About the construction of the scattering amplitudes using $M^8 - H$ duality

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Abstract

In TGD, point-like particles are replaced with 3-surfaces and these in turn with the analogs of Bohr orbits. $M^8 - H$ duality is the generalization of momentum-position duality and is now rather well understood. It however remains a mere academic mathematical construct unless it can be used to achieve some practical goal. The construction of scattering amplitudes is the basic dream of TGD and $M^8 - H$ duality gives hope of achieving this goal in terms of the TGD counterparts for the momentum space Feynman diagrams.

The notion of exotic smooth structure, having interpretation as an ordinary smooth structure with 3-D defects and possible only in 4-D space-time, is crucial. Fermions in H are free but fermion pair creation is possible at the defects at which fermion lines can turn backwards in time. Also a more general change of direction is possible. This makes the counterpart of fermionic Feynman diagrammatic extremely simple at the level of H. Only fermionic 2-vertices associated with 3-D geometric defects are needed. Fermionic interactions reduce to an 8-D Brownian motion in the induced classical fields and the singularities of the space-time surfaces at which minimal surface property fails define the location of the vertices.

The interactions of two space-time surfaces, identified in holography = holomorphy vision as 4-D generalized Bohr orbits, correspond geometrically to contact interactions at their intersections. If the Hamilton-Jacobi structures are the same, the intersections are 2-D strings world sheets. The edges of these string world sheets would contain the vertices.

In this article an attempt to formulate this picture at M^8 level by using a precise formulation of M^8-H duality is made.

1 Introduction

$M^8 - H$ duality is the generalization of momentum-position duality to TGD, where point-like particles are replaced with 3-surfaces and these in turn with the analogs of Bohr orbits. $M^8 - H$ duality is now rather well understood [L16]. It however remains a mere academic mathematical construct unless it can be used to achieve some practical goal. The construction of scattering amplitudes is the basic dream of TGD and during years a lot of progress has occurred and $M^8 - H$ duality gives hopes of achieving this goal.

1. In standard QFTs the construction of scattering amplitudes reduces to Feynman rules formulated in momentum space. The divergences of the scattering amplitudes are the key problem and the needed regulation makes the application of the theory extremely difficult technically.

   In the TGD framework, holography = holomorphy principle [L6, L9, L7, L15, L3] allows to get rid of the mathematically ill-defined path integral but there is a weak failure of determinism associated with space-time surfaces as 4-D analogs of Bohr orbits. The counterparts of Feynman diagrams with lines identified as Bohr orbits appear at the level of H.

2. The geometric interactions are identified as contact interactions. The intersection of two space-time surfaces having a common Hamilton-Jacobi structure [L3] consists of 2-D string world sheets rather than being a discrete set of points. These string world sheets should provide the description of the scattering and also of self-interactions at the level of H.

3. The fermions are free at the level of H and X^4 but exotic smooth structures, possible only in space-time dimension $D = 4$ [A3, A4, A1], are conjectured to make possible fermion pair creation as turning of fermion backwards in time and also more general fermionic 2-vertices [L8, L2, L13] in the induced classical gauge fields.

   The minimal view is that there are only fermionic 2-vertices as defects of the standard smooth structure and fermionic scattering correspond to 8-D Brownian motion in H. The smoothness would fail at the edges of the fermion lines associated with 3-D edges of the space-time surfaces at which the minimal surface property and standard smoothness fails.

4. By $M^8 - H$ duality, space-time surfaces $X^4 \subset H$ have counterparts $Y^4 \subset M^8$ having interpretation in terms of dispersion relations in 4-D momentum space. By translational invariance, all translates of a given space-time surface $X^4 \subset H$ are mapped to the same 4-surface $Y^4 \subset M^8$ so that a huge simplification takes place in the construction fermionic part of the scattering amplitude.

At fermionic 2-vertices the correspondence $H \rightarrow M^8$ is two-valued. In particular, the string world sheets are mapped to their counterparts in Y^4 and serve as a natural seat for the virtual momenta. In fact, the edges of string world sheets at which standard smoothness fails correspond, in accordance with the Uncertainty Principle, to the seat vertices in H and for virtual momenta in M^8.

In the following the challenge is to understand the detailed construction of the fermionic scattering amplitudes at the level of M^8.

