

Introduction to "Towards M-Matrix"

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March 7, 2018

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1 Basic Ideas Of Topological Geometroynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

1.1 Basic Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).
2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in imbedding space metric and parallel translation using spinor connection of imbedding space.

The induction procedure applies to octonionic structure and the conjecture is that for preferred extremals the induced octonionic structure is quaternionic: again one just projects the octonion units. I have proposed that one can lift space-time surfaces in H to the Cartesian product of the twistor spaces of M^4 and CP_2 , which are the only 4-manifolds allowing twistor space with Kähler structure. Now the twistor structure would be induced in some sense, and should co-incide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of space-time surface. This poses constraint on the imbedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows

a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogeneity in background space-time surface which looks like the space-time of general relativity in long length scales.

One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. A possible resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics. A possible objection is that the Kähler-Dirac gamma matrices do not define an integrable distribution of 2-planes defining string world sheet.

An even stronger condition would be that the induced classical gauge fields at string world sheet vanish: this condition is allowed by the topological description of particles. The CP_2 projection of string world sheet would be 1-dimensional. Also the number theoretical condition that octonionic and ordinary spinor structures are equivalent guaranteeing that fermionic dynamics is associative leads to the vanishing of induced gauge fields.

The natural action would be given by string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant would be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality condition. The hierarchy of Planck constants $\hbar_{eff}/\hbar = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of super string theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations

for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether also induced spinor fields associated with Kähler-Dirac action and de-localized inside entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using imbeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces serve are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [?, ?, ?]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.
4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the imbedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.
5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are

introduced: Kähler geometry and associated spinor structure are enough. “Topological” in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [?]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of everyday world represent non-trivial topology of space-time in TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerges as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchies of Planck constants $h_{eff} = n \times h$ reducing to the quantum criticality of TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.2 Two Vision About TGD And Their Fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H -metric to four-surface define classical electro-weak, color gauge fields and metric in X^4 .

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs

of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models. Scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a “topological condensate” containing matter as particle like 3-surfaces “glued” to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the “topological condensate” there could be “vapor phase” that is a “gas” of particle like 3-surfaces and string like objects (counterpart of the “baby universes” of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see **Fig.** <http://tgdtheory.fi/appfigures/manysheeted.jpg> or **Fig.** ?? in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell’s theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at

which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other things this leads to models for cell membrane, nerve pulse, and EEG.

1.4 P-Adic Variants Of Space-Time Surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. One might say that TGD space-time is adelic. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.5 The Threads In The Development Of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.
2. The discussions with Tony Smith initiated a fourth thread which deserves the name “TGD as a generalized number theory”. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the “physics as generalized number theory” thread.
3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called “physics as a generalized number theory”. In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K22, K14, K9, K29, K18, K28, K27, K17] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K20, K3, K11, K2, K5, K6, K7, K16, K26] are warmly recommended to the interested reader.

1.5.1 *Quantum TGD as spinor geometry of World of Classical Worlds*

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was “Do not quantize”. The basic ingredients to the new

approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH (“world of classical worlds”, WCW) consisting of all possible 3-surfaces in H . “All possible” means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.
2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!
3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory ¹
4. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense.

In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of

¹There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as Kähler action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also Kähler Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K24]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. \circ tells that S acts at the active boundary of CD only. It turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix. S-matrix has interpretation as exponential of the Virasoro generator L_{-1} of the Virasoro algebra associated with super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The Kähler-Dirac gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.
6. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time

(recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only “almost” since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.5.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space (“world of classical worlds”, WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name “TGD as a generalized number theory”. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. *p-Adic TGD and fusion of real and p-adic physics to single coherent whole*

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduce the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of

this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, imbedding space, and WCW.

The notion of p-adic manifold [K30] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheet as chart map provided a possible solution of the basic challenge of relating real and p-adic classical physics. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, “thought bubbles”). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach and this leads to problems with symmetries: canonical identification does not commute with symmetries.

It is now clear that much more elegant approach based on abstraction exists [K31]. The map of real preferred extremals to p-adic ones is not induced from a local correspondence between points but is global. Discretization occurs only for the parameters characterizing string world sheets and partonic 2-surfaces so that they belong to some algebraic extension of rationals. Restriction to these 2-surfaces is possible by strong form of holography. Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** <http://tgdtheory.fi/appfigures/cat.jpg> or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces

(imaginings) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K8].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative (“quaternionic” is equivalent with “associative”) surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K19]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.
2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K19] ? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4 . This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.
3. $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H . One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.
4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.
5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8

or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K19] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2 . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the “free” gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.
2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.
3. For gamma matrix option induced rather than Kähler-Dirac gamma matrices must be in question since Kähler-Dirac gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields.

