

The considerations of this article were inspired by an interview of Edward Frenkel relating to the Langlands correspondence and led to a considerably more detailed understanding of how number theoretic and geometric Langlands correspondences emerge in the TGD framework from number theoretic universality, holography = holomorphy vision leading to a general solution of field equations based on the generalization of holomorphy, and $M^8 - H$ duality relating geometric and number theoretic visions of TGD.

The space-time surfaces are realized as roots for a pair (P_1, P_2) of holomorphic polynomials of four generalized complex coordinates of $H = M^4 \times CP_2$. In this view space-time surfaces are representations of the function field of generalized polynomial pairs in H and can be regarded as numbers with arithmetic operations induced from those for the polynomial pairs. Product is always well defined by inverse is ill-defined if either function vanishes.

A proposal for how to count the number of roots of the $(P_1, P_2) = (0, 0)$, when the arguments are restricted to a finite field in terms of modular forms defined at the hyperboloid $H^3 \times CP_2 \subset M^4 \times CP_2$. The geometric variant of the Galois group as a group mapping different roots for a polynomial pair (P_1, P_2) identifiable as regions of the space-time surface (minimal surface) would be in terms of holomorphisms of H .

The interpretation of space-time surfaces as numbers leads to a general construction recipe for quantum states in terms of a geometric analog of a tensor product with the property that the product involves automatically interaction terms resulting in the multiplication of space-time surfaces as numbers. One can say that TGD is exactly solvable at both classical space-time level and quantum level. The geometric Langlands correspondence extends to a trinity between number theory, geometry and physics.