

Introduction to ”p-Adic Length Scale Hypothesis”

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1 Basic Ideas Of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. The CMAP files at my homepage provide an overview about ideas and evolution of TGD and make easier to understand what TGD and its applications are about (<http://tgdtheory.fi/cmaphtml.html> [L1]).

1.1 Basic Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).
2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in imbedding space metric and parallel translation using spinor connection of imbedding space.

The induction procedure applies to octonionic structure and the conjecture is that for preferred extremals the induced octonionic structure is quaternionic: again one just projects the octonion units. I have proposed that one can lift space-time surfaces in H to the Cartesian product of the twistor spaces of M^4 and CP_2 , which are the only 4-manifolds allowing twistor space with Kähler structure. Now the twistor structure would be induced in some sense, and should co-incide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of space-time surface. This poses constraint on the imbedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogeneity in background space-time surface which looks like the space-time of general relativity in long length scales.

One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. A possible resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics. A possible objection is that the Kähler-Dirac gamma matrices do not define an integrable distribution of 2-planes defining string world sheet.

An even stronger condition would be that the induced classical gauge fields at string world sheet vanish: this condition is allowed by the topological description of particles. The CP_2 projection of string world sheet would be 1-dimensional. Also the number theoretical condition that octonionic and ordinary spinor structures are equivalent guaranteeing that fermionic dynamics is associative leads to the vanishing of induced gauge fields.

The natural action would be given by string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant would be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality condition. The hierarchy of Planck constants $\hbar_{eff}/\hbar = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of super string theories. This option allows the natural interpretation of M^4 type

vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether also induced spinor fields associated with Kähler-Dirac action and de-localized inside entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using imbeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces serve are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.
4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the imbedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. “Topological” in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [?]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of everyday world represent non-trivial topology of space-time in TGD Universe.
6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerges as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchies of Planck constants $h_{eff} = n \times h$ reducing to the quantum criticality of TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.2 Two Vision About TGD And Their Fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A2, A5, A1, A4].

The identification of the space-time as a sub-manifold [A3, A7] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H -metric to four-surface define classical electro-weak, color gauge fields and metric in X^4 .

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models. Scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a “topological condensate” containing matter as particle like 3-surfaces “glued” to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the “topological condensate” there could be “vapor phase” that is a “gas” of particle like 3-surfaces and string like objects (counterpart of the “baby universes” of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see **Fig.** <http://tgdtheory.fi/appfigures/manysheeted.jpg> or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell’s theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other things this leads to models for cell membrane, nerve pulse, and EEG.

1.4 P-Adic Variants Of Space-Time Surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. One might say that TGD space-time is adelic. Also the hierarchy of Planck constants forces

a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.5 The Threads In The Development Of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.
2. The discussions with Tony Smith initiated a fourth thread which deserves the name “TGD as a generalized number theory”. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the “physics as generalized number theory” thread.
3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called “physics as a generalized number theory”. In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K20, K12, K8, K26, K16, K25, K24, K15] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K18, K3, K9, K2, K5, K6, K7, K14, K23] are warmly recommended to the interested reader.

1.5.1 Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was “Do not quantize”. The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH (“world of classical worlds”, WCW) consisting of all possible 3-surfaces in H . “All possible” means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A6, A8, A9]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.
2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!
3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory ¹
4. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense.

In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

¹There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as Kähler action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also Kähler Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K21]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. \circ tells that S acts at the active boundary of CD only. It turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix. S-matrix has interpretation as exponential of the Virasoro generator L_{-1} of the Virasoro algebra associated with super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The Kähler-Dirac gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.
6. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only “almost” since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.5.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space (“world of classical worlds”, WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name “TGD as a generalized number theory”. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. *p-Adic TGD and fusion of real and p-adic physics to single coherent whole*

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduce the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, imbedding space, and WCW.

