About the Relationship of Electroweak and Strong Interactions in the TGD Universe: Part II

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Abstract

The purpose of the second part of the chapter is to develop a detailed model for the relationship between electroweak and strong interactions at the level of the embedding space H developed in the first part of the chapter. The initiative came from Martin Ebner who wanted to find clearcut predictions related to the standard model, in particular Weinberg angle. This led to a project involving Martin and two AI agents, who did extremely valuable work identifying the numerous errors in my paper and pencil calculations and also in the earlier view about Weinberg angle estimated in my thesis dating back to 1982. The project led to a rather dramatic increase in the understanding of the relationship between electroweak and strong interactions in the TGD framework.

TGD explains the coupling pattern of electroweak interactions but gluon couplings do not appear in the embedding space and space-time variants of the Dirac action. The fact that the holonomy group of CP_2 is identifiable as a subgroup of the color group acting as isometries of CP_2 motivates the conjecture that color interactions and electroweak interactions are parts

of the same pattern. The pleasant surprise was that the induced gauge couplings included not only electroweak gauge potentials but also a $U(1)$ gauge potential assignable to strong interactions and coupling to strong isospin. The 3-D generalization of the Weinberg mixing and conformal invariance for the photon emission vertex relate the Weinberg angle and the strong interaction coupling strength α_s to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing.

A generalization of the Weinberg mixing is proposed. The assumption that the emission of photons having wrong helicity with respect to the direction of fermion momentum – light-like in 8-D sense – vanishes, relates the Weinberg angle and the strong interaction coupling strength α_s to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing. Electric-magnetic duality states that $U(1)$ interaction at X^4 level corresponds to color interaction of QCD such that the two coupling strengths are inverses of each other. A model for WCW wave functions as monopole harmonics at partonic 2-surfaces leads to a 3-D generalization of Weinberg rotation and an estimate of the Weinberg angle, identification of strong and weak interactions as parts of color interaction and estimate of the strong coupling strength. What is required is charge fractionation and $SL(2,C)$ acting as global conformal symmetries at partonic 2-surface allow representations with fractional spin.

This view would give at the space-time level a concrete realization for the idea that strong *resp.* weak interactions represent orbital angular momentum-like *resp.* spin-like aspects of color interactions and that strong interactions relate deeply to electroweak interactions as the old-fashioned hadron physics assumed. I have speculated for decades about a kind of duality between QCD description and low energy hadronic description in terms of strong isospin and it seems that this finding makes possible a precise quantitative realization of this vision.

TGD also predicts the M^4 counterpart of Kähler force as a new $U(1)$ interaction possibly related to gravitation. If this interaction is strong as compared to the gravitational force, it could relate to the hadronic strings with transversal M^4 degrees of freedom. Perhaps the more plausible option is that M^4 Kähler gauge potential generalizes the Newtonian gravitational potential.

1 Introduction

The purpose of this work is to develop a more detailed view of the relationship between electroweak and strong interactions at the level of the embedding space H. The initiative came from Martin Ebner who wanted to find clearcut predictions related to the standard model, in particular Weinberg angle. This led to a project involving Martin and two AI agents, who did extremely valuable work identifying the numerous errors in my paper and pencil calculations and also in the earlier view about Weinberg angle estimated in my thesis [K1] dating back to 1982. The project led to a rather dramatic increase in the understanding of the relationship between electroweak and strong interactions in the TGD framework.

Concrete applications are the best way to make progress and in this article a model for Weinberg angle is developed. This leads to a nice overall vision consistent with basic standard model understanding of electroweak coupling constants and providing insights to the relationship between weak and strong interactions.

1. The components of the Higgs field can be seen as CP_2 parts of the electroweak gauge potentials. For W and Z^0 the space-time projections of CP_2 parts would give rise to the longitudinal polarization required by the massivation. For photons the space-time projections of fermion lines would be light-like geodesics of M^4 (no warping) and the projection of CP_2 gauge potential would be pure gauge and allow photons to have only two polarizations. The Higgs field as a dynamical degree of freedom would be the CP_2 part of the gauge potential coupling to the vector isospin.

2. p-Adic mass calculations predict the fermion masses and estimates for the boson masses. No potential model is needed to generate Higgs field vacuum expectation value. Fermion masses are predicted rather than fed into the model as fermion-Higgs couplings. Higgs vacuum expectation is however generated.

3. The warping of space-time surfaces distinguishing between GRT and TGD provides a space-time level description for the fermion massivation. Warping means that fermion lines X

at light-like partonic orbits X^3 are light-like geodesics of both H and X^3 but massive as geodesics of M^4. The fermion mass is proportional to the frequency parameter ω associated with X via the correspondence $m \propto \hbar_{eff}\omega$.

The generous help of Martin Ebner made it possible to identify numerous mistakes in the calculations and led to a dramatic increase in the understanding of the theory. The problems were not only related to my poor computational skills. It eventually became clear that the physical picture of TGD related to the standard model was not quite correct.

In the TGD framework, the geometric interaction vertices are partonic 2-surfaces identified as intersections of 3-D parton orbits and 3-D edges of space-time surface. Fermion lines are identified as boundaries of string world sheets as self-intersections of space-time surfaces. They intersect partonic 2-surfaces at a discrete set of points. This description makes it possible to express the basic coupling parameters of the standard model in terms of the wave function for the position of the intersection of the fermion line at the partonic 2-surface proposed to be analogous to the magnetic harmonic oscillator states.

TGD explains the coupling pattern of electroweak interactions but gluon couplings do not appear in the embedding space and space-time variants of the Dirac action. The fact that the holonomy group of CP_2 is identifiable as a subgroup of the color group acting as isometries of CP_2 motivates the conjecture that color interactions and electroweak interactions are parts of the same pattern. The pleasant surprise was that the induced gauge couplings included not only electroweak gauge potentials but also a $U(1)$ gauge potential assignable to strong interactions and coupling to strong isospin so that the old-fashioned hadron physics makes a come back in the space-time description of strong interactions whereas the embedding space description relies on the notion of color partial waves.

Conformal invariance relates the Weinberg angle to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing. It states that the matrix elements for the emission of photons with holomorphic polarization fields vanish. For partonic 2-surface X^2 the coordinates z of $CP_1 \subset CP_2$ and w of $CP_1 \subset \delta M^4_+$ are holomorphically related: $z = z(w)$ and $\gamma_w = \gamma_z dz/dw$. The postulate is that $\langle \Gamma^w \gamma_w \rangle = 0$ for any holomorphic map $z = z(w)$ whereas $\langle \Gamma^{\overline{w}} \gamma_{\overline{w}} \rangle$ is in general non-vanishing. The matrix element is between holomorphic scalar harmonics (WCW wave function) $\Psi_n \propto (z/(1+r^2))^n$ for the position of the fermion line at X^2 defining the vertex.

$\langle \Gamma^w \gamma_w \rangle = 0$ condition forces a 3-D generalization of Weinberg rotation and a space-time description of strong interaction as a $U(1)$ interaction. Electric-magnetic duality states that $U(1)$ interaction at X^4 level is the counterpart of the color interaction of QCD realized in TGD at the level of embedding space and WCW: the two coupling strengths are inverses of each other. Also an estimate for the strong interaction coupling strength $\alpha_s(U(1))$ is obtained from electric-magnetic duality.

A generalization of the Weinberg mixing is proposed. The assumption that the emission of photons having wrong helicity with respect to the direction of fermion momentum – lightlike in 8-D sense – vanishes, relates the Weinberg angle and the strong interaction coupling strength α_s to the choice of the monopole harmonic and to a second parameter of the generalized Weinberg mixing. Electric-magnetic duality states that $U(1)$ interaction at X^4 level corresponds to color interaction of QCD such that the two coupling strengths are inverses of each other. A model for WCW wave functions as monopole harmonics at partonic 2-surfaces leads to a 3-D generalization of Weinberg rotation and an estimate of the Weinberg angle, identification of strong and weak interactions as parts of color interaction and estimate of the strong coupling strength. What is required is charge fractionation and $SL(2,C)$ acting as global conformal symmetries at partonic 2-surface allow representations with fractional spin.

This view would give at the space-time level a concrete realization for the idea that strong *resp.* weak interactions represent orbital angular momentum-like *resp.* spin-like aspects of color interactions and that strong interactions relate deeply to electroweak interactions as the old-fashioned hadron physics assumed. I have speculated for decades about a kind of duality between QCD description and low energy hadronic description in terms of strong isospin and it seems that this finding makes possible a precise quantitative realization of this vision.

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relate to the hadronic strings with transversal M^4 degrees of freedom. Perhaps the more plausible option is that M^4 Kähler gauge potential generalizes the Newtonian gravitational potential.

2 From a model of Weinberg angle to a model relating electroweak and strong interactions

The following discussion started from an attempt to understand Weinberg angle but it turned out the original picture was not quite correct: the modified Dirac equation contains more than the standard electroweak couplings. There are couplings to fields coupling to vectorial isospin and the natural interpretation is in terms of strong interactions. This realization led to a generalization of the Weinberg mixing and to a concrete proposal of how the classical induced fields related to electroweak and strong interactions relate and even to the formulas for the basic coupling parameters.

The general vision is already described in the previous section. In this section a model for the wave function for the position of the edge of the fermion line at the partonic 2-surface based on holomorphic monopole harmonics is developed. They can be divided to holomorphic scalar monopole harmonics (ScMHs) and spinor monopole harmonics (SpMHs).

2.1 A WCW model for the TGD counterpart of the bosonic propagator

One must consider the analog of a bosonic propagator as a WCW correlation function.

1. Here the holography = holomorphy (H-H) vision and the assumption that fermion vertices are associated with the intersections of the 3-D edges of the space-time surface with light-like partonic 3-surfaces. If the interaction is contact interaction then it is associated with the string world sheets as the intersection and it intersects partonic orbit along fermion line intersection partonic 2-surface at point.

2. The WCW counterpart of the gauge boson propagator must be studied and its ends correspond to two fermion vertices associated with to separate space-time edges. One can study either t-channel exchange or s-channel resonance.

3. The partonic 2-surface has a complex 2-surface of CP_2 as projection and a homologically non-trivial geodesic sphere corresponding to the lowest generation without topological mixing inducing CKM mixing is a natural simplifying assumption. For higher generations complex surfaces with higher genus are needed.

   It is natural to assign to the position of the fermion vertex (or by ZEO the position of the associated light-like fermion line) a wave function. The first guess was that it corresponds to the spinor monopole harmonic (SpMH) at the CP_2 projection of the partonic 2-surface associated with the vertex. For a sphere CP_1, the harmonics can be either scalar monopole harmonics of form $(z/F)^n$, $F = 1 + r^2$ and $r^2 = z\overline{z}$, or spin monopole harmonics of form $(z/\sqrt{F})(z/F)^n$. The volume element of CP_1 is $dA = dzd\overline{z}/F^2$.

4. Conformal invariance could determine to a high degree the massless bosonic propagators and coupling constants. On the other hand, the breaking of electroweak symmetries means breaking of conformal invariance. This suggests that photon is conformally invariant in some sense.

5. What suggests itself is that matrix elements for the emission of the bosons involve the averages of wave functions for the position of fermion line at the partonic orbit X^3 determining in turn the bosonic vertices at partonic 2-surfaces X^2 as analogs of matrix elements of $\gamma^k A_k$ in standard model. Preferred extremal property and holography = holomorphy principle are expected to give rise to strong correlations between the two partonic 2-surfaces. It took a long time to realize that conformal invariance suggests that the vertex for the emission of holomorphic photons vanishes unlike that for antiholomorphic photons. This condition would determine the Weinberg angle.

2.1.1 The arduous path to the condition predicting the Weinberg angle

From the very beginning it was intuitively clear that the vanishing of the expectation or matrix element of the classical photon field at the parton surface, defined in some sense, predicts the value of the Weinberg angle.

1. The original proposal was that the correlation $\langle \gamma, Z_0 \rangle$ between partonic wave functions for the vertices at the ends of the photon propagator identified as WCW correlation function vanishes. The local version of this argument was used already in my thesis to get an estimate for the Weinberg angle from the counterpart of Yang-Mills action from the condition that there are no $\gamma - Z^0$ cross terms in the action.

   It was assumed that correlation function $\langle \gamma, Z_0 \rangle$ reduces to a product $\langle \gamma \rangle \langle Z_0 \rangle$. This looks odd. Either $\langle \gamma \rangle$ or $\langle Z^0 \rangle$ must vanish. The choice $\langle \gamma \rangle = 0$ would determine the Weinberg angle.

2. Then came the observation that the vanishing of $\langle \gamma \rangle$ can be regarded as an independent assumption stating that the photon field corresponds to fluctuations around the vanishing value of the classical gauge potential γ.

3. It also became clear that the condition $\langle \gamma \rangle = 0$ and Weinberg rotation must be applied in the M^4 sector for K-C-S action, which is necessary for obtaining non-vanishing gauge boson emission vertices.

4. The detailed study of the condition for the monopole harmonics led to the conclusion that it is satisfied by ScMHs universally provided that the vertex at which γ contracted with gamma matrices appears depends only on the radial coordinate r of CP_1. No dependence of the condition on the angle coordinate ϕ of CP_1 is allowed. This is automatically true only for the holomorphic part of the induced spinor connection but not for its conjugate. This observation led to the realization that the condition can be interpreted in terms of conformal invariance natural in the case of massless particles like photon. The emission vertex for a conformal photon vanishes!

   By requiring that the condition holds true for all surfaces $X^2 \subset CP_1 \times CP_2 \subset \delta M^4_+ \times CP_2$ one obtains a condition fixing the Weinberg angle. The condition generalizes to more general surfaces satisfying holomorphy $z = z(w)$ locally. The complex structure need not be induced from that of H so that also Lagrangian minimal surfaces can satisfy it.

1. Conditions for the conformal invariance

Consider now the condition for the conformal invariance in more detail.

1. The expectation $\langle \Gamma^w \gamma_w \rangle$ for the matrix elements $\Gamma^w \gamma_w$ as analog of $\gamma^k \epsilon_k$ between modes of the induced spinor fields for the emission of holomorphic photons is considered. WCW wave function for the position of the fermion line at partonic orbit X^3 and vertex at partonic 2-surface X^2 defines the expectation. The partonic 2-surface with $z = z(w)$ defines induced $\gamma_w = \gamma_z dz/dw$. The condition $\langle \Gamma^w \gamma_w \rangle = 0$ for any surface of this kind realizes conformal invariance and determines the value of the Weinberg angle once the generalized 3-D Weinberg rotation is specified.

2. The averaging of $\langle \Gamma^w \gamma_w \rangle$, for conformal photon $\gamma_w = \gamma_z dz/dw$, expressed in the complex coordinate z for $CP_1 \subset CP_2$, should not modify the prediction for p and would therefore effectively reduce to a constant. If $\Gamma^w \gamma_w$ brings in new r-dependence it must correspond to powers z^n, $n > 0$, for which the averages vanish so that effectively there is only the dependence of induced gauge potentials on r. New dependence on r could change the values of Z and p.

   There would be non-trivial contributions from $\gamma_w = \gamma_z dz/dw, dz/dw = \sum_n a_n z^n$ and the z^n, $n > 1$ give a vanishing contribution. Therefore the holomorphic correspondence $z = z(w)$, also possible for Lagrangian minimal surfaces, guarantees the condition for p is not affected.

3. p-Adic thermodynamics allows a small mass for photons. Massivation would mean warping at space-level reducing the M^4 light-velocity from c to $c_\# < c$: warping would be a gravitational effect due to constant gravitational potential and would not involve curvature. Warping implies the reduction of light velocity in condensed matter implying reflection and transmission at boundaries. Could warping imply also a failure of conformal invariance and explain why Weinberg angle differs slightly from $p = 1/4$? Interesting question relates to the fact that for the genera $g > 0$ there is a moduli space of conformal structures. Can this affect the situation?