2 What could be the role of $M^8 - H$ duality?

M^8 is identifiable as an 8-D momentum space and if M^8_H duality is useful it should make possible the description of the scattering amplitudes at the level of momentum space. The quaternionic and thus associative 4-surfaces $Y^4 \subset M^8$ satisfying the additional condition that they contain a complex and thus commutative 2-surface, are analogous to a representation of dispersion relation for both on- and off-mass-shell fermions. If so, it should be possible to realize the TGD analogies of Feynman graphs in terms of $M^8 - H$ duality.

2.1 The role of M^8?

How should one proceed?

1. The most ambitious goal involves two tasks.

   (a) Construct M^8 counterpart both for the representation of the initial and final states by means of the second quantized spinor fields of M^8. The 8-dimensional masslessness would be realized at both sides and the spectrum of M^8 Dirac operator should be the same as that for H Dirac opeartor.

   (b) Construct the scattering amplitudes associated with analog of the quark-gluon phase associated with Y^4.

   The basic problem is that in M^8 it is not possible to realize differential geometry and the couplings to induced gauge fields. Only completely free spinor fields in M^8 are possible and correspond to octonionic spinors. Could the free Dirac equation in M^8 be enough for the description of scattering amplitudes?

2. This suggests a more modest goal based on a division of labor between M^8 and H levels.

   (a) The M^8 level could be an elegant way to construct scattering amplitudes in the quark-gluon phase that is at the level of Y^4? The reduction of the X^4 level description to Y^4 would conform with the basic idea of holography. At the level of Y^4 the geometry would reduce to algebra.

   (b) The H level could provide an elegant description for the description of initial and final states and also for classical dynamics for the spacetime surfaces necessary for the physical interpretation.

2.2 $H \rightarrow M^8$ correspondence is very-many-to-one and 1-to-2 at vertices

The original expectation that M^8H correspondence is 1-1 has turned out to be too simple.

1. The correspondence $H \rightarrow M^8$ is very-many-to-one. All causal diamonds $cd \times CP_2$ (CDs) [L4] obtained as translates of each other in H are mapped to the same in M^8 analog of CD consisting of opposite light-cones with opposite sign of energy and energies below some maximum energy. This is consistent with the Uncertainty Principle and means a huge simplification in calculating scattering amplitudes if Y^4 can be interpreted as momentum space.

2. This also makes it possible to understand the CP_2 extremals [K1] [L5] as equivalents of M^8 singularities for which the quaternionic or Minkowskian normal space is not unique. It would be a singularity analogous to a vertex of an algebraic surface. Every CD would go to the same CD equivalent in M^8 and should correspond to $f(o)$.

3. On the H side, the analogs of plane waves occur in the moduli space of CDs [L4]. The plane waves have as argument the M^4 position of the CD containing X^4. This guarantees the translational invariance so that the amplitudes of M^8 at the limit of an infinitely large CD are Poincare invariant and the various Poincare charges are preserved.

4. It is important to notice that even in the direction $H \rightarrow M^8$ the $M^8 - H$ duality can fail to be a one-to-one correspondence. In the creation of a fermion pair as defect of a standard smooth structure, the same point in H corresponds to two different momenta in M^8. This is true also for the 8-D Brownian motion giving rise to the fermion line with edges which are holomorphic singularities.

   This failure of one-to-one correspondence is essential for the understanding of what the vertices of ordinary Feynman diagrams mean in TGD. The vertex would correspond to the point H to which 2 M^8 points are assigned. The 1-to-2 vertex is the simplest one and analogous to the vertices giving rise to the massivation of fermion in the Higgs field.

5. Are n-vertices with $n \geq 2$ needed? For fermions these vertices do not make sense in QFTs since for instance a four-fermion vertex leads to a non-renormalizable theory. This suggests that only Brownian motion allowing pair creation as turning back in time direction is enough for fermions. Higher vertices emerge in a finite measurement resolution and would do so at the QFT limit of TGD. Note however that at the geometric side the situation is different: the splitting of a 3-surface into two can be considered and would be involved with the geometric description of the particle reactions which conforms with the classical view of particle reactions expressed in terms of Feynman diagrams.

2.3 About the identification of the surfaces Y^3 and Y^4

A generalization of momentum space dispersion relation would be a natural interpretation for the surface $Y^4 \subset M^8$. 8 options for the realization of associativity can be considered and the challenge is to identify the correct option or options.

1. There are two basic interpretations for what associativity means. For option T the tangent space T of Y^4 is associative. For option N the normal space N is associative. The number theoretic metric signature of Y^4 is Minkowskian for T and Euclidian for N.