The notion of p-adic manifold [K27] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheet as chart map provided a possible solution of the basic challenge of relating real and p-adic classical physics. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, “thought bubbles”). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach and this leads to problems with symmetries: canonical identification does not commute with symmetries.

It is now clear that much more elegant approach based on abstraction exists [K28]. The map of real preferred extremals to p-adic ones is not induced from a local correspondence between points but is global. Discretization occurs only for the parameters characterizing string world sheets and partonic 2-surfaces so that they belong to some algebraic extension of rationals. Restriction to these 2-surfaces is possible by strong form of holography. Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** <http://tgdtheory.fi/appfigures/cat.jpg> or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world

sheets and partonic 2-surfaces. Also a generalization of p-adic length scale hypothesis emerges from NMP.

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative (“quaternionic” is equivalent with “associative”) surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K17]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.
2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K17] ? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4 . This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.
3. $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H . One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.
4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.
5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to

surfaces in $M^4 \times CP_2$ [K17] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2 . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the “free” gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.
2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.
3. For gamma matrix option induced rather than Kähler-Dirac gamma matrices must be in question since Kähler-Dirac gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The

interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.6 Hierarchy Of Planck Constants And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.6.1 Dark matter as large \hbar phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of \hbar_{gr} . Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that \hbar_{gr} would be much smaller. Large \hbar_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K13].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $\hbar_{eff} = n \times \hbar_{gr}$. The large value of \hbar_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $\hbar_{eff}/\hbar = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n . Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that fermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $\hbar_{eff}/\hbar = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = \hbar f_{high} = \hbar_{eff} f_{low}$) of bunch of n low energy gravitons.

1.6.2 *Hierarchy of Planck constants from the anomalies of neuroscience and biology*

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K10, K11, K22]) support the view that dark matter might be a key player in living matter.

1.6.3 *Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?*

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant \hbar_0 is assigned with a multiple singular covering of the imbedding space [K4]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the “world of classical worlds”. If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invari-

ance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see **Fig.** <http://tgdtheory.fi/appfigures/planckhierarchy.jpg> or **Fig. ??** the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n .

1.6.4 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff} .

1.7 Twistors And TGD

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K19]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2 . The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric. This condition would define the dynamics, and the conjecture is that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

The condition that the basic formulas for the twistors in M^8 serving as tangent space of imbedding space generalize. This is the case if one introduces octonionic sigma matrices allowing twistor representation of 8-momentum serving as dual for four-momentum and color quantum numbers. The conditions that octonionic spinors are equivalent with ordinary requires that the induced gamma matrices generate quaternionic sub-algebra at given point of string world sheet. This is however not enough: the charge matrices defined by sigma matrices can also break associativity and induced gauge fields must vanish: the CP_2 projection of string world sheet would be one-dimensional at most. This condition is symplectically invariant. Note however that for the interior dynamics of induced spinor fields octonionic representations of Clifford algebra cannot be equivalent with the ordinary one.

One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of superconformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this is nothing but a concretization of Equivalence Principle. Also a connection with string model emerges.

Twistor approach developed rapidly during years. Witten's twistor string theory generalizes: the most natural counterpart of Witten's twistor strings is partonic 2-surface. The notion of positive Grassmannian has emerged and TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the

only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings. Braid interpretation gives further support for the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

2 Bird's Eye Of View About The Topics Of The Book

The book is devoted to the applications of p-adic length scale hypothesis and dark matter hierarchy.

1. p-Adic length scale hypothesis states that primes $p \simeq 2^k$, k integer, in particular prime, define preferred p-adic length scales. Physical arguments supporting this hypothesis are based on the generalization of Hawking's area law for blackhole entropy so that it applies in case of elementary particles.
2. A much deeper number theory based justification for this hypothesis is based on the generalization of the number concept fusing real number fields and p-adic number fields among common rationals or numbers in their non-trivial algebraic extensions. This approach also justifies the notion of multi-p-fractality and allows to understand scaling law in terms of simultaneous $p \simeq 2^k$ - and 2-fractality.
3. Certain anomalous empirical findings inspire in TGD framework the hypothesis about the existence of entire hierarchy of phases of matter identifiable as dark matter. The levels of dark matter hierarchy are labeled by the values of dynamical quantized Planck constant. The justification for the hypothesis provided by quantum classical correspondence and the fact the sizes of space-time sheets identifiable as quantum coherence regions can be arbitrarily large.

