The Relationship Between TGD and GRT

M. Pitkänen,

December 26, 2016

Email: matpitka@luukku.com.
http://tgdtheory.com/public_html/.
Recent postal address: Karkinkatu 3 I 3, 00360, Karkkila, Finland.

Contents

Abstract

In this chapter the recent view about TGD as Poincare invariant theory of gravitation is discussed. Radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

1. Equivalence Principle and GRT limit of TGD

The views about Equivalence Princple (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

2. The problem of cosmological constant

A further implication of dark matter hierarchy is that astrophysical systems correspond to stationary states analogous to atoms and do not participate to cosmic expansion in a continuous manner but via discrete quantum phase transitions in which gravitational Planck constant increases. By quantum criticality of these phase transitions critical cosmologies are excellent candidates for the modeling of these transitions. Imbeddable critical (and also over-critical) cosmologies are unique apart from a parameter determining their duration and represent accelerating cosmic expansion so that there is no need to introduce cosmological constant.

It indeed turns out possible to understand these critical phases in terms of quantum phase transition increasing the size of large modeled in terms of cosmic strings. A possible mechanism driving the strings to the boundaries of large voids could be repulsive interaction due to net charges of strings. Also repulsive gravitational acceleration could do this. In this framework cosmological constant like parameter does not characterize the density of dark energy but that of dark matter identifiable as quantum phases with large Planck constant.

A concrete interpretation for the dark matter is as Kähler magnetic energy of Kähler magnetic flux tubes, which are outcome of the expansion of primordial cosmic strings. Dark matter in turn corresponds to particles with non-standard value of Planck constant given by $h_{eff} = n\times$ residing at the Kähler magnetic flux tubes. The GRT limit of TGD allows a description of dark energy in terms of cosmological constant in Einstein's equations.

A further problem is that the naive estimate for the cosmological constant is predicted to be by a factor 10^{120} larger than its value deduced from the accelerated expansion of the Universe. In TGD framework the resolution of the problem comes naturally from the fact that large voids are quantum systems which follow the cosmic expansion only during the quantum critical phases.

p-Adic fractality predicting that cosmological constant is reduced by a power of 2 in phase transitions occurring at times $T(k) \propto 2^{k/2}$, which correspond to p-adic time scales. These phase transitions would naturally correspond to quantum phase transitions increasing the size of the large voids during which critical cosmology predicting accelerated expansion naturally applies. On the average $\Lambda(k)$ behaves as $1/a^2$, where a is the light-cone proper time. This predicts correctly the order of magnitude for observed value of Λ.

3. Topics of the chapter

The topics discussed in the chapter are following.

1. The basic principles of GRT (General Coordinate Invariance, Equivalence Principle, and Machian Principle) are discussed from TGD point of view.
2. The theory assuming that the most important solutio is applied to the vacuum extremal embeddings of Reissner-Nordström and Schwartschild metric.
3. A model for the final state of star indicates that Z^0 force, presumably created by dark matter, might have an important role in the dynamics of the compact objects. During year 2003, more than decade after the formulation of the model, the discovery of the connection between supernovas and gamma ray bursts provided strong support for the predicted axial magnetic and Z^0 magnetic flux tube structures predicted by the model for the final state of a rotating star. Two years later the interpretation of the predicted long range weak forces as being caused by dark matter emerged.

   The progress in understanding of hadronic mass calculations has led to the identification of what I call super-symplectic bosons and their super-counterparts as basic building blocks of hadrons. This notion leads also to a microscopic description of neutron stars and black-holes in terms of highly entangled string like objects in Hagedorn temperature and in very precise sense analogous to gigantic hadrons.
4. There is a brief summary about cosmic strings, which form a corner stone of TGD inspired cosmology.
5. The idea of entropic gravity is not consistent with what is already known about the quantal behavior of neutrons in the Earth's gravitational field. The discussion of entropic gravity in TGD framework however leads to fresh ideas about GRT limit of TGD and is therefore included.

1 Introduction

In this chapter the recent view about TGD as Poincare invariant theory of gravitation is discussed. It must be admitted that the development of the proper interpretation of the theory has been rather slow and involved rather weird twists motivated by conformist attitudes. Typically these attempts have brought into theory ad hoc identifications of say gravitational four-momentum although theory itself has from very beginning provided completely general formulas.

Perhaps the real problem has been that radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

1.1 Does Equivalence Principle Hold True In TGD Universe?

The motivation for TGD as a Poincare invariant theory of gravitation was that the notion of four-momentum is poorly defined in curved space-time since corresponding Noether currents do not exist. There however seems to be a fundamental obstacle against the existence of a Poincare invariant theory of gravitation related to the notions of inertial and gravitational energy.

1. The conservation laws of inertial energy and momentum assigned to the fundamental action would be exact in this kind of a theory. Gravitational four-momentum can be assigned to the curvature scalar as Noether currents and is thus completely well-defined unlike in GRT. Equivalence Principle requires that inertial and gravitational four-momenta are identical. This is satisfied if curvature scalar defines the fundamental action principle crucial for the definition of quantum TGD. Curvature scalar as a fundamental action is however non-physical and had to be replaced with so called Kähler action.

2. One can question Equivalence Principle because the conservation of gravitational four-momentum seems to fail in cosmological scales. Zero Energy Ontology however implies that four-momentum is length scale dependent notion so that the problem disappers.

3. For the extremals of Kähler action the Noether currents associated with curvature scalar are well-defined but non-conserved. Also for vacuum extremals satisfying Einstein's equations gravitational four-momentum fails to be conserved and non-conservation becomes large for

small values of cosmic time. This looks fine but the problem is whether the possible failure of Equivalence Principle is so serious that it leads to conflict with experimental facts.

TGD view about quantum gravity led first to a technical understanding of what the equality of inertial and gravitational four-momenta means. This equality is something ill-definable in GRT context whether the conservation laws for four-momentum is purely local stating the vanishing of the covariant divergences of energy momentum-tensor.

The path leading to the recent view about GRT limit of TGD and EP at classical level has been long and tortuous. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$. It turned out however that the problems related to TGD-GRT relationship and EP have been basically pseudo-problems due to the too restricted vision about what TGD limit of TGD could be. One must introduce GRT space-time as a fictive notion naturally obtained from Minkowski space by replacing its metric with effective metric describing gravitation: this is also the spirit of perturbation theoretic approach to quantum gravity. GRT emerges from TGD as a simplified concept replacing many-sheeted space-time.

1. The replacement of superposition of fields with superposition of their effects on test particle simultaneously topologically condensed to the space-time sheet means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets. This resolves also the objections due to the lacking superposition fields at given space-time sheet and strong correlations between different induced quantities (metric and spinor connection).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of flat metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.

   The fact that Maxwell's electrodynamics, gauge theories and GRT work so well suggests that many-sheetedness is really present and only in special situations becomes manifest. Example of this kind of situation is represented by the propagation of light signal along different space-time sheets so that it spends different times on the travel. Encouragingly, two neutrino bursts followed by photon burst arrived from SN1987 supernova.

This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

1.2 Zero Energy Ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state.

The vacuum extremals are absolutely essential for the TGD based view about long length scale limit about gravitation but involve the assumption that solutions of Einstein's equations allowing imbedding as vacuum extremal are in physically preferred role. Already the Kähler action defined by CP_2 Kähler form J allows enormous vacuum degeneracy: any four-surface having Lagrangian sub-manifold of CP_2 as its CP_2 projection is a vacuum extremal. The dimension of these sub-manifolds is at most two. Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K15, K5] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

Zero energy states decompose into positive and negative energy parts having identification as initial and final states of particle reaction in time scales of perception longer than the geometro-temporal separation T of positive and negative energy parts of the state. If the time scale of perception is smaller than T, the usual positive energy ontology applies.

In zero energy ontology inertial four-momentum is a quantity depending on the temporal time scale T used and in time scales longer than T the contribution of zero energy states with parameter $T_1 < T$ to four-momentum vanishes. This scale dependence alone implies that it does not make sense to speak about conservation of inertial four-momentum in cosmological scales. Hence it would be in principle possible to identify inertial and gravitational four-momenta and achieve strong form of Equivalence Principle. It however seems that this is not the correct approach to follow.

The concept of negative potential energy is completely standard notion in physics. Perhaps so standard that physicists have begun to regard it as understood. The precise physical origin of the negative potential energy is however complete mystery, and one is forced to take the potential energy as a purely phenomenological concept deriving from quantum theory as an effective description.

In TGD framework topological field quantization leads to the hypothesis that quantum concepts should have geometric counterparts. Also potential energy should have precise correlate at the level of description based on topological field quanta. This could be the case. As already explained, ZEO allows space-time sheets to have both positive and negative time orientations. This in turn implies that also the sign of energy can be also negative. This suggests that the generation of negative energy space-time sheets representing virtual gravitons together with energy conservation makes possible the generation of huge gravitationally induced kinetic energies and gravitational collapse. In this process inertial energy would be conserved since instead, of positive energy gravitons, the inertial energy would go to the energy of matter.

This picture has a direct correlate in quantum field theory where the exchange negative energy virtual bosons gives rise to the interaction potential. The gravitational red-shift of microwave background photons is the strongest support for the non-conservation of energy in General Relativity. In TGD it could have concrete explanation in terms of absorption of negative energy virtual gravitons by photons leading to gradual reduction of their energies. This explanation is consistent with the classical geometry based explanation of the red-shift based on the stretching of electromagnetic wave lengths. This explanation is also consistent with the intuition based on Feynman diagram description of gravitational acceleration in terms of graviton exchanges.

1.3 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the imbedding space was inspired both by empirical input (Bohr quantization of planetary orbits) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence.

Quantum classical correspondence suggests that Jones inclusions [A1] have space-time correlates [K30, K12]. There is a symplectic hierarchy of Jones inclusions labeled by finite subgroups of SU(2) [A6] This leads to a generalization of the imbedding space obtained by gluing an infinite number of copies of H regarded as singular bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$. Gluing occurs along a factor for which the group is same. The generalized imbedding space has clearly a book like structure with pages of books intersecting along

4-D sub-manifold $M^2 \times S^2$, S^2 a geodesic sphere of CP_2 characterizing the choice of quantization axies. Entire configuration space is union over "books" corresponding to various choices of this sub-manifold.

The groups in question define in a natural manner the direction of quantization axes for for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a\hbar_0$ and $\hbar_b = n_b\hbar_0$ appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2, is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i. The hierarchy of Planck constants is interpreted in terms of dark matter hierarchy [K12]. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

G_a would correspond directly to the observed symmetries of visible matter induced by the underlying dark matter [K12]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K12, K4] and recently reported strange properties of graphene [D1]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K10]. [D2].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [E4] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K23] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

Since the gravitational Planck constant is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

In the gravitational case the order of G_a is gigantic and at least GM_1m/v_0, $v_0 = 2^{-11}$ the favored value. The natural interpretation is as a discrete rotational symmetry of the gravitational field body of the system having both gravimagnetic and gravi-electric parts. The subgroups of G_a for which order is a divisor of the order of G_a define broken symmetries at the lower levels of dark matter hierarchy, in particular symmetries of visible matter.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5- multiples should be important. Note that in the structure of chromosomes p-adic length scale $L(151) \simeq 10$ characterizes beads-on-string structure of DNA whereas the length scale $3L(151)$ appears in the coiling of this structure.

It has turned that there are good hopes of reducing the hierarchy of Planck constants to the basic TGD [K3]. By the extreme non-linearity of the Kähler action the correspondence between the time derivatives of the imbedding space coordinates and canonical momentum densities is many-to-one. This leads naturally to the introduction of covering spaces of $CD \times CP_2$, which are singular in the sense that the sheets of the covering co-incide at the ends of CD and at wormhole throats. One can say that quantum criticality means also the instability of the 3-surfaces defined by throats and ends against the decay to several space-time sheets and consequence charge fractionization. The interpretation is as an instability caused by too strong density of mass and making perturbative description possible since the matter density at various branches is reduced. The situation can be described mathematically either by using effectively only single sheet but an integer multiple of Planck constant or many-sheeted covering and ordinary value of Planck constant. In [K12] the argument that this indeed leads to hierarchy of Planck constants including charge fractionization is developed in detail. The restriction to singular coverings is consistent with the experimental constraints and means that only integer valued Planck constants are possible. A given value of Planck constant corresponds only to a finite number of the pages of the Big Book and that the evolution by quantum jumps is analogous to a diffusion at half-line and tends to increase the value of Planck constant.

1.4 The Problem Of Cosmological Constant

A further implication of dark matter hierarchy is that astrophysical systems correspond to stationary states analogous to atoms and do not participate to cosmic expansion in a continuous manner but via discrete quantum phase transitions in which gravitational Planck constant increases. By quantum criticality of these phase transitions critical cosmologies are excellent candidates for the modelling of these transitions. Imbeddable critical cosmologies are unique apart from a parameter determining their duration and represent accelerating cosmic expansion so that there is no need to introduce cosmological constant.

It indeed turns out possible to understand these critical phases in terms of quantum phase transition increasing the size of large modeled in terms of "big" cosmic strings with negative gravitational mass whose repulsive gravitation drives "galactic" cosmic strings with positive gravitational mass to the boundaries of the void. In this framework cosmological constant like parameter does not characterize the density of dark energy but that of dark matter identifiable as quantum phases with large Planck constant.

A further problem is that the naive estimate for the cosmological constant is predicted to be by a factor 10^{120} larger than its value deduced from the accelerated expansion of the Universe. In TGD framework the resolution of the problem comes naturally from the fact that large voids are quantum systems which follow the cosmic expansion only during the quantum critical phases.

p-Adic fractality predicting that cosmological constant is reduced by a power of 2 in phase transitions occurring at times $T(k) \propto 2^{k/2}$, which correspond to p-adic time scales. These phase transitions would naturally correspond to quantum phase transitions increasing the size of the large voids during which critical cosmology predicting accelerated expansion naturally applies. On the average $\Lambda(k)$ behaves as $1/a^2$, where a is the light-cone proper time. This predicts correctly the order of magnitude for observed value of Λ.

1.5 Topics Of The Chapter

The notion of many-sheeted space-time has been extensively discussed in the previous chapters [K14, K15] and is therefore left out from this chapter. The topics included in this chapter are following.

The first three sections are devoted to the general theoretical picture.

1. There is a discussion of General Coordinate Invariance, Equivalence Principle, and Machian Principle in TGD context with a special emphasis on the views about the relationship of inertial and gravitational masses, the zero energy ontology, and dark matter hierarchy. A special emphasis is given to the notion of four-momentum and the latest (year 2013) vision about the situation is discussed.

2. The vacuum extremal imbeddings of Reissner-Nordström and Schwartschild metric are studied. The interpretational problems involved were responsible for much of the tension which eventually led to the recent understanding of Equivalence Principle in TGD framework.

The remaining sections are devoted to examples about applications.

1. A simple model for the final state of a star is proposed. The model indicates that Z^0 force, presumably created by dark matter, might have an important role in the dynamics of the compact objects. During year 2003, more than decade after the formulation of the model, the discovery of the connection between supernovas and gamma ray bursts [E5] provided strong support for the predicted axial magnetic and Z^0 magnetic flux tube structures predicted by the model for the final state of a rotating star. Two years later the interpretation of the predicted long range weak forces as being caused by dark matter emerged.

   The progress in the understanding of hadronic mass calculations [K18] has led to the identification of so called super-symplectic bosons and their super-counterparts as basic building blocks of hadrons. This notion suggests also a microscopic description of neutron stars and black-holes in terms of highly entangled string like objects in Hagedorn temperature and in very precise sense analogous to gigantic hadrons.

2. The idea of entropic gravity is not consistent with what is already known about the quantal behavior of neutrons in the Earth's gravitational field. The discussion of entropic gravity in TGD framework however leads to fresh ideas about GRT limit of TGD and is therefore included.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L2]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L3]. The topics relevant to this chapter are given by the following list.

- General Coordinate Invariance [L4]
- Zero Energy Ontology (ZEO) [L8]
- Geometrization of fields [L5]
- TGD and GRT [L6]
- TGD inspired cosmology [L7]

2 Basic Principles Of General Relativity From TGD Point Of View

General Coordinate Invariance, Equivalence Principle are corner stones of general relativity and one expects that they hold true also in TGD some sense. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

Objections against TGD have turned out to be the best route to the correct interpretation of the theory. A very general objection against TGD relies on the notion of induced gauge fields and metric implying extremely strong constraints between classical gauge fields for preferred extremals. These constraints cannot hold true for gauge fields in the usual sense. Also linear superposition is lost. The solution of the problem comes from simple observation: it is not fields which superpose but their effects on test particle topologically condensed to space-time sheets carrying the classical fields. Superposition is replaced with set theoretic union. This leads also naturally to explicit identification of the effective metric and gauge potentials defined in M^4 and defining GRT limit of TGD.

Finite length scale resolution is central notion in TGD and implies that the topological inhomogenities (space-time sheets and other topological inhomogenities) are treated as point-like objects and described in terms of energy momentum tensor of matter and various currents coupling to effective YM fields and effective metric important in length scales above the resolution scale. Einstein's equations with coupling to gauge fields and matter relate these currents to the Einstein tensor and metric tensor of the effective metric of M^4. The topological inhomogenities below cutoff scale serve determine the curvature of the effective metric.

The original proposal, which I called smoothed out space-time, took into account the topological inhomogenities but neglected many-sheetedness in length scales above resolution scale. I also identified the effective metric can be identified as induced metric: this is very strong assumption although the properties of vacuum extremals support this identification at least in some important special cases.

The attempts to understand Kähler-Dirac (or Kähler-Dirac-) action has provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions: for instance, if is far from clear how em charge can be well-defined for the modes of the induced spinor field and how the effective absence of weak bosons above weak scale is realized at classical level for Kähler-Dirac action.

2.1 General Coordinate Invariance

General Coordinate Invariance plays in the formulation of quantum TGD even deeper role than in that of GRT. Since the fundamental objects are 3-D surfaces, the construction of the geometry of the configuration space of 3-surfaces (the world of classical worlds, WCW) requires that the definition of the geometry assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$. This space-time surface is completely analogous to Bohr orbit, which means a completely unexpected connection with quantum theory.

General Coordinate Invariance is analogous to gauge symmetry and requires gauge fixing. The definition assigning $X^4(X^3)$ to given X^3 must be such that the outcome is same for all 4-diffeomorphs of X^3. This condition is highly non-trivial since $X^4(X^3) = X^4(Y^3)$ must hold true if X^3 and Y^3 are 4-diffeomorphs. One manner to satisfy this condition is by assuming quantum holography and weakened form of General Coordinate Invariance: there exists physically preferred 3-surfaces X^3 defining $X^4(X^3)$, and the 4-diffeomorphs Y^3 of X^3 at $X^4(X^3)$ provide classical holograms of X^3: $X^4(Y^3) = X^4(X^3)$ is trivially true. Zero energy ontology allows to realize this form of General Coordinate Invariance.

1. In ZEO WCW decomposes into a union of sub-WCWs associated with causal diamonds $CD \times CP_2$ (CD denotes the intersection of future and past directed light-cones of M^4), and the intersections of space-time surface with the light-light boundaries of $CD \times CP_2$ are excellent candidates for preferred space-like 3-surfaces X^3. The 3-surfaces at $\delta CD \times CP_2$ are indeed physically special since they carry the quantum numbers of positive and negative energy parts of the zero energy state.

2. Preferred 3-surfaces could be also identified as light-like 3-surfaces X_l^3 at which the Euclidian signature of the induced space-time metric changes to Minkowskian. Also light-like boundaries of X^4 can be considered. These 3-surfaces are assumed to carry elementary particle quantum numbers and their intersections with the space-like 3-surfaces X^3 are 2-dimensional partonic surfaces so that effective 2-dimensionality consistent with the conformal symmetries of X_l^3 results if the identifications of 3-surfaces are physically equivalent. Light-like 3-surfaces are identified as generalized Feynman diagrams and due to the presence of 2-D partonic 2-surfaces representing vertices fail to be 3-manifolds. Generalized Feynman diagrams could be also identified as Euclidian regions of space-time surface.

3. General Coordinate Invariance in minimal form requires that the slicing of $X^4(X_l^3)$ by light light 3-surfaces Y_l^3 "parallel" to X_l^3 predicted by number theoretic compactification gives rise to quantum holography in the sense that the data associated with any Y_l^3 allows an equivalent formulation of quantum TGD. This poses a strong condition on the spectra of the Kähler-Dirac operator at Y_l^3 and thus to the preferred extremals of Kähler action since the WCW Kähler functions defined by various choices of Y_l^3 can differ only by a sum of a holomorphic function and its conjugate [K31, K5] .

2.2 The Basic Objection Against TGD

The basic objection against TGD is that induced metrics for space-time surfaces in $M^4 \times CP_2$ form an extremely limited set in the space of all space-time metrics appearing in the path integral formulation of General Relativity. Even special metrics like the metric of a rotating black hole fail to be imbeddable as an induced metric. For instance, one can argue that TGD cannot reproduce the post-Newtonian approximation to General Relativity since it involves linear superposition of gravitational fields of massive objects. As a matter fact, Holger B. Nielsen- one of the very few colleagues who has shown interest in my work - made this objection for at least two decades ago in some conference and I remember vividly the discussion in which I tried to defend TGD with my poor English.

The objection generalizes also to induced gauge fields expressible solely in terms of CP_2 coordinates and their gradients. This argument is not so strong as one might think first since in standard model only classical electromagnetic field plays an important role.

1. Any electromagnetic gauge potential has in principle a local imbedding in some region. Preferred extremal property poses strong additional constraints and the linear superposition of massless modes possible in Maxwell's electrodynamics is not possible.

2. There are also global constraints leading to topological quantization playing a central role in the interpretation of TGD and leads to the notions of field body and magnetic body having non-trivial application even in non-perturbative hadron physics. For a very large class of preferred extremals space-time sheets decompose into regions having interpretation as geometric counterparts for massless quanta characterized by local polarization and momentum directions. Therefore it seems that TGD space-time is very quantal. Is it possible to obtain from TGD what we have used to call classical physics at all?

The imbeddability constraint has actually highly desirable implications in cosmology. The enormously tight constraints from imbeddability imply that imbeddable Robertson-Walker cosmologies with infinite duration are sub-critical so that the most pressing problem of General Relativity disappears. Critical and over-critical cosmologies are unique apart from a parameter characterizing their duration and critical cosmology replaces both inflationary cosmology and cosmology characterized by accelerating expansion. In inflationary theories the situation is just the opposite of this: one ends up with fine tuning of inflaton potential in order to obtain recent day cosmology.

Despite these and many other nice implications of the induced field concept and of sub-manifold gravity the basic question remains. Is the imbeddability condition too strong physically? What about linear superposition of fields which is exact for Maxwell's electrodynamics in vacuum and a good approximation central also in gauge theories. Can one obtain linear superposition in some sense?

1. Linear superposition for small deformations of gauge fields makes sense also in TGD but for space-time sheets the field variables would be the deformations of CP_2 coordinates which are scalar fields. One could use preferred complex coordinates determined about SU(3) rotation to do perturbation theory but the idea about perturbations of metric and gauge fields would be lost. This does not look promising. Could linear superposition for fields be replaced with something more general but physically equivalent?

2. This is indeed possible. The basic observation is utterly simple: what we know is that the *effects* of gauge fields superpose. The assumption that fields superpose is un-necessary! This is a highly non-trivial lesson in what operationalism means for theoreticians tending to take these kind of considerations as mere "philosophy".

