefore define a complex structure.

The conformal transformations ξ of X^2 are defined as the transformations leaving invariant the angles between the vectors of X^2 tangent space invariant: the angle between the vectors X and Y at point x is same as the angle between the images of the vectors under Jacobian map at the image point $\xi(x)$. These transformations need not be globally defined and in each coordinate patch they correspond to holomorphic (anti-holomorphic) mappings as is clear from the diagonal form of the metric in the local complex coordinates. A distinction should be made between local conformal transformations and globally defined conformal transformations, which will be referred to as conformal symmetries: for instance, for hyper-elliptic surfaces the group of the conformal symmetries contains two-element group Z_2.

Using the complex structure one can decompose one-forms to linear combinations of one-forms of type $(1,0)$ $(f(z,\bar{z})dz)$ and $(0,1)$ $(f(z,\bar{z})d\bar{z})$. $(1,0)$ form ω is holomorphic if the function f is holomorphic: $\omega = f(z)dz$ on each coordinate patch.

There are g independent holomorphic one forms ω_i known also as Abelian differentials Alvarez,Farkas,Mumford and one can fix their normalization by the condition

$$\int_{a_i} \omega_j = \delta_{ij} \quad . \tag{2.8}$$

This condition completely specifies ω_i.

Teichmueller parameters Ω_{ij} are defined as the values of the forms ω_i for the homology generators b_j

$$\Omega_{ij} = \int_{b_j} \omega_i \quad . \tag{2.9}$$

The basic properties of Teichmueller parameters are the following:

1. The $g \times g$ matrix Ω is symmetric: this is seen by applying the formula (2.3) for $\theta = \omega_i$ and $\eta = \omega_j$.

2. The imaginary part of Ω is positive: $Im(\Omega) > 0$. This is seen by the application of the same formula for $\theta = \eta$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.

3. The space of Teichmueller parameters can be regarded as a coset space $Sp(2g,R)/U(g)$ [A4] : the action of $Sp(2g,R)$ is of the same form as the action of $Sp(2g,Z)$ and $U(g) \subset Sp(2g,R)$ is the isotropy group of a given point of Teichmueller space.

4. Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.

5. Teichmueller parameters specify completely the conformal structure of Riemann surface [A5] .

Although Teichmueller parameters fix the conformal structure of the 2-surface completely, they are not in one-to-one correspondence with the conformal equivalence classes of the two-surfaces:
i) The dimension for the space of the conformal equivalence classes is $D = 3g - 3$, when $g > 1$ and smaller than the dimension of Teichmueller space given by $d = (g \times g + g)/2$ for $g > 3$: all Teichmueller matrices do not correspond to a Riemann surface. In TGD approach this does not produce any problems as will be found later.
ii) The action of the topologically nontrivial diffeomorphisms on Teichmueller parameters is non-trivial and can be deduced from the action of the diffeomorphisms on the homology ($Sp(2g,Z)$ transformation) and from the defining condition $\int_{a_i} \omega_j = \delta_{i,j}$: diffeomorphisms correspond to elements $[A,B;C,D]$ of $Sp(2g,Z)$ and act as generalized Möbius transformations

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{2.10}$$

All Teichmueller parameters related by $Sp(2g,Z)$ transformations correspond to the same Riemann surface.
iii) The definition of the Teichmueller parameters is not unique since the definition of the canonical homology basis involves an arbitrary numbering of the homology basis. The permutation S of the handles is represented by same $g \times g$ orthogonal matrix both in the basis $\{a_i\}$ and $\{b_i\}$ and induces a similarity transformation in the space of the Teichmueller parameters

$$\Omega \rightarrow S\Omega S^{-1} \ . \tag{2.11}$$

Clearly, the Teichmueller matrices related by a similarity transformations correspond to the same conformal equivalence class. It is easy to show that handle permutations in fact correspond to $Sp(2g,Z)$ transformations.

2.4.3 *Hyper-ellipticity*

The motivation for considering hyper-elliptic surfaces comes from the fact, that $g > 2$ elementary particle vacuum functionals turn out to be vanishing for hyper-elliptic surfaces and this in turn will be later used to provide a possible explanation the non-observability of $g > 2$ particles.

Hyper-elliptic surface X can be defined abstractly as two-fold branched cover of the sphere having the group Z_2 as the group of conformal symmetries (see [A6, A5, A4] . Thus there exists a map $\pi : X \rightarrow S^2$ so that the inverse image $\pi^{-1}(z)$ for a given point z of S^2 contains two points except at a finite number (say p) of points z_i (branch points) for which the inverse image contains only one point. Z_2 acts as conformal symmetries permuting the two points in $\pi^{-1}(z)$ and branch points are fixed points of the involution.

The concept can be generalized [A6] : g-hyper-elliptic surface can be defined as a 2-fold covering of genus g surface with a finite number of branch points. One can consider also p-fold coverings instead of 2-fold coverings: a common feature of these Riemann surfaces is the existence of a discrete group of conformal symmetries.

A concrete representation for the hyper-elliptic surfaces [A4] is obtained by studying the surface of C^2 determined by the algebraic equation

$$w^2 - P_n(z) = 0 , \tag{2.12}$$

where w and z are complex variables and $P_n(z)$ is a complex polynomial. One can solve w from the above equation

$$w_{\pm} = \pm\sqrt{P_n(z)} , \tag{2.13}$$

where the square root is determined so that it has a cut along the positive real axis. What happens that w has in general two roots (two-fold covering property), which coincide at the roots z_i of $P_n(z)$ and if n is odd, also at $z = \infty$: these points correspond to branch points of the hyper-elliptic surface and their number r is always even: $r = 2k$. w is discontinuous at the cuts associated with the square root in general joining two roots of $P_n(z)$ or if n is odd, also some root of P_n and the point $z = \infty$. The representation of the hyper-elliptic surface is obtained by identifying the two branches of w along the cuts. From the construction it is clear that the surface obtained in this manner has genus $k-1$. Also it is clear that Z_2 permutes the different roots $w_{\pm}$ with each other and that $r = 2k$ branch points correspond to fixed points of the involution.

The following facts about the hyper-elliptic surfaces [A5, A4] turn out to be important in the sequel:
i) All $g < 3$ surfaces are hyper-elliptic.
ii) $g \geq 3$ hyper-elliptic surfaces are not in general hyper-elliptic and form a set of codimension 2 in the space of the conformal equivalence classes [A4] .

2.4.4 *Theta functions*

An extensive and detailed account of the theta functions and their applications can be found in the book of Mumford [A4] . Theta functions appear also in the loop calculations of string [J1] [A2] . In the following the so called Riemann theta function and theta functions with half integer characteristics will be defined as sections (not strictly speaking functions) of the so called Jacobian variety.

For a given Teichmueller matrix Ω, Jacobian variety is defined as the $2g$-dimensional torus obtained by identifying the points z of C^g (vectors with g complex components) under the equivalence

$$z \sim z + \Omega m + n , \tag{2.14}$$

where m and n are points of Z^g (vectors with g integer valued components) and Ω acts in Z^g by matrix multiplication.

The definition of Riemann theta function reads as

$$\Theta(z|\Omega) = \sum_n exp(i\pi n \cdot \Omega \cdot n + i2\pi n \cdot z) . \tag{2.15}$$

Here $\cdot$ denotes standard inner product in C^g. Theta functions with half integer characteristics are defined in the following manner. Let a and b denote vectors of C^g with half integer components (component either vanishes or equals to 1/2). Theta function with characteristics $[a, b]$ is defined through the following formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a) + i2\pi(n+a)\cdot(z+b)\right] . \tag{2.16}$$

A brief calculation shows that the following identity is satisfied

$$\Theta[a,b](z|\Omega) = exp(i\pi a\cdot\Omega\cdot a + i2\pi a\cdot b)\times\Theta(z+\Omega a+b|\Omega) \tag{2.17}$$

Theta functions are not strictly speaking functions in the Jacobian variety but rather sections in an appropriate bundle as can be seen from the identities

$$\begin{aligned}\Theta[a,b](z+m|\Omega) &= exp(i2\pi a\cdot m)\Theta[a,b](z\Omega)\ ,\\ \Theta[a,b](z+\Omega m|\Omega) &= exp(\alpha)\Theta[a,b](z|\Omega)\ ,\\ exp(\alpha) &= exp(-i2\pi b\cdot m)exp(-i\pi m\cdot\Omega\cdot m-2\pi m\cdot z)\ .\end{aligned} \tag{2.18}$$

The number of theta functions is 2^{2g} and same as the number of nonequivalent spinor structures defined on two-surfaces. This is not an accident [A2] : theta functions with given characteristics turn out to be in a close relation to the functional determinants associated with the Dirac operators defined on the two-surface. It is useful to divide the theta functions to even and odd theta functions according to whether the inner product $4a\cdot b$ is even or odd integer. The numbers of even and odd theta functions are $2^{g-1}(2^g+1)$ and $2^{g-1}(2^g-1)$ respectively.

The values of the theta functions at the origin of the Jacobian variety understood as functions of Teichmueller parameters turn out to be of special interest in the following and the following notation will be used:

$$\Theta[a,b](\Omega) \equiv \Theta[a,b](0|\Omega)\ , \tag{2.19}$$

$\Theta[a,b](\Omega)$ will be referred to as theta functions in the sequel. From the defining properties of odd theta functions it can be found that they are odd functions of z and therefore vanish at the origin of the Jacobian variety so that only even theta functions will be of interest in the sequel.

An important result is that also some *even* theta functions vanish for $g>2$ hyper-elliptic surfaces : in fact one can characterize $g>2$ hyper-elliptic surfaces by the vanishing properties of the theta functions [A5, A4] . The vanishing property derives from conformal symmetry (Z_2 in the case of hyper-elliptic surfaces) and the vanishing phenomenon is rather general [A6] : theta functions tend to vanish for Riemann surfaces possessing discrete conformal symmetries. It is not clear (to the author) whether the presence of a conformal symmetry is in fact equivalent with the vanishing of some theta functions. As already noticed, spinor structures and the theta functions with half integer characteristics are in one-to-one correspondence and the vanishing of theta function with given half integer characteristics is equivalent with the vanishing of the Dirac determinant associated with the corresponding spinor structure or equivalently: with the existence of a zero mode for the Dirac operator Alvarez . For odd characteristics zero mode exists always: for even characteristics zero modes exist, when the surface is hyper-elliptic or possesses more general conformal symmetries.

2.5 Elementary Particle Vacuum Functionals

The basic assumption is that elementary particle families correspond to various elementary particle vacuum functionals associated with the 2-dimensional boundary components of the 3-surface. These functionals need not be localized to a single boundary topology. Neither need their dependence on the boundary component be local. An important role in the following considerations is played by the fact that the minimization requirement of the Kähler action associates a unique 3-surface to each boundary component, the "Bohr orbit" of the boundary and this surface provides a considerable (and necessarily needed) flexibility in the definition of the elementary particle vacuum functionals. There are several natural constraints to be satisfied by elementary particle vacuum functionals.

2.5.1 *Extended Diff invariance and Lorentz invariance*

Extended Diff invariance is completely analogous to the extension of 3-dimensional Diff invariance to four-dimensional Diff invariance in the interior of the 3-surface. Vacuum functional must be invariant not only under diffeomorphisms of the boundary component but also under the diffeomorphisms of the 3- dimensional "orbit" Y^3 of the boundary component. In other words: the value of the vacuum functional must be same for any time slice on the orbit the boundary component. This is guaranteed if vacuum functional is functional of some two-surface Y^2 belonging to the orbit and defined in $Diff^3$ invariant manner.

An additional natural requirement is Poincare invariance. In the original formulation of the theory only Lorentz transformations of the light cone were exact symmetries of the theory. In this framework the definition of Y^2 as the intersection of the orbit with the hyperboloid $\sqrt{m_{kl}m^km^l} = a$ is $Diff^3$ and Lorentz invariant.

1. Interaction vertices as generalization of stringy vertices

For stringy diagrams Poincare invariance of conformal equivalence class and general coordinate invariance are far from being a trivial issues. Vertices are now not completely unique since there is an infinite number of singular 3-manifolds which can be identified as vertices even if one assumes space-likeness. One should be able to select a unique singular 3-manifold to fix the conformal equivalence class.

One might hope that Lorentz invariant invariant and general coordinate invariant definition of Y^2 results by introducing light cone proper time a as a height function specifying uniquely the point at which 3-surface is singular (stringy diagrams help to visualize what is involved), and by restricting the singular 3-surface to be the intersection of $a = constant$ hyperboloid of M^4 containing the singular point with the space-time surface. There would be non-uniqueness of the conformal equivalence class due to the choice of the origin of the light cone but the decomposition of the configuration space of 3-surfaces to a union of WCWs characterized by unions of future and past light cones could resolve this difficulty.

2. Interaction vertices as generalization of ordinary ones

If the interaction vertices are identified as intersections for the ends of space-time sheets representing particles, the conformal equivalence class is naturally identified as the one associated with the intersection of the boundary component or light like causal determinant with the vertex. Poincare invariance of the conformal equivalence class and generalized general coordinate invariance follow trivially in this case.

2.5.2 *Conformal invariance*

Conformal invariance implies that vacuum functionals depend on the conformal equivalence class of the surface Y^2 only. What makes this idea so attractive is that for a given genus g WCW becomes effectively finite-dimensional. A second nice feature is that instead of trying to find coordinates for the space of the conformal equivalence classes one can construct vacuum functionals as functions of the Teichmueller parameters.

That one can construct this kind of functions as suitable functions of the Teichmueller parameters is not trivial. The essential point is that the boundary components can be regarded as sub-manifolds of $M^4_+ \times CP_2$: as a consequence vacuum functional can be regarded as a composite function:

2-surface $\rightarrow$ Teichmueller matrix Ω determined by the induced metric $\rightarrow \Omega_{vac}(\Omega)$

Therefore the fact that there are Teichmueller parameters which do not correspond to any Riemann surface, doesn't produce any trouble. It should be noticed that the situation differs from that in the Polyakov formulation of string models, where one doesn't assume that the metric of the two-surface is induced metric (although classical equations of motion imply this).

2.5.3 *Diff invariance*

Since several values of the Teichmueller parameters correspond to the same conformal equivalence class, one must pose additional conditions on the functions of the Teichmueller parameters in order to obtain single valued functions of the conformal equivalence class.

The first requirement of this kind is the invariance under topologically nontrivial Diff transformations inducing $Sp(2g, Z)$ transformation $(A, B; C, D)$ in the homology basis. The action of these transformations on Teichmueller parameters is deduced by requiring that holomorphic one-forms satisfy the defining conditions in the transformed homology basis. It turns out that the action of the topologically nontrivial diffeomorphism on Teichmueller parameters can be regarded as a generalized Möbius transformation:

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \quad . \tag{2.20}$$

Vacuum functional must be invariant under these transformations. It should be noticed that the situation differs from that encountered in the string models. In TGD the integration measure over WCW is Diff invariant: in string models the integration measure is the integration measure of the Teichmueller space and this is not invariant under $Sp(2g, Z)$ but transforms like a density: as a consequence the counterpart of the vacuum functional must be also modular covariant since it is the product of vacuum functional and integration measure, which must be modular invariant.

It is possible to show that the quantities

$$(\Theta[a, b]/\Theta[c, d])^4 \quad . \tag{2.21}$$

and their complex conjugates are $Sp(2g, Z)$ invariants [A4] and therefore can be regarded as basic building blocks of the vacuum functionals.

Teichmueller parameters are not uniquely determined since one can always perform a permutation of the g handles of the Riemann surface inducing a redefinition of the canonical homology basis (permutation of g generators). These transformations act as similarities of the Teichmueller matrix:

$$\Omega \rightarrow S\Omega S^{-1} \quad , \tag{2.22}$$

where S is the $g \times g$ matrix representing the permutation of the homology generators understood as orthonormal vectors in the g- dimensional vector space. Therefore the Teichmueller parameters related by these similarity transformations correspond to the same conformal equivalence class of the Riemann surfaces and vacuum functionals must be invariant under these similarities.

It is easy to find out that these similarities permute the components of the theta characteristics: $[a, b] \rightarrow [S(a), S(b)]$. Therefore the invariance requirement states that the handles of the Riemann surface behave like bosons: the vacuum functional constructed from the theta functions is invariant under the permutations of the theta characteristics. In fact, this requirement brings in nothing new. Handle permutations can be regarded as $Sp(2g, Z)$ transformations so that the modular invariance alone guarantees invariance under handle permutations.

2.5.4 *Cluster decomposition property*

Consider next the behavior of the vacuum functional in the limit, when boundary component with genus g splits to two separate boundary components of genera g_1 and g_2 respectively. The splitting into two separate boundary components corresponds to the reduction of the Teichmueller matrix Ω^g to a direct sum of $g_1 \times g_1$ and $g_2 \times g_2$ matrices $(g_1 + g_2 = g)$:

$$\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2} \quad , \tag{2.23}$$

when a suitable definition of the Teichmueller parameters is adopted. The splitting can also take place without a reduction to a direct sum: the Teichmueller parameters obtained via $Sp(2g, Z)$ transformation from $\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2}$ do not possess direct sum property in general.

The physical interpretation is obvious: the non-diagonal elements of the Teichmueller matrix describe the geometric interaction between handles and at this limit the interaction between the handles belonging to the separate surfaces vanishes. On the physical grounds it is natural to require that vacuum functionals satisfy cluster decomposition property at this limit: that is they reduce to the product of appropriate vacuum functionals associated with the composite surfaces.

Theta functions satisfy cluster decomposition property [A2, A4] . Theta characteristics reduce to the direct sums of the theta characteristics associated with g_1 and g_2 ($a = a_1 \oplus a_2$, $b = b_1 \oplus b_2$) and the dependence on the Teichmueller parameters is essentially exponential so that the cluster decomposition property indeed results:

$$\Theta[a,b](\Omega^g) = \Theta[a_1,b_1](\Omega^{g_1})\Theta[a_2,b_2](\Omega^{g_2}) \ . \tag{2.24}$$

Cluster decomposition property holds also true for the products of theta functions. This property is also satisfied by suitable homogenous polynomials of thetas. In particular, the following quantity playing central role in the construction of the vacuum functional obeys this property

$$Q_0 = \sum_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 \ , \tag{2.25}$$

where the summation is over all even theta characteristics (recall that odd theta functions vanish at the origin of C^g).

Together with the $Sp(2g,Z)$ invariance the requirement of cluster decomposition property implies that the vacuum functional must be representable in the form

$$\Omega_{vac} = P_{M,N}(\Theta^4, \bar{\Theta}^4)/Q_{MN}(\Theta^4, \bar{\Theta}^4) \tag{2.26}$$

where the homogenous polynomials $P_{M,N}$ and $Q_{M,N}$ have same degrees (M and N as polynomials of $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$.

2.5.5 *Finiteness requirement*

Vacuum functional should be finite. Finiteness requirement is satisfied provided the numerator $Q_{M,N}$ of the vacuum functional is real and positive definite. The simplest quantity of this type is the quantity Q_0 defined previously and its various powers. $Sp(2g,Z)$ invariance and finiteness requirement are satisfied provided vacuum functionals are of the following general form

$$\Omega_{vac} = \frac{P_{N,N}(\Theta^4, \bar{\Theta}^4)}{Q_0^N} \ , \tag{2.27}$$

where $P_{N,N}$ is homogenous polynomial of degree N with respect to $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$. In addition $P_{N,N}$ is invariant under the permutations of the theta characteristics and satisfies cluster decomposition property.

2.5.6 *Stability against the decay* $g \rightarrow g_1 + g_2$

Elementary particle vacuum functionals must be stable against the genus conserving decays $g \rightarrow g_1 + g_2$. This decay corresponds to the limit at which Teichmueller matrix reduces to a direct sum of the matrices associated with g_1 and g_2 (note however the presence of $Sp(2g,Z)$ degeneracy). In accordance with the topological description of the particle reactions one expects that this decay doesn't occur if the vacuum functional in question vanishes at this limit.

In general the theta functions are non-vanishing at this limit and vanish provided the theta characteristics reduce to a direct sum of the odd theta characteristics. For $g < 2$ surfaces this condition is trivial and gives no constraints on the form of the vacuum functional. For $g = 2$ surfaces the theta function $\Theta(a,b)$, with $a = b = (1/2, 1/2)$ satisfies the stability criterion identically (odd theta functions vanish identically), when Teichmueller parameters separate into a direct sum. One can however perform $Sp(2g, Z)$ transformations giving new points of Teichmueller space describing the decay. Since these transformations transform theta characteristics in a nontrivial manner to each other and since all even theta characteristics belong to same $Sp(2g, Z)$ orbit [A2, A4] , the conclusion is that stability condition is satisfied provided $g = 2$ vacuum functional is proportional to the product of fourth powers of all even theta functions multiplied by its complex conjugate.

If $g > 2$ there always exists some theta functions, which vanish at this limit and the minimal vacuum functional satisfying this stability condition is of the same form as in $g = 2$ case, that is proportional to the product of the fourth powers of all even Theta functions multiplied by its complex conjugate:

$$\Omega_{vac} = \prod_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 / Q_0^N \ , \qquad (2.28)$$

where N is the number of even theta functions. The results obtained imply that genus-generation correspondence is one to one for $g > 1$ for the minimal vacuum functionals. Of course, the multiplication of the minimal vacuum functionals with functionals satisfying all criteria except stability criterion gives new elementary particle vacuum functionals: a possible physical identification of these vacuum functionals is most naturally as some kind of excited states.