   For the N option the integrability conditions are satisfied for any distribution of normal spaces N. This is somewhat worrying since the dynamics induced in H by $M^8 - H$ duality might be quite too non-deterministic. The identification of this option as the correct one initiated an Odysseia that lasted for years.

   The option N has also other potential problems: the four-momenta identified as points of E^4 would be co-quaternionic, non-associative, and tachyonic. Can they be allowed as off-mass-shell momenta that appear in ordinary Feynman graphs or could the tachyonicity emerge from dispersion relations for option T?

2. There are two alternative conditions on the octonion analytic function $f(o)$ possibly defining Y^3 as a representation of holographic data.

   Option A: For octonion real-analytic functions the condition $Re(f(o)) = 0$ ($Re(f)$ and $Im(f)$ are understood in octonionic sense) is invariant under local G_2, which as octonionic automorphisms commute with f and do not affect $Re(f)$.

   Option B: If one assumes $Im(f) = 0$, local G_2 is expected to reduce to a subgroup leaving $Im(f) = 0$ condition invariant. This condition however defines a 6-sphere S^6 and since G_2 is a subgroup of $SO(7)$, the local G_2 is also now a dynamical symmetry.

3. One can also consider two alternative identifications of Y^4.

   Option I: Y^4 is obtained by holography by using 3-D holographic data Y^3 satisfying the condition $Im(f) = 0$. The simplest surfaces Y^4 would be regions of M^4 or E^4 consistent with the holography using Y^3 defining the boundary of the surfaces. More complex surfaces Y^4 would be generated by the action of the local G_2 transformations commuting with the condition $Im(f) = 0$.

   Option II: Y^4 is a union of 3-D surfaces Y^3 satisfying $f = c$, where c is a real octonion. Also this identification is invariant under local G_2 and would give Y^4 as a union $Y^4 = \cup_c Y^3(c)$. The additional condition $Im(f) = 0$ gives rise to 3-D union of spheres $\cup_c S^2(c)$. A possible interpretation is as a 3-D holographic data. This would correspond to a slicing of Y^4 by 3-surfaces identifiable as orbits of partonic 2-surfaces. The slicing by partonic 2-surfaces would correspond to the $H - J$ structure [L3].

The basic questions are the following.

1. Which combinations of the above options, if any, are physically plausible?
2. How to interpret the surfaces Y^3 and the surface Y^4.
3. Are both quaternionic tangent space (Y^4 is Minkowskian) and normal space (Y^4 is Euclidean) physically possible or can we limit ourselves to the tangent space option for which Y^4 is Minkowskian?

2.4 General observations about the dispersion relation defined by Y^3

For both options mentioned above, the surface Y^3 is defined by the condition $Re(f) = 0$ or $Im(f) = 0$ and it is useful to first make some general observations related to the physical interpretation.

1. For the general points of Y^3, the dispersion relation $E^2 - p^2 = m^2$ is modified but would remain rotation invariant so we can talk about mass, energy and momentum. Could this 2-sphere S^2 (or its local G_2 deformation) for a given energy be the counterpart of the Fermi sphere? Would the violation of the dispersion relation physically correspond to the counterpart of the effect of gauge couplings in the induced Dirac equation for $X^4 \subset H$, which would modify the dispersion relation to be non-local.

2. 8-D masslessness would hold true as additional condition for on-mass-shell states and select a set of momenta for which the energy E would be quantized and the Lorentz invariant dispersion relation $E^2 - p^2 = m^2$ would hold tue. The energy would vary along the orbit of $S^2(E)$ and correspond to a definite 3-momentum squared. Could these Lorentz invariant points correspond to free on-mass-shell states and incoming and outgoing particles, most naturally fermions? The M^4 mass spectrum of the on-mass-shell states should be the spectrum of these states as given by the Dirac equation in H [L12, L11].

3. For a given $f(o)$, the mass spectrum is determined. In particular, the spectrum of incoming states and outgoing on-mass-shell states, corresponds in H to a set of 2-D subsets of hyperbolic 3-space H^3 as hyperboloid with a fixed light-cone proper coordinate a, which is determined by the inverse of the mass and defines the analog of cosmic time. The interpretation of these 2-surfaces as partonic 2-surfaces representing counterparts of vertices for incoming particles is suggestive.

