TOPOLOGICAL GEOMETRODYNAMICS
TGD and von Neumann Algebras

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Clifford algebra of world of classical worlds is hyperfinite factor of type II_1

- ! Infinite-dimensional Clifford algebra is Hyperfinite factor of type II_1 . One particular kind of von Neumann algebra. Finite-dimensional approximation for this factor is excellent.

- Factors of type I correspond to quantum mechanics and factors of type III to quantum field theory in 4-D space-time.

- Tr(Id)=1 the defining property of hyper-finite factor of type II_1. Unit matrix of infinite-dimensional Clifford algebra has unit trace: elimination of fermionic divergences.

- Hyperfinite factors of type II_1 have intriguing connections with quantum groups, knot and braid invariants, topological quantum field theories, and conformal field theories. Reference: Was von Neumann Right After All?.

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Ideas related to hyperfinite factors of type II_1

- Tr(Id)=1. Unit matrix of infinite-dimensional Clifford algebra has unit trace. No fermionic divergences. S-matrix as entanglement coefficients for zero energy states. No problems with normalization: $Tr(SS^+)=1$.

- Generalization of quantum measurement theory. Jones inclusion N∑ M: N represents measurement resolution and M/N understood as N module represents measurable degrees of freedom. Quantum measurement projects to N. S-matrix consistent with Connes tensor product for which N takes the role of complex numbers for ordinary tensor product. Crossing symmetry for elements of N.

- Number theoretic braids result when induced spinor fields anticommute only in a discrete subset of points of number theoretic string at partonic 2-surface. Reason: functions appearing as coefficients of oscillator operators become N valued and thus non-commutative in M→M/N reduction.

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- TGD emerges from infinite-dimensional Clifford algebra made local by multiplying it with quantal variant of complex octonions. Dynamics from associativity. Space-time as a surface M^8 or equivalently in H= $M^4 \times CP_2$. CP_2 has number-theoretic interpretation.
- Further generalization of notion of imbedding space. Based on Jones inclusions characterized by subgroups of SU(2). Quantum groups emerge naturally. Jones inclusions characterize also resolution of quantum measurement.
- Quantization of Planck constants associated with M^4 and CP_2 degrees of freedom from Poincare and color invariance. Dark matter as phase with Planck constants differing from ordinary Planck constant. Living matter as ordinary matter quantum controlled by dark matter.
- TGD is able to emulate almost any ADE gauge theory or ADE type theory with Kac-Moody symmetry.

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Jones inclusions and quantum measurement theory with finite measurement resolution

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- Projector to a complex ray of state space have vanishing trace. Ordinary quantum measurement theory does not work.

- Jones inclusion N ∑ ∑ M defines quantum measurement theory: N characterizes measurement resolution. States related by the action of N indistuinguishable.

- M/N generates physical states modulo resolution. Finite-dimensional matrix algebra with N-valued commuting matrix elements.

- Complex numbers replaced with algebra N. N-rays, N-unitarity, N-hermiticity, state vectors with N-valued components. Non-commutative physics corresponds to finite measurement resolution. State function reduction to N-ray.

- S-matrix has N-valued elements. Probabilities as moduli squared are now hermitian operators which should commute. Eigen value spectrum means a collection of S-matrices.

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- **Fuzzy quantum states**. Moduli squared for the components of quantum spinors commuting operators with spectrum having interpretation as probabilities. For finite n pure states impossible.

- **Fuzziness equivalent with number theoretic statistical ensemble with quantized probabilities.**

- **Indications for anomalies in EPR-Bohm experiments for correlation function of photon polarizations**. Correlation function measured in experiment has maximum and minimum scaled down in magnitude by factor .9. Fuzziness for polarization states equivalent with ensemble averaging over polarizations. Data reproduced with P1=.9 and P2=.1?

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How number theoretic braids emerge from Jones inclusions?

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- Number theoretic braids result when induced spinor fields anticommute only in a discrete subset of points of number theoretic string at partonic 2-surface.
- M→M/N reduction implies that the number of spinor modes becomes finite
- Complex coordinates z associated with geodesic spheres of CP_2 and lightcone boundary become N-valued and non-commutative and commute only at points of braid.
- Coordinates z appear in the generalized eigenvalues for the modes of induced spinor field so that also induced spinor field anticommutes only at these points.
- Physical states coherent states for z and eigenmodes of the complex coordinates. Eigenvalues expressible in terms of zeros of zeta.
- Bosonic quantization at imbedding space level as a descrip[...]n finite measurement resolution!