3. The hypothesis is that the superposition of effects of gauge fields occurs when the M^4 projections of space-time sheets carrying gauge and gravitational fields intersect so that the sheets are extremely near to each other and can touch each other (CP_2 size is the relevant scale).

A more detailed formulation goes as follows.

1. One can introduce common M^4 coordinates for the space-time sheets. A test particle (or real particle) is identifiable as a wormhole contact and is therefore point-like in excellent approximation. In the intersection region for M^4 projections of space-time sheets the particle forms topological sum contacts with all the space-time sheets for which M^4 projections intersect.

2. The test particle experiences the sum of various gauge potentials of space-time sheets involved. For Maxwellian gauge fields linear superposition is obtained. For non-Abelian gauge fields gauge fields contain interaction terms between gauge potentials associated with different space-time sheets. Also the quantum generalization is obvious. The sum of the fields induces quantum transitions for states of individual space time sheets in some sense stationary in their internal gauge potentials.

3. The linear superposition applies also in the case of gravitation. The induced metric for each space-time sheet can be expressed as a sum of Minkowski metric and CP_2 part having interpretation as gravitational field. The natural hypothesis that in the above kind of situation the effective metric is sum of Minkowski metric with the sum of the CP_2 contributions from various sheets. The effective metric for the system is well-defined and one can calculate a curvature tensor for it among other things and it contains naturally the interaction terms between different space-time sheets. At the Newtonian limit one obtains linear superposition of gravitational potentials. One can also postulate that test particles moving along geodesics in the effective metric. These geodesics are not geodesics in the metrics of the space-time sheets.

4. This picture makes it possible to interpret classical physics as the physics based on effective gauge and gravitational fields and applying in the regions where there are many space-time sheets which M^4 intersections are non-empty. The loss of quantum coherence would be due to the effective superposition of very many modes having random phases.

The effective superposition of the CP_2 parts of the induced metrics gives rise to an effective metric which is not in general imbeddable to $M^4 \times CP_2$. Therefore many-sheeted space-time makes possible a rather wide repertoire of 4-metrics realized as effective metrics as one might have expected and the basic objection can be circumvented In asymptotic regions where one can expect single sheetedness, only a rather narrow repertoire of "archetypal" field patterns of gauge fields and gravitational fields defined by topological field quanta is possible.

The skeptic can argue that this still need not make possible the imbedding of a rotating black hole metric as induced metric in any physically natural manner. This might be the case but need of course not be a catastrophe. We do not really know whether rotating blackhole metric is realized in Nature. I have indeed proposed that TGD predicts new physics new physics in rotating systems. Unfortunately, gravity probe B could not check whether this new physics is there since it was located at equator where the new effects vanish.

2.3 How GRT And Equivalence Principle Emerge From TGD?

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book) . This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?",

> How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?

[K21].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

2.3.1 *TGD and GRT*

Concerning GRT limit the basic questions are the following ones.

1. Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 imbedding space coordinates (actually only 4 by general coordinate invariance?

2. What happens to Einstein equations?

3. What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?

4. What about Equivalence Principle (EP)

5. Can one predict the value of gravitational constant?

6. What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **??** in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.

3. This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.

4. One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.

5. Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.

6. The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

1. EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.

2. The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

1. Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to imbedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.

2. TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [K28]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.

3. Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.

4. The notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar et al [E8, E13] [K21].

For few ago entropic gravity [B2, B8] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [K28].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

1. In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave functions having interpretation as square roots of thermal distributions in ZEO. The quantum model for cell membrane [K9] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.

2. A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

2.3.2 *Equivalence Principle*

Equivalence Principle has several interpretations.

1. The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.

2. The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

1. In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.

2. Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.

3. In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

1. Inertial momentum is defined as Noether charge for Kähler action.

2. One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [K31]. Its conservation is however not at all all trivial since imbedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [K31]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.

3. Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.

4. QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the imbedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

1. Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.

2. Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.

3. By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

1. In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?

2. Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other manners. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

2.3.3 *EP as quantum classical correspondence*

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This view might be equivalent with coset space view. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K28].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

2.4 The Recent View About Kähler-Dirac Action

The understanding of Kähler-Dirac action and equation have provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of howe EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

2.4.1 *Kähler-Dirac action*

2.5 Kähler-Dirac Action

2.5.1 *Kähler-Dirac equation*

2.6 Kähler-Dirac Equation In The Interior Of Space-Time Surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

1. Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.

2. Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.

3. What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of imbedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that imbedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the imbedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the imbedding space isometries?

1. Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.

2. Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to imbedding space coordinates would not vanish in general.

3. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig. ??** also in the Appendix).

2.7 Boundary Terms For Kähler-Dirac Action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

2.7.1 *What one wants?*

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.

The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

2. If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

2.7.2 *Chern-Simons Dirac action from mathematical consistency*

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{2.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

2.8 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K28]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K28, K35]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of ZEO (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K34] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K27]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian [A4] [B6, B3, B4] variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [?]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

2.8.1 *Scale dependent notion of four-momentum in zero energy ontology*

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K24], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of ZEO (ZEO) [K2, K33], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K33]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K31].

2.8.2 *Are the classical and quantal four-momenta identical?*

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

2.8.3 *What Equivalence Principle (EP) means in quantum TGD?*

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K35]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

   The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

   The objection against the identification h in the decomposition $g = t + h$ of the symplectic algebra as Kac-Moody algebra is that this does not make sense mathematically. The strong form of holography implied by strong form of General Coordinate Invariance however implies that the action of Kac-Moody algebra for the maxima of Kähler function induces unique action of sub-algebra of symplectic algebra so that the identification makes sense after all [K6].

3. Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K34] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors would be extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep, and it seems that the coset option is definitely wrong: the reason is that for H in G/H decomposition the four-momentum vanishes.

2.8.4 *TGD-GRT correspondence and Equivalence Principle*

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig. ??** in the Appendix).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in ZEO (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

2.8.5 *How translations are represented at the level of WCW?*

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

1. Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2. The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

2.8.6 *Yangian and four-momentum*

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [?]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K27], where also references to the work of pioneers can be found.

1. Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K27] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 super-conformal Yang-Mills theory* [B3] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2. How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A2] and Virasoro algebras [A3] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

2. The formal generalization looks surprisingly straightforward at the formal level. In ZEO one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in ZEO and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

3 Imbedding Of The Reissner-NordströOm Metric

The recent view about how GRT relates to TGD differs considerably from that during period 1980-1990 when the calculations related to the imbeddings of the basic solutions of Einstein's equations were carried out. At that time I believed that the physically most relevant space-times of GRT would allow imbedding as sub-manifolds of $M^4 \times CP_2$. This is certainly not true for all of them: consider only rotating blackholes. Year 2014 - 37 years from discovery of TGD - was the year when I finally realized that the problems related to this relationship were mostly pseudo-problems. As described in previous section, the space-time of GRT emerges elegantly as statistical concept from quantum TGD and the notion of many-sheeted space-time. Space-time sheets represent the microscopic level of gravity not seen in GRT and preferred extremal property makes them extremely simple objects. The experimental challenge is to reveal the presence of these simpled building bricks.

One can of course consider seriously the possibility that the solutions of Einstein's equations imbeddable as vacuum extremals might be physically special. This justifies the discussions of this section as also the section devoted to the final state of the star.

In the following the imbedding of electromagnetically neutral Reissner-Nordström metric to $M^4_+ \times CP_2$ will be studied. The imbedding generalizes to an imbedding of any spherically symmetric metric. The imbeddings as vacuum extremals reduce to imbeddings into 6-dimensional $M^4 \times Y^2$, Y^2 Lagrange manifold (vanishing induced Kähler form). Any vacuum extremal defines a solution of Einstein's equations if energy momentum tensor is defined by Einstein's equations. Non-vacuum imbeddings of Reissner-Nordström solutions would correspond to homologically non-trivial geodesic sphere of CP_2, and it is implausible that non-vacuum imbeddings could be extremals. Whether the imbeddings of the metrics believed to describe rotating objects in GRT Universe are possible at all, is not known but it might well be that the dimension of the imbedding space is too low to allow them. This would mean that the predictions of TGD concerning gravi-magnetism can differ from those of GRT.

3.1 Two Basic Types Of Imbeddings

One can construct a large number of imbeddings for Reissner-Nordström metric. These imbeddings need not be extremals of Kähler action except when they are represent vacua.

1. X^4 could be a sub-manifold of $M^4 \times S^2_i$, $i = I, II$, where S^2_i is one of the geodesic spheres of CP_2. For $i = II$ the imbeddings are vacuum extremals but this is not the case for $i = I$. The properties of these imbeddings are essentially those associated with the spherically symmetric stationary extremals of the Kähler action. Long range electromagnetic and Z^0 fields assignable to dark matter [K14] are present but the corresponding forces are by a factor 10^{-4} weaker than gravitational force, when the parameter ωR is of order one.

2. The vacuum extremals of the Kähler action are the physically most interesting candidates for the imbeddings of solutions of Einstein's equations. For these imbeddings electro-weak fields are in general non-vanishing. Em neutrality is possible to achieve only for $p = sin^2(\theta_W) = 0$. Long ranged W^+ and W^- fields can be present and they induce a small mixing between charged dark lepton and corresponding neutrino spinors.

3.2 The Condition Guaranteeing The Vanishing Of Em, Z^0, Or Kähler Fields

In order to obtain imbedding with vanishing em, Z^0, or Kähler field, one must pose the condition guaranteeing the vanishing of corresponding field (see the Appendix of the book). For extremals of Kähler action em Z^0 fields are always simultaneously present unless Weinberg angle vanishes. In practice only the condition guaranteeing vanishing of Kähler field is thus interesting.

Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2 the surfaces in question can be expressed as

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D|k+u|^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ D = \frac{r_0^2}{1+r_0^2} \times \frac{1}{C} \ , \quad C = |k + cos(\Theta_0)|^{\epsilon} \ . \end{aligned} \tag{3.1}$$

Here C and D are integration constants. The value of the parameter ϵ characterizes which field vanishes:

$$\begin{aligned} a) \ \epsilon = \tfrac{3+p}{3+2p} \ , \quad b) \ \epsilon = \tfrac{1}{2} \ , \quad c) \ \epsilon = 1 \ , \\ p = sin^2(\Theta_W) \ . \end{aligned} \tag{3.2}$$

Here a/b/c corresponds to the vanishing of em/Z^0/Kähler field.

$0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{\epsilon}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\epsilon} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

These imbeddings obviously possess a 2-dimensional CP_2 projection. The generation of long range vacuum weak and color electric fields is a purely TGD based phenomenon related to the fact that gauge fields are not primary dynamical variables.

For future purposes it is convenient to list the explicit expressions of relevant gauge field when em or Kähler field vanishes.

1. Using coordinates $(u = cos(\Theta), \Phi)$ the expressions for the Kähler form and Z^0 field for space-time surfaces with vanishing em field read as

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \quad X = D|k+u|^{\frac{3+p}{3+2p}} \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{3.3}$$

2. For vacuum extremals ($\epsilon = 1$) classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \quad X = D|k+u| \ , \\ \gamma &= -\frac{p}{2} Z^0 \ . \end{aligned} \tag{3.4}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible. The only reasonable physical interpretation seems to be in terms of a hierarchy of electro-weak physics with arbitrarily light weak boson mass scales.

The effective form of the CP_2 metric is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ . \end{aligned} \tag{3.5}$$

This expression is useful in the construction of electromagnetically neutral imbedding of, say Schwartchild metric. For $k \neq 1$ $u = \pm 1$ corresponds in general to circle rather than single point as is clear from the fact that $s^{eff}_{\Phi\Phi}$ is non-vanishing at $u = \pm 1$ so that u and Φ parameterize a piece of cylinder.

3.3 Imbedding Of Reissner-Nordström Metric

The imbedding of R-N metric to be discussed generalizes with minor modifications to an imbedding of a spherically symmetric star model characterized by a mass density $\rho(r_M)$ and pressure $p(r_M)$ since the corresponding line element can be written in the form $ds^2 = A(r_M)dt^2 - B(r_M)dr_M^2 - r_M^2 d\Omega^2$ [E11]. For vacuum extremal a solution of field equations results.

Denote the coordinates of M^4_+ by (m^0, r_M, θ, ϕ) and those of X^4 by (t, r_M, θ, ϕ). The expression for Reissner-Nordström metric reads as

$$\begin{aligned} ds^2 &= Adt^2 - Bdr_M^2 - r_M^2 d\Omega^2 \ , \\ A &= 1 - \frac{a}{r_M} - \frac{b}{r_M^2} \ , \quad B = \frac{1}{A} \ , \\ a &= 2GM \ , \quad b = G\pi q^2 \ . \end{aligned} \tag{3.6}$$

The imbedding is given by the expression

$$\begin{aligned} \Phi &= \omega_1 t + f(r_M) \ , \\ \Psi &= k\Phi = \omega_2 t + kf(r_M) \ , \\ m^0 &= \lambda t + h(r_M) \ , \\ \lambda &= \sqrt{1 + \frac{R^2\omega_1^2}{4} s^{eff}_{\Phi\Phi}(\infty)} \ , \quad k = \frac{\omega_2}{\omega_1} \ . \end{aligned} \tag{3.7}$$

The components of s^{eff} are given by Eq. 3.5 and general form of imbedding by Eqs. 3.1 and 3.2.

The functions $f(r_M)$ and $h(r_M)$ are determined by the condition

$$\lambda \partial_{r_M} h = \frac{R^2}{4} s^{eff}_{\Phi\Phi} \omega_1 \partial_{r_M} f \tag{3.8}$$

resulting from the requirement $g_{tr_M} = 0$ and from the expression for $g_{r_M r_M} = -B$:

$$\begin{aligned} h &= \int dr_M \sqrt{Y} \ , \quad Y = \frac{Y_1}{Y_2} \ , \\ Y_1 &= -B + 1 + \frac{R^2}{4} s^{eff}_{\Theta\Theta} \frac{(\partial_{r_M} u)^2}{(1-u^2)} \ , \\ Y_2 &= 1 - \frac{4\lambda^2}{R^2\omega_1^2} \frac{s^{eff}_{\Theta\Theta}}{s^{eff}_{\Phi\Phi}} \ . \end{aligned} \tag{3.9}$$

The condition $Y > 0$ at the limit $r \to \infty$ gives non-trivial conditions. Y_1 is positive at large values of r_M and this gives

$$Y_1 = -B + 1 + s^{eff}_{\Theta\Theta}\frac{(\partial_{r_M} u)^2}{(1-u^2)} \geq 0$$

for the allowed values of r_M. Y_1 can change sign at some critical radius above Schwartschild radius $r_S = 2GM$ since B becomes infinite at r_S: this can be avoided only provided one has $u \to 1$ at $r_M \to r_S$. Y_2 must preserve its sign and this is possible if the value of $R\omega_1$ is sufficiently large. Below $r = r_S$ Y_1 has positive and also Y_2 can be positive down to some critical radius. At $r = r_S$ Y_1 has infinite discontinuity in case that Y_1 approaches finite value from above and CP_2 coordinates are continuous. It is easy to see that square root singularity of Θ as a function of $r_M - r_S$ is in question so that the function h is continuous so that the solution is well-defined.

The dependence of $u \equiv cos(\Theta)$ on radial coordinate r_M is determined by the expression for $g_{tt} = A$ giving the condition

$$A = \lambda^2 - \frac{R^2\omega_1^2}{4}s^{eff}_{\Phi\Phi}\omega_1^2 \ . \tag{3.10}$$

The asymptotic behavior of the coordinate $u = cos(\Theta)$ is of form

$$u \simeq u_\infty + \frac{K}{r_M} \ , \tag{3.11}$$

u_∞ is fixed by the condition $A(\infty) = 1$:

$$\begin{aligned} \lambda^2 &- \frac{R^2\omega_1^2}{4}s^{eff}_{\Phi\Phi}(\infty) = 1 \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \quad X = D|k+u|^\epsilon \ . \end{aligned} \tag{3.12}$$

The value of K is given by

$$K = \frac{8GM}{R^2\omega_1^2}\left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \ . \tag{3.13}$$

The values of K and u_∞ depend on parameters $\lambda, R\omega_1, k, D$.

For definiteness one can assume that the value of u at infinity is non-negative:

$$u_\infty \geq 0 \ . \tag{3.14}$$

There are two different solution types depending on the sign of the parameter K.

1. For $K < 0$ u decreases and approaches to $u_{min} \geq 0$ as r_M decreases.

2. For $K > 0$ u increases and approaches to $u_{max} \leq 1$. The requirement that the solution can be continued below Schwartshild radius allows only this option. Below Schwartshild radius u must transform to a solution of type a).

3.3.1 Imbeddability breaks for a critical value of the radial coordinate

The imbeddability breaks for some critical value of the coordinate r_M. The extremal value of u and the radius r_c below which the imbedding fails corresponds to the maximum possible value of $s^{eff}_{\Phi\Phi}$. This value corresponds either to $u = 0, 1$ or to a vanishing derivative of $s^{eff}_{\Phi\Phi}$

$$\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u} = 0 \ . \tag{3.15}$$

For $\epsilon = 1$ corresponding to vacuum extremals s_{eff} is a fourth order polynomial as a function of u depending on external parameters. One has

$$s^{eff}_{\Phi\Phi} = D|k+u| \times \left[(1-D|k+u|)(k+u)^2 + 1 - u^2\right] \ . \tag{3.16}$$

s_{eff} becomes negative for very large values of u. Hence a restriction of the standard form of the dual of the cusp catastrophe to the range $u \in (0,1)$ results. Depending on the values of external parameters there are either 2 maxima or single maximum. For $k = 1$ the positive extremum correspond to $u = 1/|D|$.

In the case of the Schwartshild metric this gives for the critical radius the expression

$$\begin{aligned} r_c &= \frac{r_S}{\delta} \ , \\ \delta &= 1 - \lambda^2 + \frac{R^2\omega_1^2}{4} s^{eff}_{\Phi\Phi}(max) \ , \\ r_S &= 2GM \ . \end{aligned} \tag{3.17}$$

The existing evidence for black hole like objects suggests that it would be better to have $\delta >> 1$ in order to get imbeddings of the Schwartschild metric containing also horizon and part of the interior region. A sufficiently large value of $R\omega_1$ indeed allows to have arbitrarily small value of r_c. There the experimental evidence for the existence of black hole like objects leads to no problems.

3.3.2 The vacuum extremal imbeddings of Schwartschild metric possess electro-weak charges

The vacuum imbeddings of Reissner-Nodrström and Scwartshild metric necessarily possess some non-vanishing electro-weak charges. Consider first vacuum extremals. Z^0 electric field Z^0_{tr} is proportional to ω_1

$$Z^0_{tr_M} = \omega_1(k+u)\partial_{r_M} u \ . \tag{3.18}$$

The gauge flux through a sphere with radius r_M depends on r_M so that Z^0 vacuum charge density is necessarily present.

The condition $\theta \propto \sqrt{r - r_S}$ allowing to continue the imbedding below $r_M < r_S$ implies that gauge fluxes, which are proportional to $sin(\Theta)\partial_{r_M}\Theta$, are finite at $r = r_S$ so that the renormalizations of gauge couplings remain finite at least down to Schwartschild radius.

At large distances the gauge flux approaches to

$$\begin{aligned} Q_Z(\infty) &= \frac{1}{g_Z}\int_{r_M\rightarrow\infty} Z^0_{tr_M} r_M^2 d\Omega \ , \\ &= \frac{4\pi}{g_Z}\omega_1(k+u_\infty)K = \frac{4\pi}{g_Z}(k+u_\infty)\frac{8GM}{R^2\omega_1}\left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \end{aligned} \tag{3.19}$$

at the limit $r_M \rightarrow \infty$. Z^0 charge is proportional to the gravitational mass. The gauge flux grows at small distances in accordance with the general wisdom about the coupling constant evolution of $U(1)$ gauge field.

The requirement that Z^0 force is weaker than gravitational force expressed as the condition

$$\frac{Q_Z^2}{GM^2} \ll 1$$

implies

$$\frac{32\pi}{R\omega_1 g_Z}(k+u_\infty)\left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \ll \frac{R}{\sqrt{G}} \quad . \tag{3.20}$$

It seems that a sufficiently large value of $R\omega_1$ allows arbitrarily small values for both the Z^0 charge and the critical radius r_c. In the earliest scenario, which was based on the assumption that CP_2 radius is of order Planck length the situation was different. It is clear that the larger radius of CP_2 makes it possible to avoid too strong classical electro-weak forces.

The non-extremal imbedding to S^2_I studied in detail here is Kähler charged and therefore also Z^0 charged since the condition $Z^0 = 6J/p$ holds true by electromagnetic neutrality. The value of the Kähler charge for non-vacuum imbedding depends on the distance from the origin

$$\begin{aligned} Q_K(r_M) &= \frac{1}{g_K}\int_{r_M=const} J_{tr_M} r_M^2 d\Omega \ , \\ J_{tr_M} &= -\frac{p}{2(3+p)}\omega_1 |k+u|^{\frac{3+p}{3+2p}}\partial_{r_M} u \ , \end{aligned} \tag{3.21}$$

The expression for the charge differs only in minor details from that for Z^0 charge for vacuum extremals. Essentially similar conclusions about the behavior of the gauge charges hold true also in the case of vacuum extremals and the expressions differ only by the value of the parameter ϵ characterizing whether em, Z^0, of Kähler field vanishes.

3.3.3 Equivalence Principle and critical radius

When one considers Equivalence Principle (EP), one must keep in mind that the Kähler charged imbeddings of Reissner Nodström and Scwartschild metrics are *not* extremals of Kähler action. Second thing that one must forget is that the most feasible realization of EP at quantum level seems is based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance [K28]. With this backgrounds the problems due to EP reduce to pseudo-problems due to too narrow interpretation caused by the attempt to identify GRT space-time with single space-time sheet.

1. Equivalence Principle and imbeddings as vacuum extremals

In the case of vacuum extremals the naive interpretation is that net inertial energy density of the space-time outside the topologically condensed space-time sheet representing charged system is vanishing but the density of gravitational energy is non-vanishing and non-conserved in general. The gravitational mass of the topologically condensed space-time sheet however consists of both inertial and purely gravitational contribution. For RN solution it is natural to interpret the gravitational mass as the gravitational energy of the classical gauge fields. For genuine RN case the densities of inertial color gauge charges vanish but those for gravitational color gauge charges in $SO(3) \subset SU(3)$ are in general non-vanishing. Schwartschild metric possesses necessarily a vacuum densities of some electro-weak gauge charges but the contribution to the gravitational energy momentum tensor vanishes.

EP obviously fails if applied to vacuum extremal. The proper interpretation is that for vacuum extremals induced metric can be identified with the effective metric of GRT space-time obtained by lumping the sheets of many-sheeted space-time to Minkowski space with effective metric. EP is true for GRT space-time satisfying Einstein equations reflecting Poincare invariance and also the vanishing of divergence of the energy momentum tensor for the proposed general ansatz to solutions of field equations.

2. Equivalence Principle and imbeddings as non-vacuum extremals

One can consider Equivalence Principle in the case of Kähler charged imbeddings only if one believes that the imbedding is in a reasonable approximation an extremal. Equivalence Principle requires that the Kähler mass of the solution should be smaller than its gravitational mass. This does not pose any conditions on the critical radius since the density of Kähler charge can change sign inside the critical radius (meaning that antimatter dominates inside the critical radius). Thus no constraints results.