One of the questions posed in the beginning was related to the experimental absence of $g > 0$, possibly massless, elementary bosons. The proposed stability criterion suggests a nice explanation. The point is that elementary particles are stable against decays $g \rightarrow g_1 + g_2$ but not with respect to the decay $g \rightarrow g + \ sphere$. As a consequence the direct emission of $g > 0$ gauge bosons is impossible unlike the emission of $g = 0$ bosons: for instance the decay muon $\rightarrow$ electron $+(g = 1)$ photon is forbidden.

2.5.7 *Stability against the decay* $g \rightarrow g - 1$

This stability criterion states that the vacuum functional is stable against single particle decay $g \rightarrow g - 1$ and, if satisfied, implies that vacuum functional vanishes, when the genus of the surface is smaller than g. In stringy framework this criterion is equivalent to a separate conservation of various lepton numbers: for instance, the spontaneous transformation of muon to electron is forbidden. Notice that this condition doesn't imply that that the vacuum functional is localized to a single genus: rather the vacuum functional of genus g vanishes for all surfaces with genus smaller than g. This hierarchical structure should have a close relationship to Cabibbo-Kobayashi-Maskawa mixing of the quarks.

The stability criterion implies that the vacuum functional must vanish at the limit, when one of the handles of the Riemann surface suffers a pinch. To deduce the behavior of the theta functions at this limit, one must find the behavior of Teichmueller parameters, when i:th handle suffers a pinch. Pinch implies that a suitable representative of the homology generator a_i or b_i contracts to a point.

Consider first the case, when a_i contracts to a point. The normalization of the holomorphic one-form ω_i must be preserved so that that ω_i must behaves as $1/z$, where z is the complex coordinate vanishing at pinch. Since the homology generator b_i goes through the pinch it seems obvious that the imaginary part of the Teichmueller parameter $\Omega_{ii} = \int_{b_i} \omega_i$ diverges at this limit (this conclusion is made also in [A4]): $Im(\Omega_{ii}) \rightarrow \infty$.

Of course, this criterion doesn't cover all possible manners the pinch can occur: pinch might take place also, when the components of the Teichmueller matrix remain finite. In the case of torus topology one finds that $Sp(2g, Z)$ element $(A, B; C, D)$ takes $Im(\Omega) = \infty$ to the point C/D of real axis. This suggests that pinch occurs always at the boundary of the Teichmueller space: the imaginary part of Ω_{ij} either vanishes or some matrix element of $Im(\Omega)$ diverges.

Consider next the situation, when b_i contracts to a point. From the definition of the Teichmueller parameters it is clear that the matrix elements Ω_{kl}, with $k, l \neq i$ suffer no change. The matrix element Ω_{ki} obviously vanishes at this limit. The conclusion is that i:th row of Teichmueller matrix vanishes at this limit. This result is obtained also by deriving the $Sp(2g, Z)$ transformation permuting a_i and b_i with each other: in case of torus this transformation reads $\Omega \rightarrow -1/\Omega$.

Consider now the behavior of the theta functions, when pinch occurs. Consider first the limit, when $Im(\Omega_{ii})$ diverges. Using the general definition of $\Theta[a, b]$ it is easy to find out that all theta functions for which the i:th component a_i of the theta characteristic is non-vanishing (that is $a_i = 1/2$) are proportional to the exponent $exp(-\pi\Omega_{ii}/4)$ and therefore vanish at the limit. The theta functions with $a_i = 0$ reduce to $g-1$ dimensional theta functions with theta characteristic obtained by dropping i:th components of a_i and b_i and replacing Teichmueller matrix with Teichmueller matrix obtained by dropping i:th row and column. The conclusion is that all theta functions of type $\Theta(a, b)$ with $a = (1/2, 1/2, ..., 1/2)$ satisfy the stability criterion in this case.

What happens for the $Sp(2g, Z)$ transformed points on the real axis? The transformation formula for theta function is given by [A2, A4]

$$\Theta[a, b]((A\Omega + B)(C\Omega + D)^{-1}) \quad = \quad exp(i\phi)det(C\Omega + D)^{1/2}\Theta[c, d](\Omega) \ , \tag{2.29}$$

where

$$\left(\begin{array}{c} c \\ d \end{array}\right) = \left(\begin{array}{cc} A & B \\ C & D \end{array}\right)\left(\left(\begin{array}{c} a \\ b \end{array}\right) - \left(\begin{array}{c} (CD^T)_d/2 \\ (AB^T)_d/2 \end{array}\right)\right) \ . \tag{2.30}$$

Here ϕ is a phase factor irrelevant for the recent purposes and the index d refers to the diagonal part of the matrix in question.

The first thing to notice is the appearance of the diverging square root factor, which however disappears from the vacuum functionals (P and Q have same degree with respect to thetas). The essential point is that theta characteristics transform to each other: as already noticed all even theta characteristics belong to the same $Sp(2g, Z)$ orbit. Therefore the theta functions vanishing at $Im(\Omega_{ii}) = \infty$ do not vanish at the transformed points. It is however clear that for a given Teichmueller parameterization of pinch some theta functions vanish always.

Similar considerations in the case $\Omega_{ik} = 0$, i fixed, show that all theta functions with $b = (1/2, ..., 1/2)$ vanish identically at the pinch. Also it is clear that for $Sp(2g, Z)$ transformed points one can always find some vanishing theta functions. The overall conclusion is that the elementary particle vacuum functionals obtained by using $g \to g_1 + g_2$ stability criterion satisfy also $g \to g - 1$ stability criterion since they are proportional to the product of all even theta functions. Therefore the only nontrivial consequence of $g \to g - 1$ criterion is that also $g = 1$ vacuum functionals are of the same general form as $g > 1$ vacuum functionals.

A second manner to deduce the same result is by restricting the consideration to the hyper-elliptic surfaces and using the representation of the theta functions in terms of the roots of the polynomial appearing in the definition of the hyper-elliptic surface [A4] . When the genus of the surface is smaller than three (the interesting case), this representation is all what is needed since all surfaces of genus $g < 3$ are hyper-elliptic.

Since hyper-elliptic surfaces can be regarded as surfaces obtained by gluing two compactified complex planes along the cuts connecting various roots of the defining polynomial it is obvious that the process $g \to g - 1$ corresponds to the limit, when two roots of the defining polynomial coincide. This limit corresponds either to disappearance of a cut or the fusion of two cuts to a single cut. Theta functions are expressible as the products of differences of various roots (Thomae's formula [A4])

$$\Theta[a, b]^4 \propto \prod_{i<j\in T} (z_i - z_j) \prod_{k<l\in CT} (z_k - z_l) \ , \tag{2.31}$$

where T denotes some subset of $\{1, 2, ..., 2g\}$ containing $g + 1$ elements and CT its complement. Hence the product of all even theta functions vanishes, when two roots coincide. Furthermore, stability criterion is satisfied only by the product of the theta functions.

Lowest dimensional vacuum functionals are worth of more detailed consideration.

i) $g = 0$ particle family corresponds to a constant vacuum functional: by continuity this vacuum functional is constant for all topologies.

ii) For $g = 1$ the degree of P and Q as polynomials of the theta functions is 24: the critical number

of transversal degrees of freedom in bosonic string model! Probably this result is not an accident.
ii) For $g = 2$ the corresponding degree is 80 since there are 10 even genus 2 theta functions.

There are large numbers of vacuum functionals satisfying the relevant criteria, which do not satisfy the proposed stability criteria. These vacuum functionals correspond either to many particle states or to unstable single particle states.

2.5.8 *Continuation of the vacuum functionals to higher genus topologies*

From continuity it follows that vacuum functionals cannot be localized to single boundary topology. Besides continuity and the requirements listed above, a natural requirement is that the continuation of the vacuum functional from the sector g to the sector $g+k$ reduces to the product of the original vacuum functional associated with genus g and $g = 0$ vacuum functional at the limit when the surface with genus $g+k$ decays to surfaces with genus g and k: this requirement should guarantee the conservation of separate lepton numbers although different boundary topologies suffer mixing in the vacuum functional. These requirements are satisfied provided the continuation is constructed using the following rule:

Perform the replacement

$$\Theta[a,b]^4 \quad \rightarrow \quad \sum_{c,d} \Theta[a \oplus c, b \oplus d]^4 \tag{2.32}$$

for each fourth power of the theta function. Here c and d are Theta characteristics associated with a surface with genus k. The same replacement is performed for the complex conjugates of the theta function. It is straightforward to check that the continuations of elementary particle vacuum functionals indeed satisfy the cluster decomposition property and are continuous.

To summarize, the construction has provided hoped for answers to some questions stated in the beginning: stability requirements explain the separate conservation of lepton numbers and the experimental absence of $g > 0$ elementary bosons. What has not not been explained is the experimental absence of $g > 2$ fermion families. The vanishing of the $g > 2$ elementary particle vacuum functionals for the hyper-elliptic surfaces however suggest a possible explanation: under some conditions on the surface X^2 the surfaces Y^2 are hyper-elliptic or possess some conformal symmetry so that elementary particle vacuum functionals vanish for them. This conjecture indeed might make sense since the surfaces Y^2 are determined by the asymptotic dynamics and one might hope that the surfaces Y^2 are analogous to the final states of a dissipative system.

2.6 Explanations For The Absence Of The $g > 2$ ElementaryParticles From Spectrum

The decay properties of the intermediate gauge bosons [C12] are consistent with the assumption that the number of the light neutrinos is $N = 3$. Also cosmological considerations pose upper bounds on the number of the light neutrino families and $N = 3$ seems to be favored [C12]. It must be however emphasized that p-adic considerations [K9] encourage the consideration the existence of higher genera with neutrino masses such that they are not produced in the laboratory at present energies. In any case, for TGD approach the finite number of light fermion families is a potential difficulty since genus-generation correspondence suggests that the number of the fermion (and possibly also boson) families is infinite. Therefore one had better to find a good argument showing that the number of the observed neutrino families, or more generally, of the observed elementary particle families, is small also in the world described by TGD.

It will be later found that also TGD inspired cosmology requires that the number of the effectively massless fermion families must be small after Planck time. This suggests that boundary topologies with handle number $g > 2$ are unstable and/or very massive so that they, if present in the spectrum, disappear from it after Planck time, which correspond to the value of the light cone proper time $a \simeq 10^{-11}$ seconds.

In accordance with the spirit of TGD approach it is natural to wonder whether some geometric property differentiating between $g > 2$ and $g < 3$ boundary topologies might explain why only $g < 3$ boundary components are observable. One can indeed find a good candidate for this kind of property: namely hyper-ellipticity, which states that Riemann surface is a two-fold branched covering of sphere possessing two-element group Z_2 as conformal automorphisms. All $g < 3$ Riemann surfaces are hyper-elliptic unlike $g > 2$ Riemann surfaces, which in general do not posses this property. Thus it is natural to consider the possibility that hyper-ellipticity or more general conformal symmetries might explain why only $g < 2$ topologies correspond to the observed elementary particles.

As regards to the present problem the crucial observation is that some even theta functions vanish for the hyper-elliptic surfaces with genus $g > 2$ [A4] . What is essential is that these surfaces have the group Z_2 as conformal symmetries. Indeed, the vanishing phenomenon is more general. Theta functions tend to vanish for $g > 2$ two-surfaces possessing discrete group of conformal symmetries [A6] : for instance, instead of sphere one can consider branched coverings of higher genus surfaces.

From the general expression of the elementary particle vacuum functional it is clear that elementary particle vacuum functionals vanish, when Y^2 is hyper-elliptic surface with genus $g > 2$ and one might hope that this is enough to explain why the number of elementary particle families is three.

2.6.1 *Hyper-ellipticity implies the separation of $g \leq 2$ and $g > 2$ sectors to separate worlds*

If the vertices are defined as intersections of space-time sheets of elementary particles and if elementary particle vacuum functionals are required to have Z_2 symmetry, the localization of elementary particle vacuum functionals to $g \leq 2$ topologies occurs automatically. Even if one allows as limiting case vertices for which 2-manifolds are pinched to topologies intermediate between $g > 2$ and $g \leq 2$ topologies, Z_2 symmetry present for both topological interpretations implies the vanishing of this kind of vertices. This applies also in the case of stringy vertices so that also particle propagation would respect the effective number of particle families. $g > 2$ and $g \leq 2$ topologies would behave much like their own worlds in this approach. This is enough to explain the experimental findings if one can understand why the $g > 2$ particle families are absent as incoming and outgoing states or are very heavy.

2.6.2 *What about $g > 2$ vacuum functionals which do not vanish for hyper-elliptic surfaces?*

The vanishing of all $g \geq 2$ vacuum functionals for hyper-elliptic surfaces cannot hold true generally. There must exist vacuum functionals which do satisfy this condition. This suggest that elementary particle vacuum functionals for $g > 2$ states have interpretation as bound states of g handles and that the more general states which do not vanish for hyper-elliptic surfaces correspond to many-particle states composed of bound states $g \leq 2$ handles and cannot thus appear as incoming and outgoing states. Thus $g > 2$ elementary particles would decouple from $g \leq 2$ states.

2.6.3 *Should higher elementary particle families be heavy?*

TGD predicts an entire hierarchy of scaled up variants of standard model physics for which particles do not appear in the vertices containing the known elementary particles and thus behave like dark matter [K19] . Also $g > 2$ elementary particles would behave like dark matter and in principle there is no absolute need for them to be heavy.

The safest option would be that $g > 2$ elementary particles are heavy and the breaking of Z_2 symmetry for $g \geq 2$ states could guarantee this. p-Adic considerations lead to a general mass formula for elementary particles such that the mass of the particle is proportional to $\frac{1}{\sqrt{p}}$ [K21] . Also the dependence of the mass on particle genus is completely fixed by this formula. What remains however open is what determines the p-adic prime associated with a particle with given quantum numbers. Of course, it could quite well occur that p is much smaller for $g > 2$ genera than for $g \leq 2$ genera.

3 Non-Topological Contributions To Particle masses From P-Adic Thermodynamics

In TGD framework p-adic thermodynamics provides a microscopic theory of particle massivation in the case of fermions. The idea is very simple. The mass of the particle results from a thermal mixing of the massless states with CP_2 mass excitations of super-conformal algebra. In p-adic thermodynamics the Boltzmann weight $exp(-E/T)$ does not exist in general and must be replaced with p^{L_0/T_p} which exists for Virasoro generator L_0 if the inverse of the p-adic temperature is integer valued $T_p = 1/n$. The expansion in powers of p converges extremely rapidly for physical values of p, which are rather large. Therefore the three lowest terms in expansion give practically exact results. Thermal massivation does not not necessarily lead to light states and this drops a large number of exotic states from the spectrum of light particles. The partition functions of N-S and Ramond type representations are not changed in TGD framework despite the fact that fermionic super generators carry fermion numbers and are not Hermitian. Thus the practical calculations are relatively straightforward albeit tedious.

In free fermion picture the p-adic thermodynamics in the boson sector is for fermion-anti-fermion states associated with the two throats of the bosonic wormhole. The question is whether the thermodynamical mass squared is just the sum of the two independent fermionic contributions for Ramond representations or should one use N-S type representation resulting as a tensor product of Ramond representations.

The overall conclusion about p-adic mass calculations is that fermionic mass spectrum is predicted in an excellent accuracy but that the thermal masses of the intermediate gauge bosons come 20-30 per cent to large for $T_p = 1$ and are completely negligible for $T_p = 1/2$. The bound state character of the boson states could be responsible for $T_p < 1$ and for extremely small thermodynamical contribution to the masses (present also for photon).

This forces to consider seriously the possibility that thermal contribution to the bosonic mass is negligible and that TGD can, contrary to the original expectations, provide dynamical Higgs field as a fundamental field and that even Higgs mechanism could contribute to the particle masses.

Higgs mechanism is probably the only viable description of Higgs mechanism in QFT approach, where particles are point-like but not in TGD, where particles are replaced by string like objects consisting of two wormhole contacts with monopole Kähler magnetic flux flowing between "upper" throats and returning back along "lower" space-time sheets. In this framework the assumption that fermion masses would result from p-adic thermodynamics but boson masses from Higgs couplings looks like an ugly idea. A more plausible vision is that the dominating contribution to gauge boson masses comes from the two flux tubes connecting the two wormhole contacts defining boson. This contribution would be present also for fermions but would be small. The correct W/Z mass ratio is obtained if the string tension is proportional to weak gauge coupling squared. The nice feature of this scenario is that naturalness is not lost: the dimensional gradient coupling of fermion to Higgs is same for all fermions.

The stringy contribution to mass squared could be expressed in terms of the deviation of the ground state conformal weight from negative half integer.

The problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.

3.1 Partition Functions Are Not Changed

One must write Super Virasoro conditions for L_n and *both* G_n and $G_n^\dagger$ rather than for L_n and G_n as in the case of the ordinary Super Virasoro algebra, and it is a priori not at all clear whether the partition functions for the Super Virasoro representations remain unchanged. This requirement is however crucial for the construction to work at all in the fermionic sector, since even the slightest changes for the degeneracies of the excited states can change light state to a state with mass of order m_0 in the p-adic thermodynamics.

3.1.1 *Super conformal algebra*

Super Virasoro algebra is generated by the bosonic the generators L_n (n is an integer valued index) and by the fermionic generators G_r, where r can be either integer (Ramond) or half odd integer (NS). G_r creates quark/lepton for $r > 0$ and antiquark/antilepton for $r < 0$. For $r = 0$, G_0 creates lepton and its Hermitian conjugate anti-lepton. The defining commutation and anti-commutation relations are the following:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{2}m(m^2-1)\delta_{m,-n} \ , \\
[L_m, G_r] &= (\frac{m}{2} - r)G_{m+r} \ , \\
[L_m, G_r^\dagger] &= (\frac{m}{2} - r)G_{m+r}^\dagger \ , \\
\{G_r, G_s^\dagger\} &= 2L_{r+s} + \frac{c}{3}(r^2 - \frac{1}{4})\delta_{m,-n} \ , \\
\{G_r, G_s\} &= 0 \ , \\
\{G_r^\dagger, G_s^\dagger\} &= 0 \ .
\end{aligned} \tag{3.1}$$

By the inspection of these relations one finds some results of a great practical importance.

1. For the Ramond algebra G_0, G_1 and their Hermitian conjugates generate the $r \geq 0, n \geq 0$ part of the algebra via anti-commutations and commutations. Therefore all what is needed is to assume that Super Virasoro conditions are satisfied for these generators in case that G_0 and $G_0^\dagger$ annihilate the ground state. Situation changes if the states are *not* annihilated by G_0 and $G_0^\dagger$ since then one must assume the gauge conditions for both L_1, G_1 and $G_1^\dagger$ besides the mass shell conditions associated with G_0 and $G_0^\dagger$, which however do not affect the number of the Super Virasoro excitations but give mass shell condition and constraints on the state in the cm spin degrees of freedom. This will be assumed in the following. Note that for the ordinary Super Virasoro only the gauge conditions for L_1 and G_1 are needed.

2. NS algebra is generated by $G_{1/2}$ and $G_{3/2}$ and their Hermitian conjugates (note that $G_{3/2}$ cannot be expressed as the commutator of L_1 and $G_{1/2}$) so that only the gauge conditions associated with these generators are needed. For the ordinary Super Virasoro only the conditions for $G_{1/2}$ and $G_{3/2}$ are needed.

3.1.2 *Conditions guaranteeing that partition functions are not changed*

The conditions guaranteeing the invariance of the partition functions in the transition to the modified algebra must be such that they reduce the number of the excitations and gauge conditions for a given conformal weight to the same number as in the case of the ordinary Super Virasoro.

1. The requirement that physical states are invariant under $G \leftrightarrow G^\dagger$ corresponds to the charge conjugation symmetry and is very natural. As a consequence, the gauge conditions for G and $G^\dagger$ are not independent and their number reduces by a factor of one half and is the same as in the case of the ordinary Super Virasoro.

2. As far as the number of the thermal excitations for a given conformal weight is considered, the only remaining problem are the operators $G_n G_n^\dagger$, which for the ordinary Super Virasoro reduce to $G_n G_n = L_{2n}$ and do not therefore correspond to independent degrees of freedom. In present case this situation is achieved only if one requires

$$(G_n G_n^\dagger - G_n^\dagger G_n)|phys\rangle = 0 \ . \tag{3.2}$$

It is not clear whether this condition must be posed separately or whether it actually follows from the representation of the Super Virasoro algebra automatically.

3.1.3 *Partition function for Ramond algebra*

Under the assumptions just stated, the partition function for the Ramond states not satisfying any gauge conditions

$$Z(t) = 1 + 2t + 4t^2 + 8t^3 + 14t^4 + ... \ , \tag{3.3}$$

which is identical to that associated with the ordinary Ramond type Super Virasoro.