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TGD Universe from local version of infinite- dimensional Clifford algebra?

- Localized version of hyper-finite factor of type II_1 as fundamental structure from which TGD Universe emerges. Functions from imbedding space to factor. Generalization of conformal field: z replaced with complex number, hyper-quaternion, or hyper-octonion or matrix representation (representation as $x= x^k\sigma_k$) and field has values in hyperfinite factor of type II_1. [Hyper-quaternions and –octonions (hyper-octonions linear space with basis $1, ie_k, k=1,..7$, i commuting imaginary unit) required by Minkowski signature.]

- Localization only possible in hyper-octonionic case: otherwise the localized version isomorphic to the original algebra.

- Space-time surface associative/hyper-quaternionic surface of hyper octonionic space-time $HO=M^8$. Also co-HQ/co-associative property possible. Electric magnetic duality?

- Why hyper-octonions rather than octonions? Quantum 8-space with non-hermitian coordinates (analog of complexified octonions): hyper-octonionic (rather than octonionic!) space-time corresponds to a maximal set of commuting observables having coordinate values as its eigenvalues (Was von Neumann Right After All?).

- **How $H=M^4\sum CP_2$ emerges?** HO-H duality or number theoretical "compactification": **M^8=HO as imbedding space is equivalent with $H=M^4\sum CP_2$ as imbedding space**. One can assign to any hyper-quaternionic 4-surface in HO a 4-surface in H in the following manner.

  a) M^4 coordinates correspond to first 4 coordinates of M^8 point.

  b) Fix complex structure i.e. preferred imaginary unit e_1. $SU(3) \sum G_2$ remaining octonionic automorphisms. **The hyper-quaternionic tangent planes containing e_1 are parameterized by CP_2** so that one can assign to a given point of 4-surface in **HO=M^8** unique CP_2 point characterizing its tangent plane. 4-surface in **M^8** <-> 4-surface in **$M^4\sum CP_2$**.

- CP_2 indeed parameterizes the choices of hyper-quaternionic planes going through a point of **M^8** . SU(3) leaves **$1,e_1$** and its complement consisting of 3 and 3bar invariant. Quaternion plane corresponds to **1, e_1, and e_2, e_3**. **U(1)** acts as rotations in plane **e_2,e_3** and **SU(2)** as automorphisms in **normal space**. Hence **CP_2 =SU(3)/U(2)** la
choices.

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Some conjectures

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- Absolute minima/maxima of Kähler action in H picture correspond in HO picture to HQ/co-HQ <-> associative/co-associative space-time sheets. More precisely, a region of space-time where Kähler action density has fixed sign correspond to absolute extremum of Kähler action.

- There is a connection with the notion of calibration.

a) For calibration volume form defines the action. For extrema volume form reduces to a restriction of a closed 4-form of imbedding space. Absolute extrema as representatives of cohomology equivalence classes for calibrations.

b) Kähler calibration. Kähler action density defines an integrating factor for calibration. Kähler action restricted to absolute extrema = restriction of a closed 4-form of HO to extrema. Or something analogous. This would correspond to almost TQFT property of partonic formulation. Reference: TGD as a Generalized Number Theory II: Quaternions, Octonions and their Hyper Counterparts.

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Jones inclusions and their geometric interpretation

- Jones inclusion N∑ M of hyperfinite factors. The trace of the projector P to N characterizes partially the inclusion. Index of inclusion N∑ M defined as M:N= 1/Tr(P) . M:N= B_n= $4\cos^2(\pi/n)$, n ∑ 3, Beraha numbers.∑

- Space M/N as quantum Clifford algebra acting on quantum spinor space with fractal dimension $2\cos(\pi/n)$. Quantum plane would be the space of quantum spinors. Connection with quantum groups. Quantum phase $q= \exp(i\pi/n)$.

- Canonical hierarchy of Jones inclusions characterized by finite subgroups G∑ SU(2). Elements of N∑ M invariant under G-automorphisms. n is the order of the maximal cyclic sub-group of G. McKay correspondence: sub-groups G <->Dynkin diagrams of ADE Lie algebras (D_{2n+1}, E_7 (cube) are not allowed). E_6 (tedrahedron) and E_8 (dodecahedron) are exceptional.

- In TGD N as G invariant elements for the Clifford algebra of CH. G could correspond to subgroup of isometries of M^4 or CP_2 or of electro

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Generalization of the notion of imbedding space.