The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.6 Hierarchy Of Planck Constants And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.6.1 Dark matter as large \hbar phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of \hbar_{gr} . Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that \hbar_{gr} would be much smaller. Large \hbar_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K15].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $\hbar_{eff} = n \times \hbar_{gr}$. The large value of \hbar_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $\hbar_{eff}/\hbar = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n . Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that fermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $\hbar_{eff}/\hbar = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = \hbar f_{high} = \hbar_{eff} f_{low}$) of bunch of n low energy gravitons.

1.6.2 Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K12, K13, K25]) support the view that dark matter might be a key player in living matter.

1.6.3 Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant \hbar_0 is assigned with a multiple singular covering of the imbedding space [K4]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invari-

ance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see **Fig.** <http://tgdtheory.fi/appfigures/planckhierarchy.jpg> or **Fig. ??** the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n .

1.6.4 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff} .

1.7 Twistors And TGD

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K21]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2 . The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric. This condition would define the dynamics, and the conjecture is that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

The condition that the basic formulas for the twistors in M^8 serving as tangent space of imbedding space generalize. This is the case if one introduces octonionic sigma matrices allowing twistor representation of 8-momentum serving as dual for four-momentum and color quantum numbers. The conditions that octonionic spinors are equivalent with ordinary requires that the induced gamma matrices generate quaternionic sub-algebra at given point of string world sheet. This is however not enough: the charge matrices defined by sigma matrices can also break associativity and induced gauge fields must vanish: the CP_2 projection of string world sheet would be one-dimensional at most. This condition is symplectically invariant. Note however that for the interior dynamics of induced spinor fields octonionic representations of Clifford algebra cannot be equivalent with the ordinary one.

One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of superconformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this is nothing but a concretization of Equivalence Principle. Also a connection with string model emerges.

Twistor approach developed rapidly during years. Witten's twistor string theory generalizes: the most natural counterpart of Witten's twistor strings is partonic 2-surface. The notion of positive Grassmannian has emerged and TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the

only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings. Braid interpretation gives further support for the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

2 Bird's Eye Of View About The Topics Of The Book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

1. Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.
3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.
4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.
5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle

carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

2. Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M_+^4 \times CP_2$, where δM_+^4 is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

3. Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

4. The organization of the book

The first part of the book describes basic quantum TGD in its recent form.

1. The properties of the preferred extremals of Kähler action are crucial for the construction and the discussion of known extremals is therefore included.
2. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD.
3. In zero energy ontology S-matrix is replaced with M-matrix and identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones. M-matrix is a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal.

Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.

4. A category theoretical formulation of quantum TGD is considered. Finite measurement resolution realized in terms of a fractal hierarchy of causal diamonds inside causal diamonds leads to a stringy formulation of quantum TGD involving effective replacement of the 3-D light-like surface with a collection of braid strands representing the ends of strings. A formulation in terms of category theoretic concepts is proposed and leads to a hierarchy of algebras forming what is known as operads.
5. Twistors emerge naturally in TGD framework and could allow the formulation of low energy limit of the theory in the approximation that particles are massless. The replacement of massless plane waves with states for which amplitudes are localized are light-rays is suggestive in twistor theoretic framework. Twistors could allow also a dual representation of space-time surfaces in terms of surfaces of $X \times CP_2$, where X is 8-D twistor space or its 6-D projective variant. These surfaces would have dimension higher than four in non-perturbative phases meaning an analogy with branes. In full theory a massive particles must be included but represent a problem in approach based on standard twistors. The interpretation of massive particles in 4-D sense as massless particles in 8-D sense would resolve the problem and requires a generalization of twistor concept involving in essential manner the triality of vector and spinor representations of $SO(7, 1)$.
6. In TGD Universe bosons are in well-defined sense bound states of fermion and anti-fermion. This leads to the notion of bosonic emergence meaning that the fundamental action is just Dirac action coupled to gauge potentials and bosonic action emerges as part of effective action as one functionally integrates over the spinor fields. This kind of approach predicts the evolution of all coupling constants if one is able to fix the necessary UV cutoffs of mass and hyperbolic angle in loop integrations. The guess for the hyperbolic cutoff motivated by the geometric view about finite measurement resolution predicts coupling constant evolution which is consistent with that predicted by standard model. The condition that all N-vertices defined by fermionic loops vanish for $N > 3$ when incoming particles are massless gives hopes of fixing completely the hyperbolic cutoff from fundamental principles.

3 Sources

The eight online books about TGD [K22, K14, K29, K18, K9, K28, K27, K17] and nine online books about TGD inspired theory of consciousness and quantum biology [K20, K3, K11, K2, K5, K6, K7, K16, K26] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (<http://tinyurl.com/ybv8dt4n>) contains a lot of material about TGD. In particular, a TGD glossary at <http://tinyurl.com/yd6jf3o7>.