The organization of the book is following.

1. The first part of the book is devoted to the description of elementary particle massivation in terms of p-adic thermodynamics.
2. In second part is devoted to the detailed calculation of masses of elementary particles and hadrons, and to various new physics suggested or predicted by the resulting scenario.

3 Sources

The eight online books about TGD [K20, K12, K26, K16, K8, K25, K24, K15] and nine online books about TGD inspired theory of consciousness and quantum biology [K18, K3, K9, K2, K5, K6, K7, K14, K23] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (<http://www.tgdtheory.com/curri.html>) contains a lot of material about TGD. In particular, there is summary about TGD and its applications using CMAP representation serving also as a TGD glossary [L1, L2] (see <http://tgdtheory.fi/cmaphtml.html> and <http://tgdtheory.fi/tgdglossary.pdf>).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (<http://journals.sfu.ca/jnonlocality/index.php/jnonlocality>) founded by Lian Sidorov and in *Prespacetime Journal* (<http://prespacetime.com>), *Journal of Consciousness Research and Exploration* (<https://www.createspace.com/4185546>), and *DNA Decipher Journal* (<http://dnadecipher.com>), all of them founded by Huping Hu. One can find the list about the articles published at <http://www.tgdtheory.com/curri.html>. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

4 The contents of the book

4.1 Part I: p-Adic description of particle massivation

In the first part of the book a p-adic description of particle massivation using p-adic thermodynamics and TGD variant of Higgs mechanism is developed.

4.1.1 Overall view about TGD from particle physics perspective

Topological Geometroynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the imbedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces.

What GRT limit of TGD and Equivalence Principle mean in TGD framework have are problems which found a solution only quite recently (2014). GRT space-time is obtained by lumping together the sheets of many-sheeted space-time to single piece of M^4 provided by an effective metric defined by the sum of Minkowski metric and the deviations of the induced metrics of space-time sheets from Minkowski metric. Same description applies to gauge potentials of gauge theory limit. Equivalence Principle as expressed by Einstein's equations reflects Poincare invariance of TGD.

Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves four Dirac operators.

1. The Kähler Dirac equation holds true in the interior of space-time surface: the well-definedness of em charge as quantum number of zero modes implies localization of the modes of the induced spinor field to 2-surfaces. It is quite possible that this localization

is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

2. Assuming measurement interaction term for four-momentum, the boundary condition for Kähler-Dirac operator gives essentially massless M^4 Dirac equation in algebraic form coupled to what looks like Higgs term but gives a space-time correlate for the stringy mass formula at stringy curves at the space-like ends of space-time surface.
 3. The ground states of the Super-Virasoro representations are constructed in terms of the modes of imbedding space spinor fields which are massless in 8-D sense.
 4. The fourth Dirac operator is associated with super Virasoro generators and super Virasoro conditions defining Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S -matrix to a collection of what I call M -matrices defining the rows of unitary U -matrix defining unitary process.
- Twistor approach has inspired several ideas in quantum TGD during the last years. The basic finding is that M^4 resp. CP_2 is in a well-defined sense the only 4-D manifold with Minkowskian resp. Euclidian signature of metric allowing twistor space with Kähler structure. It seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness: contrary to original expectations the stringy character of the amplitudes seems necessary to guarantee finiteness.

4.2 The recent vision about preferred extremals and solutions of the modified Dirac equation

During years several approaches to what preferred extremals of Kähler action and solutions of the Kähler-Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other. It is good to list various approaches first.