For a Super Virasoro representation with $N = 5$ sectors, of main interest in TGD, one has

$$\begin{aligned} Z_N(t) &= Z^{N=5}(t) = \sum D(n)t^n \\ &= 1 + 10t + 60t^2 + 280t^3 + ... \ . \end{aligned} \tag{3.4}$$

The degeneracies for the states satisfying gauge conditions are given by

$$d(n) = D(n) - 2D(n-1) \ . \tag{3.5}$$

corresponding to the gauge conditions for L_1 and G_1. Applying this formula one obtains for $N = 5$ sectors

$$d(0) = 1 \ , \quad d(1) = 8 \ , \quad d(2) = 40 \ , \quad d(3) = 160 \ . \tag{3.6}$$

The lowest order contribution to the p-adic mass squared is determined by the ratio

$$r(n) = \frac{D(n+1)}{D(n)} \ ,$$

where the value of n depends on the effective vacuum weight of the ground state fermion. Light state is obtained only provided the ratio is integer. The remarkable result is that for lowest lying states the ratio is integer and given by

$$r(1) = 8 \ , \quad r(2) = 5 \ , \quad r(3) = 4 \ . \tag{3.7}$$

It turns out that $r(2) = 5$ gives the best possible lowest order prediction for the charged lepton masses and in this manner one ends up with the condition $h_{vac} = -3$ for the tachyonic vacuum weight of Super Virasoro.

3.1.4 *Partition function for NS algebra*

For NS representations the calculation of the degeneracies of the physical states reduces to the calculation of the partition function for a single particle Super Virasoro

$$Z_{NS}(t) = \sum_n z(n/2)t^{n/2} \ . \tag{3.8}$$

Here $z(n/2)$ gives the number of Super Virasoro generators having conformal weight $n/2$. For a state with N active sectors (the sectors with a non-vanishing weight for a given ground state) the degeneracies can be read from the N-particle partition function expressible as

$$Z_N(t) = Z^N(t) \ . \tag{3.9}$$

Single particle partition function is given by the expression

$$Z(t) = 1 + t^{1/2} + t + 2t^{3/2} + 3t^2 + 4t^{5/2} + 5t^3 + ... \ . \tag{3.10}$$

Using this representation it is an easy task to calculate the degeneracies for the operators of conformal weight Δ acting on a state having N active sectors.

One can also derive explicit formulas for the degeneracies and calculation gives

$$\begin{array}{ll} D(0,N)=1 \ , & D(1/2,N)=N \ , \\ D(1,N)=\frac{N(N+1)}{2} \ , & D(3/2,N)=\frac{N}{6}(N^2+3N+8) \ , \\ D(2,N)=\frac{N}{2}(N^2+2N+3) \ , & D(5/2,N)=9N(N-1) \ , \\ D(3,N)=12N(N-1)+2N(N-1) & . \end{array} \tag{3.11}$$

as a function of the conformal weight $\Delta=0,1/2,...,3$.

The number of states satisfying Super Virasoro gauge conditions created by the operators of a conformal weight Δ, when the number of the active sectors is N, is given by

$$d(\Delta,N) \quad = \quad D(\Delta,N)-D(\Delta-1/2,N)-D(\Delta-3/2,N) \ . \tag{3.12}$$

The expression derives from the observation that the physical states satisfying gauge conditions for $G^{1/2}$, $G^{3/2}$ satisfy the conditions for all Super Virasoro generators. For $T_p=1$ light bosons correspond to the integer values of $d(\Delta+1,N)/d(\Delta,N)$ in case that massless states correspond to thermal excitations of conformal weight Δ: they are obtained for $\Delta=0$ only (massless ground state). This is what is required since the thermal degeneracy of the light boson ground state would imply a corresponding factor in the energy density of the black body radiation at very high temperatures. For the physically most interesting nontrivial case with $N=2$ two active sectors the degeneracies are

$$d(0,2)=1 \ , \quad d(1,2)=1 \ , \quad d(2,2)=3 \ , \quad d(3,2)=4 \ . \tag{3.13}$$

N,Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 1: Degeneracies $d(\Delta,N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation for bosons assuming that they correspond to N-S representations are listed.

3.2 Fundamental Length And Mass Scales

The basic difference between quantum TGD and super-string models is that the size of CP_2 is not of order Planck length but much larger: of order $10^{3.5}$ Planck lengths. This conclusion is forced by several consistency arguments, the mass scale of electron, and by the cosmological data allowing to fix the string tension of the cosmic strings which are basic structures in TGD inspired cosmology.

3.2.1 *The relationship between CP_2 radius and fundamental p-adic length scale*

One can relate CP_2 "cosmological constant" to the p-adic mass scale: for $k_L=1$ one has

$$m_0^2 \quad = \quad \frac{m_1^2}{k_L}=m_1^2=2\Lambda \ . \tag{3.14}$$

$k_L=1$ results also by requiring that p-adic thermodynamics leaves charged leptons light and leads to optimal lowest order prediction for the charged lepton masses. Λ denotes the "cosmological constant" of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta}=\Lambda g^{\alpha\beta}$ with cosmological term).

The real counterpart of the p-adic thermal expectation for the mass squared is sensitive to the choice of the unit of p-adic mass squared which is by definition mapped as such to the real unit in canonical identification. Thus an important factor in the p-adic mass calculations is the correct identification of the p-adic mass squared scale, which corresponds to the mass squared unit and hence to the unit of the p-adic numbers. This choice does not affect the spectrum of massless states but can affect the spectrum of light states in case of intermediate gauge bosons.

1. For the choice

$$M^2 = m_0^2 \leftrightarrow 1 \tag{3.15}$$

the spectrum of L_0 is integer valued.

2. The requirement that all sufficiently small mass squared values for the color partial waves are mapped to real integers, would fix the value of p-adic mass squared unit to

$$M^2 = \frac{m_0^2}{3} \leftrightarrow 1 \ . \tag{3.16}$$

For this choice the spectrum of L_0 comes in multiples of 3 and it is possible to have a first order contribution to the mass which cannot be of thermal origin (say $m^2 = p$). This indeed seems to happen for electro-weak gauge bosons.

p-Adic mass calculations allow to relate m_0 to electron mass and to Planck mass by the formula

$$\begin{aligned} \frac{m_0}{m_{Pl}} &= \frac{1}{\sqrt{5+Y_e}} \times 2^{127/2} \times \frac{m_e}{m_{Pl}} \ , \\ m_{Pl} &= \frac{1}{\sqrt{\hbar G}} \ . \end{aligned} \tag{3.17}$$

For $Y_e = 0$ this gives $m_0 = .2437 \times 10^{-3} m_{Pl}$.

This means that CP_2 radius R defined by the length $L = 2\pi R$ of CP_2 geodesic is roughly $10^{3.5}$ times the Planck length. More precisely, using the relationship

$$\Lambda = \frac{3}{2R^2} = M^2 = m_0^2 \ ,$$

one obtains for

$$L = 2\pi R = 2\pi \sqrt{\frac{3}{2}} \frac{1}{m_0} \simeq 3.1167 \times 10^4 \sqrt{\hbar G} \ \text{for} \ Y_e = 0 \ . \tag{3.18}$$

The result came as a surprise: the first belief was that CP_2 radius is of order Planck length. It has however turned out that the new identification solved elegantly some long standing problems of TGD. **Table 2** gives the value of the scale parameter K_R.

The value of top quark mass favors $Y_e = 0$ and $Y_e = .5$ is largest value of Y_e marginally consistent with the limits on the value of top quark mass.

3.2.2 *CP_2 radius as the fundamental p-adic length scale*

The identification of CP_2 radius as the fundamental p-adic length scale is forced by the Super Virasoro invariance. The pleasant surprise was that the identification of the CP_2 size as the fundamental p-adic length scale rather than Planck length solved many long standing problems of older TGD.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{\hbar G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{\hbar G}) \times 10^{-4}$	3.1167	3.1167	3.1167
K_R/K	1.0267	1.1293	1.1868

Table 2: Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times3\times5\times7\times11\times13\times17\times19\times23\times2^{-3}*(15/17)$ is rational number producing R^2/G approximately is given.

1. The earliest formulation predicted cosmic strings with a string tension larger than the critical value giving the angle deficit 2π in Einstein's equations and thus excluded by General Relativity. The corrected value of CP_2 radius predicts the value k/G for the cosmic string tension with k in the range $10^{-7} - 10^{-6}$ as required by the TGD inspired model for the galaxy formation solving the galactic dark matter problem.

2. In the earlier formulation there was no idea as how to derive the p-adic length scale $L \sim 10^{3.5}\sqrt{\hbar G}$ from the basic theory. Now this problem becomes trivial and one has to predict gravitational constant in terms of the p-adic length scale. This follows in principle as a prediction of quantum TGD. In fact, one can deduce G in terms of the p-adic length scale and the action exponential associated with the CP_2 extremal and gets a correct value if α_K approaches fine structure constant at electron length scale (due to the fact that electromagnetic field equals to the Kähler field if Z^0 field vanishes).

   Besides this, one obtains a precise prediction for the dependence of the Kähler coupling strength on the p-adic length scale by requiring that the gravitational coupling does not depend on the p-adic length scale. p-Adic prime p in turn has a nice physical interpretation: the critical value of α_K is same for the zero modes with given p. As already found, the construction of graviton state allows to understand the small value of the gravitational constant in terms of a de-coherence caused by multi-p fractality reducing the value of the gravitational constant from L_p^2 to G.

3. p-Adic length scale is also the length scale at which super-symmetry should be restored in standard super-symmetric theories. In TGD this scale corresponds to the transition to Euclidian field theory for CP_2 type extremals. There are strong reasons to believe that sparticles are however absent and that super-symmetry is present only in the sense that super-generators have complex conformal weights with $Re(h) = \pm 1/2$ rather than $h = 0$. The action of this super-symmetry changes the mass of the state by an amount of order CP_2 mass.

4 Color Degrees Of Freedom

The ground states for the Super Virasoro representations correspond to spinor harmonics in $M^4 \times CP_2$ characterized by momentum and color quantum numbers. The correlation between color and electro-weak quantum numbers is wrong for the spinor harmonics and these states would be also hyper-massive. The super-symplectic generators allow to build color triplet states having negative vacuum conformal weights, and their values are such that p-adic massivation is consistent with the predictions of the earlier model differing from the recent one in the quark sector. In the following the construction and the properties of the color partial waves for fermions and bosons are considered. The discussion follows closely to the discussion of [A3] .

4.1 SKM Algebra And Counterpart Of Super Virasoro Conditions

There have been a considerable progress also in the understanding of super-conformal symmetries [K20, K3].

1. Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW.

2. One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one. Note that the geometric part of SKM algebra must respect the light-likeness of the partonic 3-surface.

3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

The requirement that symplectic generators have well defined radial conformal weight with respect to the light-like coordinate r of X^3 restricts M^4 conformal transformations to the group $SO(3) \times E^3$. This involves choice of preferred time coordinate. If the preferred M^4 coordinate is chosen to correspond to a preferred light-like direction in $\delta M^4_\pm$ characterizing the theory, a reduction to $SO(2) \times E^2$ more familiar from string models occurs. SKM algebra contains also $U(2)_{ew}$ Kac-Moody algebra acting as holonomies of CP_2 and having no bosonic counterpart.

p-Adic mass calculations require $N = 5$ sectors of super-conformal algebra. These sectors correspond to the 5 tensor factors for the $SO(3) \times E^3 \times SU(3) \times U(2)_{ew}$ (or $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$) decomposition of the SKM algebra to gauge symmetries of gravitation, color and electroweak interactions.

For symplectic isometries (Super-Kac-Moody algebra) fermionic algebra is realized in terms second quantized induced spinor field Ψ and spinor modes with well-defined em charge restricted to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. The full symplectic algebra is realized in terms of Ψ and covariantly constant right handed neutrino mode. One can consider also the possiblity of extended the symplectic isometries of $\delta M^4_\pm = S^2 \times R_+$ to include all isometries which act as conformal transformations of S^2 and for which conformal scaling of the metric is compensated by S^2 local scaling of the light-like radial coordinate r_M of R_+.

The algebra differs from the standard one in that super generators $G(z)$ carry lepton and quark numbers are not Hermitian as in super-string models (Majorana conditions are not satisfied). The counterparts of Ramond representations correspond to zero modes of a second quantized spinor field with vanishing radial conformal weight.

The Ramond or N-S type Virasoro conditions satisfied by the physical states in string model approach are replaced by the formulas expressing mass squared as a conformal weight. The condition is not equivalent with super Virasoro conditions since four-momentum does not appear in super Virasoro generators. It seems possible to assume that the commutator algebra $[SKM, SC]$ and the commutator of $[SKMV, SSV]$ of corresponding Super Virasoro algebras annihilate physical states. This would give rise to the analog of Super Virasoro conditions which could be seen as a Dirac equation in the world of classical worlds.

4.1.1 CP_2 *CM degrees of freedom*

Important element in the discussion are center of mass degrees of freedom parameterized by imbedding space coordinates. By the effective 2-dimensionality it is indeed possible to assign to partons momenta and color partial waves and they behave effectively as free particles. In fact, the technical problem of the earlier scenario was that it was not possible to assign symmetry transformations acting only on the light-like 3-surfaces at which the signature of the induced metric transforms from Minkowskian to Euclidian.

The original assumption was that 3-surface has boundary components to which elementary particle quantum numbers were assigned. It however became clear that boundary conditions at boundaries probably fail to be satisfied. Hence the above described light-like 3-surfaces took the role the boundary components. Space-time sheets were replaced with surfaces looking like double-sheeted (at least) structures from M^4 perspective with sheets meeting along 3-D surfaces. Sphere in Euclidian 3-space is the simplest analog for this kind of structure.

One can assign to each eigen state of color quantum numbers a color partial wave in CP_2 degrees of freedom. Thus color quantum numbers are not spin like quantum numbers in TGD framework except effectively in the length scales much longer than CP_2 length scale. The correlation between color partial waves and electro-weak quantum numbers is not physical in general: only the covariantly constant right handed neutrino has vanishing color.

4.1.2 *Mass formula, and condition determining the effective string tension*

Mass squared eigenvalues are given by

$$M^2 = m^2_{CP_2} + kL_0 \ . \qquad (4.1)$$

The contribution of CP_2 spinor Laplacian to the mass squared operator is in general not integer valued.

The requirement that mass squared spectrum is integer valued for color partial waves possibly representing light states fixes the possible values of k determining the effective string tension modulo integer. The value $k = 1$ is the only possible choice. The earlier choice $k_L = 1$ and $k_q = 2/3$, $k_B = 1$ gave integer conformal weights for the lowest possible color partial waves. The assumption that the total vacuum weight h_{vac} is conserved in particle vertices implied $k_B = 1$.

4.2 General Construction Of Solutions Of Dirac Operator Of H

The construction of the solutions of massless spinor and other d'Alembertians in $M^4_+ \times CP_2$ is based on the following observations.

1. d'Alembertian corresponds to a massless wave equation $M^4 \times CP_2$ and thus Kaluza-Klein picture applies, that is M^4_+ mass is generated from the momentum in CP_2 degrees of freedom. This implies mass quantization:

   $$M^2 = M_n^2 \ ,$$

   where M_n^2 are eigenvalues of CP_2 Laplacian. Here of course, ordinary field theory is considered. In TGD the vacuum weight changes mass squared spectrum.

2. In order to get a respectable spinor structure in CP_2 one must couple CP_2 spinors to an odd integer multiple of the Kähler potential. Leptons and quarks correspond to $n = 3$ and $n = 1$ couplings respectively. The spectrum of the electromagnetic charge comes out correctly for leptons and quarks.

3. Right handed neutrino is covariantly constant solution of CP_2 Laplacian for $n = 3$ coupling to Kähler potential whereas right handed "electron" corresponds to the covariantly constant solution for $n = -3$. From the covariant constancy it follows that all solutions of the spinor Laplacian are obtained from these two basic solutions by multiplying with an appropriate solution of the scalar Laplacian coupled to Kähler potential with such a coupling that a correct total Kähler charge results. Left handed solutions of spinor Laplacian are obtained simply by multiplying right handed solutions with CP_2 Dirac operator: in this operation the eigenvalues of the mass squared operator are obviously preserved.

4. The remaining task is to solve scalar Laplacian coupled to an arbitrary integer multiple of Kähler potential. This can be achieved by noticing that the solutions of the massive CP_2 Laplacian can be regarded as solutions of S^5 scalar Laplacian. S^5 can indeed be regarded as a circle bundle over CP_2 and massive solutions of CP_2 Laplacian correspond to the solutions of S^5 Laplacian with $exp(is\tau)$ dependence on S^1 coordinate such that s corresponds to the coupling to the Kähler potential:

$$s = n/2 \ .$$

Thus one obtains

$$D_5^2 = (D_\mu - iA_\mu\partial_\tau)(D^\mu - iA^\mu\partial_\tau) + \partial_\tau^2 \quad (4.2)$$

so that the eigen values of CP_2 scalar Laplacian are

$$m^2(s) = m_5^2 + s^2 \quad (4.3)$$

for the assumed dependence on τ.

5. What remains to do, is to find the spectrum of S^5 Laplacian and this is an easy task. All solutions of S^5 Laplacian can be written as homogenous polynomial functions of C^3 complex coordinates Z^k and their complex conjugates and have a decomposition into the representations of $SU(3)$ acting in natural manner in C^3.

6. The solutions of the scalar Laplacian belong to the representations $(p, p+s)$ for $s \geq 0$ and to the representations $(p+|s|, p)$ of $SU(3)$ for $s \leq 0$. The eigenvalues $m^2(s)$ and degeneracies d are

$$\begin{aligned} m^2(s) &= \frac{2\Lambda}{3}[p^2 + (|s|+2)p + |s|] \ , \quad p > 0 \ , \\ d &= \frac{1}{2}(p+1)(p+|s|+1)(2p+|s|+2) \ . \end{aligned} \quad (4.4)$$

Λ denotes the "cosmological constant" of CP_2 ($R_{ij} = \Lambda s_{ij}$).

4.3 Solutions Of The Leptonic Spinor Laplacian

Right handed solutions of the leptonic spinor Laplacian are obtained from the asatz of form

$$\nu_R = \Phi_{s=0}\nu_R^0 \ ,$$

where u_R is covariantly constant right handed neutrino and Φ scalar with vanishing Kähler charge. Right handed "electron" is obtained from the ansats

$$e_R = \Phi_{s=3}e_R^0 \ ,$$

where e_R^0 is covariantly constant for $n = -3$ coupling to Kähler potential so that scalar function must have Kähler coupling $s = n/2 = 3$ a in order to get a correct Kähler charge. The d'Alembert equation reduces to

$$\begin{aligned} (D_\mu D^\mu - (1-\epsilon)\Lambda)\Phi = -m^2\Phi \ , \\ \epsilon(\nu) = 1 \ , \quad \epsilon(e) = -1 \ . \end{aligned} \quad (4.5)$$

The two additional terms correspond to the curvature scalar term and $J_{kl}\Sigma^{kl}$ terms in spinor Laplacian. The latter term is proportional to Kähler coupling and of different sign for ν and e, which explains the presence of the sign factor ϵ in the formula.

Right handed neutrinos correspond to (p, p) states with $p \geq 0$ with mass spectrum

$$\begin{aligned} m^2(\nu) &= \frac{m_1^2}{3}\left[p^2 + 2p\right] \quad , \quad p \geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \tag{4.6}$$

Right handed "electrons" correspond to $(p, p+3)$ states with mass spectrum

$$m^2(e) = \frac{m_1^2}{3}\left[p^2 + 5p + 6\right] \quad , \quad p \geq 0 \ . \tag{4.7}$$

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions and have the same mass spectrum and representational content as right handed leptons with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p = 0, p = 0)$ state) annihilates it.

4.4 Quark Spectrum

Quarks correspond to the second conserved H-chirality of H-spinors. The construction of the color partial waves for quarks proceeds along similar lines as for leptons. The Kähler coupling corresponds to $n = 1$ (and $s = 1/2$) and right handed U type quark corresponds to a right handed neutrino. U quark type solutions are constructed as solutions of form

$$U_R = u_R \Phi_{s==1} \ ,$$

where u_R possesses the quantum numbers of covariantly constant right handed neutrino with Kähler charge $n = 3$ ($s = 3/2$). Hence Φ_s has $s = -1$. For D_R one has

$$D_R = d_r \Phi_{s=2} \ .$$

d_R has $s = -3/2$ so that one must have $s = 2$. For U_R the representations $(p + 1, p)$ with triality one are obtained and $p = 0$ corresponds to color triplet. For D_R the representations $(p, p + 2)$ are obtained and color triplet is missing from the spectrum ($p = 0$ corresponds to $\overline{6}$).

The CP_2 contributions to masses are given by the formula

$$\begin{aligned} m^2(U, p) &= \frac{m_1^2}{3}\left[p^2 + 3p + 2\right] \quad , \quad p \geq 0 \ , \\ m^2(D, p) &= \frac{m_1^2}{3}\left[p^2 + 4p + 4\right] \quad , \quad p \geq 0 \ . \end{aligned} \tag{4.8}$$

Left handed quarks are obtained by applying Dirac operator to right handed quark states and mass formulas and color partial wave spectrum are the same as for right handed quarks.

The color contributions to p-adic mass squared are integer valued if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since canonical identification does not commute with a division by integer. More precisely, the images of number xp in canonical identification has a value of order 1 when x is a non-trivial rational whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes. This choice does not however affect the spectrum of massless states but can affect the spectrum of light states in case of electro-weak gauge bosons.

4.5 Spectrum Of Elementary Particles

The assumption that $k = 1$ holds true for all particles forces to modify the earlier construction of quark states. This turns out to be possible without affecting the p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (which can be negative).

4.5.1 *Leptonic spectrum*

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod \ 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p + 3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2 + 2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i \sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Right handed neutrino remains essentially massless. $p = 0$ right handed neutrino does not however generate $N = 1$ space-time (or rather, imbedding space) super symmetry so that no sparticles are predicted. The breaking of the electro-weak symmetry at the level of the masses comes out basically from the anomalous color electro-weak correlation for the Kaluza-Klein partial waves implying that the weights for the ground states of the fermions depend on the electromagnetic charge of the fermion. Interestingly, TGD predicts lepto-hadron physics based on color excitations of leptons and color bound states of these excitations could correspond topologically condensed on string like objects but not fundamental string like objects.

