

# New results about causal diamonds from the TGD point of view

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### Abstract

This chapter was inspired by two interesting results related to the notion of causal diamond (CD) playing a central role in quantum TGD. One interpretation is as a quantization volume and the second interpretation is as a geometric representation of the perceptive field of conscious entity. CDs can be said to define the backbone of the "world of classical worlds" (WCW) central for quantum TGD.

For these reasons it is interesting to ask the precise mathematical definition of the moduli space of CDs. TGD suggests a definition as the semidirect product  $D \rtimes P/SO(3)$  of scaling group and Poincare group divided by  $SO(3)$  subgroup leaving the CD invariant: this gives 8-D space. The definition that inspired this article is based on conformal group and gives also 8-D space  $SO(2,4)/SO(1,3) \times SO(1,1)$ . The metric signature is (4,4) for both spaces and they could be identical. These definitions are compared and one can consider the conditions under which both identification can give rise to representations of the Poincare group as expected with the scaling group reduced to a discrete subgroup.

Second result relates to the finding that special conformal transformations in the time direction defined by CD leave CD invariant. The corresponding hyperbolic flows correspond to a motion with constant acceleration to which the so-called Unruh effect is associated. One can consider an  $SL(2,R)$  algebra assignable to a conformal quantum mechanics and assign a hyperbolic time evolution operator to this flow. The conformal 2-point functions associated with this operator correspond to thermal partition functions with thermal mass defined by the temperature which is essentially the inverse of the CD scale.

Holography does not allow us to consider these flows for the space-time surfaces inside CD but the action of the hyperbolic evolution operator on quantum states at the boundaries of CD is well-defined. This also raises interesting questions related to TGD inspired consciousness, where subsequent scalings of CD in state function reductions (SFRs) give rise to the correlation of subjective time and geometric time defined as the distance between the tips of CD. The SFRs associated with the hyperbolic time evolution operator would not affect CD and would correspond to "timeless" state of consciousness. One cannot avoid reconsidering the details of "small" SSFRs defining the subjective time flow correlating with the flow of geometric time assigned with the increase of CD.

## 1 Introduction

This work is a response to two very interesting articles related to the notion of causal diamond (CD).

### 1.1 The metric for the space of causal diamonds

Dainis Zeps sent article [B1] (<https://rb.gy/gcfvus>) related to the space of causal diamonds (cd) of  $M^4$  ( $CD$  is the Cartesian product  $cd \times CP_2$  of  $cd$ , defined as intersection of future and past directed light-cones, and of  $CP_2$ ).

Although it remains unclear whether the proposal for the identification of the moduli space is realistic in the TGD framework, where also a simpler realization of the moduli space can be considered, the article led to a clarification of many aspects related to ZEO and the space of CDs as the backbone of the "world of classical worlds" (WCW).

### 1.2 The existence of a hyperbolic generator of conformal group defining a flow, which maps causal diamond to itself

Gary Ehlenberger sent a link to a very interesting article [B4] (<https://rb.gy/z7o7wj>) related to causal diamonds (cd) of 4-D Minkowski space.

The article is very interesting from the TGD point view since causal diamond (CD) of  $H = M^4 \times CP_2$  is the key object in zero energy ontology defining the ontology of TGD. CD is defined as the Cartesian product  $CD = cd \times CP_2$  of the causal diamond  $cd \subset M^4$ , identified as an intersection of future and past directed light cones, and of  $CP_2$ .

The basic findings of [B4] are as follows.

1. The generator of  $K$  of special conformal transformation in time direction, scaling generator  $D$  and generator  $P_0$  of time translation generate an  $SL(2, R)$  algebra. A suitable combination of

these 3 generators defines a generator of what generates hyperbolic time translations, call it  $S$ . Its exponentiation generates an analog of time evolution in conformal quantum mechanics. What is remarkable is that this flow takes the CD to itself so that the time range of time evolution can be said to correspond to the distance between the tips of the CD.

The flow lines of the exponentiated Killing vector would have an interpretation as the world lines of accelerated observables with constant acceleration determined by the initial value of the radial  $M^4$  coordinate.

The hyperbolic time evolution is time-like only in the interior of the second light-cone associated with CD. Only a single arrow of time is allowed by time-likeness. It is time-like also outside the light-cones.

2. The time evolution of a static CD observer is equivalent to a time evolution of conformal QM defined by a hyperbolic Hamiltonian. The flow states for a fixed value of radial coordinate  $r$  correspond to a time evolution from the value of hyperbolic time  $\tau = -\infty$  to  $\tau = \infty$ . This evolution corresponds to a finite time interval with respect to the ordinary Minkowski time  $t$ .

One obtains a family of states corresponding to time evolutions for various values of  $t$ . These states are so-called coherent states, which are not orthogonal and their inner product for values  $t = t_1$  and  $t = t_2$  defines a correlation function of conformal QM.

The behaviour of 2-points functions of conformal QM in CD is equivalent to that for thermal QFT in  $M^4$ . One can say that static CD observers falling freely along the flow lines of the Killing vector observe the vacuum state of the inertial  $M^4$  system as a thermal state characterized by Unruh temperature  $T = \hbar/2\pi L$  (<https://rb.gy/qxyp8q>), where  $L$  is the size of CD .

### 1.3 Could these findings have significance for TGD

An obvious question is whether these findings could have significance for TGD, in particular for the zero energy ontology (ZEO) [L5, L16, L12] [K8], which replaces the standard ontology as a foundation of quantum theory and in this way solves the basic paradox of quantum measurement theory.

#### 1.3.1 Zero energy ontology (ZEO) and CDs

Causal diamond (CD) defines the analog of quantization volume in the embedding space  $H = M^4 \times CP_2$  inside which the zero energy states are superpositions of space-time surfaces, which connect the light-like boundaries of CD. In the TGD inspired theory of consciousness, CD corresponds to a 4-D perceptive field of self.

The allowed space-time surfaces are preferred extremals (PEs) analogous to Bohr orbits and satisfy almost completely deterministic holography forced by the general coordinate invariance. Quantum states can be also regarded as superpositions of pairs of 3-D states assignable the opposite boundaries of CD and constructed using second quantized fermions of  $H$  satisfying free Dirac equation or its modification in the case that also  $M^4$  allows the analog of Kähler structure suggested by the twistor lift of TGD. At the limit of large CDs also the Poincare quantum numbers are opposite at the opposite boundaries of CD.

The TGD based proposal is that the time evolution at the level of "world of classical worlds" (WCW), which consists of these preferred extremals for given CD, is defined by a conformal scaling leaving the passive boundary of causal diamond and states at it invariant but affect the active boundary and 3-D states at it. The gradual increase of CD would correspond to the growth of geometric time correlating with the flow of subjective time defined by "small" state function reductions (SSFRs) as analogs of weak measurements and this time evolution replaces the trivial time evolution of Zeno effect in TGD and gives rise to self as a conscious entity. In "big" SFRs (BSFRs) the roles of the active and passive boundaries change and the arrow of time changes.

Under certain conditions the evolution by scaling can be approximated by a time translation. The scalings commute with the Lorentz transformations mapping the passive boundary of CD to itself. This time evolution allows us to understand spin-glass type systems [L11]).

From the point of view of ordinary time evolution defined by energy, the difference is that the relaxation processes obey fractal power law rather than exponential decay. This is a characteristic feature of the N-point functions of conformal field theories (CFTs) as opposed to those of massive quantum field theories (QFTs). The conformal invariance of 2-D conformal theories generalizes since the light-cone boundary  $\delta M_{\pm}^4 \times CP_2$  and light-like 3-surfaces in general allow extended conformal invariance.  $\delta M_{\pm}^4 \times CP_2$  allows also supersymplectic invariance for light-like radial coordinate, which takes the role of complex coordinate.

One can say that in the TGD framework the 2-D conformal field theories generalize to dimension  $D = 4$ . In particular, the boundaries of space-time surfaces and the 3-D surfaces at the boundaries of CDs define 3-D holographic data for the generalized conformal field theory.

### 1.3.2 Poincare group acts in WCW rather than at the level of space-time and $M^4$

In the TGD framework, the great conceptual leap is made possible by the moduli space of CDs is that 4-D Poincare and conformal groups would not act at the level of space-time or of CD but at the level moduli space of CDs forming the backbone of WCW.