2. Objections

One can invent objections against the proposed picture.

1. The spinor modes appearing in the matrix element for the photon emission can contain dependence on r since spinor modes are restricted from partonic orbit to the fermion line and vertex. Suppose that the dependence is expressible as a vierbein rotation in the tangent space of the partonic 2-surface.

   What matters is the M^4 part of the tangent space rotation acting in $CP_1 \subset \delta M^4_+$. One could assume that the rotations are inverses for the initial and final state spinors $\overline{\Psi}_i$ and $\overline{\Psi}_f$. The M^4 rotations do not commute with Γ^w and this can modify the expectation given the value of Z and p

2. There is also a second objection. If the dependence of the spinor modes on z can be neglected and if the vertex for the emission of holomorphic photons vanishes, why would this not be the case also for the antiholomorphic photons. In the earlier argument the powers of z in $\Gamma^w dz/dw$ would be only replaced with powers of $\overline{z}$ in $\Gamma^{\overline{w}} dz/d\overline{w}$. There would be no emission of photons!

How to overcome these objections?

1. One can also now expand spinor modes $\overline{\Psi}_i$ and Ψ_f as power series in powers $\overline{z}^{n_i}$ and z^{n_f} with opposite phase angles and $\Gamma^w dz/dw$ in powers z^n. In integral over CP_1 the only surviving contributions come from products $\overline{z}^{n_i} z^{n_f} z^n$ satisfying $n_f - n_i + n = 0$ and they give additional power r^{2n_i} to the integrand. Therefore the radial dependence of the integrand in the average is modified and values of Z and p are modified.

   Notice that the modification implies that without further conditions (such as $n_i = 0$) the values of Z for different ScMHs are different. Can this be interpreted in terms of a violation of conformal invariance?

2. This could make possible the reduction of the value of p from its value $p = 1/4$. This also predicts that the integrands are different for $\Gamma^w dz/dw$ and $\Gamma^{\overline{w}} dz/d\overline{w}$ the condition guaranteeing the vanishing of the amplitude holomorphic photon give different values of p and either option must be selected.

Even a third objection can be found. $\Gamma^w(CKS, M^4)$ appearing in the conformal vertex could provide dangerous r-dpendent terms spoiling the argument for the conformal invariance. This also forces us to ask whether the argument determining the Weinberg angle is physically solid.

1. Since topological QFT is in question, the metric determinant disappears from $L(KCS, M^4)$ and CP_2 and metric is not the induced metric but naturally that of CP_1 projection. There are two options to consider :$CP_1 \subset M^4$ and $CP_1 \subset CP_2$. Denote the radial coordinates of for the two CP_1:s by $|w|$ and $r = |z|$.

   The dangerous part of the matrix element is determined by $g^{w\overline{w}} A_w = g^{w\overline{w}} A_z dz/dw$. The contributions from positive powers of dz/dw vanish in angular integration if the dependence coming from z dependence of spinor modes can be neglected.

2. There is however a problem. A simple calculation, using $L(KCS, \delta M^4) = \epsilon^{uw\overline{w}} A_u J_{w\overline{w}}$, shows that $\Gamma^w(KCS, M^4)$ is proportional to

$$\sqrt{det(g_2(CP_1))}\gamma^w(CP_1) = \sqrt{det(g_2(cP_1))}g^{w\overline{w}}(CP_1)\gamma_{\overline{w}}(CP_1)$$

$$= \gamma_{\overline{w}}(CP_1) \propto e_3(CP_1) \propto |w|/(1+|w|^2) \ .$$

This factor is problematic since it modifiies the integrals involved dramatically unless $CP_1(M^4)$ is homologically trivial making it possible for e_3 to be constant in a good approximation.

3. Another way to get rid from this term is to assume that the normalization factors of the incoming and outgoing spinor modes cancel this term. Perhaps spinors as half-densities are naturally characterized by the square root of E as a factor just as the metric is quadratic in E and connection linear in it.

   If this is possible and if the integration measure for WCW wave function is for the geodesic sphere of $CP_1 \subset CP_2$ the conformal invariance is violated only by the r-dependence terms coming from the spinor modes. One way to state this speculation is to say that the matrix element has vanishing conformal weight and this reflects the masslessness of the photon.

4. Suppose that both Ψ_{in} and Ψ_{out} have no w-dependence apart from that coming from the complex normalization factor proportional to $\sqrt{e^3_w}$ *resp.* $\sqrt{e^3_{\overline{w}}}$. The radial dependence from these factors should be cancelled by that for e^3_w and $e^3_{\overline{w}}$. For instance, for γ_w the phases of e^3_w and those coming from Ψ_{in} and Ψ_{out} should sum up to zero or to a non-trivial phase to give a non-vanishing matri

   (a) Photon has spin and helicity and the phase of e^3 could correspond to this. The spin direction of massless fermion changes by helicity (angular momentum) conservation. So that there are two possible emission vertices. For an ordinary massless fermion with a fixed chirality helicity conservation allows for the emitted photon only the second helicity parallel to the direction of the fermion momentum. Is the situation same now?

   (b) Warping reduces light velocity and makes fermions massive. Massivation by warping means that fermions remain massless in 8-D sense along fermion lines as light-like geodesics of H. Could this mean that only the second helicity is possible also for photons emitted by fermions massive in this sense? Could the vanishing condition for the emission of the wrong helicity photon determine the Weinberg angle? If so, the Weinberg angle would depend on warping. The wrong helicity photon would mean that the change of the spin for the fermion is wrong. The sign of the change of spin would affect the matrix element.

3. $L(KCS, H)$ or $L(KCS, M^4) + L(KCS, CP_2)$?

The basic existential question have been about whether Kähler-Chern-Simons action for partonic orbits determining the modified gamma matrices is **a)** $L(KCS, H = M^4 \times CP_2)$ defined for the sum $J_K(M^4) + J_K(CP_2)$ of induced Kähler forms as the dynamical variable or **b)** by the sum $L(KCS, M^4) + L(KCS, CP_2)$ for $J_K(M^4)$ and $J_K(CP_2)$.

1. $J_K(M^4)$ is nonvanishing only in the transversal degrees of freedom defined by Hamilton-Jacobi structure [L11]. This requires that $B_K(M^4)$ is a pure gauge in the longitudinal degrees of freedom. A possible interpretation is as a counterpart of Newtonian gravitational potential. A stronger assumption is that $B_K(M^4)$ is a non-vanishing constant at the fermion line. Constancy is however not a necessary assumption and the identification as gravitational potential is attractive. This option corresponds to the earlier option in the model for p.

2. If $B_K(M^4)_u = 0$ then only option **a)** is possible and corresponds to $B_u = B_u(CP_2) \propto r^2/(1+r^2)$. The presence of this factor modifies dramatically the expectation values determining the values of Z and p since this factor multiplies the parts of photon γ appearing in the vanishing condition for the emission of conformal photon, physically this would correspond to a photon with a wrong helicity. One implication is that the universality of Z and p for ScMHs is lost.

3. For the option **a)** involving the sum $J(M^4)+J(CP_2)$ it is important to clarify the importance of relative sizes of $J(M^4)$ and $J(CP_2)$. Smallness of $J(CP_2)$ relative to the transversal (and non-vanishing) part of $J(M^4)$ is what matters at the M^4 projection of the partonic orbit. The transversal part of $J(M^4)$ is for $CP_1 \subset \delta M^4_+$ proportional to the square of the M^4 radius of order of Compton length of fermion whereas $CP_1 \subset CP_2$ has scale of 10^4 Planck lengths. The ratio of these scales determines which contribution is large at $CP_1 \subset \delta M^4_+$.

   If $B_u(M^4) = 0$, which is a natural option since the longitudinal part of $J(M^4)$ vanishes, then $B(CP_2)_u$ is large in relative sense. $B_u(M^4)$ could be nonvanishing if it is gradient. The interpretation of $B_u(M^4)$ could be as Newtonian gravitational potential. In this case it would be very small. Its value is about $r_s/R_E \simeq 10^{-9}$ in the gravitational field of the Earth at its surface but this is not a problem for option **b)** for which $B_u(M^4)$ does not appear in $L(KCS, CP_2)$.

4. Estimating the condition for the holomorphic scalar monopole harmonics (ScMHs) One can estimate the condition for the vanishing for the holomorphic scalar monopole harmonics (ScMHs) $\Psi_n = [z/F]^n$ and for holomorphic spinor monopole harmonics (SpMHs) $\Psi_n = [z/F^{1/2}][z/F]^n$.

1. The formula for the Weinberg angle depends on the details of the model for the Weinberg mixing and 3-D mixing is required by unitarity. The value of p is determined by the ratio $Z = \langle 1/F\rangle/\langle 2(F-1)/F\rangle$, $F = 1+r^2$. For the ScMHs the ratio is equal to $Z = 1/2$ so that the value of the Weinberg angle is universal. For the proposed model of Weinberg rotation, the vanishing condition gives universally the same value of Weinberg angle $p = sin^2(\theta_W) = 1/4$. For the spinor monopole harmonics the values $Z = 1/2$ and $p = 1/4$ are obtained at the limit $n \to \infty$.

2. Partonic 2-surfaces carry an analog of magnetic monopole field, which suggests that WCW wave functions could be analogous to 2-D harmonic oscillator wave functions in the Kähler magnetic field for the intersection of the fermion line with X^2 and forming an edge with it.

   Their identification as modes of the modified Dirac equation for light-like partonic orbits determined by Kähler-Chern-Simons (K-C-S) action with coupling to the monopole magnetic field defined by the Kähler form at partonic 2-surface meets the objections already given: in particular, p would depend on spinor mode if monopole harmonics are in question. This does not favor the reduction of WCW wave functions to spinor modes to the modes for induced spinor fields.

3. As will be found, for ScMHs the condition $\langle \Gamma^w \gamma_w \rangle = 0$ determining the value of p does not depend strongly on the conformal details or even on the homology of the partonic surface and on the light-like geodesic associated with the fermion line. Hence p should be in a good approximation independent of fermion state and can depend only on the length scale. The case of a general partonic surface can be treated by considering the partonic 2-surface as covering space of a geodesic sphere CP_1 of CP_2 so that it can be defined to local regions having CP_1 as analog of coordinate chart (see the Appendix).

2.1.2 Could $\langle \Gamma^w \gamma_w \rangle = 0$ condition make sense for the spinor modes?

Although the application of $\langle \Gamma^w \gamma_w \rangle = 0$ condition to spinor modes at the partonic surface strongly suggests the dependence of p on the spinor mode, one can discuss also this option as an example of a failed trial.

One should solve the modified Dirac equation for L_{K-C-S} explicitly at the partonic orbits. The explicit solution reduces to conformal symmetry as in string models. The solutions are holomorphic spinor modes whereas the model based on the analogs of harmonic oscillator states reduces in Bargmann representation to holomorphic scalar modes. The basic difference is that the analog of the harmonic oscillator Gaussian is an imaginary exponent for the spinor modes and a real exponent for harmonic oscillator states.

I have deduced the explicit solution spectrum involving mass spectrum only at the level of H and spacetime level in general form [L19, L18]. The modified Dirac equation for partonic orbits can be solved by holomorphic ansatz. The universality of the Weinberg angle condition allows us

to consider the simplest situation, that is the geodesic sphere $CP_1 \subset CP_2$. For more general cases one expects that the value of Weinberg angles can be different but this can be understood as an outcome of coupling constant evolution.

1. The modified Dirac equation and modified gamma matrices are completely fixed from the condition

$$\Gamma^\mu = \frac{\partial L(K-C-S)}{\partial_{\partial_\mu h^k}}\Gamma_k \ . \tag{2.1}$$

   defining modified gamma matrices and implying analog of supersymmetry so that conserved isometry currents have fermionic super counterparts. Note that for $L(K-C-S)$ the modified gamma matrix Γ^μ is well-defined since the singular induced metric does not appear in $L(K-C-S)$.

   The Chern-Simons Dirac could — and actually must — contain the M^4 part coming from the Hamilton-Jacobi structure as a generalized complex/Kähler structure. Also the twistor lift suggests this. The M^4 Kähler form has only the transverse part: the proposal is that this is the case inside causal diamonds. At the level of the entire H there would be no M^4 Kähler form.

2. The naive expectation was that one obtains a hierarchy of harmonic oscillator states from the transversal part of the Dirac operator. The M^4 mass squared would correspond to the analog of magnetic energy coming from CP_1 degrees of freedom. By harmonic oscillator analogy the spectrum would come as integer multiples of the basic value analogous. This would give an analogy with stringy mass formula.

2.1.3 $\langle \Gamma^w \gamma_w \rangle = 0$ condition for monopole harmonics?

Martin Ebner has calculated the value of the parameter $Z = \langle 1/(1+r^2)\rangle / \langle 2r^2/(1+r^2)\rangle$ for the wave functions of the analog of magnetic monopole harmonic oscillator [C3, C2]. The wave functions are known as monopole harmonics.

1. Monopole harmonics $Y_{q,l,m}$ are a generalization of spherical harmonics labelled by angular momentum l and monopole flux q having an interpretation as a monopole flux identifiable in TGD as homology charge of the partonic 2-surface. The spin quantum number $m = 2k - q$ varies in the range $[-l, l]$. In the general case $Y_{q,l,m}$ depends on variables $u = \cos(\frac{\theta}{2})e^{i\phi/2}$ and $v = \sin(\frac{\theta}{2})e^{-i\phi/2}$.

   Spinor harmonics with spin 1/2 give rise to more general monopole harmonics. In this case l must be replaced with $j = l \pm 1/2$ and the minimum value $l_{min} = q$ splits to $j_{min} = q \pm 1/2$.

2. The lowest energy states with $q = l$ for ScMHs correspond to Landau levels with lowest energy and minimal angular momentum. In this case the monopole harmonics are holomorphic sections of the monopole bundle and depend only on $tan(\theta/2)exp(i\phi)$. They are favored by holography = holomorphy vision [L14, L16] suggesting that space-time physics is holomorphic. In the TGD framework, q has an interpretation as the quantized monopole flux assignable to the homologically non-trivial CP_2 projection of the partonic 2-surface. l has interpretation as quantized angular momentum (isospin) and m as its component along quantization axis. For holomorphic ScMHs conformal weight is the natural quantum number and corresponds to mass squared in string model picture applied also in p-adic mass calculations [L17] [K5].

3. The system at CP_1 can be mapped by stereographic projection to the plane. The magnetic binding in the case of the plane corresponds to the geometric binding at the sphere due to the homological non-triviality. The presence of the ratio of $(2(q-k)+1)/(2k+1)$ in $Z(q,k)$ reflects the fact that $SU(2) \times SU(2)$ is the dynamical symmetry group in the case of CP_2. For the magnetic harmonic oscillator in plane $SO(2,3)$ appears as the dynamical symmetry group.

4. Monopole harmonics appear in the modelling of fractional Quantum Hall Effect (FQHE) and the TGD proposal for the charge fractionation is in terms of space-time surfaces defining multiple coverings of M^4 and CP_2 so that single 2π rotation brings the particle to a different space-time sheet and the particle has effectively fractional spin or isospin.

   In the case of FQHE the explanation is that when a unit of monopole flux is added to the McLaughlin state representing a highly correlated many-electron state, the state changes as if the flux quantum had fractional electric charge. In TGD, the many particle-state would make the space-time surface many-sheeted and induce the effect. The braiding group as a covering group of permutations provides a mathematical description of what happens with permutations represented as flows along the space-time surface with rotation as an M^4 or CP_2 projection.