The strongest form of Equivalence Principle in TGD context (rather than applying at GRT limit as reflection of Poincare invariance) would require that the Kähler mass of the solution equals to its gravitational mass. It is difficult to see how this could be implied by any deep principle. This requirement poses a lower limit to the critical radius since the Kähler energy outside the critical radius should be smaller than the gravitational mass of the system. In the lowest order approximation this energy is given by the expression

$$\begin{aligned} \frac{E_K}{M} &= \frac{1}{8\pi\alpha_K M}\int_{r_M\geq r_c}\lambda E_K^2 dV \\ &= \frac{\lambda Q_K^2}{GM^2}\frac{r_S}{r_c} \ . \end{aligned} \tag{3.22}$$

The requirement that electro-weak interactions are much weak than gravitational interaction imply the condition $Q_K^2/GM^2 \ll 1$ so that the ratio can be equal to 1 as Equivalence Principle requires only if $r_S/r_c \gg 1$ holds true.

3.3.4 Gravitational energy is not conserved for vacuum imbedding of Reissner-Nordström metric

The inertial energy associated with Kähler action inside a ball of given radius is not conserved for Reissner-Nordström metric imbedded as a non-vacuum extremal since extremal of Kähler action is not in question. This follows from the dependence $m^0 = \lambda t + h(r_M)$ implying that energy current has a radial component and from the non-vanishing of $T^{r_M r_M}$. The non-conservation is not due to the outflow of energy but due to the fact that in the case of Kähler charged imbedding field equations are not satisfied. The basic reason is that the contraction of the energy momentum tensor with the second fundamental form is non-vanishing.

For vacuum extremals it is gravitational energy which fails to be conserved. For instance, for the imbedding of Reissner-Nordström this happens. Only at the limit of Schwartshild metric gravitational energy is conserved. The vacuum extremals which are extremals of Einstein-Hilbert action for the induced metric conserve gravitational four momenta and color charges and are excellent candidates for models of the asymptotic state of star.

The simplest interpretation for the non-conservation of gravitational energy without losing Equivalence Principle is in terms of zero energy ontology. In zero energy ontology the extremals of curvature scalar have interpretation in terms of infinitely long time scale associated with the causal diamond.

The non-stationarity of the vacuum extremal imbedding ($m^0 = \lambda t + h(r_M)$) of R-N metric leads to the following expression for the rate of the change of gravitational energy per time inside a sphere of radius r

$$\begin{aligned} \frac{dE_{vap}/dt}{E(r_M)} &= \frac{dE(r_M)/dt}{E(r_M)} + X \ , \\ X &= \frac{\int T^{r_M r_M}\partial_{r_M} m^0\sqrt{g}d\Omega}{E(r_M)} \ , \\ E(r_M) &= \int T^{tt}\partial_0 m^0\sqrt{g}dV \ . \end{aligned} \tag{3.23}$$

The latter term depending on $T^{r_M r_M}$ takes into account the flow of gravitational energy through boundaries of the sphere and is in general non-vanishing for Reissner-Nordström metric.

Since the proposed solution ansatz works also in the more general case of a stationary spherically symmetric star model, characterized by the pressure $p(r_M)$ and the energy density $\rho(r_M)$, one can write a general order of magnitude estimate for the gravitational energy transfer associated with the boundary of the sphere approximating $h(r_M)$ with the corresponding function for the Schwartshild metric for large values of r_M as

$$X \simeq -\partial_{r_M} h(r_M)\frac{4\pi p r_M^2}{M} . \qquad (3.24)$$

The explicit expression for $\partial_{r_M} h(r_M)$ is given by

$$\begin{aligned}\partial_{r_M} h(r_M) &\simeq \frac{1}{\lambda}\sqrt{\frac{Y_1}{Y_2}} ,\\ Y_1 &= -B+1+\frac{R^2}{4}s^{eff}_{\Theta\Theta}\frac{(\partial_{r_M}u)^2}{(1-u^2)} ,\\ 1-\frac{4\lambda^2}{R^2\omega_1^2}\frac{s^{eff}_{\Theta\Theta}}{s^{eff}_{\Phi\Phi}} & .\end{aligned} \qquad (3.25)$$

Here B is determined by the Einstein equations defining the star model and can be approximated with its value for Schwartshild metric.

At the surface of the Sun ($r_M \simeq 6\cdot 10^8\ m$, particle density $n \simeq 10^{21}/m^3$, $T \simeq 0.5\ eV$, $M \simeq 10^{57}m_p$ and pressure $p \simeq nT$) the order of magnitude of this term is about $X/E \simeq 2\pi K 10^{-13}/year$. For $K \sim 1$ (obtained if the radius of CP_2 is of order Planck length) the loss would be of the same order of magnitude as the inertial energy loss associated with the solar wind: $K \sim 1/k$, $10^4 < k << 10^8$, however implies that the loss is roughly four orders of magnitudes smaller.

It should be noticed that in the case of matter dominated cosmology shows that the rate for the reduction of the gravitational energy is of the order of $(dE/da)/E \simeq 1/a \simeq 10^{-11}/year$, which is of the same order as the fusion energy production of Sun. Thus it would seem that the rate for the change of gravitational energy in cosmological length scales is same as that for the inertial energy in the solar length scale.

4 A Model For The Final State Of The Star

Also this chapter was written decades ago and reflects different views about how TGD space-time relates to GRT space-time.

1. The assumption that single preferred extremal describes the final state might be over-simplifying since many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) having GRT space-time as a model is the correct description, at least according to the recent view.

2. At the time of writing the role of classical electroweak gauge fields was poorly understood although above weak scale it was clear that somehow their effects should be small. In living matter parity breaking effects are large and it is quite possible that classical Z^0 forces is present. This is allowed also by the recent view about Kähler-Dirac equation stating that spinor modes are restricted to 2-D string world sheets carrying vanishing W fields (to guarantee well-defined em charge) and possibly also vanishing Z^0 fields to guarantee vectorial couplings and experimental absence of Z^0 coupling in long length scales. An interesting possibility is that string world sheets can also carry Z^0 field below weak scale which for dark phase with large Planck constant is scaled up.

The best one can hope, is that the general features of the model as preferred extremal could be reflected also in the properties of GRT space-time.

As found, the energy production by fusion inside stars is of the same order of magnitude as the rate of change for the gravitational energy associated with the recent matter dominated cosmology. Since no energy is produced in the final state of the star, the stationary solutions provide a natural model for the final state of the star.

Besides stationarity, there is also a second new element, namely color and electro-weak long ranged forces coupling to the dark matter. For instance, for Kähler charged extremals one necessary has classical Z^0 force even when classical em force can vanish. For Schwartshild solution this force becomes very strong at small values of the radial distance. Therefore the presence of the Z^0 force, and presumably also other classical electro-weak forces, *might* play crucial role in the dynamics of the compact objects. The most plausible physical interpretation would be in terms of dark matter.

The topics to be discussed in the following are:

1. Spherically symmetric stationary model for the final state of the star. It is found that the model cannot be completely realistic since the stationarity assumption fails at the origin and at the surface of the star.

2. Generalization of the model to what could be called dynamo model in order to achieve stationarity.

3. The possible consequences of long range weak and color forces associated with dark matter, in particular the Z^0 force, concerning the dynamics of the compact objects.

The original discussion was based on a different view about energy and motivated the study of Kähler charged solutions with the stationarity property. These 4-surfaces are *not* extremals of the Kähler action. The replacement of the stationary solutions with vacuum extremals requires however only the replacement of the geodesic sphere S^2_I with S^2_{II} implying that both em and Z^0 fields are unavoidably present (or even $W^{\pm}$ fields, depending on vacuum extremal). A serious limitation of the model is that it is single-sheeted. Indeed, the fact that the rotation axis and magnetic axis of super novae are different can be seen as a signal of many-sheeted-ness: the dominantly em and Z^0 fields would reside at different space-time sheets and would correspond to ordinary and dark matter. Of course, entire hierarchy of space-time sheets are expected to be present.

4.1 Spherically Symmetric Model

The simplest model for the final state of the star that one can imagine is obtained by assuming time translation invariance plus spherical symmetry and imbeddability to $M^4 \times S^2_i$, where S^2_i, $i = I, II$ is the geodesic sphere of CP_2. For the homologically non-trivial sphere S^2_I the solution is *not* an extremal whereas S^2_{II} gives an extremal with a vanishing density of inertial energy. In the original discussion cosmological constant was assumed to vanish. There are excellent reasons to assume that this constant is so small that it does not have any appreciable effects in the scale of the star and can thus be neglected. The nice feature of this kind of model is that symmetry assumptions plus stationarity requirement fix almost completely the model: no assumptions about the equation of state for the matter inside the star are needed.

The solution ansatz giving rise to vacuum extremal corresponds to a surface $X^4 \subset M^4_+ \times S^2_{II}$, where S^2_{II} is the homologically trivial geodesic sphere of CP_2. The solution ansatz has the same general form as the imbedding of spherically symmetric metric.

$$\begin{aligned} m^0 &= \lambda t + h(r) \ , \\ \Theta &= \Theta(r) \ , \\ \Phi &= \omega t + k(r) \ . \end{aligned} \tag{4.1}$$

The requirement that g_{tr} vanishes, implies a relationship between the functions $h(r)$ and $k(r)$. One might think that the simplest model is obtained, when the functions $h(r)$ and $k(r)$ vanish identically. One doesn't however obtain physically acceptable solutions in this manner: this is seen by expressing the g_{rr} component of the metric in terms of the mass function

$$-g_{rr} = 1 + \frac{R^2}{4}(\partial_r\Theta)^2 = \frac{1}{1 - \frac{2GM(r)}{r}} \ .$$

At the radii of order star radius (larger than Schwartshild radius $r_S = 2GM$) the gradient of Θ must be of the order of 1/R and this is inconsistent with the finite range of possible values for Θ.

As already shown the field equations $G^{\alpha\beta} D_\beta \partial_\alpha h^k = 0$ are obtained by varying the integral of the curvature scalar over the space time surface. Field equations reduce to conservation conditions for suitably chosen conserved current: for instance the relevant components of the gravitational 4-momentum and gravitational color currents and express the conservation of gravitational four-momentum current and corresponding color currents.

The expression for the induced metric is given by

$$\begin{aligned} ds^2 &= Bdt^2 - Adr^2 - r^2 d\Omega^2 \ , \\ B &= \lambda^2 - \frac{R^2\omega^2}{4} sin^2\Theta \ , \\ A &= 1 + \frac{R^2}{4}(\partial_r\Theta)^2 + \frac{R^2}{4} sin^2\Theta(\partial_r k)^2 - (\partial_r h)^2 \ . \end{aligned} \tag{4.2}$$

The vanishing of the g_{tr} component of the metric implies the condition

$$\lambda\partial_r h - \frac{R^2}{4}\sin^2\Theta\omega\partial_r k = 0 \ . \tag{4.3}$$

The expressions for the components of Einstein tensor for spherically symmetric stationary metric are given by

$$\begin{aligned} G^{rr} &= \frac{1}{A^2}(-\frac{\partial_r B}{Br} + \frac{(A-1)}{r^2}) \ , \\ G^{\theta\theta} &= \frac{1}{r^2}[-\frac{\partial_r^2 B}{2BA} + \frac{1}{2Ar}(\frac{\partial_r A}{A} - \frac{\partial_r B}{B}) \\ &+ \frac{\partial_r B}{4AB}(\frac{\partial_r A}{A} + \frac{\partial_r B}{B}] \ , \\ G^{tt} &= \frac{1}{AB}(-\frac{\partial_r A}{Ar} + \frac{(1-A)}{r^2}) \ . \end{aligned} \tag{4.4}$$

A solution of the field equations with one-dimensional CP_2 projection and vanishing gauge fields is obtained by specifying the solution ansatz in the following manner

$$\begin{aligned} \Theta &= \frac{\pi}{2} \ , \\ h(r) &= hr \ , \\ k(r) &= kr \ . \end{aligned} \tag{4.5}$$

The requirement that g_{rt} vanishes gives the condition

$$h\lambda = R^2\omega k/4 \ .$$

The functions A and B are in this case just constants. Since A differs from unity, the resulting metric is however non-flat and the non-vanishing components of the Einstein tensor are given by the expressions

$$\begin{aligned} G^{tt} &= \frac{(1-A)}{ABr^2} \ , \\ G^{rr} &= -\frac{(1-A)}{A^2r^2} \ . \end{aligned} \tag{4.6}$$

Field equations can be written as conservation conditions, say for the components of gravitational 4-momentum and the conserved "gravitational" color charges associated with the symmetry $\Phi \rightarrow \Phi + \varepsilon$. Quite generally, "gravitational" isometry currents have only time and radial components and radial component represent radial flow to or from the origin. Since the time component is time independent, the field equations state that radial flow is constant so that radial component of the current must behave as $1/r^2$. This is guaranteed provided the condition

$$\partial_r(G^{rr}\sqrt{g}) \quad = \quad 0 \tag{4.7}$$

holds true: this is indeed the case since G^{rr} is proportional to $1/r^2$.

The radial flow of gravitational energy is non-vanishing due to $m^0 = \lambda tt + h(r)$ behavior and given by the expression

$$J^r = \frac{G^{rr}h}{16\pi G} \quad .$$

The conservation condition for J^r fails to be satisfied at origin, which acts as a source or a sink for the gravitational energy. Conservation law fails also at the surface of the star.

One can consider several interpretations.

1. Gravitational mass could be genuinely non-conserved at these locations. Dark particles *resp.* antiparticles with positive *resp.* negative energy would be created in the center of star such that the net density of inertial energy remains zero. Positive and negative energy particles would flow along their own space-time sheets to the outer surface and annihilate there so that there would be no net growth of the gravitational mass. The simplest possibility is that # contacts, which correspond to bound states of parton and negative energy antiparton [K14], split to give rise to particles of opposite inertial energy. At the outer surface # contacts fuse together again.

2. Second option assumes the conservation of gravitational four-momentum. At the surface the non-conservation could result from the flow of the gravitational 4-momentum to a larger space-time sheet via flux tubes. No net flow of inertial energy would be involved since positive and negative energy flows must cancel each other. For example, for a physically acceptable solution the gravitational energy might flow radially from or towards the z-axis, flow to say north pole at the surface of the object and return back along z-axis. Gravitational energy could also flow at origin to a second space-time sheet and return back at the surface of the star.

The (gravitational) mass function of the solution is given by the expression

$$M(r) \quad = \quad \frac{\lambda}{16\pi G}\frac{(A-1)}{AB}\sqrt{AB}r \quad . \tag{4.8}$$

Mass is positive for Minkowskian metric with $B > 0$ and Euclidian metric but negative for the interior black hole metric ($A < 0, B < 0$). Mass is proportional to the radius of the star and in order to obtain an object with about Schwartshild radius one must assume that the parameter k is of the order of $1/R$.

Concerning the physical interpretation of the $\Theta = \pi/2$ solution following remarks are in order:

1. Various gauge fields vanish since CP_2 projection is actually a geodesic circle. The interpretation is that various gauge charges vanish. Note that one-dimensional CP_2 projection conforms with the similar property of Robertson Walker cosmologies.

2. Both gravitational, color and weak forces vanish inside the star and the motion along radial geodesics takes place with constant velocity. This is consistent with the radial flow of gravitational energy.

3. Solution ansatz allows generalizations. For example, the following modification is stationary with respect to energy: $m^0 = \lambda t$, $\Phi = \omega_1 t + k_1 r$, $\Psi = \omega_2 t + k_2 r$, $u = constant < \infty$, $\Theta = \pi/2$. By choosing the values of the parameters suitably all the field equations are satisfied but stationarity is not achieved.

4. The solution should allow gluing to the Schwartshild metric at $\Theta = \pi/2$. As found, for the imbedding of Schwartshild metric the $\Theta = 0$ correspond to the Schwartchild radius so that $\Theta = \pi/2$ would most naturally correspond to $r_M < r_S$. Since radial gauge fluxes are non-vanishing and finite at Schwartshild radius, they must be non-vanishing $\Theta = \pi/2$ surface too, so that the star would carry surface charges and behave somewhat like a conducting sphere.

4.2 Dynamo Model

The previous considerations have shown that the spherically symmetric solution is probably not physically realistic as such and it seems also clear that spherical symmetry must be given up and be replaced with a symmetry with respect to rotations around z-axis in order to obtain more realistic solutions. Since realistic stars rotate and have strong magnetic fields it is natural to ask whether rotation and magnetic fields might provide remedy for the pathological features of the solution. The rotation of the gauge charged matter (in "gravitational" sense) indeed creates classical gauge magnetic fields, which become very strong near the surface of the star, where the condition $\Theta \simeq \pi/2$ holds. If matter is approximately gauge neutral in the interior, the gauge fields should vanish to a very good approximation in the interior and the previous solution should be a good approximation to the actual situation. The rotating star could therefore be regarded as a rotating electro-weak conductor. Both Z^0 and em fields are present for a vanishing Kähler field and the ratio of field strengths is $\gamma/Z^0 = -sin^2(\theta_W)/2 \simeq -1/8$ (see Appendix) so that Z^0 field dominates.

The generation of strong em and Z^0 electric and magnetic fields suggests a mechanism guaranteeing the stability of the solution: star behaves like a dynamo. For solutions with a 2-dimensional CP_2 projection em and Z^0 electric and magnetic fields are automatically orthogonal. For $\Theta \simeq \pi/2$ they are very strong and dominate over gravitation and centrifugal force. Therefore the stability of the surface region naturally results from the cancelation of the electric and magnetic em and Z^0 forces ($\bar{E} + \bar{v} \times \bar{B} = 0$), which takes place, when the velocity field of the matter is suitably chosen. This condition is completely analogous to the vanishing of Kähler Lorentz 4-force which seems to be a general property of the solutions of field equations [K3] and there are reasons to hope non-vacuum extremals describing rotating star can be found.

4.2.1 Conditions for the vanishing of the induced Kähler field

Although the situation becomes too complicated in order to allow the finding of exact solutions describing rotating star, one can identify some general properties of the solution ansatz describing the rotating configuration with Kähler electric and magnetic fields. In order to study the general properties of the solution ansatz in its most general form the explicit expressions for the line element and Kähler form of CP_2 given by the expression

$$\begin{aligned}
ds^2 &= \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos(\Theta)d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2(\Theta)d\Phi^2) \ , \\
J &= \frac{r}{2F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta \wedge d\Phi \ , \\
F &= 1 + r^2 \ , \qquad (4.9)
\end{aligned}$$

0
are needed.

The vanishing of Kähler field is can be guaranteed by the conditions (not the most general ones, symplectic transformations generate new solutions)

$$\begin{aligned}
\Phi &= q\Psi \ , \\
\frac{dr}{d\theta} &= -qrF\frac{sin(\theta)}{1 + qcos(\theta)} \ . \qquad (4.10)
\end{aligned}$$

Note that this ansatz excludes the case $qcos(\Theta) = -1$ for which only $W^{\pm}$ fields are non-vanishing. For this ansatz the expressions for em and Z^0 fields (see Appendix for general formulas) are

$$\begin{aligned}
&\gamma = -sin^2(\Theta_W)R_{03} \ , \qquad Z^0 = 2R_{03} \ , \\
&R_{03} = -qr^2Fsin(\theta)d\Theta \wedge d\Psi \ . \qquad (4.11)
\end{aligned}$$

Here R_{03} denotes a component of spinor curvature.

4.2.2 Topological quantum numbers

The crucial point is that the expansions for the angle coordinates Φ and Ψ using spherical coordinates contain linear terms in t, r and ϕ

$$\begin{aligned} \Phi &= n_1\phi + \omega_1 t + k_1 \ , \\ \Psi &= n_2\phi + \omega_2 t + k_2 \ . \end{aligned} \qquad (4.12)$$

The functions k_1 and k_2 corresponds to Fourier expansion in terms of the plane waves $exp(in\phi)$ with coefficients depending on the coordinates (t, r, θ).

The terms depending linearly on ϕ imply a nontrivial topological structure for the gauge fields not present for the ordinary Maxwell fields. What happens is that space-time divides into regions, which correspond to different values of the topological quantum numbers (n_1, n_2). In the boundaries of these regions the values of the coordinates u and Θ must be such that different values of Φ and Ψ correspond to same point of CP_2. From the expression of the line element one finds that for Ψ the point $u = 0$ and the sphere $u = \infty$ corresponds to these kinds of points. For Φ the surfaces $u = 0$ and $u = \infty$, $\Theta = 0$ correspond to these kinds of surfaces. The form of Φ and Ψ implies that both electric and magnetic gauge fields are nontrivial and rather closely related as is clear from the expression for the Kähler form. Therefore the non-triviality of the winding numbers n_1 and n_2 is what seems to be the crucial, purely TGD based feature of rotating gauge field structures.

4.2.3 Stationary, axially symmetric ansatz with a non-vanishing Kähler field

To make the discussion more concrete, let us assume that the induced metric is invariant with respect to rotations around z-axis and time translations. This is achieved if CP_2 coordinates (apart from linear dependence on ϕ) depend on the coordinates r_M and θ only.

$$\begin{aligned} r &= r(r_M, \theta) \\ \Theta &= \Theta(r_M, \theta) \ , \\ k_i &= k_i(r_M, \theta) \ , \quad i = 1, 2 \ . \end{aligned} \qquad (4.13)$$

This kind of ansatz is clearly consistent: field equations reduce to four equations since second fundamental form is orthogonal to the four-surface and there are four free functions of r and θ: one has effectively two dimensional field theory. Since the general solution ansatz for field equations relies on the vanishing of the Lorentz Kähler force central for the dynamo mechanism, it is of interest to study the general properties of the solution ansatz with a non-vanishing Kähler field. This ansatz can give as special cases space-time sheets carrying Z^0 and em fields with magnetic fields having different rotation axis.

In order to further simplify the discussion let us assume that X^4 corresponds to a sub-manifold of $M^4 \times S^2_I$. For instance, the ansatz

$$\begin{aligned} r &= \infty \ , \\ \Theta &= \Theta(r_M, \theta) \ , \\ \Phi &= n\phi + \omega t + k(r_M, \theta) \ . \end{aligned} \qquad (4.14)$$

is consistent with this assumption. A simpler ansatz is obtained by assuming $k(r_M, \theta) = 0$. This ansatz has the following properties.

1. Induced Kähler (and Z^0-) electric and magnetic fields are automatically orthogonal since CP_2 projection is two-dimensional. In fact, the orthogonality holds to an excellent approximation also for the values of u different but near to $u = \infty$ since the resulting additional components of the Kähler field are extremely small. Kähler electric and magnetic fields are given by

$$
\begin{aligned}
E_{r_M} &= J_{r_M t} = -\partial_{r_M} cos(\Theta)\omega/2 \ , \\
E_\theta &= -\partial_\theta cos(\Theta)\omega/2 \ , \\
B_\theta &= -\partial_{r_M} cos(\Theta) n/2 \ , \\
B_{r_M} &= -\partial_\theta cos(\Theta) n/2 \ .
\end{aligned}
\tag{4.15}
$$

The field strengths are related by

$$
\begin{aligned}
E &= vB \ , \\
v &= \frac{\omega}{n}\sqrt{-\frac{g_{\phi\phi}}{g_{tt}}} \simeq \frac{\omega}{n}\rho \ ,
\end{aligned}
\tag{4.16}
$$

where ρ denotes radial distance from the rotation axis. v can be interpreted as a velocity type parameter. The requirement that $v < 1$ gives a lower bound for the value of n: $n > \omega r_0$, where r_0 denotes the radius of the star: the condition implies that n must be larger than the mass of the star using Planck mass as unit. Somewhat counter intuitively, small rotation velocities seem to correspond to large values of n.