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (<http://tinyurl.com/ycyrxj4o>) founded by Lian Sidorov and in *Prespacetime Journal* (<http://tinyurl.com/ycvktjhn>), *Journal of Consciousness Research and Exploration* (<http://tinyurl.com/yba4f672>), and *DNA Decipher Journal* (<http://tinyurl.com/y9z52khg>), all of them founded by Huping Hu. One can find the list about the articles published at <http://tinyurl.com/ybv8dt4n>. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

4 The contents of the book

4.1 Part I: The recent view about field equations

4.1.1 Basic extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at

fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the “world of classical worlds” (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and associativity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting them, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute “preferred” is not relevant for them.
3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = \exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.
4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $\exp(K)$ is analogous to the exponent $\exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.
5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2 , gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.
2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.
3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariance from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

4.1.2 About Identification of the Preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody

type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K23]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n -multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

4.2 WCW Spinor Structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space (“world of classical worlds” (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the “orbital” degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.
3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$ -dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.
4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vanish at these surfaces.

The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.
3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covariantly constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

awcwspin

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2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to

the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$ -dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.
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Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.
3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covariantly constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with

left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

4.3 Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the Kähler-Dirac action.

4.3.1 Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occam's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Kähler action in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent imbedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At imbedding space level the modes of imbedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at imbedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

4.4 Part II: General Theory

4.4.1 Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. In fact, the general definition of geometry is as a structure characterized by given symmetries. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the “world of the classical worlds” (WCW) identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. *Physics as infinite-dimensional Kähler geometry*

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. Kähler-Dirac equation (or Kähler-Dirac equation) gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

A crucial feature of the Kähler-Dirac equation is the localization of the modes to 2-D surfaces with vanishing induced W fields (this in generic situation and for all modes but covariantly constant right-handed neutrino): this is needed in order to have modes with well-defined em charge. Also Z^0 fields can be vanish and is expected to do so - at least above weak scale. This implies that all elementary particles are string like objects in very concrete sense.

2. *p*-adic physics and *p*-adic variants of basic symmetries

p-Adic mass calculations relying on *p*-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and *p*-adic thermodynamics. The need to fuse real physics and various *p*-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and *p*-adics together along common rationals and algebraics. The interpretation of *p*-adic space-time sheets is as correlates for cognition and intentionality. *p*-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

3. *Hierarchy of Planck constants and dark matter hierarchy*

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzle leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an *n*-fold singular covering or factor space of CP_2 or of a causal diamond (*CD*) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups. The extension of imbedding space can be seen as a formal tool allowing an elegant description of the multi-sheetedness due to the non-determinism of Kähler action. At the space-like ends the sheets fuse together so that a singular covering is in question.

The original intuition was the the space-time would be *n*-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have *n* conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by *n*. This defines infinite number of inclusion hierarchies $\dots \subset C(n_1) \subset C(n_3) \dots$ such that n_{i+1} divides n_i . These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

4. *Number theoretical symmetries*

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids - ends of string world sheets - can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times \dots$ of the completion of S_∞ .
2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, $SU(3)$ acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4 , one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4 . One ends up with quantum TGD by

considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H . so that TGD could be seen as a generalized number theory.

4.4.2 Zero Energy Ontology and Matrices

During years the basic mathematical and conceptual building bricks of quantum TGD have become rather obvious.

One important building brick is Zero Energy Ontology (ZEO). ZEO forces to generalize the notion of S-matrix by introducing M-matrices and U-matrix and allows a new view about observer based on TGD inspired theory of consciousness.

Second building brick consists of various hierarchies and connections between them. There is the hierarchy of quantum criticalities for super-symplectic algebra and its Yangian extension acting as a spectrum generating algebra. This hierarchy is closely related to the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchies of criticalities correspond also to fractal hierarchies of breakings of super-symplectic gauge conformal symmetry: only the sub-algebra isomorphic to the original gauge algebra acts as gauge algebra after the breaking. At each step one criticality is reduced and the number of physical degrees of freedom increases. There is also a natural connection between these hierarchies with hierarchies of hyperfinite factors of type II_1 (HFFs) and their inclusions providing a description for the notion of measurement resolution. Also the construction of zero energy states using super-symplectic Yangian provides a concrete realization for the notion of finite measurement resolution in the structure of zero energy states and manifesting in the structure of space-time surfaces serving as classical correlates of quantum states.

There are also other important building bricks but in this chapter only ZEO and hyper-finite factors are discussed.

4.4.3 What Scattering Amplitudes Should Look Like?

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the ZEO applied already earlier in TGD inspired cosmology. ZEO motivated the replacement of the term "S-matrix" with "M-matrix".