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- The subgroup G_b ∑ ∑ SU(2) ∑ SU(3)∑ defines a covering of M^4 by G-related points. In the similar manner covering of CP_2 by G_a ∑ SU(2) ∑ SL(2,C) related M^4 points is defined.

- Preferred point of CP_2 as fixed point of SU(2). In M^4 degrees of freedom timelike plane M^2 containing quantization axes of angular momentum as singularity remaining invariant under G_a unless it corresponds to E6 or E8 (in this case only timelike axis M^1 as singular set, quantization axes not completely unique).

- Coverings can be regarded as a singular bundles having points of G_a∑ G_b orbit as fiber and G_a∑ G_b orbit (G_a∑ G_b equivalence class) as base point.

- The inclusion of base to covering is geometric counterpart for Jones inclusion!

- Planck constants in M^4 resp. CP_2 degrees of freedom scaled by n_a resp. n_b. The copies with different choice of bundle structure are not physically equivalent!

- Infinite-hierarchy of copies of imbedding space.

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- Isometric identification of M^4 factors for $G_{a1} = G_{a2}$ and $G_{b1} \neq G_{b2}$: $n_{b1}m_1 \leftarrow > n_{b2}m_2$ at CP_2 orbifold point in Minkowski coordinates.

- Isometric identification of CP_2 factors for $G_{b1} = G_{b2}$ and $G_{a1} \neq G_{a2}$. No scaling in CP_2 metric. Kähler action invariant under over all scaling of H metric. Scale H metric by $1/n_b^2$ so that all CP_2:s identical and M^4 metric is scaled by $(n_b/n_a)^2$. Ordinary Planck constant scaled by $(n_a/n_b)^2$ and its scaling has purely geometric interpretation.

- G invariant of states in fermionic degrees of freedom but finite-dimensional representations of G in covering space. Group algebra of G. G degrees of freedom are transformed from fermionic to bosonic ones.

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Is TGD able to mimick ADE physics?

- McKay correspondence relates subgroups of SU(2) to Lie-groups and restricted ADE hierarchy. Sub-groups G ←>Dynkin diagrams of ADE Lie algebras (D_{2n+1}, E_7 (cube) are not allowed). E_6 (tedrahedron) and E_8 (dodecahedron) are exceptional. Could have deep physical correspondence.

- The sheets of multiple covering make it possible to construct representations of gauge group G using G-singlets formed from fundamental fermions and representations of G in group algebra. Is TGD Universe able to emulate all gauge theories appearing in this hierarchy?

- G=SU(2) gives also rise to ADE hierarchy with extended ADE diagrams: now interpretation would be in terms of Kac-Moody algebras. Quantum group would correspond to monodromy groups of corresponding conformal field theories. TGD would be able to mimic also almost any "stringy theory".

- Analogy of G-covering with a stack consisting of N branes infinitesimally near to each other. AdS/CFT correspondence.

- Large N limit of SU(N) gauge theory g^2N= constant replaced with

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Quantization of Planck constants

- Separate Planck constants in M^4 and CP_2 degrees of freedom. Different Planck constants correspond to different branches of imbedding space which can be glued together along identical M^4 or CP_2 factor.

- $h(M^4)= n_a h_0$ and $h(CP_2)= n_b h_0$. Observed Planck constant $h_{eff} =(n_a/n_b)h_0$. Can have all rational values in principle. n_a and n_b orders of maximal cyclic subgroups of G_a and G_b defining Jones inclusions.

- n_b-fold cyclic covering of M^4 by CP_2 points means that angular momentum projection m is fractionized:

  $m \rightarrow m/n_b$. n_b turns of 2π before original sheet reached.
  Unit of angular momentum is fractionized: $hbar_0 \rightarrow (na/nb)hbar_0$.

- Elementary particle mass spectrum universal and unchanged in the scaling of h.

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- Are Planck constants same for all particles in Feynman diagram so that no direct interactions in this sense? This seems to be the generic case. Phases with different Planck constants would be relatively dark.

- Phase transitions in which particles leak to another copy of imbedding space if M^4 or CP_2 factor is common must occur. In leakage either CP_2 becomes point or M^4 projection piece of preferred light-like geodesic at δM^4_+ and these degrees of freedom disappear from dynamics. $h(CP_2)$ or $h(M^4)$ or can be different for particles in same Feynman diagram in this kind of situation.