   The mass spectrum of the incoming states would therefore have a direct representation in the geometry of the spacetime surface. The $a = constant$ surface brings in mind the 3-D equivalent of the celestial sphere, whose points correspond to the directions of the allowed wave vectors, appearing in crystallography.

4. Do the other Y^3 points correspond to momenta for which interactions have changed the dispersion relation? Could one say that at the H level the interaction of the particle with classical fields causes this and also leads to the breaking of Lorentz invariance into rotational invariance?

5. It is noteworthy that when the local G_2 transformation is non-trivial, the situation remains the same by the commutativity of G_2 with the conditions posed on $f(o)$.

2.5 Surfaces Y^4 with quaternionic tangent space (option T)

For the option T, surfaces Y^4 have an associative tangent space T. Note that besides integrable distribution of tangent spaces also an integrable distribution of complex sub-spaces is necessary in order to have M^8-H duality. These conditions are non-trivial and here local G_2 invariance would come to rescue. Integrability conditions are true for the local G_2 transforms and are suggested to generate Hamilton-Jacobi structures from the simplest H-J structure defined by the decomposition $M^4 = M^2 \times E^2$.

Let's first forget the local G_2 transformations and limit ourselves to the data that determines the simplest holography corresponding to a region of M^4. The questions depend on the option considered.

The key questions concern the interpretation of Y^3 for various options.

Option A: The dispersion relation for the points of Y^3 is not Lorentz invariant $E^2 - p^2 = m^2 = r_7^2$ but is more general but still rotationally invariant. Could one interpret Y^3 as a generalized mass-shell? Could Y^3 be regarded as an analog of the Fermi ball and $E = constant$ 2-spheres as Fermi surfaces? What about the 4-momenta in the interior of Y^4: do they represent off-mass shell states? Could the failure of Lorentz invariance be interpreted as reflecting the effects of the interactions with classical fields?

Option B: In this case the on-mass-hell states would naturally correspond to a Lorentz invariant dispersion relation $E^2 - m^2 = 0$, $m^2 = r_7^2$ (r_7 is the radius of S^6), which is natural for the incoming and outgoing states. The points of $Y^3(c)$ satisfying it define a 2-sphere $S^2(c)$.

Option I: For $Re(f) = 0$ option the values of masses for on-mass-shell states depend on c so that the spheres $S^2(c)$ belong to different mass shells. Could these 2-spheres correspond in H to partonic 2-surfaces identified as generalized vertices in which incoming particles appear.

Option II: For $Im(f) = 0$, the masses do not depend on c and the union $\cup_c S^2(c)$ defines a subset of the mass shell, that is hyperbolic 3-space H^3. The remaining points of Y^4 could be regarded as off-mass-shell states satisfying a dispersion relation which is only rotationally invariant. Note that tachyonic 4-momenta are possible also now and would be due to the modification of the dispersion relation.

At the level of H, the second hypercomplex coordinate is dynamically passive: could this tranlate to the condition $Im(f) = 0$ implying passivity at the level of M^8?

The parameter c would make itself physically visible only at the points $f = 0$ if they are identified as vertices. The virtual momenta assignable to the vertices would depend on c and the unions of vertices defined in this way could define $Y^4 \subset M^8$ images of the fermion lines of $X^4 \subset H$. In H they define boundaries of string worlds sheets and the interpretation of vertices as edges of fermion lines would suggest that these lines define edges of string world sheets.

The options TBI and TBII seem to be physically the most promising ones. TBI allows richer mass spectrum but allows only a spheres as subset of H^3 for a given mass. A couple of remarks are in order:

1. The governing equations for Y^3 have several roots corresponding to different radii r_7 for S^6. This gives nested spheres S^6 and S^2. Could tachyonic propagator lines connect the $M^4 \subset H$ images of such such spheres?

2. Local G_2 does not change the roots for $f(o)$ nor the dispersion relation for Y^3.

Consider now the interpretation of the interior of Y^4 for option TBI.

1. The interior of Y^4 correspond to both on-mass-shell states and off-mass-shell states and for a given mass only the energy shell $S^2 \subset H^3$ of on-mass shell states is realized.

2. Two interacting space-time surfaces X_1^4 and X_2^4 in H with the same H-J structure intersect along string world sheets X^2 defining a contact interaction for Bohr orbits. This applies also to self-interactions. The M^8 images Y_1^4 and Y_2^4 should meet along the 2-D surface Y^2. For the option TBI, the condition $E_1 = E_2$ and $p_1 = p_2$ are satisfied in the intersection $Y_1^4 \cap Y_2^4$, so that interaction region consists of 2-dimensional string world sheets and virtual momenta would naturally belong to them. Therefore the option TBI is consistent with H picture.