The general mathematical concepts are not enough to get to the level of concrete scattering amplitudes. The notion of preferred extremal inspiring the notion of generalized Feynman diagram is central in bringing in this concretia. The very notion of preferred extremals means that ordinary Feynman diagrams providing a visualization of path integral are not in question. Generalized Feynman diagrams have 4-D Euclidian space-time regions (wormhole contacts) as lines, and light-like partonic orbits of 2-surfaces as 3-D lines. String world sheets carrying fermions are also present and have 1-D boundaries at the light-like orbits of partonic 2-surfaces carrying fermion number and light-like 8-momenta suggesting strongly 8-D generalization of twistor approach.

The resulting objects could be indeed seen as generalizations of twistor diagrams rather than Feynman diagrams. The preferred extremal property strongly encourages the old and forgotten TGD inspired idea as sequences of algebraic operations with product and co-product representing 3-vertices. The sequences connect given states at the opposite boundaries of CD and have minimal length. The algebraic structure in question would be the Yangian of the super-symplectic algebra with generators identified as super-symplectic charges assignable to strings connecting partonic 2-surfaces.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction requires the introduction of generalization of twistor approach to 8-D context.

4.4.4 Does Riemann Zeta Code for Generic Coupling Constant Evolution?

A general model for the coupling constant evolution is proposed. The analogy of Riemann zeta and fermionic zeta $\zeta_F(s)/\zeta_F(2s)$ with complex square root of a partition function natural in Zero Energy Ontology suggests that the poles of $\zeta_F(ks)$, $k = 1/2$, correspond to complexified critical temperatures identifiable as inverse of Kähler coupling strength itself having interpretation as inverse of critical temperature. One can actually replace the argument s of ζ_F with Möbius transformed argument $w = (as + b)/(cs + d)$ with a, b, c, d real numbers, rationals, or even integers. For α_K $w = (s + b)/2$ is proper choices and gives zeros of $\zeta(s)$ and $s = 2 - b$ as poles. The identification $\alpha_K = \alpha_{U(1)}$ leads to a prediction for α_{em} , which deviates by .7 per cent from the experimental value at low energies (atomic scale) if the experimental value of the Weinberg angle is used. The conjecture generalizes also to weak, color and gravitational interactions when general Möbius transformation leaving upper half-plane invariant is allowed. One ends up with a general model predicting successfully the entire electroweak coupling constant evolution successfully from the values of fine structure constant at atomic or electron scale and in weak scale.

4.4.5 Could $\mathcal{N} = 2$ Super-conformal Theories Be Relevant For TGD?

TGD has as is symmetries super-conformal symmetry (SCS), which is a huge extension of the ordinary SCS. For instance, the infinite-dimensional symplectic group plays the role of finite-dimension Lie-group as Kac-Moody group and the conformal weights for the generators of algebra corresponds to the zeros of fermionic zeta and their number of generators is therefore infinite.

The relationship of TGD SCS to super-conformal field theories (SCFTs) known as minimal models has remained without definite answer. The most general super-conformal algebra (SCA) assignable to string world sheets by strong form of holography has \mathcal{N} equal to the number of spin states of leptonic and quark type fundamental spinors but the space-time SUSY is badly broken for it. Covariant constancy of the generating spinor modes is replaced with holomorphy - kind of "half covariant constancy". Right-handed neutrino and antineutrino are excellent candidates for generating $\mathcal{N} = 2$ SCS with a minimal breaking of the corresponding space-time SUSY.

$\mathcal{N} = 2$ SCS has also some inherent problems. The critical space-time dimension is $D = 4$ but the existence of complex structure seems to require the space-time has metric signature different from Minkowskian: here TGD suggests a solution. $\mathcal{N} = 2$ SCFTs are claimed also to reduce to topological QFTs under some conditions: this need not be a problem since TGD can be characterized as almost topological QFT. What looks like a further problem is that p-adic mass calculations require half-integer valued negative conformal weight for the ground state (and vanishing weight for massless states). One can however shift the scaling generator L_0 to get rid of problem: the shift has physical interpretation in TGD framework and must be half integer valued which poses the constraint $h = K/2$, $K = 0, 1, 2, ..$ on the representations of SCA.

$\mathcal{N} = 2$ SCA allows a spectral flow taking Ramond representations to Neveu-Schwartz variant of algebra. The physical interpretation is as super-symmetry mapping fermionic states to bosonic states. The representations of $\mathcal{N} = 2$ SCA allowing degenerate states with positive central charge c and non-vanishing ground state conformal weight h give rise to minimal models allowing ADE classification, construction of partition functions, and even of n-point functions. This could make S-matrix of TGD exactly solvable in the fermionic sector. The ADE hierarchy suggests a direct interpretation in terms of orbifold hierarchy assignable to the hierarchy of Planck constants associated with the super-symplectic algebra: primary fields would correspond to orbifolds identified as coset spaces of ADE groups. Also an interpretation in terms of inclusions of hyper-finite factors is highly suggestive.

4.5 Part III: Twistors and TGD

4.5.1 TGD Variant of Twistor Story

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D imbedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its

Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2, 2)/SU(2, 1) \times U(1)$ and F_3 defines twistor space for the imbedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD and classical TGD defined by the extremals of Kähler action. In the following I summarize the background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams.