   I have proposed a model of charge and spin fractionation based on the notion of many-sheeted space-time. Many-sheetedness implies that the 2π rotations in M^4 and CP_2 lifted to the space-time sheet do not bring the point to the original point. If there are m sheets a rotation of $m2\pi$ is needed. This means that the particle has effectively fractional spin (Kähler charge, strong isospin or em charge) [K8] [L3] [D1].

To what kind of system could the wave functions of a monopole harmonic oscillator be assigned? There are two candidates: partonic spinor modes and WCW wave functions.

1. Could the states in question be conformal spinor modes at a partonic 2-surface having a vanishing half-covariant derivative? These states cannot be analogous to harmonic oscillator wave functions. There would be an analog of oscillator Gaussian defined by the exponent of the Kähler function of CP_2 but the exponent as a candidate for ground state would be imaginary, just like the action exponential in QFTs. Furthermore, the estimate for $\langle \Gamma^w \gamma_w \rangle$ and therefore for p is expected to depend on the quantum numbers of the spinor mode. This does make sense physically.

2. If this argument is correct, only the interpretation as a wave function in WCW. Effectively one would have a wave function at the partonic 2-surface for the position of the intersection of fermion line and partonic 2-surface where it has edge. Real (rather than imaginary) exponent defined by the sum of CP_2 abd M^4 Kähler functions (associated with the Hamilton-Jacobi structure [L11]) is an excellent candidate for the analog of the oscillator Gaussian and the coupling to harmonic oscillator potential is analogous to Kähler gauge potential. Furthermore, only ScMHs allow a universal prediction for the Weinberg angle.

The assignment of a wave function to the location of the vertex, which exists instantaneously or only for a brief period of time, can be criticized.

1. This wave function is not a property of particle/parton/fermion as 3-D/2-D/1-D entity but a local property of a 4-D/3-D/1-D analog of Bohr orbit. This makes sense in zero energy ontology (ZEO) forced by the classical non-determinism, present already for the ordinary minimal surfaces at the spanning frames [L7].

2. One can also assign a wave function at the partonic orbit with the fermion line identified as the intersection of string world sheet and partonic orbit and characterized by the intersection points if a light-like geodesic line of H is in question. In this case, wave function in the usual sense would be in question. In this case one can assume that the homology charge is non-trivial and the monopole harmonics (q, l) with $k \neq 0$ are a reasonable guess.

3. The quantum numbers assigned with the position of the vertex do exist only instantaneously and cannot define conserved quantum numbers unless the sum of these quantum numbers associated with the particle reaction vertex vanishes. This would be an analog of confinement.

4. The model of the elementary particles in terms of closed monopole flux loops having pieces along parallel space-time sheets connected by two wormhole contacts (WCs) allows to concretize the situation. The opposite throats at a given WC are at a distance of CP_2 size scale and must be extremely strongly correlated. A boson has at least two fermion lines, which

are either at the opposite throats of a single WC or at the throats associated with different WCs. Also now a strong correlation is expected but in elementary particle Compton length scale.

5. Particle vertices are proposed to involve splitting of the closed string-like objects formed by the flux tubes via reconnection. The creation of a fermion pair as turning of a fermion backwards in time from a boson can be modelled as a process in which the flux tubes at opposite sheets touch and a homologically trivial WC is created. This state can continuously transform to a pair of homologically non-trivial wormhole contacts with opposite homology charges. The instantaneous state defining the vertex would have a trivial homology charge. The edge of the fermion line turning backwards in time would have a wave function for its position at this surface. If so it could correspond to $(q, l = 0)$ state. The monopole angular momenta of the fermion and antifermion would sum up to zero.

2.1.4 Is FQHE realized at the level of parton surfaces?

Both M^4 and CP_2 degrees of freedom are present and if the H-J structure defines slicing of light-cone interior by light-cone boundaries with the tip removed, homology is non-trivial also in the M^4 sector. If the wave functions for the edge of the fermion line at partonic 2-surface are products of monopole harmonics associated with ΔM^4_+ which is metrically and by conformal gauge invariance sphere, the products of monopole harmonics are in question.

It would not be surprising if there were strong correlations between the parameters (q, k). The first intuitive guess is that the values of monopole fluxes are the same. The values of k would be the same if holomorphic sections are assumed.

The idea that partonic surfaces could realize analog of FQHE sounds strange since it is assigned with many-electron systems. On the other hand, many particle systems force the space-time surface to be many-sheeted and also the hierarchy of Planck constants h_{eff} implies that the sizes of M^4 projections of partonic surfaces can be very large. The projections to M^4 and CP_2 could give rise to spin *resp.* charge fractionation. What is fascinating is that the genus of the partonic surface is given as the bidegree $(d_1 - 1)(d_2 - 1)$ of the polynomial of two complex variables so that at least for H-H options the model becomes very concrete. For spheres either $d_1 = 1$ or $d_2 = 1$ is true and for higher genera higher degree polynomials and higher homological charges are needed.

As already found, for the simplest TGD based interpretation of spin and charge fractionation in terms of many-sheeted coverings of M^4 and CP_2, the charge fractionation would occur for a single sheet of the covering but there would be no fractionation for the many-sheeted structure. The fractionation would reflect the higher homology charges of the partonic 2-surface identifiable as monopole fluxes.

2.2 An estimate for the Weinberg angle from the vanishing of the emission vertex for holomorphic photons

An estimate for the Weinberg angle θ_W can be deduced by considering the TGD counterpart of a bosonic emission vertex. The semiclassical estimate carried in my thesis (1982) [K1] [L1] for the Weinberg angle $p \equiv sin^2(\theta_W)$ came from the condition that the electroweak YM action for the induced gauge fields contains no γZ_0 cross term. The simplest estimate gave $p = 9/28$, which is somewhat smaller than $p = 3/8 = 9/24$ of a typical GUT. In the recent formulation of quantum TGD, the classical electroweak YM action is replaced with Kähler action plus volume term so that the estimate is lost. The earlier estimate contained also computational mistake.

2.2.1 Preliminaries

Before starting, it is advantageous to introduce basic differental geometry of CP_2. The vielbein vector e^3 to which the neutral part of induced gauge potential is proportional to, is proportional to r/F, $F = 1 + r^2$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2+r^2\sigma_3^2)}{F^2}+\frac{r^2(\sigma_1^2+\sigma_2^2)}{F} \ , \tag{2.2}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2-\xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2-\xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1+\xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{2.3}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2\sum_A e_k^A e_l^A \ , \tag{2.4}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{2.5}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{2.6}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2}+\frac{r^2}{4F^2}(d\Psi+cos\Theta d\Phi)^2+\frac{r^2}{4F}(d\Theta^2+sin^2\Theta d\Phi^2) \ . \tag{2.7}$$

From this expression one finds that at coordinate infinity $r=\infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2+sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

It is convenient to introduce 3 functions:

$$\begin{aligned} &U = 2r^2/(1+r^2) = 2(F-1)/F \ , && (B_K), \\ &V = (r^2-1)/(1+r^2) = (F-2)/F && (V_{03}, Z^0 \text{ and } gG_1) \ , \\ &W = (2r^2+1)/(1+r^2) = (2F-1)/F \ , && (V_{12} \text{ and } g_s G_2) \ . \end{aligned} \tag{2.8}$$

In this shorthand notation, Kähler gauge potential is given as $B_K = U$.The axially coupling gauge potential V_{03} and vectorially coupling gauge potentials V_{12} re given as $V_{03} = V$ and $V_{12} = W$.

In the sequel, the first aim is to describe the results of the earlier estimate and to demonstrate the picture about mixing implies the predictions of the Weinberg theory for the electroweak gauge couplings.

1. The basic difference between TGD and standard model is that the TGD counterpart of the standard model gauge potential A is A_{ind}/g, where A_{ind} is the induced gauge potential and g is the corresponding coupling constant.

   The identification of the induced gauge potentials proposed in the thesis [K1] turned out to contain mistakes. The corrected identification is found by requiring that the neutral part of

the gauge field is the same before the Weinberg mixing and after it. Charge matrices for γ and Z^0 are known. This condition requires that the terms proportional to $p = sin^2(\theta_W)$ cancel and this fixes the expressions for γ and Z^0. Note that e^3 is proportional to $r/(1+r^2)$.

The standard model formulas for the ratios of electroweak couplings in terms of the Weinberg angle are predicted. The value of $U(1)$ coupling g_1 as Kähler coupling would determine by the quantum criticality condition the overall scale of couplings and Weinberg angle would determine their ratios.

2. A second new element is the presence of M^4 Kähler form $J(M^4)$ suggested both by the twistor lift [L8, L9] of TGD and Hamilton-Jacobi (H-J) structure [L11]. For partonic surfaces one obtains gauge boson propagators only if $J(M^4)$ is present in $L(K-C-S)$ and defines a separate contribution so that induced $J(M^4)$ and $J(CP_2)$ do not sum up.

3. A third new element is the notion of warping distinguishing between GRT and TGD [L22, L20]. It leads to the generalization of H-J structure [L11] involving a mixing of M^4 and CP_2 degrees of freedom. Warping gives a concrete space-time counterpart for the massivation of fermions.

4. A further new element is the assignment of the analog of classical Higgs field to the part of $\Gamma^\alpha D_\alpha$ involving CP_2 gamma matrices. It couples automatically the left- and right-handed M^4 spinors and in this sense behaves like scalar field. Since the Higgs couplings are not proportional to the electromagnetic charge but to the vector isospin, the neutral part of Higgs would be the part of $\Gamma^\alpha D_\alpha$ involving CP_2 gamma matrices and coupling to vector isospin.

The second aim is to formulate a quantum version for the classical argument determining the Weinberg angle.

1. As found in the previous section, the correct identification for the condition determining the Weinberg angle is based on conformal invariance for massless particles, in particular photons. The vanishing of the emission vertex for the holomorphic photons is the condition. For antiholomorphic parts of photons the emission vertex does not vanish.

2. The averages are estimated as expectation values of the classical induced fields A_{ind} acting as local multiplicative operators acting on the induced spinor field at the position of the vertex. The average is for the matrix element of the of $\Gamma^w\gamma_w$ between fermion modes at the partonic 2-surface and there is averaging over the WCW wave function for the position of the fermion line at the partonic orbit X^3 restricted to the partonic 2-surface X^2. γ_w is the induced photon gauge potential and one considers all partonic 2-surfaces, which are holomorphic so that the complex coordinate z for $CP_1 \subset CP_2$ is a holomorphic function of the complex coordinate w for $CP_1 \subset \delta M^4_+$. Holography = holomorphy hypothesis is not necessary since all 2-surfaces allow a conformal structure so that also Lagrangian minimal surfaces can be holomorphic and homologically trivial geodesic sphere of CP_2 can be considered.

   It turns out that there are strong conditions from the unitarity, which force to generalize the notion of Weinberg rotation to 3-D rotation mixing γ, the vectorial part left from the axial V_{03} proportional to $Z^0 \propto (F-1)/F$ when the left-handed Z^0 is extracted and the vectorial V_{12} defining counterpart of strong $U(1)$ field coupling to vectorial part of isospin identifiable as strong isospin. The presence of this term in the gauge couplings came as a surprise.

3. The generalized Weinberg mixing involves 2 angle parameters: the Weinberg angle θ_W and a second angle ϕ_W, near to $\pi/2$, which determines the strong interaction coupling strength $\alpha_s(U(1))$ identified by electric-magnetic duality as the inverse of the QCD coupling strength $\alpha_s(QCD)$.

   Partonic dynamics involve the geometrodynamics of the partonic giving also rise to a topological description of CKM mixing in terms of different topological mixings of partonic 2-topologies [K3] and also the transitions between the modes of modified Dirac equation inducing changes of the Weinberg angle describing Weinberg mixing.

4. The modified Dirac equation associated with Kähler-Chern-Simons (K-C-S) action at the partonic orbit is analogous to the Dirac equation in a monopole field. K-C-S action must be taken to be the sum of M^4 and CP_2 parts so that $J(M^4)$ and $J(CP_2)$ give rise to independent induced gauge fields. $J(M^4)$ possibly relates to gravitation or what might be called strong gravitation.

The intuitive expectation is that the analogs of harmonic oscillator states in the magnetic field are obtained and they are localized at radii corresponding to Bohr orbit radii. One can consider two alternative interpretations.

Option 1: Fermionic spin harmonics are in question. This option suggests that holomorphic spinor monopole harmonics (SpMHs) describe the situation. This option fails to predict a universal value of Weinberg angle.

Option 2: A WCW wave function for the position of the fermion vertex at the projection of the parton surface to CP_2, interpreted as CP_2 twistor sphere. Its projection or lift to M^4 would have a natural interpretation as M^4 twistor sphere defining its multiple covering [L8, L9]. For the twistor lift of TGD, these twistor spheres correspond to fibers of twistor spaces $T(M^4)$ and $T(CP_2)$.

At the level of H these spheres would correspond to a geodesic sphere of CP_2 and a sphere at the boundary of light-cone with constant value of light-like radial coordinate. This sphere is a geodesic sphere in the metric of the light-cone and defines a physical realization of the twistor sphere. Since the line connecting the tips of CD is effectively a hole in CD, this sphere is homologically non-trivial. The homologically non-trivial geodesic sphere of CP_2 would define a concrete representation of the twistor sphere of the twistor bundle of CP_2.

An interesting conjecture deserving a closer study is that the WCW wave functions correspond to a unitary representation of the Lorentz group at either twistor sphere. The Lorenz group could also relate to the Virasoro algebra of conformal symmetries at the partonic 2-surfaces or be identifiable as Lorentz transformations acting as isometries of the light-cone proper time constant hyperboloids inside CD. Using Google with the prompt "Unitary representations of Lorentz group at sphere" one obtains a nice summary of these representations (see the Appendix). There are two kinds of representations.

1. The Principal Series of unitary representations (PSUR) are labelled by a single continuous parameter $1+i\eta$ and half integer m having interpretation as spin (see this). If fermion modes are in question (the first option), one would have $m = 1/2$. η is analogous to the hyperbolic counterpart of angular momentum. The spinor modes are spin harmonics and the inner product is local.

2. For the exceptional series of unitary representations (ESUR) for which η is imaginary, which is strictly spinless ($m = 0$) and involves all angular momenta, has standard spherical harmonics as basis and has bilocal rather than local inner product. This representation could be natural for the WCW option and is attractive also by its uniqueness. One can argue that the states of the ER are analogous to the bound states of the hydrogen atom and therefore more strongly localized around the origin than the states of the PSUR. The ESUR is discussed in the Appendix in more detail and it relates to a conformal field theory (CFT) at sphere.

2.2.2 The earlier view of the Weinberg angle

I proposed in my thesis published 1982 a model predicting the value of the Weinberg angle assuming Weinberg mixing of the TGD counterparts of the $U(1)$ gauge potential identified as a multiple of Kähler gauge potential and B_3 component of $SU(2)_L$ gauge potential.

The basic formulas of the standard model are

$$\gamma = cos(\theta_W)B_0 + sin(\theta_W)B_3 \ , \quad Z^0 = -sin(\theta_W)B_0 + cos(\theta_W)B_3 \ . \tag{2.9}$$

The model determines the ratios of standard model couplings so that only the $U(1)$ coupling g_1 is free. In TGD it corresponds to Kähler coupling and its discrete spectrum would be determined by quantum criticality. The analogy of Kähler coupling strength with critical temperature is important

since in TGD path integral is replaced by functional integral and the mathematical analogy with thermodynamics becomes strong.