In TGD, Poincare invariance need not be a symmetry of the action at the space-time level as has become clear already earlier. Indeed, the twistor lift of TGD suggests that  $M^4$  has a Kähler form contributing to Kähler action of  $H$  so that Poincare and Lorentz symmetries would be broken at the level of  $M^4$ .

This picture would fit nicely with the breaking of Lorentz invariance implied by the momentum and polarization vectors assignable to massless particles and also with the view of hadrons based on quarks characterized by a place carrying 2-D longitudinal momenta. This kind of flexibility gives a strong support for the WCW approach.

The quantum numbers of the zero energy states assignable to a given CD must correlate or be equal with the quantum numbers, such as four-momentum and angular momentum assignable to the wave function in the moduli space of CD.

1. The value of mass squared for CD should correspond to that for the generator of conformal scalings realized in super symplectic and super-Kac-Moody type degrees of freedom in the interior of CD.
2. One can assign to the modes of the second quantized spinor field of  $H$  4-momenta and the total momentum associated with these at either half-cone of CD would naturally correspond to the momentum assignable to CD.
3. Classical charges are conserved for action and also these should be identical with those assignable to CD and with the wave function in the moduli space of CDs. Here interesting questions relate to the violation of translational invariance and Lorentz invariance induced by the  $M^4$  Kähler form.
4. In case of TGD, CD means actually  $cd \times CP_2$ : also the total quantum numbers assignable to  $CP_2$  should be the same for CD and the wave function in the space of CDs.

### 1.3.3 Problem and questions

In the TGD framework, the existence of a hyperbolic generator  $S$  mapping cd to itself and the properties of the corresponding flow raise some questions.

1. The exponentiation of a hyperbolic generator defines a diffeomorphism of CD, which would deform the space-time surface in the interior of the CD. One would have a problem since this deformation is not expected respect the preferred extremal property.

In TGD, hyperbolic generator  $S$ , just like supersymplectic generators, can act only at the boundaries of CD and affect the data at them. These data define boundary conditions for holography, which extends this action to an action on the space-time surface inside the CD. If the hyperbolic flow is modified at all at the boundaries of CD, this induces a modification of the preferred extremal at the level of the space-time surface.

2. The flow induced by  $S$  corresponds to a flow that maps the states for a fixed value of radial coordinate  $r$  ( $r = \text{constant}$  means that system is "at rest") from time  $\tau = -\infty$  to  $\tau = \infty$  and this corresponds to relativistic at the level of CD accelerate motion with acceleration  $a = 1/L$ ,  $L$  the size of CD. The initial and final points of motion correspond to point  $(t_{\pm}(r), r)$  at the boundaries of CD. Somewhat disappointingly, the flow does not affect the points at the boundaries at all.
3. What is however possible is that the time evolution as an exponentiation of the hyperbolic generator labelled by  $t$  affects the quantum states at the boundaries of CD for each value value of  $t$ . One can say that time evolution is represented as unitary time evolution at the boundary of CD.

This would relate the states at the points of boundary with different values of light-like radial coordinate. This correlation could be interpreted in terms of radial conformal invariance and could be manifested in terms of conformal correlation functions. This is just what one expects since the light-like radial coordinate for the light-cone boundary is analogous to the complex coordinate for ordinary conformal invariance.

Hyperbolic time evolution would define unitary action on zero energy states and could define "small" SFRs (SSFRs), which does not scale CD so that the geometric time defined by the distance between the tips of CD is not changed. In TGD inspired theory of consciousness this kind of SSFRs would correspond to "timeless" conscious experience. Could the sequences of these time evolutions followed by SSFRs correspond to meditative states, which are reported to be timeless? SSFRs involving scalings of CD would in turn correspond to ordinary ordinary conscious experience involving the sensation of time flow.

These findings force to reconsider the basic assumptions of ZEO.

1. The assumption has been that only SSFRs scaling the size of CD possible? Could one think that only the SSFRs corresponding to the hyperbolic time evolution are possible. The size of CD remains unaffected in the sequence of SSFRs.

Could the scalings of CD correspond to SSFRs or could they precede BSFRs in which the arrow of time changes? Note that the scalings would affect the WCW spinor fields representing zero energy states at the level of moduli space of CDs whereas hyperbolic time evolutions would affect only the WCW spinor field in "internal" CD degrees of freedom.

2. One can assign to the 2-point functions at the boundary of the CD a temperature given by  $T = \hbar/L$ , where  $L$  is the size of the CD. Could this temperature and thus the size of CD correlate with the physical temperature of the environment? In fact, I have already ended up with the view that  $L$  codes for the analog of cosmic temperature gradually reduces during the sequence of SFRs.

## 2 Two possible identifications for the space of CDs and its metric

The space of causal diamonds (CDs) forms the backbone of the "world of classical worlds" (WCW). Each CD interpreted as a geometric correlate for the perceptive field corresponds to a sub-WCW. Zero energy states are realized as superpositions of space-time surfaces inside CD and pairs of fermionic Fock states at boundaries of CD. The space of CDs would define cm degrees of freedom in WCW.

A possible interpretation [B3] (<https://rb.gy/m1dvwq>) of CDs assigned with, say geodesic lines, is that the size of the CD (temporal distance between its tips) defines the duration for a "moment of consciousness". This interpretation makes no distinction between subjective and geometric time. This distinction is however very real as many examples from everyday life demonstrate.

The increase of the size of CD in "small" state function reduction (SSFR) inducing a small scaling of CD would correspond to the increment of geometric time identified as the distance between the tips of CD. The basic TGD based proposal is that this corresponds to the duration

of experienced moment of subjective time in SSFR. The duration of the moment of consciousness between two SSFRs would thus correspond to an increase of the size of the CD rather than to its size.

One can consider two very different identifications of the moduli space of CDs.

## 2.1 TGD in inspired option for the moduli space of CDs

Consider first the TGD inspired identification for the moduli space of CDs.

1. It is easy to see that the space of CDs is 8-D. The position of either tip or of center of the CD gives  $M^4$ . The Lorentz transformations  $SO(3,1)$  leaving either tip of CD invariant and the group  $D$  of scalings leaving the cm of CD invariant give new CDs. The isotropy group of the CD is  $SO(3)$ . This gives 8-D space.
2. The first guess is that the space of CDs is the coset space  $D \times P/SO(3)$ . The division by  $SO(3)$  is due to the fact that a given CD corresponds to a unique rest system and is invariant under rotations. Translations of  $M^4$ , that is  $M^4$ , code for the position of the cm or tip of CD, and  $D$  corresponds to the size of CD. Locally one would have an 8-D product  $D \times M^4 \times H^3$ ,  $H^3$  is 3-D hyperbolic space analogous to mass shell. There might be some delicate effects changing the topology.

The naive guess that the signature of the metric is  $(1,7)$ , is wrong. For the Lorentz group  $SO(1,3)$  the signature is  $(3,3)$  and for  $H^3$  as coset space  $H^3 = SO(1,3)/SO(3)$  the signature is  $(3,0)$  rather than  $(0,3)$  as one might expect. This has no physical implications. If  $D$  has space-like signature,  $D \times M^4 \times H^3$  has signature  $(0,1) + (1,3) + (3,0) = (4,4)$ . This is what one obtains also for the  $SO(2,4)/SO(1,3) \times SO(1,1)$  option so that options could be equivalent.

## 2.2 The identification of the moduli space of CDs inspired by conformal invariance

An identification of the moduli space of CDs inspired by conformal invariance is considered in [B1] (<https://rb.gy/gcfvvs>). The 5-D  $AdS_5 = SO(2,4)/SO(1,3)$  appears in AdS/CFT correspondence and is associated with  $4 \rightarrow 5$  holography whereas the space-time surfaces inside CD realize  $3 \rightarrow 4$  holography. This might have served as a motivation for the proposal: maybe the idea has been that one could generalize holography by introducing 8-D space  $SO(2,4)/SO(1,3) \times SO(1,1)$  as analog of AdS and its "boundary" as a 7-D space.

### 2.2.1 The proposal for the space of CDs

The proposal of the article is that the space of CDs is given by  $SO(2,4)/SO(1,3) \times SO(1,1)$  8-D space.

1.  $SO(2,4)$  is the 15-D conformal group of  $M^4$  including Poincare transformations, scalings and also the 4-D group of special conformal transformations.  $SO(2,4)$  acts linearly in 2+4-D Minkowski space  $M(2,4)$ . The action of special conformal transformation is however singular and this might mean difficulties. CD is defined by a 2-D sphere of  $M^4$  defining the maximal ball of CD and thus CD.