2. Kähler electric and magnetic fields indeed provide a possible mechanism guaranteeing the stability of the star at the surface, where Z^0 forces dominate over gravitation and centrifugal force. Star behaves like a dynamo: matter rotates with a velocity guaranteeing the vanishing of the Z^0 force. It should be noticed that no upper bound for the rotation velocity except that resulting from causality is obtained ($\Omega < 1/r_0$). Therefore this mechanism might explain the observed very large rotation velocities (for instance in Super Nova SN1987A), which are hard to understand in GRT based models [E12].

3. The ansatz indeed describes a rotating object. First, the dynamo mechanism for the stability necessitates the presence of rotation and determines rotation velocity also. Secondly, the presence of Kähler magnetic field can be understood as being created by the rotation of gauge charges. Thirdly, the $g_{t\phi}$ component of the induced metric and therefore the angular momentum density $J^t_z \propto G^{t\phi} r^2 sin^2\theta$ is non-vanishing. A rough order of magnitude estimate for the angular momentum gives $J \simeq M\sqrt{G}n$. In order to obtain angular momentum of order $MR \simeq GM^2$ the order of magnitude for the parameter n must be $n \simeq M\sqrt{G}$ or the mass of the star using Planck mass as unit or: notice that also the Kähler charge of the star is of the same order of magnitude.

4. The gluing of the solution to Schwartshild solution realized as a vacuum extremal is possible at a surface $\Theta = 0$, which corresponds to Scwartschild radius, since at this surface different values of Φ correspond to same point of CP_2. The gluing condition gives additional constraint $u = \infty$ at $r_M = r_S$.

5. The experience with the radially symmetric solution ansatz suggests that Θ is very nearly constant $\Theta \simeq \pi/2$ in the interior and varies considerably only at the surface of the star where Θ must go to zero in order to allow gluing to Schwartshild metric at $r = r_S$. A possible picture is therefore the following. On z-axis there is a Z^0 charged vortex creating radial Z^0 electric field and Z^0 magnetic field in the direction of the vortex. In order to obtain cyclic energy flow matter velocity near the surface of the star must have besides the rotational component a component in θ direction (Z^0 force vanishes in this direction).

6. An interesting possibility is that the vortex actually corresponds to a Kähler charged cosmic string which has gradually lost its enormous inertial mass by a generating pairs of positive and negative energy particles, such that positive energy particles have left the string and participated in the formation of the star. The weakening of the magnetic field would have forced a gradual thickening of the cosmic string to an ordinary magnetic flux tube. This

"stars as pearls in necklace" picture would be consistent with the idea that cosmic strings serve as seeds of galaxy and star formation. Both negative and positive energy strings should be present in order to guarantee vanishing of net inertial energy and one can wonder whether the axis of Z^0 and em magnetic fields correspond to these two kinds of strings.

4.2.4 Does Sun have a solid surface?

The model for the asymptotic state of star predicts that mass at given space-time sheet is concentrated in a spherical shell so that star would have a multi-sheeted onion-like structure. This brings in mind the model for the formation of planetary systems in which spherical layers of quantum coherent dark matter serve as templates for the formation of visible matter which eventually condensed to planets [K23, K8]. It would not be surprising if also younger stars and also planets would possess similar structure. This picture is in conflict with the simplest model of Sun as a gas sphere.

Recently new satellites have begun to provide information about what lurks beneath the photosphere. The pictures produced by Lockheed Martin's Trace Satellite and YOHKOH, TRACE and SOHO satellite programs are publicly available in the web. SERTS program for the spectral analysis suggest a new picture challenging the simple gas sphere picture [E9]. The visual inspection of the pictures combined with spectral analysis has led Michael Moshina to suggests that Sun has a solid, conductive spherical surface layer consisting of calcium ferrite. The article of [E9] [E9] provides impressive pictures, which in my humble non-specialist opinion support this view. Of course, I have not worked personally with the analysis of these pictures so that I do not have the competence to decide how compelling the conclusions of Moshina are. In any case, I think that his web article [E9] deserves a summary.

Before SERTS people were familiar with hydrogen, helium, and calcium emissions from Sun. The careful analysis of SERTS spectrum however suggest the presence of a layer or layers containing ferrite and other heavy metals. Besides ferrite SERTS found silicon, magnesium, manganese, chromium, aluminum, and neon in solar emissions. Also elevated levels of sulphur and nickel were observed during more active cycles of Sun. In the gas sphere model these elements are expected to be present only in minor amounts. As many as 57 different types of emissions from 10 different kinds of elements had to be considered to construct a picture about the surface of the Sun.

Moshina has visually analyzed the pictures constructed from the surface of Sun using light at wave lengths corresponding to three lines of ferrite ions (171, 195, 284 Angstroms). On basis of his analysis he concludes that the spectrum originates from rigid and fixed surface structures, which can survive for days. A further analysis shows that these rigid structure rotate uniformly.

The existence of rigid structures idealizable as spherical shells in the first approximation would conform with the model for the final state of star extrapolated to a qualitative picture about the structure younger stars.

4.3 Z^0 Force And Dynamics Of Compact Objects

The fact that long ranged color fields and weak fields, in particular Z^0 electric fields, could become strong under certain conditions and in fact dominate over gravitation might have interesting consequences in the physics of compact objects. Besides the dynamo mechanism guaranteeing the stability of the compact object the following ideas come immediately into mind.

1. In GRT based models Super Nova explosion is explained in terms of the pressure of the collapsed matter. Numerical simulations however fail to produce the explosion [E12] and it might even be that GRT based models in fact predict the collapse to black hole. Z^0 and em electric fields created by dark matter plus the existence of particles with Z^0 charge, which are suggested by TGD based model of nucleus and condensed matter to be present already in ordinary condensed matter, might provide a natural mechanism preventing the formation of the black hole (also excluded by the failure of complete imbeddability). When matter collapses to a sufficiently small volume the value of Θ approaches $\pi/2$ in the surface region and very strong repulsive radial Z^0 force is generated and could indeed lead to the explosion. Very light exotic variants of Higgs bosons identified as wormhole contacts having left handed weak charge provides a possible mechanism generating Z^0 charge.

2. The strong Z^0 fields at the surface of the star might provide energy source and acceleration mechanism for very high energy cosmic rays and a mechanism producing very high energy X-rays. These rays would be dark matter particles but could transform to ordinary matter by the mechanism discussed in [K10, K8]. For instance, one can imagine the ejection of a particle beam from the surface of a compact object: particles in dark matter phase gain very high energies in the Z^0 electric field and emit brehmstrahlung in the direction of their motion: most intense emission appears in the region very near the surface of the star, where the Z^0 electric field is strongest. This kind of mechanism might provide alternative explanation for the pulsars. In standard explanations the emission takes place in the direction of the magnetic axis, which does not coincide with the rotation axis. In present case the emission point could be anywhere on the surface of the star and magnetic and rotation axes might well coincide as they do in the simplest model. What one has to do is to invent a mechanism creating the surface instability pushing the matter from the surface of the star to the Kähler electric field.

3. The topological character of the magnetic structures might have applications also in the physics of the ordinary stars. It is known that solar magnetic fields correspond to definite isolated structures [E2]. Since electromagnetic fields must be accompanied by Kähler fields it is tempting to assume that these structures indeed correspond to the structures predicted by TGD. At the surface of the Sun the value of Θ near $\pi/2$ are possible and therefore Z^0 force can be very strong inside the magnetic structures.

4.4 Correlation Between Gamma Ray Bursts And Supernovae And Dynamo Model ForThe Final State Of The Star

The correlation between gamma ray bursts and supernovae is certainly the cosmological discovery of the year 2003 [E5, E6].

1. The first indications for supernova gamma ray burst connection came 1998 when a supernova was seen few days after the gamma ray burst in the same region of sky. In this case the intensity of the burst was however by four orders of magnitude weaker than for the typical gamma ray bursts so that the idea about the correlation was not taken seriously. On 29 March, observers recorded a burst christened as GRB030329. On 6 April, theorists at the Technion Institute of Technology in Israel and CERN in Geneva predicted that there would be signs of a supernova in the visible light and infrared spectra on 8 April [E5]. On cue, two days later, observers picked up the telltale spectrum of a type Ic supernova in the same region of sky, triggered as the collapsing star lost hydrogen from its surface. It has now become clear that a large class of gamma ray bursts correlate with supernovae of type Ib and Ic [E1], and that they could thus be powered by the mere core collapse leading to supernova. Recall that supernovae of type II involve hydrogen lines unlike those of type I. Supernovae of type Ib shows Helium lines, and Ic shows neither hydrogen nor helium but intermediate mass elements instead. Supernovae of type Ib and Ic are thought to result as core collapse of massive stars.

2. One of the most enigmatic findings were the "mystery spots" accompanying supernova SN1987A at a distance of few light weeks at the symmetry axis at opposite sides of the supernova [E7]. Their luminosity was nearly 5 per cent of the maximal one. SN1987A was also accompanied by an expanding axi-symmetric remnant surrounded by three concentric rings.

3. The latest finding [E3] is that the radiation associated with the gamma ray bursts is maximally polarized. The polarization degree is the incredible 80 ± 20 per cent, which tells that it must be generated in an extremely strong magnetic field rather than in a simple explosion. The magnetic field must have a strong component parallel to the eye sight direction.

According to the updated model discussed in detail in [K7], cosmic strings transform in topological condensation to magnetic flux tubes about which they represent a limiting case. Primordial magnetic flux tubes forming ferro-magnet like structures become seeds for gravitational condensation leading to the formation of stars and galaxies. The TGD based model for the asymptotic state of a rotating star as dynamo leads to the identification of the predicted magnetic flux tube at the rotation axis of the star as Z^0 magnetic flux tube of primordial origin and assignable to dark matter. Besides Z^0 magnetic flux tube structure also magnetic flux tube structure exists at different space-time sheet but is in general not parallel to the Z^0 magnetic structure. This structure cannot have primordial origin (the magnetic field of star can even flip its polarity).

The flow of matter along Z^0 magnetic (rotation) axis generates synchrotron radiation, which escapes as a precisely targeted beam along magnetic axis and leaves the star. The identification is as the rotating light beam associated with ordinary neutron stars. During the core collapse leading to the supernova this beam becomes gamma ray burst. The mechanism is very much analogous to the squeezing of the tooth paste from the tube.

TGD based models of nuclei [K25] and condensed matter [K10] suggests that the nuclei of dense condensed matter develop anomalous color and weak charges coupling to dark weak bosons having Compton length L_w of order atomic size. Also lighter copies of weak bosons can be important in living matter. This weak charge is vacuum screened above L_w and by dark particles below it. Dark neutrinos, which according to TGD based explanation of tritium beta decay anomaly [K25] should have the same mass scale as ordinary neutrinos, are good candidates for screening dark particles. The Z^0 charge unbalance caused by the ejection of screening dark neutrinos hinders the gravitational collapse. The strong radial compression amplifies the tooth paste effect in this kind of situation so that there are hopes to understand the observed incredibly high polarization of 80 ± 20 per cent [E3].

4.5 Z^0 Force And Super Nova Explosion

The mechanism behind Super Nova explosion is not completely understood. The general picture is roughly the following.

1. The formation of iron means the end of the nuclear processes. The inner parts of the star contract and the degeneracy pressure of the non-relativistic electrons ($E_F \propto \rho^{2/3}$) increases and compensates the gravitational force. The equilibrium state is not stable. When the mass of the iron core approaches Chandrasekhar mass $1.4M_{Sun}$ electrons become relativistic. The milder dependence of the electron Fermi energy on density $E_F \propto \rho^{1/3}$ at the relativistic limit leads to the loss of stability. The high Fermi energy of the electrons allows also the reactions $p + e^- \rightarrow n + \nu_e$ implying decrease of the electronic pressure and neutronization of nuclear matter in the core. Gravitational collapse starts.

2. Collapse stops, when the density of the core reaches the density of the nuclear matter. The degeneracy pressure of the neutrons stops contraction, a shock wave is created and the shock wave and neutrino radiation blow the outer regions of the star away so that Super Nova explosion results.

The problem of this scenario is that numerical simulations do not lead to a strong enough Super Nova explosion and the star tends to collapse into a black hole. A repulsive long ranged Z^0 force predicted by TGD based model of atomic nuclei [K25] generating an additional pressure provides a possible mechanism hindering the collapse and leading to the explosion.

1. The TGD based model for nuclei [K25] and condensed matter [K10] suggests that the nuclei of dense condensed matter develop anomalous color and weak charges coupling to dark weak bosons having Compton length L_w of order atomic size. Weak charge is due to the charged color bonds between nucleons: for instance, tetraneutron can be understood as an alpha particle containing two negatively charged color bonds [K25]. This weak charge is vacuum screened above L_w and by dark particles below it. The charged bonds could exist and also generated between nucleons of different nuclei during collapse.

   Dark neutrinos, which according to the TGD based explanation of tritium beta decay anomaly [K25] should have the same mass scale as ordinary neutrinos, are good candidates for screening dark weak force partially below length scale L_w. In equilibrium color force compensates the partially screened Z^0 force in the bonds. For the ordinary condensed matter densities vacuum screening effectively eliminates the force between neighboring nuclei, and the force makes it visible only via low compressibility. The gravitational collapse could be hindered by the strong additional pressure created by the repulsive L_w-ranged weak interaction between nucleons becoming manifest in the resulting dense phase.

2. In the initial state Z^0 charge is screened by dark neutrinos below L_w so that the repulsive Z^0 force is weaker than gravitational force and attractive color force associated with the bonds. Neutronization reactions $p + e^- \rightarrow n + \nu_e$ trigger the collapse. During collapse density increases so that dark neutrinos are not able to screen the anomalous Z^0 charge density. The dark neutrino radiation escaping from the star can also reduce the Z^0 screening. The resulting repulsive weak force implies a rapid increase of pressure with increasing density and thus a very low compressibility as it is proposed to imply also in the case of ordinary condensed matter [K10]. The repulsive weak force thus stops the collapse to black-hole.

3. The study of the spherically symmetric star models as 4-surfaces imbedded in $M^4_+ \times CP_2$ shows that the extreme nonlinearity of Kähler action implies that Z^0 force dominates over gravitation near the surface of the star.

4.6 Microscopic Description Of Black-Holes In TGD Universe

In TGD framework the imbedding of the metric for the interior of Schwartshild black-hole fails below some critical radius. This strongly suggests that only the exterior metric of black-hole makes sense in TGD framework and that TGD must provide a microscopic description of black-holes. Somewhat unexpectedly, I ended up with this description from a model of hadrons.

Super-symplectic algebra is a generalization of Kac-Moody algebra obtained by replacing the finite-dimensional group G with the group of symplectic transformations of $\delta M^4_\pm \times CP_2$. This algebra defines the group of isometries for the "world of classical worlds" and together with the Kac-Moody algebra assignable to the deformations of light-like 3-surfaces representing orbits of 2-D partonic surfaces it defines the mathematical backbone of quantum TGD as almost topological QFT.

From the point of view of experimentalist the basic question is how these super-symplectic degrees of freedom reflect themselves in existing physics and the pleasant surprise was that super-symplectic bosons explain what might be called the missing hadronic mass and spin. The point is that quarks explain only about 170 MeV of proton mass. Also the spin puzzle of proton is known for years. Also precise mass formulas for hadrons emerge.

Super-symplectic degrees of freedom represent dark matter in electro-weak sense and highly entangled hadronic strings in Hagedorn temperature are very much analogous to black-holes. This indeed generalizes to a microscopic model for black-holes created when hadronic strings fuse together in high density.

4.6.1 Super-symplectic bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K6, K31], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5, 6, 58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K18] and here only a brief summary is given.

As explained in [K18], the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution

should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.

2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.

3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C1] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C2]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

4.6.2 Are ordinary black-holes replaced with super-symplectic black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-hole like states associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \tag{4.17}$$

   where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \tag{4.18}$$

   $m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of $\hbar_0$ for all values of $\hbar$.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \tag{4.19}$$

   For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K23] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

   $v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \ \ v_0 = 1/2 \ . \tag{4.20}$$

   The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K23]. Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0, say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.

7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

4.6.3 Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic. $\hbar = GM^2/v_0$, $v_0 = 1/4$, would hold true for an ideal black hole with Planck length $(\hbar G)^{1/2}$ equal to Schwartshild radius $2GM$. Since black hole entropy is inversely proportional to $\hbar$, this would predict black hole entropy to be of order single bit. This of course looks totally non-sensible if one believes in standard thermodynamics. For the star with mass equal to 10^{40} Planck masses the entropy associated with the initial state of the star would be roughly the number of atoms in star equal to about 10^{60}. Black hole entropy proportional to $GM^2/\hbar$ would be of order 10^{80} provided the standard value of $\hbar$ is used as unit. This stimulates some questions.

1. Does second law pose an upper bound on the value of $\hbar$ of dark black hole from the requirement that black hole has at least the entropy of the initial state. The maximum value of $\hbar$

would be given by the ratio of black hole entropy to the entropy of the initial state and about 10^{20} in the example consider to be compared with $GM^2/v_0 \sim 10^{80}$.

2. Or should one generalize thermodynamics in a manner suggested by zero energy ontology by making explicit distinction between subjective time (sequence of quantum jumps) and geometric time? The arrow of geometric time would correlate with that of subjective time. One can argue that the geometric time has opposite direction for the positive and negative energy parts of the zero energy state interpreted in standard ontology as initial and final states of quantum event. If second law would hold true with respect to subjective time, the formation of ideal dark black hole would destroy entropy only from the point of view of observer with standard arrow of geometric time. The behavior of phase conjugate laser light would be a more mundane example. Do self assembly processes serve as example of non-standard arrow of geometric time in biological systems? In fact, zero energy state is geometrically analogous to a big bang followed by big crunch. One can however criticize the basic assumption as ad hoc guess. One should really understand the the arrow of geometric time. This is discussed in detail in [L1].

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

5 TGD Based Model For Cosmic Strings

The model for cosmic strings has forced to question all cherished assumptions including positive energy ontology, Equivalence Principle, and positivity of gravitational energy.

5.1 Zero Energy Ontology And Cosmic Strings

There are two kinds of cosmic strings: free and topological condensed ones.

1. Free cosmic strings need not be as such preferred extremals of Kähler action. This statement of course depends on what one means with "preferred extremal". The original belief was that preferred extremal corresponds to an absolute minima of Kähler action and does not look a promising identification anymore except perhaps in Euclidian regions where Kähler action is positive definite. There are several other proposals based on some abstract geometric property. One proposal assumes the existence of so called Hamilton-Jacobi structure [K28]. A proposal inspired by number theoretc vision assumes quaternionic structure in some sense as a characterized of preferred extremal [K26]. Since the magnetic field of cosmic string corresponds to CP_2 degrees of freedom with Euclidian signature electric duals do not probably exist.

   Also criticality could be the defining property of preferred extremals. In the new formulation preferred extremals correspond to quantum criticality identified as the vanishing of the second variation of Kähler action at least for the deformations defining symmetries of Kähler action [K31, K15]. The symmetries very probably correspond to conformal symmetries acting as or almost as gauge symmetries. The number of conformal equivalence classes of space-time sheets with same Kähler action and conserved charges is expected to be finite and correspond to n in $h_{eff} = n \times h$ defining the hierarchy of Planck constants labelling phases of dark matter (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book).

2. In long enough length and time scales Kähler action per volume must vanish so that the idealization of cosmology as a vacuum extremal becomes possible and there must be some mechanism compensating the positive action of the free cosmic strings. The mechanism need not be local.

The original picture was based on dynamical cancellation mechanism involving generation of strong Kähler electric fields in the topological condensation such that the Kähler electric action compensates the Kähler magnetic action. Maybe this picture is the most realistic considered hitherto.

One possible cancellation mechanism relies on zero energy ontology. If the sign of the Kähler action is assumed to depend on time orientation - this assumption is vulnerable to criticism - t would be opposite for positive and negative energy space-time sheets and the actions associated with them would cancel if the field configurations are identical. Hence zero energy states would naturally have small Kähler action. Obviously this mechanism is non-local and looks rather questionable.

In this framework zero energy states correspond to cosmologies leading from big bang to big crunch separated by some time interval T of geometric time. Quantum jumps can gradually increase the value T and TGD inspired theory of consciousness suggests that the increase of T might relate to the shift for the contents of conscious experience towards geometric future. In particular, what is usually regarded as cosmology could have started from zero energy state with a small value of T.

5.2 Topological Condensation Of Cosmic Strings

1. Exterior metrics of topologically condensed $g > 1$ strings

If the sign of the gravitational string tension is negative, the simple imbedding of the metric existing for positive string tension ceases to exist. There exists however a different imbedding for which angle excess is in a good approximation same as for the flat solution. The solution is not flat anymore and this implies outwards radial gravitational acceleration. The imbedding of the exterior metric also fails beyond a critical radius. This is not the only possible exterior metric: also non-flat exterior metric are possible and look actually more plausible and also this metric implies radial outwards acceleration as one might indeed expect. What remains to be shown that these metrics do not only yield small angle defect but are also consistent with Newtonian intuitions as the constant velocity spectrum for distant stars around galaxies suggests.

The natural interpretation would be as a mechanism generating large void around a central cosmic string having $g > 1$ and negative string tension and containing at its boundary $g = 1$ positive energy cosmic strings with string tension equal to Kähler string tension. Since angle surplus instead of angle deficit is predicted for $g > 1$ strings, lense effect transforms in this case to angle divergence and one avoids the basic objection against big cosmic strings. The emergence of preferred axes defined by $g > 1$ strings in the scale of large void could relate to the anomalies observed in Cosmic Microwave Background.

Negative gravitational energy of $g > 1$ cosmic strings could be regarded as that part of gravitational energy which causes the accelerated cosmic expansion by driving galactic strings to the boundaries of large voids which then induces phase transition increasing the size of the voids. This kind of acceleration is encountered already at the level of Newton's equations when some of the gravitational masses are negative.

2. Exterior metrics of topologically condensed $g = 1$ strings

One cannot assume that the exterior metric of the galactic $g = 1$ strings is the one predicted by assuming $G = 0$ in the exterior region. This would mean that metric decomposes as $g = g_2(X^2)+g_2(Y^2)$. $g(X^2)$ would be flat as also $g_2(Y^2)$ expect at the position of string. The resulting angle defect due to the replacement of plane Y^2 with cone would be large and give rise to lense effect of same magnitude as in the case of GUT cosmic strings. This lensing has not been observed.

The constant velocity spectrum for distant stars of galaxies and the fact that galaxies are organized along strings suggests that these string generate in a good approximation Newtonian potential. This potential predicts constant velocity spectrum with a correct value velocity.

In the stationary situation one expects that the exterior metric of galactic string corresponds to a small deformation of vacuum extremal of Kähler action which is also extremal of the curvature scalar in the induced metric. This allows a solution ansatz which conforms with Newtonian intuitions and for which metric decomposes as $g = g_1 + g_3$, where g_1 corresponds to axis in the direction of string and g_3 remaining $1 + 2$ directions.

5.3 Dark Energy Is Replaced With Dark Matter In TGD Framework

The first thing that comes in mind is that negative gravitational energy could be the TGD counterpart for the positive dark vacuum energy known to dominate over the mass density in cosmological length scales and believed to cause the accelerated cosmic expansion. This argument is wrong.