4.5.2 *Spectrum of quarks*

Earlier arguments [K12] related to a model of CKM matrix as a rational unitary matrix suggested that the string tension parameter k is different for quarks, leptons, and bosons. The basic mass formula read as

$$M^2 = m_{CP_2}^2 + kL_0 \ .$$

The values of k were $k_q = 2/3$ and $k_L = k_B = 1$. The general theory however predicts that $k = 1$ for all particles.

1. By earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p \ mod \ = \pm 1$ for U type quark belonging to $(p + 1, p)$ type representation and obeying $h_c(U) = (p^2 + 3p + 2)/3$ and for $p \ mod \ 3 = 1$ for D type quark belonging $(p, p + 2)$ type representation and obeying $h_c(D) = (p^2 + 4p + 4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

2. In the recent case $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This forces the super-symplectic operator O to compensate the anomalous color to have a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SSV: this would also explain why h_{sc} would be same for all fermions.

3. It would seem that the tensor product of the spinor harmonic of quarks (as also leptons) with Hamiltonians gives rise to a large number of exotic colored states which have same

thermodynamical mass as ordinary quarks (and leptons). Why these states have smaller values of p-adic prime that ordinary quarks and leptons, remains a challenge for the theory. Note that the decay widths of intermediate gauge bosons pose strong restrictions on the possible color excitations of quarks. On the other hand, the large number of fermionic color exotics can spoil the asymptotic freedom, and it is possible to have and entire p-adic length scale hierarchy of QCDs existing only in a finite length scale range without affecting the decay widths of gauge bosons.

Table 3 summarizes the color conformal weights and super-symplectic vacuum conformal weights for the elementary particles.

	L	ν_L	U	D	W	γ, G, g
h_{vac}	-3	-3	-3	-3	-2	0
h_c	2	1	2	3	2	0

Table 3: The values of the parameters h_{vac} and h_c assuming that $k = 1$. The value of $h_{vac} \leq -h_c$ is determined from the requirement that p-adic mass calculations give best possible fit to the mass spectrum.

4.5.3 *Photon, graviton and gluon*

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac} = 0$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain several physically more or less identical photons and this would be seen in the spectrum of black-body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

Masslessness or approximate masslessness requires low enough temperature $T_p = 1/n$, $n > 1$ at least and small enough value of the possible contribution coming from the ground state conformal weight.

In NS thermodynamics the only possibility to get exactly massless states in thermal sense is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. For neutral gauge bosons this is indeed achieved. For $T_p = 1/2$, which is required by the mass spectrum of intermediate gauge bosons, the thermal contribution to the mass squared is however extremely small even for W boson.

5 Modular Contribution To The Mass Squared

The success of the p-adic mass calculations gives convincing support for the generation-genus correspondence. The basic physical picture is following.

1. Fermionic mass squared is dominated by partonic contribution, which is sum of cm and modular contributions: $M^2 = M^2(cm)+M^2(mod)$. Here "cm" refers to the thermal contribution. Modular contribution can be assumed to depend on the genus of the boundary component only.

2. If Higgs contribution for diagonal (g, g) bosons (singlets with respect to "topological" $SU(3)$) dominates, the genus dependent contribution can be assumed to be negligible. This should be due to the bound state character of the wormhole contacts reducing thermal motion and thus the p-adic temperature.

3. Modular contribution to the mass squared can be estimated apart from an overall proportionality constant. The mass scale of the contribution is fixed by the p-adic length scale hypothesis. Elementary particle vacuum functionals are proportional to a product of all even theta functions and their conjugates, the number of even theta functions and their conjugates

being $2N(g) = 2^g(2^g + 1)$. Also the thermal partition function must also be proportional to $2N(g)$:th power of some elementary partition function. This implies that thermal/ quantum expectation $M^2(mod)$ must be proportional to $2N(g)$. Since single handle behaves effectively as particle, the contribution must be proportional to genus g also. The success of the resulting mass formula encourages the belief that the argument is essentially correct.

The challenge is to construct theoretical framework reproducing the modular contribution to mass squared. There are two alternative manners to understand the origin modular contribution.

1. The realization that super-symplectic algebra is relevant for elementary particle physics leads to the idea that two thermodynamics are involved with the calculation of the vacuum conformal weight as a thermal expectation. The first thermodynamics corresponds to Super Kac-Moody algebra and second thermodynamics to super-symplectic algebra. This approach allows a first principle understanding of the origin and general form of the modular contribution without any need to introduce additional structures in modular degrees of freedom. The very fact that super-symplectic algebra does not commute with the modular degrees of freedom explains the dependence of the super-symplectic contribution on moduli.

2. The earlier approach was based on the idea that he modular contribution could be regarded as a quantum mechanical expectation value of the Virasoro generator L_0 for the elementary particle vacuum functional. Quantum treatment would require generalization the concepts of the moduli space and theta function to the p-adic context and finding an acceptable definition of the Virasoro generator L_0 in modular degrees of freedom. The problem with this interpretation is that it forces to introduce, not only Virasoro generator L_0, but the entire super Virasoro algebra in modular degrees of freedom. One could also consider of interpreting the contribution of modular degrees of freedom to vacuum conformal weight as being analogous to that of CP_2 Laplacian but also this would raise the challenge of constructing corresponding Dirac operator. Obviously this approach has become obsolete.

The thermodynamical treatment taking into account the constraints from that p-adicization is possible might go along following lines.

1. In the real case the basic quantity is the thermal expectation value $h(M)$ of the conformal weight as a function of moduli. The average value of the deviation $\Delta h(M) = h(M) - h(M_0)$ over moduli space $\mathcal{M}$ must be calculated using elementary particle vacuum functional as a modular invariant partition function. Modular invariance is achieved if this function is proportional to the logarithm of elementary particle vacuum functional: this reproduces the qualitative features basic formula for the modular contribution to the conformal weight. p-Adicization leads to a slight modification of this formula.

2. The challenge of algebraically continuing this calculation to the p-adic context involves several sub-tasks. The notions of moduli space $\mathcal{M}_p$ and theta function must be defined in the p-adic context. An appropriately defined logarithm of the p-adic elementary particle vacuum functional should determine $\Delta h(M)$. The average of $\Delta h(M)$ requires an integration over $\mathcal{M}_p$. The problems related to the definition of this integral could be circumvented if the integral in the real case could be reduced to an algebraic expression, or if the moduli space is discrete in which case integral could be replaced by a sum.

3. The number theoretic existence of the p-adic Θ function leads to the quantization of the moduli so that the p-adic moduli space is discretized. Accepting the sharpened form of Riemann hypothesis [K14] , the quantization means that the imaginary *resp.* real parts of the moduli are proportional to integers *resp.* combinations of imaginary parts of zeros of Riemann Zeta. This quantization could occur also for the real moduli for the maxima of Kähler function. This reduces the problematic p-adic integration to a sum and the resulting sum defining $\langle \Delta h \rangle$ converges extremely rapidly for physically interesting primes so that only the few lowest terms are needed.

5.1 Conformal Symmetries And Modular Invariance

The full SKM invariance means that the super-conformal fields depend only on the conformal moduli of 2-surface characterizing the conformal equivalence class of the 2-surface. This means that all induced metrics differing by a mere Weyl scaling have same moduli. This symmetry is extremely powerful since the space of moduli is finite-dimensional and means that the entire infinite-dimensional space of deformations of parton 2-surface X^2 degenerates to a finite-dimensional moduli spaces under conformal equivalence. Obviously, the configurations of given parton correspond to a fiber space having moduli space as a base space. Super-symplectic degrees of freedom could break conformal invariance in some appropriate sense.

5.1.1 *Conformal and SKM symmetries leave moduli invariant*

Conformal transformations and super Kac Moody symmetries must leave the moduli invariant. This means that they induce a mere Weyl scaling of the induced metric of X^2 and thus preserve its non-diagonal character $ds^2 = g_{z\overline{z}}dzd\overline{z}$. This is indeed true if

1. the Super Kac Moody symmetries are holomorphic isometries of $X^7 = \delta M^4_\pm \times CP_2$ made local with respect to the complex coordinate z of X^2, and
2. the complex coordinates of X^7 are holomorphic functions of z.

Using complex coordinates for X^7 the infinitesimal generators can be written in the form

$$J^{An} = z^n j^{Ak} D_k + \overline{z}^n j^{A\overline{k}} D_{\overline{k}} \ . \tag{5.1}$$

The intuitive picture is that it should be possible to choose X^2 freely. It is however not always possible to choose the coordinate z of X^2 in such a manner that X^7 coordinates are holomorphic functions of z since a consistency of inherent complex structure of X^2 with that induced from X^7 is required. Geometrically this is like meeting of two points in the space of moduli.

Lorentz boosts produce new inequivalent choices of S^2 with their own complex coordinate: this set of complex structures is parameterized by the hyperboloid of future light cone (Lobatchevski space or mass shell), but even this is not enough. The most plausible manner to circumvent the problem is that only the maxima of Kähler function correspond to the holomorphic situation so that super-symplectic algebra representing quantum fluctuations would induce conformal anomaly.

5.1.2 *The isometries of δM^4_+ are in one-one correspondence with conformal transformations*

For CP_2 factor the isometries reduce to $SU(3)$ group acting also as symplectic transformations. For $\delta M^4_+ = S^2 \times R_+$ one might expect that isometries reduce to Lorentz group containing rotation group of $SO(3)$ as conformal isometries. If r_M corresponds to a macroscopic length scale, then X^2 has a finite sized S^2 projection which spans a rather small solid angle so that group $SO(3)$ reduces in a good approximation to the group $E^2 \times SO(2)$ of translations and rotations of plane.

This expectation is however wrong! The light-likeness of δM^4_+ allows a dramatic generalization of the notion of isometry. The point is that the conformal transformations of S^2 induce a conformal factor $|df/dw|^2$ to the metric of δM^4_+ and the local radial scaling $r_M \rightarrow r_M/|df/dw|$ compensates it. Hence the group of conformal isometries consists of conformal transformations of S^2 with compensating radial scalings. This compensation of two kinds of conformal transformations is the deep geometric phenomenon which translates to the condition $L_{SC} - L_{SKM} = 0$ in the sub-space of physical states. Note that an analogous phenomenon occurs also for the light-like CDs X^3_l with respect to the metrically 2-dimensional induced metric.

The X^2-local radial scalings $r_M \rightarrow r_M(z,\overline{z})$ respect the conditions $g_{zz} = g_{\overline{z}\overline{z}} = 0$ so that a mere Weyl scaling leaving moduli invariant results. By multiplying the conformal isometries of δM^4_+ by z^n (z is used as a complex coordinate for X^2 and w as a complex coordinate for S^2) a conformal localization of conformal isometries would result. Kind of double conformal transformations would be in question. Note however that this requires that X^7 coordinates are holomorphic functions of X^2 coordinate. These transformations deform X^2 unlike the conformal transformations of X^2. For X^3_l similar local scalings of the light like coordinate leave the moduli invariant but lead out of X^7.

5.1.3 *Symplectic transformations break the conformal invariance*

In general, infinitesimal symplectic transformations induce non-vanishing components $g_{zz}, g_{\overline{zz}}$ of the induced metric and can thus change the moduli of X^2. Thus the quantum fluctuations represented by super-symplectic algebra and contributing to the WCW metric are in general moduli changing. It would be interesting to know explicitly the conditions (the number of which is the dimension of moduli space for a given genus), which guarantee that the infinitesimal symplectic transformation is moduli preserving.

5.2 The Physical Origin Of The Genus Dependent Contribution To The Mass Squared

Different p-adic length scales are not enough to explain the charged lepton mass ratios and an additional genus dependent contribution in the fermionic mass formula is required. The general form of this contribution can be guessed by regarding elementary particle vacuum functionals in the modular degrees of freedom as an analog of partition function and the modular contribution to the conformal weight as an analog of thermal energy obtained by averaging over moduli. p-Adic length scale hypothesis determines the overall scale of the contribution.

The exact physical origin of this contribution has remained mysterious but super-symplectic degrees of freedom represent a good candidate for the physical origin of this contribution. This would mean a sigh of relief since there would be no need to assign conformal weights, super-algebra, Dirac operators, Laplacians, etc.. with these degrees of freedom.

5.2.1 *Thermodynamics in super-symplectic degrees of freedom as the origin of the modular contribution to the mass squared*

The following general picture is the simplest found hitherto.

1. Elementary particle vacuum functionals are defined in the space of moduli of surfaces X^2 corresponding to the maxima of Kähler function. There some restrictions on X^2. In particular, p-adic length scale poses restrictions on the size of X^2. There is an infinite hierarchy of elementary particle vacuum functionals satisfying the general constraints but only the lowest elementary particle vacuum functionals are assumed to contribute significantly to the vacuum expectation value of conformal weight determining the mass squared value.

2. The contribution of Super-Kac Moody thermodynamics to the vacuum conformal weight h coming from Virasoro excitations of the $h = 0$ massless state is estimated in the previous calculations and does not depend on moduli. The new element is that for a partonic 2-surface X^2 with given moduli, Virasoro thermodynamics is present also in super-symplectic degrees of freedom.

   Super-symplectic thermodynamics means that, besides the ground state with $h_{gr} = -h_{SC}$ with minimal value of super-symplectic conformal weight h_{SC}, also thermal excitations of this state by super-symplectic Virasoro algebra having $h_{gr} = -h_{SC} - n$ are possible. For these ground states the SKM Virasoro generators creating states with net conformal weight $h = h_{SKM} - h_{SC} - n \geq 0$ have larger conformal weight so that the SKM thermal average h depends on n. It depends also on the moduli M of X^2 since the Beltrami differentials representing a tangent space basis for the moduli space $\mathcal{M}$ do not commute with the super-symplectic algebra. Hence the thermally averaged SKM conformal weight h_{SKM} for given values of moduli satisfies

$$h_{SKM} = h(n, M) \ . \tag{5.2}$$

3. The average conformal weight induced by this double thermodynamics can be expressed as a super-symplectic thermal average $\langle \cdot \rangle_{SC}$ of the SKM thermal average $h(n, M)$:

$$h(M) = \langle h(n, M) \rangle_{SC} = \sum p_n(M) h(n) \ , \tag{5.3}$$

where the moduli dependent probability $p_n(M)$ of the super-symplectic Virasoro excitation with conformal weight n should be consistent with the p-adic thermodynamics. It is convenient to write $h(M)$ as

$$h(M) = h_0 + \Delta h(M) \ , \tag{5.4}$$

where h_0 is the minimum value of $h(M)$ in the space of moduli. The form of the elementary particle vacuum functionals suggest that h_0 corresponds to moduli with $Im(\Omega_{ij}) = 0$ and thus to singular configurations for which handles degenerate to one-dimensional lines attached to a sphere.

4. There is a further averaging of $\Delta h(M)$ over the moduli space $\mathcal{M}$ by using the modulus squared of elementary particle vacuum functional so that one has

$$h = h_0 + \langle \Delta h(M) \rangle_{\mathcal{M}} \ . \tag{5.5}$$

Modular invariance allows to pose very strong conditions on the functional form of $\Delta h(M)$. The simplest assumption guaranteeing this and thermodynamical interpretation is that $\Delta h(M)$ is proportional to the logarithm of the vacuum functional Ω:

$$\Delta h(M) \propto -log(\frac{\Omega(M)}{\Omega_{max}}) \ . \tag{5.6}$$

Here Ω_{max} corresponds to the maximum of Ω for which $\Delta h(M)$ vanishes.

5.2.2 *Justification for the general form of the mass formula*

The proposed general ansatz for $\Delta h(M)$ provides a justification for the general form of the mass formula deduced by intuitive arguments.

1. The factorization of the elementary particle vacuum functional Ω into a product of $2N(g) = 2^g(2^g+1)$ terms and the logarithmic expression for $\Delta h(M)$ imply that the thermal expectation values is a sum over thermal expectation values over $2N(g)$ terms associated with various even characteristics (a, b), where a and b are g-dimensional vectors with components equal to 1/2 or 0 and the inner product $4a \cdot b$ is an even integer. If each term gives the same result in the averaging using Ω_{vac} as a partition function, the proportionality to $2N_g$ follows.

2. For genus $g \geq 2$ the partition function defines an average in $3g-3$ complex-dimensional space of moduli. The analogy of $\langle \Delta h \rangle$ and thermal energy suggests that the contribution is proportional to the complex dimension $3g-3$ of this space. For $g \leq 1$ the contribution the complex dimension of moduli space is g and the contribution would be proportional to g.

$$\begin{aligned} \langle \Delta h \rangle &\propto g \times X(g) \ \text{ for } \ g \leq 1 \ , \\ \langle \Delta h \rangle &\propto (3g-3) \times X(g) \ \text{ for } \ g \geq 2 \ , \\ X(g) &= 2^g(2^g+1) \ . \end{aligned} \tag{5.7}$$

If X^2 is hyper-elliptic for the maxima of Kähler function, this expression makes sense only for $g \leq 2$ since vacuum functionals vanish for hyper-elliptic surfaces.

3. The earlier argument, inspired by the interpretation of elementary particle vacuum functional as a partition function, was that each factor of the elementary particle vacuum functional gives the same contribution to $\langle \Delta h \rangle$, and that this contribution is proportional to g since each handle behaves like a particle:

$$\langle \Delta h \rangle \quad \propto \quad g \times X(g) \ . \tag{5.8}$$

The prediction following from the previous differs by a factor $(3g-3)/g$ for $g \geq 2$. This would scale up the dominant modular contribution to the masses of the third $g=2$ fermionic generation by a factor $\sqrt{3/2} \simeq 1.22$. One must of course remember, that these rough arguments allow $g-$ dependent numerical factors of order one so that it is not possible to exclude either argument.

5.3 Generalization Of Θ Functions And Quantization Of P-Adic Moduli

The task is to find p-adic counterparts for theta functions and elementary particle vacuum functionals. The constraints come from the p-adic existence of the exponentials appearing as the summands of the theta functions and from the convergence of the sum. The exponentials must be proportional to powers of p just as the Boltzmann weights defining the p-adic partition function. The outcome is a quantization of moduli so that integration can be replaced with a summation and the average of $\Delta h(M)$ over moduli is well defined.

It is instructive to study the problem for torus in parallel with the general case. The ordinary moduli space of torus is parameterized by single complex number τ. The points related by $SL(2,Z)$ are equivalent, which means that the transformation $\tau \rightarrow (A\tau+B)/(C\tau+D)$ produces a point equivalent with τ. These transformations are generated by the shift $\tau \rightarrow \tau+1$ and $\tau \rightarrow -1/\tau$. One can choose the fundamental domain of moduli space to be the intersection of the slice $Re(\tau) \in [-1/2, 1/2]$ with the exterior of unit circle $|\tau|=1$. The idea is to start directly from physics and to look whether one might some define p-adic version of elementary particle vacuum functionals in the p-adic counterpart of this set or in some modular invariant subset of this set.

Elementary particle vacuum functionals are expressible in terms of theta functions using the functions $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ as a building block. The general expression for the theta function reads as

$$\Theta[a,b](\Omega) \quad = \quad \sum_n exp(i\pi(n+a)\cdot\Omega\cdot(n+a))exp(2i\pi(n+a)\cdot b) \ . \tag{5.9}$$

The latter exponential phase gives only a factor $\pm i$ or ± 1 since $4a \cdot b$ is integer. For $p \ mod \ 4 = 3$ imaginary unit exists in an algebraic extension of p-adic numbers. In the case of torus (a,b) has the values $(0,0)$, $(1/2,0)$ and $(0,1/2)$ for torus since only even characteristics are allowed.

Concerning the p-adicization of the first exponential appearing in the summands in Eq. 5.9, the obvious problem is that π does not exists p-adically unless one allows infinite-dimensional extension.

1. Consider first the real part of Ω. In this case the proper manner to treat the situation is to introduce and algebraic extension involving roots of unity so that $Re(\Omega)$ rational. This approach is proposed as a general approach to the p-adicization of quantum TGD in terms of harmonic analysis in symmetric spaces allowing to define integration also in p-adic context in a physically acceptable manner by reducing it to Fourier analysis. The simplest situation corresponds to integer values for $Re(\Omega)$ and in this case the phase are equal to $\pm i$ or ± 1 since a is half-integer valued. One can consider a hierarchy of variants of moduli space characterized by the allowed roots of unity. The physical interpretation for this hierarchy would be in terms of a hierarchy of measurement resolutions. Note that the real parts of Ω can be assumed to be rationals of form m/n where n is constructed as a product of finite number of primes and therefore the allowed rationals are linear combinations of inverses $1/p_i$ for a subset $\{p_i\}$ of primes.

2. For the imaginary part of Ω different approach is required. One wants a rapid convergence of the sum formula and this requires that the exponents reduces in this case to positive powers of p. This is achieved if one has

$$Im(\Omega) = -n\frac{log(p)}{\pi)} \ , \quad (5.10)$$

Unfortunately this condition is not consistent with the condition $Im(\Omega) > 0$. A manner to circumvent the difficulty is to replace Ω with its complex conjugate. Second approach is to define the real discretized variant of theta function first and then map it by canonical identification to its p-adic counterpart: this would map phase to phases and powers of p to their inverses. Note that a similar change of sign must be performed in p-adic thermodynamics for powers of p to map p-adic probabilities to real ones. By rescaling $Im(\Omega) \rightarrow \frac{log(p)}{\pi)}Im(\Omega)$ one has non-negative integer valued spectrum for $Im(\Omega)$ making possible to reduce integration in moduli space to a summation over finite number of rationals associated with the real part of Ω and powers of p associated with the imaginary part of Ω.