The space of CDs would be 8-D have signature  $(4,4)$  also now. This can be seen by noticing that boosts correspond to time-like directions and rotations to space-like directions so that the signature of  $SO(2,4)$  is  $(8,7)$ .  $SO(1,3)$  has signature  $(3,3)$  and  $SO(1,1)$  has signature  $(1,0)$ . Signature  $(4,4)$  is indeed the outcome. Maybe the two candidates for the space of CDs are identical.

2. Why  $SO(2,4)$  would be a correct choice for the group considered. The hyperbolic time evolution as an exponential of special conformal transformation in the time direction defined by CD maps the lower boundary of CD to the upper boundary. Special conformal transformation can be composed from an inversion with respect to the middle point of the CD followed by an ordinary translation followed by inversion. Inversion  $x^\mu \rightarrow x^\mu/x^2$  performs a local scaling

of the point of the point  $(\pm t, r = T \mp t)$  of the CD boundary to  $(\pm t, r = T \mp t)/(-T^2 \pm 2Tt)$  and scales the CD boundary. This scaled boundary is translated and preserves its shape. The inversion scales this CD down. Therefore special conformal transformations can leave the shape of CD invariant but induce Lorentz transformation, scaling, and translation.

3. The intuitive expectation is that the subgroup dividing  $SO(2, 4)$  should leave CD invariant. If so,  $SO(1, 1)$  should map the CD to itself. A possible identification is as the group of special conformal transformations in the time direction defined by CD. If  $SO(1, 3)$  corresponds to the usual Lorentz group, it does not leave the CD invariant unlike  $SO(3)$ .

This suggests a different interpretation. Perhaps the division means that  $SO(1, 3) \times SO(1, 1)$  consists of Lorentz boosts and hyperbolic special conformal transformations leaving the center point of CD invariant. Only  $SO(3) \times SO(1, 1)$  would leave the CD unaffected.

4. The basic distinction from the TGD inspired proposal is that there is no holography involved with CD. Rather, it seems that one starts from the idea that one has  $7 \rightarrow 8$  holography is associated with the 8-D  $SO(2, 4)/SO(1, 3) \times SO(1, 1)$ . 7-D subset of CDs would provide the holographic data. One can, of course, consider this kind of holography in WCW cm degrees of freedom.

For this option, the identification  $SO(3, 1) \times SO(1, 1)$  could have the same role as  $SO(3)$  in the case of the Poincare group or Lorentz group. This would give for CD Lorentz degrees of freedom and assignable with the  $SO(1, 1)$  mapping CD to itself.

### 2.2.2 How representations of the Poincare group could be realized in $SO(2, 4)/O(1, 3) \times SO(1, 1)$ ?

The representations of the Poincare group should be realized in the moduli space of CDs. How this is achieved is not obvious for the conformal group option.

1. It would seem that instead of finite-D possibly non-unitary representations of the rotation group  $SO(3)$  as in the case of  $P$ , non-unitary finite-D representations of  $SO(1, 3) \times SO(1, 1)$  characterize what might be called generalized spin degrees of freedom? The situation would resemble that in Poincare invariant QFTs for which one has non-unitary finite-D representations of  $SO(3, 1)$  given by say spinors. It is not clear whether the induction gives rise to a direct sum of state spaces associated with mass shells  $H^3$  as the physical intuition would suggest.
2. What seems like a paradox is that the Poincare group allows 4-D translations as Cartan algebra whereas  $SO(2, 4)$  has 3-D Cartan algebra. Constant mass squared for Poincare irreps however means that only 3 momentum generators are dynamical.  $SO(2, 4)$  Lie algebra contains 3-D Cartan algebra, which consists of commuting rotation generators of  $SO(4)$  and  $SO(2)$  having a discrete spectrum. Momenta are not expected to have a similar discrete spectrum without special assumptions. Intriguingly, the number theoretical vision of TGD, involving the notion of Galois confinement TGD, predicts that the spectrum of physical stats is integer valued in suitable units.
3. For the TGD inspired option the moduli space is identified as  $D \rtimes P/SO(3)$  and the situation is much simpler and differs from the construction of the representations of the Poincare group only by the presence of the scaling group with defines the analogs of unitary time evolutions preceding SSFRs. The scaling group would reduce by a symmetry breaking forced by the number theoretical picture to a discrete subgroup of scalings giving a discrete mass squared spectrum.
4.  $SO(2, 4)$  acts as symmetries of massless theories to which ordinary twistor approach applies so that the proposed picture is attractive in the framework of massless QFTs in  $M^4$ . Masslessness in  $M^4$  sense conforms with the invariance of the state space of massless particles under scalings.

In the TGD framework, the masslessness holds true in the 8-D sense and also the twistor space of  $CP_2$  enters the game so that masslessness in the  $M^4$  sense is not absolutely necessary

and the physical expectation is that the mass square spectrum is integer valued using as the momentum unit defined by CD size scale. Note however that the ground states of super symplectic representations are massless in this sense.

5. The space of CDs would be non-compact and would have 4 time-like directions. Holography analogous to AdS/CFT correspondence, with AdS interpreted as space-time, does not look plausible.

### 2.3 Questions about the dynamics in the space of CDs

CDs define a decomposition of WCW to sub-WCWs. The intuitive picture is that one has a network of CDs acting as analogs of interaction volumes and that "particle lines" connect the CDs to each other. One would approach standard ontology as these networks analogous to Feynman diagrams increase. In standard ontology one can speak not only about events, but something that exists. This would be like a transition from Eastern to Western world view.

Should one try to describe these particle lines by modifying the space of CDs and by introducing interactions between CDs? Does it make sense to assume that overlapping CDs interact in the sense that the space-time surfaces belonging to two different CDs touch? If CDs are interpreted as perceptive fields, this does not look like an attractive idea. The emergence of larger Feynman diagram-like structures would only mean emergence of larger CDs containing sub-CDs. Of course, understanding the interaction between CD and sub-CDs remains a challenge.

For the simplest option based one has the space of CDs, in which CDs are like particles with internal degrees of freedom. How can one construct transition amplitudes in the space of CDs?

1. If one can assign representations of the Poincare group to CDs, they would be analogous to particles characterized by momenta and angular momenta having also conformal weight as a quantum number associated with scalings. They would also have internal dynamics, which have been the main target of attention hitherto.
2. Transitions are expected to occur between CDs with different positions, sizes and different rest systems (direction of time line defined by the tips). In the recent picture, state function reductions are assumed to correspond to scalings only.

One would expect that the transition amplitude between quantum states for the moduli space propagation between two different CDs has a kinematic part, which one might hope to reduce to symmetry considerations just as for the propagation of particles in Minkowski space.

3. There is also an inner product of zero energy states related to CDs. The basis of the zero energy states characterized by holography are expected to differ by the action of an element  $g(CD_1, CD_2)$  of the group  $SO(4, 2)$  or of  $D \times P$  transforming the CDs to each other.

The transition amplitude should be proportional to the overlap of these states and therefore to the matrix element of  $g(CD_1, CD_2)$  between the zero energy states associated with  $CD_1$  and  $CD_2$ .

4. Physical intuition suggests that the transition amplitudes are small for "large" transformations  $g(CD_1, CD_2)$  and that in good approximation small translations, Lorentz boosts, and scalings are preferred. In the approximation that translations and Lorentz boosts affecting the center point of CD are trivial, only scalings and hyperbolic evolutions remain under consideration.

## 3 TGD inspired questions and ideas related to the interpretation of the hyperbolic flow

The interpretation of the findings related to hyperbolic time evolution in the TGD framework inspire several questions and ideas.

### 3.1 The flow lines of time-like special conformal transformation as a motion with a constant acceleration

The exponentiated hyperbolic generator  $S$  corresponds to a relativistic motion in  $M^4$  with a constant acceleration  $a$ , which is essentially the inverse of the size  $L$  of CD:  $a = \hbar c/L$  for  $c = 1$ , where  $L$  is the size of CD identified as distance between its tips. Could this constant acceleration correspond to a representation for an acceleration of the system defined by CD in an external gravitational field, which is constant in the first approximation?

Note that this acceleration is huge when compared with that assignable to macroscopic systems. Gravitational acceleration  $g$  at the surface of Earth corresponds to a thermal energy of order  $10^{-21}$  eV. The size of the CD would be in this case of order  $10^{15}$  m.

