Coupling Constant Evolution in Quantum TGD

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Contents

Abstract

How to calculate or at least "understand" the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Basically the unability to calculate is of course due to the lack of understanding.

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions providing a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries. p-Adic length scale hypothesis suggests that continuous coupling constant evolution is replaced by discrete p-adic coupling constant evolution and that number theoretical constraints are of crucial importance. These are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules. This was the picture behind previous updating.

Several steps of progress have however occurred since then.

1. A crucial step in progress has been the understanding of how GRT space-time emerges from the many-sheeted space-time of TGD. At classical level Equivalence Principle (EP) follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

2. Second powerful idea is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

   This idea can be formulated more convincingly in terms of a generalization of the AdS/CFT duality to TGD framework motivated by the generalization of conformal symmetry. In full generality this principle would state that all predictions of the theory can be expressed either in terms of classical fields in the interior of the space-time surface or in terms of scattering amplitudes formulated in terms of fundamental fermions defining the building bricks of elementary particles. The implication would that correlation functions can be also identified as those for classical induce gauge and gravitational fields.

3. Third powerful vision inspired by the notion of preferred extremal - I gave up the vision for years as too crazy - is that scattering amplitudes correspond to sequences of computations and that all computations connecting collectings of algebraic objects produce same scattering amplitudes [K24]. All scattering amplitudes could be reduced to minimal tree diagrams by moving the ends of the lines and snipping away the loops. The 8-D generalization of twistor approach to TGD allows to identify the arithmetics as that of super-symplectic Yangian and basic vertices in the construction correspond to product and co-product in Yangian.

4. The fourth new ingredient is the dramatic increase in the understanding of the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchy corresponds to hierarchy of quantum criticalities at which the sub-algebra of super-symplectic algebra with natural conformal structure changes. Sub-algebras are labelled by integer n: the conformal weights of the sub-algebra come as multiples of n. One has infinite number of hierarchies $n_{i+1} = \prod_{k<i+1} m_k$ which relate naturally to the hierarchies of inclusions of hyper-finite factors. The sub-algebra acts as gauge symmetries whereas the other generators of the full algebra fail to do so. Therefore the increase of n means that gauge degrees of freedom become physical ones. One can assign coupling constant evolution also with these hierarchies and

the natural conjecture is that coupling constants for given value of n are renormalization group invariances.

Especially intereting are the implications for the understanding of gravitational binding assuming that strings connecting partonic 2-surfaces are responsible for the formation of bound states. This leads together with the generalization of AdS/CFT corresponds and localization of fermions to string world sheets to a prediction that Kähler action is expressible as string area in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices. This predicts that the size scale of bound states scales as h_{eff} and it is possible to obtain bound states of macroscopic size unlike for ordinary string area action for which their sizes would be given by Planck length.

1 Introduction

In quantum TGD two kinds of discrete coupling constant evolutions emerge. p-Adic coupling constant evolution is with respect to the discrete hierarchy of p-adic length scales and p-adic length scale hypothesis suggests that only the length scales coming as half octaves of a fundamental length scale are relevant here. Second coupling constant evolution corresponds to hierarchy of Planck constants requiring a generalization of the notion of imbedding space. One can assign this evolution with angle resolution in number theoretic approach. It is now clear that the two evolutions can be understood as different aspects of number theoretic evolution defined by a hierarchy of algebraic extensions of rationals.

This picture is inspired by quantum criticality of TGD Universe realized concretely as a hierarchy of supersymplectic symmetry breakings with sub-algebra of the entire super-symplectic algebra with conformal weights coming as n-multiples of those of the entire algebras acting as conformal gauge symmetries. Number theoretic coupling constant evolution is discrete: various coupling constant parameters depend on algebraic extension but are RG invariant for a given extension. Phase transitions between extensions give rise to number theoretic RG evolution. It should be possible to express number theoretic coupling constant evolution in terms of the parameters of extension: such as ramified prime defining p-adic primes (and p-adic length scales) and the degree of the polynomial defining the extension and defining angle resolution.

The continuous coupling constant evolution of quantum field theories follows at GRT limit when many-sheeted space-time is approximated by GRT space-time by replacing sheets with single slightly curved region of Minkowski space with gravitational and gauge fields identified as sums of those for the sheets.

The notion of zero energy ontology allows to justify p-adic length scale hypothesis and formulate the discrete coupling constant evolution at fundamental level. WCW would consists of sectors associated with causal diamonds (CDs) identified as intersections of future and past directed light-cones. If the sizes of CDs come in powers of 2^n, p-adic length scale hypothesis emerges, and coupling constant evolution is discrete provided RG invariance holds true inside CDs for space-time evolution of coupling constants defined in some sense to be defined It is however clear that all integer scalings of CDs are allowed and p-adic length scale hypothesis is prediction rather than input. In this chapter arguments supporting this conclusion are given by starting from a detailed vision about the basic properties of preferred extremals of Kähler action.

1.1 New Ingredients Helping To Understand Coupling Constant Evolution

How to calculate or at least "understand" the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Basically the in-ability to calculate is of course due to the lack of understanding.

ZEO, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions providing a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries. As already mentioned, continuous coupling constant evolution is replaced by a discrete number theoretical coupling constant evolution.

These ideas were seen as the most important pieces of the puzzle. Their combination was thought to make possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

This was the picture behind previous updating. Several steps of progress have however occurred since then.

1. A crucial step in progress has been the understanding of how GRT space-time emerges from the many-sheeted space-time of TGD. At classical level Equivalence Principle (EP) follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

   The coupling constant evolution in QFT sense is in this framework an approximate notion emerging when TGD space-time is replaced with GRT space-time.

2. Second powerful (possibly too strong) idea is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

   This idea can be formulated more convincingly in terms of a generalization of the AdS/CFT duality to TGD framework motivated by the generalization of conformal symmetry. In full generality this principle would state that all predictions of the theory can be expressed either in terms of classical fields in the interior of the space-time surface or in terms of scattering amplitudes formulated in terms of fundamental fermions defining the building bricks of elementary particles. The implication would that correlation functions can be also identified as those for classical induce gauge and gravitational fields.

3. The third ingredient is ZEO leading to a rather concrete picture about the architecture of scattering amplitudes. The basic notions are U-, M, and S-matrix. M-matrix is defined between positive and negative energy parts of zero energy states and essential for the definition of zero energy states. M-matrix is a product of hermitian square root of density matrix and of unitary S-matrix, whose powers corresponds to the standard S-matrix with positive integer exponent taking the role of discretized time. U-matrix is realized betweeen zero energy states and is the analog of unitary time evolution operator acting in the moduli space of CDs and associated zero energy states.

   U-matrix is expressible in M-matrices [K30] so that the basic matrix to be constructed is S-matrix. S-matrix should be constructible by a generalization the twistorial approach possible only for $M^4 \times CP_2$, whose Cartesian factors are the only 4-D manifolds for which twistor spaces are Kähler manifolds [K24]. The huge symmetries and the close analogies with ordinary Grassmann twistorial program raise hopes about quite concrete construction.

4. Fourth powerful vision inspired by the notion of preferred extremal - I gave up the vision for years as too crazy - is that scattering amplitudes correspond to sequences of computations and that all computations connecting collections of algebraic objects produce same scattering amplitudes [K24, K1]. All scattering amplitudes could be reduced to minimal tree diagrams by moving the ends of the lines and snipping away the loops: this means a huge generalization of the duality symmetry of hadronic string models. The 8-D generalization of twistor

approach to TGD allows to identify the arithmetics as that of super-symplectic Yangian and basic vertices in the construction correspond to product and co-product in Yangian.

5. The fifth new ingredient is the dramatic increase in the number theoretical understanding of the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchy corresponds to hierarchy of quantum criticalities at which the sub-algebra of super-symplectic algebra with natural conformal structure changes. Sub-algebras are labelled by integer n: the conformal weights of the sub-algebra come as multiples of n. One has infinite number of hierarchies $n_{i+1} = \prod_{k<i+1} m_k$ which relate naturally to the hierarchies of inclusions of hyper-finite factors. The sub-algebra acts as gauge symmetries whereas the other generators of the full algebra fail to do so. Therefore the increase of n means that gauge degrees of freedom become physical ones. One can assign coupling constant evolution also with these hierarchies and the natural conjecture is that coupling constants for given value of n are renormalization group invariances.