1. The gigantic value of gravitational Planck constant implies that dark matter makes TGD Universe a macroscopic quantum system even in cosmological length scales. Astrophysical systems become stationary quantum systems which participate in cosmic expansion only via quantum phase transitions increasing the value of gravitational Planck constant. Critical cosmologies, which are determined apart from a single parameter in TGD Universe, are natural during all quantum phase transitions, in particular the phase transition periods increasing the size of large voids and having interpretation in terms of an increase of gravitational Planck constant. Cosmic expansion is predicted to be accelerating during these periods. The mere criticality requires that besides ordinary matter there is a contribution $\Omega_\Lambda \simeq .74$ to the mass density besides visible matter and dark matter.

2. The essential characteristic of dark energy is its negative pressure. Negative gravitational energy could effectively create this negative pressure during phase transitions increasing the size of large voids. Since negative gravitational mass would be basically responsible for the accelerated expansion, one can assume that dark energy is actually dark matter.

3. Note that the pressure is negative during critical period. This is however interpreted as a correlate for the expansion caused by the phase transition increasing Planck constant rather than being due to positive cosmological constant or quintessence with negative pressure.

5.4 The Values For The TGD Counterpart Of Cosmological Constant

One can introduce a parameter characterizing the contribution of dark mass to the mass density during critical periods and call it cosmological constant recalling however that the contribution does not correspond to negative pressure now. The value of this parameter is same as in the standard cosmology from mere criticality assumption.

What is new that p-adic fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing Λ. The order of magnitude for the recent value of the cosmological constant comes out correctly. The gravitational energy density assignable to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

The naive expectation would be the density of cosmic strings would behave as $1/a^2$ as function of M^4_+ proper time. The vision about dark matter as a phase characterized by gigantic Planck constant however implies that large voids do not expand in continuous manner during cosmic evolution but in discrete quantum jumps increasing the value of the gravitational Planck constant and thus increasing the size of the large void as a quantum state. Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids.

Classically one can understand the occurrence of the phase transitions increasing the size of the void as resulting when the galactic strings end up to the boundary of the large void in the repulsive gravitational field of the big string.

5.5 Matter-Antimatter Asymmetry And Cosmic Strings

Despite huge amount of work done during last decades (during the GUT era the problem was regarded as being solved!) matter-antimatter asymmetry remains still an unresolved problem of cosmology. A possible resolution of the problem is matter-antimatter asymmetry in the sense that cosmic strings contain antimatter and their exteriors matter. The challenge would be to understand the mechanism generating this asymmetry. The vanishing of the net gauge charges of cosmic string allows this symmetry since electro-weak charges of quarks and leptons can cancel each other.

The challenge is to identify the mechanism inducing the CP breaking necessary for the matter-antimatter asymmetry. Quite a small CP breaking inside cosmic strings would be enough.

1. The key observation is that vacuum extremals as such are not physically acceptable: small deformations of vacuum extremals to non-vacua are required. This applies also to cosmic strings since as such they do note present preferred extremals. The reason is that the preferred extremals involve necessary regions with Euclidian signature providing four-dimensional representations of generalized Feynman diagrams with particle quantum numbers at the light-like 3-surfaces at which the induced metric is degenerate.

2. The simplest deformation of vacuum extremals and cosmic strings would be induced by the topological condensation of CP_2 type vacuum extremals representing fermions. The topological condensation at larger space-time surface in turn creates bosons as wormhole contacts.

3. This process induces a Kähler electric fields and could induce a small Kähler electric charge inside cosmic string. This in turn would induce CP breaking inside cosmic string inducing matter antimatter asymmetry by the minimization of the ground state energy. Conservation of Kähler charge in turn would induce asymmetry outside cosmic string and the annihilation of matter and antimatter would then lead to a situation in which there is only matter.

4. Either galactic cosmic strings or big cosmic strings (in the sense of having large string tension) at the centers of galactic voids or both could generate the asymmetry and in the recent scenario big strings are not necessary. One might argue that the photon to baryon ratio $r \sim 10^{-9}$ characterizing matter asymmetry quantitatively must be expressible in terms of some fundamental constant possibly characterizing cosmic strings. The ratio $\epsilon = G/\hbar R^2 \simeq 4 \times 10^{-8}$ is certainly a fundamental constant in TGD Universe. By replacing R with $2\pi R$ would give $\epsilon = G/(2\pi R)^2 \simeq 1.0 \times 10^{-9}$. It would not be surprising if this parameter would determine the value of r.

The model can be criticized.

1. The model suggest only a mechanism and one can argue that the Kähler electric fields created by topological condensates could be random and would not generate any Kähler electric charge. Also the sign of the asymmetry could depend on cosmic string. A CP breaking at the fundamental level might be necessary to fix the sign of the breaking locally.

2. The model is not the only one that one can imagine. It is only required that antimatter is somewhere else. Antimatter could reside also at other p-adic space-time sheets and at the dark space-time sheets with different values of Planck constant.

The needed CP breaking is indeed predicted by the fundamental formulation of quantum TGD in terms of the Kähler-Dirac action associated with Kähler action and its generalization allowing include instanton term as imaginary part of Kähler action inducing CP breaking [K31, K20] .

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal, whose proper identification becomes a challenge. In ZEO (ZEO) 3-surfaces are pairs of space-like 3-surfaces assignable to the boundaries of causal diamond (CD) and for deterministic action principle this suggests that the extremals are unique. In presence of non-determinism the situation changes.

2. The huge vacuum degeneracy of Kähler action suggests that for given pair of 3-surfaces at the boundaries of CD there is a continuum of extremals with the same Kähler action and conserved charges obtained from each other by conformal transformations acting as gauge symmetries and respecting the light-likeness of wormhole throats (as well as the vanishing of the determinant of space-time metric at them). The interpretation is in terms of quantum criticality with the hierarchy of symmetries defining a hierarchy of criticalities analogous to the hierarchy defined by the rank of the matrix defined by the second derivatives of potential function in Thom's catastrophe theory.

3. The number of gauge equivalence classes is expected to be finite integer n and the proposal is that it corresponds to the value of the effective Planck constant $h_{eff} = n \times h$ so that a connection with dark matter hierarchy labelled by values of n emerges [K12].

4. This representation generalizes - at least formally. One could add an imaginary instanton term to the Kähler function and corresponding Kähler-Dirac operator D_K so that the generalized eigenvalues assignable to D_K become complex. The generalized eigenvalues correspond to the square roots of the eigenvalues of the operator $DD^\dagger = (p^k\gamma_k + \Gamma^n)(p^k\gamma_k + \Gamma^n)^\dagger$ acting at the boundaries of string world sheets carrying fermion modes and it seems that only space-like 3-surfaces contribute. Γ^n is the normal component of the vector defined by Kähler-Dirac gamma matrices. One can define Dirac determinant formally as the product of the eigenvalues of $DD^\dagger$.

   The conjecture is that the resulting Dirac determinant equals to the exponent of Kähler action and imaginary instanton term for the preferred extremal. The instanton term does not contribute to the WCW metric but could provide a first principle description for CP breaking and anyonic effects. It also predicts the dependence of these effects on the page of the book like structure defined by the generalized imbedding space realizing the dark matter hierarchy with levels labeled by the value of Planck constant.

5. In the case of cosmic strings CP breaking could be especially significant and force the generation of Kähler electric charge. Instanton term is proportional to $1/h_{eff}$ so that CP breaking would be small for the gigantic values of h_{eff} characterizing dark matter. For small values of h_{eff} the breaking is large provided that the topological condensation is able to make the CP_2 projection of cosmic string four-dimensional so that the instanton contribution to the complexified Kähler action is non-vanishing and large enough. Since instanton contribution as a local divergence reduces to the contributions assignable to the light-like 3-surfaces X_l^3 representing topologically condensed particles, CP breaking is large if the density of topologically condensed fermions and wormhole contacts generated by the condensation of cosmic strings is high enough.

6 Entropic Gravity In TGD Framework

Entropic gravity (EG) introduced by Verline [B2] has stimulated a great interest. One of the most interesting reactions is the commentary of Sabine Hossenfelder [B8]. The article of Kobakhidze [B1] relies on experiments supporting the existence of Schrödinger amplitudes of neutron in the gravitational field of Earth develops an argument suggesting that EG hypothesis in the form in which it excludes gravitons is wrong. The following arguments represent TGD inspired view about what entropic gravity (EG) could be if one throws out the unnecessary assumptions such as the emerging dimensions and absence of gravitons. Before continuing I want to express my gratitude to Prof. Masud Chaichian for the stimulus which led to a re-evaluation and reformulation of EG hypothesis. I want also to represent my thanks to Archil Kobakhidze for his clarifications concerning his argument against Verlinde's entropic gravity.

1. If one does not believe in TGD, one could start from the idea that stochastic quantization or something analogous to it might imply something analogous to entropic gravity (EG). What is required is the replacement of the path integral with functional integral. More precisely, one has functional integral in which the real contribution to Kähler action of the preferred extremal from Euclidian regions of the space-time surface to the exponent represents

Kähler function and the imaginary contribution from Minkowskian regions serves as a Morse function so that the counterpart of Morse theory in WCW is obtained on stationary phase approximation in accordance with the vision about TGD as almost topological QFT [K32]. The exponent of Kähler function is the new element making the functional integral well-defined and the presence of phase factor gives rise to the interference effects characteristic for quantum field theories although one does not integrate over all space-time surfaces. In zero energy ontology one has however pairs of 3-surfaces at the opposite light-like boundaries of CD so that something very much analogous to path integral is obtained.

2. Holography requires that everything reduces to the level of 3-metrics and more generally, to the level of 3-D field configurations. Something like this happens if one can approximate the functional integral with the integral over small deformations for the minima of the action. This happens in precise sense in completely integral quantum field theories.

   The basic vision behind quantum TGD is that this approximation is much nearer to reality than the original theory. In other words, holography is realized in the sense that to a given 3-surface the metric of WCW assigns a unique space-time and this space-time serves as the analog of Bohr orbit and allows to realize 4-D general coordinate invariance in the space of 3-surfaces so that classical theory becomes an exact part of quantum theory. This point of view will be adopted in the following also in the framework of general relativity where one considers abstract 4-geometries instead of 4-surfaces: functional integral should be over 3-geometries with the definition of Kähler metric assigning to 3-geometry a unique 4-geometry.

3. A powerful constraint is that the functional integral is free of divergences. Both 4-D path integral and stochastic quantization for gravitation fail in this respect due to the local divergences (in super-gravity situation might be different). The TGD inspired approach reducing quantum TGD to almost topological QFT with Chern-Simons term and a constraint term depending on metric associated with preferred 3-surfaces allows to circumvent this difficulty. This picture will applied to the quantization of GRT and one could see the resulting theory as a guess for what GRT limit of TGD could be. The first guess that Kähler function corresponds to Einstein-Maxwell action for this kind of preferred extremal turns out to be correct. An essential and radically new element of TGD is the possibility of space-time regions with Euclidian signature of the induced metric replacing the interiors of blackholes: this element will be assumed also now. The conditions that CP_2 represents and extremal of EYM action requires cosmological constant in Euclidian regions determined by the constant curvature of CP_2 and one can ask whether the average value of cosmological constant over 3-space could correspond to the cosmological constant explaining accelerating cosmic expansion.

4. Before going to a more precise formulation it is better to discuss how the phenomenology of EG with gravitons and without the fuzzy assumption about the emergence of space-time could be understood in TGD framework. This article is kind of continuation to the earlier article published in http://www.scribd.com/doc/45928480/PSTJ-V1-10-More-Possible-Games-in- Town-ContinuedPrespace-Time Journal [B7], where the proposal that Quantum TGD as a hermitian square root of thermodynamics might imply something analogous to entropic gravity since S-matrix is replaced with the analog of thermal S-matrix. The article of Hossenfelder [B8] has been of great help. Entropic gravity is generalized in TGD framework so that all interactions are entropic: the reason is that in zero energy ontology (ZEO) the $S-$matrix is replaced with $M-$matrix defining a square root of thermodynamics in a well defined sense.

6.1 The Phenomenology Of EG In TGD Framework

In TGD framework one can consider a modification of EG allowing gravitons. In this framework thermodynamics is assigned with the virtual gravitons (and also real) flowing along the flux tubes mediating gravitational interaction. The entropy proportional to the length of flux tube corresponds to the entropy assigned with the holographic screen and temperature is the temperature of gravitons decreasing with distance just like the temperature of the radiation from Sun decreases as $1/r^2$: this is due to the absence of gravitonic heat sources in empty space.

TGD based view about EG leads also to new views. The basic objection against EG is that it applies also to electromagnetic interactions and leads to negative temperatures. In zero energy ontology the resolution of the problem could be that matter and antimatter correspond to opposite arrow sof geometric time and therefore different causal diamonds and space-time sheets: this could explain also the apparent absence of antimatter.

6.1.1 EG with gravitons and without emergence of space-time

The following arguments explain how the basic formulas of EG follow from TGD framework assuming that virtual gravitons reside at flux tubes connecting interacting systems.

1. The argument originally to Kobakhidze [?]uggests that EG in the strong sense predicting the absence of gravitons is inconsistent with experimental facts. The argument does not mention gravitons but relies on the experimental fact that neutron bound states in Earth's gravitational field exist. Chaichian et al [?]owever claim the argument contains an error because the formula (8) of [?]or the density matrix of neutron plus screen reading as

$$\rho_S(z + \Delta z) = \rho_N(z + \Delta z) \times \rho_{S/N}(z)$$

   gives constant density matrix for screen when one removes neutron and this is certainly not true. According to Kobakhidze (private communication) the theory of Verlinde implies that the removal of neutron effectively removes the screen from $z + \Delta z$ to z. I leave it for the reader to decide what is the truth. Second challengable assumption of Kobakhidze used before equation (10) of [?]s the additivity of the entropies of the screen and neutron: the interaction with the screen implies interaction entropy and the question is whether it can be neglected.

2. According to Chaichian et al [?]hat there exist transitions between the excited states suggest that the emission of gravitons must be involved (one can of course consider also electromagnetic transitions). This assumption is not testable since the rate of graviton induced transitions is extremely low. This result together with the vision about quantum theory as a square root of thermodynamics suggests that one must consider a modification of EG such that it allows gravitons and try to assign entropy and temperature to some real systems.

3. Suppose that one takes EG formulas seriously but accepts the existence of gravitons. EG should be understandable in terms of the classical space-time correlates of gravitational interaction assignable to virtual gravitons with space-like momenta. Could virtual gravitons mediating the gravitational flux through a hologram surface be responsible for the gravitational entropy?

   Could one assign entropy to the gravitons inside flux tube like structures from the source and traversing the holographic screen and carrying virtual gravitons with wave length much shorter than the distance to source so that quantum coherence for gravitons is lost? If the density of entropy per unit length of the flux tube is constant, gravitational entropy is proportional to the length of flux tube from the source to the constant potential surface so that $S \propto \Phi_{gr} A$ hypothesis would follow as a consequence.

4. Why the temperature of graviton carrying flux tubes should be reduced as $1/r^2$ with distance in the case of a spherically symmetric source? Could the masses serve as heat sources creating thermal ensemble of gravitons? The virtual gravitons emitted at the source would be at certain temperature just as ordinary photons created in Sun. The gravitons flowing along the flux tubes would cool- maybe by the expansion of the transversal cross section of the flux tube- and the condition that heat is not created or absorbed in the empty space would imply $1/r^2$ behavior. The flux tubes carrying virtual gravitons would serve as counterparts of long strings in holographic argument. In TGD the string like objects indeed appear quite concretely.

5. By using reduced mass, gravitational temperature and entropy become symmetric as functions of the masses of two objects. This assumption makes sense only in many-sheeted

space-time for which each pair of systems is characterized by its own flux tubes (space-time sheets) mediating the gravitational interaction. Also the notion of gravitational Planck constant proportional to GMm makes sense only if it characterizes the flux tubes.

6. Unless the special nature of gravitational force as inertial force distinguishes gravitation from other interactions representing genuine forces, EG argument applies also in electrodynamics. The temperature in this case is proportional to the projection of the electric field which is in the direction of the normal of constant potential surface and has wrong sign for the second sign of the charge. Could the negative temperature implying instability relating somehow to the matter antimatter asymmetry? Antimatter and antimatter could not appear in same space-time region because either of them would give rise to negative temperature for flux tubes carrying virtual photons. In TGD framework similar outcome results also from totally different arguments and states that matter and antimatter should reside at different space-time sheets. Antimatter could be also dark in TGD sense. This point will be discussed in detail below and will be related to the generation of thermodynamical arrow of time which would be different for particles and antiparticles. In this case the reduced mass must be replaced with reduced charge $Q_1Q_2/(Q_+Q_2)$ to achieve symmetry.

7. Could one say that in the GRANIT experiment [C3] giving support for the description of the neutron in Earth's gravitational field using Schrödinger equation the entropy of neutron plus screen is just the entropy associated with the Coulomb potential of Earth and neutron obtainable as $S(r) \propto (\phi_{gr,Earth} + \Phi_{gr,neutron})A$? The gravitational potential appearing in the Schrödinger equation would be expressible essentially as the entropy per surface area and -as already noticed- this could be a mere accident having nothing to do with the real nature of gravitational force.

The assignment of entropy with the lines of generalized Feynman graphs is consistent with the replacement of S-matrix with $M-$matrix identified as a product of S-matrix and a Hermitian square root of density matrix commuting with $S-$matrix. These Hermitian square roots commute with $S-$matrix and generate infinite-D symmetry algebra of $S-$matrix defining a generalization of Yangian [A4] [B6, B3, B4] in ZEO since they are multi-local with respect to the partonic 2-surfaces located at the two light-like boundaries of CD. This algebra generated by zero energy states generalizes the twistorial Yangian and allowing CDs with integer multiples of basic scale one obtains a generalization of Kac-Moody algebra in which the non-commutative phase S^n generalizes the commutative phase factor $exp(in\phi)$ of Kac-Moody algebra. Also vacuum functional can be interpreted as a complex square root of density matrix for ground states with Minkowskian part of Kähler action defining the phase and the exponent of Euclidian part defining the modulus.

6.1.2 Could gravity reduce to entropic force in long length scales?

The pessimistic view is that the possibility to regard gravitation as an entropic force is purely accidental and follows from the fact that gravitational potential happens to represent the density of gravitonic entropy per surface area and gravitonic temperature happens to be proportional to the normal component of the gravitational acceleration. On the other hand, one can develop an argument in which the absorption l of virtual gravitons with wavelength must shorter than the distance between the two systems is analogous to radiation pressure and describable in terms of entropic gravity.

The proposal that both virtual and real gravitons are characterized by temperature and entropy is questionable in standard quantum theory. It however makes sense in ZEO in which $S-$matrix is replaced with $M-$matrix identifiable as a Hermitian square root of density matrix so that thermodynamics emerges even at the level of virtual particles. That it does so conforms with the fact that the basic building blocks of virtual particles are on mass shell massless particles. Allowing negative energies one can have also space-like net values of virtual momenta and virtual particles differ from incoming ones only in that the bound state conditions for masses is given up. The resulting powerful constraints on virtual momenta allowing to avoid both UV and IR divergences and justify twistorial description for both on mass shell particles and virtual particles.

6.1.3 Flux tube picture for gravitational interaction

Consider now the emission of gravitational radiation and its absorpion allowing also virtual gravitons. In the picture about flux tubes as space-time sheets carrying gravitons between two objects there are two cases as I have discussed earlier but without realizing that these cases could correspond to to non-entropic and entropic gravitation respectively.

Remark: The flux tube picture emerged from the attempt to understand why the gravitational Planck constant introduced by Nottale and taken seriously by me as characteristics of dark gravitons is proportional to the masses of Sun and planet: the explanation is that $\hbar_{gr}$ is associated with flux tubes connecting these objects. It follows also from fractal string picture with string like objects identified as flux tubes.

In the minimal formulation the hierarchy of Planck constants coming as integer multiples of ordinary Planck constant and assigned to dark matter can be understood as an effective hierarchy due to the possibility of many-sheeted classical solutions of field equations with identical canonical momentum densities at various sheets implied by the huge vacuum degeneracy of Kähler action.

1. When the wavelength of gravitons is longer than that of flux tube, the graviton serves as a string connecting the systems (say ends of long bar, of the receiving system and source-not in practice) together and induces at classical level coherent oscillations of the relative distance.

   In the detection of gravitational waves this kind arrangement should appear and typically appears. For instance, for millisecond pulsar the graviton wavelength is about 10^5 meters. This would represent quantum realm in which entropic gravity does not apply. Classical description however works in accordance with quantum classical correspondence.

   Remark: If one is ready to take seriously the idea about large gravitational Planck constant, the wavelengths would be very long and one would be practically always in this realm.

2. When the wave length of gravitons is shorter than flux tube, the graviton beam losses its coherence and is characterized by temperature and entropy and generates on the receiver something analogous to gravitational radiation pressure induced by virtual particles (this pressure is however negative for gravitation!). This would generate entropic force with definite direction since the momentum of virtual gravitons is of the same sign.

   (a) This would suggest that gravitational waves with wavelengths shorter than the size of the detector should not be detectable via standard empirical arrangements.

   (b) A stronger condition would be that gravitational waves with wave lengths shorter than the distance between source and receiver cannot be detected: this would effectively conform with EG and predict that gravitational waves ill not be detected. This should have no practical consequences since even in the case of neutrons of GRANIT experiment the wavelength seems to be of order 10^5 meters from the peV energy scale of the bound states in Earth's gravitational field.

3. Entropic gravity is not in conflict with the geometrization of gravitational interaction since also thermodynamics should have space-time correlates by quantum classical correspondence. In accordance with stringy vision about short range gravitation, gravitational interaction in non-entropic realm is mediated by flux tubes connecting the masses involved and acting like strings.

6.1.4 The identification of the temperature and entropy

One can look at the situation also at more quantitative level. The natural guess for the temperature parameter would be as Unruh temperature

$$T_{gr} = \frac{\hbar}{2\pi} a \ , \tag{6.1}$$

where a is the projection of the gravitational acceleration along the normal of the gravitational potential = constant surface. In the Newtonian limit it would be acceleration associated with the relative coordinates and correspond to the reduced mass and equal to $a = G(m_1 + m_2)/r^2$.

One could identify T_{gr} also as the magnitude of gravitational acceleration. In this case the definition would involved only be purely local. This is in accordance with the character of temperature as intensive property.

The general relativistic objection against the generalization is that gravitation is not a genuine force: only a genuine acceleration due to other interactions than gravity should contribute to the Unruh temperature so that gravitonic Unruh temperature should vanish. On the other hand, any genuine force should give rise to an acceleration. The sign of the temperature parameter would be different for attractive and repulsive forces so that negative temperatures would become possible. Also the lack of general coordinate invariance is a heavy objection against the formula.

1. Gravitonic temperature in TGD Universe

In TGD Universe the situation is different. In this case the definition of temperature as magnitude of local acceleration is more natural.

1. Space-time surface is sub-manifold of the imbedding space and one can talk about acceleration of a point like particle in imbedding space $M^4 \times CP_2$. This acceleration corresponds to the trace of the second fundamental form for the imbedding and is completely well-defined and general coordinate invariant quantity and vanishes for the geodesics of the imbedding space. Since acceleration is a purely geometric quantity this temperature would be same for flux sheets irrespective of whether they mediate gravitational or some other interactions so that all kinds of virtual particles would be characterized by this same temperature.

