The notion of Hamilton-Jacobi structure is becoming more and more important in the TGD framework. Hamilton-Jacobi structure would define a 4-D generalization of 2-D complex structure at the space-time level. The deep idea is that space-time surfaces, as analogs of Bohr orbits realizing almost deterministic holography, are preferred extremals of some action defining classical TGD. The preferred extremals in $H = M^4 \times CP_2$ are simultaneous extremals of both volume action and Kähler action apart from lower-dimensional singularities. This is true if the space-time surface is determined by the simultaneous vanishing of 2 complex valued functions of H coordinates and thus defining the space-time as a 4-surface. In the simplest situation, the functions are generalizations of functions of several complex variables and are functions of a light-like coordinate of $M^2 \subset M^4$ as an analog of complex coordinate, a complex coordinate of E^2 orthogonal to M^2, and 2 complex coordinates of CP_2.

2-D Riemann surfaces allow several complex structures parameterized by the moduli space (Teichmüller parameters). The same is expected to be true in the 4-D case, and the natural expectation that Hamilton-Jacobi structure, defining an integrable distribution of partonic 2-surfaces and string world sheets, defines a slicing of the space-time surface by partonic 2-surfaces and string world sheets such that the complex structure for the partonic 2-surface depends on space-time point as also its possibly existing analog for the string world sheet. Space-time could be seen as a 4-D surface in the product of Teichmüller space for the partonic 2-surfaces and its analog for string world sheets.