In standard model one obtains conditions on the ratios of coupling constants g_1, g, g_Z and e associated with the gauge potentials for $U(1)$, charged $SU(2)_L$ bosons ,Z_0 and γ.

$$\frac{g_1}{g} = tan(\theta_W) \quad , \quad \frac{g_Z}{g} = \frac{1}{cos(\theta_W)} \quad , \quad \frac{e}{g} = sin(\theta_W) \quad . \tag{2.10}$$

2.2.3 Basic formulas of TGD for gauge potentials and fields in relation to standard model formulas

In TGD the induced gauge potentials B_{ind} correspond to the standard model gauge potentials gB_{st}, where g is the corresponding coupling constant. The reason is that in the purely geometric approach it is not possible to identify the couplings g at the level of the action principle.

In TGD, gauge potentials and fields differ from the their standard model counterparts in that the gauge couplings are adsorbed to them so that no gauge couplings appear explicitly: therefore the rule is $gA_{st} \leftrightarrow A_{TGD}$.

The induced gauge potentials for the neutral part F_{nc} of the electro-weak gauge field coming from CP_2 (M^4 Kähler form gives an additional contribution to the Kähler gauge potential and it will be that it is necessary) are given by the formulas

$$\begin{aligned} & g_1 B_0 \leftrightarrow Be^3 = 2re^3 \quad , && gB_3 \leftrightarrow V_{12} = (2r + 1/r)e^3 \quad , \\ & g_Z Z^0 \leftrightarrow V_{03} = (r - 1/r)e^3 \quad , && e\gamma \leftrightarrow [2(1-p)r - sin^2(\theta_W)/r]e^3 \quad . \end{aligned} \tag{2.11}$$

Note that the Kähler form is given by $J = dB$, $B = 2re^3$. e^3 is proportional to $r/(1+r^2) = r/F$ and a part depending on angles and their differentials for Eguchi-Hanson coordinates. Therefore the dependence r relevant for calculations, can be expressed by shorthand notations $B = (F-1)/F$, $V_{03} = (F-2)/F$ and $V_{12} = (2F-1)/F$.

One must also study the expressions for the curvature R_{AB} given in [L13].

1. The charged part of the CP_2 curvature form is given by

$$R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \quad , \quad R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \quad . \tag{2.12}$$

   $R_0 1\Sigma^{01} + R_{23}\Sigma^{12} = R_{01}(\Sigma^{01} - \Sigma^{12})$ gives the left-handed part of W_1 and the right-handed part vanishes. Same is true for W_2.

2. The non-charged parts of curvature tensor are given by

$$R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \quad , \quad R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \quad . \tag{2.13}$$

   R_{03} is the axial part and R_{12} is the vectorial part, which mixes with Kähler form in Weinberg mixing.

   The non-charged part $R_{nc} = R_{03}\Sigma^{03} + R_{12}\Sigma^{12}$ can be written as sum of left and right handed contributions as

$$R_{nc} = R_{nc,L}(\Sigma^{03} - \Sigma^{12}) + R_{nc,R}(\Sigma^{03} + \Sigma^{12}) \quad . \tag{2.14}$$

   By comparing the two expressions, one finds

$$R_{nc,R} = 3(e^0 \wedge e^3 + e^1 \wedge e^2) \quad , \quad R_{nc,L} = e^0 \wedge e^3 - e^1 \wedge e^2. \tag{2.15}$$

The charge matrices in question [L13] are defined as

$$Q_{em} = \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \quad , \quad Q_Z = I_L^3 - pQ_{em} \quad , \quad I_L^3 = \frac{(\Sigma^{12} - \Sigma^{03})}{2} \quad . \tag{2.16}$$

Also the formulas for vierbein connection and vierbein of CP_2 [L13] are needed and are added below for convenience.

1. The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{ll} V_{01} = -\frac{e^1}{r} \ , & V_{23} = \frac{e^1}{r} \ , \\ V_{02} = -\frac{e^2}{r} \ , & V_{31} = \frac{e^2}{r} \ , \\ V_{03} = (r - \frac{1}{r})e^3, & V_{12} = (2r + \frac{1}{r})e^3. \end{array} \tag{2.17}$$

2. The vielbein vectors are given by the following formulas.

$$\begin{array}{lllllll} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 & = & \frac{r\sigma_2}{\sqrt{F}} \ , & e^3 & = & \frac{r\sigma_3}{F} \ . \end{array} \tag{2.18}$$

Here the quantities σ_i are defined as

$$\begin{array}{lll} r^2\sigma_1 & = & Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 & = & -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 & = & -Im(\xi^1 d\bar{\xi^1} + \xi^2 d\bar{\xi^2}) \ . \end{array} \tag{2.19}$$

Here the Eguchi-Hanson coordinates for CP_2 are defined as $(\xi^1, \xi^2) = (rcos(\Theta/2)exp(i(\Psi + \Phi)), rsin(\Theta/2)exp(i(\Psi - \Phi)))$ [L13] .

2.2.4 Weinberg mixing in standard model *viz.* TGD

In the standard model, Z^0 and γ emerge via the Weinberg mixing of the gauge potentials A_0 and B_3. The first guess is that one can translate this mixing to the TGD level for fields gV_{12}, g_1B_K giving g_ZZ^0 and $e\gamma$. It turned out that the actual picture is more complex.

The Weinberg mixing for the standard model fields is given by

$$\gamma = c_W B_0 + s_W A_3 \ , \quad Z^0 = -s_W B_0 + c_W A_3 \ . \tag{2.20}$$

c_W and s_W are shorthands for the $cos(\theta_W)$ and $sin(\theta_W)$.

The description of Weinberg mixing in the standard model must be translated to TGD description. The basic difference is that in the TGD coupling constants are absorbed to classical gauge potentials: $gA_{st} \leftrightarrow A_{TGD}$ and one must dig them out to make possible the comparison between the two views.

In the standard model the mixing occurs between $U(1)$ gauge potential and Z^0. In TGD, $g_ZZ^0I_L^3$ corresponds to $g_ZZ^0 \propto 2V$, $F = 1 + r^2$. It has a vectorial partner $gG_1 \propto V$ with the same functional form left when $g_ZZ^0I_L^3$ is extracted from $V_{03}I_A^3$.

$$gA_3 = \propto (r - \frac{1}{r})e^3 \ , \quad g_1B_0 = B_K = 2re^3 \ . \tag{2.21}$$

2.3 What does one mean with Weinberg rotation?

The goal of this section is to derive an estimate for the Weinberg angle based on a generalization of Weinberg rotation (WR). This involves the identification of the two vector fields gG_1 and g_sG_2. g_sG_2 can be assigned with the space-time description of strong interactions analogous to that provided by old-fashioned hadron physics. The original view was that after WR, parts of $e\gamma$ must be transferred to Z^0 and to g_sG_2. Both fields get coupled to photons. In the case of g_sG_2 the coupling has interpretation in terms of vector boson dominance. The realization that the dual of Weinberg rotation must be performed for the charges allowsed to get rid of the ad hoc notion of transfer but changed dramatically the identification of p.

2.3.1 A surprising connection between Weinberg mixing and strong interactions

One can try the deduction of Weinberg mixing from scratch without trying to use old and possibly erratic formulas. The optimistic expectation is that deduction of standard models goes through as such and gives all the required basic predictions since the mixing itself does not depend at all on the representation of fields as classical fields. One obtains also very simple explicit expressions for $e\gamma$ and g_ZZ^0 without any dependence on Weinberg angle, which is also very nice. Only charge matrices change in the mixing. M^4 would correspond to $r - \frac{1}{r}$ appearing in the axial part of the neutral part of connection V_{03}.

For a fleeting moment, I thought that the situation is now clear but then I realized that the vectorial parts couplings from V_{03} and V_{12} give rise to an anomalous term coupling to vector isospin. It seems that the only physically plausible interpretation is as vectorially coupling strong interaction. Also the necessary $U(1)$ term coming from M^4 could relate to strong interaction. The basic crazy proposal indeed is that electroweak interactions reduce to strong interactions or vice versa. Hence the problems were not only technical and not totally due to my miserable computational skills. This motivated a fresh loo at the Weinberg mixing from the beginning instead of using formulas deduced almost half a century ago.

2.3.2 The identification of the classical "gluon" field

The field $gG_1 == VI_V^3/2$ is left when $g_ZZ^0 = 2VI_L^3$ (here the sign is not completely certain) is extracted from $V_{03}I_A^3$. Second functionally independent vectorially coupling field is $g_sG_2 = V_{12}I_V^3 = W$. The tentative interpretation of these fields is as weak and strong fields. The generalized 3-D WR should mix these fields.

1. The condition $\langle e\gamma \rangle = 0$ is highly sensitive to the detailed identification of the fields gG_1 and g_sG_2. Even normalization matters. The way to fix these fields is to assume that for $p = 0$ $e\gamma$ reduces the superposition $B_KQ_K + agG_1I_V^3 + bg_sG_2I_V^3$ to $B_KQ_{em} = 2r(Q_K + I_V^3)$. This fixes the coefficients a and b in $agG_1 + bg_sG_2$.

2. This conditions gives for $p = 0$

$$aW + bV = 0 \ . \tag{2.22}$$

This gives

$$(a, b) = \frac{1}{3}(1, 1) \ . \tag{2.23}$$

The expressions of fields gG_1 and g_sG_2 are

$$gG_1 = \tfrac{1}{3}V \ , \quad g_sG_2 = \tfrac{1}{3}W \ . \tag{2.24}$$

Contrary to the first guess, it turns out that g_sG_2 allows interpretation in terms of strong interaction and gG_1 having the same form as g_ZZ^0 field plays the same role as Z^0 field in the case of ordinary Weinberg mixing. This should have been obvious from the beginning.

Warning: in the formula for gG_1, one cannot exclude the possible presence of a sign factor related to the identification of gG_1 as classical induced gauge potential $g_Z Z^0$.

2.3.3 Is Weinberg rotation 3-dimensional in TGD framework?

Consider now the crucial steps related to the improved understanding of WR in the TGD framework.

1. As one extracts the left handed $Z^0 \propto V$ term from the axial $V_{03}I_A^3$ term $\propto V = (F - 2/F)$, where $F = 1 + r^2$ comes from e^3 this leaves a part coupling to vectorial isospin I_V^3, which is also proportional to V and thus to Z^0. This vectorial part proportional to Z^0 would naturally replace Z^0 in Weinberg mixing, identified in the standard model as mixing of B_K and Z^0. This part emerges in Weinberg mixing to photon and must be transferred to Z^0 to give it its coupling to Q_{em}.

2. The presence of $V_{12}I_V^3 = WI_V^3$, $W = (2F-1)/F$ is something totally new. A cautious suggestion is that TGD predicts a new field coupling to vector isospin I_V^3 and the interpretation in terms of old-fashioned hadron physics is highly suggestive. One has also M^4 Kähler force also (without it gauge boson propagators are impossible). Could these 2 new vectorial long range forces without parity breaking provide a description of strong interactions at space-level in the TGD counterpart of the quark gluon phase?

For years I have applied CVC and PCAC thinking to TGD (see for instance [K9]) and proposed a duality between QCD type description and old-fashioned hadron physics. This duality would correspond to the duality between H phase description and X^4 phase descriptions in the TGD view as a generalization of the QCD description and TGD inspired quantum measurement theory. One must specify precisely the physical conditions involved with WR. The following is a modest attempt to be quantitative.

1. The situation in absence of mixing

One should get the correct charge matrix Q_{em} for $p = 0$ state for which the mixing is trivial. For $p = 0$ $e\gamma$ must be a linear combination of $B_K Q_K = UQ_K$, $U = 2(F - 1)/F$ of the vector part $gG_1 I_V^3$ of $V_A = V_{03}I_A^3 \propto Z^0$ left after the extraction of $Z^0 I_L^3$ and $V_{12}I_V^3 \equiv g_s G_2 I_V^3$ cancelling the $1/r$ term to get Q_{em} correctly.

$V_A = V_{03}I_A^3 = VI_A^3$ can be expressed as

$$V_{03}I^3A = g_Z Z^0 I_L^3 + gG_1 I_V^3 \quad , \quad g_Z Z^0 = 2gG_1 \quad . \tag{2.25}$$

In $p = 0$ state $gG_1 \propto VI_V^3$ and $g_s G_2$ must be added to $e\gamma = UQ_K$ so that their sum reduces to UQ_{em} for $p = 0$:

$$e\gamma = B_K Q_K + (2/3)(gG_1 + g_s G_2) = UQ_{em} \text{ for } p = 0 \quad . \tag{2.26}$$

2. The vectorial part of the gauge spinor connection contains two contributions

TGD predicts the symmetries of the electroweak coupling structure. My long-held belief that TGD predicts only the counterparts of electroweak fields as gauge potentials coupling to H spinors turned out to be wrong.

Consider first the situation without Weinberg mixing ($p = 0$).

1. In this case $e\gamma$ reduces to $e\gamma = B_K = 2r$. This gives a strong constraint on the vector part of $e\gamma$ for $p = 0$ since the additional contributions must sum up to zero in this case. One can deduce the charge matrices and couplings from the vierbein connection. Charge matrix should emerge from the neutral part of the spinor connection automatically.

2. The extraction of the left handed part form the axial part $V_{03}I_A^3$ of the connection leaves a vectorial part $gG_1 = g_Z Z^0 I_L^3 \propto 2VI_L^3$ proportional to Z^0. Z^0 must have coupling to Q_{em}.

The first proposal was that after WR the part of $e\gamma$ proportional to gG_1 can be added to Z^0 to give it a coupling to em charge. This is admittedly an ad choce procedure and this "transfer" occurs if the charges associatd with gauge potentials experience the dual of the Weinberg rotation.

3. What I failed to realize is that the second term $g_sG_2 \propto WI_V^3$ coupling to vector isospin has no counterpart in the standard model. This changes the formula for Q_{em} unless one interprets the new vectorial term as a new intaction. An obvious guess is as the spacetime level representation of strong interaction! Also $B_K(M^4)$ gives a new vectorial interaction, which is vectorial. Could g_sG_2 and $B_K(M^4)$ correspond to the gauge potentials in the Cartan algebra of the color group?

4. gG_1 and g_sG_2 are functionally independent classical fields. The Weinberg rotation gives for $e\gamma$ a part proportional to pG_1 and it can be transferred to Z^0 to give it a coupling to em charge. $g_sG_2I_V^3$ would give rise to a new classical field having an interpretation in terms of strong force coupling to vectorial isospin. There would be no parity violation. This conforms with the fact that there is no coupling of classical gluons to fermions at fundamental level. The strong force would emerge independently of whether the $\langle\gamma\rangle = 0$ argument, which could have an interpretation in terms of masslessness of the photon, makes sense.

5. The belief held hitherto has been that TGD predicts only the counterparts of electroweak gauge potentials. There would be no classical gluons coupling to fermions. The surprise was that the vector part $gG_1I_V^3$ and $g_sG_2I_V^3$ contain two functionally independent parts corresponding to $2r$ and $1/r$ and $g_sG_2I_V^3$ is something totally new.

6. Color algebra corresponds to the holonomy algebra of CP_2 to $U(2)$ subalgebra of color group. Could the neutral gluons somehow relate to these two parts? Could $g_sG_2I_V^3$ be identified as space-time as an analogs of classical gluon field in the electromagnetically neutral 2-D Cartan algebra of $SU(3)$? Note that in TGD colored gluons correspond to fermion-antifermion bound states at the level of the embedding space H?

   g_sG_2 corresponds to the first counterpart of the second Cartan algebra color gauge potential coupling to I_c^3. What about the second Cartan algebra gauge potential coupling to Y_c. It took a long time to realize that electric-magnetic duality, which is an old idea of TGD [K2] described in the previous section, could come to the rescue. The second color gauge potential could correspond to magnetic/holonomy charge.

