In this chapter the goal is to find whether the general mathematical structures associated with twistor approach, superstring models and M-theory could have a generalization or a modification in TGD framework. The contents of the chapter is an outcome of a rather spontaneous process, and represents rather unexpected new insights about TGD resulting as outcome of the comparisons.

\vm {\it 1. Infinite primes, Galois groups, algebraic geometry, and TGD}\vm

In algebraic geometry the notion of variety defined by algebraic equation is very general: all number fields are allowed. One of the challenges is to define the counterparts of homology and cohomology groups for them. The notion of cohomology giving rise also to homology if Poincare duality holds true is central. The number of various cohomology theories has inflated and one of the basic challenges to find a sufficiently general approach allowing to interpret various cohomology theories as variations of the same motive as Grothendieck, who is the pioneer of the field responsible for many of the basic notions and visions, expressed it.

Cohomology requires a definition of integral for forms for all number fields. In p-adic context the lack of well-ordering of p-adic numbers implies difficulties both in homology and cohomology since the notion of boundary does not exist in topological sense. The notion of definite integral is problematic for the same reason. This has led to a proposal of reducing integration to Fourier analysis working for symmetric spaces but requiring algebraic extensions of p-adic numbers and an appropriate definition of the p-adic symmetric space. The definition is not unique and the interpretation is in terms of the varying measurement resolution.

The notion of infinite has gradually turned out to be more and more important for quantum TGD. Infinite primes, integers, and rationals form a hierarchy completely analogous to a hierarchy of second quantization for a super-symmetric arithmetic quantum field theory. The simplest infinite primes representing elementary particles at given level are in one-one correspondence with many-particle states of the previous level. More complex infinite primes have interpretation in terms of bound states.

\begin{enumerate}

\item What makes infinite primes interesting from the point of view of algebraic geometry is that infinite primes, integers and rationals at the n:th level of the hierarchy are in 1-1 correspondence with rational functions of n arguments. One can solve the roots of associated polynomials and perform a root decomposition of infinite primes at various levels of the hierarchy and assign to them Galois groups acting as automorphisms of the field extensions of polynomials defined by the roots coming as restrictions of the basic polynomial to planes $x_n=0$, $x_n=x_{n-1}=0$, etc...

\item These Galois groups are suggested to define non-commutative generalization of homotopy and homology theories and non-linear boundary operation for which a geometric interpretation in terms of the restriction to lower-dimensional plane is proposed. The Galois group G_k would be analogous to the relative homology group relative to the plane $x_{k-1}=0$ representing boundary and makes sense for all number fields also geometrically. One can ask whether the invariance of the complex of groups under the permutations of the orders of variables in the reduction process is necessary. Physical interpretation suggests that this is not the case and that all the groups obtained by the permutations are needed for a full description.

\item The algebraic counterpart of boundary map would map the elements of G_k identified as analog of homotopy group to the commutator group $[G_{k-2},G_{k-2}]$ and therefore to the unit element of the abelianized group defining cohomology group. In order to obtains something analogous to the ordinary homology and cohomology groups one must however replaces Galois groups by their group algebras with values in some field or ring. This allows to define the analogs of homotopy and homology groups as their abelianizations. Cohomotopy, and cohomology would emerge as duals of homotopy and homology in the dual of the group algebra.

\item That the algebraic representation of the boundary operation is not expected to be unique turns into blessing when on keeps the TGD as almost topological QFT vision as the guide line. One can include all boundary homomorphisms subject to the condition that the anticommutator $\delta^i_k\delta^j_{k-1}+\delta^j_{k}\delta^i_{k-1}$ maps to the group algebra of the commutator group $[G_{k-2},G_{k-2}]$. By adding dual generators one obtains what looks like a generalization of anticommutative fermionic algebra and what comes in mind is the spectrum of quantum states of a SUSY algebra spanned by bosonic states realized as group algebra elements and fermionic states realized in terms of homotopy and cohomotopy and in abelianized version in terms of homology and cohomology. Galois group action allows to organize quantum states into multiplets of Galois groups acting as symmetry groups of physics. Poincare duality would map the analogs of fermionic creation operators to annihilation operators and vice versa and the counterpart of pairing of k:th and $n-k$:th homology groups would be inner product analogous to that given by Grassmann integration. The interpretation in terms of fermions turns however to be wrong and the more appropriate interpretation is in terms of Dolbeault cohomology applying to forms with homomorphic and antiholomorphic indices.