   Especially interesting are the implications for the understanding of gravitational binding assuming that strings connecting partonic 2-surfaces are responsible for the formation of bound states. This leads together with the generalization of AdS/CFT corresponds and localization of fermions to string world sheets to a prediction that Kähler action is expressible as string area in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices. This predicts that the size scale of bound states scales as h_{eff} and it is possible to obtain bound states of macroscopic size unlike for ordinary string area action for which their sizes would be given by Planck length.

6. The original picture was that there are two separate evolutions: one associated with p-adic length scale hierarchy and second associated with angle resolution. It is now clear that these two evolutions can be unified to a number theoretic evolution in terms of increasing complexity of an algebraic extension of rational numbers inducing also the extensions of p-adic number fields. Space-time and quantum physics become adelic. The algebraic extensions are associated with the parameters characterizing partonic 2-surfaces and string world sheets, which by strong form of holography determine space-time surfaces as preferred extremals of Kähler action. In this framework the crucial number theoretical universality necessary for adelization is almost trivially realized by algebraic continuation from the intersection of realities and p-adicities defined by the 2-surfaces with parameters in algebraic extensions of rationals.

   The existence of preferred p-adic primes can be understood in this picture: they correspond to the so called ramified primes of the algebraic extension. One can also deduce a generalization of p-adic length scale hypothesis in terms of Negentropy Maximization Principle (NMP) [K12]. One might hope that all basic building bricks have been identified.

1.2 A Sketch For The Coupling Constant Evolution

The following summarizes the basic vision about coupling constant evolution. Needless to say that it involved a lot of guesses and should be taken only as a sketch.

1.2.1 p-Adic evolution in phase resolution and the spectrum of values for Planck constants

The quantization of Planck constant has been the basic theme of TGD. The basic idea was that different values of Planck constant could relate to the evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1.2.2 The most recent view about coupling constant evolution

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the the definition of classical fields absorbs them as normalization constants. Hence the notion of continuous coupling constant evolution at space-time level is not needed nor makes sense in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

Discrete p-adic coupling constant evolution replacing in TGD the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination might make possible a concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

The work during 2016-2017 with adelic TGD [**?**, K35, K37] has led to a purely number theoretic view about coupling constant evolution. Coupling constant evolution is discrete and the phase transitions changing the values of coupling parameters correspond to changes for the extension of rationals inducing the extensions of p-adic number fields defining together with reals the adele. $h_{eff}/h = n$ can be identified as the dimension of the extension dividind the order of its Galois group.

The earlier proposal discussed also below is that gravitational coupling could be understood in terms of Kähler coupling strength and p-adic length scale hypothesis. The twistor lift of TGD however introduces Planck length as fundamental length scale assignable to the twistor sphere of twistor bundle $M^4 \times S^2$ of M^4. The value of the ratio $l_P^2/R^2(CP_2)$ remains to be predicted and could follow from quantum criticality.

1.2.3 p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation [K3] . M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scal-

ing invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

The chapter decomposes into sections. In the first part basic notions are introduced and a general vision about coupling constant evolution is introduced. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered. In the second part quantitative predictions involving some far from rigorous arguments, which I however dare to take half-seriously, are discussed. It must be emphasized that this chapter like many others is more like a still continuing story about development of ideas - not a brief summary about a solution of a precisely defined problem. What I take very seriously is the general vision discussed above, addition of details to end up with formulas tends to lead to all kinds of fuzziness. There are many ad hoc ideas and conflicting views. These books are just lab note books - nothing more.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L2].

2 Summary Of Basic Ideas Of Quantum TGD

2.1 General Ideas Of Quantum TGD

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

2.1.1 Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCDthey define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum

TGD. The assumption that the commutator algebra of these super-symplectic and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of configuration space Dirac equation.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. WCW spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds.

2.1.2 p-Adic physics as physics of cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of configuration space spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K22]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

2.1.3 Hierarchy of Planck constants and dark matter hierarchy

The identification of dark matter as phases having large value of Planck constant [K19, K8, K5] led to a vigorous evolution of ideas. Entire dark matter hierarchy with levels labelled by increasing values of Planck constant is predicted, and in principle TGD predicts the values of Planck constant if physics as a generalized number theory vision is accepted [K8].

The original vision was that the hierarchy of Planck constants demands a generalization of quantum TGD. This would have required a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. It however became clear that the hierarchy of Planck constants labels a hierarchy of quantum criticalities characterized by sub-algebras of super-symplectic algebras possessing a natural conformal structure. The sub-algebra for which the conformal weights come as n-ples of those for the entire algebra is isomorphic to the full algebra and acts as a conformal gauge algebra at given level of criticality.

In particular, the classical symplectic Noether charges for preferred extremals connecting 3-surfaces at the ends of CD vanish. and this defines preferred extremal property. There would be n conformal gauge equivalence classes of preferred extremals which would correspond to n sheets of a covering of the space-time surface serving as base space. There is very close similarity with the Riemann surfaces. Therefore coverings would be generated dynamically and there is no need for actual coverings of the imbedding space.

The gauge degeneracy corresponds to the non-determinism associated with the criticality having interpretation in terms of non-determinism of Kähler action and with strong form of holography. The extremely strong super-symplectic gauge conditions would guarantee that the continuation of string world sheets and partonic 2-surface to preferred extremals is possible at least for somes value of p-adic prime. A good guess is that this is the case for the so called ramified primes associated with the algebraic extension in question at least. These ramified primes would characterize physical system and the weak form of NMP would allow to understand how p-adic length scale hypothesis follows [K34]. The continuation could be possible for all p-adic primes due to the possibility of p-adic pseudo-constants having vanishing derivative.

The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K15]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

A further strong prediction is that the phase transitions increasing h_{eff} and thus reducing criticality (TGD Universe is like hill at the top of the hill at....) occur spontaneously [K33]. This conforms with NMP and suggests that evolution occurs spontaneously. The state function reduction increasing h_{eff} means however the death of a sub-self so that selves are fighting to stay at the criticality. The metabolic energy bringing inNE allows to satisfy the needs of NMP so that the system survives and provides a garden in which subselves can are born and die and gradually generate negentropic entanglement. Living systems are thus negentropy gatherers and each death and re-incarnation generates new negentropy.

All particles in the vertices of Feynman diagrams have the same value of Planck constant so that the particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to h_{eff} changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory. These observations motivate the tentative identification of the macroscopic quantum phases in terms of dark matter and also of dark energy with gigantic "gravitational" Planck constant.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

2.1.4 Identification of symplectic and Kac-Moody symmetries

The basic symmetries are isometries of "world of classical worlds" (WCW) proposed to be realized as symplectic transformations of the boundaries of causal diamonds (CD) locally identifiable as $\delta M^4_\pm \times CP_2$. These symplectic symmetries contains as algebra symplectic isometries which are expected to be of special importance. These transformations are expected to have continuation to deformations of the entire preferred extremal. They cannot be symmetries of Kähler action.

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric. These symmetries would act as gauge symmetries and related to quantum criticality due to the non-determinism of Kähler action in turn giving rise to the hierarchy of Planck constants explaining dark matter. The recent vision looks like follows.

1. The recent interpretation is that these gauge symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

2. The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

3. A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

   A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

4. This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

2.1.5 Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrices identifiable as products of orthonormal hermitian square roots of density matrices and universal S-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. The detailed description of U- and M-matrices is considered in [K30].

2.1.6 Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects suggests that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. The original proposal that Chern-Simons action for light-like 3-surfaces defined by the regions at which the signature of the induced metric changes its sign however failed and one must use Kähler action and corresponding Kähler-Dirac action with measurement term to define the fundamental theory. At the limit when the momenta of particles vanish, the theory reduces to topological QFT defined by Kähler action and corresponding modified Dirac action. The imaginary exponent of the instanton term associated with the induced Kähler form defines the counterpart of Chern-Simons action as a phase of the vacuum functional and contributes also to Kähler-Dirac equation.