There is also a very closely analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. The landscape is replaced with twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach. Furthermore, one ends up to a formulation of the scattering amplitudes in terms of Yangian of the super-symplectic algebra relying on the idea that scattering amplitudes are sequences consisting of algebraic operations (product and co-product) having interpretation as vertices in the Yangian extension of super-symplectic algebra. These sequences connect given initial and final states and having minimal length. One can say that Universe performs calculations.

4.5.2 From Principles to Diagrams

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

4.5.3 About Twistor Lift of TGD

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner.

One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n , $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter twistor lift is studied in detail.

1. The first working hypothesis is that the values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. The first interesting finding is that allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are minimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.
2. The other working hypothesis is $\alpha_K(M^4) = \alpha_K(CP_2)$. The small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs. This option looks more natural.

Both options lead to a generalization of Chladni mechanism to a “dynamics of avoidance” meaning that at least asymptotically the two dynamics decouple. This leads to an interpretation with profound implications for the views about what happens in particle physics experiment and in quantum measurement, for consciousness theory and for quantum biology.

A related observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

4.5.4 Some Questions Related to the Twistor Lift of TGD

In this chapter I consider questions related to both classical and quantum aspects of twistorialization.

1. The first group of questions relates to the twistor lift of classical TGD. What does the induction of the twistor structure really mean? Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications? How does gravitational coupling emerge at fundamental level? Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space?

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift can be criticized and allows an alternative. This option led to a much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant: cosmological term would correspond to the *entire* 4-D action and the cancellation of Kähler action and cosmological term would lead to the small value of the effective cosmological constant.

2. Second group of questions relates to the construction of scattering amplitudes. The idea is to generalize the usual construction for massless states. In TGD all single particle states are massless in 8-D sense and this gives excellent hopes about the applicability of 8-D twistor approach. $M^8 - H$ duality turns out to be the key to the construction. Also the holomorphy of twistor amplitudes in helicity spinors λ_i and independence on $\tilde{\lambda}_i$ is crucial. The basic vertex corresponds to 4-fermion vertex for which the simplest expression can be written immediately. $n > 4$ -fermion scattering amplitudes can be also written immediately.

If scattering diagrams correspond to computations as number theoretic vision suggests, the diagrams should be reducible to tree diagrams by moves generalizing the old-fashioned hadronic duality. This condition reduces to the vanishing of loops which in terms of BCFW recursion formula states that the twistor diagrams correspond to closed objects in what might be called WCFW homology.

4.5.5 The Recent View about Twistorialization in TGD Framework

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.
2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization in terms of “cognitive representations” as space-time time points with M^8 -coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adèle. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

This must imply the reduction of “world of classical worlds” (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H , where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

4.6 Part IV: Category theory and TGD

4.6.1 Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form

and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type II₁ and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

Years after writing this chapter a very interesting new TGD related candidate for a category emerged. The preferred extremals of Kähler action would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of imbedding space respects associativity (co-associativity). The duality would allow to construct new preferred extremals of Kähler action.

4.6.2 Could categories, tensor networks, and Yangians provide the tools for handling the complexity of TGD?

TGD Universe is extremely simple locally but the presence of various hierarchies make it to look extremely complex globally. Category theory and quantum groups, in particular Yangian or its TGD generalization are most promising tools to handle this complexity. The arguments developed in the sequel suggest the following overall view.

1. Positive and negative energy parts of zero energy states can be regarded as tensor networks identifiable as categories. The new element is that one does not have only particles (objects) replaced with partonic 2-surfaces but also strings connecting them (morphisms). Morphisms and functors provide a completely new element not present in standard model. For instance, S-matrix would be a functor between categories. Various hierarchies of of TGD would in turn translate to hierarchies of categories.
2. TGD view about generalized Feynman diagrams relies on two general ideas. First, the twistor lift of TGD replaces space-time surfaces with their twistor-spaces getting their twistor structure as induced twistor structure from the product of twistor spaces of M^4 and CP_2 . Secondly, topological scattering diagrams are analogous to computations and can be reduced to tree diagrams with braiding. This picture fits very nicely with the picture suggested by fusion categories. At fermionic level the basic interaction is 2+2 scattering of fermions occurring at the vertices identifiable as partonic 2-surface and re-distributes the fermion lines between partonic 2-surfaces. This interaction is highly analogous to what happens in braiding interaction but vertices expressed in terms of twistors depend on momenta of fermions.
3. Braiding transformations take place inside the light-like orbits of partonic 2-surfaces defining boundaries of space-time regions with Minkowskian and Euclidian signature of induced metric respectively permuting two braid strands. R-matrix satisfying Yang-Baxter equation characterizes this operation algebraically.
4. Reconnections of fermionic strings connecting partonic 2-surfaces are possible and suggest interpretation in terms of 2-braiding generalizing ordinary braiding: string world sheets get

knotted in 4-D space-time forming 2-knots and strings form 1-knots in 3-D space. Reconnection induces an exchange of braid strands defined by the boundaries of the string world sheet and therefore exchange of fermion lines defining boundaries of string world sheets. A generalization of quantum algebras to include also algebraic representation for reconnection is needed. Also reconnection might reduce to a braiding type operation.