   Remark: A number-theoretic vision would suggest that the coefficients of $f(o)$ belong to an algebraic extension of rationals. If $f(o)$ is a polynomial then the momentum components are in the algebraic extension of this extension. This gives a discretization of also string world sheets.

2.6 Octonionic Dirac equation in M^8

How to generalize the induced Dirac equation in $X^4 \subset H$ [L12] to $Y^4 \subset M^8$?

1. Partial differential equations must be replaced by algebraic equations. This is what happens in free field theory for plane-wave solutions of the Dirac equation. M^8 indeed corresponds to 8-D momentum space and Y^4 to 4-D momentum space.

2. The Dirac equation in H contains partial derivatives that must be replaced by momentum components at the level of M^8. In addition, there is a coupling to the spinor connection. How to describe this coupling at the level of M^8 where differential geometry is now allowed? Do Y^3 and Y^4 give this description in terms of the dispersion relation for the virtual 4-momenta? For the states that are not incoming states, the Lorentz invariant dispersion relation reduces to rotationally invariant one and is invariant with respect to the local G_2.

3. The ordinary M^8 Dirac operator $\gamma^k \partial_k$ is replaced by $p^k \gamma_k$, where p^k as 8-momentum corresponds is restricted to be a point in $Y^4 \subset M^8$, which is quaternionic for option T. If Y^4 is Minkowskian, the condition $p^2 - m^2 = 0$, $m^2 = r_7^2$ is satisfied on-mass shell. Since only the rotationally invariant dispersion relation is satisfied, the ordinary M^4 propagator does not vanish except for incoming and outgoing states. In the Euclidean case, there is no solution to the mass shell conditions so that these momenta can appear only as virtual momenta.

How to identify octonionic gamma matrices γ_k and octonionic spinors Ψ? Here the crucial observation is that the octonionic units define analogies of Pauli spin matrices: by non-associativity ordinary matrix representation is however not possible except for quaterninic subspaces since quaternionic units allow matrix representation as ordinary Pauli spin matrices.

1. The representation of 8-momentum as octonion $p^k I_k$ is reduced to a quaternion for the T option. It is analogous to $p^k \sigma_k$. By forming the tensor product of the real unit I_0 with say σ_z and the tensor products of imaginary units I_k , $k > 0$ with σ_x, n anticommuting octonionic gamma matrices are obtained! This brings in spin degrees of freedom but by separate conservation of lepton and quark numbers the two spin components do not probably relate to quarks and leptons. A more plausible interpretation is as a counterpart of complexity of the ordinary gamma matrices.

2. Another octonionic miracle is that gamma matrices define an octonic spinor. This corresponds to the 8-D triality stating that for $SO(1,7)$ both the vector representation, spinor representation, and its conjugate are 8-dimensional [A2]! Note that octonionic spinors can have ordinary complex numbers as a coefficient field.

3. The square of M^8 Dirac operator would give the algebraic condition $p^2 - m^2 = 0$, where $m = r_7$ is the radius of S^2. The effects of gauge couplings in H should correspond in M^8 to a reduction of the Lorentz invariance of the dispersion relation to a rotation invariance and $f(o)$ would characterize these effects. The Dirac operator and its inverse can also be defined at the interior points of Y^4 and for virtual momenta the Dirac equation is not true.

3 How to construct the scattering amplitudes using $M^8 - H$ duality?

In this section the picture about $M^8 - H$ duality developed in the previous section is applied to the construction of scattering amplitudes, which in the TGD framework can be interpreted as WCW wavefunctions for a set of Bohr orbits defining the state.

3.1 The overall view of particle interactions at the level of H

Consider first the general description of particle reactions in H.

1. TGD leads to a generalization of the basic picture about hadronic reactions [L13]. Initial and final states of any particle reaction are analogous to many hadron states obeying color confonamen and correspond to many-fermion states associated with the spinor basis of H and possibly of the causal diamond (CD). Fermions are massless in the 8-D sense but apart from the covariantly constant right-handed neutrino, ther 4-D mass has CP_2 mass scale. Therefore the problem is how to obtain many-fermion states which are in 4-D sense light and in the first approximation massless.