M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Superconformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

2.1.7 Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix is consistent with Connes tensor product. This does not fix the M-matrix as was the original belief but only realizes mathematically the notion of finite measurement resolution. Together with super-conformal symmetries this constraint should fix possible M-matrices to a very high degree if one assumes the existence of universal M-matrix from which M-matrices with finite measurement resolution are obtained.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

2.2 The Construction Of U, M-, And S-Matrices

The general architecture of matrices is now rather well-understood and described in chapter [K30]. A brief summary is also given in the introduction. The key matrix is U-matrix acting in the space of zero states but leaving the states at the second boundary of CD invariant. M-matrix acts between positive and negative energy parts of given zero energy state being the product of a hermitian square root of density matrix and of a unitary S-matrix. The hermitian matrices involved would naturally form a representation of super-symplectic algebra or its sub-algebra and their "moduli squared" define a density matrix characterizing the second part of zero energy state. An open question is whether this density matrix relates to thermodynamics only formally or whether there is a deeper connection.

The recipe reduces the decisive step to a construction of S-matrix for a given CD and of a unitary time evolution operator in the moduli space of CDs providing unitary representation for a discrete subgroup of Lorentz group. The S-matrix for a given CD is n:th power of fundamental S-matrix S^n for CD whose size is n times the minimal size of CD characterized by the CP_2 time scale.

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

2.2.1 Emergence of particles as bound state of fundamental fermions, extended space-time supersymmetry, and generalized twistors

During year 2009 several new ideas emerged and give hopes about a concrete construction of M-matrix.

1. The notion of bosonic emergence [K17] follows from the fact that gauge bosons are identifiable as pairs of fermion and anti-fermion at opposite light-like throats of wormhole contact. As a consequence, bosonic propagators and vertices are generated radiatively from a fundamental action for fermions and their super partners. At QFT limit without super-symmetry this means that Dirac action coupled to gauge bosons is the fundamental action and the counterpart of YM action is generated radiatively. All coupling constants follow as predictions as they indeed must do on basis of the general structure of quantum TGD.

2. Whether the counterparts of space-time supersymmetries are possible in TGD Universe has remained a long-standing open question and my cautious belief has been that the super partners do not exist. The resolution of the problem came with the increased understanading of the dynamics of the Kähler-Dirac action [K9, K10]. In particular, the localization of the electroweakly charged modes at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces- meant an enormous simplification since the solutions of the Kähler-Dirac equation are conformal spinor modes.

   The oscillator operators associated with the modes of the induced spinor field satisfy the anti-commutation relations defining the generalization of space-time super-symmetry algebra and these oscillator operators serve as the building blocks of various super-conformal algebras. The number of super-symmetry generators is very large, perhaps even infinite. This forces a generalization of the standard super field concept. The action for chiral super-fields emerges as a generalization of the Dirac action to include all possible super-partners. The huge super-symmetry gives excellent hopes about cancelation of UV divergences. The counterpart of super-symmetric YM action emerges radiatively. This formalism works at the QFT limit. The generalization of the formalism to quantum TGD proper is yet to be carried out.

3. Twistor program has become one of the most promising approaches to gauge theories. This inspired the question whether TGD could allow twistorialization [K27]. Massive states -both real and virtual- are the basic problem of twistor approach. In TGD framework the obvious idea is that massive on mass shell states can be interpreted as massless states in 8-D sense. Massive off-mass shell states in turn could be regarded as pairs of positive and negative on mass shell states. This means opening of the black box of virtual state attempted already in the model for bosonic propagators inspired by the bosonic emergence , and one can even hope that individual loop integrals are finite and that Wick rotation is not needed. The third observation is that 8-dimensional gamma matrices allow a representation in terms of octonions (matrices are not in question anymore). If the Kähler-Dirac gamma "matrices" associated with space-time surface define a quaternionic sub-algebra of the complexified octonion algebra, they allow a matrix representation defined by octonionic structure constants. This holds true for are hyper-quaternionic space-time surfaces so that a connection with number theoretic vision emerges. This would more or less reduce the notion of twistor to its 4-dimensional counterpart.

2.2.2 Generalization of Feynman diagrams

An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests an interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD for instance, photons traveling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

2.2.3 Scattering amplitudes as computations in Yangian arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length and that all diagrams connecting the same states at the boundaries of CD produce the same scattering amplitude. This would mean enormous calculational simplification.

The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K1], Yangians [K24], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics.

The identification of universal 3-vertex as a product or co-product in Yangian looks highly promising approach to the construction of the scattering amplitude. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices.

This resembles strongly the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

2.2.4 Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K30]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K30].

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

2.3 Are Both Symplectic And Conformal Field Theories Needed In TGD Framework?

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to one possible concrete view about the generalized Feynman diagrams giving M-matrix elements and at least a resemblance with ordinary Feynman diagrammatics.

It must be of course admitted that there are several apparentely competing visions. Twistorial vision [K24] and the vision about scattering amplitudes as representations for sequences of algebraic operations in super-symplectic Yangian [A2] [B3, B1, B2] seem to be consistent views. Symplectic approach seems to be suitable to understand the integration over WCW zero mode degrees of freedom not included in the other approaches.

2.3.1 Symplectic invariance

Symplectic symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K4] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

2.3.2 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K16]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k \Phi_l = c_{kl}^m \Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1, s_2, s_3))\Phi_m(s) d\mu_s \quad . \tag{2.1}$$

   Here the coefficients c_{kl}^m are constants and $A(s_1, s_2, s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1, s_2, s)) I d d\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1, s_2, s)) d\mu_s \quad . \tag{2.2}$$

   Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1, s_2, s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

2.3.3 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{2.3}$$

   of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \quad (2.4)$$
$$c_{kl}^r c_{rm} \int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \ . \quad (2.5)$$

   Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(\Delta A(s_1,s_2,s)) d\mu_s \quad (2.6)$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

2.3.4 Generalization to quantum TGD

(Number theoretic) braids are identifiable as boundaries of string world sheets at which the modes of induced spinor fields are localized in the generic case in Minkowskian space-time regions. Fundamental fermions can be assigned to these lines. Braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the "world of classical worlds".

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW . Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy

of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [K3] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K23].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond directly to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is

not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

2.3.5 More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities could follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

UV finiteness is suggested also by the generalized Feynman rules providing a phenomenological view about what TGD predicts. According to these rules fundamental fermions propagate like massless particles. In twistor Grassmann approach residue integration is expected to reduce internal fermion lines to on mass shell propagation with non-physical helicity. The fundamental 4-fermion interaction is assignable to wormhole contact and corresponds to stringy exchange of four-momentum with propagator being defined by the inverse of super-conformal scaling generator $1/L_0$. Wormhole contacts carrying fermion and antifermion at their throats behave like fundamental bosons. Stringy propagators at wormhole contacts make TGD rules a hybrid of Feynmann and stringy rules. Stringy propagators are necessary in order to avoid logarithmic divergences. Higher mass excitations crucial for finiteness belong to the representations of super-conformal algebra and can be regarded as bound states of massless fermions. Massivation of external particles allows to avoid infrared divergences. Not only physical bosons but also physical fermions emerge from fundamental massless fermions.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-symplectic conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [K4].

1. The state construction utilizes both super-symplectic and super Kac-Moody algebras. super-symplectic algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-symplectic algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-symplectic operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the WCW Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-symplectic super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the WCW super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to WCW gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since ordinary fermionic propagator and boson-emission vertices at the ends of the line containing WCW gamma matrix and its conjugate give compensating fermion numbers [K24].

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would correspond evolution as a function of the scale of CD. It might have interpretation also in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram as the size of CD is increased. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

3 General Vision About Real And P-Adic Coupling Constant Evolution

Many new pieces of understanding have emerge since the last updating of the views about coupling constant evolution. It is now understood how GRT space-time and QFT gauge theory limit emerge from many-sheeted space-time in long length scales. Quantum classical correspondence (QCC) suggests that classical correlation functions correspond to those for elementary particles. What is new that the generalization of AdS/CFT correspondence strongly suggested by the extension of super-conformal symmetries and the possibility to express WCW Kähler metric in two manners provides support for QCC. It is now understood how the hierarchy of Planck constants relates to a hierarchy of symmetry breakings for super-symplectic algebra and to a hierarchy of quantum criticalities. The vision about scattering amplitudes as representations of sequences of arithmetic operations in the Yangian of super-sympelectic algebra [A2] [B3, B1, B2] gives hopes about the computation of scattering amplitudes and already now gives vision about their general structure and what is the counterpart for the coupling constant evolution at the fundamental level.