2. One could even generalize T_{gr} to a purely local position dependent parameter by identifying it as the magnitude of second fundamental form at given point of space-time surface. This would mean that the temperature in question would have purely geometric correlate. This temperature would be alwas non-negative. This purely local definition would also save from possible inconsistencies in the definition of temperature resulting from the assumption that its sign depends on whether the interaction is repulsive or attractive.

3. The trace of the second fundamental form -call it H- and thus T_{gr} vanishes for minimal surfaces. Examples of minimal surfaces are cosmic strings [?, ?] massless extremals, and CP_2 type vacuum extremals with M^4 projection which is light-like geodesic [?] Vacuum extremals with at most 2-D Lagrangian CP_2 projection has a non-vanishing H and this is true also for their deformations defining the counterpart of GRT space-time. Also the deformations of cosmic strings with 2-D M^4 projection to magnetic flux tubes with 4-D M^4 projection are expected to be non-minimal surfaces. Same applies to the deformations of CP_2 vacuum extremals near the region where the signature of the induced metric changes. The predicted cosmic string dominated phase of primordial cosmology [?]ould correspond to the vanishing gravitonic temperature. Also generic CP_2 type vacuum extremals have non-vanishing H.

4. Massless extremals define an excellent macroscopic space-time correlate for gravitons. The massivation of gravitons is however strongly suggested by simple considerations encouraged by twistorial picture and wormhole throats connecting parallel MEs define the basic building bricks of gravitons and would bring in non-vanishing geometric temperature, (extremely small but non-vanishing) graviton mass, and gravitonic entropy.

   (a) The M^4 projection of CP_2 type vacuum extremal is random light-like curve rather than geodesic of M^4 (this gives rise to Virasoro conditions [K3]). The mass scale defined by the second fundamental form describing acceleration is non-vanishing. I have indeed assigned this scale as well as the mixing of M^4 and CP_2 gamma matrices inducing mixing of M^4 chiralities to massivation. The original proposal was that the trace of second fundamental form could be identifiable as classical counterpart of Higgs field. One can speak of light-like randomness above a given length scale defined by the inverse of the length of the acceleration vector.

   (b) This suggests a connection with p-adic mass calculations: the p-adic mass scale m_p is proportional to the acceleration and thus could be given by the geometric temperature: $m_p = nR^{-1}p^{-1/2} \sim \hbar H = \hbar a$, where $R \sim 10^4 L_{Pl}$ is CP_2 radius, and n some numerical constant of order unity. This would determine the mass scale of the particle and relate it to the momentum exchange along corresponding CP_2 type vacuum extremal. Local graviton mass scale at the flux tubes mediating gravitational interaction would be essentially the geometric temperature.

(c) Interestingly, for photons at the flux tubes mediating Coulomb interactions in hydrogen atom this mass scale would be of order $\hbar a \sim e^2\hbar/m_p n^4 a_0^2 \sim 10^{-5}/n^4$ eV, which is of same order of magnitude as Lamb shift, which corresponds to 10^{-6} eV energy scale for $n = 2$ level of hydrogen atom. Hence it might be possible to kill the hypothesis rather easily.

(d) Note that momentum exchange is space-like for Coulomb interaction and the trace H^k of second fundamental form would be space-like vector. It seems that one define mass scale as $H = \sqrt{-H^k H_k}$ to get a real quantity.

(e) This picture is in line with the view that also the bosons usually regarded as massless possess a small mass serving as an IR cufoff. This vision is inspired by zero energy ontology and twistorial considerations [K29]. The prediction that Higgs is completely eaten by gauge bosons in massivation is a prediction perhaps testable at LHC already during year 2011.

Remark: In MOND theory of dark matter a critical value of acceleration is introduced. I do not believe personally to MOND and TGD explains galactic rotation curves without any modification of Newtonian dynamics in terms of dark matter assignable to cosmic strings containing galaxies like around it like pearls in necklace. In TGD framework the critical acceleration would be the acceleration above which the gravitational acceleration caused by the dark matter associated with the cosmic strings traversing along galactic plane orthogonally and behaving as $1/\rho$ overcomes the acceleration caused by the galactic matter and behaving as $1/\rho^2$. Could this critical acceleration correspond to a critical temperature T_{gr}- presumably determined by an appropriate p-adic length scale and coming as a power $2^{-k/2}$ by p-adic length scale hypothesis? Could critical value of H perhaps characterize also a critical magnitude for the deformation from minimal surface extremal? The critical acceleration in Milgrom's model is about 1.2×10^{-10} m/s^2 and corresponds to a time scale of 10^{12} years, which is of the order of the age of the Universe.

The formula contains Planck constant and the obvious question of the inhabitant of TGD Universe is whether the Planck constant can be identified with the ordinary Planck constant or with the *effective* Planck constant coming as integer multiple of it [**?**]

1. For the ordinary value of $\hbar$ the gravitational Unruh temperature is extremely small. To make things more concrete one can express the Unruh temperature in gravitational case in terms of Schwartschild radius $r_S = 2GMm$ at Newtonian limit. This gives

$$T_{gr} = \frac{\hbar}{4\pi r_S}\frac{M+m}{M}(\frac{r_S}{r})^2 \ . \tag{6.2}$$

   Even at Schwartschild radius the temperature corresponds to Compton length of order $4\pi r_S$ for $m << M$.

2. Suppose that Planck constant is gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where $v_0 \simeq 2^{-11}$ holds true for inner planets in solar system [**?**] This would give

$$T_{gr} = \frac{m}{8\pi v_0}\frac{M+m}{M}(\frac{r_S}{r})^2 \ .$$

   The value is gigantic so that one must assume that the temperature parameter corresponds to the minimum value of Planck constant. This conforms with the identification of the p-adic mass scale in terms of the geometric temperature.

2. Gravitonic entropy in TGD Universe

A good guess for the value of gravitational entropy (gravitonic entropy associated with the flux tube mediating gravitational interaction) comes from the observation that it should be proportional to the flux tube length.The relationship $dE = TdS$ suggests $S \propto \phi_{gr}/T_{gr}$ as the first guess in Newtonian limit. A better guess would be

$$S_{gr} = -\frac{V_{gr}}{T_{gr}} = \frac{M+m}{M}\frac{r}{\hbar m}, \ , \tag{6.3}$$

The replacement $M \rightarrow M+m$ appearing in the Newtonian equations of motion for the reduced mass has been performed to obtain symmetry with respect to the exchange of the masses.

The entropy would depend on the interaction mediated by the space-time sheet in question which suggests that the generalization is

$$S = -\frac{V(r)}{T_{gr}} \ . \tag{6.4}$$

Here $V(r)$ is the potential energy of the interaction. The sign of S depends on whether the interaction is attractive or repulsive and also on the sign of the temperature. For a repulsive interaction the entropy would be negative so that the state would be thermodynamically unstable in ordinary thermodynamics.

The integration of $dE = TdS$ in the case of Coulomb potential gives $E = V(r) - V(0)$ for both options. If the charge density near origin is constant, one has $V(r) \propto r^2$ in this region implying $V(0) = 0$ so that one obtains Coulombic interaction energy $E = V(r)$. Hence thermodynamical interpretation makes sense formally.

The challenge is to generalize the formula of entropy in Lorentz invariant and general coordinate invariant manner. Basically the challenge is to express the interaction energy in this manner. Entropy characterizes the entire flux tube and is therefore a non-local quantity. This justifies the use of interaction energy in the formula. In principle the dynamics defined by the extremals of Kähler action predicts the dependence of the interaction energy on Minkowskian length of the flux tube, which is well-defined in TGD Universe. Entropy should be also a scalar. This is achieved since the rest frame is fixed uniquely by the time direction defined by the time-like line connecting the tips of CD: the interaction energy in rest frame of CD defines a scalar. Note that the sign of entropy correlates with the sign of interaction energy so that the repulsive situation would be thermodynamically unstable and this indeed suggests that antimatter should have opposite arrow of time.

The sign of entropy for a Coulomb type interaction potential is always positive for the identification of T_{gr} as the normal component of gravitational acceleration whereas T_{gr} can be negative. If T_{gr} corresponds to the magnitude of the acceleration, entropy is negative for repulsive Coulomb interaction.

6.1.5 Negative temperatures/entropies for virtual bosons and the spontaneous generation of the arrow of time in ZEO

Negative entropies/temperatures are especially interesting from the point of ZEO in which causal diamonds (CDs) containing positive and negative energy states at their future and past light-like boundaries. Note that the term CD is used somewhat loosely about Cartesian product of CD and CP_2. Note also that CD is highly analogous to Penrose diagram and defines causal unit in quantum TGD.

There is also a fractal hierarchy CDs within CDs and the minimum number theoretically motivated assumption is that the scales of CDs come as integer multiples of CP_2 scale. Poincare transforms of CDs with respect to another tip are allowed and the position of the second tip with respect to the first one is quantized for number theoretic reasons and corresponds to a lattice like structure in the proper time constant hyperboloid of M^4_+. This has some highly non-trivial cosmological implications such as quantization of cosmic redshifts for which there is empirical evidence.

Both definitions lead to very similar predictions.

1. The identification of temperature T_{gr} as a scalar defined by the length of second fundamental form is favored in TGD framework. Entropy is defined in terms of interaction energy by the

formula $S = -V/T_{gr}$. This definition can be defended in TGD Universe by Poincare invariance and general coordinate invariance. In this case temperature is always non-negative and entropy is positive for attractive interactions but negative for repulsive interactions. Therefore systems consisting mostly from matter or antimatter and having repulsive electromagnetic Coulomb forces have negative entropy and should be thermodynamically unstable. This would suggest that the arrow of time for these systems could be non-standard one in ZEO. For charge neutral systems entropy can be positive. It does not matter whether the system consists of matter or antimatter.

2. Second definition differs from the first one only in that T_{gr} is the magnitude of acceleration with a sign factor telling whether repulsion or attraction is in question. In this temperature can have both signs but entropy is always non-negative. For systems consisting dominantly of matter or antimatter with long range Coulomb interactions the temperature would be negative but entropy positive. This would suggest that the arrow of geometric time is non-standard one. Again it does not matter whether the system consists of matter or antimatter.

The obvious idea is that the thermal instability could imply matter antimatter asymmetry. The original argument that antimatter and matter would correspond to opposite arrows of geometric time turned out to be wrong. One can however modify the argument to state that thermal instability leads to a generation of regions consisting preferentially of matter and antimatter and having non-standard arrow of geometric time so that from the point of view of standard arrow of geometric time these region are formed rather than decay as second law would dictate. For definiteness the definition of geometric temperature as trace of the second fundamental form is assumed but the argument can be easily modified to the second case.

1. Does the negative entropy mean that the time evolutions assignable to the systems consisting mostly of matter (or antimatter) obeys opposite arrow of geometric time? From the point of view of observer with standard arrow of time these systems would obey second law in reverse direction of the geometric time. Spontaneous self assembly of biomolecules representes a standard example about this and the interpretation would be in terms of formation of structures consisting preferentially of matter or antimatter. Could this lead to a separation of antimatter to separate domains in the final states identifiable as negative energy parts of zero energy states? If so then matter matter antimatter asymmetry would relate to the purely geometric thermodynamics.

2. The arrow of geometric time emerges spontaneously in TGD Universe by a not too well-understood mechanism involving arguments from TGD inspired theory of consciousness. Thermodynamics must be involved since the sequence of quantum jumps identified as moments of consciousness induces the arrow of experienced time. Could it be that the arrow of geometric time is opposite for charged particles and antiparticles from thermodynamic stability?

   (a) Quite generally, positive energy parts of at past boundary of CD energy states would have definite particle number and also other quantum numbers whereas the outcome of measurement at future boundary dictated by $M-$matrix would be a superposition of final states at the opposite end of CD. This defines the arrow of geometric time as the direction of geometric time induced by that of thermodynamical time and experienced time defined in terms of a sequence of quantum jumps.

   (b) What mechanism selects the light-like boundary of CD which corresponds to the initial prepared states with second one identified in terms of the outcome of the scattering process expressible as a superpositions of states with well defined particle numbers and also other quantum numbers? The mechanism should relate to the square root of density matrix appearing in $M-$matrix and therefore to entropy of virtual bosons consisting of basic building blocks which are on mass shell massless particles with both signs of energy assignable to what I call wormhole throats to be discussed below.

   (c) The wrong sign of the entropy for systems consisting predominantly of matter or antimatter means they have must have negative energies and second law realized as properties of zero energy states would correspond to opposite direction of geometric time allowed in TGD based generalization of thermodynamics. The arrow would emerge at scales longer than wavelength and would be therefore a macroscopic phenomenon if wavelength is taken as the borderline between microscopic and macroscopic.

6.1.6 How to circumvent the difficulties in generalizing EG to relativistic situation

One argument against EG is that it applies as such to Newtonian gravity only. The general coordinate invariant and Lorentz invariant definitions of T and S have been already considered and are favore in TGD framework and give always non-negative temperature depending only on purely local data. In the following the original definitions of T and S involving equi-potential surfaces in the case of T are considered.

To make the generalization in a coordinate invariant manner two physically preferred coordinates defined modulo diffeomorphism are required: time coordinate t allowing to identify gravitational scalar potential Φ_{gr} as the deviation of g_{tt} from unity and radial coordinate orthogonal to the equipotential surfaces of Φ_{gr} so that Φ_{gr} itself could be regarded as the second preferred coordinate. This requires a slicing of space-time by 2-dimensional surfaces parametrized by (t,r) with remaining space-time coordinates regarded as constant for a given slice. The other two coordinates could define a dual slicing.

Entropy density $s_{gr} = dS_{gr}/dA$ per unit are of flux tube would be proportional to Φ_{gr} and a unique physical identification of the radial coordinate would be as proportional to the entropy $S_{gr}(r) = \int s_{gr} dA \propto \int \Phi_{gr} dA$ proportional to radial coordinate in the Newtonian limit. One should somehow specify what one precisely means with flux tube and here the area could be identified as the area inside which Kähler magnetic flux has definite sign if monopole flux is involved. If not then Kähler flux could take this role.

A possible identification of the preferred coordinates (t,r) is in terms of stringy slicing of the space-time surface by 2-D surfaces required by general consistency conditions. Strings would connect the points of partonic 2-surfaces carrying fermion number and the braids defining the orbits of these points would define string world sheets so that a rather concrete concretization of TGD as almost topological QFT would be obtained.

The physical interpretation of stringy slicing is in terms of integrable distribution of planes of non-physical polarization directions assignable to massless fields and orthogonal dual slicing would correspond to the directions of physical polarizations. The existence of the stringy slicing is motivated also by number theoretical considerations. The general ansatz for the preferred extremals leads to an identification of time preferred coordinate as a coordinate associated with the flow lines of conserved currents defining Beltrami flow. Second stringy coordinate could correspond to the direction for the gradient of gravitational field $\Phi_{gr} = g_{tt} - 1$ in accordance with the idea that gravitons flow along string like flux tubes and the polarizations of gravitons are orthogonal to the direction of their motion.

The translation of Witten's ideas about knots to TGD framework lead to the string worlds sheets could correspond to inverse images of geodesic spheres of CP_2 for the imbedding map of space-time surface to CP_2. This would conform with the idea that wormhole throats are magnetic monopoles at the ends of stringy flux tubes.

6.2 The Conceptual Framework Of TGD

There are several reasons to expect that something analogous to thermodynamics results from quantum TGD. The following summarizes the basic picture, which will be applied to a proposal about how to quantize (or rather de-quantize!) Einstein-Maxwell system with quantum states identified as the modes of classical WCW spinor field with spinors identifiable in terms of Clifford algebra of WCW generated by second quantized induced spinor fields of H.

1. In TGD framework quantum theory can be regarded as a "complex square root" of thermodynamics in the sense that zero energy states can be described in terms of what I call $M-$matrices which are products of hermitian square roots of density matrices and unitary $S-$matrix so that the moduli squared gives rise to a density matrix. The mutually orthogonal Hermitian square roots of density matrices span a Lie algebra of a subgroup of the unitary group and the $M-$matrices define a Kac-Moody type algebra with generators proportional to

powers of S assuming that they commute with S. Therefore this algebra acts as symmetries of the theory.

What is nice that this algebra consists of generators multi-local with respect to partonic 2-surfaces and represents therefore a generalization of Yangian algebra. The algebra of $M-$matrices makes sense if causal diamonds (double light-cones) have sizes coming as integer multiples of CP2 size. $U-$matrix has as its rows the $M-$matrices. One can look how much of this structure could make sense in GRT framework.

2. In TGD framework one is forced to geometrize WCW [K16] consisting of 3-surfaces to which one can assign a unique space-time surfaces as analogs of Bohr orbits and identified as preferred extremals of Kähler action (Maxwell action essentially). The 3-surfaces could be identified as the intersections space-time surface with the future and past light-like boundaries causal diamond (CDs analogous to Penrose diagrams). The preferred extremals associated with the preferred 3-surfaces allow to realize General Coordinate Invariance (GCI) and its natural to assign quantum states with these.

   GCI in strong sense implies even stronger form of holography. Space-time regions with Euclidian signature of metric are unavoidable in TGD framework and have interpretation as particle like structure and are identified as lines of generalized Feynman diagrams. The light-like 3-surfaces at which the signature of the induced metric changes define equally good candidates for 3-surfaces with which to assign quantum numbers. If one accepts both identifications then the intersections of the ends of space-time surfaces with these light-like surfaces should code for physics. In other words, partonic 2-surfaces plus their 4-D tangent space-data would be enough and holography would be more or less what the holography of ordinary visual perception is!

   In the sequel the 3-surfaces at the ends of space-time and and light-like 3-surfaces with degenerate 4-metric will be referred to as *preferred 3-surfaces*.

3. WCW spinor fields are proportional to a real exponent of Kähler function of WCW defined as Kähler action for a preferred extremal so that one has indeed square root of thermodynamics also in this sense with Kähler essential one half of Hamiltonian and Kähler coupling strength playing the role of dimensionless temperature in "vibrational" degrees of freedom. One should be able to identify the counterpart of Kähler function also in General Relativity and if one has Einstein-Maxwell system one could hope that the Kähler function is just the Euclidian part of Maxwell action for a preferred extremal and therefore formally identical with the Kähler function in TGD framework. The phase factor from the Minkowskian contribution emerges naturally as one takes complex square root of the Boltzmann factor. The delicacies of this picture are discussed in [K32].

   Fermionic degrees of freedom correspond to spinor degrees of freedom and are representable in terms of oscillator operators for second quantized induced spinor fields [K31]. This means geometrization of fermionic statistics. There is no quantization at WCW level and everything is classical so that one has "quantum without quantum" as far as quantum states are considered.

4. The dynamics of the theory must be consistent with holography. This means that the Kähler action for preferred extremal must reduce to an integral over 3-surface. Kähler action density decomposes to a sum of two terms. The first term is $j^\alpha A_\alpha$ and second term a boundary term reducing to integral over light-like 3-surfaces and ends of the space-time surface. The first term must vanish and this is achieved if the Kähler current j^α is proportional to Abelian instanton current

$$j^\alpha \propto *j^\alpha = \epsilon^{\alpha\beta\gamma\delta} A_\beta J_{\gamma\delta} \tag{6.5}$$

   since the contraction involves A_α twice. This is at least part of the definition of preferred extremal property but not quite enough. Note that in Einstein-Maxwell system without matter j^α vanishes identically so that the action reduces automatically to a surface term.

5. The action would reduce reduce to terms which should make sense at light-like 3-surfaces. This means that only Abelian Chern-Simons term is allowed. This is guaranteed if the weak form of electric-magnetic duality [K31] stating

$$*F^{n\beta} = kF^{n\beta} \tag{6.6}$$

at preferred at light-like throats with degenerate four-metric and at the ends of space-time surface. These conditions reduce the action to Chern-Simons action with a constraint term realizing what I call weak form of electric-magnetic duality. One obtains almost topological QFT since the constraint term depends on metric. This is of course what one wants.

Here the constant k is integer multiple of basic value which is proportional to g_K^2 from the quantization of Kähler electric charge which corresponds to U(1) part of electromagnetic charge. Fractional charges for quarks require $k = ng_K^2/3$. Physical particles correspond to several Kähler magnetically charged wormhole throats with vanishing net magnetic charge but with non-vanishing Kähler electric proportional to the sum $\sum_i \epsilon_i k_i Q_{m,i}$, with $\epsilon_i = \pm 1$ determined by the direction of the normal component of the magnetic flux for i: th throat.

The first guess is that the length of magnetic flux tube associated with the particle is of order Compton length or perhaps corresponds to weak length scale as was the original proposal. The screening of weak isospin can be understood as magnetic confinement such that neutrino pair at the second end of magnetic flux tube screens the weak charged leaving only electromagnetic charge. Also color confinement could be understood in terms of flux tubes of length of order hadronic size scales. Compton length hypothesis is enough to understand color confinement and weak screening.

Note that $1/g_K^2$ factor in Kähler action is compensated by the proportionality of Chern-Simons action to g_K^2. This need not mean the absence of non-perturbative effects coming as powers of $1/g_K^2$ since the constraint expressing electric magnetic duality depends on g_K^2 and might introduce non-analytic dependence on g_K^2.

6. In TGD the space-like regions replace black holes and a concrete model for them is as deformations of CP_2 type vacuum extremals which are just warped imbeddings of CP_2 to $M^4 \times CP_2$ with random light-like random curve as M^4 projection: the light-like randomness gives Virasoro conditions. This reflects as a special case the conformal symmetries of light-like 3-surfaces and those assignable to the light-like ends of the CDs.

One could hope that this picture more or less applies for the GRT limit of quantum TGD.

6.3 What One Obtains From Quantum TGD By Replacing Space-Times As Surfaces With Abstract 4-Geometries?

It is interesting to see what one obtains when one applies TGD picture by replacing space-times as 4-surfaces with abstract geometries as in Einstein's theory and assumes holography in the sense that space-times satisfy besides Einstein-Maxwell equations also conditions guaranteeing Bohr orbit like property. The resulting picture could be also regarded as GRT type limit of quantum TGD obtained by dropping the condition that space-times are surfaces.

GRT is a more general theory than TGD in the sense that much more general space-times are allowed than in TGD - this leads also to difficulties - and one could also argue that the mathematical existence of WCW Kähler geometry actually forces the restriction of these geometries to those imbeddable in $M^4 \times CP_2$ so that the quantization of GRT type theory would lead to TGD.

6.3.1 What one wants?

What one wants is at least following.

1. Euclidian regions of the space-time should reduce to metrically deformed pieces of CP_2. Since CP_2 spinor structure does not exist without the coupling of the spinors to Kähler gauge potential of CP_2 one must have Maxwell field. CP_2 is gravitational instanton and

constant curvature space so that cosmological constant is non-vanishing unless one adds a constant term to the Maxwell action, which is non-vanishing only in Euclidian regions. It is matter of taste, whether one regards V_0 as term in Maxwell action or as cosmological constant term in gravitational part of the action. CP_2 radius is determined by the value of this term so that it would define a fundamental constant.

This raises an interesting question. Could one say that one has a small value of cosmological constant defined as the average value of cosmological constant assignable to the Euclidian regions of space-time? The average value would be proportional to the fraction of 3-space populated by Euclidian regions (particles and possibly also macroscopic Euclidian regions). The value of cosmological constant would be positive as is the observed value. In TGD framework the proposed explanation for the apparent cosmological constant is different but one must remain open minded. In fact, I have proposed the description in terms of cosmological constant also as a proper description in the approximation to TGD provided by GRT like theory. The answer to the question is far from obvious since the cosmological constant is associated with Euclidian rather than Minkowskian regions: all depends on the boundary conditions at the wormhole throats where the signature of the metric changes.