1. For preferred extremals generalization of conformal invariance to 4-D situation is very attractive approach and leads to concrete conditions formally similar to those encountered in string model. The approach based on basic heuristics for massless equations, on effective 3-dimensionality, and weak form of electric magnetic duality is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space.
2. There are also several approaches for solving the Kähler-Dirac equation. The most promising approach is assumes that other than right-handed neutrino modes are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of number theoretic vision. The conditions stating that electric charge is conserved for preferred extremals is an alternative very promising approach.

In this chapter the question whether these various approaches are mutually consistent is discussed. It indeed turns out that the approach based on the well-definedness of electric charge leads under rather general assumptions to the proposal that solutions of the Kähler-Dirac equation are localized on 2-dimensional string world sheets and/or partonic 2-surfaces. This leads to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD.

4.2.1 Elementary particle vacuum functionals

Genus-generation correspondence is one of the basic ideas of TGD approach. In order to answer various questions concerning the plausibility of the idea, one should know something about the dependence of the elementary particle vacuum functionals on the vibrational degrees of freedom for the partonic 2-surface.

The construction of the elementary particle vacuum functionals based on Diff invariance, 2-dimensional conformal symmetry, modular invariance plus natural stability requirements indeed leads to an essentially unique form of the vacuum functionals and one can understand why $g > 0$ bosonic families are experimentally absent and why lepton numbers are conserved separately.

An argument suggesting that the number of the light fermion families is three, is developed. The crux of the argument is that the partonic 2-surfaces coding for quantum states are for the maxima of Kähler action hyper-elliptic, that is possess Z_2 conformal symmetry, which for $g > 2$ implies that elementary particle vacuum functional vanishes.

Although the the original model of elementary particle have been modified and replaced with more complex one, the basic idea about the origin of three generations remains intact.

4.2.2 Massless states and particle massivation

This chapter represents the most recent view about elementary particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters. In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S -matrix to what I call M -matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the Kähler-Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected.

Since 2010 a further progress took place. These steps of progress relate closely to ZEO, bosonic emergence, the discovery of the weak form of electric-magnetic duality, the realization of the importance of twistors in TGD, and the discovery that the well-definedness of em charge forces the modes of Kähler-Dirac operator to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This allows to assign to elementary particle closed string with pieces at two parallel space-time sheets and accompanying a Kähler magnetic flux tube carrying monopole flux.

Twistor approach and the understanding of the solutions of Kähler-Dirac Dirac operator served as a midwife in the process giving rise to the birth of the idea that all fundamental fermions are massless and that both ordinary elementary particles and string like objects emerge from them. Even more, one can interpret virtual particles as being composed of these massless on mass shell particles assignable to wormhole throats. Four-momentum conservation poses extremely powerful constraints on loop integrals but does not make them manifestly finite as believed first. String picture is necessary for getting rid of logarithmic divergences.

The weak form of electric-magnetic duality led to the realization that elementary particles correspond to bound states of two wormhole throats with opposite Kähler magnetic charges with second throat carrying weak isospin compensating that of the fermion state at second wormhole throat. Both fermions and bosons correspond to wormhole contacts: in the case of fermions topological condensation generates the second wormhole throat. This means that altogether four wormhole throats are involved with both fermions, gauge bosons, and gravitons (for gravitons this

is unavoidable in any case). For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N} = 4$ SYM. At the level of effective space-time assigned to many-sheeted space-time this symmetry is broken. Besides this there is weak “stringy” contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

1. *Physical states as representations of super-symplectic and Super Kac-Moody algebras*

The basic constraint is that the super-conformal algebra involved must have five tensor factors. The precise identification of the Kac-Moody type algebra has however turned out to be a surprisingly difficult task. The latest view is as follows. Electroweak algebra $U(2)_{ew} = SU(2)_L \times U(1)$ and symplectic isometries of light-cone boundary ($SU(2)_{rot} \times SU(3)_c$) give 2+2 factors and full supersymplectic algebra involving only covariantly constant right-handed neutrino mode would give 1 factor. This algebra could be associated with the 2-D surfaces X^2 defined by the intersections of light-like 3-surfaces with $\delta M_{\pm}^4 \times CP_2$. These 2-surfaces have interpretation as partons.