It has been already noticed that in the TGD framework  $S$  can affect only the holographic data at the boundaries of CD. The action of  $S$  exponentiates to a unitary hyperbolic time evolution on quantum states at the boundaries of CD. The two-point functions of the conformal QM are thermal with temperature determined by the scale  $L$  of CD so that the acceleration  $a = 1/L$  can be said to make itself visible via Unruh effect (<https://rb.gy/qxyp8q>).

### 3.2 Can one assign thermodynamics to CD?

One can assign to the CD a temperature. I have earlier proposed that the expansion of CD by sequences of SSFRs could be interpreted as a stepwise cosmic expansion with temperature decreasing like  $\hbar/L$ . This would conform with the finding that the astrophysical objects themselves do not seem to participate in the expansion.

1. In QFT in  $M^4$ , constant acceleration corresponds to the so-called Unruh temperature proportional to  $\hbar \times a$ . For the ordinary value of  $\hbar$ , this temperature is extremely small for the accelerations encountered in macroscopic quantum systems. An accelerated system sees the vacuum of an inertial system as a black body at Unruh temperature.

Note that the blackhole temperature is analogous to Unruh temperature and proportional to  $\hbar GM/R^2 \propto \hbar/GM$  (this temperature is extremely small for astrophysical blackhole-like entities).

2. The conformal 2-point functions of a CFT inside a CD with "energy"/Hamiltonian associated with infinitesimal special conformal transition, behave like those for ordinary CFT at finite temperature  $T_{CD} = \hbar c/L$ ,  $L$  the size of CD.  $T_{CD}$  is analogous to Unruh temperature but much larger.

For a massive particle with Compton length  $L$ , the CD temperature would be of order  $T_{CD} = m = \hbar c/L$ . This would be more like ordinary temperature for a system of mass  $m$  moving in a volume defined by Compton length. I have proposed that CDs are characterized by this temperature and the expansion of CD in the sequence of SSFRs leads to the reduction of this temperature analogous to that taking place in cosmology.

What could be the interpretation of  $T_{CD}$ ?

1. For the cell scale the CD temperature  $T_{CD}$  would be of order of the energy of visible light. Could  $T_{CD}$  relate to bio-photons? Could CD temperatures correspond to those in ordinary thermodynamics and could they be interpreted as a kind of sensory/cognitive representations for the real temperatures in terms of the internal physics of CD?
2. In p-Adic thermodynamics [K2] [L19], energy is replaced with mass squared interpreted as proportional to conformal weight as in string models. The p-adic analog of the temperature is inverse of an integer and corresponds to a dimensional p-adic temperature  $T_p = \hbar \log(p)/L_p$ , where  $L_p$  is the p-adic length scale proportional to  $\sqrt{p}$ . p-Adic length scale corresponds to the length scale defined by the Compton length of the particle. p-Adic length scale hypothesis states that preferred p-adic primes are near to powers of 2, or possibly also other small primes and that these primes correspond to fixed points of discrete p-adic coupling constant evolution. Number theoretic vision of TGD suggests a concrete mechanism implying this [L21].

A possible interpretation is that p-adic massivation and p-adic temperature characterizes the density matrix for the particle entanglement with the environment [L19]. p-Adic temperature is assigned with a scaling which changes the size of the CD and could be perhaps associated with "big" CDs.

$T_{CD}$  would naturally correspond to the p-adic temperature  $T_p = \hbar \log(p)/L_p$ .

3.  $T_{CD} = \hbar/L$  generalizes to  $T_{CD} = \hbar_{eff}/L$ . For gravitational Planck constant  $\hbar_g r/\hbar = GMm/\beta_0$ ,  $\beta_0 < 1$ , where  $M$  is some large mass, say Earth mass, one has  $T \propto (GMm/\beta_0)m$ .

If the size scale of CD is expected to scale like  $\hbar_{eff}$ , one obtains a zoomed version of the system and temperature is not changed. This applies also to the temperature  $T_{CD,gr} = \hbar_g r/L$  if  $L$  scales like  $\hbar_{gr}$ . Note that for  $\hbar_{gr}(Earth)$  assignal to dark particles at gravitational flux tubes, gravitational Compton  $L$  is about .45 cm and does not depend on mass  $m$  (Equivalence Principle). For electrons and protons  $T_{CD,gr}$  would be unrealistically high.

### 3.3 Could the dynamics of CD define a sensory map of the exterior of CD?

The time evolution by a special conformation maps the CD into itself. This is a surprising result. The infinitesimal generator vanishes at the light-like boundaries and the generator is time-like for either half-cone of the cd.

In TGD, CD is identified both as the analog of quantization volume and the perceptive field of self. This raises some questions.

1. The special conformal transformation in time direction consists of an inversion  $I$  with respect to the center point of the CD followed by an ordinary time translation followed by the same inversion  $I$ .

Ordinary time translation in the exterior is mapped to the special conformal transformation inside the cd. Does this imply some kind of cognitive or sensory map of the exterior world to the interior world? Note that the same can be done also for the other special conformal transformations. Is this something that the monads of Leibniz might be doing?

2. Inversion is also involved with the  $M^8 - H$  duality and gives a semiclassical realization of Uncertainty Principle [L6, L7]. The mass shells  $H^3 \subset M^4 \subset M^8$  with mass  $m$  are mapped by inversion  $m \rightarrow a = \hbar_{eff}/m$  to cosmic time  $a = constant$  hyperboloids of  $M^4 \subset H$ .

$M^8 - H$  duality, has an interpretation as a generalization of momentum-position duality. This duality is natural since quantum TGD is essentially the generalization of wave mechanics obtained by replacing point-like particles with 4-D surfaces obeying holography. Could one interpret the special conformal transformations in terms of  $M^8 - H$  duality. If so, special conformal transformations could be seen as mirror images of translations of in momentum space.

Twistor approach [B2] to scattering amplitudes involves a duality between two Minkowski spaces in which dual conformal groups act.  $M^8 - H$  duality suggests a natural identification of these two Minkowski spaces as space-time and momentum space and could correspond to this duality.

3. In zero energy ontology (ZEO) [L5, L16, L12] [K8], the time flow corresponds to the increase of CD (at least in statistical sense) taking place in scalings of CD? For a sequence of time-like special conformal transformations realized as unitary hyperbolic time evolutions the experienced flow of time should therefore cease. Could these sequences serve as correlates for "timeless" meditative states of consciousness?

### 3.4 Could astrophysical SSFRs correspond to special conformal transformations leaving CD size unaffected?

There are also questions related to the TGD inspired cosmology, which can be quantum coherent in arbitrarily long length scales. This would explain the constancy of the temperature of CMB.

In the TGD inspired cosmology [K6, K5], the cosmic expansion does not take place for astrophysical objects (stars and planets at least) as a smooth process but as rapid "jerks" in which flux tubes as thickened cosmic strings suffer phase transitions increasing their thickness and reducing their string tension [K1, K4, K7]. This is also an empirical fact and not understood in general relativity.

On the other hand, the emergence of complex life forms in the Cambrian Explosion is a biological mystery. TGD explains it in terms of this kind of rapid expansion of the radius of Earth by factor 2 bursting highly developed life forms from underground oceans to the surface of Earth [L4, L13, L8, L20].

Should one modify the views of ZEO [L5, L16, L12] [K8]? Could one think that at quantum level the TGD analog of unitary time evolution could correspond to a sequence of "small" state function reductions (SSFRs) followed by hyperbolic time evolutions, which do not scale up the CD? This even in astrophysical scales. In this view, only "big" SFRs (BSFRs) changing the arrow of time and occurring at the level of magnetic body (MB) would correspond to the scalings of CD and a pair of BSFR would bring back the original arrow of time. This would be the case if CD size corresponds to that for a *real* physical system rather than for the perceptive field of a conscious entity as assumed hitherto.

### 3.5 About the quantum realization of hyperbolic time evolution in TGD framework

What one can say in the TGD framework about the quantum realization of the exponentiated  $S$  using holography?

1. The situation can be understood by noticing that in the lowest approximation the motion is given by  $r = r_0 + gt^2/2$  so that a parabola intersecting the light-cone boundary is obtained in a finite time.  $S$  is well-defined at the light-like boundaries of CD.
2. Holography inside CD does not allow assignment of a flow inside CD to the representation of the exponentiation of  $S$  represented as an operator.  $S$  is not parallel to the light-like boundaries, in which case the action of the superposition of 3-surfaces would be non-trivial. Therefore only the action in fermionic degrees of freedom can be non-trivial.
3. The action of  $S$  on the quantum state at the boundary makes sense and is analogous to that of a Hamiltonian in Hilbert space at time= constant hyper-surface. Zero energy state is the sum of over pairs of 3-D states located at passive and active boundaries of CD and by holography the action of exponentiation of  $S$  at the passive boundary and determines the action of  $S$  at the active boundary. There is no flow inside the CD. Holography defines the state at the opposite boundary.