The older approach was rather phenomenological and based mostly on p-adic considerations and the future challenge is to combine the new ingredients with p-adic picture. Perhaps the most important questions about p-adic coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

3.1 A General View About Coupling Constant Evolution

The following general vision about coupling constant evolution summarizes the recent understanding. The details of the picture are of course bound to fluctuate.

3.1.1 Einstein's equations, Equivalence Principle, and GRT and QFT limits of TGD

Coupling constant evolution makes sense in quantum field theory defined in fixed background space-time, say Minkowski space-time. In TGD framework imbedding space replaces this fixed space-time and in ZEO the hierarchy of causal diamonds replaces imbedding space. It is not at all clear whether at the level of basic TGD coupling constant evolution makes sense at all whereas it should make sense at QFT limit of TGD. This requires understanding of QFT and GRT limits of TGD including also Equivalence Principle.

At quantum level Equivalence Principle (EP) can be reduced to quantum classical correspondence: the conserved four-momentum associated with Kähler action equals to the eigenvalue of conserved quantal four-momentum assignable to Kähler-Dirac equation [K29]. This quantal four-momentum in turn can be associated with string world sheets which emerge naturally from Kähler-Doirac equation.

Einstein's equation give a purely local meaning for EP. How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).
.

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Similar description applies to induced electroweak gauge potentials and color gauge potentials: the sum of these gauge potentials over space-time sheets should define the classical gauge fields of QFT limit of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

What coupling constant evolution could mean in TGD framework? Kähler action and Kähler-Dirac action do not contain any fundamental couplings affecting to the dynamics. Kähler coupling strength does not affect classical dynamics and is analogous to critical temperature, and therefore invariant under renormalization group if defined in TGD framework. This suggests that the analog of renormalization group equations at space-time level does not look feasible. Continuous coupling constant evolution might be useful notion only at the QFT limit.

The natural length scale hierarchy associated with coupling constant evolution would be the hierarchy of length scales assignable to CDs. The minimal sizes of CDs assumed to be equal to secondary p-adic length scales in the case of elementary particles. More generally, number theoretical arguments suggest that the scales of CDs come as integer multiples of CP_2 radius. What is new that coupling constant evolution would be discretized, being labelled by integers. Primes and primes near powers of 2 could correspond to physically favored minimal size scales for CDs: kind of survivors in fight for survival. Discrete coupling constant evolution as evolution of various M-matrix elements as function of the size-scale of CD would look like a reasonable TGD counterpart of coupling constant evolution. For single CD one might say that system is quantum critical, and coupling constants do not evolve.

3.1.2 Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K3] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A6] known as hyperfinite factor of type II_1 (HFF) [K3, K28, K8]. HFF [A3, A5] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [A1]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A8], anyons [D1] , quantum groups and conformal field theories [A9], and knots and topological quantum field theories [A7, A4].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is.1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K3]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. The original hope was that Connes tensor product realizing mathematical the finite measurement resolution could fix M-matrix to high degree turned out be too optimistic.

3.1.3 How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

In zero energy ontology zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of the intersecting future and past directed light-cones comes as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and its generalization a slightly more general hypothesis $T_n = p^n T_0$, p prime. It however seems that these scales are dynamically favored but that also other scales are possible.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their

orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^kT_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have secondary Compton length Electron's secondary Compton time $T_e(127) = \sqrt{5}T_2(127) = .1$ seconds defines a fundamental biological rhythm. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

3.1.4 Could symplectic variant of QFT allow to understand coupling constant evolution in zero modes?

Symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-symplectic (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

Symplectic QFT might allow to calculate the coupling constant evolution in zero modes which do not contribute to the line element of sub- WCW expect as contribute a conformal factor depending on zero modes invariant under symplectic transformations.

3.2 Number Theoretical Vision About Coupling Constant Evolution

The recent progress in the understanding of TGD has led to a rather abstract number theoretical vision about coupling constant evolution.

3.2.1 Coupling constant evolution as increase in computational precision in Yangian arithmetics

One should relate the picture of the usual perturbation theory to the picture in which one identifies scattering amplitudes as sequences of arithmetical manipulations in super-symplectic Yangian [A2] [B3, B1, B2]. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture [K24].

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of M^4 and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from M^4 metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one. A good

first guess is that secondary p-adic length scales proportional to p define preferred size scales for CD among integer multiples of CP_2 scale. For electron the scale corresponds to time scale of .1 second defining a fundamental bio-rhythm.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

   For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram.

1. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations CP_2 type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total CP_2 volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

2. Convergence depends on how large the fraction of volume of CP_2 is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

3. One must be of course be very cautious in making conclusions. For instance, the presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of h_{eff} only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K33] in terms electric-magnetic duality.

Renormalization group flow would have two meanings.

1. RG flow can correspond to the increase of resolution in the sense that the number of fermionic strings per partonic 2-surfaces increases This would mean increase of the resolution and replace computational sequences with more complex ones involving more vertices. Both length scale resolution and angular resolution can increase and these resolutions should relate to the algebraic resolution.

2. Another meaning is as flow defined by the hierarchy of quantum criticalities and having interpretation in terms of hierarchy of h_{eff}. This process transforms gauge degrees of freedom to physical degrees of freedom and there is temptation to interpret this as discrete evolution such that each h_{eff} defines a plateau in the evolution somewhat like in fractional quantum Hall effect for which I have indeed proposed an explanation in terms of hierarchy of Planck constants.

3.2.2 Coupling constant evolution and ramified primes in algebraic extensions of rationals

The recent adelic vision about TGD allows totally new insights about p-adic coupling constant evolution. In particular, the origin of preferred p-adic primes can be understood and the realization of number theoretical universality by algebraic continuations becomes almost trivial.

1. The recent general picture about coupling constant evolution relies on the hierarchy of algebraic extensions of rational in which the parameters characterizing string world sheets and partonic 2-surfaces. They define the intersection of reality and various p-adicities, and strong holography is assumed to allow a continuation of these 2-surfaces to preferred extremals of Kähler action [K34].

2. By strong form of holography also scattering amplitudes are determined by the data in the intersection so that coupling constant evolution should reduce to the evolution of complexity for the hierarchy of algebraic extensions of rationals. The basic parameters characterizing the extensions are the ramified primes - the primes containing higher powers in the product expansion using the primes of the extension (to be precise, one should talk about prime ideals).

3. The product of ramified primes is a basic parameter characterizing extension. Preferred p-adic primes would naturally correspond to the ramified primes. p-Adic continuations identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K15]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

   NMP [K12] in turn could allow to understand why the p-adic primes near but below powers of primes are favored: $p \simeq p_1^k$. The original form of the p-adic length scale hypothesis corresponds to $p_1 = 2$.

   This justifies the basic picture implied by p-adic mass calculations and allows to generalize canonical identification as a map taking real values of various group invariants (inner products of four-momenta, etc..) to their p-adic counterparts as one algebraically continues scattering amplitudes from the intersection to various number fields.

4. The reasonable expectation is that coupling constant evolution reduces to the evolution of complexity for the algebraic extensions of rationals and is thus discretized and that the preferred primes serve as parameters defining p-adic length scales appearing in the p-adic length scale evolution as varying parameters.

5. Infinite primes at the lowest level representing bound states label can be mapped to polynomials, and parametrize irreducible extensions of rationals so that coupling constant evolution corresponds to evolution at the level of infinite primes too. An interesting question concerns the meaning of higher level infinite primes mappable to polynomials of several variables.

To sum up, the number theoretical vision has become rather concrete.

3.3 Could Correlation Functions, S-Matrix, And Coupling Constant Evolution Be Coded The Statistical Properties Of Preferred Extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K30]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K30].

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables

$X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \rightarrow \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

4 P-Adic Coupling Constant Evolution

p-Adic coupling constant evolution is one of the genuinely new elements of quantum TGD. In the following some aspects of the evolution will be discussed. The discussion is a little bit obsolete as far as the role of canonical identification is considered. The most recent view about p-adic coupling constant evolution is discussed at the end of the section.