Yangians look especially natural quantum algebras from TGD point of view. They are bi-algebras with co-product Δ . This makes the algebra multi-local raising hopes about the understanding of bound states. Δ -iterates of single particle system would give many-particle systems with non-trivial interactions reducing to kinematics.

One should assign Yangian to various Kac-Moody algebras (SKMAs) involved and even with super-conformal algebra (SSA), which however reduces effectively to SKMA for finite-dimensional Lie group if the proposed gauge conditions meaning vanishing of Noether charges for some sub-algebra H of SSA isomorphic to it and for its commutator $[SSA, H]$ with the entire SSA. Strong form of holography (SH) implying almost 2-dimensionality motivates these gauge conditions. Each SKMA would define a direct summand with its own parameter defining coupling constant for the interaction in question.

4.6.3 Are higher structures needed in the categorification of TGD?

The notion of higher structures promoted by John Baez looks very promising notion in the attempts to understand various structures like quantum algebras and Yangians in TGD framework. The stimulus for this article came from the nice explanations of the notion of higher structure by Urs Schreiber. The basic idea is simple: replace “=” as a blackbox with an operational definition with a proof for $A = B$. This proof is called homotopy generalizing homotopy in topological sense. n -structure emerges when one realizes that also the homotopy is defined only up to homotopy in turn defined only up...

In TGD framework the notion of measurement resolution defines in a natural manner various kinds of “=”s and this gives rise to resolution hierarchies. Hierarchical structures are characteristic for TGD: hierarchy of space-time sheet, hierarchy of p-adic length scales, hierarchy of Planck constants and dark matters, hierarchy of inclusions of hyperfinite factors, hierarchy of extensions of rationals defining adeles in adelic TGD and corresponding hierarchy of Galois groups represented geometrically, hierarchy of infinite primes, self hierarchy, etc...

In this article the idea of n -structure is studied in more detail. A rather radical idea is a formulation of quantum TGD using only cognitive representations consisting of points of space-time surface with imbedding space coordinates in extension of rationals defining the level of adelic hierarchy. One would use only these discrete points sets and Galois groups. Everything would reduce to number theoretic discretization at space-time level perhaps reducing to that at partonic 2-surfaces with points of cognitive representation carrying fermion quantum numbers.

Even the “world of classical worlds” (WCW) would discretize: cognitive representation would define the coordinates of WCW point. One would obtain cognitive representations of scattering amplitudes using a fusion category assignable to the representations of Galois groups: something diametrically opposite to the immense complexity of the WCW but perhaps consistent with it. Also a generalization of McKay’s correspondence suggests itself: only those irreps of the Lie group associated with Kac-Moody algebra that remain irreps when reduced to a subgroup defined by a Galois group of Lie type are allowed as ground states. Also the relation to number theoretic Langlands correspondence is very interesting.

4.6.4 Is Non-associative Physics and Language Possible Only in Many-Sheeted Space-Time?

Language is an essentially non-associative structure as the necessity to parse linguistic expressions essential also for computation using the hierarchy of brackets makes obvious. Hilbert space operators are associative so that non-associative quantum physics does not seem plausible without an extension of what one means with physics. Associativity of the classical physics at the level of *single* space-time sheet in the sense that tangent or normal spaces of space-time sheets are associative as sub-spaces of the octonionic tangent space of 8-D imbedding space $M^4 \times CP_2$ is one of the key conjectures of TGD. But what about many-sheeted space-time? The sheets of the many-sheeted

space-time form hierarchies labelled by p-adic primes and values of Planck constants $h_{eff} = n \times h$. Could these hierarchies provide space-time correlates for the parsing hierarchies of language and music, which in TGD framework can be seen as kind of dual for the spoken language? For instance, could the braided flux tubes inside larger braided flux tubes inside... realize the parsing hierarchies of language, in particular topological quantum computer programs? And could the great differences between organisms at very different levels of evolution but having very similar genomes be understood in terms of widely different numbers of levels in the parsing hierarchy of braided flux tubes- that is in terms of magnetic bodies as indeed proposed. If the intronic portions of DNA connected by magnetic flux tubes to the lipids of lipid layers of nuclear and cellular membranes make them topological quantum computers, the parsing hierarchy could be realized at the level of braided magnetic bodies of DNA. The mathematics needed to describe the breaking of associativity at fundamental level seems to exist. The hierarchy of braid group algebras forming an operad combined with the notions of quasi-bialgebra and quasi-Hopf algebra discovered by Drinfeld are highly suggestive concerning the realization of weak breaking of associativity.