One obtains a one parameter family of exponentiations of  $S$  analogous as unitary time evolution operators  $U(\tau)$ . The value of the parameter  $\tau$  in the exponential runs from  $-\infty$  to  $+\infty$  and corresponds to a finite range for the ordinary time coordinate  $t$ . This is indeed analogous to 1-D conformal QM since the angular and radial degrees of freedom are effectively absent. The light-like radial coordinates take the role of complex variable  $z$  in conformal field theories.

## 4 Motion of CDs in the moduli space and kinematic qualia

The proposal that CD serves as a correlate for the perceptive field of self implies that the motions of CDs in the moduli space of CDs gives rise to the kinematic qualia assignable to various kinds of motions.

The first thing to notice is that the transformations, which leave the position of CD that is the center point of CD invariant, are in a special role.

1. These transformations consist of Lorentz group  $SO(3, 1)$ , scalings, plus possibly also the subgroup  $SL(2, R)$  of special conformal transformations leaving the center point of CD invariant. This group would characterize the internal degrees of freedom analogous to the orientations of a rigid body.

2. Besides this there are transformations of  $SO(2, 4)$  affecting the center point of the CD. The moduli space  $SO(4, 2)/SO(1, 3) \times SO(1, 1)$  could correspond to these transformations in the space of generalized positions of CD.
3.  $SO(1, 3) \times SO(1, 1)$  would leave the center point of the CD invariant and code for various shapes of the CD with one point fixed. This space would consist of various shapes of CD representing the states of motion of the CD. For instance, if a CD moves in a particular direction with some velocity. The CD would have a shape characterized by a corresponding Lorentz transformation. This group includes also scalings leaving center point invariant.

## 4.1 The motion at quantum level

One can consider the situation both at quantum and classical level.

1. At the quantum level the system would be characterized by a wave function in moduli space and small state function reduction (SSFRs) would correspond to steps in the motion.

The analogues of both position and momentum measurements are possible and these could by  $M^8 - H$  duality correspond to position measurements in the space of CDs assignable to  $M^8$  and  $H$ . This duality could correspond to the duality of twistor Grassmannian amplitude realized in terms of ordinary and momentum twistors [L3].

If a localization takes place in the degrees of freedom considered as commuting degrees of freedom at each step, one obtains a discrete motion in these degrees of freedom.

2. The  $M^4$  position of CD would define naturally Cartan algebra and SSFRs involving a position measurement in  $E^3$  would define the discrete motion in  $M^4$ .
3. This discrete motion in  $H$  would give rise to kinematic qualia such as experience of motion with constant velocity and acceleration. The localization in the space of CDs with respect to scaling would fix the size of CD and therefore geometric time as a correlate of subjective time identified as a distance between the tips of CD. The localization with respect to the time coordinate of CD is impossible and does not allow to identify Minkowski time as an observable.

Scalings increasing the size of CD in a statistical sense would correspond to the growth of geometric time as distance between the tips of CD correlating with the subjective time identifiable as a sequence of SSFRs.

4.  $D$  and boost and rotation generators (call them  $K_z$  and  $J_z$ ) of the Lorentz group leaving the center point of the CD invariant can serve as observables for states localized in the moduli space.

What about momentum eigenstates in the moduli space?  $D$  does not commute with momentum so that for momentum eigenstates  $D$  is not well-defined.  $K_z$  and  $J_z$  would be analogous to helicity since their identification depends on the position of CD or momentum of momentum eigenstate. The measurement of  $D$  implies delocalization with respect to the size of CD. The dual measurement of the size of CD, call it  $L$ , means a measurement of geometric time.

For a given size  $L$  of CD one can construct momentum eigenstates as analogues of plane waves so that the measurement of  $L$  seems to commute with the measurement of momentum. Geometric time as a gradual increase of the size of CD would flow only during the measurements involving measurement of  $L$ .

### 4.1.1 Can the scaling operator $D$ correspond to the "stringy" scaling operator $L_0$ ?

There is a problem related to the identification of conformal weights as eigenvalues of the scaling operator  $L_0$  as mass squared eigenvalues. In string models, the scaling operator  $L_0$  acting in complex coordinate  $z$  of the string world sheet is proportional to the mass squared operator and commutes with it.  $L_0$  commutes with the mass square operator but the scaling operator  $D$  does not. What could be the counterpart of  $L_0$  in TGD?

One can consider several candidates for  $L_0$  in TGD. Light-like 3-surfaces appear at the level of both embedding space and space-time surface so that there are two basic types of candidates. These two kinds of scaling generators could relate to the notions of inertial and gravitational masses and therefore to the Equivalence Principle.

Consider first the embedding space level.

1. The light-like radial coordinate  $r$  for the light-cone boundary would provide the first guess for  $L_0$  as scaling  $L_0 = r\partial_r$ .  $L_0$  however scales the size of the light-like boundary and therefore CD so that this interpretation can be challenged.

On the other hand, the basic assumption is that the states at the passive boundary of CD are unaffected under SSFRs so that scaling should not affect the momenta. Note that the center point of CD must be shifted in time direction under the action of CD so that one has a combination of  $D$  and  $P_0$ .

2. The most stringy candidate for  $L_0$  would be as the scaling generator  $L_0 = zd/dz$  for the conformal transformations of  $S^2$  of light-cone boundary representable as  $S^2 \times R_+$ . Remarkably,  $L_0$  does not affect the size of the CD.

The generators of globally defined conformal transformations of  $S^2$  are representable as Möbius transformations representing  $SL(2, C)$ . One can even compensate for the conformal scaling factor associated with these transformations by a suitable radial scaling of  $r$  depending on  $z$  so that they act as isometries.

Light-like 3-surfaces appear also at the level of space-time surfaces. Both the light-like 3-surfaces defining boundaries of space-time surfaces [L17], and the boundaries between Euclidean wormhole contacts and Minkowskian space-time regions, identified as deformed  $CP_2$  type extremals, allow by their metric 3-dimensionality extended conformal invariance.

1. One assign light-like coordinate  $r$  with the light-like 3-surface and the scaling generator  $L_0 = r\partial_r$  is highly analogous to  $L_0$ .
2. Also now one has the representations  $X^2 \times R_+$  and complex coordinate for  $X^2$  defines counterpart of  $L_0$ .
3. Could  $L_0$  assignable to  $X^2$ , represent gravitational mass squared? Could holography assign momenta to the light-like 3-surfaces identifiable as gravitational momenta and by Equivalence Principle be identical with inertial momenta assignable to the boundaries of CDs. Gravitational-inertial dictotomy would reflect the space-time embedding space dichotomy.
4. Classical TGD is an exact part of quantum TGD and the conserved classical momenta assignable to the space-time surfaces are also natural candidates for gravitational momenta. The inertial momenta would be assignable to the CDs.

## 4.2 The motion at the classical level

At the classical level would have motions in moduli space consisting of small pieces, which are geodesic lines in  $D \times P$  or  $SO(4, 2)$ .

1. Causality would mean that only discretized world lines consisting of pieces which are time-like or light-like geodesic lines are considered (recall that the signature of the moduli space is  $(4, 4)$ ).
2. A small transformation of the group  $D \times P$  or of  $SO(2, 4)$ , which is representable as an action of a Lie algebra generator, would describe a basic step in the motion. In the quantum case, one must have Cartan algebra and one can argue that all classical motions consist of small translations or scaling rotation, and boost commuting with it.

The measurements of the size and position of CD are dual measurements and the measurement of the size of CD would give rise to the experience flow of time correlating with the flow of the geometric time.

3. For  $D \times P$  small translations along geodesics of  $M^4$  would be basic building bricks of translational motion and the direction of small translation would change step by step in curvilinear motion.

For  $SO(2, 4)$  also special conformal transformations affecting the center point of the CD could be considered classically. They do not however commute with translations so that at least quantum mechanically they would be excluded. The 4 special conformal transformations could however serve as a dual set of observables instead of translations.