3. Since the exponents appearing in

$$p^{(n+a)\cdot Im(\Omega_{ij,p})\cdot(n+a)} = p^{a\cdot Im(\Omega)\cdot a} \times p^{2a\cdot Im(\Omega\cdot n} \times p^{+n\cdot Im(\Omega_{ij,p})\cdot n}$$

are positive integers valued, $\Theta_{[a,b]}$ exist in R_p and converges. The problematic factor is the first exponent since the components of the vector a can have values 1/2 and 0 and its existence implies a quantization of $Im(\Omega_{ij})$ as

$$Im(\Omega) = -Kn\frac{log(p)}{p} \ , \ n \in Z \ , \ n \geq 1 \ , \quad (5.11)$$

In p-adic context this condition must be formulated for the exponent of Ω defining the natural coordinate. $K = 4$ guarantees the existence of Θ functions and $K = 1$ the existence of the building blocks $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ of elementary particle vacuum functionals in R_p. The extension to higher genera means only replacement of Ω with the elements of a matrix.

4. One can criticize this approach for the loss of the full modular covariance in the definition of theta functions. The modular transformations $\Omega \rightarrow \Omega + n$ are consistent with the number theoretic constraints but the transformations $\Omega \rightarrow -1/\Omega$ do not respect them. It seem that one can circumvent the difficulty by restricting the consideration to a fundamental domain satisfying the number theoretic constraints.

This variant of moduli space is discrete and p-adicity is reflected only in the sense that the moduli space makes sense also p-adically. One can consider also a continuum variant of the p-adic moduli space using the same prescription as in the construction of p-adic symmetric spaces [K17] .

1. One can introduce $exp(i\pi Re(\Omega))$ as the counterpart of $Re(\Omega)$ as a coordinate of the Teichmueller space. This coordinate makes sense only as a local coordinate since it does not differentiate between $Re(\Omega)$ and $Re(\Omega + 2n)$. On the other hand, modular invariance states that Ω abd $\Omega + n$ correspond to the same moduli so that nothing is lost. In the similar manner one can introduce $exp^(\pi Im(\Omega)) \in \{p^n, n > 0\}$ as the counterpart of discretized version of $Im(\Omega)$.

2. The extension to continuum would mean in the case of $Re(\Omega)$ the extension of the phase $exp(i\pi Re(\Omega))$ to a product $exp(i\pi Re(\Omega))exp(ipx) = exp(i\pi Re(\Omega) + exp(ipx)$, where x is p-adic integer which can be also infinite as a real integer. This would mean that each root of unity representing allowed value $Re(\Omega)$ would have a p-adic neighborhood consisting of p-adic integers. This neighborhood would be the p-adic counterpart for the angular integral $\Delta\phi$ for a given root of unity and would not make itself visible in p-adic integration.

3. For the imaginary part one can also consider the extension of $exp(\pi Im(\Omega))$ to $p^n \times exp(npx)$ where x is a p-adic integer. This would assign to each point p^n a p-adic neighborhood defined by p-adic integers. This neighborhood is same all integers n with same p-adic norm. When n is proportional to p^k one has $exp(npx) - 1 \propto p^k$.

The quantization of moduli characterizes precisely the conformal properties of the partonic 2-surfaces corresponding to different p-adic primes. In the real context -that is in the intersection of real and p-adic worlds- the quantization of moduli of torus would correspond to

$$\tau = K\left[\sum q + i \times n\frac{log(p)}{\pi}\right] , \quad (5.12)$$

where q is a rational number expressible as linear combination of inverses of a finite fixed set of primes defining the allowed roots of unity. $K = 1$ guarantees the existence of elementary particle vacuum functionals and $K = 4$ the existence of Theta functions. The ratio for the complex vectors defining the sides of the plane parallelogram defining torus via the identification of the parallel sides is quantized. In other words, the angles Φ between the sides and the ratios of the sides given by $|\tau|$ have quantized values.

The quantization rules for the moduli of the higher genera is of exactly same form

$$\Omega_{ij} = K\left[\sum q_{ij} + i \times n_{ij} \times \frac{log(p)}{\pi}\right] , \quad (5.13)$$

If the quantization rules hold true also for the maxima of Kähler function in the real context or more precisely- in the intersection of real and p-adic variants of the "world of classical worlds" identified as partonic 2-surfaces at the boundaries of causal diamond plus the data about their 4-D tangent space, there are good hopes that the p-adicized expression for Δh is obtained by a simple algebraic continuation of the real formula. Thus p-adic length scale would characterize partonic surface X^2 rather than the light like causal determinant X_l^3 containing X^2. Therefore the idea that various p-adic primes label various X_l^3 connecting fixed partonic surfaces X_i^2 would not be correct.

Quite generally, the quantization of moduli means that the allowed 2-dimensional shapes form a lattice and are thus additive. It also means that the maxima of Kähler function would obey a linear superposition in an extreme abstract sense. The proposed number theoretical quantization is expected to apply for any complex space allowing some preferred complex coordinates. In particular, WCW of 2-surfaces could allow this kind of quantization in the complex coordinates naturally associated with isometries and this could allow to define WCW integration, at least the counterpart of integration in zero mode degrees of freedom, as a summation.

Number theoretic vision leads to the notion of multi-p-p-adicity in the sense that the same partonic 2-surface can correspond to several p-adic primes and that infinite primes code for these primes [K20, K16] . At the level of the moduli space this corresponds to the replacement of p with an integer in the formulas so that one can interpret the formulas both in real sense and p-adic sense for the primes p dividing the integer. Also the exponent of given prime in the integer matters.

5.4 The Calculation Of The Modular Contribution $\langle \Delta H \rangle$ To The Conformal Weight

The quantization of the moduli implies that the integral over moduli can be defined as a sum over moduli. The theta function $\Theta[a,b](\Omega)_p(\tau_p)$ is proportional to $p^{a\cdot aIm(\Omega_{ij,p})} = p^{Kn_{ij}m(a)/4}$ for $a \cdot a = m(a)/4$, where $K = 1$ *resp.* $K = 4$ corresponds to the existence existence of elementary particle vacuum functionals *resp.* theta functions in R_p. These powers of p can be extracted from the thetas defining the vacuum functional. The numerator of the vacuum functional gives $(p^n)^{2K\sum_{a,b} m(a)}$. The numerator gives $(p^n)^{2K\sum_{a,b} m(a_0)}$, where a_0 corresponds to the minimum value of $m(a)$. $a_0 = (0, 0, .., 0)$ is allowed and gives $m(a_0) = 0$ so that the p-adic norm of the denominator equals to one. Hence one has

$$|\Omega_{vac}(\Omega_p)|_p = p^{-2nK\sum_{a,b} m(a)} \quad (5.14)$$

The sum converges extremely rapidly for large values of p as function of n so that in practice only few moduli contribute.

The definition of $log(\Omega_{vac})$ poses however problems since in $log(p)$ does not exist as a p-adic number in any p-adic number field. The argument of the logarithm should have a unit p-adic norm. The simplest manner to circumvent the difficulty is to use the fact that the p-adic norm $|\Omega_p|_p$ is also a modular invariant, and assume that the contribution to conformal weight depends on moduli as

$$\Delta h_p(\Omega_p) \quad \propto \quad log(\frac{\Omega_{vac}}{|\Omega_{vac}|_p}) \ . \tag{5.15}$$

The sum defining $\langle \Delta h_p \rangle$ converges extremely rapidly and gives a result of order $O(p)$ p-adically as required.

The p-adic expression for $\langle \Delta h_p \rangle$ should result from the corresponding real expression by an algebraic continuation. This encourages the conjecture that the allowed moduli are quantized for the maxima of Kähler function, so that the integral over the moduli space is replaced with a sum also in the real case, and that Δh given by the double thermodynamics as a function of moduli can be defined as in the p-adic case. The positive power of p multiplying the numerator could be interpreted as a degeneracy factor. In fact, the moduli are not primary dynamical variables in the case of the induced metric, and there must be a modular invariant weight factor telling how many 2-surfaces correspond to given values of moduli. The power of p could correspond to this factor.

6 The Contributions Of P-Adic Thermodynamics To Particle Masses

In the sequel various contributions to the mass squared are discussed.

6.1 General Mass Squared Formula

The thermal independence of Super Virasoro and modular degrees of freedom implies that mass squared for elementary particle is the sum of Super Virasoro, modular and Higgsy contributions:

$$M^2 \quad = \quad M^2(color) + M^2(SV) + M^2(mod) + M^2(Higgsy) \ . \tag{6.1}$$

Also small renormalization correction contributions might be possible.

6.2 Color Contribution To The Mass Squared

The mass squared contains a non-thermal color contribution to the ground state conformal weight coming from the mass squared of CP_2 spinor harmonic. The color contribution is an integer multiple of $m_0^2/3$, where $m_0^2 = 2\Lambda$ denotes the "cosmological constant" of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$).

The color contribution to the p-adic mass squared is integer valued only if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since the simplest form of the canonical identification does not commute with a division by integer. More precisely, the image of number xp in canonical identification has a value of order 1 when x is a non-trivial rational number whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes.

The choice of the p-adic mass squared unit are no effects on zeroth order contribution which must vanish for light states: this requirement eliminates quark and lepton states for which the CP_2 contribution to the mass squared is not integer valued using m_0^2 as a unit. There can be a dramatic effect on the first order contribution. The mass squared $m^2 = p/3$ using $m_0^2/3$ means that the particle is light. The mass squared becomes $m^2 = p/3$ when m_0^2 is used as a unit and the particle has mass of order 10^{-4} Planck masses. In the case of W and Z^0 bosons this problem is actually encountered. For light states using $m_0^2/3$ as a unit only the second order contribution to the mass squared is affected by this choice.

6.3 Modular Contribution To The Mass Of Elementary Particle

The general form of the modular contribution is derivable from p-adic partition function for conformally invariant degrees of freedom associated with the boundary components. The general form of the vacuum functionals as modular invariant functions of Teichmueller parameters was derived in [K1] and the square of the elementary particle vacuum functional can be identified as a partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g > 0$ is given by

$$N(g) = 2^{g-1}(2^g + 1) \ . \tag{6.2}$$

One has $N(1) = 3$ for muon and $N(2) = 10$ for τ.

1. Single theta function is analogous to a partition function. This implies that the modular contribution to the mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in the partition function.

2. The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g-g_1$ handles, the partition function reduces to a product of g_1 and $g-g_1$ partition functions. This implies that the contribution to the mass squared is proportional to the genus of the surface. Altogether one has

$$\begin{aligned} M^2(mod, g) &= 2k(mod)N(g)g\frac{m_0^2}{p} \ , \\ k(mod) &= 1 \ . \end{aligned} \tag{6.3}$$

   Here $k(mod)$ is some integer valued constant (in order to avoid ultra heavy mass) to be determined. $k(mod) = 1$ turns out to be the correct choice for this parameter.

Summarizing, the real counterpart of the modular contribution to the mass of a particle belonging to $g + 1$:th generation reads as

$$\begin{aligned} M^2(mod) &= 0 \ for \ e, \nu_e, u, d \ , \\ M^2(mod) &= 9\frac{m_0^2}{p(X))} \ for \ X = \mu, \nu_\mu, c, s \ , \\ M^2(mod) &= 60\frac{m_0^2}{p(X)} \ for \ X = \tau, \nu_\tau, t, b \ . \end{aligned} \tag{6.4}$$

The requirement that hadronic mass spectrum and CKM matrix are sensible however forces the modular contribution to be the same for quarks, leptons and bosons. The higher order modular contributions to the mass squared are completely negligible if the degeneracy of massless state is $D(0, mod, g) = 1$ in the modular degrees of freedom as is in fact required by $k(mod) = 1$.

6.4 Thermal Contribution To The Mass Squared

One can deduce the value of the thermal mass squared in order $O(p^2)$ (an excellent approximation) using the general mass formula given by p-adic thermodynamics. Assuming maximal p-adic temperature $T_p = 1$ one has

$$\begin{aligned} M^2 &= k(sp + Xp^2 + O(p^3)) \ , \\ s_\Delta &= \frac{D(\Delta + 1)}{D(\Delta)} \ , \\ X_\Delta &= 2\frac{D(\Delta + 2)}{D(\Delta)} - \frac{D^2(\Delta + 1)}{D^2(\Delta)} \ , \\ k &= 1 \ . \end{aligned} \tag{6.5}$$

Δ is the conformal weight of the operator creating massless state from the ground state.

The ratios $r_n = D(n+1)/D(n)$ allowing to deduce the values of s and X have been deduced from p-adic thermodynamics in [K8] . Light state is obtained only provided $r(\Delta)$ is an integer. The remarkable result is that for lowest lying states this is the case. For instance, for Ramond representations the values of r_n are given by

$$(r_0, r_1, r_2, r_3) = (8, 5, 4, \frac{55}{16}) \ . \tag{6.6}$$

The values of s and X are

$$\begin{aligned} (s_0, s_1, s_2) &= (8, 5, 4) \ , \\ (X_0, X_1, X_2) &= (16, 15, 11 + 1/2)) \ . \end{aligned} \tag{6.7}$$

The result means that second order contribution is extremely small for quarks and charged leptons having $\Delta < 2$. For neutrinos having $\Delta = 2$ the second order contribution is non-vanishing.

6.5 The Contribution From The Deviation Of Ground StateConformal Weight From Negative Integer

The interpretation inspired by p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of the Kähler-Dirac operator correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. A good guess is that induced Kähler magnetic field B_K dictates the magnitude of the eigenvalues which is thus of order $h_0 = \sqrt{B_K R}$, R CP_2 radius. The first guess is that eigenvalues in the first approximation come as $(n+1/2)h_0$. Each region where induced Kähler field is non-vanishing would correspond to different scale mass scale h_0.

1. The vacuum expectation value of Higgs is only proportional to an eigenvalue λ, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and anti-fermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this).

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of Kähler-Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign if the effective metric of X_l^3 defined by the inner products $T_K^{k\alpha} T_K^{l\beta} h_{kl}$ of the Kähler energy momentum tensor $T^{k\alpha} = h^{kl} \partial L_K / \partial h_\alpha^l$ and appearing in the Kähler-Dirac operator D_K has Minkowskian signature.

The situation changes if the effective metric has Euclidian signature. This seems to be the case for the light-like surfaces assignable to the known extremals such as MEs and cosmic strings. In this kind of situation light-like coordinate possesses Euclidian signature and real eigenvalue spectrum is replaced with a purely imaginary one. Since Dirac operator is in question both signs for eigenvalues are possible and one obtains both exponentially increasing and decreasing solutions. This is essential for having solutions extending from the past end of X_l^3 to its future end. Non-unitary time evolution is possible because X_l^3 does not strictly speaking represent the time evolution of 2-D dynamical object but actual dynamical objects (by light-likeness both interpretation as dynamical evolution and dynamical object are present). The Euclidian signature of the effective metric would be a direct analog for the tachyonicity of the Higgs in unstable minimum and the generation of Higgs vacuum expectation would correspond to the compensation of ground state conformal weight by conformal weights of Super Virasoro generators.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = \lambda_i^2 = -1/2 - n + \Delta h_c$ so that lowest ground state conformal weight would be $h_c = -1/2$ in the first approximation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

6.6 General Mass Formula For Ramond Representations

By taking the modular contribution from the boundaries into account the general p-adic mass formulas for the Ramond type states read for states for which the color contribution to the conformal weight is integer valued as

$$\begin{aligned}
\frac{m^2(\Delta = 0)}{m_0^2} &= (8 + n(g))p + Yp^2 \ , \\
\frac{m^2(\Delta = 1)}{m_0^2} &= (5 + n(g)p + Yp^2 \ , \\
\frac{m^2(\Delta = 2)}{m_0^2} &= (4 + n(g))p + (Y + \frac{23}{2})p^2 \ , \\
n(g) &= 3g \cdot 2^{g-1}(2^g + 1) \ .
\end{aligned} \tag{6.8}$$

Here Δ denotes the conformal weight of the operators creating massless states from the ground state and g denotes the genus of the boundary component. The values of $n(g)$ for the three lowest generations are $n(0) = 0$, $n(1) = 9$ and $n(2) = 60$. The value of second order thermal contribution is nontrivial for neutrinos only. The value of the rational number Y can, which corresponds to the renormalization correction to the mass, can be determined using experimental inputs.

Using m_0^2 as a unit, the expression for the mass of a Ramond type state reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p)_R &= K(\Delta, g, p)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0, g, p) &= \sqrt{\frac{n(g) + 8 + Y_R}{X}} \\
K(1, g, p) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \\
K(2, g, p) &= \sqrt{\frac{n(g) + 4 + Y_R}{X}} \quad , \\
X &= \sqrt{5 + Y(e)_R} \ .
\end{aligned} \tag{6.9}
$$

Y can be assumed to depend on the electromagnetic charge and color representation of the state and is therefore same for all fermion families. Mathematica provides modules for calculating the real counterpart of the second order contribution and for finding realistic values of Y.

6.7 General Mass Formulas For NS Representations

Using $m_0^2/3$ as a unit, the expression for the mass of a light NS type state for $T_p = 1$ ad $k_B = 1$ reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p, N)_R &= K(\Delta, g, p, N)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0, g, p, 1) &= \sqrt{\frac{n(g) + Y_R}{X}} \quad , \\
K(0, g, p, 2) &= \sqrt{\frac{n(g) + 1 + Y_R}{X}} \quad , \\
K(1, g, p, 3) &= \sqrt{\frac{n(g) + 3 + Y_R}{X}} \quad , \\
K(2, g, p, 4) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \quad , \\
K(2, g, p, 5) &= \sqrt{\frac{n(g) + 10 + Y_R}{X}} \quad , \\
X &= \sqrt{5 + Y(e)_R} \ .
\end{aligned} \tag{6.10}
$$

Here N is the number of the "active" NS sectors (sectors for which the conformal weight of the massless state is non-vanishing). Y denotes the renormalization correction to the boson mass and in general depends on the electro-weak and color quantum numbers of the boson.

The thermal contribution to the mass of W boson is too large by roughly a factor $\sqrt{3}$ for $T_p = 1$. Hence $T_p = 1/2$ must hold true for gauge bosons and their masses must have a non-thermal origin perhaps analogous to Higgs mechanism. Alternatively, the non-covariant constancy of charge matrices could induce the boson mass [K8] .

It is interesting to notice that the minimum mass squared for gauge boson corresponds to the p-adic mass unit $M^2 = m_0^2 p/3$ and this just what is needed in the case of W boson. This forces to ask whether $m_0^2/3$ is the correct choice for the mass squared unit so that non-thermally induced W mass would be the minimal $m_W^2 = p$ in the lowest order. This choice would mean the replacement

$$Y_R \rightarrow \frac{(3Y)_R}{3}$$

in the preceding formulas and would affect only neutrino mass in the fermionic sector. $m_0^2/3$ option is excluded by charged lepton mass calculation. This point will be discussed later.

6.8 Primary Condensation Levels From P-Adic Length ScaleHypothesis

p-Adic length scale hypothesis states that the primary condensation levels correspond to primes near prime powers of two $p \simeq 2^k$, k integer with prime values preferred. Black hole-elementary particle analogy [K13] suggests a generalization of this hypothesis by allowing k to be a power of prime. The general number theoretical vision discussed in [K17] provides a first principle justification for p-adic length scale hypothesis in its most general form. The best fit for the neutrino mass squared differences is obtained for $k = 13^2 = 169$ so that the generalization of the hypothesis might be necessary.

A particle primarily condensed on the level k can suffer secondary condensation on a level with the same value of k: for instance, electron ($k = 127$) suffers secondary condensation on $k = 127$ level. u, d, s quarks ($k = 107$) suffer secondary condensation on nuclear space-time sheet having $k = 113$). All quarks feed their color gauge fluxes at $k = 107$ space-time sheet. There is no deep reason forbidding the condensation of p on p. Primary and secondary condensation levels could also correspond to different but nearly identical values of p with the same value of k.

7 Fermion Masses

In the earlier model the coefficient of $M^2 = kL_0$ had to be assumed to be different for various particle states. $k = 1$ was assumed for bosons and leptons and $k = 2/3$ for quarks. The fact that $k = 1$ holds true for all particles in the model including also super-symplectic invariance forces to modify the earlier construction of quark states. This turns out to be possible without affecting the earlier p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (h_{gr} can be negative). The structure of lepton and quark states in color degrees of freedom was discussed in [K8] .

7.1 Charged Lepton Mass Ratios

The overall mass scale for lepton and quark masses is determined by the condensation level given by prime $p \simeq 2^k$, k prime by length scale hypothesis. For charged leptons k must correspond to $k = 127$ for electron, $k = 113$ for muon and $k = 107$ for τ. For muon $p = 2^{113} - 1 - 4*378$ is assumed (smallest prime below 2^{113} allowing $\sqrt{2}$ but not $\sqrt{3}$). So called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne so that all charged leptons correspond to generalized Mersenne primes.