For conformal algebra there are several candidates. For symplectic algebra radial light-like coordinate of light-cone boundary replaces complex coordinate. Light-cone boundary $S^2 \times R_+$ allows extended conformal symmetries which can be interpreted as conformal transformations of S^2 depending parametrically on the light-like coordinate of R_+ . There is infinite-D subgroup of conformal isometries with S^2 dependent radial scaling compensating for the conformal scaling in S^2 . Kähler-Dirac equation allows ordinary conformal symmetry very probably liftable to imbedding space. The light-like orbits of partonic 2-surface are expected to allow super-conformal symmetries presumably assignable to quantum criticality and hierarchy of Planck constants. How these conformal symmetries integrate to what is expected to be 4-D analog of 2-D conformal symmetries remains to be understood.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams. The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces.

2. *Particle massivation*

Particle massivation can be regarded as a generation of thermal mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The obvious objection is that Poincare invariance is lost. One could argue that one calculates just the vacuum expectation of conformal weight so that this is not case. If this is not assumed, one would have in positive energy ontology superposition of ordinary quantum states with different four-momenta and breaking of Poincare invariance since eigenstates of four-momentum are not in question. In Zero Energy Ontology this is not the case since all states have vanishing net quantum numbers and one has superposition of time evolutions with well-defined four-momenta. Lorentz invariance with respect to the either boundary of CD is achieved but there is small breaking of Poincare invariance characterized by the inverse of p-adic prime p characterizing the particle. For electron one has $1/p = 1/M_{127} \sim 10^{-38}$.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to \hbar so that it should correspond to a generator of some Lie-algebra (Virasoro generator $L_0!$)). What basically matters is the number of tensor factors

involved and five is the favored number.

2. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.
3. A natural identification of the non-integer contribution to the conformal weight is as stringy contributions to the vacuum conformal weight (strings are now “weak strings”). TGD predicts Higgs particle and Higgs is necessary to give longitudinal polarizations for gauge bosons. The notion of Higgs vacuum expectation seems to be replaced by an analog of Higgs vacuum expectation which gives space-time correlate for the stringy mass formula in case of fundamental fermions. Also gauge bosons usually regarded as exactly massless particles would naturally receive small mass from p-adic thermodynamics. The theoretical motivation for a small mass would be exact Yangian symmetry which is broken at the QFT limit of the theory using GRT limit of many-sheeted space-time.
4. Hadron massivation requires the understanding of the CKM mixing of quarks reducing to different topological mixing of U and D type quarks. Number theoretic vision suggests that the mixing matrices are rational or algebraic and this together with other constraints gives strong constraints on both mixing and masses of the mixed quarks.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $\exp(-E/kT)$ is replaced with p^{L_0/T_p} , $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.
2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5} \sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

4.3 Part II: New physics predicted by TGD

4.3.1 Higgs or something else?

The question whether TGD predicts Higgs or not has been one of the longstanding issues of TGD. For 10 years ago I would not have hesitated to tell that TGD does not predict Higgs and had good looking arguments for my claim. During years my views have been alternating between Higgs and no-Higgs option. In the light of after wisdom the basic mistake has been the lack of a conscious attempt to localize precisely the location of the problem and suggest a minimal modification of standard theory picture to solve it.

Now the situation is settled experimentally: Higgs is there. It is however somewhat too light so that Higgs mechanism is not stable against radiative corrections. SUSY cannot take care of this problem since LHC demonstrated that SUSY mass scale is too high. One has the problem known as loss of “naturalness”. Hence Higgs is not yet a fully written page in the history of physics. Furthermore, the experiments demonstrate the existence of Higgs, not the reality of Higgs mechanism. Higgs mechanism in fermionic sector is indeed an ugly duckling: the dimensionless couplings of fermions to Higgs vary in huge range: 12 orders of magnitude between neutrinos and top quark.