4. A Lorentz boost of CD would code for the velocity associated with a given step of translational motion and would be determined by the direction and size of the step. It would be coded by the shape of the CD characterized by the direction of the time-like axis connecting its tips. The shape of the CD would change during the motion.
5. Rotational motion around the center point would leave center point and CD itself invariant but affect the zero energy state of CD. More general rotational motion would correspond to steps consisting of translations.

## 5 Questions related to ZEO

ZEO involves several questions which are not completely understood. Do SSFRs correspond to repeated measurements for a set  $O$  of commuting observables? Does BSFR occur when a new set of observables not commuting with the set  $O$  are measured? What exactly happens in SSFR?

### 5.1 Questions related to SSFRS

#### 5.1.1 SSFRs as a generalization of Zeno effect and weak measurements

Consider once again the question related to the identification of SSFRs. SSFRs are identified as the TGD counterpart for weak measurements, generalizing the notion of repeated measurements giving rise to the Zeno effect.

1. The most straightforward generalization of the Zeno effect is that in the kinematic degrees of freedom for CDs the sequence of SSFRs corresponds to a sequence of measurements of commuting observables. BSFR would take place always when the set of measured observables changes to a new one, not commuting with the original set.
2.  $D, K_z$  and  $J_z$  leave the center point of CD, identified as position of CD, invariant.  $D$  does not commute with momenta. Should one just accept that momenta and  $\{D, K_z, J_z\}$  are two sets of mutually commuting observables and that the change of this set induces BSFR.

The size of CD and therefore the value of the geometric time would change in the sequence of measurements of  $D, K_z$  and  $J_z$  but not in the sequence of momentum measurements one would have superposition over different sizes of CD and time would be ill-defined as also Uncertainty Principle requires. This would conform with the original view.

#### 5.1.2 What really happens in SSFRs?

I have written a lot of what might happen in SSFRs and BSFRs but I must admit that the situation is still unclear and the proposals depend on what one takes as starting point assumptions, which can be overidealizations.

On the more general level, the sequence of SSFRs would correspond to dispersion in the moduli space of CDs and if SSFRs correspond to the measurement of same commuting observables identified as generators of  $SO(2, 4)$  or  $D \times P$  or their duals as generalized position in the moduli space, rather simple picture emerges of what can happen.

BSFR would take place when the new set of observables not commuting with the original set emerges. What are the conditions forcing this? If one assumes that sleep is induced by BSFR, it becomes clear that this does not happen at will but when metabolic energy resources are depleted and the system must rest. The dissipation of the time reversed system looks like self-organization

and the system heals during sleep. Also homeostasis would rely on BSFRs in various scales making it possible to stay near quantum criticality.

But what exactly happens in SSFR? It seems clear that the states at the passive boundary are not changed. But what happens to the passive boundary?

1. Do the contents of sensory experience assigned with the sequence of SSFRs localize

**Option a:** to the active boundary of the CD or

**Option b:** to the 3-ball at which the half-cones of the CD meet.

2. What happens to the passive boundary itself in SSFR? The scaling occurs for the entire CD but there are two basic options.

**Option 1:** The scaling leaves the *center point* of the CD invariant. Passive boundary is shifted towards past just like active boundary towards future.

If the sensory experience is assigned to the active boundary (Option a)), option 1) is consistent with what happens when we wake up. The time has been flowing during sleep but we have not been aware of this. The arrow time would be determined solely by the change of the state at the active boundary.

If the sensory experience is assigned with the 3-ball (option b)) at the center of CD (Option b)), time does not flow in the sequences of SSFRs.

**Option 2:** The scaling leaves invariant the tip of CD associated with the passive boundary so that it is not shifted at all but is scaled. This option is consistent with both option a) and b) for the localization of the experience of time flow. However, waking-up from sleep would take at the time when we fell asleep: this does not make sense.

The model for sleep favours option a)+1) for which CDs would define ever expanding sub-cosmologies changing the arrow of time repeatedly. Any conscious entity would eventually evolve to a cosmology, a kind of God-like conscious entity.

3. One can also consider other empirical inputs. There are stars and even galaxies older than the Universe. Their existence is consistent with option a)+1).

CDs form a scaling hierarchy. CDs in the distant geometric past assignable to stars and galaxies are much smaller than the cosmological CD. The scaling cosmological CD inducing the time flow takes place much faster than the scaling of the much smaller astrophysical CDs. Cosmological time runs much faster and astrophysical CDs remain in the distant geometric past.

4. A third test is based on after images, which appear repeatedly. They correspond to sub-CDs of a CD. Could the after images correspond to life cycles of the *same* sub-CD as I have proposed? This is the case if the sub-CDs are comoving in the scalings of the CD shift. This looks rather natural.

## 5.2 More questions and objections related to ZEO and consciousness

The best way to make progress is to make questions and objections against the existing view, which is often far from clear. In the following I raise some questions of this kind.

What could BSFR mean biologically?

1. I have considered the possibility that BSFR could mean as biologically birth in opposite time direction. This however leads to rather complex speculations.

The most natural assumption is that it means what it says, the emergence of a new CD [L22] as a perceptive field of a conscious entity. This does not require that biological death would be a birth in the opposite time direction although this cannot be excluded. This means one counter argument less.

2. I have considered the idea that in BSFR the size of a CD could decrease dramatically so that the reincarnated CD would be much smaller than before BSFR. This would make possible what one might call childhood. The idea is that the painful memories from the end of the

lifecycle could be deleted. This model however requires rather detailed assumptions about how the memories of life cycle are stored at the active boundary of CD. The oldest memories would reside near the tip of CD and newest nearest to the intersection of the half-cones of the CD.

Is this picture consistent with the view about SFR as a localization in the space of CDs? Since the number of CDs larger than given CD is much larger than those with size smaller than it, one can argue that the size of CD increases in statistical sense without limit in SFRs. If one can assume that death involves localization in the space-like degrees in the space of CDs ( $E^3$  position and size of CD), the reduction of CD size looks rather implausible. If the preceding SSFR involved also this kind of localization then the CD after BSFR would in statistical sense be larger than it was before BSFR.

3. Can CDs interact? For instance, can a CD catch the sub-CD defining a mental image of the CD with which it overlaps? This is not the case: it is not possible to catch the spotlight of consciousness.

CD serves as a correlate for the perceptive field of self. Self is also an active causal agent. This aspect must relate to the zero energy states defined as superpositions of space-time surfaces inside a CD.

1. CD defines a perceptive field, a kind of spotlight of consciousness, which makes it possible to sensorily perceive the space-time surface, which continues outside CD although one can also imagine a situation in which this is not the case. Saying that mental image co-moves means that the spotlight moves.
2. Self has also causal powers. SSFRs change the state at the active boundary of CD. This induces changes inside the future light-cone in turn define perturbations of CDs of the geometric future possibly inducing BSFRs.

Since the superposition of 3-surfaces at the active boundary of CD changes in SSFR, SSFRs have an effect on the geometric future. This is of course the case: our acts of free will affect the world around us but conform with causality.

Almost deterministic holography for space-time surfaces and zero energy states dramatically reduces the freedom of free will due to state function reductions. The delocalization in WCW taking place in the space of CDS during the analogues of unitary time evolutions preceding SSFR improves the situation.

One can also imagine a situation in which nothing changes at the boundaries of CD: self is completely passive: this is of course true at the passive boundary and can be true also at the active boundary in special situations. The classical time evolution for preferred extremals is not fully deterministic. Space-time surface is analogous to a 4-D soap film with frames and the case of 2-D soap films suggests that a finite non-determinism is assignable to the frames. This kind of SSFRs would not affect the space-time surface around CD at all. Pure cognition or meditative states might correspond to this kind of SSFRs.

The notion of ego is central in Eastern philosophies. How could one understand this notion in the ZEO based theory of consciousness?

1. Ego means that mental images want to survive. Self survival instinct is an analogous notion although it refers to the biological body. The quantum state at the passive boundary of the CD defines a good candidate for ego since it is indeed preserved during the sequence of SSFRs during which the set of measured observables is preserved.
2. BSFRs means death of self or subself as a sub-CD. Also the external physical perturbations arriving at the passive or active boundary can affect the quantum state at it and can induce BSFR. The self assignable to CD is exposed to perturbations, which might induce BSFR. A simple example of this kind of perturbation would be a blow in the head inducing a loss of consciousness.

Ego preservation could mean that self does its best to make the periods of time with an opposite arrow of time as short as possible. This is not in conflict with the fact that the

durations of sleep and awake states are roughly the same if a given arrow of time means that the time fraction spent in a state with this arrow of time dominates over that in a state with an opposite arrow of time.