4.1 General Considerations

One of the basic challenges of quantum TGD is to understand whether the notion of p-adic coupling constant evolution is something related to the basic TGD or whether it emerges at GRT and QFT limits only.

1. Since neither classical field equations for Kähler action nor Kähler-Dirac action depend on coupling constants except as overall multiplicative normalization factor, one expects that at the level of TGD space-time the notion of coupling constant evolution is not well-defined or at least fails to be a fundamental notion. Coupling constant evolution would characterize GRT and QFT limits of TGD and since causal diamond (CD) is the basic unit, the scale of CD would serve as a fundamental scale.

   What would give rise to the ordinary continuous coupling constant evolution at long length scales, would be the replacement of many-sheeted space-time with GRT space-time containing gauge potentials which are sums of induced gauge potentials associated with various space-time sheets. The increase in the size of CD would induce the scaling of the size of the space-time sheet. Hence the geometric correlate for coupling constant evolution would be the scaling of CD size. The original belief was that it would be scaling of the size of the space-time sheet.

2. The original view was that there are two separate coupling constant evolutions: one associated with p-adic length scale hierarchy and second associated with angle resolution and characterized by the hierarchy of Planck constants. In the recent view these evolutions are unified to a number theoretic evolution in terms of increasing complexity of an algebraic extension of rational numbers inducing also the extensions of p-adic number fields. Space-time and quantum physics become adelic. The algebraic extensions are associated with the parameters characterizing partonic 2-surfaces and string world sheets, which by strong form of holography determine space-time surfaces as preferred extremals of Kähler action. The crucial number theoretical universality necessary for the adelization is almost trivially realized by algebraic continuation from the intersection of realities and p-adicities defined by the 2-surfaces with parameters in algebraic extensions of rationals.

   Preferred p-adic primes emerge naturally in the number theoretic vision as so called ramified primes of the algebraic extension. One can also deduce a generalization of p-adic length scale hypothesis in terms of Negentropy Maximization Principle (NMP) [K12]. Hence

3. At the fundamental level this evolution is discrete by p-adic length scale hypothesis justified by zero energy ontology, where CD sizes are assumed to come as integer multiples of CP_2 mass: the discretization is for number theoretical reasons and gives hopes of number theoretical universality. The most general option is that the CD sizes come as integer multiples of CP_2 size. Discreteness means that continuous mass scale is replaced by mass scales coming square root prime multiples of CP_2 mass. Obviously continuous evolution is an excellent approximation in elementary particle p-adic mass scales. p-Adic length scale hypothesis allows only half octaves of CP_2 mass. Kähler coupling strength α_K or gravitational coupling constant is assumed to remain invariant under p-adic coupling constant evolution. The basic problem is to understand the value of α_K and here p-adic mass calculations give strong constraints.

4. The realization that well-definedness of em charge requires the localization of the modes of the induced spinor field to string world sheets or partonic 2-surfaces was an important step in the process of understanding super-symplectic and other symmetries, and has led to the recent realization that strong form of holography is realized for string world sheets and partonic 2-surfaces by continuing them to preferred extremals of Kähler action.

   Coupling constant evolution should allow a reduction to string model type description. The ordinary AdS/CFT correspondence between $n-1$-D conformal field theory in $AdS^n \times S^5$ is modified. Super-symplectic generalization of conformal field theory is associated with string world sheets and partonic 2-surfaces and 10-D bulk containing strings is replaced with space-time surface contained in $M^4 \times CP_2$. This holography looks very much like old-fashioned holography.

   This framework has powerful physical implications. Gravitationally bound states correspond to partonic 2-surfaces connected by fermionic strings. The ordinary string theory picture with string tension defined by Planck length would not allow gravitationally bound states above Planck length scale. The string tension is however dynamical since effective stringy action can be assigned to the effective metric defined by Kähler-Dirac gamma matrices appearing in the K-D equation. The value of string tension must characterize the value of Kähler action by strong form of holography, and it decreases in long length scales by its $1/h_{eff}^2$-proportionality. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K33] in terms electric-magnetic duality.

   Quantum gravitational coherence is present in astrophysical scales and assignable to the large values of $h_{eff} = h_{gr}$, where $\hbar_{gr} = GMm/v_0$, $v_0/c < 1$, with v_0 having dimensions of velocity, is gravitational Planck constant characterizing magnetic flux tubes mediating gravitational interaction between masses M and m [K19]. v_0 is of the order of magnitude for a typical rotational velocity in the system. Hadronic string tension can be also understand as this kind of string tension.

   Gravitational quantum coherence is predicted in astrophysical scales, and dark matter is macroscopically quantum coherent. In this framework superstring models give quantum gravitation in Planck length scale but the proposed QFT based vision about long length scale limit must be seen as a figment of bad imagination leading to the landscape catastrophe and total loss of predictive power.

5. If weak form of electric magnetic duality and $j \cdot A = 0$ condition for Kähler current and gauge potential in the interior of space-time sheets are satisfied, Kähler action reduces to Chern-Simons terms at light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface. Induced metric would apparently disappear from the action in accordance with the idea about TGD as almost topological QFT.

4.2 How P-Adic And Real Coupling Scattering Amplitudes Are Related To Each Other?

p-Adic and real scattering amplitudes would be obtained from the amplitudes having values in the intersection of reality and p-adicities corresponding to a particular algebraic extension of rationals inducing those of p-adic number fields.

1. The algebraic intersection of reality and various p-adicities would be defined by string world sheets and partonic 2-surfaces. The amplitudes in the intersection are algebraically universal amplitudes having the same values as numbers of algebraic extension in any number field. Unitary is satisfied and symmetries respected in their discrete versions. This requires that various quantum numbers such as momenta belong to the algebraic extension of rationals. One might say that scattering amplitudes in algebraic extension are kind of genes for scattering amplitudes in various number fields and define adelic S-matrix as the analog of finite-dimensional adelic matrices appearing in Langlands program.

2. This kind of direct identification of real and p-adic amplitudes cannot be continuous if one allows all possible values of the parameters - say various Lorentz invariants appearing in the amplitude. The possibility of p-adic pseudo-constants allows the identification via common algebraics if one poses a cutoff and performs the algebraic identification only for the discrete set of parameters in the extension defined by the cutoff. The values of amplitudes for arbitrary reals and p-adics are obtained by algebraic continuation of these amplitudes to real and p-adic sectors. Unitarity and symmetries should fix the continuation highly uniquely.

   Both UV and IR cutoffs are necessary: otherwise arbitrarily large/small algebraic numbers in p-adic topology (proportional to negative/positive power of p) could correspond to arbitrarily small/large algebraic numbers in real topology.

3. This is not the only possible definition of the scattering amplitudes in the intersection. The definition of intersection of real and p-adic variants of WCWs does not introduce discretization at space-time level but at the level of parameter space characterizing the space of string world sheets and partonic 2-surfaces. Also in the case of scattering amplitudes a more abstract manner to define the intersection would use (say) Lorentz invariant rational functions of four-momenta and of other parameters with coefficients belonging to the algebraic extension of rationals. The algebraic continuation would be reduced to that for the parameters whereas the arguments of these function could be taken to belong the appropriate number field. This is certainly the more elegant option.

Is there any need for canonical identification or some of its variants with IR and UV cutoffs mapping real momenta/Lorentz invariants formed from them to p-adic ones and vice versa?

1. Canonical identification does not seem to be absolutely necessary. The correlates of matter and mind are independent below the resolution outside the set of the parameter values belonging to the algebraic intersection with cutoffs: dynamics for cognition would not reduce to that for matter.

   p-Adic mass calculations [K31] however suggest that canonical identification maps p-adic mass squared to its real counterpart. This map would assign to real mass scale its p-adic counterpart and would be essential for the physical interpretation of p-adic mass calculations. p-Adic/real mass squared can be generalized to Lorentz invariants defined by 4-momenta. The modification of the canonical identification would co-incide with the direct identification as algebraic numbers inside the range of parameters defining the intersection and map the powers of p above a upper cutoff to their inverses to achieve continuity.

2. The minimal assumption would be that some variant of canonical identification with cutoffs applies only to Lorentz invariants (for the variants of canonical identification see [K21].