\item The intuitive idea that the Galois group is analogous to 1-D homotopy group which is the only non-commutative homotopy group, the structure of infinite primes analogous to the braids of braids of braids of ... structure, the fact that Galois group is a subgroup of permutation group, and the possibility to lift permutation group to a braid group suggests a representation as flows of 2-D plane with punctures giving a direct connection with topological quantum field theories for braids, knots and links. The natural assumption is that the flows are induced from transformations of the symplectic group acting on $\delta M^2_{\pm} \times CP_2$ representing quantum fluctuating degrees of freedom associated with WCW (\blockquote{world of classical worlds}). Discretization of WCW and cutoff in the number of modes would be due to the finite measurement resolution. The outcome would be rather far reaching: finite measurement resolution would allow to construct WCW spinor fields explicitly using the machinery of number theory and algebraic geometry.

\item A connection with operads is highly suggestive. What is nice from TGD perspective is that the non-commutative generalization homology and homotopy has direct connection to the basic structure of quantum TGD almost topological quantum theory where braids are basic objects and also to hyper-finite factors of type II_1. This notion of Galois group makes sense only for the algebraic varieties for which coefficient field is algebraic extension of some number field. Braid group approach however allows to generalize the approach to completely general polynomials since the braid group make sense also when the ends points for the braid are not algebraic points (roots of the polynomial).

\end{enumerate}

This construction would realize the number theoretical, algebraic geometrical, and topological content in the construction of quantum states in TGD framework in accordance with TGD as almost TQFT philosophy,

TGD as infinite-D geometry, and TGD as generalized number theory
visions.

\vm {\it 2. p-Adic integration and cohomology}\vm

This picture leads also to a proposal how p-adic integrals could be
defined
in TGD framework.

\begin{enumerate} \item The calculation of twistorial amplitudes
reduces
to multi-dimensional residue calculus. Motivic integration gives
excellent
hopes for the p-adic existence of this calculus   and braid
representation
would give space-time representation for the residue integrals in
terms of
the braid points representing poles of the integrand: this would
conform
with quantum classical correspondence. The power of 2π
appearing in
multiple residue integral is problematic unless it disappears from
scattering amplitudes. Otherwise one must allow an extension of p-
adic
numbers to a ring containing powers of 2π.

\item Weak form of electric-magnetic duality and the general
solution
ansatz for preferred extremals reduce the K\"ahler action defining
the
K\"ahler function for WCW to the integral of Chern-Simons 3-form.
Hence
the reduction to cohomology takes places at space-time level and
since
p-adic cohomology exists there are excellent hopes about the
existence of
p-adic variant of K\"ahler action.   The existence of the exponent
of
K\"ahler gives additional powerful constraints on the value of the
K\"ahler
fuction in the intersection of real and p-adic worlds consisting of
algebraic partonic 2-surfaces and allows to guess the general form
of the
K\"ahler action in p-adic context.

\item One also should define p-adic integration for vacuum
functional at
the level of WCW. p-Adic thermodynamics serves as a guideline
leading to
the condition that in p-adic sector exponent of K\"ahler action is
of form
$(m/n)^r$, where m/n is divisible by a positive power of p-adic
prime
p. This implies that one has sum over contributions coming as

powers of p and the challenge is to calculate the integral for K= constant surfaces using the integration measure defined by an infinite power of K\"ahler form of WCW reducing the integral to cohomology which should make sense also p-adically. The p-adicization of the WCW integrals has been discussed already earlier using an approach based on harmonic analysis in symmetric spaces and these two approaches should be equivalent. One could also consider a more general quantization of K\"ahler action as sum $K=K_1+K_2$ where $K_1=rlog(m/n)$ and $K_2=n$, with n divisible by p since $exp(n)$ exists in this case and one has $exp(K)= (m/n)^r \times exp(n)$. Also transcendental extensions of p-adic numbers involving $n+p-2$ powers of $e^{1/n}$ can be considered.