   The proposal is that in many fermion states also the solutions of the Dirac equation in H, or at least in CD, can have tachyonic mass squared values [L11, L13]. This would allow to have massless many-fermion states although the fermions have CP_2 mass scale. p-Adic thermodynamics [L10] would give them a small mass. For the CD variant of the Dirac operator the mass squared spectrum is stringy, that is integer valued.

2. Interacting phase in X^4 can be seen as a generalization of the quark-gluon plasma phase of QCD. Spinors are restrictions of H spinors to X^4 and the induced Dirac equation or the more general modified Dirac equation, determined by supersymmetry from the classical variational principle, allows explicit solutions by holomorphy = holography principle. Fermions are massless in the 4-D sense. The failure of the precise classical determinism for the Bohr orbits [?, L9, L14] plus exotic smooth structures [?]ive a non-trivial quantum dynamics in X^4. Originally, classical non-determinism, identifiable as p-adic non-determinism, was thought to be involved only with cognition [L18].

3. The transition from hadronic phase to quark-gluon phase and vice versa are essential in the QCD framework and correspond to the fragmentation of quarks and gluons to hadrons and hadronization of quarks and gluons to hadrons. These concepts are generalized in the TGD framework. These processes reduce to the overlap for the states belonging to the analogs of hadron phase and quark-gluon plasma phase. This applies also to the leptons which also move in color partial waves [L13, L10].

3.1.1 Geometrically interactions as contact interactions

Geometrically the interactions would be contact interactions.

1. Consider two spacetime surfaces in H and their interaction. The H-H principle implies that if the H-J structures are the same, the intersection consists of 2-D string world sheets X^2. This also applies to self-interactions. The interaction would be a contact interaction [L13, L10, L17].

2. The intersections $X^2 = X_1^4 \cap X_2^4$ of the spacetime surfaces should be mapped by M^8-H duality to the intersection Y^2 of the corresponding intersections $Y^2 = Y_1^4 \cap Y_2^4$ and also at M^8 side, 2-D string world sheets would be obtained. They would intersect the fermion lines Y^1 along Y^3 as images of light-like partonic orbits in H.

3.1.2 Analogs of Feynman diagrams in H

Consider first the analogs of Feynman diagrams in H.

1. The simplest proposal is that in $X^4 \subset H$ the vertices of Feynman diagrams correspond to points at which the edges of fermion lines in H meet. The vertices would be located at the 3-D defects of the standard smooth structure giving rise to exotic smooth structure making possible fermion pair creation and more generally, change of the direction of the fermion line.

   This is certainly the most elegant option and would mean that there are only 2-vertices. The extremely complex topology and combinatorics of the ordinary Feyman graphs would trivialize. This would certainly be true for fermions but for particles identified as 3-surfaces also vertices in which more than 2-surfaces meet are possible. At level of M^8 the pair of fermion lines would correspond to a pair of momenta.

2. The geometric 3-D analogs of vertices and fermion vertices would involve infinite acceleration and at the H side the failure of the minimal surface property. The 3-vertex and 4-vertex of gauge theories would, if looked at more closely, split into two 2-vertices. The pair creation at one point and the edge of the fermion line at another point.

   Fermionic Feynman graphs in H would consist of lines connected by a set of edges: the dynamics would reduce to 8-D Brownian motion. The pair creation would correspond to a V-shaped line that would turn backwards in time. Higher vertices are not needed and it can be argued that divergences can be avoided in this way.

   One can also think of a closed fermion polygons as an analogy of vacuum loops. These bring to mind twistor diagrams. However, at the H-level, the incoming particles at the vertices would correspond to classical fields and singularities of the otherwise vanishing trace $Tr(H)$ of the second fundamental form.

3.2 The analogs of Feynman graphs in M^8

What would be the TGD analogs of Feynman graphs in M^8? It was found that one can consider 8 options but TBI and TBII look the most plausible ones and only these will be considered. The basic questions concern the identification of the virtual momenta assignable to the fermion lines and the M^8 counterpart for their ordering to fermion lines in H.

For both TBI and TBII, one can ask whether the points for which $Re(f) = 0$ and $Im(f) = 0$ could define allowed virtual momenta. One could speak of singularity at which the octonion analyticity, which actually reduces to ordinary analyticity, breaks down. These points would be analogs of zeros of Riemann Zeta. The surfaces would correspond to partonic 2-surfaces identifiable as spheres S^2.