4.7 Part V: Miscellaneous topics

4.7.1 Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the Kähler-Dirac action. The introduction of a measurement interaction term to the action allows to understand how stringy propagator results and provides profound insights about physics predicted by TGD.

The appearance of the momentum (and possibly also color quantum numbers) in the measurement interaction couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation a finite-dimensional SUSY algebra or the fermionic part of super-conformal algebra with an infinite number of oscillator operators results. The addition of a fermion in particular mode would define particular super-symmetry. Zero energy ontology implies that fermions as wormhole throats correspond to chiral super-fields assignable to positive or negative energy SUSY algebra whereas bosons as wormhole contacts with two throats correspond to the direct sum of positive and negative energy algebra and fields which are chiral or antichiral with respect to both positive and negative energy theta parameters. This super-symmetry is badly broken due to the dynamics of the Kähler-Dirac operator which also mixes M^4 chiralities inducing massivation. Since righthanded neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.
2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D=10$ and $D=11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel.
3. The massivation of particles is basic problem of both SUSYs and twistor approach. The fact that particles which are massive in M^4 sense can be interpreted as massless particles in

$M^4 \times CP_2$ suggests a manner to understand super-symmetry breaking and massivation in TGD framework. The octonionic realization of twistors is one possibility in this framework and quaternionicity condition guaranteeing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

4. The first approach is based on an approximation assuming only the super-multiplets generated by right-handed neutrino or both right-handed neutrino and its antineutrino. The assumption that right-handed neutrino has fermion number opposite to that of the fermion associated with the wormhole throat implies that bosons correspond to $\mathcal{N} = (1, 1)$ SUSY and fermions to $\mathcal{N} = 1$ SUSY identifiable also as a short representation of $\mathcal{N} = (1, 1)$ SUSY algebra trivial with respect to positive or negative energy algebra. This means a deviation from the standard view but the standard SUSY gauge theory formalism seems to apply in this case.
5. A more ambitious approach would put the modes of induced spinor fields up to some cutoff into super-multiplets. At the level next to the one described above the lowest modes of the induced spinor fields would be included. The very large value of \mathcal{N} means that $\mathcal{N} \leq \exists \in$ SUSY cannot define the QFT limit of TGD for higher cutoffs. One must generalize SUSYs gauge theories to arbitrary value of \mathcal{N} but there are reasons to expect that the formalism becomes rather complex. More ambitious approach working at TGD however suggest a more general manner to avoid this problem.
 - (a) One of the key predictions of TGD is that gauge bosons and Higgs can be regarded as bound states of fermion and antifermion located at opposite throats of a wormhole contact. This implies bosonic emergence meaning that its QFT limit can be defined in terms of Dirac action. The resulting theory was discussed in detail in [K10] and it was shown that bosonic propagators and vertices can be constructed as fermionic loops so that all coupling constant follow as predictions. One must however pose cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops in order to obtain finite theory and to avoid massivation of bosons. The resulting coupling constant evolution is consistent with low energy phenomenology if the cutoffs in hyperbolic angle as a function of p-adic length scale is chosen suitably.
 - (b) The generalization of bosonic emergence that the TGD counterpart of SUSY is obtained by the replacement of Dirac action with action for chiral super-field coupled to vector field as the action defining the theory so that the propagators of bosons and all their super-counterparts would emerge as fermionic loops.
 - (c) The huge super-symmetries give excellent hopes about the cancelation of infinities so that this approach would work even without the cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops. Cutoffs have a physical motivation in zero energy ontology but it could be an excellent approximation to take them to infinity. Alternatively, super-symmetric dynamics provides cutoffs dynamically.
6. The condition that $\mathcal{N} = \infty$ variants for chiral and vector superfields exist fixes completely the identification of these fields in zero energy ontology.
 - (a) In this framework chiral fields are generalizations of induced spinor fields and vector fields those of gauge potentials obtained by replacing them with their super-space counterparts. Chiral condition reduces to analyticity in theta parameters thanks to the different definition of hermitian conjugation in zero energy ontology (θ is mapped to a derivative with respect to theta rather than to $\bar{\theta}$) and conjugated super-field acts on the product of all theta parameters.
 - (b) Chiral action is a straightforward generalization of the Dirac action coupled to gauge potentials. The counterpart of YM action can emerge only radiatively as an effective action so that the notion emergence is now unavoidable and indeed basic prediction of TGD.