3. Is the Weinberg rotation be 3-dimensional?

Could Weinberg mixing occur between Kähler gauge potential and g_1G_1 and g_sG_2? The Weinberg mixing would be 3-D rather than 2-D.

$gG_1 \propto VI_V^3$ of $V_A = V_{03} = 2V$ remaining after the subtraction of the left-handed part of $Z^0 = VI_L^3$ and the vector part g_sG_2 of $V_{12} \propto WI_V^3$ are functionally independent. This suggests that one must take both of them into account. The fact that Weinberg mixing is between Z^0 in standard model suggests that $gG_1 \propto VI_V^3$ corresponds to electroweak interactions whereas $g_sG_2 \propto WI_V^3$ could be tentatively assigned to the space-time description of strong interactions.

The mixing would be 3-D and between B_K, the vectorial part $gG_1 \propto VI_V^3$ left from $V_{03}I_A^3$ after the extraction of $Z^0 = 2VI_L^3$ and $g_sG_2 \propto V_{12}I_V^3 \propto WI_V^3$. In the geometric picture, B_K would correspond to the vertical axis and gG_1 and g_sG_2 would lie in the horizontal plane. The generalized Weinberg rotation be 3-D and parameterized by Weinberg angle θ_W characterizing the tilt of vertical axes and angle ϕ_W characterizing a rotation in the V_12, Z_V plane.

2.3.4 Generalized Weinberg rotation

The presence of a third player in the game requires a generalization of WR.

1. The rotation corresponds to a rotation in plane followed by a tilt and gives rise to a $SO(3)$ matrix R with orthogonal rows with unit length

$$R = \begin{pmatrix} c_W & s_W sin(\phi_W) & s_W cos(\phi_W) \\ -s_W & c_W sin(\phi_W) & c_W cos(\phi_W) \\ 0 & -cos(\phi_W) & sin(\phi_W) \end{pmatrix} \tag{2.27}$$

Here the shorthand notations $c_W = cos(\theta_W)$ and $s_W = sin(\theta_W)$ are used. The sign factors are chosen so that the determinant of R is unity. R acts on the vector $e\gamma = UQ_{em}$ corresponding to the situation for $p = 0$. This choice is perhaps the simplest one. The interpretation could be that the third direction corresponds to strong interactions. For $sin(\phi) = 0$ this gives an ordinary WR matrix and this case will be considered first.

2. The first component of $e\gamma$ is naturally $B_K = 2r$ corresponding to Q_K. $gG_1 = V$ and $g_sG_2 = W$ ahave 2 functionally independent parts and one can form from them a linear combination, which for $p = 0$ is simply $B_3 = UI_V^3$ and can be formed as a linear combination of $2(gV_{12} + g_sZ_V)/3 = U$. Here there is uncertainty due to the normalization factors. The sum $B_KQ_K + B_3I_V^3 = U(Q_K + I_V^3)$ has the correct charge matrix for $p = 0$.

The effect of the generalized WR defined by R is as follows.

1. Photon as vector $(B_K = U, U, 0)$ for $p = 0$ transforms to

$$e\gamma = e \begin{pmatrix} c_W & s_W cos(\phi_W) & s_W sin(\phi_W) \\ -s_W & c_W cos(\phi_W) & c_W sin(\phi_W) \\ 0 & -sin(\phi_W) & cos(\phi_W) \end{pmatrix} \begin{pmatrix} g_1B_0 \\ gG_1 \\ g_sG_2 \end{pmatrix} \tag{2.28}$$

$$\begin{pmatrix} g_1B_0 \\ gG_1 \\ g_sG_2 \end{pmatrix} = \begin{pmatrix} U \\ \frac{1}{3}V \\ \frac{1}{3}W \end{pmatrix} \tag{2.29}$$

The condition that Weinberg rotation is an orthogonal rotation fixes the normalization factors and also sign factors to a high degree.

2. The generalization of the Weinberg mixing requires a generalization of the formulas for the coupling constant ratios. The simplest generalization that one can imagine is as follows:

$$\begin{array}{lll} \frac{g_1}{g} = \frac{cot(\theta_W)}{sin(\phi_W)} \ , & \frac{e}{g_1} = cos(\theta_W) & \frac{e}{g} = sin(\theta_W)sin(\phi_W) \ , \\ \frac{e}{g_s} = sin(\theta_W)cos(\phi_W) \ , & \frac{g_s}{g} = tan(\phi_W) \ . & \end{array} \tag{2.30}$$

These ratios are consistent with each other. For instance, $e/g_1 = (e/g) \times (g/g_1)$ and $g_s/g = (e/g) \times (g/g_s)$ are satisfied.

The value of $sin(\phi_W)$ near unity implies that g_s is large, which conforms with the assignment of the field W to strong interactions. These conditions imply $g_s/g = tan(\phi_W)$. The weak coupling g of the standard model corresponds to the coupling $\tilde{g} = g/cos(\phi_W)$ in the sense that one has $e/\tilde{g} = s_W$.

3. What is remarkable that if the wave function for the position of the fermion vertex as edge on partonic 2-surface is known, the expectations for $\langle (F-1)/F \rangle$ and $\langle 1/F \rangle$ can be predicted so that p, $cos^2(\phi)$ and various Higgs type expectations can be calculated.

2.3.5 The effect of generalized Weinberg rotation

Consider now the effect of generalized Weinberg rotation (WR).

1. The matrix representation of generalized Weinberg rotation as orthogonal matrix for which the inverse matrix is transpose obtained by changing rows and columns, is given by

$$R(\theta_W,\phi_W) = \begin{pmatrix} cos(\theta_W) & sin(\theta_W)sin(\phi_W) & sin(\theta_W)cos(\phi_W) \\ -sin(\theta_W) & cos(\theta_W)sin(\phi_W) & cos(\theta_W)cos(\phi_W) \\ 0 & -cos(\phi_W) & sin(\phi_W) \end{pmatrix} \quad . \tag{2.31}$$

   The rows and columns are orthogonal and the determinant equals 1 as is easy to verify at the limit $(\theta_w,\phi_w) \to (0,0)$.

2. The effect of the WR on $e\gamma = B_K Q_{em}$ is given by

$$e\gamma \to \frac{e}{g_1} c_W \times U + + \frac{e}{g} s_W sin(\phi_W)\frac{\epsilon}{3} V + \frac{e}{g_s} s_W \frac{1}{3} cos(\phi_W) W \quad . \tag{2.32}$$

   This formula conforms with the view of the standard model about WR as rotation mixing B_K and Z^0.

3. The generalization of the formulas for the coupling constant ratios was already proposed. In the case of $e\gamma$, the proposed formulas are $e/g_1 = cos(\theta_W)$, $g/g_1 = cot(\theta_W)/sin(\phi_W)$, $e/g = sin(\theta_W)sin(\phi_W)$, and $e/g_s = sin(\theta_W)cos(\phi_W)$. The modification is small if $sin^2(\phi_W)$ is near unity. The substitution to the outcome of WR gives the expressions for $e\gamma$, gG_1 and g_sG_2 involves only $p = sin^2(\theta_W)$ and $1 - cos^2(\phi_W) \equiv 1 - c^2 \equiv s^2$.

   By using $(e/g_1, e/g_s, e/g) = (c_W, s_W sin(\phi_W), s_W cos(\phi_W))$, this formula can be cast into the form

$$\begin{aligned} e\gamma &= A \times U + B/F \quad , \\ A &= 1 - p + \tfrac{p}{6}[2c^2 + \epsilon(1-c^2)] \quad , \\ B &= \tfrac{p}{3}[c^2 - \epsilon(1-c^2)] \quad . \end{aligned} \tag{2.33}$$

   Here the shorthand notations $c^2 = cos^2(\phi_W)$ and $s^2 = sin^2(\phi_W)$ are used.

2.3.6 Dual Weinberg rotation for charges

The overall physical picture looks nice but the arguments related to the deduction of the Weinberg angle are far from convincing.

1. The notion of 3-D Weinberg rotation is rather convincing although the details can be criticized. Holomorphic scalar monopole harmonics (ScMHs) giving $Z = 1/2$ and $p = 1/4$ looks nice but might be wrong since the very idea of the massivation is that conformal invariance is broken by warping for fermions and ScMH:s are associated with partonic 2-surfaces of fermions, not of photon.

2. The transfer of pieces from $e\gamma$ to Z and to the strong $U(1)$ boson g_sG_2 is physically well-motivated but admittedly an ad hoc procedure and should be understood at a deeper level.

3. In the original model based on the transfer idea, the value $x = 1$ parametrizing the transfer for g_sG_2 is an ad hoc guess and $x = 1.1$ to get acceptable value of the Weinberg angle is even more so. The em coupling pc^2Q_{em} would conform with what is done in the case of Z^0 but would lead to a catastrophe and a value of p near unity.

Concerning the deeper mathematical interpretation of the transfer operation, the only thing that comes to mind is the following.

1. The 3 gauge potentials $e\gamma$ g_1G_1 and g_sG_2, and charge matrices multiplying them are like vectors and their duals. This suggest that the Weinberg rotation is performed for the gauge potentials whereas its inverse (transpose) rotates their charge matrices. This should give the desired modification of charge matrices.

   One might argue that the charge matrices for g_1G_1 and g_sG_2 are given by I_V^3 and identical and not orthogonal in the sense that the trace for their product would vanish. One way to achieve vanishing of the trace might be that the isospin matrices I_1^3 and I_2^3 are proportional to the unit matrix 1 and chirality operator Γ^9 for H so that the trace would vanish. At least in this case the rotation makes sense.

2. $e\gamma$ charge matrix should remain Q_{em}: this is the key aspect of the symmetry breaking mechanism.

3. Z^0 charge matrix could receive part pQ_{em}. The strong boson charge matrix should receive pc^2Q_{em} giving rise to vector boson dominance. As a matter of fact, transfer should be xps^2Q_{em}: otherwise the value of the Weinberg angle turns out to be near unity.

The transpose of the Weinberg rotation matrix, already given, is given by

$$R^T(\theta_W,\phi_W) = \begin{pmatrix} cos(\theta_W) & -sin(\theta_W) & 0 \\ sin(\theta_W)sin(\phi_W) & cos(\theta_W)sin(\phi_W) & -cos(\phi_W) \\ sin(\theta_W)cos(\phi_W) & cos(\theta_W)cos(\phi_W) & sin(\phi_W) \end{pmatrix} . \quad (2.34)$$

The entity $\sum_i A_iQ_i$ is invariant under combined rotations. This is like passive rotation in which basis vectors and coefficients of a vector change but the vector remains as such. The physically observed charge matrix and gauge potentials change. The rotation is analogous to the transition to unitary gauge in the standard model and spontaneous symmetry breaking motivates the preferred gauge.

This inverse of the Weinberg matrix should rotate the charge matrices. But what are they? The duality suggests that the charge matrices are $Q_{em}/e, I_{V,1}^3/g$ and $I_{V,2}^3/g_s$. g_1G_1 (g_sG_2) would receive the addition $sin(\theta_W)sin(\phi_W)(e/g)Q_{em}$ ($sin(\theta_W)cos(\phi_W)(e/g_s)Q_{em}$. ps^2Q_{em} is required by the Z^0 charge matrix and suggests pc^2Q_{em}. This requires $e/g = sin(\theta_W)sin(\phi_W)$ and $e/g_s = sin(\theta_W)cos(\phi_W)$. These conditions are satisfied.

Weinberg rotation for the charge matrices gives for the vector part of Z^0 and thus also Z^0 coupling to Q_{em}. This conforms with the standard model prediction. This happens also in the case of g_sG_2 and could explain vector boson dominance which is a concept of old-fashioned hadron physics. Vector boson dominance means that photons have a coupling to vector bosons. Quanta Magazine tells about the most recent twist related to the anomaly of muon magnetic moment (see this). In the city of Novosibirsk in southern Siberia, the VEPP-2000 collider has studied electrons-positron collisions since the turn of the millennium. The VEPP-2000 collider is operating at 6,000 times lower energy than LHC. The pion pair production rate was found to differ significantly from expectations. However, the so-called generalized vector dominance model explained the findings [C1].

2.4 Critical questions

The proposed scenario concretizes the proposed connection between strong and electroweak interactions. One can however pose several critical questions.

2.4.1 A more precise view of $\langle\gamma\rangle = 0$ condition

The condition is $\langle\gamma\rangle = 0$ is crucial but rather loosely defined and therefore deserves a critical scrutiny.

1. Photon is massless. What does this mean?

(a) Could the masslessness condition for photon reduce to $\langle\gamma\rangle = 0$ condition? Masslessenss means conformal invariance. Could the condition $\langle\gamma\rangle = 0$ correspond to conformal invariance. In the massivation gauge boson "eats" a Higgs component, which becomes its longitudinal mode. In TGD, this would mean that the gauge boson has a CP_2 part behaving like a scalar field in Minkowski degrees of freedom and giving rise to right-left couplings making the boson massive.

(b) Could the masslessness of photon mean that only the vacuum expectation for the M^4 part $(\Gamma^\alpha(K-C-S)\gamma_\alpha)_{|M^4}$ or the CP_2 part $(\Gamma^\alpha(K-C-S)\gamma_\alpha)_{|CP_2}$ vanishes? Either the condition $\langle\gamma\rangle_{|M^4} = 0$ or $\langle\gamma\rangle_{|CP_2} = 0$ would be true. The two conditions are in principle independent. Could this be the only mathematically consistent option?

At the level of H, photon couplings to fermion pairs are determined by its structure as a superposition of fermion pairs. At the space-time level the vectorial couplings to long range interactions appear in $(\Gamma^\alpha(K-C-S)\gamma_\alpha)_{|M^4}$ so that Weinberg mixing should occur for the M^4 part at least.

(c) If both $(\Gamma^\alpha(K-C-S)\gamma_\alpha)_{|M^4}$ can have vanishing vacuum expectation values, the CP_2 part $(\Gamma^\alpha(K-C-S)\gamma_\alpha)_{|CP_2}$ would have the same charge dependent couplings as photon. This is not true. Since the Higgs couplings depend only on the vector isospin, the neutral part of Higgs would be the CP_2 part of a vectorially coupling field, perhaps the vector part $gG_1I_V^3$ of V_{03}.

This suggests that there is no Weinberg rotation for the CP_2 part $(\Gamma^\alpha(K-C-S)B_{K,\alpha})_{|CP_2}$. The condition $\langle\gamma\rangle_{|M^4} = 0$ would determine the Weinberg angle.

2.4.2 Is the dependence of the induced gamma matrices on the point of partonic 2-surface negligible?

Also the detailed structure of the WR and its generalization must be discussed critically. It has been assumed that the dependence of the induced gamma matrices on the point of the partonic 2-surface can be neglected.

1. Is this really the case? If not, the conditions $\langle\gamma\rangle_{|M^4} = 0$, $\langle\gamma\rangle_{|CP_2} = 0$ and $\langle\gamma\rangle = 0$ are not equivalent. The dependence of the modified gamma matrices on M^4 and/or CP_2 coordinates could imply that the conditions are not equivalent and only one of the conditions can be satisfied.

2. The treatment of $\langle\gamma\rangle = 0$ condition remains exact if M^4 part $D(K-C-S)_{|M^4}$ of the modified Dirac operator does not depend on the point of the partonic surface. If the M^4 projection of the partonic 2-surface can be approximated with a plane, this is the case since M^4 parts of the induced gamma matrices are constant. However, if the complex transversal M^4 coordinate w (assuming H-H and H-J structure) as a function of complex coordinate ξ of the partonic surface defines a map with a non-trivial winding number, it is not clear how good this approximation is.

   If this approximation is good for $\langle\gamma\rangle_{|M^4} = 0$ condition, the estimate of Weinberg angle and parameter $cos^2(\phi)$ is expected to be realistic.