At the magnetic bodies carrying dark matter as phases with large  $h_{eff}$ , the interactions perturbing the boundaries of CD are expected to be rather weak. One has something analogous to a quantum computer isolated from the external world.

3. This suggests a more quantitative definition of the period with a fixed arrow of time. One expects that consciousness with a given arrow of time can have gaps. There is indeed empirical evidence suggesting that our flow of consciousness has gaps. Perhaps the wake-up-sleep ratio of the periods with different arrows of time is what matters. For a given arrow of time, the system would be dominantly in wake-up state or in sleep state.

At a given level of self-hierarchy there is some average time for a given arrow of time and it is expected to increase at the higher levels. Magnetic bodies carrying dark matter interact only weakly with lower levels of the hierarchy, in particular ordinary matter, would make possible long periods with a given arrow of time, in the first guess proportional to say  $h_{eff}$ .

4. What could biological death as a process at the level of ordinary biomatter mean? Is biological death determined by the situation at the lower hierarchy levels? On the other hand, dark matter at MBs defines a control hierarchy and is gradually thermalized as suggested in [L23] so that the ability to perform biocontrol is reduced. Also the ability to gain metabolic energy is reduced and makes it difficult to preserve the arrow of time. Since the average value of  $h_{eff}$  is reduced, the system becomes more vulnerable to perturbations inducing a BSFR changing the arrow of time.

There are also questions related to metabolism.

1. A metabolic energy feed is needed to preserve the distribution for the values of  $h_{eff}$ . The energies of quantum states increase with  $h_{eff}$  and in the absence of a metabolic energy feed, the values of  $h_{eff}$  at MBs tend to decrease. The system becomes more vulnerable to perturbations and the BSFRs changing the arrow of time occur more often. The system becomes drowsy.
2. Sun serves as a fundamental source of metabolic energy but TGD leads to a proposal that also radiation from the core of Earth, which happens to be at the same wavelength range as solar radiation could have served and maybe still serve as a source of metabolic energy.
3. I have proposed remote metabolism as a mechanism in which the system contains a subsystem with an opposite arrow which emits energy, say dissipates, in opposite time direction and thus seems to gain metabolic energy if seen from the standard arrow of time.

This is possible if there is a system able to receive the *effective* negative energy signals. For instance, a population reversed laser could serve as such a system. The second option is that the environment loses thermal energy so that the second law in its standard form would be violated. For instance, heat could be transferred from a system with a given temperature to a system with higher temperature. The dissipation for the time reversed system looks like self-organization. Sleep periods would in this picture mean gain of metabolic resources and healing.

4. Also life with the opposite arrow of time needs metabolic energy. We receive metabolic energy basically from the Sun. Could the Sun serve as a source of metabolic energy also for the time reversed systems? The answer is positive.

To understand why, one must clarify what the change of the arrow of time means. Time reversed signals have positive energy and only the reversed time direction makes them look like negative energy signals. The sum of energies for the sub systems with opposite arrows of time is conserved apart from effects due to finite sizes of CDs (Uncertainty Principle). Also life with an opposite arrow of time can use solar energy as a metabolic energy source.

5. The biological death is assumed to be due to the loss of quantum coherence at the level of MBs inducing a loss of ordinary coherence in short scales implying bodily decay. What could the situation be in the next reincarnation with the same arrow of time? Does the next life with the same arrow of time end at roughly the same time so that the size of the CD would become rather stationary. There would not be much progress.

Or could the MB be able to preserve the quantum coherence for a longer time in the next reincarnation? Since the quantum coherence of MB naturally explains the coherence of the ordinary biomatter, impossible to understand in the standard physics framework, there is no reason why MB could not achieve this feat in the next incarnation.

### 5.3 Is Negentropy Maximization Principle needed as an independent principle?

The proposal has been that Negentropy Maximization Principle (NMP) [K3] [L10, L1] serves as the basic variation principle of the dynamics of conscious experience. NMP says that the information related to the contents of consciousness increases for the whole system even though it can decrease for the subsystem. Mathematically, NMP is very similar to the second law although it states something completely opposite. Second law follows from statistical physics and is not an independent physical law. Is the situation the same with the NMP? Is NMP needed at all as a fundamental principle or does it follow from number theoretic physics?

The number theoretic evolution is such a powerful principle that one must ask whether NMP is needed as a separate principle or whether it is a consequence of number theoretical quantum physics, just like the second law follows from ordinary quantum theory.

2 additional aspects are involved. Evolution can in adelic physics [L2] be seen as an unavoidable increase in the algebraic complexity characterized by the dimension  $n = h_{eff}/h_0$  of extension of rationals associated with the polynomial define space-time surface at the fundamental level by so-called  $M^8 - H$  duality [L6, L7]. There is also the possibility to identify a quantum correlate for ethics in terms of quantum coherence: a good deed corresponds to a creation of quantum coherence and the evil deed to its destruction.

How do these two aspects relate to the NMP? Is NMP an independent dynamical principle or a consequence of number theoretic (adelic) quantum physics?

Consider in the sequel "big" state function reduction (BSFR) as the counterpart of the ordinary state function reduction. I'm not completely sure whether the following arguments can be also applied to SSFRs for which the arrow of time does not change.

One can consider two alternative formulations for NMP.

#### 5.3.1 Option I

Option I is the simpler and physically more plausible option.

1. BSFR divides the quantum entangled system at the active boundary of CD into two parts, which are analogous to the measurement apparatus and the measured system. The selection of this partition is completely free and decided by the system. This choice corresponds to an act of free will. Depending on conditions to be discussed, the action of the measurement to this pair can be trivial in which case the entanglement is not reduced. The measurement can also reduce the entanglement partially or completely and the p-adic entanglement negentropy and entropy decreases or becomes zero.
2. If the partition into two parts is completely free and if the choice is such that NMP, or whatever the principle in question is, allows BSFR, the quantum coherence decreases. Number theoretic evolution suggests that the principle telling when BSFR can occur is number theoretic.

There is a cascade of BSFRs since BSFRs are also possible for the emerging untangled subsystem and its complement. The cascade stops when the entanglement becomes stable.

3. What condition could determine whether the reduction of the entanglement takes place? What could make the entanglement stable against BSFR?

Number theoretical vision suggests an answer. Physical intuition suggests that bound states represent a typical example of stable quantum entanglement. Bound states correspond to Galois confined states [L18, L9, L14, L15] for which the momenta of fermions are algebraic integers in an extension of rationals but total momentum has integer valued components. This mechanism for the formation of the bound states would be universal.

A natural number theoretical proposal is that the entanglement is stable if the entanglement probabilities obtained by diagonalizing the density matrix characterizing the entanglement belong to an extension of rational, which is larger than the extension, call it  $E$ , defined by the polynomial  $P$  defining the space-time surface. An even stronger condition, inspired by the fact that cognition is based on rational numbers, is that BSFR can take place only if they are rational.

This kind of entanglement would be outside the number system used and one can argue that this forces the stability of the entanglement. A weaker statement is that the reduction is possible to a subspace of the state space for which the entanglement probabilities belong to  $E$  (or are rational).

4. This option could replace NMP as a criterion with a purely number theoretical principle. This does not however mean that NMP would not be preserved as a principle analogous to the second law and implied by the number theoretic evolution implied by the hierarchy of extensions of rationals.

Could free will as the ability to do evil or good deeds reduce to number theory that is to the choice of a partition, which leads to either increase or decrease of entanglement negentropy and therefore of quantum coherence?

The basic objection can be formulated as a question. How can the conscious entity know whether a given choice of partition leads to BSFR or not? Memory must be involved. Only by making this kind of choices, a system with a memory can learn the outcome of a given choice. How could the self learn, which deeds are good and which are evil? The answer is suggested by the biologically motivated view of survival instinct and origin of ego [L22] based on SSFRs as a generalization of Zeno effect.

1. Conscious entity has a self characterized by the set of observables measured in the sequence of SSFRs. BSFR as a reduction of entanglement occurs when a new set of observables not commuting with the original set are measured. In BSFR self "dies" (loses consciousness). Second BSFR means reincarnation with the original arrow of time.
2. The perturbations of the system at both boundaries of CD are expected to induce BSFRs and to occur continually. Therefore the arrow of time is fixed only in the sense that it dominates over the opposite arrow.
3. Self preserves its identity (in particular memories defining it) if the second BSFR leads to a set of observables, which does not differ too much from the original one. The notions of survival instinct and ego would reduce to an approximate Zeno effect.
4. This mechanism would allow the self to learn the distinction between good and evil and also what is dangerous and what is not. A BSFR inducing only a brief period of life with a reversed arrow of time could teach the system when the BSFR leads to a reduction of entanglement and loss of coherence.