- (c) The propagators associated with the monomials of n theta parameters behave as $1/p^n$ so that only $J = 0, 1/2, 1$ states propagate in normal manner and correspond to normal particles. The presence of monomials with number of thetas higher than 2 is necessary for the propagation of bosons since by the standard argument fermion and scalar loops cancel each other by super-symmetry. This picture conforms with the identification of graviton as a bound state of wormhole throats at opposite ends of string like object.
- (d) This formulation allows also to use Kähler-Dirac gamma matrices in the measurement interaction defining the counterpart of super variant of Dirac operator. Poincare invariance is not lost since momenta and color charges act on the tip of CD rather than the coordinates of the space-time sheet. Hence what is usually regarded as a quantum theory in the background defined by classical fields follows as exact theory. This feeds all data about space-time sheet associated with the maximum of Kähler function. In this approach WCW as a Kähler manifold is replaced by a cartesian power of CP_2 , which is indeed quaternionic Kähler manifold. The replacement of light-like 3-surfaces with number theoretic braids when finite measurement resolution is introduced, leads to a similar replacement.
- (e) Quantum TGD as a “complex square root” of thermodynamics approach suggests that one should take a superposition of the amplitudes defined by the points of a coherence region (identified in terms of the slicing associated with a given wormhole throat) by weighting the points with the Kähler action density. The situation would be highly analogous to a spin glass system since the Kähler-Dirac gamma matrices defining the propagators would be analogous to the parameters of spin glass Hamiltonian allowed to have a spatial dependence. This would predict the proportionality of the coupling strengths to Kähler coupling strength and bring in the dependence on the size of CD coming as a power of 2 and give rise to p-adic coupling constant evolution. Since TGD Universe is analogous to 4-D spin glass, also a sum over different preferred extremals assignable to a given coherence regions and weighted by $\exp(K)$ is probably needed.
- (f) In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processes involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of stringy curve so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest CD present must be actively involved. At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit. It turns out that surprisingly simple candidate for the bi-local action exists.

This statement has become somewhat misleading. It has turned out that all elementary particle in TGD framework are bi-local objects: one can assign to them both closed magnetic flux tubes behaving like strings and closed strings carrying fermion number. For other elementary particles than graviton second wormhole contact carries only neutrino pair neutralizing electroweak-isospin so that above weak scale they correspond to single em charged wormhole contact.

4.7.2 Coupling Constant Evolution in Quantum TGD

How to calculate or at least “understand” the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Basically the inability to calculate is of course due to the lack of understanding.

Zero energy ontology, the construction of M -matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1 , the realization that symplectic invariance of N-point functions providing a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries. p-Adic length scale hypothesis suggests that continuous coupling constant evolution is replaced by discrete p-adic coupling constant

evolution and that number theoretical constraints are of crucial importance. These are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules. This was the picture behind previous updating.

Several steps of progress have however occurred since then.

1. A crucial step in progress has been the understanding of how GRT space-time emerges from the many-sheeted space-time of TGD. At classical level Equivalence Principle (EP) follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.
2. Second powerful idea is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

This idea can be formulated more convincingly in terms of a generalization of the AdS/CFT duality to TGD framework motivated by the generalization of conformal symmetry. In full generality this principle would state that all predictions of the theory can be expressed either in terms of classical fields in the interior of the space-time surface or in terms of scattering amplitudes formulated in terms of fundamental fermions defining the building bricks of elementary particles. The implication would that correlation functions can be also identified as those for classical induce gauge and gravitational fields.

3. Third powerful vision inspired by the notion of preferred extremal - I gave up the vision for years as too crazy - is that scattering amplitudes correspond to sequences of computations and that all computations connecting collectings of algebraic objects produce same scattering amplitudes [K21]. All scattering amplitudes could be reduced to minimal tree diagrams by moving the ends of the lines and snipping away the loops. The 8-D generalization of twistor approach to TGD allows to identify the arithmetics as that of super-symplectic Yangian and basic vertices in the construction correspond to product and co-product in Yangian.
4. The fourth new ingredient is the dramatic increase in the understanding of the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchy corresponds to hierarchy of quantum criticalities at which the sub-algebra of super-symplectic algebra with natural conformal structure changes. Sub-algebras are labelled by integer n : the conformal weights of the sub-algebra come as multiples of n . One has infinite number of hierarchies $n_{i+1} = \prod_{k < i+1} m_k$ which relate naturally to the hierarchies of inclusions of hyper-finite factors. The sub-algebra acts as gauge symmetries whereas the other generators of the full algebra fail to do so. Therefore the increase of n means that gauge degrees of freedom become physical ones. One can assign coupling constant evolution also with these hierarchies and the natural conjecture is that coupling constants for given value of n are renormalization group invariances.

Especially interesting are the implications for the understanding of gravitational binding assuming that strings connecting partonic 2-surfaces are responsible for the formation of bound states. This leads together with the generalization of AdS/CFT corresponds and localization of fermions to string world sheets to a prediction that Kähler action is expressible as

string area in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices. This predicts that the size scale of bound states scales as h_{eff} and it is possible to obtain bound states of macroscopic size unlike for ordinary string area action for which their sizes would be given by Planck length.

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