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod \ 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p + 3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2 + 2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Using CP_2 mass scale m_0^2 [K8] as a p-adic unit, the mass formulas for the charged leptons read as

$$\begin{aligned} M^2(L) &= A(\nu)\frac{m_0^2}{p(L)} \ , \\ A(e) &= 5 + X(p(e)) \ , \\ A(\mu) &= 14 + X(p(\mu)) \ , \\ A(\tau) &= 65 + X(p(\tau)) \ . \end{aligned} \tag{7.1}$$

$X(\cdot)$ corresponds to the yet unknown second order corrections to the mass squared.

Table 4 gives the basic parameters as determined from the mass of electron for some values of Y_e. The mass of top quark favors as maximal value of CP_2 mass which corresponds to $Y_e = 0$.

Y_e	0	.5	.7798
$(m_0/m_{Pl}) \times 10^3$	.2437	.2323	.2266
$K \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954

Table 4: Table gives the values of CP_2 mass m_0 using Planck mass $m_{Pl} = 1/\sqrt{G}$ as unit, the ratio $K = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$.

Table 5 lists the lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value. The values of lepton masses are $m_e = .510999$ MeV, $m_\mu = 105.76583$ MeV, $m_\tau = 1775$ MeV.

$$\begin{aligned} \frac{m(\mu)_+}{m(\mu)} &= \sqrt{\frac{15}{5}2^7\frac{m_e}{(\mu)}} \simeq 1.0722 \ , \\ \frac{m(\mu)_-}{m(\mu)} &= \sqrt{\frac{14}{6}2^7\frac{m_e}{m(\mu)}} \simeq 0.9456 \ , \\ \frac{m(\tau)_+}{m(\tau)} &= \sqrt{\frac{66}{5}2^{10}\frac{m_e}{m(\tau)}} \simeq 1.0710 \ , \\ \frac{m(\tau)_-}{m(\tau)} &= \sqrt{\frac{65}{6}2^{10}\frac{m_e}{m(\tau)}} \simeq .9703 \ . \end{aligned} \tag{7.2}$$

Table 5: Lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value.

For the maximal value of CP_2 mass the predictions for the mass ratio are systematically too large by a few per cent. From the formulas above it is clear that the second order corrections to mass squared can be such that correct masses result.

τ mass is least sensitive to $X(p(e)) \equiv Y_e$ and the maximum value of $Y_e \equiv Y_{e,max}$ consistent with τ mass corresponds to $Y_{e,max} = .7357$ and $Y_\tau = 1$. This means that the CP_2 mass is at least a fraction .9337 of its maximal value. If Y_L is same for all charged leptons and has the maximal value $Y_{e,max} = .7357$, the predictions for the mass ratios are

$$\begin{aligned} \frac{m(\mu)_{pr}}{m(\mu)} &= \sqrt{\frac{14+Y_{e,max}}{5+Y_{e,max}}} \times 2^7\frac{m_e}{m(\mu)} \simeq .9922 \ , \\ \frac{m(\tau)_{pr}}{m(\tau)} &= \sqrt{\frac{65+Y_{e,max}}{5+Y_{e,max}}} \times 2^{10}\frac{m_e}{m(\tau)} \simeq .9980 \ . \end{aligned} \tag{7.3}$$

The error is .8 per cent *resp.* .2 per cent for muon *resp.* τ.

The argument leading to estimate for the modular contribution to the mass squared [K8] leaves two options for the coefficient of the modular contribution for $g = 2$ fermions: the value of coefficient is either $X = g$ for $g \leq 1$, $X = 3g - 3$ for $g \geq 2$ or $X = g$ always. For $g = 2$ the predictions are $X = 2$ and $X = 3$ in the two cases. The option $X = 3$ allows slightly larger maximal value of Y_e equal to $Y^{1)}_{e,max} = Y_{e,max} + (5 + Y_{e,max})/66$.

7.2 Neutrino Masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible and neutrino mixing cannot yet be predicted from the basic principles. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the LSND data on neutrino mass measurement of [C22, C10] and the explanation of the atmospheric ν-deficit in terms of $\nu_\mu - \nu_\tau$ mixing [C13, C11] one can deduce that the most plausible condensation level of μ and τ neutrinos is $k = 167$ or $k = 13^2 = 169$ allowed by the more general form of the p-adic length scale hypothesis suggested by the blackhole-elementary particle analogy. One can also deduce information about the mixing matrix associated with the neutrinos so that mass predictions become rather precise. In particular, the mass splitting of μ and τ neutrinos is predicted correctly if one assumes that the mixing matrix is a rational unitary matrix.

7.2.1 *Super Virasoro contribution*

Using $m_0^2/3$ as a p-adic unit, the expression for the Super Virasoro contribution to the mass squared of neutrinos is given by the formula

$$\begin{aligned} M^2(SV) &= (s + (3Yp)_R/3)\frac{m_0^2}{p} \ , \\ s &= 4 \text{ or } 5 \ , \\ Y &= \frac{23}{2} + Y_1 \ , \end{aligned} \tag{7.4}$$

where m_0^2 is universal mass scale. One can consider two possible identifications of neutrinos corresponding to $s(\nu) = 4$ with $\Delta = 2$ and $s(\nu) = 5$ with $\Delta = 1$. The requirement that CKM matrix is sensible forces the asymmetric scenario in which quarks and, by symmetry, also leptons correspond to lowest possible excitation so that one must have $s(\nu) = 4$. Y_1 represents second order contribution to the neutrino mass coming from renormalization effects coming from self energy diagrams involving intermediate gauge bosons. Physical intuition suggest that this contribution is very small so that the precise measurement of the neutrino masses should give an excellent test for the theory.

With the above described assumptions and for $s = 4$, one has the following mass formula for neutrinos

$$\begin{aligned} M^2(\nu) &= A(\nu)\frac{m_0^2}{p(\nu))} \ , \\ A(\nu_e) &= 4 + \frac{(3Y(p(\nu_e)))_R}{3} \ , \\ A(\nu_\mu) &= 13 + \frac{(3Y(p(\nu_\mu)))_R}{3} \ , \\ A(\nu_\tau) &= 64 + \frac{(3Y(p(\nu_\tau)))_R}{3} \ , \\ 3Y &\simeq \frac{1}{2} \ . \end{aligned} \tag{7.5}$$

The predictions must be consistent with the recent upper bounds [C7]
of order 10 eV, 270 keV and 0.3 MeV for ν_e, ν_μ and ν_τ respectively. The recently reported results

of LSND measurement [C10] for $\nu_e - > \nu_\mu$ mixing gives string limits for $\Delta m^2(\nu_e, \nu_\mu)$ and the parameter $sin^2(2\theta)$ characterizing the mixing: the limits are given in the figure 30 of [C10]. The results suggests that the masses of both electron and muon neutrinos are below 5 eV and that mass squared difference $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$ is between $.25 - 25$ eV^2. The simplest possibility is that ν_μ and ν_e have common condensation level (in analogy with d and s quarks). There are three candidates for the primary condensation level: namely $k = 163, 167$ and $k = 169$. The p-adic prime associated with the primary condensation level is assumed to be the nearest prime below 2^k allowing p-adic $\sqrt{2}$ but not $\sqrt{3}$ and satisfying $p \ mod \ 4 = 3$. The **Table 6** gives the values of various parameters and unmixed neutrino masses in various cases of interest.

k	p	$(3Y)_R/3$	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau)/eV$
163	$2^{163} - 4*144 - 1$	1.36	1.78	3.16	6.98
167	$2^{167} - 4*144 - 1$	.34	.45	.79	1.75
169	$2^{169} - 4*210 - 1$	.17	.22	.40	.87

Table 6: The values of various parameters and unmixed neutrino masses in various cases of interest.

7.2.2 *Could neutrino topologically condense also in other p-adic length scales than $k = 169$?*

One must keep mind open for the possibility that there are several p-adic length scales at which neutrinos can condense topologically. Biological length scales are especially interesting in this respect. In fact, all intermediate p-adic length scales $k = 151, 157, 163, 167$ could correspond to metastable neutrino states. The point is that these p-adic lengths scales are number theoretically completely exceptional in the sense that there exist Gaussian Mersenne $2^k \pm i$ (prime in the ring of complex integers) for all these values of k. Since charged leptons, atomic nuclei ($k = 113$) , hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes, it would not be surprising if the biologically important Gaussian Mersennes would correspond to length scales giving rise to metastable neutrino states. Of course, one can keep mind open for the possibility that $k = 167$ rather than $k = 13^2 = 169$ is the length scale defining the stable neutrino physics.

7.2.3 *Neutrino mixing*

Consider next the neutrino mixing. A quite general form of the neutrino mixing matrix D given by **Table 7** will be considered.

	ν_e	ν_μ	ν_τ
ν_e	c_1	s_1c_3	s_1s_3
ν_μ	$-s_1c_2$	$c_1c_2c_3 - s_2s_3exp(i\delta)$	$c_1c_2s_3 + s_2c_3exp(i\delta)$
ν_τ	$-s_1s_2$	$c_1s_2c_3 + c_2s_3exp(i\delta)$	$c_1s_2s_3 - c_2c_3exp(i\delta)$

Table 7: General form of neutrino mixing matrix.

Physical intuition suggests that the angle δ related to CP breaking is small and will be assumed to be vanishing. Topological mixing is active only in modular degrees of freedom and one obtains for the first order terms of mixed masses the expressions

$$\begin{aligned} s(\nu_e) &= 4 + 9|U_{12}|^2 + 60|U_{13}|^2 = 4 + n_1 \ , \\ s(\nu_\mu) &= 4 + 9|U_{22}|^2 + 60|U_{23}|^2 = 4 + n_2 \ , \\ s(\nu_\tau) &= 4 + 9|U_{32}|^2 + 60|U_{33}|^2 = 4 + n_3 \ . \end{aligned} \tag{7.6}$$

The requirement that resulting masses are not ultra heavy implies that $s(\nu)$ must be small integers. The condition $n_1+n_2+n_3=69$ follows from unitarity. The simplest possibility is that the mixing matrix is a rational unitary matrix. The same ansatz was used successfully to deduce information about the mixing matrices of quarks. If neutrinos are condensed on the same condensation level, rationality implies that $\nu_\mu-\nu_\tau$ mass squared difference must come from the first order contribution to the mass squared and is therefore quantized and bounded from below.

The first piece of information is the atmospheric ν_μ/ν_e ratio, which is roughly by a factor 2 smaller than predicted by standard model [C13]. A possible explanation is the CKM mixing of muon neutrino with τ-neutrino, whereas the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [C13] are in accordance with the mixing $m^2(\nu_\tau)-m^2(\nu_\mu)\simeq 1.6\cdot 10^{-2}\ eV^2$ and mixing angle $sin^2(2\theta)=1.0$: also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If mixing matrix is assumed to be rational then only $k=169$ condensation level is allowed for ν_μ and ν_τ. For this level $\nu_\mu-\nu_\tau$ mass squared difference turns out to be $\Delta m^2\simeq 10^{-2}\ eV^2$ for $\Delta s\equiv s(\nu_\tau)-s(\nu_\mu)=1$, which is the only acceptable possibility and predicts $\nu_\mu-\nu_\tau$ mass squared difference correctly within experimental uncertainties! The fact that the predictions for mass squared differences are practically exact, provides a precision test for the rationality assumption.

What is measured in LSND experiment is the probability $P(t,E)$ that ν_μ transforms to ν_e in time t after its production in muon decay as a function of energy E of ν_μ. In the limit that ν_τ and ν_μ masses are identical, the expression of $P(t,E)$ is given by

$$\begin{aligned} P(t,E) &= sin^2(2\theta)sin^2(\frac{\Delta Et}{2}) \ , \\ sin^2(2\theta) &= 4c_1^2s_1^2c_2^2 \ , \end{aligned} \tag{7.7}$$

where ΔE is energy difference of ν_μ and ν_e neutrinos and t denotes time. LSND experiment gives stringent conditions on the value of $\sin^2(2\theta)$ as the figure 30 of [C10] shows. In particular, it seems that $sin^2(2\theta)$ must be considerably below 10^{-1} and this implies that s_1^2 must be small enough.

The study of the mass formulas shows that the only possibility to satisfy the constraints for the mass squared and $sin^2(2\theta)$ given by LSND experiment is to assume that the mixing of the electron neutrino with the tau neutrino is much larger than its mixing with the muon neutrino. This means that s_3 is quite near to unity. At the limit $s_3=1$ one obtains the following (nonrational) solution of the mass squared conditions for $n_3=n_2+1$ (forced by the atmospheric neutrino data)

$$\begin{aligned} s_1^2 &= \frac{69-2n_2-1}{60} \ , \\ c_2^2 &= \frac{n_2-9}{2n_2-17} \ , \\ sin^2(2\theta) &= \frac{4(n_2-9)}{51}\frac{(34-n_2)(n_2-4)}{30^2} \ , \\ s(\nu_\mu)-s(\nu_e) &= 3n_2-68 \ . \end{aligned} \tag{7.8}$$

The study of the LSND data shows that there is only one acceptable solution to the conditions obtained by assuming maximal mass squared difference for ν_e and ν_μ

$$\begin{aligned} n_1 &= 2 \ \ n_2=33 \ \ n_3=34 \ , \\ s_1^2 &= \frac{1}{30} \ \ c_2^2=\frac{24}{49} \ , \\ sin^2(2\theta) &= \frac{24}{49}\frac{2}{15}\frac{29}{30}\simeq .0631 \ , \\ s(\nu_\mu)-s(\nu_e)) &= 31\leftrightarrow .32\ eV^2 \ . \end{aligned} \tag{7.9}$$

That c_2^2 is near 1/2 is not surprise taking into account the almost mass degeneracy of ν_{mu} and ν_τ. From the figure 30 of [C10] it is clear that this solution belongs to 90 per cent likelihood region of LSND experiment but $sin^2(2\theta)$ is about two times larger than the value allowed by Bugey reactor experiment. The study of various constraints given in [C10] shows that the solution is consistent with bounds from all other experiments. If one assumes that $k > 169$ for ν_e $\nu_\mu - \nu_e$ mass difference increases, implying slightly poorer consistency with LSND data.

There are reasons to hope that the actual rational solution can be regarded as a small deformation of this solution obtained by assuming that c_3 is non-vanishing. $s_1^2 = \frac{69-2n_2-1}{60-51c_3^2}$ increases in the deformation by $O(c_3^2)$ term but if c_3 is positive the value of $c_2^2 \simeq \frac{24-102c_1^0c_2^0s_2^0c_3}{49} \sim \frac{24-61c_3}{49}$ decreases by $O(c_3)$ term so that it should be possible to reduce the value of $sin^2(2\theta)$. Consistency with Bugey reactor experiment requires $.030 \leq sin^2(2\theta) < .033$. $sin^2(2\theta) = .032$ is achieved for $s_1^2 \simeq .035$,$s_2^2 \simeq .51$ and $c_3^2 \simeq .068$. The construction of U and D matrices for quarks shows that very stringent number theoretic conditions are obtained and as in case of quarks it might be necessary to allow complex CP breaking phase in the mixing matrix. One might even hope that the solution to the conditions is unique.

For the minimal rational mixing one has $s(\nu_e) = 5$, $s(\nu_\mu) = 36$ and $s(\nu_\tau) = 37$ if unmixed ν_e corresponds to $s = 4$. For $s = 5$ first order contributions are shifted by one unit. The masses ($s = 4$ case) and mass squared differences are given by **Table 8**.

k	$m(\nu_e)$	$m(\nu_\mu)$	$m(\nu_\tau)$	$\Delta m^2(\nu_\mu - \nu_e)$	$\Delta m^2(\nu_\tau - \nu_\mu)$
169	.27 eV	.66 eV	.67 eV	.32 eV^2	.01 eV^2

Table 8: Mass squared differences for neutrinos.

Predictions for neutrino masses and mass squared splittings for $k = 169$ case.

7.2.4 *Evidence for the dynamical mass scale of neutrinos*

In recent years (I am writing this towards the end of year 2004 and much later than previous lines) a great progress has been made in the understanding of neutrino masses and neutrino mixing. The pleasant news from TGD perspective is that there is a strong evidence that neutrino masses depend on environment [C16]. In TGD framework this translates to the statement that neutrinos can suffer topological condensation in several p-adic length scales. Not only in the p-adic length scales suggested by the number theoretical considerations but also in longer length scales, as will be found.

The experiments giving information about mass squared differences can be divided into three categories [C16].

1. There along baseline experiments, which include solar neutrino experiments [C9, C14, C15] and [C18] as well as earlier studies of solar neutrinos. These experiments see evidence for the neutrino mixing and involve significant propagation through dense matter. For the solar neutrinos and KamLAND the mass splittings are estimated to be of order $O(8 \times 10^{-5})$ eV^2 or more cautiously 8×10^{-5} eV$^2 < \delta m^2 < 2 \times 10^{-3}$ eV^2. For K2K and atmospheric neutrinos the mass splittings are of order $O(2 \times 10^{-3})eV^2$ or more cautiously $\delta m^2 > 10^{-3}$eV^2. Thus the scale of mass splitting seems to be smaller for neutrinos in matter than in air, which would suggest that neutrinos able to propagate through a dense matter travel at space-time sheets corresponding to a larger p-adic length scale than in air.

2. There are null short baseline experiments including CHOOZ, Bugey, and Palo Verde reactor experiments, and the higher energy CDHS, JARME, CHORUS, and NOMAD experiments, which involve muonic neutrinos (for references see [C16]. No evidence for neutrino oscillations have been seen in these experiments.

3. The results of LSND experiment [C10] are consistent with oscillations with a mass splitting greater than $3 \times 10^{-2}eV^2$. LSND has been generally been interpreted as necessitating a mixing with sterile neutrino. If neutrino mass scale is dynamical, situation however changes.

If one assumes that the p-adic length scale for the space-time sheets at which neutrinos can propagate is different for matter and air, the situation changes. According to [C16] a mass 3×10^{-2} eV in air could explain the atmospheric results whereas mass of of order .1 eV and $.07eV^2 < \delta m^2 <$

$.26eV^2$ would explain the LSND result. These limits are of the same order as the order of magnitude predicted by $k = 169$ topological condensation.

Assuming that the scale of the mass splitting is proportional to the p-adic mass scale squared, one can consider candidates for the topological condensation levels involved.

1. Suppose that $k = 169 = 13^2$ is indeed the condensation level for LSND neutrinos. $k = 173$ would predict $m_{\nu_e} \sim 7 \times 10^{-2}$ eV and $\delta m^2 \sim .02$ eV^2. This could correspond to the masses of neutrinos propagating through air. For $k = 179$ one has $m_{\nu_e} \sim .8 \times 10^{-2}$ eV and $\delta m^2 \sim 3 \times 10^{-4}$ eV^2 which could be associated with solar neutrinos and KamLAND neutrinos.

2. The primes $k = 157, 163, 167$ associated with Gaussian Mersennes would give $\delta m^2(157) = 2^6 \delta m^2(163) = 2^{10} \delta m^2(167) = 2^{12} \delta m^2(169)$ and mass scales $m(157) \sim 22.8$ eV, $m(163) \sim 3.6$ eV, $m(167) \sim .54$ eV. These mass scales are unrealistic or propagating neutrinos. The interpretation consistent with TGD inspired model of condensed matter in which neutrinos screen the classical Z^0 force generated by nucleons would be that condensed matter neutrinos are confined inside these space-time sheets whereas the neutrinos able to propagate through condensed matter travel along $k > 167$ space-time sheets.

7.2.5 *The results of MiniBooNE group as a support for the energy dependence of p-adic mass scale of neutrino*

The basic prediction of TGD is that neutrino mass scale can depend on neutrino energy and the experimental determinations of neutrino mixing parameters support this prediction. The newest results (11 April 2007) about neutrino oscillations come from MiniBooNE group which has published its first findings [C6] concerning neutrino oscillations in the mass range studied in LSND experiments [C5].

1. The motivation for MiniBooNE

Neutrino oscillations are not well-understood. Three experiments LSND, atmospheric neutrinos, and solar neutrinos show oscillations but in widely different mass regions (1 eV^2 , 3×10^{-3} eV^2, and 8×10^{-5} eV^2).

In TGD framework the explanation would be that neutrinos can appear in several p-adically scaled up variants with different mass scales and therefore different scales for the differences Δm^2 for neutrino masses so that one should not try to try to explain the results of these experiments using single neutrino mass scale. In single-sheeted space-time it is very difficult to imagine that neutrino mass scale would depend on neutrino energy since neutrinos interact so extremely weakly with matter. The best known attempt to assign single mass to all neutrinos has been based on the use of so called sterile neutrinos which do not have electro-weak couplings. This approach is an ad hoc trick and rather ugly mathematically and excluded by the results of MiniBooNE experiments.

2. The result of MiniBooNE experiment

The purpose of the MiniBooNE experiment was to check whether LSND result $\Delta m^2 = 1eV^2$ is genuine. The group used muon neutrino beam and looked whether the transformations of muonic neutrinos to electron neutrinos occur in the mass squared region $\Delta m^2 \simeq 1$ eV^2. No such transitions were found but there was evidence for transformations at low neutrino energies.

What looks first as an over-diplomatic formulation of the result was *MiniBooNE researchers showed conclusively that the LSND results could not be due to simple neutrino oscillation, a phenomenon in which one type of neutrino transforms into another type and back again.* rather than direct refutation of LSND results.