   (a) This kind of variant would map the coefficients of powers of p^M to itself but invert the powers. It could also map rationals m/n to $I(m)/I(n)$. This kind of map would allow to assign to real scattering amplitudes p-adic scattering amplitudes with the same real values of Lorentz invariants defined by canonical identification.

   (b) A stronger correspondence would be obtained by mapping also the resulting p-adic scattering amplitudes to real ones by some variant of the canonical identification. The deviations between the two amplitudes should not be large if p-adic physics allows precise cognitive representations. p-Adic description mapped to real context might mean huge simplification as it indeed does in the case of p-adic mass calculations.

3. There is no need to apply canonical identification at space-time level. It has indeed become clear that canonical identification is ugly at space-time level although I was for some time enthusiastic about it [K32]. Strong form of holography however allows to realize the correspondence between real and p-adic space-time surfaces as a non-local correspondence since string world sheets and partonic 2-surfaces serve as "genes of space-time" and the correspondence between p-adicities and realities identified as space-time surfaces becomes non-local.

A further objection against the continuation from the algebraic intersection is based on the fact that non-algebraic transcendentals like π appear in the real scattering amplitudes. Could the counterpart of 2π in algebraic extension be defined as $N \times sin(2\pi/N)$, where $sin(2\pi/N)$ define the smallest angle in the abelian algebraic extension containing roots of unity? In the real sector the limit could be taken to give $2/pi$. One can wonder whether the positivity of the real numbers as Cartesian factor of ideles (invertible adeles) could somehow relate to the positive Grassmannians encountered in the twistor approach.

4.2.1 How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parameterization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p\ mod\ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

1. Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^\dagger D$.

2. Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional

algebraic extension involving various roots of rationals. Also in this case the product $U^\dagger D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p,k)$.

4.3 How Could P-Adic Coupling Constant Evolution And P-Adic Length Scale Hypothesis Emerge From Quantum TGD Proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

4.3.1 *M-matrix and coupling constant evolution*

A breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in ZEO [K3] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

As discussed in [K18] and in the earlier chapter about number theoretical vision, it is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and a slightly more general hypothesis $T_n = p^n T_0$, p prime. It seems that these scales are dynamically favored but that also other scales are possible.

Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

4.3.2 *The origin of the preferred p-adic length scales*

This question was posed already for two decades ago has but remained without a convincing answer. Quite recently however the number theoretical vision allowed to understand both the origin of preferred p-adic number fields and the emergence of p-adic length scale hypothesis in a generalized form. Preferred primes are near but below powers prime which can be also larger that $p = 2$.

The preferred primes could correspond to so called ramified rational primes, which split in to products of the primes of the extension. If some prime appears as higher than first power, one has ramification. The number of ramified primes is finite.

In strong form of holography p-adic continuations of 2-surfaces to preferred extremals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K15]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

Weak form of NMP allows to understand the emergence of preferred p-adic length scales. NMP favors ramified primes, for which the integer n is power of single prime p. If n is a prime slightly below $n_{max} = p^n$ defining the dimension of the sub-space corresponding to maximal negentropy gain, weak form of NMP favors its selection since the p-adic topology is farthest from the discrete topology assignable to formal p-adic topology characterized by $p = 1$ [K34].

5 Quantitative Guesses For The Values Of Coupling Constants

This focus of attention in this section is in quantitative for the p-adic evolution of couplings constants obtained by combining information coming from p-adic mass calculations with number theoretic constraints and general formula for gravitational constant inspired by a simple physical picture.

Only educated (if even this) guesses are in question since real understanding of coupling constant evolution has begun to emerge only rather recently (2014) as the relationship between TGD and GRT and QFT was finally clarified.

The quite recent powerful results following from strong form of holography, adelic vision about space-time allowing to realize number theoretical universality, and from the vision that scattering amplitudes can be seen as representations for computational sequences in in Yangian [A2] [B3, B1, B2] of super-symplectic algebra have not been taken account at all.

The most recent idea is that exponent of Kähler action is number theoretically universal for preferred extremals and that also coupling constants are number theoretically universal [K34]. These conditions are extremely powerful and force to modify the earlier ad hoc ideas about number theoretic anatomy of Kähler coupling strength. Therefore this section is not a summary of final results but summary of various approaches to a difficult problem.

The view about coupling constant evolution has changed radically during 2016-2017 [K7, K36, K35, K37] as the number theoretic vision about TGD as adelic physics and the vision about twistor lift of TGD have co-evolved. Number theoretic vision has extremely powerful consequences and has led to amazingly simple proposals for the scattering amplitudes and coupling constant evolution. The following arguments trying to guess values of coupling constants are in the light of the new vision simply wrong so that this section can be regarded more or less as a curiosity.

5.1 A Revised View About Coupling Constant Evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

1. The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime M_{127}. Later I replaced fine structure constant with

electro-weak U(1) coupling strength at this length scale. The recent discussion returns back to the roots in both aspects.

2. The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [K29]. What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the Kähler-Dirac operator. This formula dictates the number theoretical anatomy of g_K^2 and also of other coupling constants: the most general option is that α_K is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.

3. A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of $\sqrt{2}$. This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether $p \simeq 2^k$ should be replaced with 2^k in all formulas as the recent view about quantum TGD suggests.

4. The prediction is that Kähler coupling strength α_K is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared and corresponds to the largest Mersenne prime (M_{127}), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter R^2/G p-adicization program allows to consider two options: either this constant is of form e^q or 2^q: in both cases q is rational number. $R^2/G = exp(q)$ allows only M_{127} gravitons if number theory is taken completely seriously. $R^2/G = 2^q$ allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.

5. A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

It seems fair to conclude that the attempts to understand the implications of p-adicization for coupling constant evolution have begun to bear fruits.

5.1.1 General formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to $1/\alpha_K$ since the matrices $\hat{\Gamma}^\alpha$ have this proportionality. This gives the formula

$$exp(\frac{S_{K,R}(X^4(X^3))}{2g_K^2}) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{(g_K)^{2N}} . \quad (5.1)$$

Here $\lambda_{0,i}$ by definition corresponds to $g_K^2 = 4\pi\alpha_K = 1$. $S_{K,R} = \int J^* J$ is the reduced Kähler action.

For $S_{K,R} = 0$, which might correspond to so called massless extremals [K2] one obtains the formula

$$g_K^2 = (\prod_i \lambda_{0,i})^{1/N} . \quad (5.2)$$

Thus for $S_{K,R} = 0$ extremals one has an explicit formula for g_K^2 having interpretation as the geometric mean of the eigenvalues $\lambda_{0,i}$. Several values of α_K are in principle possible.

p-Adicization suggests that $\lambda_{0,i}$ are rational or at most algebraic numbers. This would mean that g_K^2 is N: th root of this kind of number. $S_{K,R}$ in turn would be

$$S_{K,R} = 2g_K^2 log(\frac{\prod_i \lambda_{0,i}}{g_K^{2N}}) \ . \tag{5.3}$$

so that the reduced Kähler action $S_{K,R}$ would be expressible as a product N: th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and S_K.

For CP_2 type vacuum extremal one would have $S_{K,R} = \frac{\pi^2}{2}$ in apparent conflict with the above result. The conflict is of course only apparent since topological condensation of CP_2 type vacuum extremal generates a hole in CP_2 having light-like wormhole throat as boundary so that the value of the action is modified.

5.1.2 Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(S_K(CP_2))$ defining vacuum functional and thus the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} \ . \tag{5.4}$$

   Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \tag{5.5}$$

   $a < 1$ conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127}-1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

1. The formula for the gravitational constant

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r\hbar_0 G = L_p^2 \times exp(-2aS_K(CP_2)) \ , \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{5.6}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. What was noticed before is that this relationship allows even constant value of G if a has appropriate dependence on p.

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

1. I assumed that modulus squared for vacuum functional is in question: hence the factor $2a$ in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement $2a \rightarrow a$ is necessary.

2. Second wrong assumption was that graviton corresponds to CP_2 type vacuum extremal- that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by CP_2 vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor $2a$ in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to $exp(-aS_K(CP_2))$.

The basic constraint to the coupling constant evolution comes for the invariance of g_K^2 in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p,r)\pi^2}{log(pK)} \ , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r}\frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} \ . \end{aligned} \tag{5.7}$$

2. How to guarantee that g_K^2 is RG invariant and N: th root of rational?

Suppose that g_K^2 is N: th root of rational number and invariant under p-adic coupling constant evolution.