\item If the Galois group algebras indeed define a representation for WCW spinor fields in finite measurement resolution, also WCW integration would reduce to summations over the Galois groups involved so that integrals would be well-defined in all number fields. \end{enumerate}

\vm {\it 3. Floer homology, Gromov-Witten invariants, and TGD}\vm

Floer homology defines a generalization of Morse theory allowing to deduce symplectic homology groups by studying Morse theory in loop space of the symplectic manifold. Since the symplectic transformations of the boundary of $\delta M^4_{\pm}\times CP_2$ define isometry group of WCW, it is very natural to expect that K\"ahler action defines a generalization of the Floer homology allowing to understand the symplectic aspects of quantum TGD. The hierarchy of Planck constants implied by the one-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates leads naturally to singular coverings of the imbedding space and the resulting symplectic Morse theory could characterize the homology of these coverings.

One ends up to a more precise definition of vacuum functional: K\"ahler action reduces Chern-Simons terms (imaginary in Minkowskian regions and real in Euclidian regions) so that it has both phase and real exponent which makes the functional integral well-defined. Both the phase factor and its conjugate must be allowed and the resulting degeneracy of ground state could allow to understand qualitatively the delicacies of CP breaking and its sensitivity to the parameters of the system.   The critical points with respect to zero modes correspond to those for K\"ahler function. The critical points with respect to complex coordinates associated with quantum fluctuating degrees of freedom are not allowed by the positive definiteness of K\"ahler metric of WCW. One can say that K\"ahler and Morse functions define the real and imaginary parts of the exponent of vacuum functional.

The generalization of Floer homology inspires several new insights. In particular, space-time surface as hyper-quaternionic surface could define the 4-D counterpart for pseudo-holomorphic 2-surfaces in Floer homology. Holomorphic partonic 2-surfaces could in turn correspond to the extrema of K\"ahler function with respect to zero modes and holomorphy would be accompanied by   super-symmetry.

Gromov-Witten invariants appear in Floer homology and topological string theories and this inspires the attempt to build an overall view about their role in TGD. Generalization of topological string theories of type A and B to TGD framework is proposed. The TGD counterpart of the mirror symmetry would be the equivalence of formulations of TGD in $H=M^4\times CP_2$ and in $CP_3\times CP_3$ with space-time surfaces replaced with 6-D sphere bundles.

\vm {\it 4. K-theory, branes, and TGD }\vm

K-theory and its generalizations play a fundamental role in super-string models and M-theory since they allow a topological classification of branes. After representing some physical objections against the notion of brane more technical problems of this approach are discussed briefly and it is proposed how TGD allows to overcome these problems. A more precise formulation of the weak form of electric-magnetic duality emerges: the original formulation was not quite correct for space-time regions with Euclidian signature of the induced metric. The question about possible TGD counterparts of R-R and NS-NS fields and S, T, and U dualities is discussed.

\vm {\it 5. p-Adic space-time sheets as correlates for Boolean cognition}\vm

p-Adic physics is interpreted as physical correlate for cognition. The so called Stone spaces are in one-one correspondence with Boolean algebras and have typically 2-adic topologies. A generalization to p-adic case with the interpretation of p pinary digits as physically representable Boolean statements of a Boolean algebra with $2^n>p>p^{n-1}$ statements is encouraged by p-adic length scale hypothesis. Stone spaces are synonymous with profinite spaces about which both finite and infinite Galois groups represent basic examples. This provides a strong support for the connection between Boolean cognition and p-adic space-time physics. The Stone space character of Galois groups suggests also a deep connection between number theory and cognition and some arguments providing support for this vision are discussed.