3. LSND and MiniBooNE are consistent in TGD Universe

The habitant of the many-sheeted space-time would not regard the previous statement as a mere diplomatic use of language. It is quite possible that neutrinos studied in MiniBooNE have suffered topological condensation at different space-time sheet than those in LSND if they are in different energy range (the preferred rest system fixed by the space-time sheet of the laboratory or Earth). To see whether this is the case let us look more carefully the experimental arrangements.

1. In LSND experiment 800 MeV proton beam entering in water target and the muon neutrinos resulted in the decay of produced pions. Muonic neutrinos had energies in 60-200 MeV range [C5].

2. In MiniBooNE experiment [C6] 8 GeV muon beam entered Beryllium target and muon neutrinos resulted in the decay of resulting pions and kaons. The resulting muonic neutrinos had energies the range 300-1500 GeV to be compared with 60-200 MeV.

Let us try to make this more explicit.

1. Neutrino energy ranges are quite different so that the experiments need not be directly comparable. The mixing obeys the analog of Schrödinger equation for free particle with energy replaced with $\Delta m^2/E$, where E is neutrino energy. The mixing probability as a function of distance L from the source of muon neutrinos is in 2-component model given by

$$P = sin^2(\theta)sin^2(1.27\Delta m^2 L/E) \ .$$

The characteristic length scale for mixing is $L = E/\Delta m^2$. If L is sufficiently small, the mixing is fifty-fifty already before the muon neutrinos enter the system, where the measurement is carried out and no mixing is detected. If L is considerably longer than the size of the measuring system, no mixing is observed either. Therefore the result can be understood if Δm^2 is much larger or much smaller than E/L, where L is the size of the measuring system and E is the typical neutrino energy.

2. MiniBooNE experiment found evidence for the appearance of electron neutrinos at low neutrino energies (below 500 MeV) which means direct support for the LSND findings and for the dependence of neutron mass scale on its energy relative to the rest system defined by the space-time sheet of laboratory.

3. Uncertainty Principle inspires the guess $L_p \propto 1/E$ implying $m_p \propto E$. Here E is the energy of the neutrino with respect to the rest system defined by the space-time sheet of the laboratory. Solar neutrinos indeed have the lowest energy (below 20 MeV) and the lowest value of Δm^2. However, atmospheric neutrinos have energies starting from few hundreds of MeV and $\Delta; m^2$ is by a factor of order 10 higher. This suggests that the growth of Δm^2 with E^2 is slower than linear. It is perhaps not the energy alone which matters but the space-time sheet at which neutrinos topologically condense. For instance, MiniBooNE neutrinos above 500 MeV would topologically condense at space-time sheets for which the p-adic mass scale is higher than in LSND experiments and one would have $\Delta m^2 >> 1$ eV^2 implying maximal mixing in length scale much shorter than the size of experimental apparatus.

4. One could also argue that topological condensation occurs in condensed matter and that no topological condensation occurs for high enough neutrino energies so that neutrinos remain massless. One can even consider the possibility that the p-adic length scale L_p is proportional to E/m_0^2, where m_0 is proportional to the mass scale associated with non-relativistic neutrinos. The p-adic mass scale would obey $m_p \propto m_0^2/E$ so that the characteristic mixing length would be by a factor of order 100 longer in MiniBooNE experiment than in LSND.

7.2.6 *Comments*

Some comments on the proposed scenario are in order: some of the are written much later than the previous text.

1. Mass predictions are consistent with the bound $\Delta m(\nu_\mu, \nu_e) < 2\ eV^2$ coming from the requirement that neutrino mixing does not spoil the so called r-process producing heavy elements in Super Novae [C17].

2. TGD neutrinos cannot solve the dark matter problem: the total neutrino mass required by the cold+hot dark matter models would be about 5 eV. In [K4] a model of galaxies based on string like objects of galaxy size and providing a more exotic source of dark matter, is discussed.

3. One could also consider the explanation of LSND data in terms of the interaction of ν_μ and nucleon via the exchange of $g=1$ W boson. The fraction of the reactions $\bar{\nu}_\mu + p \rightarrow e^+ + n$ is at low neutrino energies $P \sim \frac{m_W^4(g=0)}{m_W^4(g=1)} sin^2(\theta_c)$, where θ_c denotes Cabibbo angle. Even if the condensation level of $W(g=1)$ is $k=89$, the ratio is by a factor of order .05 too small to explain the average $\nu_\mu \rightarrow \nu_e$ transformation probability $P \simeq .003$ extracted from LSND data.

4. The predicted masses exclude MSW and vacuum oscillation solutions to the solar neutrino problem unless one assumes that several condensation levels and thus mass scales are possible for neutrinos. This is indeed suggested by the previous considerations.

7.3 Quark Masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

7.3.1 *Basic mass formulas*

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p,p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p=1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-symplectic operators O with a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

Consider now the mass squared values for quarks. For $h(D) = 0$ and $h(U) = -1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D) = 8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{7.10}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark. When m_0^2 is used as a unit X is replaced by $X = (Y_p)_R$.

With the above described assumptions one has the following mass formula for quarks

$$\begin{aligned} &M^2(q) = A(q)\frac{m_0^2}{p(q)} \ , \\ &A(u) = 5 + X_U(p(u) \ , \quad A(c) = 14 + X_U(p(c)) \ , \quad A(t) = 65 + X_U(p(t)) \ , \\ &A(d) = 8 + X_D(p(d)) \ , \quad A(s) = 17 + X_D(p(s)) \ , \quad A(b) = 68 + X_D(p(b)) \ . \end{aligned} \tag{7.11}$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulomb energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$. The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$(n_d, n_s, n_b) = (5, 5, 59) \quad , \quad (n_u, n_c, n_t) = (5, 6, 58) \quad . \tag{7.12}$$

   fixing completely the quark masses apart possible Higgs contribution [K12] . Note that top quark mass is still rather near to its maximal value.

2. The constraint that valence quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. $u - d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

7.3.2 *The p-adic length scales associated with quarks and quark masses*

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model [K12]) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\bar{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.

2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L_e(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *allb* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux fed to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is fed to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

   One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be fed at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

2. The requirement that the masses of diagonal pseudo-scalar mesons of type $M = q\bar{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.

   Top quark mass tends to be slightly too low as compared to the most recent experimental value of $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [C19] . The optimal situation corresponds to $Y_e = 0$ and $Y_t = 1$ and happens to give top mass exactly equal to the most probable experimental value. It must be emphasized that top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top.

In the case of light quarks there are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula [K12] .

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. **Table 10** summarizes constituent quark masses as predicted by this model.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [**?**]ives the value ranges for current quark masses depicted in **Table 9** together with TGD predictions for the spectrum of current quark masses.

Some comments are in order.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8×10^3

Table 9: The experimental value ranges for current quark masses [**?**]nd TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, and $Y_e = 0$. For top quark $Y_t = 0$ is assumed. $Y_t = 1$ would give 169.2 GeV.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.

2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4 \times 31, 3 \times 41, 2 \times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2 \times 61, 11^2, 3 \times 5 \times 8)$.

3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [K15, L1] , [L1] .

4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [C1] but not that of b.

5. The mass of top quark has been slightly below the experimental estimate for long time. The experimental value has been coming down slowly and the most recent value obtained by CDF [C20] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV and consistent with the TGD prediction for $Y_e = Y_t = 0$.

One can talk about constituent and current quark masses simultaneously only if they correspond to dual descriptions. $M^8 - H$ duality [K8] has been indeed suggested to relate the old fashioned low energy description of hadrons in terms of $SO(4)$ symmetry (Skyrme model) and higher energy description of hadrons based on QCD. In QCD description the mass of say baryon would be dominated by the mass associated with super-symplectic quanta carrying color. In $SO(4)$ description constituent quarks would carry most of the hadron mass.

7.3.3 *Can Higgs field develop a vacuum expectation in fermionic sector at all?*

An important conclusion following from the calculation of lepton and quark masses is that if Higgs contribution is present, it can be of second order p-adically and even negligible, perhaps even vanishing. There is indeed an argument forcing to consider this possibility seriously. The recent view about elementary particles is following.

1. Fermions correspond to CP_2 type vacuum extremals topologically condensed at positive/negative energy space-time sheets carrying quantum numbers at light-like wormhole throat. Higgs and gauge bosons correspond to wormhole contacts connecting positive and negative energy space-time sheets and carrying fermion and anti-fermion quantum numbers at the two light-like wormhole throats.

2. If the values of p-adic temperature are $T_p = 1$ and $T_p = 1/n$, $n > 1$f or fermions and bosons the thermodynamical contribution to the gauge boson mass is negligible.

3. Different p-adic temperatures and Kähler coupling strengths for fermions and bosons make sense if bosonic and fermionic partonic 3-surfaces meet only along their ends at the vertices of generalized Feynman diagrams but have no other common points [K2] . This forces to consider the possibility that fermions cannot develop Higgs vacuum expectation value although they can couple to Higgs. This is not in contradiction with the modification of sigma model of hadrons based on the assumption that vacuum expectation of σ field gives a small contribution to hadron mass [K9] since this field can be assigned to some bosonic space-time sheet pair associated with hadron.

4. Perhaps the most elegant interpretation is that ground state conformal is equal to the square of the eigenvalue of the modified Dirac operator. The ground state conformal weight is negative and its deviation from half odd integer value gives contribution to both fermion and boson masses. The Higgs expectation associated with coherent state of Higgs like wormhole contacts is naturally proportional to this parameter since no other parameter with dimensions of mass is present. Higgs vacuum expectation determines gauge boson masses only apparently if this interpretation is correct. The contribution of the ground state conformal weight to fermion mass square is near to zero. This means that λ is very near to negative half odd integer and therefore no significant difference between fermions and gauge bosons is implied.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 10: Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

8 About The Microscopic Description Of Gauge Boson Massivation

The conjectured QFT limit allows to estimate the quantitative predictions of the theory. This is not however enough. One should identify the microscopic TGD counterparts for various aspects of gauge boson massivation. There is also the question about the consistency of the gauge theory limit with the ZEO inspired view about massivation. The basic challenge are obvious: one should translate notions like Higgs vacuum expectation, massivation of gauge bosons, and finite range of weak interactions to the language of wormhole throats, Kähler magnetic flux tubes, and string world sheets. The proposal is that generalization of super-conformal symmetries to their Yangian counterparts is needed to meet this challenge in mathematically satisfactory manner.

8.1 Can P-Adic Thermodynamics Explain The Masses Of Intermediate Gauge Bosons?

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \ .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Contrary to the original optimistic beliefs based on calculational error, it turned out impossible to predict W/e and Z/e mass ratios correctly in the original p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they seemed to exclude the explanation for the massivation of gauge bosons using p-adic thermodynamics.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

2. Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. For $T_p = 1/2$ thermal masses are completely negligible.

3. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$.

The conclusion is that p-adic thermodynamics does not produce a natural description for the massivation of weak bosons. For $p = M_{89}$ the mass scale is somewhat too small even if the second order contribution is maximal. If it is characterized by small integer, the contribution is extremely small. An explanation for the value of Weinberg angle is also missing. Hence some additional contribution to mass must be present. Higgsy contribution is not natural in TGD framework but stringy contribution looks very natural.

8.2 The Counterpart Of Higgs Vacuum Expectation In TGD

The development of the TGD view about Higgs involves several wrong tracks involving a lot of useless calculation. All this could have been avoided with more precise definition of basic notions. The following view has distilled through several failures and might be taken as starting point.

The basic challenge is to translate the QFT description of gauge boson massivation to microscopic description.

1. One can say that gauge bosons "eat" the components of Higgs. In unitary gauge one gauge rotates Higgs field to electromagnetically neutral direction defined by the vacuum expectation value of Higgs. The rotation matrix codes for the degrees of freedom assignable to non-neutral part of Higgs and they are transferred to the longitudinal components of Higgs in gauge transformation. This gives rise to the third polarization direction for gauge boson. Photon remains massless because em charge commutes with Higgs.

2. The generation of vacuum expectation value has two functions: to make weak gauge bosons massive and to define the electromagnetically neutral direction to which Higgs field is rotated in the transition to the unitary gauge. In TGD framework only the latter function remains for Higgs if p-adic thermodynamics takes care of massivation. The notion of induced gauge field together with CP_2 geometry uniquely defines the electromagnetically neutral direction so that vacuum expectation is not needed. Of course, the essential element is gauge invariance of the Higgs gauge boson couplings. In twistor Grassmann approach gauge invariance is replaced with Yangian symmetry, which is excellent candidate also for basic symmetry of TGD.

3. The massivation of gauge bosons (all particles) involves two contributions. The contribution from p-adic thermodynamics in CP_2 scale (wormhole throat) and the stringy contribution in weak scale more generally, in hadronic scale. The latter contribution cannot be calculated yet.

The generalization of p-adic thermodynamics to that for Yangian symmetry instead of mere super-conformal symmetry is probably necessary to achieve this and the construction WCW geometry and spinor structure strongly supports the interpretation in terms of Yangian.

One can look at the situation also at quantitative level.

1. W/Z mass ratio is extremely sensitive test for any model for massivation. In the recent case this requires that string tension for weak gauge boson depends on boson and is proportional to the appropriate gauge coupling strength depending on Weinberg angle. This is natural if the contribution to mass squared can be regarded as perturbative.

2. Higgs mechanism is characterized by the parameter m_0^2 defining the originally tachyonic mass of Higgs, the dimensionless coupling constant λ defining quartic self-interaction of Higgs. Higgs vacuum expectation is given by $\mu^2 = m_0^2/\lambda$, Higgs mass squared by $m_0^2 = \mu^2\lambda$, and weak boson mass squared is proportional $g^2\mu^2$. In TGD framework λ takes the role of g^2 in stringy picture and the string tensions of bosons are proportional to g_w^2, g_Z^2, λ respectively.

3. Whether λ in TGD framework actually corresponds to the quartic self-couplingof Higgs or just to the numerical factor in Higgs string tension, is not clear. The problem of Higgs mechanism is that the mass of observed Higgs is somewhat too low. This requires fine tuning of the parameters of the theory and SUSY, which was hoped to come in rescue, did not solve the problem. TGD approach promises to solve the problem.

8.3 Elementary Particles In ZEO

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [K6].

2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the Kähler-Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is spartner of left-handed neutrino.

It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

8.4 Virtual And Real Particles And Gauge Conditions In ZEO

ZEO and twistor Grassmann approach force to build a detailed view about real and virtual particles. ZEO suggests also new approaches to gauge conditions in the attempts to build detailed connection between QFT picture and that provided by TGD. The following is the most conservative one. Of course, also this proposal must be taken with extreme cautiousness.

1. In ZEO all wormhole throats - also those associated with virtual particles - can be regarded as massless. In twistor Grassmann approach [K23] this means that the fermionic propagators can be by residue integration transformed to their inverses which correspond to online massless states but having an unphysical polarization so that the internal lines do not vanish identically.

2. This picture inspired by twistorial considerations is consistent with the simplest picture about Kähler-Dirac action. The boundary term for K-D action is $\sqrt{g_4}\overline{\Psi}\Gamma^n_{K-D}\Psi d^3x$ and due to the localization of spinor modes to 2-D surfaces reduces to a term localized at the boundaries of string world sheets. The normal component Γ^n_{K-D} of the Kähler-Dirac gamma matrices defined by the canonical momentum currents of Kähler action should define the inverse of massless fermionic propagator. If the action of this operator on the induced spinor mode at stringy curves satisfies

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi \ ,$$

this reduction is achieved. One can pose the condition $g_4 = constant$ as a coordinate condition on stringy curves at the boundaries of CD and the condition would correlate the spinor modes at stringy curve with incoming quantum numbers. This is extremely powerful simplification giving hopes about calculable theory. The residue integral for virtual momenta reduces the situation to integral over on mass shell momenta and only non-physical helicities contribute in internal lines. This would generalize twistorial formulas to fermionic context.

One however ends up with an unexpected prediction which has bothered me for a long time. Consider the representation of massless spin 1 gauge bosons as pairs as wormhole throat carrying fermion and antifermion having net quantum numbers of the boson. Neglect the effects of the second wormhole throat. The problem is that for on-mass shell massless fermion and antifermion with physical helicities the boson has spin 0. Helicity 1 state would require that second fermion has unphysical helicity. What does this mean?

1. Are all on mass shell gauge bosons - including photon - massive? Or is on mass shell massless propagation impossible? Massivation is achieved if the fermion and antifermion have different momentum directions: for instance opposite 3-momen but same sign of energy. Higher order contributions in p-adic thermodynamics could make also photon massive. The 4-D world-lines of fermion and antifermion would not be however parallel, which does not conform with the geometric optics based prejudices.

2. Or could on mass shell gauge bosons have opposite four-momenta so that the second gauge boson would have negative energy? In this manner one could have massless on mass shell states. ZEO ontology certainly allows the identification massless gauge bosons as on mass shell states with opposite directions of four-momenta. This would however require the weakening of the hypothesis that all incoming (outgoing) fundamental fermions have positive (negative) energies to the assumption that only the incoming (outgoing) particles have positive (negative) energies. In the case of massless gauge boson the gauge condition $p \cdot \epsilon = 0$ would be satisfied by the momenta of both fermion and antifermion. With opposite 3-momenta (massivation) but same energy the condition $p_{tot} \cdot \epsilon = 0$ is satisfied for three polarization since in cm system p_{tot} has only time component.

3. The problem is present also for internal lines. Since by residue argument only the unphysical fermion helicities contribute in internal lines, both fermion and antifermion must have unphysical helicity. For the same sign of energy the wormhole throat would behave as scalar particle. Therefore it seems that the energies must have different sign or momenta cannot be strictly parallel. This is required also by the possibility of space-like momenta for virtual bosons.

8.5 The Role Of String World Sheets And Magnetic Flux Tubes In Massivation

What is the role of string world sheets and flux tubes in the massivation? At the fundamental level one studies correlation functions for particles and finite correlation length means massivation.

1. String world sheets define as essential element in 4-D description. All particles are basically bi-local objects: pairs of string at parallel space-time sheets extremely near to each other and connected by wormhole contacts at ends. String world sheets are expected to represent correlations between wormhole throats.

2. Correlation length for the propagator of the gauge boson characterizes its mass. Correlation length can be estimated by calculating the correlation function. For bosons this reduces to the calculation of fermionic correlations functions assignable to string world sheets connecting the upper and lower boundaries of CD and having four external fermions at the ends of CD. The perturbation theory reduces to functional integral over space-time sheets and deformation of the space-time sheet inducing the deformation of the induced spinor field expressible as convolution of the propagator associated with the Kähler-Dirac operator with vertex factor defined by the deformation multiplying the spinor field. The external vertices are braid ends at partonic 2-surfaces and internal vertices are in the interior of string world sheet. Recall that the conjecture is that the restriction to the wormhole throat orbits implies the reduction to diagrams involving only propagators connecting braid ends. The challenge is to understand how the coherent state assigned to the Euclidian pion field induces the finite correlation length in the case of gauge bosons other than photon.

3. The non-vanishing commutator of the gauge boson charge matrix with the vacuum expectation assigned to the Euclidian pion must play a key role. The study of the Kähler-Dirac operator suggests that the braid strands contain the Abelianized variant of non-integrable phase factor defined as $exp(i\int Adx)$. If A is identified as string world sheet Hodge dual of Kac-Moody charge the opposite edges of string world sheet with geometry of square given contributions which compensate each other by conservation of Kac-Moody charge if A commutes with the operators building the coherent Higgs state. For photon this would be true. For weak gauge bosons this would not be the case and this gives hopes about obtaining destructive interference leading to a finite correlation length.

One can also consider try to build more concrete manners to understand the finite correlation length.

1. Quantum classical correspondence suggests that string with length of order $L \sim \hbar/E$, $E = \sqrt{p^2+m^2}$ serves as a correlate for particle defined by a pair of wormhole contacts. For massive particle wave length satisfies $L \leq \hbar/m$. Here (p, m) must be replaced with (p_L, m_L) if one takes the notion of longitudinal mass seriously. For photon standard option gives $L = \lambda$ or $L = \lambda_L$ and photon can be a bi-local object connecting arbitrarily distant objects. For the second option small longitudinal mass of photon gives an upper bound for the range of the interaction. Also gluon would have longitudinal mass: this makes sense in QCD where the decomposition $M^4 = M^2 \times E^2$ is basic element of the theory.

2. The magnetic flux tube associated with the particle carries magnetic energy. Magnetic energy grows as the length of flux tube increases. If the flux is quantized magnetic field behaves like $1/S$, where S is the area of the cross section of the flux tube, the total magnetic energy behaves like L/S. The dependence of S on L determines how the magnetic energy depends

on L. If the magnetic energy increases as function of L the probability of long flux tubes is small and the particle cannot have large size and therefore mediates short range interactions. For $S \propto L^\alpha \sim \lambda^\alpha$, $\alpha > 1$, the magnetic energy behaves like $\lambda^{-\alpha+1}$ and the thickness of the flux tube scales like $\sqrt{\lambda^\alpha}$. In case of photon one might expect this option to be true. Note that for photon string world sheet one can argue that the natural choice of string is as light-like string so that its length vanishes.

What kind of string world sheets are possible? One can imagine two options.

1. All strings could connect only the wormhole contacts defining a particle as a bi-local object so that particle would be literally the geometric correlate for the interaction between two objects. The notion of free particle would be figment of imagination. This would lead to a rather stringy picture about gauge interactions. The gauge interaction between systems S_1 and S_2 would mean the emission of gauge bosons as flux tubes with charge carrying end at S_1 and neutral end. Absorption of the gauge boson would mean that the neutral end of boson and neutral end of charge particle fuse together line the lines of Feynman diagram at 3-vertex.