3.2.1 Some confusions to be avoided

In the framework considered, one can consider the construction of scattering amplitudes by calculating them on the M^8 side by using an analog of the usual Feynman diagrammatics.

1. To avoid confusion, it is worth noting that a ordinary Feynman graphs are generalized to a spacetime surface in H describing geometrically contact interactions of 4-D Bohr orbits. In the classical picture, a four-momentum corresponds to a geodesic line in $M^4 \subset H$. In M^8, there are momenta, which in H are associated with vertices but there are no vertices.

2. The octonionic fermion propagators of M^8 are well-defined. For the incoming and outgoing states, $E^2 - p^2 = m^2$ holds. $f(o)$ can be constructed so that it contains the given squares of the masses of the desired states as their zeros. A polynomial is sufficient if there are a finite number of states. The masses involved would determine the roots of $f(o)$, which in turn define the vertices and scattering amplitudes.

3.2.2 What are the allowed virtual momenta?

What are the allowed virtual momenta occurring in the interaction and what guarantees momentum conservation?

1. For both TBI and TBII, the conditions $Re(f) = 0$ and $Im(f) = 0$ could pose constraints on the allowed virtual momenta. One would have a singularity at which octonion analyticity, which actually reduces to ordinary analyticity, breaks down. These points would be analogs for the zeros of Riemann Zeta. The surfaces would correspond to partonic 2-surfaces identifiable as spheres S^2.

2. If the interactions are contact interactions, the virtual momenta belong to Y^2, the space of virtual momenta is reduced from Y^4 to 2-dimensional Y^2. The QFT picture suggests that this is crucial in terms of finiteness.

3. If the points of Y^2 satisfying the condition $f = 0$ correspond to virtual momenta, there would be a finite number of them for a given $Y^3(c)$ and vertices would correspond to strings

identifiable as edges of string world sheets Y^2. If f is a polynomial with coefficients in an extension of rationals, the components of the momenta would be algebraic numbers. For c in an extension of rationals, the string is discretized.

4. The momentum conservation would not hold true for 2-vertices but would be forced for the entire particle reaction by the translational invariance at the level of H involving integrations over all translates of the H images of E^4 under $M^8 - H$ duality.

5. An interesting question is what Y^2 and Y^1 could correspond to in condensed matter physics. Same question can be asked about partonic 2-surfaces.

3.2.3 Identification of the vertex factors

What determines the vertex factors?

1. On the H side, the condition $Tr(H) = 0$ stating the vanishing of the second fundamental form having interpretation as a generalization acceleration, gives a minimal surface [L7, L1]. In 3-D geometric vertices apart from singularities at which $Tr(H)$ has a delta function singularity. However, more general field equations are valid: the volume term and Kähler contributions cancel each other and the conservation laws hold. Acceleration in a generalized sense diverges.

   At the level of H, the vertex could be assigned to the divergence of fermion current [L13] stating the fact that different M^4 chiralities of the fermion are not conserved in the vertex although the total fermion number is conserved. This is what the violation of conformal invariance and effective massivation means. The non-vanishing of $Tr(H)$ as an analog of the Higgs field states the same fact.

2. There are no gauge potentials available on the M^8 side. The equivalent of the diverging acceleration would be the difference of 8-momentum Δp at a V-type vertex. The difference of E^4 part momentum can also come into play and would correspond to the Higgs singularity. The contraction $\Delta p^k \gamma_k$ would have the same dimension as the contraction of the gauge boson at the vertex. Δp could be seen as a counterpart for the discontinuity of the derivative part of the covariant derivative: the discontinuity of the gauge potential would be finite. The product of vertex factor and propagator would be scaling invariant and dimensionless.

   The vertex involves dimensionless coupling constant, which might relate to the Kähler coupling constant g_K, whose value is proposed to depend on the algebraic extension for the Taylor coefficients of f and to obey a discrete coupling constant evolution. Note that α_K is analogous to a critical temperature.

   Besides this there are integrals over the edges of the string world sheets parameterized by a hypercomplex coordinate v, whose coordinate curves have a light-like tangent vector. The sum of $x = u + v$ could define a space-like coordinate coordinate as analog of the real coordinate of the complex plane and dx/x could define a dimensionless scale invariant integration measure.

3. There would be no dimensionless coupling constants in the vertex and CP_2 radius would be the only dimensional coupling constant! As a matter of fact, the situation is the same on the H side since the gauge couplings appear as scaling factors of the induced gauge potentials.