The harmless BSFRs could provide a mechanism of imagination making survival possible. Intelligent systems could do this experimentation at the level of a self representation of a system rather than in real life and the development of complex self representations would distinguish higher life forms from those at a lower evolutionary level.

### 5.3.2 Option II

Option II is stronger than Option I but looks rather complex. I have considered it already before. NMP would select a partition for which the negentropy gain is maximal in BSFR or at least, the

decrease of the negentropy is minimal. One must however define what one means with negentropy gain.

Before considering whether this condition can be precise, it is good to list some objections.

1. Is the selection of this kind of optimal partition possible? How can the system know which partition is optimal without trying all alternatives? Doing this would reduce the situation to the first option.
2. Free will as ability to do also evil deeds seems to be eliminated as a possibility to either increase or decrease entanglement negentropy and therefore quantum coherence by choosing the partition of the system so that it reduces negentropy.
3. If the BSFR cascade would lead to a total loss of quantum entanglement, the entanglement negentropy would always be zero and NMP would not say anything interesting. On the other hand, if the selection of the partition is optimal and the number theoretic criterion for the occurrence of the reduction holds true, it could imply that nothing happens for the entanglement. Again the NMP would be trivial.
4. What does one mean with the maximal negentropy gain?

### 5.3.3 What does one mean with a maximal negentropy gain?

Option II for NMP says that for a given partition BSFR occurs if the entanglement negentropy increases maximally. What does one mean with entanglement negentropy gain? This notion is also useful for Option I although it is not involved with the criterion.

1. Entanglement negentropy refers to the negentropy related to the *passive* edge of the CD (Zeno effect). Passive boundary involves negentropic entanglement because NMP does not allow a complete elimination of quantum entanglement (bound state entanglement is stable). The new passive boundary of CD emerging in the BSFR corresponds to the previously active boundary of CD.
2. For option I for which the concept of good/bad is meaningful, the number theoretical criterion could prevent BSFR and stop the BSFR cascade. There is however no guarantee that the total entanglement negentropy would increase in the entire BSFR cascade. This would make the term "NMP" obsolete unless NMP follows in a statistical sense from number theoretic evolution: this looks however plausible.

The unavoidable increase of the number theoretical complexity would force the increase of p-adic entanglement negentropy and NMP as an analog of the second law would follow from the hierarchy of extensions of rationals.

## 6 Appendix: About the action of $SL(2, R)$ inside causal diamond

$SL(2, R)$  appearing as conformal symmetries of conformal field theories and mapping the upper half plane of the complex plane to itself by Möbius transformations. I have collected the basic facts about how  $SL(2, R)$  is realized for causal diamonds. These facts can be found also from [B4].

The Lie algebra of  $SL(2, R)$  is spanned by the generator  $D_0 r \partial_r + t \partial_t$  of scaling with respect to the center point of CD, by the generator  $P_0 = \partial_t$  of time translation in the direction defined by the line connecting the tips of CD, and by the generator  $K_0 = 2tr \partial_r + (t^2 + r^2) \partial_t = 2tD_0 + (r^2 - t^2)P_0$  of a special conformal transformation in time direction obtained as  $IP_0I$ , where  $I$  is the inversion with respect to the center point of CD. Conformal Killing vector is linear combination of form

$$\xi = aK_0 + bD_0 + cT_0 . \quad (6.1)$$

For  $b = 0$  the center of point of the CD is at origin ( $t = 0, r = 0$ ). The generators obey the Lie-algebra

$$[P_0, D_0] = P_0 \quad , \quad [K_0, D_0] = -K_0 \quad , \quad [P_0, K_0] = m2D_0 \quad . \quad (6.2)$$

The time evolution of conformal QM is defined by a Hamiltonian which in its most general form is given as  $G = i(uP_0 + vD_0 + wK_0)$ . The conformal Hamiltonians  $G$  can be classified by the Casimir invariant

$$C = D_0^2 - \frac{1}{2}(K_0P_0 + P_0K_0) \quad . \quad (6.3)$$

This corresponds to the invariance of the determinant of the matrix  $(b, 2c; 2a, b)$  given by  $\Delta = b^2 - 4ac$ .

1. For  $\Delta < 0$  one has generators of elliptic transformations analogous to rotations in the Lorentz group.

$$R = \frac{1}{2}(\alpha P_0 + \frac{K_0}{\alpha}) \quad . \quad (6.4)$$

is a representative of this class. One has  $\alpha = L$ , where  $L$  is the radius of CD defined by the maximal radial distance from the time axis of CD. In the sequel will use the notation  $\alpha$  used also in [B4] The distance between the tips of the CD is  $2L$ . The radial conformal Killing vector is everywhere time-like.

2. Generators with  $\Delta = 0$  generate parabolic transformations, null rotations. Also now the radial conformal Killing vector is time-like everywhere except for the light-cone emanating from  $t = -b/2a, r = 0$
3. Generators with  $\Delta > 0$  generate hyperbolic transformations analogous to Lorentz group perspective. Dilation  $D$  and the generator

$$S_0 = \frac{1}{2}(\alpha P_0 - \frac{K_0}{\alpha}) \quad . \quad (6.5)$$

serves as a representative for this class. In this case the conformal Killing vector is null at the tips of the light-cone ( $t = t_{\pm}, r = 0$ ),  $t_{\pm}0 - b + \sqrt{\Delta}/2a$ , time-like inside either light-cone or outside both light-cones, and space-like everywhere else.

One can assign to  $S_0$  resp.  $R_0$  time coordinates  $T$  resp.  $\tau$  in such a way that one has

$$R_0 = \partial_T \quad , \quad S_0 = \partial_{\tau} \quad . \quad (6.6)$$

These time coordinates are related to Minkowski time  $t$  by

$$t = \alpha \tan(T/2) = \alpha \tanh(\tau/2) \quad . \quad (6.7)$$

One finds that the range  $(-\alpha, \alpha)$  for  $t$  corresponds to the range  $(-\infty, \infty)$  for  $\tau$  and to the range  $(-\pi, \pi)$  for  $T$ .

$S_0$  has a representation as a differential operator

$$S_0 = \frac{1}{2\alpha}[(\alpha^2 - t^2 - r^2)\partial_t - 2tr\partial_r] \quad . \quad (6.8)$$

$S_0$  maps CD identifiable as the region  $|t| + |vertr| < \alpha$  to itself.

One can identify so-called diamond coordinates via the formulas

$$t = \alpha \frac{\sinh(\tau)}{\cosh(x) + \cosh(\tau)} \quad , \quad r = \frac{\sinh(x)}{\cosh(x) + \cosh(\tau)} \quad . \quad (6.9)$$

The ratio of the equations gives  $t/r = \sinh(\tau)/\sinh(x)$ . For small values of  $\tau$  and  $x$  this gives ( $t \simeq \alpha\tau, r \simeq \alpha x$ ). From  $t = \alpha \tanh(\tau/2)$  one can solve  $\sinh(\tau)$  and use it to the expression of  $t/r$  to get

$$\sinh(x) = 2\frac{t}{\alpha}/(1 - (\frac{t}{\alpha})^2) . \quad (6.10)$$

$\sinh(x)$  is constant for the flow lines.

In diamond coordinates, the line element of Minkowski metric reads as

$$ds^2 = \frac{\alpha^2}{(\cosh(x) + \cosh(\tau))^2} (-d\tau^2 + dx^2 + \sinh(x)^2 \Omega^2) . \quad (6.11)$$

and the flow lines correspond to a particle at rest. Diamond coordinates are analogous to the natural coordinates of a freely falling system.

The integral curves of the Killing flows defined by  $S_0$  are obtained as

$$t^2 - (r - \alpha\omega)^2 = \alpha^2(1 - \omega^2) , \quad \omega = \frac{1}{\tanh(x)} . \quad (6.12)$$

$\omega$  is constant along these curves and varies in the range  $[1, \infty)$ . This equation should be equivalent with the equation already obtained. The integral curves correspond to a relativistic motion with constant acceleration given by  $a(x) = \sinh(x)/\alpha$  which is constant along each integral curve for which  $x$  indeed remains constant. For the line passing through origin one has  $a = 0$ .

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