%\begin{abstract}

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. In fact, the general definition of geometry is as a structure characterized by given symmetries. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the \blockquote{world of the classical worlds} (WCW) identified as the infinite-dimensional WCW of light-like 3-surfaces of $H=M^4\times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

\vm {\it 1. Physics as infinite-dimensional K\"ahler geometry} \vm

\begin{enumerate}\item The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW K\"ahler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the K\"ahler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

\item WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation

relations
for the gamma matrices of the WCW. WCW gamma matrices
contracted with Killing vector fields give rise to a super-symplectic
algebra which together with Hamiltonians of the WCW forms what I have used
to call super-symplectic algebra.

Super-symplectic degrees of freedom represent completely new degrees of
freedom and have no electroweak couplings. In the case of hadrons
super-symplectic quanta correspond to what has been identified as
non-perturbative sector of QCD: they define TGD correlate for the degrees
of freedom assignable to hadronic strings. They are responsible for the
most of the mass of hadron and resolve spin puzzle of proton.

\item Besides super-symplectic symmetries there are Super-Kac Moody
symmetries assignable to light-like 3-surfaces and together these algebras
extend the conformal symmetries of string models to dynamical conformal
symmetries instead of mere gauge symmetries. The construction of the
representations of these symmetries is one of the main challenges of
quantum TGD. Modular invariance is one aspect of conformal symmetries and
plays a key role in the understanding of elementary particle vacuum
functionals and the description of family replication phenomenon in terms
of the topology of partonic 2-surfaces.

\item K\"ahler-Dirac equation (or K\"ahler-Dirac equation) gives also rise to a
hierarchy super-conformal algebras assignable to zero modes. These algebras
follow from the existence of conserved fermionic currents. The
corresponding deformations of the space-time surface correspond to
vanishing second variations of K\"ahler action and provide a realization
of quantum criticality. This led to a breakthrough in the understanding of
the K\"ahler-Dirac action via the addition of a measurement interaction
term to the action allowing to obtain among other things stringy propagator
and the coding of quantum numbers of super-conformal representations to the
geometry of space-time surfaces required by quantum classical correspondence.

A crucial feature of the K\"ahler-Dirac equation is the localization of the

modes to 2-D surfaces with vanishing induced W fields (this in generic
situation and for all modes but covariantly constant right-handed
neutrino): this is needed in order to have modes with well-defined em
charge. Also Z^0 fields can be vanish and is expected to do so - at least
above weak scale. This implies that all elementary particles are string
like objects in very concrete sense.

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{\it 2. p-adic physics and p-adic variants of basic symmetries}

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p-Adic mass calculations relying on p-adic length scale hypothesis led to
an understanding of elementary particle masses using only super-conformal
symmetries and p-adic thermodynamics. The need to fuse real physics and
various p-adic physics to single coherent whole led to a generalization of
the notion of number obtained by gluing together reals and p-adics together
along common rationals and algebraics. The interpretation of p-adic
space-time sheets is as correlates for cognition and intentionality. p-Adic
and real space-time sheets intersect along common rationals and algebraics
and the subset of these points defines what I call number theoretic braid
in terms of which both WCW geometry and S-matrix elements should be
expressible. Thus one would obtain number theoretical discretization which
involves no adhoc elements and is inherent to the physics of TGD.

\vm {\it 3. Hierarchy of Planck constants and dark matter hierarchy}
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The realization for the hierarchy of Planck constants proposed as a
solution to the dark matter puzzle leads to a profound generalization of
quantum TGD through a generalization of the notion of imbedding space to
characterize quantum criticality. The resulting space has a book like

structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups. The extension of imbedding space can be seen as a formal tool allowing an elegant description of the multi-sheetednes due to the non-determinism of K\"ahler action. At the space-like ends the sheets fuse together so that a singular covering is in question.

The original intuition was the the space-time would be n-sheeted for $h_{eff}=n$. Quantum criticality expected on basis of the vacuum degeneracy of K\"ahler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $..\subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_{i}. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

\vm {\it 4. Number theoretical symmetries} \vm

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

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\item There are good reasons to believe that the strands
of number theoretical braids - ends of string world sheets - can be
assigned with the roots of a polynomial
with suggests the interpretation corresponding Galois groups as purely
number theoretical symmetries of quantum TGD. Galois groups are subgroups
of the permutation group S_{∞} of infinitely manner objects acting
as the Galois group of algebraic numbers. The group algebra of
S_{∞} is HFF which can be mapped to the HFF defined by configuration
space spinors. This picture suggest a number theoretical gauge invariance
stating that S_{∞} acts as a gauge group of the theory and that
global gauge transformations in its completion correspond to the elements
of finite Galois groups represented as diagonal groups of $G\times G\times
....$ of the completion of S_{∞}.

\item HFFs inspire also   an idea about how entire TGD emerges from
classical number fields, actually their complexifications. In particular,
SU(3) acts as subgroup of octonion automorphisms leaving invariant
preferred imaginary unit. If space-time surfaces are hyper-quaternionic
(meaning that the octonionic counterparts of the K\"ahler-Dirac gamma matrices
span complex quaternionic sub-algebra of octonions) and contain at each
point a preferred plane M^2 of M^4, one ends up with M^8-H duality
stating that space-time surfaces can be equivalently regarded as surfaces
in M^8 or $M^4\times CP_2$. One can actually generalize M^2 to a
two-dimensional Minkowskian sub-manifold of M^4. One ends up with
quantum TGD by considering associative sub-algebras of the local octonionic
Clifford algebra of M^8 or H. so that TGD could be seen as a
generalized number theory.

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