2.4.3 How to estimate the Weinberg angle for a partonic surface of higher genus?

If the CP_2 projection of the partonic 2-surface is a geodesic sphere CP_1 of CP_2, the calculations are easy. However, the CP_2 projection of the partonic 2-surface is not a geodesic sphere in general. How can one evaluate the Weinberg angle from the condition $\langle e\gamma\rangle = 0$ involving an integral over a partonic 2-surface? How strongly the result depends on the genus of the partonic 2-surface? Here comes a pleasant surprise.

1. The genera $g = 0, 1, 2$ to which the observed fermions can be assigned, are special in that they allow a Z_2 involution projecting to sphere proving therefore local coordinates for the parton surface. A geodesic sphere of CP_2 can define a local coordinate chart for the partonic 2-surface. The Z_2 involution from a given part of the partonic 2-surface to the geodesic sphere defines the local chart map.

2. The CP_2 projection of the partonic 2-surface can be chosen to be a geodesic sphere, which is homologically trivial for holography = holomorphy vision. Homological triviality is possible for super-minimal surfaces, which are Lagrangian and thus have a vanishing induced Kähler form are allowed. This aspect is discussed in Appendix.

   Conformal field theory at a partonic 2-surface can be formulated at CP_1 having a lift to its covering provided by the partonic 2-surface of higher genus $g \leq 2$. The existence of the covering provides powerful constraints of the partonic 2-surface in H-H vision fixing the bidegree of the polynomial fixing the partonic 2-surface in $\delta M^4_+ \times CP_1$. This is discussed in detail in Appendix.

2.4.4 Cartan algebra "gluons" viz. standard model gluons

One could also worry about the interpretation of gG_1 and g_sG_2, or at least g_sG_2, as space-time analogs of the Cartan algebra gluons.

1. That only the Cartan algebra is involved actually conforms with the general idea about quantum classical correspondence assuming that only the conserved classical charges and gauge fields for the Cartan algebra sector have a physical meaning at the space-time level. This is in fact realized also in the proposed model for the partonic 2-surfaces predicting vanishing W gauge fields.

2. The color couplings of fermions, which in the standard model do not depend on charge, transform to couplings of space-time gluons, which depend on charge. This charge dependence has a counterpart at the level of Dirac equation in H [L19]. The color multiplets for leptons and quarks and also for their different charge states are different so that there is a breaking of isospin symmetry already at this level so that color and electroweak interactions do not separate completely as in the standard model. This feature conforms with the old fashioned view of hadron interactions.

3. The CP_2 parts of B_K, and "gluons" gG_1, g_sG_2 would give rise to neutral Higgs-like fields. This conforms with the non-vanishing of the expectations $\langle g_1 \rangle_{|M^4}$ and $\langle g_1 \rangle_{|M^4}$. The physical interpretation is that the space-time "gluons" g_1 and g_2 are massive and have a finite interaction range. This conforms with the empirical facts and the idea that confinement and screening are dual ways to describe confinement and massivation.

2.5 Estimate for the Weinberg angle

The estimate for the Weinberg angle from the condition $\langle \gamma \rangle = 0$ involves the following basic steps.

1. The identification the classical fields $gG_1 \propto V_{12} = V$ and $g_sG_2 \propto W$ gG_1 and g_sG_2 is the first step. g_sG_2 is left when the axially coupling g_ZZ^0 is extracted from $V_{03}I^3_A = VI^3_A$.

2. In the state $p = 0$ without WR $e\gamma B_K Q_K + (gG_1 + g_sG_2)I^V_3$ reduces to $e\gamma = UQ_{em}$. This fixes the normalizations of gG_1 and g_sG_2 apart from the sign ambiguity of g_ZZ^0, which is due to my miserable arithmetic skills. One has $gG_1 = 2\epsilon V$ and $g_sG_2 = -2\epsilon W$. Note that the $1/r$ parts cancel in the sum and $gG_1 + g_sG_2 = -\epsilon U$. $\epsilon = 1$ is determined from the condition that Q_{em} comes out correctly.

3. WR is generalized to a 3-D rotation represented by an orthogonal rotation matrix R having as parameters Weinberg angle θ_W characterizing the tilt of the vertical axis defined by B_K and the angle ϕ_W of planar rotation. It took time to realized that the dual (inverse) of this rotation for charges is needed to explain the charge transfer without ad hoc assumptions.

4. A generalization of the formulas for the coupling constant ratios is proposed. The formula $(e/g, e/g_s) = (s_W sin(\phi_W), s_W cos(\phi_W))$ looks like a plausible generalization of $e/g = s_W$. WR should give $e\gamma = cos^2(\theta_W)B_K + ...$ just as in the standard model. This requires $g/g_1 = cot(\theta_W) = (e/g_1)/(e/g) \rightarrow \frac{cot(\theta_W)}{sin(\phi_W)}$.

The modification should be small so that $sin(\phi_W)$ should be near unity. Hence $cos^2(\phi_W)$ must be small and this conforms with the view that gG_1 corresponds to weak interactions and g_sG_2 to strong interactions with a large coupling strength $\alpha_s \propto 1/cos(\phi_W)$.

In the case of $e\gamma$, the proposed formulas are $e/g_1 = \frac{cot(\theta_W)}{sin(\phi_W)}$, $e/g = sin(\theta_W)sin(\phi_W)$ and $e/g_s = sin(\theta_W)cos(\phi_W)$: note that g_s is large for small $cos(\phi_W)$. The substitution to the outcome of WR gives the expressions for $e\gamma$, gG_1 and g_sG_2 involving only $p = sin^2(\theta_W)$ and $1 - cos^2(\phi_W) \equiv 1 - c^2 \equiv s^2$.

After WR, one can express $e\gamma$ in the form $e\gamma = A \times U + B/F$: $A = 1 - ap$ and $B = -bp$ and the condition $\langle e\gamma \rangle = 0$ provides estimate for p.

2.5.1 Monopole harmonics as a model for the evaluation of the Weinberg angle

One can consider the magnetic monopole harmonic oscillator as a model for the states of the fermions at the partonic vertex [C3, C2]. The most plausible interpretation is that these states can be regarded as a wave function for the position of the edge of the fermion line at the partonic 2-surface.

The proposal is that these wave functions are identifiable as monopole harmonics.

1. Monopole harmonics $Y_{q,l,m}$ define a generalization of spherical harmonics labelled by angular momentum l and q having an interpretation as a monopole charge. For a given value of l the $2l + 1$ values of m define an "ordinary" $SU(2)$ multiplet and the l *resp.* $l + 1$ values of k for odd *resp.* even $q + l$ define a second $SU(2)$ multiplet so that one has $SU(2) \times SU(2)$ symmetry.

   A more precise defined due to Martin Ebner goes as follows. The monopole harmonics $Y_{q,l,m}$ have $l = q, q + 1, q + 2, ...$ and $m = -l, ..., l$. Here q is Wu and Yang's charge: the monopole adds the angular momentum $-q\hat{r}$, its radial projection cannot exceed l, and the flux is $2q$ quanta. The lowest multiplet $l = q$ has $2q+1$ states, $\overline{u}^{2q-K}\overline{v}^K = e^{-iq\phi}z^K/F^q$ with $K = 0, 1, ..., 2q$ and $m = K - q$, all of them holomorphic sections. The label k of $Z(q, k)$ is $k = K/2 = (q+m)/2$ and takes the values $0, 1/2, 1, ..., q$. All these states have $l = q$: $k < q/2$ means $m < 0$, not $l < q$, so that $Z(q, k)$ is exact for them and consistent with $l \geq q$.

2. What does the attribute "monopole" mean? Dirac monopole q gives rise to anomalous spin $q/2$. This implies the formula $m = 2k - q$. The value range $m \in [-l, l]$ implies the value range $k \in [(q - l)/2, (q + l)/2]$ for k. Since k and m are linearly dependent, for given q there are only $2l + 1$ states and this conforms with the fact that monopole harmonics are defined on CP_1.

3. Monopole harmonics are closely related to the representations of the rotation group $SU(2)$ (S^3 in its group algebra consisting of functions $f(g) : SU(2) \to G$. $SU(2)$ can act on the argument of the w functions $f(g)$ both from right and left so that the symmetry group is $SU(2)_R \times SU(2)_R$. The irreducible representations are labelled by angular momentum j and the same representation defined by spherical harmonics is associated with both $SU(2)_R$ and $SU(2)_R$. The representation function $D^l_{m,n}(g)$ is characterized by spin values (m, n) associated with the two angle coordinates (Φ, Ψ) of $SO(3)$. These wave functions represent the quantum states of a spherically symmetric rigid body.

4. The restriction of $D^l_{m,n}(g)$ to S^3 with $S^3/O(2) = CP_1$ means the loss of the second angle degree of freedom. Not all harmonics are possible and the formula $m = 2k - q$ expresses this for a given q. When q varies over all its values, all harmonics are however obtained. One can say that q effectively completes CP_1 to $SO(3) = SU(2)$ or that the missing states associated with ψ are represented in terms of magnetic/ homology charge q. One has a homological multiplet formed by partonic 2-surfaces with different homologies. This situation is impossible in standard view but fits nicely to the WCW framework. In the general case $Y_{q,l,m}$ depends on variables $u = \cos(\frac{\theta}{2})e^{i\phi/2}$ and $v = \sin(\frac{\theta}{2})e^{-i\phi/2}$.

5. The monopole harmonics for which the spin projection m is maximal have $m = l = q$ and $k = (q + l)/2 = q$) are holomorphic. The harmonics for which the spin projection is minimal

have $m = -l$ and $k = (q-l)/2 = 0$) are antiholomorphic. For both cases $l = q$ is satisfied. The holomorphic monopole harmonics are especially interesting candidates for wave functions.

6. Google Gemini argues that the condition $l \geq q$ holds true and the argument is that the spin projections of S^3 harmonics to the two directions defined by angles ψ and ϕ are both limited by l. If the interpretation of q is as spin projection, $q > l$ leads to a contradiction.

7. q gives rise to q-fold covering of the partonic 2-surface and this means that a rotation of 2π in at partonic 2-surface means rotation of $2\pi/q$ at its M^4 or CP_2 projection (note that there are two monopole charges involved corresponding to M^4 and CP_2). This implies spin fractionation since the unit of spin from the point of view of M^4 or CP_2 is $1/q$.

   Furthermore, the fact that the total magnetic charge vanishes for the physical particle implies that magnetic charges do not affect the particle's quantum numbers at all. For $q > 0$ there would be anomalous spin or (isospin). Fractional Quantum Hall effect [D1] could be understood in terms of $q = l > 0$ at elementary particle scales for the M^4 projections of partonic 2-surfaces [K8]. The covering space picture suggests that the monopole harmonics are obtained by replacing z with $Z^{1/q}$ in the argument of ordinary spherical harmonics.

8. Monopole harmonics would define a wave function for the position of the vertex as an edge of the fermion line at the partonic surface. Monopole charge would correspond to monopole flux through the partonic 2-surface.

   The question whether this wave function characterizes the position of the fermion line at the orbit of the partonic 2-surface or the position of the singular edge of the fermion line at the partonic 2-surface defining the vertex. Also the interpretation option makes sense in zero energy ontology (ZEO) as wave function in WCW in which the space-time surface as an analog of Bohr orbit is the basic entity. The physical meaning of this difference has been discussed in the previous section.

Martin Ebner has calculated the values of the parameter Z for the monopole harmonic harmonics.

1. The values are given by

$$Z(q,k) = \frac{\langle 1/F \rangle}{\langle U \rangle} = \frac{2(q-k)+1}{2(2k+1)} \quad . \tag{2.35}$$

   As explained, the condition $m = 2k - q$ holds true. The minimal energy situation $[q, m] = [l, -l]$ corresponds to $k = 0$. Physical intuition tells that also antiholomorphic situation corresponds to minimal energy.

2. The minimal option $q = l$, gives $Z = 1/2$ irrespective of the value of q. This however requires that l is integer valued (half-odd integer values of $j = l \pm 1/2$ are possible for spinor monopole harmonics). The option requires holomorphic scalar monopole harmonics (ScMHs), which universally give $p = 1/4$ irrespective of the value of q. These states correspond to holomorphic *resp.* antiholomorphic sections of the monopole bundle. Holomorphy is one of the key assumptions at the space-time level, universality makes this option attractive.

3. The values of Z for the multiplet q, k, $k \in [(q-l)/2, (q+l)/2]$ are in the range $[(q+l+1)/2(l-q+1), (l-q+1)/2(l+q+1)]$. The allowed states form a slice of 2-D lattice structure of q and l values. For given q there are l *resp.* $l+1$ values of k for odd *resp.* even $q+l$. The band of allowed states for a given q is along the line $q = l$ and has width l *resp.* $l+1$ for odd *resp.* even $q+l$. For a given q, there is no upper bound for l. There is a second multiplet of $2l+1$ values of m associated with l. This corresponds to the dynamical $SU(2) \times SU(2)$ symmetry.

2.5.2 Weinberg angle in the model based on dual Weinberg rotation

The dual rotation for the 3 charges involved allows us to get rid of the original idea that the part of charges of g_1G_1 and g_sG_2 proportional to Q_{em} are "transferred" from $e\gamma$. This however destroys the earlier scenario which assumed an anomalously large "transfer" to g_1G_1 and g_sG_2. The condition determining Weinberg angle changes rather profoundly since these transfers do not modify $e\gamma$.

The assumption that the inverse of the Weinberg rotation acts on charge matrices Q_{em} kills also the idea that $x = 1.1$ could allow to have $p = .2315$ universally for the holomorphic ScMHs. In this model, the prediction of the Weinberg angle becomes $p \sim 1$ for the holomorphic ScMHs and is catastrophically large. Therefore one must seriously ask whether the condition $\langle e\gamma \rangle = 0$ produces mere trouble. In any case, if the prediction of the Weinberg angle is possible it requires some new physical insight.

1. The Weinberg rotated $e\gamma$ would be

$$\begin{aligned} e\gamma &= 2(1-p)(F-1)/F + ps^2 \times 2(F-2)/F + pc^2 \times (2F-1)/F \\ &= 2(F-1)/F - p(2-3c^2)/F \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{2.36}$$

   The $2r$ part is not changed at all. Notice that the vectorial part of Z^0 left when left-handed Z^0 is extracted from the axial V_{03} is $2Z^0$.

2. The condition $\langle e\gamma \rangle = 0$ reads as

$$p(2-3c^2) = \frac{\langle 2(F-1)/F \rangle}{\langle 1/F \rangle} = 1/Z \ . \tag{2.37}$$

   This gives the formulas

$$\frac{1}{p} = Z \times (2-3c^2) \ , c^2 = (2/3)(1-p_1/p) \ , \quad p_1 = 1/2Z \tag{2.38}$$

   for p and c^2. At least this formula is incredibly simple. For $c^2 = 0$ this gives $p = p_1 = 1/2Z$. $p \leq 1$ requires $Z \geq 1/2$.

3. $Z = 1/2$ gives $p = 1$ and holomorphic ScMHs giving a universal prediction for p are lost. This could be seen as disappointing. One could however argue that the basic aspect of massivation is violation of conformal symmetry and this occurs for fermions rather than photons and reflets itself in the form of ScMHs. The condition $\langle e\gamma \rangle = 0$ could be regarded as a remnant of conformal invariance and say that the emission of photons with helicity opposite to fermion helicity is zero. Note that warping is important.

   The condition $l \geq q$ guarantees that the rotational energies involving a positive contribution proportional to $l(l+1)$ and a negative contribution proportional to q^2 are positive. It does not exclude $Z \geq 1$.