1. In TGD framework Higgs mechanism is replaced by p-adic thermodynamics. The couplings of Higgs to fermions are by dimensional arguments very naturally gradient couplings with coupling constant, which has dimensions of inverse mass. This dimensional coupling is same for all fermions so that naturalness is achieved.

2. Massivation of gauge bosons combines Higgs components and weak gauge bosons to massive particles in unitary gauge but leaves photon massless apart from small higher order corrections from p-adic thermodynamics. Unitary gauge is in TGD uniquely fixed by CP_2 geometry. This trivial observation means that there is no need for Higgs vacuum expectation value to define the em neutral direction in gauge algebra. Furthermore, the absence of covariantly constant holomorphic CP_2 vector fields strongly suggests that Higgs vacuum expectation does not make sense. This does not exclude the existence of Higgs like particle as the original wrong conclusion was.
3. W/Z mass squared ratio - the source of troubles in p-adic thermodynamics based approach - is expressible in terms of corresponding gauge coupling strengths g_i^2 , $i = W, Z$, if the string tension of the flux tube connecting the two wormhole contacts assignable to gauge boson is proportional to g_i^2 . This is definitely a new element in the physical picture and replaces Higgs vacuum energy with the energy of string.
4. p-Adic thermodynamics relying on super-conformal invariance can describe in its recent form only the contributions of wormhole contacts to the particle masses. The contributions from "long strings" connecting different wormhole contacts cannot be calculated. To achieve this one must generalize conformal invariance to include two conformal weights: the conformal weight assignable to the conformal weight for the light-like radial coordinate of light-cone boundary and the spinorial conformal weight assignable to the induced spinor fields at string world sheets. It seems that also an extension to Yangian algebra containing poly-local generators with locus defined as partonic 2-surface is needed: the number of partonic 2-surface would define a quantum number. p-Adic thermodynamics for the representations of Yangian with states labeled by these three integers could provide the complete description of the states.

The recent construction of WCW geometry indeed leads to a picture allowing interpretation in terms of Yangian extension of super-conformal invariance. The matrix elements of WCW metrix are labelled by two conformal weights assignable to the light-like radial coordinate of light-cone boundary and to the coordinate along string defining the boundary of string world sheet at which fermions are located from the condition that spinor modes have a well-defined value of em charge.

In this chapter the recent view about Higgs is described and reader is saved from the many alternatives that I have considered during last years.

4.3.2 SUSY in TGD Universe

Contrary to the original expectations, TGD seems to allow a generalization of the space-time SUSY to its 8-D variant with masslessness in 4-D sense replaced with masslessness in 8-D sense. The algebra in question is the Clifford algebra of fermionic oscillator operators associated with given partonic 2-surface. In terms of these algebras one can in turn construct generators super-symplectic algebra as stringy Noether charges and also other super-conformal algebras and even their Yangians used to create quantum states. This also forces to generalize twistor approach to give 8-D counterparts of ordinary 4-D twistors.

The 8-D analog of super Poincare algebra emerges at the fundamental level through the anti-commutation relations of the fermionic oscillator operators. For this algebra $\mathcal{N} = \infty$ holds true. Most of the states in the representations of this algebra are massive in $4 - D$ sense. The restriction to the massless sector gives the analog of ordinary SUSY with a finite value of \mathcal{N} - essentially as the number of massless states of fundamental fermions to be distinguished from elementary fermions. The addition of a fermion in particular mode defines particular super-symmetry. This super-symmetry is broken due to the dynamics of the Kähler-Dirac operator, which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a restriction to a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of a fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D=10$ and $D=11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

In superstring theory the hermiticity of super generator G_0 giving as its square scaling generator L_0 is strong argument in favor if Majorana spinors since G_0 appears as a propagator. In TGD framework the counterparts of G_0 in quark and lepton sector carry fermion number so that identification as a propagator does not make sense. The recent formulation of scattering amplitudes in terms of Yangian algebra allows to circumvent the problem. Fundamental propagators are fermion propagators for fermions massless in 8-D sense.