1. The most general manner to guarantee the expressibility of g_K^2 as N: th root of rational is guaranteed for both options by the condition

$$a(p,r) = \frac{g_K^2}{\pi^2} log(\frac{pK_0}{r}) \ . \tag{5.8}$$

That a would depend logarithmically on p and $r = \hbar/\hbar_0$ looks rather natural. Even the invariance of G under p-adic coupling constant evolution can be considered.

2. The condition

$$\frac{r}{p} < K_0(p) \ . \tag{5.9}$$

must hold true to guarantee the condition $a > 0$. Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition $a < 1$ is guaranteed by the condition

$$\frac{r}{p} > exp(-\frac{\pi^2}{g_K^2}) \times K_0(p) \ . \tag{5.10}$$

The condition implies that for very large values of p the value of Planck constant must be larger than $\hbar_0$.

3. The two conditions are summarized by the formula

$$K_0(p) \times exp(-\frac{\pi^2}{g_K^2}) < \frac{r}{p} < K_0(p) \tag{5.11}$$

characterizing the allowed interval for r/p. If G does not depend on p, the minimum value for r/p is constant. The factor $exp(-\frac{\pi^2}{g_K^2})$ equals to 1.8×10^{-47} for $\alpha_K = \alpha_{em}$ so that $r > 1$ is required for $p \geq 4.2 \times 10^{-40}$. $M_{127} \sim 10^{38}$ is near the upper bound for p allowing $r = 1$.

The constraint on r would be roughly $r \geq 2^{k-131}$ and $p \simeq 2^{131}$ is the first p-adic prime for which $\hbar > 1$ is necessarily. The corresponding p-adic length scale is.1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for r behaves roughly as $r < 2.3 \times 10^7 p$. This condition becomes relevant for gravitational Planck constant GM_1M_2/v_0 having gigantic values. For Earth-Sun system and for $v_0 = 2^{-11}$ the condition gives the rough estimate $p > 6 \times 10^{63}$. The corresponding p-adic length scale would be of around $L(215) \sim 40$ meters.

4. p-Adic mass calculations predict the mass of electron as $m_e^2 = (5 + Y_e)2^{-127}/R^2$ where $Y_e \in [0, 1)$ parameterizes the not completely known second order contribution. Top quark mass favors a small value of Y_e (the original experimental estimates for m_t were above the range allowed by TGD but the recent estimates are consistent with small value Y_e [K14]). The range $[0, 1)$ for Y_e restricts $K_0 = R^2/\hbar_0 G$ to the range $[2.3683, 2.5262] \times 10^7$.

5. The best value for the inverse of the fine structure constant is $1/\alpha_{em} = 137.035999070(98)$ and would correspond to $1/g_K^2 = 10.9050$ and to the range $(0.9757, 0.9763)$ for a for $\hbar = \hbar_0$ and $p = M_{127}$. Hence one can seriously consider the possibility that $\alpha_K = \alpha_{em}(M_{127}$ holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that α_K corresponds to electro-weak $U(1)$ coupling strength in this length scale. The fact that M_{127} defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, this view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that g_K^2 is root of rational number, possibly even rational, and can be assumed to be equal to e^2. Also $R^2/\hbar G$ could be rational. The new element is that G need not be proportional to p and can be even invariant under coupling constant evolution since the parameter a can depend on both p and r. An unexpected constraint relating p and r for space-time sheets mediating gravitation emerges.

5.1.3 Algebraic universality and the value of Kähler coupling strength

With the development of the vision about number theoretically universal view about functional integration in WCW [K34], a concrete vision about the exponent of Kähler action in Euclidian and Minkowskian space-time regions. The basic requirement is that exponent of Kähler action belongs to an algebraic extension of rationals and therefore to that of p-adic numbers and does not depend on the ordinary p-adic numbers at all - this at least for sufficiently large primes p. Functional integral would reduce in Euclidian regions to a sum over maxima since the troublesome Gaussian determinants that could spoil number theoretic universality are cancelled by the metric determinant for WCW.

The adelically exceptional properties of Neper number e, Kähler metric of WCW, and strong form of holography posing extremely strong constraints on preferred extremals, could make this possible. In Minkowskian regions the exponent of imaginary Kähler action would be root of unity. In Euclidian space-time regions expressible as power of some root of e which is is unique in sense that e^p is ordinary p-adic number so that e is p-adically an algebraic number - p:th root of e^p.

These conditions give conditions on Kähler coupling strength $\alpha_K = g_K^2/4\pi$ $(\hbar = 1))$ identifiable as an analog of critical temperature. Quantum criticality of TGD would thus make possible number theoretical universality (or vice versa).

1. In Euclidian regions the natural starting point is CP_2 vacuum extremal for which the maximum value of Kähler action is

$$S_K = \frac{\pi^2}{2g_K^2} = \frac{\pi}{8\alpha_K} \quad .$$

The condition reads $S_K = q$ if one allows roots of e in the extension. If one requires minimal extension of involving only e and its powers one would have $S_K = n$. One obtains

$$\frac{1}{\alpha_K} = \frac{8q}{\pi} ,$$

where the rational $q = m/n$ can also reduce to integer. One cannot exclude the possibiity that q depends on the algebraic extension of rationals defining the adele in question [K34].

For CP_2 type extremals the value of p-adic prime should be larger than $p_{min} = 53$. One can consider a situation in which large number of CP_2 type vacuum extremals contribute and in this case the condition would be more stringent. The condition that the action for CP_2 extremal is smaller than 2 gives

$$\frac{1}{\alpha_K} \leq \frac{16}{\pi} \simeq 5.09 .$$

It seems there is lower bound for the p-adic prime assignable to a given space-time surface inside CD suggesting that p-adic prime is larger than $53 \times N$, where N is particle number.

This bound has not practical significance. In condensed matter particle number is proportional to $(L/a)^3$ - the volume divided by atomic volume. On basis p-adic mass calculations [K11] p-Adic prime can be estimated to be of order $(L/R)^2$. Here a is atomic size of about 10 Angstroms and R CP_2 "radius". Using $R \simeq 10^4 L_{Planck}$ this gives as upper bound for the size L of condensed matter blob a completely super-astronomical distance $L \leq a^3/R^2 \sim 10^{25}$ ly to be compared with the distance of about 10^{10} ly travelled by light during the lifetime of the Universe. For a blackhole of radius $r_S = 2GM$ with $p \sim (2GM/R)^2$ and consisting of particles with mass above $M \simeq \hbar/R$ one would obtain the rough estimate $M > (27/2) \times 10^{-12} m_{Planck} \sim 13.5 \times 10^3$ TeV trivially satisfied.

2. The physically motivated expectation from earlier arguments - not necessarily consistent with the recent ones - is that the value α_K is quite near to fine structure constant at electron length scale: $\alpha_K \simeq \alpha_{em} \simeq 137.035999074(44)$.

   The latter condition gives $n = 54 = 2 \times 3^3$ and $1/\alpha_K \simeq 137.51$. The deviation from the fine structure constant is $\Delta\alpha/\alpha = 3 \times 10^{-3}$ – .3 per cent. For $n = 53$ one obtains $1/\alpha_K = 134.96$ with error of 1.5 per cent. For $n = 55$ one obtains $1/\alpha_K = 150.06$ with error of 2.2 per cent. Is the relatively good prediction could be a mere accident or there is something deeper involved?

What about Minkowskian regions? It is difficult to say anything definite. For cosmic string like objects the action is non-vanishing but proportional to the area A of the string like object and the conditions would give quantization of the area. The area of geodesic sphere of CP_2 is proportional to π. If the value of g_K is same for Minkowskian and Euclidian regions, $g_K^2 \propto \pi^2$ implies $S_K \propto A/R^2\pi$ so that $A/R^2 \propto \pi^2$ is required.

This approach leads to different algebraic structure of α_K than the earlier arguments.

1. α_K is rational multiple of π so that g_K^2 is proportional to π^2. At the level of quantum TGD the theory is completely integrable by the definition of WCW integration(!) [K34] and there are no radiative corrections in WCW integration. Hence α_K does not appear in vertices and therefore does not produce any problems in p-adic sectors.