2. One can also consider the addition of Higgs term to the action in the hope that this could allow to get rid of constant term which is non-vanishing only in Euclidian regions. It turns turns out that only free action for Higgs field is possible from the condition that the sum of Higgs action and curvature scalar reduces to a surface term and that one must also now add to the action the constant term in Euclidian regions. Conformal invariance requires that Higgs is massless.

   The conceptual problem is that the surface term from Higgs does not correspond to topological action since it is expressible as as flux of $\Phi\nabla\Phi$. Hence the simplest possibility is that Kähler action contains a constant term in Euclidian regions just as in TGD, where curvature scalar is however absent. Einstein-Maxwell field equations however apply that it vanishes and is effectively absent also in GRT quantized like TGD.

3. Reissner-Nordström solutions are obtained as regions exterior to CP_2 type regions. In black hole horizons (when they exist) the 3- metric becomes light-like but 4-metric remains non-degenerate. Hence R-N solution cannot be directly glued to a deformed CP_2 type region at horizon but a transition region in which the determinant of 4-metric becomes zero must be present. The simplest possibility is that R-N metric is deformed slightly so that one has $g_{tt} = 0$ and $g_{rr} < \infty$ at the horizon. This surface would correspond to a wormhole throat in TGD framework. Most of the blackhole interior would be replaced with CP_2 type region. In TGD black hole solutions indeed fail to be imbeddable at certain radius so that deformed CP_2 type vacuum extremal is much more natural object than black hole. In the recent framework the finite size of CP_2 means that macroscopic size for the Euclidian regions requires large deformation of CP_2 type solution. For masses $M < Q/\sqrt{G}$ R-N metric has no horizons so that in the case of elementary particles the situation is more complex than this.

   Remark: In TGD framework large value of $\hbar$ and space-time as 4-surface property changes the situation. The generalization of Nottale's formula for gravitational Planck constant in the case of self gravitating system gives $\hbar_{gr} = GM^2/v_0$, where $v_0/c < 1$ has interpretation as velocity type parameter perhaps identifiable as a rotation velocity of matter in black hole horizon [K23, K19]. This gives for the Compton length associated with mass M the value $L_C = \hbar_{gr}/M = GM/v_0$. For $v_0 = c/2$ one obtains Scwartschild radius as Compton length. The interpretation would be that one has CP_2 type vacuum extremal in the interior up to some macroscopic value of Minkowski distance. One can whether even the large voids containing galaxies at their boundaries could correspond to Euclidian blackhole like regions of space-time surface at the level of dark matter.

4. The geometry of CP_2 allows to understand standard model symmetries when one considers space-times as surfaces [K17]. This is not necessarily the case for GRT limit.

   (a) In the recent case one has different situation color quantum numbers make sense only inside the Euclidian regions and momentum quantum numbers in Minkowskian regions.

This is in conflict with the assumption that quarks can carry both momentum and color. On the other, color confinement could be used to argue that this is not a problem.

(b) One could assume that spinors are actually 8-component $M^4 \times CP_2$ spinors but this would be somewhat ad hoc assumption in general relativistic context. Also the existence of this kind of spinor structure is not obvious for general solutions of Einstein-Maxwell equations unless one just assumes it.

(c) It is far from clear whether the symplectic transformations of CP_2 could be interpreted as isometries of WCW in general relativity like theory [K16, K6, K13]. These symmetries certainly act in non-trivial manner on Euclidian regions but it is highly questionable whether this could give rise to a genuine symmetry. Same applies to Kac-Moody symmetries assigned to isometries of $M^4 \times CP_2$ in TGD framework. These symmetries are absolutely essential for the existence of WCW Kähler geometry in infinite-D context as already the uniqueness of the loop space Kähler geometries demonstrates [A5] (maximal group of isometries is required by the existence of Riemann connection).

6.3.2 What GRT limit of TGD could mean?

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

This picture reduces at classical level Equivalence Principle to its formulation in GRT - Einstein's equations for the effective metric of M^4. Similar description applies to gauge interactions. One can also consider other limits. At elementary particle levels CP_2 would be natural background for GRT type theory with cosmological constant. Also cosmic strings $X^2 \times S^2$, where X^2 is string orbit in M^4 and S^2 a homologically non-trivial geodesic sphere in CP_2 can serve as backgrounds for GRT like theory. Now one has two cosmological constants.

At quantum level the equality of gravitational and inertial masses reduces to quantum classical correspondence [K28]. A further approach reduces EP to strong form of general coordinate invariance (GCI) implying strong form of holography. The equivalence of super-conformal representations associated with space-like and light-like 3-surfaces implies equality of corresponding four-momenta and if they can be identified as inertial and gravitational momenta, EP follows. A more convincing manner to obtain EP is by identifying classical Kähler four-momentum with the eigenvalues of quantal four-momentum associated with Kähler-Dirac action.

It seems that this picture can cope with the most obvious objections.

1. One could argue that GRT limit does not make sense since in Minkowskian regions the theory knows nothing about the color and electroweak quantum numbers: there is only metric and Maxwell field. This is true for single space-time sheet but many-sheetedness changes the situation. The interpretation of GRT limit as effective theory lumping the many-sheeted space-time to M^4 with effective metric and effective gauge fields indeed resolves this problem.

2. One could also argue that Einstein-Maxwell equations are un-realistic since also weak and color gauge fields are present as also particles represented as regions allowing representation as map from $M^4 \rightarrow CP_2$. One can however argue that in TGD one has color confinement and weak screening by magnetic confinement so that in macroscopic scales this approach might be realistic. If the functional integral over Euclidian regions representing generalized Feynman diagrams is enough to construct scattering amplitudes, pure Einstein-Maxwell system in Minkowskian regions might be enough. All experimental data is expressible in terms of classical em and gravitational fields. If Weinberg angle vanishes in Minkowskian regions, electromagnetic field reduces to Kähler form and the interpretation of the Maxwell field as em field should make sense. The very tight empirical constraints on the value of Kähler coupling strength α_K indeed allow its identification as fine structure constant at electron length scale.

3. One can also worry about the almost total disappearance of the metric from the basic TGD. This is not a problem in TGD framework since all elementary particles correspond to many-fermion states. For instance, gauge bosons are identified as pairs of fermion and anti-fermion associated with opposite throats of a wormhole connecting two space-time sheets with Minkowskian signature of the induced metric. Similar picture should make sense also now.

4. TGD possesses also approximate super-symmetries and one can argue that also these symmetries should be possessed by the GRT limit. All modes of induced spinor field generate a badly broken SUSY with rather large value of $\mathcal{N}$ (number of spinor modes) and right-handed neutrino and its antiparticle give rise to $\mathcal{N} = \in$ SUSY with R-parity breaking induced by the mixing of left- and right handed neutrinos induced by the Kähler-Dirac equation. This picture is consistent with the existing data from LHC and there are characteristic signatures -such as the decay of super partner to partner and neutrino- allowing to test it. These super-symmetries might make sense if one replaces ordinary space-time spinors with 8-D spinors.

   Note that the possible inconsistency of Minkowskian and Euclidian 4-D spinor structures might force the use of 8-D Minkowskian spinor structure.

6.3.3 Basic properties of Reissner-Nordström metric

Denote the coordinates of M^4_+ by (m^0, r_M, θ, ϕ) and those of X^4 by (t, r_M, θ, ϕ). The expression for Reissner-Nordström metric reads as

$$\begin{aligned} ds^2 &= Adt^2 - Bdr_M^2 - r_M^2 d\Omega^2 \ , \\ A &= 1 - \frac{r_s}{r_M} + \frac{r_Q^2}{r_M^2} \ , \quad B = \frac{1}{A} \ , \\ r_s &= 2GM \ , \quad r_Q^2 = Q^2 G \ . \end{aligned} \tag{6.7}$$

Here the charge $Q^2 = g^2 q^2 = 4\pi\alpha\hbar q^2$ contains gauge coupling g for the Maxwell field. For Kähler field one would have $g = g_K$.

The metric has two horizons for large enough mass values corresponding to the vanishing of function A implying that the sphere at which the vanishing takes place becomes metrically effectively 2-dimensional light-like 3-surface analogous to the boundary of light-cone. Note however that the determinant of the 4-metric is non-vanishing but just the finiteness of the radial component of the metric (something rather natural) would make it vanishing if g_{tt} remains zero. The horizon radii are given by

$$r_{\pm} = \frac{r_s}{2}\left[1 \pm \sqrt{1-(\frac{r_Q}{r_s})^2}\right] . \tag{6.8}$$

$r_{\pm}$ is real for

$$M \geq M_Q = \frac{Q}{\sqrt{G}} . \tag{6.9}$$

For smaller masses one has no horizons and naked singularity at origin. The imbeddability condition however implies that the imbedding fails below some critical radius.

Some general comments about the relation to TGD are in order [K28].

1. Reissner-Nordström metric has imbedding as a vacuum extremal but not as non-vacuum extremal for which induced Kähler field would appear as Maxwell field. Vacuum extremals which are very important in TGD framework have no counterpart in Maxwell-Einstein system, which forces to question the assumption that Einstein-Maxwell system could serve as a GRT type limit of TGD except at macroscopic scales defined by the mass condition.

2. The solution is not expected to describe the exterior metric of objects with $M < M_Q$ at short distances. For elementary particles one expects different space-time correlates of gravitational interaction. One might optimistically guess that this is the realm where TGD replaces General Relativity.

3. The determinant of the four metric is non-vanishing at the horizons so that they cannot correspond to wormhole throats. There must be a transition region within which the determinant of the metric goes to zero at both Euclidian and Minkowskian region. The transition region could be around either horizon of the Reissner-Nordström metric when these exist. For elementary particles the situation is different since R-N metric has no horizon in this case. The critical mass corresponds to a condensed matter blob with size scale of living cell and one can ask whether it might be possible to test experimentally whether something happens in the transition region.

4. Non-vacuum extremals of Kähler action are relevant near wormhole throats and an interesting and the behavior of radially symmetric extremal of Kähler action with induced Kähler form defining the Maxwell field is still an open question. This kind of extremal would serve as the first guess for a model of the exterior space-time of elementary particle but could be quite too simple. In fact, the light-likeness of wormhole throats suggests a more complex zitterbegung like behavior so that stationarity and spherical symmetry would be quite too strong conditions on the metric.

It is interesting to apply the formula for the gravitational Planck constant [K23] to the lower bound for M. The formula reads as

$$\hbar_{gr} = \frac{GMm}{v_0} , \quad \frac{v_0}{c} < 1 . \tag{6.10}$$

The parameter v_0 has dimensions of velocity and for the space-time sheets mediating gravitational interaction between Sun and the three inner planets one has $v_0 \simeq 2^{-11}$. By writing the expression for M_Q as $M_Q = q\sqrt{\alpha_K}\hbar_{gr}\sqrt{G}$, where α_K can be assumed to be equal to fine structure constant, one finds that horizons exist only if the condition

$$q \leq \frac{v_0}{\sqrt{G}m\sqrt{\alpha_K}} . \tag{6.11}$$

Therefore solar system would represent a genuine elementary particle like realm in which Reissner-Nordstöm like metric does not apply unless the electromagnetic charge is so small that it vanishes by its quantization, which is of course a non-realistic condition. This idealized argument suggests the smallness of the electric charge as a condition for the applicability of GRT type description and this indeed guarantees that space-time sheets are near vacuum extremals so that small deformation of Scwartschild metric should apply.

6.3.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{6.12}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [K1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K9, K22]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. Einstein-Maxwell limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

6.3.5 Preferred extremal property for Einstein-Maxwell system

Consider now the preferred extremal property defined to be such that the action reduces to Chern-Simons action at space-like 3-surfaces at the ends of space-time surface and at light-like wormhole throats.

1. In Maxwell-Einstein system the field equations imply

$$j^\alpha = 0 \ . \tag{6.13}$$

   so that the Maxwell action for extremals reduces automatically to a surface term assignable to the preferred 3-surfaces. Note that Higgs field could in principle serve as a source of Kähler field but its presence does not look like a good idea since it is not present in the field equations of TGD and because the resulting boundary term is not topological.

2. The condition

$$J = k \times *J \tag{6.14}$$

at preferred 3-surfaces guarantees that the surface term from Kähler action reduces to Abelian Chern-Simons term and one has hopes about almost topological QFT.

Since CP_2 type regions carry magnetic monopole charge and since the weak form of electric-magnetic duality implies that electric charge is proportional to the magnetic charge, one has electric charge without electric charge as Wheeler would express it. The identification of elementary building blocks as magnetic monopoles leads in TGD context to the picture about particle as Kähler magnetic flux tubes having opposite magnetic charges at their ends. It is not quite clear what the length of the tubes is. One possibility is Compton length and second possibility is weak length scale and the color confinement length scale. Note that in TGD the physical charges reside at the wormhole throats and correspond to massless fermions.

3. CP_2 is constant curvature space and satisfies Einstein equations with cosmological constant. The simplest manner to realize this is to add to the action constant volume term which is non-vanishing only in Euclidian regions. This term could be also interpreted as part of Maxwell action so that it is somewhat a matter of taste whether one speaks about cosmological constant or not. In any case, this would mean that the action contains a constant potential term

$$V = V_0 \times \frac{(1+sign(g))}{2} \ , \tag{6.15}$$

where $sign(g) = -1$ holds true in Minkowskian regions and $sign(g) = 1$ holds true in Euclidian regions.

Note that for a piece of CP_2 V_0 term can be expressed is proportional to Maxwell action and by self-duality this is proportional to instanton action reducible to a Chern-Simons term so that V_0 is indeed harmless from the point of view of holography.

4. For Einstein-Maxwell system with similar constant potential in Euclidian regions curvature scalar vanishes automatically as a trace of energy momentum tensor so that no interior or surface term results and the only surface term corresponds to a pure Chern-Simons term for Maxwell field. This is exactly the situation also in quantum TGD. The constraint term guaranteeing the weak form of electric-magnetic duality implies that the metric couples to the dynamics and the theory does not reduce to a purely topological QFT.

5. In TGD framework a non-trivial theory is obtained only if one assumes that Kähler function corresponds apart from sign to either the Kähler action in the Euclidian regions or its negative in Minkowskian regions. This is required also by number theoretic vision. This implies a beautiful duality between field descriptions and particle descriptions.

This also guarantees that the Kähler function reducing to Chern-Simons term is negative definite: this is essential for the existence of the functional integral and unitarity of the theory. This is due to the fact that Kähler action density as a sum of magnetic and electric energy densities is positive definite in Euclidian regions. This duality would be very much analogous to that implied by the possibility to perform Wick rotation in QFTs. Therefore it seems natural to postulate similar duality also in the proposed variant of quantized General Relativity.

6. The Kähler function of the WCW would be given by Chern-Simons term with a constraint expressing the weak form of electric-magnetic duality both in TGD and General Relativity. One should be able regard also in GRT framework WCW as a union of symmetric spaces with Kähler structure possessing therefore a maximal group of isometries. This is an absolutely essential prerequisite for the existence of WCW Kähler geometry. The symmetric spaces in

the union are labelled by zero modes which do not contribute to the line element and would represent classical degrees of freedom essential for quantum measurement theory. In TGD the induced CP_2 Kähler form would represents such degrees of freedom and the quantum fluctuating degrees of freedom would correspond to symplectic group of $\delta M^4_\pm \times CP_2$.

The difference between TGD and GRT would be that light-like 3-surfaces for all possible space-times containing Euclidian and Minkowskian regions would be considered for GRT type theory. In TGD these space-times are representable as surfaces of $M^4 \times CP_2$. In TGD framework the imbeddability assumption is crucial for the mathematical existence of the theory since it eliminates space-times with non-physical characteristics. The problem posed by arbitrarily large values of cosmological constants is one of the basic problems solved by this assumption. Also mass density is sub-critical for cosmologies with infinite duration and critical cosmologies are unique apart from their duration and quantum critical cosmologies replace inflationary cosmologies.

7. Note that one could consider assigning the gravitational analog of Chern-Simons term with the preferred 3-surfaces: this kind of term is discussed by Witten in this classic work about Jones polynomial. This term is a non-abelian version of Chern-Simons term and one must replace curvature tensor with its contraction with sigma matrices so that 4-D spinor structure is necessarily involved. The objection is that this term contains second derivatives. In TGD spinor structure is induced from that of $M^4 \times CP_2$ and this kind of term need not make sense as such since gamma matrices are expressed in terms of imbedding space gamma matrices: among other things this resolves the problems caused by the non-existence of spinor structure for generic 4-geometries. The coupling to the metric however results from the constraint term expressing weak form of electric-magnetic duality.

   The difference between TGD and GRT would be basically due to the factor of scattering amplitudes coming from the duality expressing electric-magnetic duality and due to the fact that induced metric in terms of H-coordinates and Maxwell potential is expressible in terms of CP_2 coordinates. The latter implies topological field quantization and many-sheeted space-time crucial for the interpretation of quantum TGD.

6.3.6 Could the action contain also Higgs part?

One could criticize Maxwell-Einstein action with cosmological constant non-vanishing only in Euclidian regions and ask whether a coupling to Higgs field could change the situation. This is not the case.

1. If the action contains also Higgs part, Einstein-Higgs part of the action must reduce to a surface term. The trace $G^{\alpha\beta}$ equals to the trace of the Higgs energy momentum tensor and one obtains

$$-kG = kR = -T \ ,$$

   and

$$T = -(\nabla\Phi)^2 + 4V(\Phi) = -L_H + 2V(\Phi) \ .$$

   This gives

$$L_H + kR = 2L_H - 2V(\Phi) \ .$$

2. The kinetic term of Higgs field can be written as

$$(\nabla\Phi)^2 = \nabla \cdot (\Phi\nabla\Phi) - \Phi\nabla^2\Phi \ .$$

   The first term reduces to a surface term and second term can be expressed as

$$\Phi\nabla^2\Phi = -\Phi\frac{\partial V}{\partial \phi} \ .$$

Similar formula applies also if the number of Higgs components is higher than one.

The condition that only the surface term remains gives

$$-2V + \Phi\frac{\partial V}{\partial \Phi} = 0$$

giving

$$V(\Phi) = \frac{m^2}{2}\Phi^2 \ . \tag{6.16}$$

3. The presence of constant term in V does not matter in field equations for Φ so that one can have

$$V(\Phi) = V_0 + \frac{m^2}{2}\Phi^2 \ . \tag{6.17}$$

In order to have both CP_2 like Euclidian regions and Reissner-Nordström type exterior solutions one must allow the Higgs potential to depend on the signature of the metric so that for massless Higgs favored by conformal invariance one would have

$$V(\Phi) \quad = \quad V_0 \times \frac{(1 + sign(g))}{2} \ , \tag{6.18}$$

where one has $sign(g) = -1$ for Minkowskian regions and $sign(g) = 1$ for Euclidian regions. V_0 would be a constant of nature coding for CP_2 radius about 10^4 Planck lengths.

Since the introduction of Higgs field does not allow to circumvent the introduction of a term having interpretation in terms of cosmological constant and since one loses topological QFT property, it seems that the idea about Higgs is not good.

6.3.7 Could ZEO and the notion of CD make sense in GRT framework?

The notion of CD is crucial in ZEO and one can ask whether the notion generalizes to GRT context. In the previous arguments related to EG the notion of ZEO plays a fundamental role since it allows to replace $S-$matrix with $M-$matrix defining "complex square root" of density matrix.

1. In TGD framework CDs are Cartesian products of Minkowskian causal diamonds of M^4 with CP_2. The existence of double light-cones in curved space-time would be required and its is not clear whether this makes sense generally. TGD suggest that the scales of these diamonds defined in terms of the proper time distance between the tips are integer multiples of CP_2 scale defined in terms of the fundamental constant V_0 (the more restrictive assumption allowing only 2^n multiples would explain p-adic length scale hypothesis but would not allow the generalization of Kac-Moody algebra spanned by M-matrices). The difference between boundaries of GRT CDs and wormhole throats would be that four-metric would not be degenerate at CDs.

2. The conformal symmetries of light-cone boundary and light-like wormhole throats generalize also now since they are due to the metric 2-dimensionality of light-like 3-surfaces. It is however far from clear whether one can have anything something analogous to conformal variants of symplectic algebra of $\delta M^4_\pm \times CP_2$ and isometry algebra of $M^4 \times CP_2$.

Could one perhaps identify four-momenta as parameters associated with the representations of the conformal algebras involved? This hope might be unrealistic in TGD framework: the basic idea behind TGD indeed is that Poincare invariance lost in GRT is retained if space-times are surfaces in $H = M^4 \times CP_2$. The reason is that that super-Kac-Moody symmetries correspond to localized isometries of H whereas the super-conformal algebra associated with the symplectic group is assignable to the light-like boundaries $\delta M^4_\pm \times CP_2$ of CD of H rather than space-time surface.

3. One could of course argue that some physical conditions on GRT -most naturally just the highly non-trivial mathematical existence of WCW Kähler geometry and spinor structure-could force the representability of physically acceptable 4-geometries as surfaces $M^4 \times CP_2$. If so, then also CDs would the same CDs as in TGD and quantization of GRT would lead to TGD and all the huge symmetries would emerge from quantum GRT alone.

   The first objection is that the induced spinor structure in TGD is not consistent with that natural in GRT. Second objection is that in TGD framework Einstein-Maxwell equations are not true in general and Einstein's equations can be assumed only in long length scales for the vacuum extremals of Kähler action. The Einstein tensor would characterize the energy momentum tensor assignable to the topologically condensed matter around these vacuum extremals and neither geometrically nor topologically visible in the resolution defined by very long length scale. If Maxwell field corresponds to em field in Minkowskian regions, the vacuum extremal property would make sense in scales where matter is electromagnetic neutral and em radiation is absent.

6.4 What Can One Conclude?

The previous considerations suggest that a surprisingly large piece of TGD can be applied also in GRT framework and raise the possibility about quantization of Einstein-Maxwell system in terms of Kähler geometry of WCW consisting of 3-geometries instead of 3-surfaces. One can even consider a new manner to understand TGD as resulting from the quantization of GRT in terms of WCW Kähler geometry in the space of 3-metrics realizing holography and making classical theory an exact part of quantum theory. Since the space-times allowed by TGD define a subset of those allowed by GRT one can ask whether the quantization of GRT leads to TGD or at least sub-theory of TGD. The arguments represented above however suggest that this is not the case. The generalization of S-matrix to a complex of U-matrix, S-matrix and algebra of M-matrices forced by ZEO gives a natural justification for the modification of EG allowing gravitons and giving up the rather nebulous idea about emergent space-time. Whether ZEO crucial for EG makes sense in GRT picture is not clear. A promising signal is that the generalization of EG to all interactions in TGD framework leads to a concrete interpretation of gravitational entropy and temperature, to a more precise view about how the arrow of geometric time emerges, to a more concrete realization of the old idea that matter antimatter asymmetry could relate to different arrows of geometric time (not however for matter and antimatter but for space-time sheets mediating attractive and repulsive long range interactions), and to the idea that the small value of cosmological constant could correspond to the small fraction of non-Euclidian regions of space-time with cosmological constant characterized by CP_2 size scale.