2. The spinor components of imbedding space spinors identifiable with physical helicities and with fixed fermion number correspond to the generators of the SUSY algebra at QFT limit. This SUSY is broken due to electroweak and color interactions. Right-handed neutrinos do not have these interactions but there is a mixing with left-handed neutrinos due to the mixing of M^4 and CP_2 gamma matrices in the Kähler-Dirac gamma matrices appearing in the K-D action. Therefore also the $\mathcal{N} = 2$ sub-SUSY generated by right-handed neutrinos is broken.

In this chapter the details of the above general picture are discussed. Also the existing experimental constraints on SUSY are discussed.

4.3.3 p-Adic Particle Massivation: New Physics: part I

TGD predicts a lot of new physics and it is quite possible that this new physics becomes visible at LHC. Although the calculational formalism is still lacking, p-adic length scale hypothesis allows to make precise quantitative predictions for particle masses by using simple scaling arguments.

The basic elements of quantum TGD responsible for new physics are following.

1. The new view about particles relies on their identification as partonic 2-surfaces (plus 4-D tangent space data to be precise). This effective metric 2-dimensionality implies generalization of the notion of Feynman diagram and holography in strong sense. One implication is the notion of field identity or field body making sense also for elementary particles and the Lamb shift anomaly of muonic hydrogen could be explained in terms of field bodies of quarks.
2. The topological explanation for family replication phenomenon implies genus generation correspondence and predicts in principle infinite number of fermion families. One can however develop a rather general argument based on the notion of conformal symmetry known as hyper-ellipticity stating that only the genera $g = 0, 1, 2$ are light. What “light” means is however an open question. If light means something below CP_2 mass there is no hope of observing new fermion families at LHC. If it means weak mass scale situation changes.

For bosons the implications of family replication phenomenon can be understood from the fact that they can be regarded as pairs of fermion and antifermion assignable to the opposite wormhole throats of wormhole throat. This means that bosons formally belong to octet and singlet representations of dynamical $SU(3)$ for which 3 fermion families define 3-D representation. Singlet would correspond to ordinary gauge bosons. Also interacting fermions suffer topological condensation and correspond to wormhole contact. One can either assume that the resulting wormhole throat has the topology of sphere or that the genus is same for both throats.