$Z(q,k) > 1$ is possible for real states and it is interesting to see whether a realistic value of p and c^2 are possible assuming that also the charges suffer Weinberg rotation. For ScMHs the basic formula is

$$Z(q,k) = \frac{2(q-k)+1}{2(2k+1)} \ . \tag{2.39}$$

1. The condition $p \leq 1$ gives $Z \geq 1/2$, which implies

$$q \geq 3k + \frac{1}{2} + 2(2k-1)c^2 \quad . \tag{2.40}$$

For $c = 0$ this gives $q \geq 2k$. For $k = 0$ this gives $q \geq 0$. For $k = 1$ this gives $q \geq 2$ at $c = 0$. For $k = 0$ the values of $p = 1/2Z$ are $p = 1/(2n+1)$ giving $p = 1/3, 1/5, 1/7, 1/9, ...$

2. $2Z(7,1) = 13/3$ gives $p = 3/13 = .2308$ deviating from the experimental value at Z^0 pole by .3 percent. The value of c^2 giving the empirical value $p = .2315$ at Z^0 pole would be $c^2 = 2.104 \times 10^{-3}$. This gives $\alpha(U(1)) = \alpha_{em}/pc^2$ ($\alpha_{em} \simeq 1/128$) and $\alpha_s = 1/\alpha(U(1)) = .0624$, which is 47 percent smaller than the empirical value $\alpha_s = .1187 \pm .0011$ at Z^0 pole. One must remember that holography eliminates the path integral so that the naive comparison is somewhat dangerous.

   The large monopole charge is of course a disturbing feature when one recalls the connection with genus and monopole charge. There are however 2 monopole charges and $q = 7$ could also correspond to M^4_+ monopole charge. The large value of Z conforms with the original intuition that $\langle (F-1)/F \rangle = \langle r^2/1 + r^2 \rangle$ is small. The larger the q, the smaller the value of $\langle (F-1) \rangle$.

2.5.3 Why the anomalous charges of monopole harmonics are hidden?

The physical interpretation of the monopole harmonics is problematic. The quantum numbers in question cannot characterized physical particles and must be hidden. In the previous section the connection with FQHE was mentioned. The geometric picture is that magnetic/homology charge q as winding number means that partonic 2-surface is many-sheeted and has branch points so that the rotation around them in M^4 or CP_2 leads to a new sheet and it takes q turns to get to the original point. This makes possible the fractionation of angular momentum in the case of δM^4_+ and charge fractionization in the case CP_2. At the level of the partonic orbit there is no fractionization.

Could the presence of a branch point or pinch allow the fractionation not possible for ordinary 2-surfaces. Could charge fractionation with a unit of $1/q$ take place and could the condition $l \geq q$ state that only the states without charge fractionation are included. In condensed matter systems this occurs. Could the spectrum of the angular momentum and color isospin projections be quantized in units of $\hbar/q$?

2.5.4 Why the monopole charge $q = 7$?

Why would the monopole charge be equal to $q = 7$? The assumption that the CP_2 radius squared is "dark" effective radius squared and satisfies $R^2 = (7!)^2 l_P^2$ corresponds to $\hbar/\hbar_{min} = (7!)^2$ implies that the "real" CP_2 radius is Planck length. Could the origin of the "7" be here. One can consider two different models.

1. I have proposed that the two 7!:s correspond to the number of sheets over CP_1 associated with δM^4_+ and $CP_1 \subset CP_2$ for parton surfaces $X^2 \subset CP_1 \times CP_1$. Suppose that the states with non-fractional angular momentum at the parton level corresponds in M^4 sector to $q = 6!$ so that one has $\hbar_{eff} = \hbar/7$ and that fractional states have $\hbar_{eff} = \hbar$ for $q = 6!$ and correspond to a 7-fold covering of X^2 over $CP_1 \subset \delta M^4_+$.

   7-fold winding would give rise to angular momentum fractionation in the proposed sense. As already found, the states $(q,k) = (7,1)$ predict in excellent accuracy the $p = 3/13$ for $c^2 = 0$ and the value of $c^2 = 2.104 \times 10^{-3}$ predicts $\alpha_s = .0624$. The deviation from $\alpha_s = .1187$ is 47 per cent.

2. The second option assumes that $\hbar_{eff}/\hbar_{min}$ corresponds to the number of space-time sheets. For $\hbar_{eff} = \hbar$ it would be $(7!)^2$ giving $7! = 5040$ sheets over each CP_1. Identify q as $n_1 = 7! = 5040$ over $CP_1 \subset \delta M^4_+$. To get the value $p = 3/13$ approximately from the formula $1/p = Z(2 - 3c^2)$ for $c^2 = 0$ using the general formula $2Z(q,k) = [2(q-k)+1]/(2k+1)$, one must assume

$$2(q-k)+1 \simeq 13n \quad , \quad 2k+1 \simeq 3n \quad . \tag{2.41}$$

The second equation gives $k \simeq \frac{3n-1}{2}$. The first equation gives $n \simeq 7!/8+1/8$ having $n = 630$ as an integer solution. The deviation is $\Delta n/n \simeq 1/7! = 1/5040 \simeq 2 \times 10^{-4}$. For k this gives $k = 944.5 \simeq 944$.

3. One can calculate the prediction $p_1 = 1/2Z$ for $c^2 = 0$, estimate the parameter $c^2 = (2/3)(1-p_1/p)$ giving the correct value for p and $\alpha(U(1)) = \alpha_{em}/(p \times c^2)$ for $\alpha_{em} = 1/128$ at Z^0 pole for $k \in \{944, 945, 947\}$. The results for k in the range $[942, ..., 947]$ are following

k	p	10^3c^2	$10\alpha_s$	$10^2\Delta\alpha_s/\alpha_s$
942	.2301	3.997	1.184	$-.24$
943	.2304	3.132	.928	-22
944	.2307	2.267	.671	-43
945	.2310	1.401	.415	-65
946	.2313	.535	.158	-87
947	.2316	$-.331$	$-.098$	-108

(2.42)

The experimental value of α_s is $\alpha_s = .1187$ at Z^0 pole. $k = 942$ gives the best estimate for α_s. The error for p is .5 percent and .2 percent for α_s. For $k = 947$ the sign of c^2 becomes negative. At the value 946 the estimate for p is best.

2.5.5 Interpretation of charge fractionation for the monopole harmonics

The successful predictions motivate the question whether it is possible to define these harmonics as fractal monopole harmonics concretely obtained by replacements $(l, m) \to (l/q, m/q)$.

1. In the TGD view, the many-sheeted physics explains geometrically what charge fractionation means. It occurs when one looks from the perspective of M^4 or CP_2 projections. Not if one looks from the perspective of space-time surface or partonic 2-surface.

   In the fractionation the units of angular momentum and charge are scaled down by factor $1/q$ at these levels. The total fractional contribution to the charges vanishes since the monopole charges vanish for physical particles so that this is a hidden layer: Bohmists might like to talk about hidden variables. In physics one would talk about finite measurement resolution. Color confinement is a good analogy.

2. Fractionation of angular momentum might mean that one replaces the argument z of monopole spherical harmonics $Y^l_{m,q}(z)$ with $l \geq q$ with $z^{1/q}$ and obtains $Y^{l/q}_{m/q,1}(z)$ identifiable as fractal scalar harmonics and making sense also when l/q and m/q are fractional. Every harmonic is replaced with q harmonics corresponding to the roots of $z^{1/q}$ realized as different sheets of partonic 2-surface over M^4. q would naturally correspond to the integer n characterizing quantum groups.

3. $z = 0$ corresponds to the branching point or pinch at which sheets meet. I l is multiple of q the wave function vanishes at it and space-time sheets are isolated. In the multi-sheeted case it cannot vanish. The question is how to obtain well-defined monopole harmonics which do not vanish at $z = 0$.

   Monopole charge makes the bundle nontrivial, allowing the interpretation of the harmonics as non-trivial sections. The section should be non-vanishing and could be well-defined also at $z = 0$. The phase however makes the value ill-defined unless the modulus of the section is zero, infinite or the phase vanishes, which requires $m = 0$. Itc could be also infinite at the branching point or pinch but the square integrability should not be lost.

4. Monopole harmonics are matrix elements of harmonics $Y^l_{m,q}$ in group $SO(3)$ representing the degree of freedom for a spherically symmetric rigid body having therefore a wave function in $SO(3)$. How the monopole harmonics as harmonics in S^3 with Euler angles (α, ϕ, psi) are realized at CP_1? Are their projections obtained by p fixing angle ϕ or ψ?

   In CP_2 one has an analogous situation. CP_2 spinor harmonics are obtained from those at S^5 by adding a coupling to the Kähler form of CP_2 and putting the angle coordinate of circle S^1 of S^5 as a circle bundle over CP_2 constant. This raises the question whether also CP_2 allows fractal scalar harmonics making the color isospin and hypercharge fractal? For a partonic 2-surface, CP_2 could be reduced to CP_1.

5. One might try to understand this in terms of fractality: The higher level in the hierarchy associated with $h_{eff} = h$ corresponds to sheets in the 7-layered hierarchy consisting of sheets consisting of sheets consisting of ... sheets. There are 7 sheets at the highest level and each of these sheets decomposes to 6 sheets which each decomposes to 5 sheets.

The physical interpretation of the proposal is highly interesting.

1. The total charges for particles are still integer valued since total homology charges vanish. This is what happens also in the case of the anyons. An interesting question is whether the fractional charges could be seen at the level of parton physics in somewhat the same way as color charges. Whether this might have something to do with the problems of QCD with spin of the proton is also an interesting question. An alternative guess is that the proposed M_{89} hadron physics corresponds to the level $h_{eff} = (6!)^2$.

2. The 7-layered structure suggested by the formula for $\hbar/\hbar_m in = (7!)^2$ suggests in both M^4 and CP_2 sectors a 7-sheeted fractal hierarchy of sheets, which are many particle systems with fermion lines in the role of particles. There is a strong analogy with anyonic systems. The notion of length scale resolution would be important: for instance the genus for a parton surface would characterize only the 7:th layer. The real topology, hidden by finite measurement resolution, would be much more complex. This would of course be anything new.

3. An interesting possibility is that the almost copies of hadronic physics at p-adic length scales corresponding to the 7 Mersenne primes $k \in \{107, 89, 61, 31, 19, 17, 13\}$ correspond to these 7 layers. The quarks of M_{107} hadron physics would contain hadrons of M_{89} hadron physics and the quarks of M_{89} hadron physics would contain hadrons of M_{61} hadron physics, etc.

   The levels 6,7, and 8 could correspond to M_{89}, M_{107}, M_{127} copies of standard model physics and after M_{127} there would be no non-super-astronomical p-adic length scales labelled by Mersenne primes.

   What about the primes 2,3,5,7: why wouldn't they define hierarchy levels? A possible hint is that they define both Mersenne primes and their Gaussian counterparts. Are only the scales defining only ordinary Mersenne primes included in the 7-leveled hierarchy?

4. There is also a hierarchy of Gaussian Mersennes [K11, K6] [L15] corresponding to $k \in \{2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113, 151, 157, 163, 167, ...\}$. For instance, muon and nuclear physics length scale correspond to Gaussian Mersenne $M_{G,113}$. The miracle Gaussian primes corresponding to $k \in \{151, 157, 163, 167\}$ play a key role in TGD inspired quantum biology in subcellular scales [L21]. Could Gaussian Mersennes define their own hierarchy of effective CP_2 scales?

This would also suggest a concretization of the vision about the role of various hierarchies in TGD.

1. The hierarchy of Jones inclusions for hyperfinite factors of type II (HFFs) [K12, K4, K7, K10] [L15] is in a key role in TGD. The included factor would define the measurement resolution: these degrees of freedom would reduce to gauge degrees of freedom in finite resolution. All details at the space-time sheets representing lower layers would represent analogs of Bohm's hidden variables not visible at the level of total quantum numbers. For instance, genus would correspond to genus at the highest level, all topology below it would be below measurement resolution.

2. Jones inclusions are parameterized by index $|M : N| = r = 4cos^2(\pi/n)$, $n \geq 3$ having interpretation as fractal dimension for operator algebra. One can say that it is the ratio of fractal dimensions of the factors appearing in inclusion $N \to M$ of operator algebras. Could the integer n correspond to the monopole charge q?

The hierarchy of inclusions has several connections to TGD.

1. The inclusions can be paired with finite subgroups of $SU(2)$. In particular, the integers $n \in \{6, 7, 8\}$ are exceptional [K7, K4] in the sense that the orbits of the corresponding finite subgroups of $SU(2)$ are genuinely 3-D; they correspond to the isometry groups of tetrahedron, cube, and dodecahedron.

2. McKay correspondence [**?**] relates these integers also to the Lie groups and Kac-Moody algebras associated with exceptional Lie groups E_6, E_7, and E_8 with ranks $n \in \{6, 7, 8\}$ [L4, L5, L6]. The Coxeter number has values is $m = 12, 18, 30$ in these cases and defines the value of Jones index via the formula $3cos^2(\pi/m)$. Could the integer n in $(n!)^2$ correspond to the rank n of the Lie group appearing in the hierarchy.

   These groups relate closely to octonions playing a key role in $M^8 - H$ duality [L12] as one aspects of the TGD counterpart of Langlands duality [L14, L2] relating number theoretic and geometric views of TGD.

   Cube has the same symmetries as octahedron and icotetrahedal tessellation of hyperbolic 3-space involving all these Platonic solids plays a key role in the TGD inspired model of the genetic code [L10].

3. What is intriguing is that the number of Mersenne primes larger than 7 defining non-super-astrophysical p-adic length scales is 8. Could $n = 6, 7, 8$ defining the scales of M_{89}, M_{107} and M_{127} variants of standard model physics would correspond to the exceptional subgroups of $SU(2)$. Could these Mersennes define completely exceptional levels in the hierarchy of scaled variants of standard physics predicted by TGD [L20].

   Could the levels $n = 6, 7, 8$ in the hierarchy of $h_{eff} = (n!)^2$ correspond to these 3 exceptional inclusions with ranks 6,7,8? Could $n = 6$ (M_{89}) correspond to the tetrahedron, could $n = 7$ (M_{107}) correspond to the octahedron and $n = 8$ to M_{127}, which corresponds to the electron mass scale?

   $n = 8$ level would correspond to $\hbar_{eff} = 64 \times \hbar$. Corresponding p-adic length scale corresponds to scale which is 8 times the p-adic length scale $L(127)$ of electron equal to $8L(127) = L(133) = 2^{-9}L(151)$. Icosa tetrahedral tessellation of hyperbolic 3-space is in a key role in TGD inspired quantum biology [L10] and provides a highly predictive model for the genetic code. Could genetic code relate to $n = 8$ level? Amusingly, 64 is the number of genetic codons!

4. There is a hierarchy of Galois groups and their generalizations related to the composition of maps $f = (f_1, f_2) : H \to C^2$ [L15]. The maps $g : C^2 \to C^2$ define hierarchies of dynamical symmetries and this leads to an explanation of the p-adic length scale hypothesis in terms of the function field analogs of p-adic numbers. $f_1 = f_2 = 0$ gives space-time surface X^4 and for u, v constant the roots reduce to partonic 2-surfaces.

   By adding a further function f_3,the roots $f_1 = f_2 = f_3 = 0$ give roots as partonic 2-surface and fermionic lines as their orbits. The compositions $g \circ f_3$ give inclusion hierarchies of ordinary Galois groups. Note that also the 4-D variants of Galois groups permuting space-time regions as roots $f_1 = f_2 = 0$ make sense.