7 Emergent gravity and dark Universe

Eric Verlinde has published article with title *Emergent Gravity and the Dark Universe* [B5] (see `http://tinyurl.com/grwc3fz`). The article represents his recent view about gravitational force as thermodynamical force described earlier in [B2] and suggests an explanation for the constant velocity spectrum of distant stars around galaxies and for the recently reported correlation between the real acceleration of distant stars with corresponding acceleration caused by baryonic matter [E10]. In the following I discuss Verlinde's argument and compare the physical picture with that provided by TGD. I have already earlier discussed Verlinde's entropic gravity from TGD view point [K28].

Before continuing it is good to recall the basic argument against the identification of gravity as entropic force. The point is that neutron diffraction experiments [B1] suggests that gravitational potential appears in the Schrödinger equation. This cannot be the case if gravitational potential has thermodynamic origin and therefore follows from statistical predictions of quantum theory: to my opinion Verlinde mixes apples with oranges.

7.1 Verlinden's argument

Consider now Verlinde's argument.

1. Verlinde wants to explain the recent empirical finding that the observed correlation between the acceleration of distant stars around galaxy with that of baryonic matter [E10] (see `http://tinyurl.com/jd2m9ll`) in terms of apparent dark energy assigned with entanglement entropy proportional to volume rather than horizon area as in Bekenstein-Hawking formula. This means giving up the standard holography and introducing entropy proportional to volume.

   To achieve this he replaces anti-de-Sitter space (AdS) to which AdS/CFT duality is usually assigned with de-Sitter space(dS) space with cosmic horizon expressible in terms of Hubble constant and assign it with long range entanglement since in AdS only short range entanglement is believed to be present (area law for entanglement entropy). This would give rise to an additional entropy proportional to the volume rather than area. Dark energy or matter would corresponds to a thermal energy assignable to this long range entanglement. One can of course criticize this assumption as ad hoc hypothesis.

2. Besides this Verlinde introduces tensor nets as justification for the emergence of gravitation: this is just a belief. All arguments that I have seen about this are circular (one introduces 2-D surfaces and thus also 3-space from beginning) and also Verlinde uses dS space. What is to my opinion alarming that there is no fundamental approach really explaining how space-time and gravity emerges. Emergence of space-time should lead also to the emergence of spinor structure of space-time and this seems to me something impossible if one really starts from mere Hilbert space.

3. Verlinde introduces also analogy with the thermodynamics of glass involving both short range crystal structure and amorphous long range behaviour that would correspond to entanglement entropy in long scales long range structure. Above the horizon size the contribution proportional to volume would begin to dominate in entropy. Also analogy with elasticity is introduced. Below Hubble scale the microscopic states do not thermalize below the horizon and display memory effects. Dark gravitational force would be analogous to elastic response due to what he calls entropy displacement.

4. Verlinde admits that this approach does not say much about cosmology or cosmic expansion, and even less about inflation.

7.2 The long range correlations of Verlinde correspond to hierarchy of Planck constants in TGD framework

The physical picture has analogies with my own approach [L10] to the explanation of the correlation between baryonic acceleration with observed acceleration of distant stars. In particular, long range entanglement has the identification of dark matter in terms of phases labelled by the hierarchy of Planck constants as TGD counterpart.

1. Concerning the emergence of space and gravitation TGD leads to a different view. It is not 3-space but the experience about 3-space - proprioception -, which would emerge via tensor nets realized in TGD in terms of magnetic flux tubes emerging from 3-surfaces defining the nodes of the tensor net [L9]. This picture leads to a rather attractive view about quantum biology (see for instance `http://tinyurl.com/q4jyoc5`).

2. Twistor lift of TGD has rapidly become a physically convincing formulation of TGD [K39] (see `http://tinyurl.com/zjgmax6`). One replaces space-time surfaces in $M^4 \times CP_2$ with the 12-D product $T(M^4 \times CP_2)$ of the twistor spaces $T(M^4)$ and $T(CP_2)$ and Kähler action with its 6-D variant. This requires that $T(M^4)$ and $T(CP_2)$ have Kähler structure. This is true but only for M^4 (and its variants E^4 and S^4) and CP_2. Hence TGD is completely unique also mathematically and physically (providing a unique explanation for the standard model symmetries). The preferred extremal property for Kähler action could reduce to the property that the 6-D surface as an extremal of 6-D Kähler action is twistor space of space-time surface and thus has the structure of S^2 bundle. That this is indeed the case for the preferred extremals of dimensionally reduced 4-D action expressible as a sum of Kähler action and volume term remains to be rigorously proven.

3. Long range entanglement even in cosmic scales would be crucial and give the volume term in entropy breaking the holography in the usual sense. In TGD framework hierarchy of Planck constants $h_{eff} = n \times h$ satisfying the additional condition $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ (M and m are masses and v_0 is a parameter with dimensions of velocity) is the gravitational Planck constant introduced originally by Nottale [E4], and assignable to magnetic flux tubes mediating gravitational interaction makes [K23, K19, K36]. This makes possible quantum entanglement even in astrophysical and cosmological long length scales since h_{gr} can be extremely large [K36, K37]. In TGD however most of the the galactic dark matter and energy is associated with cosmic strings having galaxies along it (like pearls in necklace) [K24, K7]. Baryonic dark matter could correspond to the ordinary matter which has resulted in the decay of cosmic strings taking the role of inflaton field in very early cosmology. This gives automatically a logarithmic potential giving rise to constant spectrum velocity spectrum modified slightly by baryonic matter and a nice explanation for the correlation, which served as the motivation of Verlinde. In particular, the only parameter of the model is string tension and TGD allows to estimate also this and the value is completely fixed for the ideal cosmic strings and reduces as they thicken.

4. Also glass analogy has TGD counterpart. Kähler action has 4-D spin glass degeneracy giving rise to 4-D spin-glass degeneracy. In twistor lift of TGD cosmological term appears and reduces the degeneracy by allowing only minimal surfaces rather than all vacuum extremals. This removes the non-determinism. Cosmological constant is however extremely small implying non-perturbative behavior in the sense that the volume term for the action is extremely small and depends very weakly on the preferred extremal. This suggests that spin glass in 3-D sense remains as Kähler action with varying sign is added: the space-time regions dominated by electric or magnetic fields give contributions with different sign and one can obtain the characteristic fractal spin glass energy landscape with valleys inside valleys.

5. The mere Kähler action for the Minkowskian (at least) regions of the preferred extremals reduces to a Chern-Simons terms at light-like 3-surfaces at which the signature of the induced metric of the space-time surface changes from Minkowskian to Euclidian. The interpretation could be that TGD is almost topological quantum field theory. Also the interpretation in terms of holography can be considered.

   Volume term proportional to cosmological constant given by the twistorial lift of TGD [K39] (see `http://tinyurl.com/zjgmax6`) could mean a small breaking of holography in the sense that it cannot be reduced to a 3-D surface term. One must however be very cautious here because TGD strongly suggests strong form of holography meaning that data at string world sheets and partonic 2-surfaces (or possibly at their metrically 2-D light-like orbits for which only conformal equivalence class matters) fix the 4-D dynamics.

   Certainly volume term means a slight breaking of the flatness of the 3-space in cosmology since 3-D curvature scalar cannot vanish for Robertson-Walker cosmology imbeddable as a minimal surface except at the limit of infinitely large causal diamond (CD) implying that cosmological constant, which is proportional to the inverse of the p-adic length scale squared, vanishes at this limit. Note that the dependence $\Lambda \propto 1/p$, p p-adic prime, allows to solve the problem caused by the large value of cosmological constant in very early cosmology. Quite generally, volume term would describe finite volume effects analogous to those encountered in thermodynamics.

7.3 The argument against gravitation as entropic force can be circumvented in zero energy ontology

Could TGD allow to resolve the basic objection against gravitation as entropic force or generalize this notion?

1. In Zero Energy Ontology (ZEO) quantum theory can be interpreted as "complex square root of thermodynamics". Vacuum functional is an exponent of the action determining preferred extremals - Kähler action plus volume term present for twistor lift. This brings in gravitational constant G and cosmological Λ constant as fundamental constants besides CP_2 size scale R and Kähler coupling strength α_K [K39]. Vacuum functional would be analogous to an exponent of $E_c/2$, where E_c would be complexified energy. I have also considered the possibility that vacuum functional is analogous to the exponent of free energy but following argument favors the interpretation as exponent of energy.

2. The variation of Kähler action would give rise to a 4-D analog of TdS term and the variation of cosmological constant term to the analog of $-pdV$ term in $dE = TdS - pdV$. Both T and p would be complex and would receive contributions from both Minkowskian and Euclidian regions. The contributions of Minkowskian and Euclidian regions to the action would differ by a multiplication with imaginary unit and it is possible that Kähler coupling strength is complex as suggested in [K11].

   If the inverse of the Kähler coupling is strength is proportional to the zero of Riemann zeta at critical line, it is complex, and the coefficient of the volume term must have the same phase: otherwise space-time surfaces are extremals of Kähler action and minimal surfaces simultaneously. In fact, the known non-vacuum extremals of Kähler action are surfaces of this kind, and one cannot exclude the possibility that preferred extremals have quite generally this property. The physical picture below does not favor this idea.

   Note: One can consider also the possibility that the values of Kähler coupling strength correspond to the imaginary part for the zero of Riemann zeta.

3. Suppose that both terms in the action are proportional to the same phase factor. The part of the variation of the Kähler action with respect to the imbedding space coordinates giving the analog of TdS term would give the analog of entropic force. Since the variation of the entire action vanishes, the variation of Kähler action would be equal to the negative of the variation of the volume term with respect to the induced metric given by $-pdV$. Since the variations of Kähler action and volume term cancel each other, the entropic force would be non-vanishing only for the extremals for which Kähler action density is non-vanishing. The variation of Kähler action contains variation with respect to the induced metric and induced Kähler form so that the sum of gravitational and U(1) force would be equal to the analog of entropic force and Verlinde's proposal would not generalize as such.

   The variation of the volume term gives rise to a term proportional to the trace of the second fundamental form, which is 4-D generalization of ordinary force and vanishes for the vacuum extremals of Kähler action in which case one has analog of geodesic line. More generally, Kähler action gives rise to the generalization of U(1) force on particle so that the field equations give a 4-D generalization of equations of motion for a point like particle in U(1) force having also interpretation as a generalization of entropic force.

4. In Zero Energy Ontology (ZEO) TGD predicts a dimensional hierarchy of basic objects analogous to the brane hierarchy in M-theory: space-time surfaces as 4-D objects, 3-D light-like orbits of partonic 2-surfaces as boundaries of Minkowskian and Euclidian regions plus space-like 3-surfaces defining the ends of space-time surface at the opposite boundaries of CD, 2-D partonic surfaces and string world sheets, and 1-D boundaries of string world sheets. The natural idea is to identify the dynamics D-dimensional objects in terms of action consisting of D-dimensional volume in induced metric and D-dimensional analog of Kähler action. The surfaces at the ends of space-time should be freely choosable apart from the conditions related to to super-symplectic algebra realizing strong form of holography since they correspond to initial values.

For the light-like orbits of partonic 2-surfaces 3-volume vanishes and one has only Chern-Simons type topological term. For string world sheets one has area term and magnetic flux, which is topological term reducing to a mere boundary term so that minimal surface equations are obtained. For the dynamical boundaries of string world sheets one obtains 1-D volume term as the length of string world line and the boundary term from string world sheet. This gives 1-D equation of motion in U(1) force just like in Maxwell's theory but with induced Kähler form defining the U(1) gauge field identifiable as the counterpart of classical U(1) field of standard model. Induced spinor fields couple at boundaries only to the induced em gauge potential since induced classical W-boson gauge fields vanish at string world sheets in order to achieve a well-defined and conserved spinorial em charge (here the absolutely minimal option would be that the W and Z gauge potentials vanish only at the time-like boundaries of string world sheet). Should world-line geometry couple to the induced em gauge field instead of induced Kähler form? The only logical option is however that geometry couples to the U(1) charge perhaps identifiable in terms of fermion number.

5. There however an objection against this picture. All known extremals of Kähler action are minimal surfaces and there are excellent number theoretical arguments suggesting that all preferred extremals of Kähler action are also minimal surfaces so that the original picture would be surprisingly near to the truth. The separate vanishing of variation implies that the solutions do not depend at all on coupling parameters as suggested by number theoretical universality and universality of the dynamics at quantum criticality. The discrete coupling constant evolution makes it however visible via boundary conditions classically. This would however predicts that the analogs to TdS and pdV vanish identically in space-time interior.

   The variations however involve also boundary terms, which need not vanish separately since the actions in Euclidian and Minkowskian regions differ by multiplication with $\sqrt{-1}$! The variations reduce to terms proportional to the normal component of the canonical momentum current contracted with the deformation at light-like 3-surfaces bounding Euclidian and Minkowskian space-time regions. These must vanish. If Kähler coupling strength is real, this implies decoupling of the dynamics due to the volume term and Kähler action also at light-like 3-surfaces and therefore also exchange of charges - in particular four-momentum - becomes impossible. This would be a catastrophe.

   If α_K is complex as quantum TGD as a square root of thermodynamics and the proposal that the spectrum of $1/\alpha_K$ corresponds to the spectrum of zeros of zeta require [K11], the normal component of the canonical momentum current for Kähler action equals to that for the volume term at the other side of the bounding surface. The analog of $dE = TdS - pdV = 0$ would hold true in the non-trivial sense at light-like 3-surfaces and thermodynamical analogy holds true (note that energy is replaced with action). The reduction of variations to boundary terms would also conform with holography. Strong form of holography would even suggest that the 3-D boundary term in turn reduces to 2-D boundary terms.

   A possible problem is caused by the variation of volume term: $\sqrt{g_4}$ vanishes at the boundary and g^{nn} diverges. The overall result should be finite and should be achieved by proper boundary conditions. What I have called weak form of electric-magnetic duality [K38] allows to avoid similar problems for Kähler action, and implies self-duality of the induced Kähler form at the boundary. A weaker form of boundary conditions would state that the sum of the variations of Kähler action and volume term is finite.

Physically this picture is very attractive and makes cosmological constant term emerging from the twistorial lift rather compelling. What is nice that this picture follows from the field equations of TGD rather than from mere heuristic arguments without underlying mathematical theory.

REFERENCES

Mathematics

[A1] Atyiah-Singer index-theorem. Available at: http://en.wikipedia.org/wiki/

Atiyah-Singer_index_theorem.

[A2] Kac-Moody algebra. Available at: http://en.wikipedia.org/wiki/KacMoody_algebra.

[A3] Super Virasoro algebra. Available at: http://en.wikipedia.org/wiki/Super_Virasoro_algebra.

[A4] Yangian symmetry. Available at: http://en.wikipedia.org/wiki/Yangian.

[A5] Freed DS. The Geometry of Loop Groups, 1985.

[A6] Jones V. In and around the origin of quantum groups. Available at: http://arxiv.org/abs/math/0309199, 2003.

Theoretical Physics

[B1] Kobakhidze A. Gravity is not an entropic force. Available at: http://arxiv.org/abs/1009.5414, 2010.

[B2] Verlinde A. On the Origin of Gravity and the Laws of Newton. Available at: http://arxiv.org/abs/1001.0785, 2010.

[B3] Witten E Dolan L, Nappi CR. Yangian Symmetry in $D = 4$ superconformal Yang-Mills theory. Available at: http://arxiv.org/abs/hep-th/0401243, 2004.

[B4] Plefka J Drummond J, Henn J. Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory. Available at: http://cdsweb.cern.ch/record/1162372/files/jhep052009046.pdf, 2009.

[B5] Verlinde E. Emergent gravity and the dark universe. arXiv:hep-th/1611.02269.Available at:http://tinyurl.com/grwc3fz, 2016.

[B6] Arkani-Hamed N et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM. Available at: http://arxiv.org/find/hep-th/1/au:+Bourjaily_J/0/1/0/all/0/1, 2010.

[B7] Pitkänen M. On Verlinde's entropic gravity. *Prespacetime Journal.* Available at: http://www.scribd.com/doc/45928480/PSTJ-V1-10-More-Possible-Games-in-Town-Continued, 1(10):1582–1585, 2010.

[B8] Hossenfelder S. Comments on and comments on comments on verlindes paper on the origin of gravity and the laws of newton. Available at: http://tinyurl.com/ondxowl, 2011.

Particle and Nuclear Physics

[C1] Back BB et al. *Phys Rev Lett* .. Available at: http://arxiv.org/abs/hep-ex/0208004, 89(22), November 2002.

[C2] Derrick M et al. *Phys Lett B*, 315:481, 1993.

[C3] Nesvizhevsky VV et al. Measurement of quantum states of neutrons in the Earth's gravitational field. Available at: http://arxiv.org/abs/hep-ph/0306198, 2003.

Condensed Matter Physics

[D1] Novoselov KS et al. Two-dimensional gas of massless Dirac fermions in graphene. *Nature*, 438, November 2005.

[D2] Glashow SL. Can Sci Save the World? Available at: http://www.hypothesis.it/nobel/nobel99/eng/pro/pro_2.htm, 1999.

Cosmology and Astro-Physics

[E1] Middleditch A. A White Dwarf Merger Paradigm for Supernovae and Gamma -Ray Bursts. Available at: http://arxiv.org/abs/astro-ph/0311484, 2003.

[E2] Durrant CJ. *The Atmosphere of the Sun.* IOP Publishing Ltd., 1988.

[E3] Boggs SE Coburn W. Polarization of Prompt gamma-ray emission from the gamma-ray burst of 6 December 2002. *Nature* . Available at: http://arxiv.org/abs/astro-ph/0305377, May 2003.

[E4] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity. Available at: http://arxiv.org/abs/astro-ph/0310036, 2003.

[E5] Rujula De A Dado S, Dar A. The Supernova associated with GRB030329. Available at: http://arxiv.org/abs/astro-ph/0304106, 2003.

[E6] Hjorth J et al. A very energetic supernova associated with the gamma-ray burst of 29 March 2003. *Nature*, 423, 2003.

[E7] Meikle W et al. *Nature*, 329, 1987.

[E8] Tajmar M et al. Experimental Detection of Gravimagnetic London Moment. Available at: http://arxiv.org/abs/gr-gc0603033, 2006.

[E9] Moshina M. The surface ferrite layer of Sun. Available at: http://www.thesurfaceofthesun.com/TheSurfaceOfTheSun.pdf, 2005.

[E10] Lelli F McGaugh SS and Schombert JM. The radial acceleration relation in rotationally supported galaxies. Available at:https://arxiv.org/pdf/1609.05917v1.pdf, 2016.

[E11] Weinberg S. *Gravitation and Cosmology.* Wiley, New York, 1967.

[E12] Teukolsky SA Shapiro SL. *Black Holes, White Dwarfs and Neutron Stars.* John Wiley, 1986.

[E13] Matos de CJ Tajmar M. Local Photon and Graviton Mass and Its Consequences. Available at: http://arxiv.org/abs/gr-gc0603032, 2006.

Physics of Earth

Fringe Physics

Neuroscience and Consciousness

Books related to TGD

[K1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Towards M-matrix.* Onlinebook. Available at: http://tgdtheory.fi/public_html/pdfpool/append.pdf, 2006.

[K2] Pitkänen M. About Nature of Time. In *TGD Inspired Theory of Consciousness.* Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#timenature, 2006.

[K3] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time.* Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#class, 2006.

[K4] Pitkänen M. Bio-Systems as Super-Conductors: part I. In *Quantum Hardware of Living Matter*. Onlinebook. Available at: http://tgdtheory.fi/public_html/bioware/bioware.html#superc1, 2006.

[K5] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#quthe, 2006.

[K6] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#compl1, 2006.

[K7] Pitkänen M. Cosmic Strings. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cstrings, 2006.

[K8] Pitkänen M. Dark Forces and Living Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#darkforces, 2006.

[K9] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#eegdark, 2006.

[K10] Pitkänen M. Dark Nuclear Physics and Condensed Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#exonuclear, 2006.

[K11] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#fermizeta, 2006.

[K12] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#Planck, 2006.

[K13] Pitkänen M. Does the Modified Dirac Equation Define the Fundamental Action Principle? In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#Dirac, 2006.

[K14] Pitkänen M. General Ideas about Many-Sheeted Space-Time: Part I. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#topcond, 2006.

[K15] Pitkänen M. General Ideas about Many-Sheeted Space-Time: Part II. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#newviews, 2006.

[K16] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#kahler, 2006.

[K17] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Onlinebook. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#mless, 2006.

[K18] Pitkänen M. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#mass3, 2006.

[K19] Pitkänen M. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#qastro, 2006.

[K20] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.#anyontgd, 2006.

[K21] Pitkänen M. Quantum Model for Bio-Superconductivity: II. In *TGD and EEG*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#biosupercondII, 2006.

[K22] Pitkänen M. Quantum Model for Nerve Pulse. In *TGD and EEG*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#pulse, 2006.

[K23] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#astro, 2006.

[K24] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cosmo, 2006.

[K25] Pitkänen M. TGD and Nuclear Physics. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#padnucl, 2006.

[K26] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionb, 2006.

[K27] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory, 2006.

[K28] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#tgdgrt, 2006.

[K29] Pitkänen M. Twistors, N=4 Super-Conformal Symmetry, and Quantum TGD. In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistor, 2006.

[K30] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

[K31] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#cspin, 2006.

[K32] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#infmotives, 2011.

[K33] Pitkänen M. Topological Geometrodynamics: Basic Visions. In *TGD based view about living matter and remote mental interactions*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdlian/tgdlian.html#lianPTGD, 2012.

[K34] Pitkänen M. *p-Adic length Scale Hypothesis*. Onlinebook. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html, 2013.

[K35] Pitkänen M. What are the counterparts of Einstein's equations in TGD? In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#EinsteinTGD, 2013.

[K36] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#qcritdark, 2014.

[K37] Pitkänen M. Quantum gravity, dark matter, and prebiotic evolution. In *Genes and Memes*. Onlinebook. Available at: http://tgdtheory.fi/public_html/genememe/genememe.html#hgrprebio, 2014.

[K38] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#prext, 2015.

[K39] Pitkänen M. From Principles to Diagrams. Onlinebook.Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#diagrams, 2016.

Articles about TGD

[L1] Pitkänen M. About the Nature of Time. Available at: http://tgdtheory.fi/articles/time.pdf, 2008.

[L2] Pitkänen M. CMAP representations about TGD. Available at: http://www.tgdtheory.fi/cmaphtml.html, 2014.

[L3] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: http://www.tgdtheory.fi/tgdglossary.pdf, 2014.

[L4] Pitkänen M. General Coordinate Invariance.Available at: http://www.tgdtheory.fi/webCMAPs/GeneralCoordinateInvariance.html. 2014.

[L5] Pitkänen M. Geometrization of fields.Available at: http://www.tgdtheory.fi/webCMAPs/Geometrizationoffields.html. 2014.

[L6] Pitkänen M. TGD and GRT.Available at: http://www.tgdtheory.fi/webCMAPs/TGDandGRT.html. 2014.

[L7] Pitkänen M. TGD inspired cosmology.Available at: http://www.tgdtheory.fi/webCMAPs/TGDinspiredcosmology.html. 2014.

[L8] Pitkänen M. Zero Energy Ontology (ZEO).Available at: http://www.tgdtheory.fi/webCMAPs/ZeroEnergyOntology(ZEO).html. 2014.

[L9] Pitkänen M. Holography and Quantum Error Correcting Codes: TGD View. Available at: http://tgdtheory.fi/public_html/articles/tensornet.pdf, 2016.

[L10] Pitkänen M. TGD interpretation for the new discovery about galactic dark matter. Available at: http://tgdtheory.fi/public_html/articles/darknew.pdf, 2016.