2. Second option allows also string world sheets connecting wormhole contacts of different particles so that there is no flux tube accompanying the string world sheet. In this case particles would be independent entities interacting via string world sheets. In this case one could consider the possibility that photon corresponds to string world sheet (or actually parallel pair of them) not accompanied by a magnetic flux tube and that this makes the photon massless at least in excellent approximation.

The first option represents the ontological minimum.

Super-conformal symmetry involves two conformal weight like integers and these correspond to the conformal weight assignable to the radial light-like coordinate appearing in the role of complex coordiante in super-symplectic Hamiltonians and to the spinorial conformal weight assignable to the solutions of Kähler Dirac equation localized to string world sheets. These conformal weights are independent quantum numbers unless one can use the light-like radial coordinate as string coordinate, which is certainly not possible always. The latter conformal weight should correspond to the stringy contribution to the masses of elementary particles and hadron like states. In fact, it is difficult to distinguish between elementary particles and hadrons at the fundamental level since both involve the stringy aspect.

The Yangian symmetry variant of conformal symmetry is highly suggestive and brings in poly-locality with respect to partonic 2-surfaces. This integer would count the number of partonic 2-surfaces to which the generator acts and need not correspond to spinorial conformal weight as one might think first. In any case, Yangian variant of p-adic termodynamics provides an attractive approach concerning the mathematical realization of this vision.

8.6 Weak Regge Trajectories

The weak form of electric-magnetic duality suggests strongly the existence of weak Regge trajectories.

1. The most general mass squared formula with spin-orbit interaction term $M^2_{L-S} L\cdot S$ reads as

$$M^2 = nM_1^2 + M_0^2 + M^2_{L-S} L\cdot S \ , \quad n = 0,2,4 \ \text{or} \ n = 1,3,5,..., \ . \tag{8.1}$$

M_1^2 corresponds to string tension and M_0^2 corresponds to the thermodynamical mass squared and possible other contributions. For a given trajectory even (odd) values of n have same parity and can correspond to excitations of same ground state. From ancient books written about hadronic string model one vaguely recalls that one can have several trajectories (satellites) and if one has something called exchange degeneracy, the even and odd trajectories define single line in $M^2 - J$ plane. As already noticed TGD variant of Higgs mechanism combines together $n = 0$ states and $n = 1$ states to form massive gauge bosons so that the trajectories are not independent.

2. For fermions, possible Higgs, and pseudo-scalar Higgs and their super partners also p-adic thermodynamical contributions are present. M_0^2 must be non-vanishing also for gauge bosons and be equal to the mass squared for the $n = L = 1$ spin singlet. By applying the formula to $h = \pm 1$ states one obtains

$$M_0^2 = M^2(boson) . \tag{8.2}$$

The mass squared for transversal polarizations with $(h, n, L) = (\pm 1, n = L = 0, S = 1)$ should be same as for the longitudinal polarization with $(h = 0, n = L = 1, S = 1, J = 0)$ state. This gives

$$M_1^2 + M_0^2 + M_{L-S}^2 L \cdot S = M_0^2 . \tag{8.3}$$

From $L \cdot S = [J(J+1) - L(L+1) - S(S+1)]/2 = -2$ for $J = 0, L = S = 1$ one has

$$M_{L-S}^2 = -\frac{M_1^2}{2} . \tag{8.4}$$

Only the value of weak string tension M_1^2 remains open.

3. If one applies this formula to arbitrary $n = L$ one obtains total spins $J = L + 1$ and $L - 1$ from the tensor product. For $J = L - 1$ one obtains

$$M^2 = (2n+1)M_1^2 + M_0^2 .$$

For $J = L + 1$ only M_0^2 contribution remains so that one would have infinite degeneracy of the lightest states. Therefore stringy mass formula must contain a non-linear term making Regge trajectory curved. The simplest possible generalization which does not affect n=0 and n=1 states is of from

$$M^2 = n(n-1)M_2^2 + (n - \frac{L \cdot S}{2})M_1^2 + M_0^2 . \tag{8.5}$$

The challenge is to understand the ratio of W and Z^0 masses, which is purely group theoretic and provides a strong support for the massivation by Higgs mechanism.

1. The above formula and empirical facts require

$$\frac{M_0^2(W)}{M_0^2(Z)} = \frac{M^2(W)}{M^2(Z)} = cos^2(\theta_W) . \tag{8.6}$$

in excellent approximation. Since this parameter measures the interaction energy of the fermion and anti-fermion decomposing the gauge boson depending on the net quantum numbers of the pair, it would look very natural that one would have

$$M_0^2(W) = g_W^2 M_{SU(2)}^2 , \quad M_0^2(Z) = g_Z^2 M_{SU(2)}^2 . \tag{8.7}$$

Here $M_{SU(2)}^2$ would be the fundamental mass squared parameter for $SU(2)$ gauge bosons. p-Adic thermodynamics of course gives additional contribution which is vanishing or very small for gauge bosons.

2. The required mass ratio would result in an excellent approximation if one assumes that the mass scales associated with $SU(2)$ and $U(1)$ factors suffer a mixing completely analogous to the mixing of $U(1)$ gauge boson and neutral $SU(2)$ gauge boson W_3 leading to γ and Z^0. Also Higgs, which consists of $SU(2)$ triplet and singlet in TGD Universe, would very naturally suffer similar mixing. Hence $M_0(B)$ for gauge boson B would be analogous to the vacuum expectation of corresponding mixed Higgs component. More precisely, one would have

$$\begin{aligned} M_0(W) &= M_{SU(2)} \ , \\ M_0(Z) &= cos(\theta_W)M_{SU(2)} + sin(\theta_W)M_{U(1)} \ , \\ M_0(\gamma) &= -sin(\theta_W)M_{SU(2)} + cos(\theta_W)M_{U(1)} \ . \end{aligned} \tag{8.8}$$

The condition that photon mass is very small and corresponds to IR cutoff mass scale gives $M_0(\gamma) = \epsilon cos(\theta_W)M_{SU(2)}$, where ϵ is very small number, and implies

$$\begin{aligned} \frac{M_{U(1)}}{M(W)} &= tan(\theta_W) + \epsilon \ , \\ \frac{M(\gamma)}{M(W)} &= \epsilon \times cos(\theta_W) \ , \\ \frac{M(Z)}{M(W)} &= \frac{1 + \epsilon \times sin(\theta_W)cos(\theta_W)}{cos(\theta_W)} \ . \end{aligned} \tag{8.9}$$

There is a small deviation from the prediction of the standard model for W/Z mass ratio but by the smallness of photon mass the deviation is so small that there is no hope of measuring it. One can of course keep mind open for $\epsilon = 0$. The formulas allow also an interpretation in terms of Higgs vacuum expectations as it must. The vacuum expectation would most naturally correspond to interaction energy between the massless fermion and anti-fermion with opposite 3-momenta at the throats of the wormhole contact and the challenge is to show that the proposed formulas characterize this interaction energy. Since CP_2 geometry codes for standard model symmetries and their breaking, it woul not be surprising if this would happen. One cannot exclude the possibility that p-adic thermodynamics contributes to $M_0^2(boson)$. For instance, ϵ might characterize the p-adic thermal mass of photon.

If the mixing applies to the entire Regge trajectories, the above formulas would apply also to weak string tensions, and also photons would belong to Regge trajectories containing high spin excitations.

3. What one can one say about the value of the weak string tension M_1^2? The naive order of magnitude estimate is $M_1^2 \simeq m_W^2 \simeq 10^4$ GeV^2 is by a factor 1/25 smaller than the direct scaling up of the hadronic string tension about 1 GeV^2 scaled up by a factor 2^{18}. The above argument however allows also the identification as the scaled up variant of hadronic string tension in which case the higher states at weak Regge trajectories would not be easy to discover since the mass scale defined by string tension would be 512 GeV to be compared with the recent beam energy 7 TeV. Weak string tension need of course not be equal to the scaled up hadronic string tension. Weak string tension - unlike its hadronic counterpart- could also depend on the electromagnetic charge and other characteristics of the particle.

8.7 Low Mass Exotic Mesonic Structures As Evidence For Dark Scaled Down Variants Of Weak Bosons?

During last years reports about low mass exotic mesonic structures have appeared. It is interesting to combine these bits of data with the recent view about TGD analog of Higgs mechanism and find whether new predictions become possible. The basic idea is to derive understanding of the low mass exotic structures from LHC data by scaling and understanding of LHC data from data about mesonic structures by scaling back.

1. The article *Search for low-mass exotic mesonic structures: II. attempts to understand the experimental results* by Taticheff and Tomasi-Gustafsson (see http://tinyurl.com/ybq323yy) [C21] mentions evidence for exotic mesonic structures. The motivation came from the observation of a narrow range of dimuon masses in $\Sigma^+ \rightarrow pP^0$, $P^0 \rightarrow \mu^-\mu^+$ in the decays of P^0 with mass of $214.3 \pm .5$ MeV: muon mass is 105.7 MeV giving $2m_\mu = 211.4$ MeV. Mesonlike exotic states with masses $M = 62, 80, 100, 181, 198, 215, 227.5$, and 235 MeV are reported. This fine structure of states with mass difference 20-40 MeV between nearby states is reported for also for some baryons.

2. The preprint *Observation of the E(38) boson* by Kh.U. Abraamyan et al (see http://tinyurl.com/y7zer8dw) [C2, C3, C8] reports the observation of what they call E(38) boson decaying to gamma pair observed in d(2.0 GeV/n)+C,d(3.0 GeV/n)+Cu and p(4.6 GeV)+C reactions in experiments carried in JINR Nuclotron.

If these results can be replicated they mean a revolution in nuclear and hadron physics. What strongly suggests itself is a fine structure for ordinary hadron states in much smaller energy scale than characterizing hadronic states. Unfortunately the main stream, in particular the theoreticians interested in beyond standard model physics, regard the physics of strong interactions and weak interactions as closed chapters of physics, and are not interested on results obtained in nuclear collisions.

In TGD framework situation is different. The basic characteristic of TGD Universe is fractality. This predicts new physics in all scales although standard model symmetries are fundamental unlike in GUTs and are reduced to number theory. p-Adic length scale hypothesis characterizes the fractality.

1. In TGD Universe p-adic length scale hypothesis predicts the possibility of scaled versions of both strong and weak interactions. The basic objection against new light bosons is that the decay widths of weak bosons do not allow them. A possible manner to circumvent the objection is that the new light states correspond to dark matter in the sense that the value of Planck constant is not the standard one but its integer multiple [K5].

   The assumption that only particles with the same value of Planck constant can appear in the vertex, would explain why weak bosons do not decay directly to light dark particles. One must however allow the transformation of gauge bosons to their dark counterparts. The 2-particle vertex is characterized by a coupling having dimensions of mass squared in the case of bosons, and p-adic length scale hypothesis suggests that the primary p-adic mass scale characterizes the parameter (the secondary p-adic mass scale is lower by factor $1/\sqrt{p}$ and would give extremely small transformation rate).

2. Ordinary strong interactions correspond to Mersenne prime M_n, $n = 2^{107}-1$, in the sense that hadronic space-time sheets correspond to this p-adic prime. Light quarks correspond to space-time sheets identifiable as color magnetic flux tubes, which are much larger than hadron itself. M_{89} hadron physics has hadronic mass scale 512 times higher than ordinary hadron physics and should be observed at LHC. There exist some pieces of evidence for the mesons of this hadron physics but masked by the Higgsteria. The expectation is that Minkowskian M_{89} pion has mass around 140 GeV assigned to CDF bump (see http://tinyurl.com/yc98cau6) [C4].

3. In the leptonic sector there is evidence for lepto-hadron physics for all charged leptons labelled by Mersenne primes M_{127}, $M_{G,113}$ (Gaussian Mersenne), and M_{107} [K18]. One can ask whether the above mentioned resonance P^0 decaying to $\mu^-\mu^+$ pair motivating the work described in [C21] could correspond to pion of muon-hadron physics consisting of a pair of color octet excitations of muon. Its production would presumably take place via production of virtual gluon pair decaying to a pair of color octet muons.

Returning to the observations of [C21]: the reported meson-like exotic states seem to be arranged along Regge trajectories but with string tension lower than that for the ordinary Regge trajectories with string tension $T = .9$ GeV^2. String tension increases slowly with mass of meson like state and has three values $T/GeV^2 \in \{1/390, 1/149.7, 1/32.5\}$ in the piecewise linear fit discussed in the article. The TGD inspired proposal is that IR Regge trajectories assignable to the color magnetic flux tubes accompanying quarks are in question. For instance, in hadrons u and d quarks - understood as constituent quarks - would have $k = 113$ quarks and string tension would be by naive scaling by a factor $2^{107-113} = 1/64$ lower: as a matter of fact, the largest value of the string tension is twice this value. For current quark with mass scale around 5 MeV the string tension would be by a factor of order $2^{107-121} = 2^{-16}$ lower.

Clearly, a lot of new physics is predicted and it begins to look that fractality - one of the key predictions of TGD - might be realized both in the sense of hierarchy of Planck constants (scaled variants with same mass) and p-adic length scale hypothesis (scaled variants with varying masses). Both hierarchies would represent dark matter if one assumes that the values of Planck constant and p-adic length scale are same in given vertex. The testing of predictions is not however expected to be easy since one must understand how ordinary matter transforms to dark matter and vice versa. Consider only the fact, that only recently the exotic meson like states have been observed and modern nuclear physics regarded often as more or less trivial low energy phenomenology was born born about 80 years ago when Chadwick discovered neutron.

8.8 Cautious Conclusions

The discussion of TGD counterpart of Higgs mechanism gives support for the following general picture.

1. p-Adic thermodynamics for wormhole contacts contributes to the masses of all particles including photon and gluons: in these cases the contributions are however small. For fermions they dominate. For weak bosons the contribution from string tension of string connecting wormhole contacts as the correct group theoretical prediction for the W/Z mass ratio demonstrates. The mere spin 1 character for gauge bosons implies that they are massive in 4-D sense unless massless fermion and anti-fermion have opposite signs of energy. Higgs provides the longitudinal components of weak bosons by gauge invariance and CP_2 geometry defines unitary gauge so that Higgs vacuum expectation value is not needed. The non-existence of covariantly constant CP_2 vector field does not mean absence of Higgs like particle as believed first but only the impossibility of Higgs vacuum expectation value.

   The usual space-time SUSY associated with imbedding space in TGD framework is not needed, and there are strong arguments suggesting that it is not present [?] For space-time regarded as 4-surfaces one obtains 2-D super-conformal invariance for fermions localized at 2-surfaces and for right-handed neutrino it extends to 4-D superconformal symmetry generalizing ordinary SUSY to infinite-D symmetry.

2. The basic predictions to LHC are following. M_{89} hadron physics, whose pion was first proposed to be identifiable as Higgs like particle, will be discovered. The findings from RHIC and LHC concerning collisions of heavy ions and protons and heavy ions already provide support for the existence of string like objects identifiable as mesons of M_{89} physics. Fermi satellite has produced evidence for a particle with mass around 140 GeV and this particle could correspond to the pion of M_{89} physics. This particle should be observed also at LHC and CDF reported already earlier evidence for it. There has been also indications for other mesons of M_{89} physics from LHC discussed in [K9].

3. Fermion and boson massivation by Higgs mechanism could emerge unavoidably as a theoretical artefact if one requires the existence of QFT limit leading unavoidably to a description in terms of Higgs mechanism. In the real microscopic theory p-adic thermodynamics for wormhole contacts and strings connecting them would describe fermion massivation, and might describe even boson massivation in terms of long parts of flux tubes. Situation remains open in this respect. Therefore the observation of decays of Higgs at expected rate to fermion pairs cannot kill TGD based vision.

The new view about Higgs combined with the stringy vision about twistor Grassmannian [K23] allows to see several conjectures related to ZEO in new light and also throw away some conjectures such as the idea about restriction of virtual momenta to plane $M^2 \subset M^4$.

1. The basic conjecture related to the perturbation theory is that wormhole throats are massless on mass shell states in imbedding space sense: this would hold true also for virtual particles and brings in mind what happens in twistor program. The recent progress [K20] in the construction of n-point functions leads to explicit general formulas for them expressing them in terms of a functional integral over four-surfaces. The deformation of the space-time surface fixes the deformation of basis for induced spinor fields and one obtains a perturbation theory in which correlation functions for imbedding space coordinates and fermionic propagator defined by the inverse of the Kähler-Dirac operator appear as building bricks and the electroweak gauge coupling of the Kähler-Dirac operator define the basic vertex. This operator is indeed 2-D for all other fermions than right-handed neutrino.

2. The functional integral gives some expressions for amplitudes which resemble twistor amplitudes in the sense that the vertices define polygons and external fermions are massless although gauge bosons as their bound states are massive. This suggests a stringy generalization of twistor Grassmannian approach [K23]. The residue integral would replace 4-D integrations of virtual fermion momenta to integrals over massless momenta. The outcome would be non-vanishing for non-physical helicities of virtual fermion. Also the problem due to the fact that fermionic Super Virasoro generator carries fermion number in TGD framework disappears.

3. There are two conformal weights involved. The conformal weight associated with the light-like radial coordinate of $\delta M^4_\pm$ and the spinorial conformal weight associated with the fermionic string connecting wormhole throats and throats of wormhole contact. Are these conformal weights independent or not? For instance, could one use radial light-like coordinate as string coordinate in the generic situation so that the conformal weights would not define independent quantum numbers? This does not look feasible. The Yangian variant of conformal algebra [A1] [**?**, **?**, **?**] involves two integers. Second integer would naturally be the number of partonic 2-surfaces acted by the generator characterizing the poly-locality of Yangian generators, and it is not clear whether it has anything to do with the spinorial conformal weight. One can of course consider also three integers! This would be in accordance with the idea that the basic objects are 3-dimensional.

   If the conjecture that Yangian invariance realized in terms of Grassmannians makes sense, it could allow to deduce the outcome of the functional integral over four-surfaces and one could hope that TGD can be transformed to a calculable theory. Also p-adic mass calculations should be formulated using p-adic thermodynamics assuming Yangian invariance and enlargened conformal algebra.

9 Calculation Of Hadron Masses And Topological Mixing Of Quarks

The calculation of quark masses is not enough since one must also understand CKM mixing of quarks in order to calculate hadron massess. A model for CKM matrix and hadron masses is constructed in [K12] and here only a brief summary about basic ideas involved is given.

9.1 Topological Mixing Of Quarks

In TGD framework CKM mixing is induced by topological mixing of quarks (that is 2-dimensional topologies characterized by genus). The strongest number theoretical constraint on mixing matrices would be that they are rational. Perhaps a more natural constraint is that they are expressible in terms of roots of unity for some finite dimensional algebraic extension of rationals and therefore also p-adic numbers.

Number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium subject to constraints from the integer valued modular contributions remaing integer valued in the mixing. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as direct sum of essentially unique 2×2 and 1×1 matrices. The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of integers n_1 and n_2 characterizing the lowest order contribution to quark mass. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n(u), n(c)) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n(u), n(c)) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n(d) = n(s) = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

9.2 Higgsy Contribution To Fermion Masses Is Negligible

There are good reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. Thus p-adic thermodynamics would explain fermion masses completely. This together with the fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it, fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very strong constraint on the model.

9.3 The P-Adic Length Scale Of Quark Is Dynamical

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses using only quark masses and the assumption mass squared is additive for quarks with same p-adic length scale and mass for quarks labelled by different primes p. This conforms with the idea that pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between hadrons containing different numbers of strange and heavy quarks can be understood if s, b and c quarks appear as several scaled up versions.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in the case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

It should be however made clear that the notion of quark mass is problematic. One can speak about current quark masses and constituent quark masses. For u and d quarks constituent quark masses have scale 10^2 GeV are much higher than current quark masses having scale 10 GeV. For current quarks the dominating contribution to hadron mass would come from super-symplectic bosons at quantum level and at more phenomenological level from hadronic string tension. The open question is which option to choose or whether one should regard the two descriptions as duals of each other based on $M^8 - H$ duality. M^8 description would be natural at low energies since $SO(4)$ takes the role of color group. One could also say that current quarks are created in de-confinement phase transition which involves change of the p-adic length scale characterizing the quark. Somewhat counter intuitively but in accordance with Uncertainty Principle this length scale would increase but one could assign it the color magnetic field body of the quark.

9.4 Super-Symplectic Bosons At Hadronic Space-Time Sheet Can Explain The Constant Contribution To Baryonic Masses

Current quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD which could be characterized in terms of constituent quark masses in M^8 picture and in terms of current quark masses and string tension or super-symplectic bosons in $M^4 \times CP_2$ picture.

Super-symplectic gluons provide an attractive description of this contribution. They need not exclude more phenomenological description in terms of string tension. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-symplectic gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-symplectic boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-symplectic boson need not to be assumed. The preferred role of pion would relate to the fact that its mass scale is below QCD Λ.

9.5 Description Of Color Magnetic Spin-Spin Splitting In Terms Of Conformal Weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are contributions coming from both ordinary gluons and super-symplectic gluons and the latter is expected to dominate by the large value of color coupling strength.

Conformal weight replaces energy as the basic variable but group theoretical structure of color magnetic contribution to the conformal weight associated with hadronic space-time sheet ($k = 107$) is same as in case of energy. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

The comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-symplectic particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-symplectic gluons. The next challenge would be to predict the correlation of hadron spin with super-symplectic particle content in case of long-lived hadrons.

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