- Preferred values of integers n correspond to Fermat polygons constructible using ruler and compass. Quantum phase involves also iterated square rooting and so that quantum phase is very simple p-adically. Preferred values of n correspond to n-polygons constructible using compass and ruler number theoretically suggestive:

  $$n= 2^k \prod_s F_s \ , \quad F_s = 2^{2^s} +1 \ , \ s=0,1,2,3,4,$$

  Given F_s at most once. The known Fermat primes 3,5,17, 257, $2^{16}+1$.

- $n=2^{k11}$, k=0,1,2,... seem to be favored in living matter. 2^{11} fundamental constant in TGD.

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Dark matter as a phase with nonstandard values of Planck constants

- Dark matter as phase with non-standard (large?) Planck constants.
- Infinite hierarchy of dark matters.
- Dark matter quantum could be coherent in astrophysical length and time scales.
  Evidence for Bohr quantization of planetary orbits with gigantic value of gravitational Planck constant identifiable as CP_2 Planck constant.
- Living matter as ordinary matter quantum controlled by dark matter.
- Phases with $h_{eff}/h_0<n_a/n_b$ also possible! Hydrogen atom binding energy scaled up by $(n_b/n_a)^2$. Could findings of Mills about hydrogen atoms with energy scaled up by k2, k=2,3,4,5,6,7,10 be example of $n_b=k$, $n_a=1$ phases?

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S-matrix in zero energy ontology

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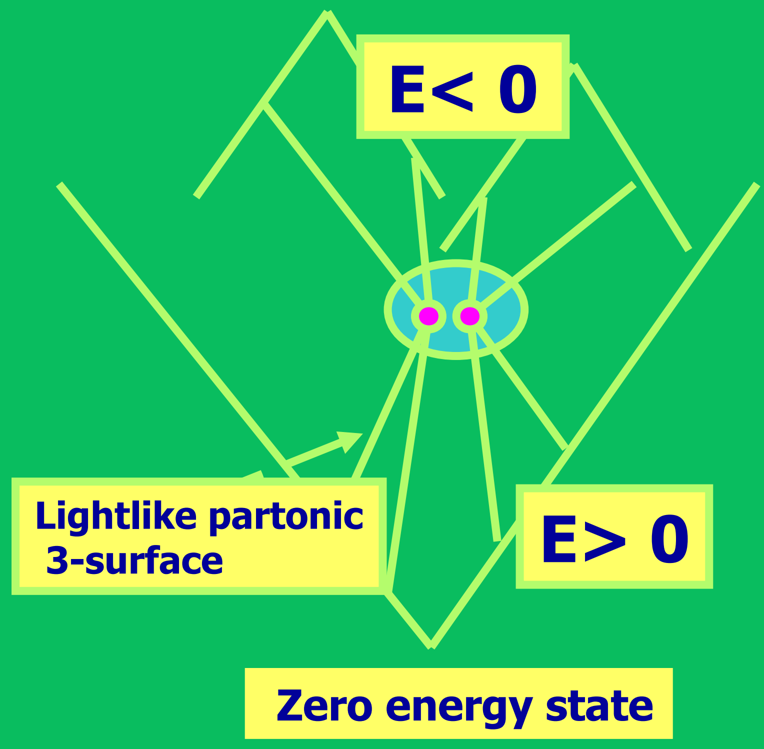

- Partonic 2-surface X^2 =intersection of incoming lightlike partonic 4-surfaces (!). Note that their interiors do not intersect! Necessary for realizing quantum classical correspondence.

S-matrix unitary entanglement matrix: SS^{Σ} =Id, Tr(Id)=1.

Generalization of the notion of imbedding space

- **(m,s) ⟶ {(m,gs)}, g $\sum$ G $\sum$ SU(2) $\sum$ SU(3) defines n(G)-fold covering of M^4 by CP_2 points.**
- **Similar covering defined for CP_2 by M^4 points. In general case double covering by group $G_a \sum G_b \sum SL(2,C) \sum SU(3)$.**
- **<u>Analogy with n-sheeted Riemann surface</u> where n is order of maximal cyclic subgroup of G. Helix for which points obtained after n turns are identified as geometric analog.**
- **Orbifold structure. Points gs=s defined singular points.**
- **Infinite number of copies of H= $M^4 \sum CP_2$ results. Metric scaled for M^4 (CP2) faztor by n(G_b) (n(G_a)).**
- **Two factors glued together along M^4 if G_b is same for them, and along CP_2 if G_a is same for them.**