5. The inclusion hierarchies for extensions of rationals defining hierarchies of Galois groups relate naturally to the hierarchy of inclusions for HFFs. The many-sheets partonic surfaces defined by polynomials with high degree expressed as composites of lower level polynomials could realize this hierarchy and provide a physical realization in terms of a hierarchy of standard model physics. In particular, the 7-layered hierarchy for h_{eff} (there would also one additional layers corresponding to M_{127}) might relate to the inclusion hierarchies of extensions or rationals and HFFs.

2.5.6 What about the Principal Series representations of $SL(2,C)$?

The exceptional representations of $SL(2,C)$ [A1], discussed also in the Appendix but from a different perspective, can be considered as a possible way to integrate the unitary representations of $SO(3)$ to a single unitary representation of $SL(2,C)$ is naturally associated with δM^4_+ containing the CP_1. Lorentz group $SL(2,C)$ acts also as global conformal transformations of CP_1 and appears naturally both in string models and TGD.

Could the ScMHs define the representation space for the exceptional representation? Could these harmonics be defined in δM^4_+ instead of CP_1? Could the light-like conformal invariance reduce δM^4_+ to CP_1? These wave functions would be in spirit with the Gromov–Witten theory in which intersections of partonic 2-surfaces correspond to strings along δM^4_+ connecting the vertices at them. Could exceptional Principal Series representations for which the kernel is holomorphic provide an explicit realization for the holomorphic monopole harmonics? Are explicit fractal representations possible?

Google Gemini gives a nice summary of the Principal Series representations.

1. The bilocal inner product based on the kernel can be is realized in C^2 rather than in CP_1 and the properties of the kernel defining the bilocal inner product guarantee the inner product reduces to an effectively local inner product in CP_1.

   Projective invariance allows complex scaling by z_1 or z_2 (North and South hemispheres) giving $z_1 \to \xi_1 = z_1/z_2, z_2 \to 1$.

2. The representations of the Principal Series are labelled by parameters $m = (n + i\nu)/2$ and $m = (-n + i\nu))/2$ ($n \in Z$ and $\nu \in C$). Single-valuedness requires that n is integer. In the general case, the kernel of the inner product is given by $K(z_1, z_2) = (z_1 - z_2)^{-2k}(\overline{z}_1 - \overline{z}_2)^{-2m}$.

   For the exceptional points one has $(k, m) = (m, 0)$ in the holomorphic sector and $(k, m) = (0, n)$ in the antiholomorphic sector so that the kernel is holomorphic or antiholomorphic. The kernel reduces to $K \propto \partial^n_{z_1}\delta(z_1 - z_2)$ and the inner product becomes local. A finite-dimensional non-unitary representation of $SL(2,C)$ appearing in quantum fields theories splits out. The representation space can be assumed to consist of spherical harmonics but also holomorphic monopole harmonics can be considered.

   The cautious conclusion is that the representation space consists of monopole harmonics and the results already obtained remain true almost as such. A stronger conclusion is that only the holomorphic harmonics remain. Martin Ebner has found that $Z = 1/2$ for the ordinary harmonics of the exceptional series, and that $Z(q, k)$ unchanged by any invariant kernel.

Google mentions other kinds of Principal Series representations, which correspond to a real and rational value of $i\nu$. $\nu = -i/2$ is discussed in the Appendix. In this case one has $(k, m) = (n - 1/2)/2, (-n-1/2)/2] = n/2-1/4, -n/2-1/4$. Because the spins are fractional, the representation remains irreducible and the kernel remains bilocal. The kernel is

$$\begin{aligned} K(z_1, z_2) &= (z_1 z_2)^{-n+1/2}(\overline{z}_1 - \overline{z}_2)^{n+1/2} \\ &= [\tfrac{\overline{z}_1 - \overline{z}_2}{z_1 - z_2}]^{-n} \times |z_1 - z_2)| \ . \end{aligned} \tag{2.43}$$

The first factor is a phase and the second factor is a power of $|z_1 - z_2|$. For $\nu = -i/2$ it becomes $1/|z_1 - z_2|$ mentioned in the Appendix. This occurs also for $\nu = ir/s$ and gives rise to fractal spins $(k, m) = (n - r/s)/2, (-n - r/s)/2] = n/2 - r/2s, -n/2 - r/2s$.

The erratic belief that it is not possible to obtain values $Z > 1$ motivated the attempt to imagine how bilocality could increase the values of Z.

1. Suppose that the representation function is localized near $z_1 = 0$. $f_1 = r^2/(1 + r^2)$ is small near origin, and $f_2 = 1/(1 + r^2)$ is nearly constant. $K(z_1, z_2)$ diverges at $z_1 = 0$ so that the contribution of f_2 is large. This increases the value of Z and raises hopes of obtaining $p < 1$ under the constraint $l \geq q$. $p \leq 1$ needs only $Z \geq 1/2$, which the harmonics with $k \leq q/2$ have at $l = q$, and an invariant kernel cannot change it.

2. This raises also the question whether the speculated fractal monopole harmonics could correspond to fractal representations of Principal Series. The integers s would define the monopole charge $q = 2s$ and the counterparts of fractional angular momenta would correspond $l_1 = (ns - r)/2s = n/2 - r/2s$ and $l_2 = (-n/2 - r/2s)$. There are two angular momentum type parameters. The integer part for $l_1 + l_2$ vanishes and the fractional part is $-r/s$. Positivity condition for l_2 forces $r \leq -ns$ and implies $l_1 \geq n$. $r = -ns$ this gives the non-fractal situation $l_1 = n$ and $l_2 = 0$. This picture fits nicely with the already discussed fractalization.

3. If the representation functions are ScMHs, the notion of $Z(q, k)$ makes sense also now with $q \leq l$ interpreted as spin projection. They could also be identified as holomorphic harmonics. The averages $f_1 = r^2/(1+r^2)$ and $f_2 = 1/(1+r^2)$ are also now between monopole harmonics. The argument of f_1 and f_2 can be taken to correspond to the second complex coordinate z_1. The formula $1/p = Z(2 - 3c^2)$ remains the same.

2.6 About holomorphic monopole harmonics

2.6.1 Spinor monopole harmonics *viz.* scalar monopole harmonics and the precise interpretation of $\langle \gamma \rangle = 0$ condition

The conformal scalar *resp.* spinor monopole harmonics $\Psi_n = (z/F)^n$ *resp.* $\Psi_n = (z/F^{1/2}) \times (z/F)^n$ associated with Landau levels minimizing energy and angular momentum are the natural first guess for the wave functions for the position of the intersection of the fermion line at the partonic 2-surface.

For the ScMHs with $q = l$ with integer valued l produce a Weinberg angle $p = 1/4$ nearest to the measured value. This is not the case also for SpMHs with $j = l \pm 1/2$ (the natural interpretation would be in terms of conformal weight): in this case the minimum Landau state splits to two states $j = q \pm 1/2$. Could the two representations correspond to Ramond and N-S representation for the conformal algebra. Note that the special exceptional representation of the Lorentz group discussed in the Appendix is analogous to scalar representation and one can ask whether it could predict a value of p nearer to the experimental value.

Martin Ebner has calculated the expectation values. This allows one to see concretely whether the physics inspired approach based on magnetic harmonic oscillator modes predicts universal Weinberg angle for ScMHs. The prediction of the Weinberg angle depends on SpMH.

This also forced a more detailed study of the assumptions behind and the physical meaning of the condition $\langle e\gamma \rangle = 0$ and forced a rather radical re-interpretation.

The outcome of these considerations have been already explained in the previous section.

1. In CP_2 sector the condition that the averaged quantity in $\langle e\gamma \rangle = 0$ does not depend on the angle ϕ is true automatically. In the M^4 sector this is actually not true. This was observed already earlier but was hoped to give rise to small effects only.

2. The interpretation of the conditions as vanishing of vacuum expectation value is questionable since vertex is in question. The vertex must be non-vanishing for physical polarizations. How is the polarization visible in the vertex? And how does this relate to the expected conformal invariance holding true for massless particles such as photons.

   This forced the view that the somewhat mysterious condition $\langle e\gamma \rangle = 0$ in M^4 sector must be replaced with the condition that the emission vertex for holomorphic photons vanish. This is nothing but an expression of conformal invariance. For the antiholomorphic photons this is not the case. Furthermore, the partonic 2-surfaces are holomorphic in the sense that the complex coordinate z for $CP_1 \subset CP_2$ can be expressed locally as a holomorphic function of the complex coordinate w for $CP_1 \subset \delta M^4_+$. The induce holomorphic photon field is $\gamma_w = \gamma_z dz/dw$ and the precise form of the condition is $\langle \Gamma^w \gamma_w \rangle = 0$. This must hold true for all holomorphic partonic surfaces and this is true if the conditions hold true for all functions $z(w)$ guaranteed if one has $\langle \Gamma^w \gamma_w w^n \rangle = 0$.

2.6.2 The calculation of the parameters determining the value of Z for the holomorphic ScMHs

The calculation of the parameters determining the value of Z for the holomorphic ScMHs.

1. One must calculate the expectation values of re_3 *resp.* e_3/r. e_3 is proportional to factor $r/(1+r^2)$ so that this gives expectation value of $r^2/(1+r^2)$ *resp.* $1/(1+r^2)$. The area element dS of CP_1 is proportional to $rdr/(1+r^2)^2$. The wave functions for the intersection as a free particle are proportional to $r/\sqrt{1+r^2}$ so that the expectation value gives factor $r^2/(1+r^2)$.

2. For $n=1$ harmonic, the expectation value of re_3 *resp.* e_3/r gives rise to a radial integral of $I_{5,4}=\int_0^\infty r^5/(1+r^2)^4$ *resp.* $I_{3,4}=\int_0^\infty r^3/(1+r^2)^4$. The normalization factor associated with the spinor harmonic gives a radial integral $I_{3,3}=\int_0^\infty r^3/(1+r^2)^3$.

3. In the general case, the integrals can be calculated by partial integration giving the recursion formula

$$\begin{aligned} I_{n,m} &= \frac{n-1}{2(m-1)} I_{n-2,m-1} \ , \\ I_{1,m} &= \int \frac{r}{(1+r^2)^m} dr = \frac{1}{2(m-1)} \ . \end{aligned} \tag{2.44}$$

This gives

$$\begin{aligned} &I_{5,4} = (2/3)I_{3,3} = (2/3)(2/4)I_{1,2} = (2/3)(2/4)(1/2) = 1/6 \ , \\ &I_{3,4} = (1/3)I_{1,3} = 1/12 = I_{5,4}/2 \ , \\ &I_{3,3} = (1/2)I_{1,2} = 1/4 \ . \end{aligned} \tag{2.45}$$

REFERENCES

Mathematics

[A1] Shapiro ZYa Gelfand IM, Minklos RA. *Representations of the rotation and Lorentz groups and their applications*. Pergamon Press, 1963.

Particle and Nuclear Physics

[C1] A. Ignatov et al. VEPP-2000 collaboration. Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector, 2026. Available at: `https://arxiv.org/abs/2302.08834`.

[C2] Haldane F.T. *PRL*, 51(605), 1983.

[C3] Wu T.T and Yang C.N. Dirac monopole without strings: monopole harmonics. *Nuclear Physics B*, 107(365), 1976.

Condensed Matter Physics

[D1] Fractional quantum Hall Effect. Available at: `https://en.wikipedia.org/wiki/Fractional_quantum_Hall_effect`.

Books related to TGD

[K1] Pitkänen M. *Topological Geometrodynamics*. 1983. Thesis in Helsinki University 1983.

[K2] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/prext.pdf, 2023.

[K3] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/elvafu.pdf, 2023.

[K4] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Topological Geometrodynamics: Overview: Part II. https://tgdtheory.fi/tgdhtml/Btgdoverview2*. Available at: https://tgdtheory.fi/pdfpool/vNeumannnew, 2023.

[K5] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2023.

[K6] Pitkänen M. More about TGD Inspired Cosmology. In *Physics in Many-Sheeted Space-Time: Part II. https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/cosmomore.pdf, 2023.

[K7] Pitkänen M. Number theoretic vision, Hyper-finite Factors and S-matrix. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/UandM.pdf, 2023.

[K8] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter. https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K9] Pitkänen M. The Recent Status of Lepto-hadron Hypothesis. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/leptc.pdf, 2023.

[K10] Pitkänen M. Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole. In *TGD as a Generalized Number Theory: Part II. https://tgdtheory.fi/tgdhtml/Btgdnumber2.html*. Available at: https://tgdtheory.fi/pdfpool/fusionTGD.pdf, 2023.

[K11] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/numbervision.pdf, 2023.

[K12] Pitkänen M. Was von Neumann Right After All? In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Topological Geometrodynamics. *Int J Theor Phys*, 22(7), 1983.

[L2] Pitkänen M. Langlands conjectures in TGD framework. Available at: https://tgdtheory.fi/public_html/articles/Langlandsagain.pdf., 2011.

[L3] Pitkänen M. Updated view about the hierarchy of Planck constants. *Prespacetime J.* Available at: https://prespacetime.com/index.php/pst/issue/view/34, 3(11), 2012.

[L4] Pitkänen M. Could McKay correspondence generalize in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2017.

[L5] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, and Inclusions of Hyperfinite Factors. Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2019.

[L6] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors. Available at: https://tgdtheory.fi/public_html/articles/McKaygeneral.pdf., 2019.

[L7] Pitkänen M. What could 2-D minimal surfaces teach about TGD? https://tgdtheory.fi/public_html/articles/minimal.pdf., 2021.

[L8] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L9] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L10] Pitkänen M. About tessellations in hyperbolic 3-space and their relation to the genetic code . https://tgdtheory.fi/public_html/articles/tessellationH3.pdf., 2023.

[L11] Pitkänen M. Holography and Hamilton-Jacobi Structure as 4-D generalization of 2-D complex structure. https://tgdtheory.fi/public_html/articles/HJ.pdf., 2023.

[L12] Pitkänen M. New findings related to the number theoretical view of TGD. https://tgdtheory.fi/public_html/articles/M8Hagain.pdf., 2023.

[L13] Pitkänen M. Reduction of standard model structure to CP_2 geometry and other key ideas of TGD. https://tgdtheory.fi/public_html/articles/cp2etc.pdf., 2023.

[L14] Pitkänen M. About Langlands correspondence in the TGD framework. https://tgdtheory.fi/public_html/articles/Frenkel.pdf., 2024.

[L15] Pitkänen M. TGD as it is towards the end of 2024: part I. https://tgdtheory.fi/public_html/articles/TGD2024I.pdf., 2024.

[L16] Pitkänen M. A more detailed view about the TGD counterpart of Langlands correspondence. https://tgdtheory.fi/public_html/articles/Langlands2025.pdf., 2025.

[L17] Pitkänen M. A refined view of the phenomenology of hadron physics and p-adic mass calculations. https://tgdtheory.fi/public_html/articles/padmass2025.pdf., 2025.

[L18] Pitkänen M. About Dirac equation in $H = M^4 \times CP_2$ assuming Kähler structure for M^4 . https://tgdtheory.fi/public_html/articles/HJdireq.pdf., 2025.

[L19] Pitkänen M. About the structure of Dirac propagator in TGD. https://tgdtheory.fi/public_html/articles/dirprop.pdf., 2025.

[L20] Pitkänen M. Comparing the S-matrix descriptions of fundamental interactions provided by standard model and TGD. https://tgdtheory.fi/public_html/articles/hadroQCDTGD.pdf., 2025.

[L21] Pitkänen M. Answers to the questions of Vasileios Basios and Marko Manninen in Hypothesis Refinery session of Galileo Commission. https://tgdtheory.fi/public_html/articles/MarkoBasios.pdf., 2026.

[L22] Pitkänen M. Allais effect again. 2026. Available at: https://tgdtheory.fi/public_html/articles/Allaisagain.pdf. .