2. This approach is consistent with the proposed formula relating gravitational constant and p-adic length scale. G/L_p^2 for $p = M_{127}$ would be rational power of e now and number theoretically universally. A good guess is that G does not depend on p. As found this could be achieved also if the volume of CP_2 type extremal depends on p so that the formula holds for all primes. α_K could also depend on algebraic extension of rationals to guarantee the independence of G on p. Note that preferred p-adic primes correspond to ramified primes of the extension so that extensions are labelled by collections of ramified primes, and the ramimified prime corresponding to gravitonic space-time sheets should appear in the formula for G/L_p^2.

3. Also the speculative scenario for coupling constant evolution could remain as such. Could the p-adic coupling constant evolution for the gauge coupling strengths be due to the breaking of number theoretical universality bringing in dependence on p? This would require mapping of p-adic coupling strength to their real counterparts and the variant of canonical identification used is not unique.

4. A more attractive possibility is that coupling constants are algebraically universal (no dependence on number field). Even the value of α_K, although number theoretically universal, could depend on the algebraic extension of rationals defining the adele. In this case coupling constant evolution would reflect the evolution assignable to the increasing complexity of algebraic extension of rationals. The dependence of coupling constants on p-adic prime would be induced by the fact that so called ramified primes are physically favored and characterize the algebraic extension of rationals used.

5. One must also remember that the running coupling constants are associated with QFT limit of TGD obtained by lumping the sheets of many-sheeted space-time to single region of Minkowski space. Coupling constant evolution would emerge at this limit. Whether this evolution reflects number theoretical evolution as function of algebraic extension of rationals, is an interesting question.

5.1.4 Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. One can consider two options.

1. The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings would approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

   Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \qquad (5.12)$$

   The relationship between $U(1)$ and em coupling strengths is

$$\begin{aligned} \alpha_{U(1)} &= \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 . \end{aligned} \qquad (5.13)$$

   Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [E1] is used. Note however that the previous argument implying $\alpha_K = \alpha_{em}(M_{127})$ excludes $\alpha = \alpha_{U(1)}(M_{127})$ option.

2. Second option is obtained by replacing $U(1)$ with electromagnetic gauge $U(1)_{em}$.

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \quad (5.14)$$

Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

1. In TGD framework coupling constant is discrete and comes as powers of $\sqrt{2}$ corresponding to p-adic primes $p \simeq 2^k$. Number theoretic considerations suggest that coupling constants g_i^2 are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have $g_i^2 = g_i^2(k)$. g_K^2 is predicted to be N: th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if $sin(\theta_W)$ and $cos(\theta_W)$ are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas $sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$ and $cos(\theta_W) = 2rs(r^2 + s^2)$.

2. A very strong prediction is that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.

3. $\alpha(M_{127}) = \alpha_K$ implies that M_{127} defines the confinement length scale in which the sign of α_s becomes negative. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [K20, L1], [L1]. Hence one can argue that color coupling strength indeed diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha(M_{127})$. Therefore the precise knowledge of $\alpha(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron.

4. $\alpha_s(M_{89})$ is predicted to be $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$. $sin^2(\theta_W) = .23120$, $\alpha_{em}(M_{89}) \simeq 1/127$, and $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ give $1/\alpha_{U(1)}(M_{89}) = 97.6374$. $\alpha = \alpha_{em}$ option gives $1/\alpha_s(M_{89}) \simeq 10$, which is consistent with experimental facts. $\alpha = \alpha_{U(1)}$ option gives $\alpha_s(M_{89}) = 0.1572$, which is larger than QCD value. Hence $\alpha = \alpha_{em}$ option is favored.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

5.1.5 Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

1. The natural guess for the general formula would be as

$$g^2(p,r) = kg_K^2 \times exp[-a_g(p,r) \times S_K(CP_2)] . \quad (5.15)$$

here k is a numerical constant.

2. The condition
$g_K^2 = e^2(M_{127})$ fixes the value of k if it's value does not depend on the character of gauge interaction:

$$k = exp[a_{gr}(M_{127}, r = 1) \times S_K(CP_2)] \ . \tag{5.16}$$

Hence the general formula reads as

$$g^2(p,r) = g_K^2 \times exp[(-a_g(p,r) + a_{gr}(M_{127}), r = 1)) \times S_K(CP_2)] \ . \tag{5.17}$$

The value of $a(M_{127}, r = 1)$ is near to its maximum value so that the exponential factor tends to increase the value of g^2 from e^2. The formula can reproduce α_s and various electro-weak couplings although it is quite possibile that Weinberg angle corresponds to a group theoretic factor not representable in terms of $a_g(p,r)$. The volume of the CP_2 type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

3. α_{em} in very long length scales would correspond to

$$e^2(p \to \infty, r = 1) = e^2 \times exp[(-1 + a(M_{127}), r = 1)) \times S_K(CP_2)] = e^2 x \ , \tag{5.18}$$

where x is in the range $[0.6549, 0.6609]$.

5.1.6 Formula relating v_0 to α_K and $R^2/\hbar G$

The parameter $v_0 = 2^{-11}$ plays a key role in the formula for gravitational Planck constant and can be also seen as a fundamental constant in TGD framework. As a matter, factor v_0 has interpretation as velocity parameter and is dimensionless when $c = 1$ is used.

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{\hbar G} \ . \end{aligned} \tag{5.19}$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 5.19 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi} \frac{2^{2m-1}}{K} . \quad (5.20)$$

For both $K = e^q$ with $q = 17$ and $K = 2^q$ option with $q = 24 + 1/2$ $m = 10$ is the smallest integer giving $b < 1$. $K = e^q$ option gives $b = .3302$ (.0826) and $K = 2^q$ option gives $b = .3362$ (.0841) for $m = 10$ ($m = 11$).

$m = 10$ corresponds to one third of the action of free cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor rather near 1/12. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, suggests the identification of the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

5.2 Why Gravitation Is So Weak As Compared To Gauge Interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2 \times 3 \times 5.... \times 23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-magnetic coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{em}$ [K8]. The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

5.2.1 Why gravitational interaction is weak?

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of WCW vacuum functional would give $exp[-2S_K(CP_2)]$ factor cancelling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

1. As already found, the key feature of CP_2 type vacuum extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

2. All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X_V^3 but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

3. The contributions of CP_2 type vacuum extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the WCW containing N CP_2 type extremals has the approximate structure $CH(N) = CH(0) \times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes

about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

If one takes absolutely seriously the number theoretic argument based on $R^2/G = exp(q)$ ansatz then M_{127} is selected uniquely as the space-time sheet of gravitons and the predicted gravitational coupling strength is indeed weak.

5.2.2 What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [K8], this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [K20]. TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [K25]. Also gravitons corresponding to smaller Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2}L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [K11]. In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1 Q_2 m_e^2/M_1 M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [K5, K6]. This prediction is in principle testable.

5.3 Super-Symplectic Gluons And Non-Perturbative Aspects Of Hadron Physics

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [K26] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [K19, K8]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [K14].

According to the model of hadron masses [K14], in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom ("world of classical worlds") in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. It turns out the model assuming same topological mixing of super-symplectic bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \ .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-symplectic colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [K8]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

5.4 Why Mersenne Primes Should Label A Fractal Hierarchy Of Physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

5.4.1 First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection against this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-symplectic bosons or the interaction between super-symplectic bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-symplectic color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labeling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have flux tubes directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for flux tubes of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

5.4.2 Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labeled by primes near power of two couple strongly with Mersenne primes.

5.5 The Formula For The Hadronic String Tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

1. One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \ . \tag{5.21}$$

   a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \ . \tag{5.22}$$

2. One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \ . \tag{5.23}$$

   This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9 \times 10^{-4}L(k)^2 \ . \tag{5.24}$$

   The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

5.6 Large Values Of Planck Constant And Electro-Weak And Strong Coupling Constant Evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action.

Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)}+1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

6 Appendix: Identification Of The Electro-Weak Couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B4] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{6.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{6.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{6.3}$$

and

$$B = 2re^3 \ , \tag{6.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{6.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{6.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{6.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{6.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{6.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{6.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{6.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{6.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{6.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{6.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{6.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{6.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{6.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{6.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{6.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{6.20}$$

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