3.2.4 How to organize the propagators and vertex factors to sequences corresponding to fermion lines in $X^4 \subset H$?

In standard Feynman diagrammatics one assigns to each vertex a subset of momenta and momentum conservation holds true in each vertex. If only 2-vertices are possible, the translational invariance applies only at the level of M^8 only to the entire set of momenta?

The scattering amplitude however depends on how one orders the propagators. Can one order them in a unique way? It should be possible to order the virtual momenta so that the scattering amplitude reflects the structure due to the decomposition to Brownian orbits at the level of H.

The H picture involving fermion lines indeed suggests that this could be the case.

1. There is a time ordering in H with respect to the proper time a for the vertices in $X^4 \subset CD \subset H$. This induces time ordering for the fermion-antifermion pairs associated with the vertices in X^4 and one can assign to each vertex a pair of propagators with a vertex factor between them. The situation in which the value of a is same for two vertices is problematic. Similar time ordering with respect to the linear Minkowski time induced by M^8-H duality could solve this problem partially for a given fermion line with edges.

2. $M^8 - H$ duality transforms the time ordering in H to energy defined by the real part of octonion to at the level of M^8. The choice of the octonion structure meaning the identification of the octonionic real axis as a time axis in M^8 reduces Lorentz group $SO(1,7)$ to G_2 so that there is no problem with momenta for which the difference is space-like. If one uses ordering with respect to the value of a, there can be a momenta whose difference is space-like.

3. The time ordering is not enough. At the level H there are several fermion lines. The ordering of the propagators to sequences at the level of M^8 must be induced by the fermion lines in $X^4 \subset H$. Each such sequence of fermion propagators has at its ends a contraction by octonionic spinors so that it is mappable to ordinary complex numbers commuting with the amplitudes associated with other fermion lines. Only linear structures connecting on mass shell states and giving rise to a complex number would be obtained in this way.

4. What happens when one allows strings as edges of 2-D string world sheets as seats of the virtual momenta. As proposed, the set of possible virtual momenta would be organized to a sequence of strings appearing as edges of the string world sheets. If so the above organization would involve only integrals over the time-ordered strings. At the H level strings world sheets are very simple since by hypercomplex analyticity, the embedding to H depends on the second hypercomplex coordinate only. The same simplicity should be inherited at the M^8 side and mildly favors option IIB. Note that the hypercomplex coordinates of M^4 have light-like tangent vectors.

3.3 Summary about the construction of the scattering amplitude in M^8

The proposed picture gives a rather concrete view of the construction of the scattering amplitudes.

1. The simplest option suggested by the notion of smooth exotic structure is that the graph contains only 2-vertices: Brownian motion in M^8 including the creation of fermion-antifermion pairs as a special case.

   Vertices could correspond to the points of H for which the map $H \to M^8$ is 2-valued: a point in H is mapped to as pair of momenta associated with the meeting fermion lines and there are no vertices in M^8.

2. The vertex factors would correspond to momentum difference. When the edge becomes trivial the vertex factor vanishes.

3. The organization of propagators and vertices and their time ordering would be inherited from the fermion lines in H.

4. There would be an integral over the positions of various propagator momenta belonging to the 1-D edges of the 2-D string world sheets $Y^2 = Y_1^4 \cap Y_2^4$. If the condition $f = 0$ expressing the failure of octonion analyticity, selects the vertices, the lines parameterized by the parameter c in the condition $Re(f) - c = 0$ would give rise to 1-D lines for vertices for the Option IIA. The parameter c would correspond to a dynamically passive coordinate not appearing in the embedding $X^4 \subset H$. Does this mean that one only the Option IIB with $Im(f) = 0$ is acceptable? The translational invariance at H side would take care of the conservation of total momentum. There would be no local conservation at vertices (which are absent in M^8).

5. Bosonic propagators should emerge at the QFT limit. On the H side, the counterparts of the bosonic propagators would be obtained as correlations of classical induced boson fields

by functionally integrating over WCW. On the M^8 side, the result should be similar if this guess is correct.

6. The octonionic fermion propagators of M^8 are well-defined. For the incoming and outgoing states, $E^2 - p^2 = m^2$ holds and the mass spectrum should correspond to that for H spinors. $f(o)$ can be constructed so that it contains the given squares of the masses of the desired states as their zeros. A polynomial is sufficient if there are a finite number of states.

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