3. The view about space-time supersymmetry differs from the standard view in many respects. First of all, the super symmetries are not associated with Majorana spinors. Super generators correspond to the fermionic oscillator operators assignable to leptonic and quark-like induced spinors and there is in principle infinite number of them so that formally one would have $\mathcal{N} = \infty$ SUSY. I have discussed the required modification of the formalism of SUSY theories and it turns out that effectively one obtains just $\mathcal{N} = 1$ SUSY required by experimental constraints. The reason is that the fermion states with higher fermion number define only short range interactions analogous to van der Waals forces. Right handed neutrino generates this super-symmetry broken by the mixing of the M^4 chiralities implied by the mixing of M^4 and CP_2 gamma matrices for induced gamma matrices. The simplest assumption is that particles and their superpartners obey the same mass formula but that the p-adic length scale can be different for them.
4. The new view about particle massivation based on p-adic thermodynamics raises the question about the role of Higgs field. The vacuum expectation value (VEV) of Higgs is not feasible in TGD since CP_2 does not allow covariantly constant holomorphic vector fields. The original too strong conclusion from this was that TGD does not allow Higgs. Higgs VEV is not needed for the selection of preferred electromagnetic direction in electro-weak gauge algebra (unitary gauge) since CP_2 geometry does that. p-Adic thermodynamics explains fermion masses but the masses of weak bosons cannot be understood on basis of p-adic thermodynamics alone giving extremely small second order contribution only and failing to explain W/Z mass ratio. Weak boson mass can be associated to the string tension of the strings connecting the throats of two wormhole contacts associated with elementary particle (two of them are needed since the monopole magnetic flux must have closed field lines).
5. One of the basic distinctions between TGD and standard model is the new view about color.
 - (a) The first implication is separate conservation of quark and lepton quantum numbers implying the stability of proton against the decay via the channels predicted by GUTs. This does not mean that proton would be absolutely stable. p-Adic and dark length scale hierarchies indeed predict the existence of scale variants of quarks and leptons and proton could decay to hadrons of some zoomed up copy of hadrons physics. These decays should be slow and presumably they would involve phase transition changing the value of Planck constant characterizing proton. It might be that the simultaneous increase of Planck constant for all quarks occurs with very low rate.
 - (b) Also color excitations of leptons and quarks are in principle possible. Detailed calculations would be required to see whether their mass scale is given by CP_2 mass scale. The so called lepto-hadron physics proposed to explain certain anomalies associated with both electron, muon, and τ lepton could be understood in terms of color octet excitations of leptons.
6. Fractal hierarchies of weak and hadronic physics labelled by p-adic primes and by the levels of dark matter hierarchy are highly suggestive. Ordinary hadron physics corresponds to $M_{107} = 2^{107} - 1$. One especially interesting candidate would be scaled up hadronic physics which would correspond to $M_{89} = 2^{89} - 1$ defining the p-adic prime of weak bosons. The corresponding string tension is about 512 GeV and it might be possible to see the first signatures of this physics at LHC. Nuclear string model in turn predicts that nuclei correspond to nuclear strings of nucleons connected by colored flux tubes having light quarks at their ends. The interpretation might be in terms of M_{127} hadron physics. In biologically most interesting length scale range 10 nm-2.5 μ m there are four Gaussian Mersennes and the conjecture is that these and other Gaussian Mersennes are associated with zoomed up variants of hadron physics relevant for living matter. Cosmic rays might also reveal copies of hadron physics corresponding to M_{61} and M_{31} .

The well-definedness of em charge for the modes of induced spinor fields localizes them at 2-D surfaces with vanishing W fields and also Z^0 field above weak scale. This allows to avoid undesirable parity breaking effects. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by

Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

7. Weak form of electric magnetic duality implies that the fermions and antifermions associated with both leptons and bosons are Kähler magnetic monopoles accompanied by monopoles of opposite magnetic charge and with opposite weak isospin. For quarks Kähler magnetic charge need not cancel and cancellation might occur only in hadronic length scale. The magnetic flux tubes behave like string like objects and if the string tension is determined by weak length scale, these string aspects should become visible at LHC. If the string tension is 512 GeV the situation becomes less promising.

In this chapter some aspects of the predicted new physics and possible indications for it are discussed. The evolution of the TGD based view about possible existing Higgs like particle and about space-time SUSY are discussed in separate chapters.

4.3.4 p-Adic Particle Massivation: New Physics: part II

In this chapter the focus is on the hadron physics. The applications are to various anomalies discovered during years.

1. Application of the many-sheeted space-time concept in hadron physics

The many-sheeted space-time concept involving also the notion of field body can be applied to hadron physics to explain findings which are difficult to understand in the framework of standard model.

1. The spin puzzle of proton is a two decades old mystery with no satisfactory explanation in QCD framework. The notion of hadronic space-time sheet which could be imagined as string like rotating object suggests a possible approach to the spin puzzle. The entanglement between valence quark spins and the angular momentum states of the rotating hadronic space-time sheet could allow natural explanation for why the average valence quark spin vanishes.
2. The notion of Pomeron was invented during the Bootstrap era preceding QCD to solve difficulties of Regge approach. There are experimental findings suggesting the reincarnation of this concept. The possibility that the newly born concept of Pomeron of Regge theory might be identified as the sea of perturbative QCD in TGD framework is considered. Geometrically Pomeron would correspond to hadronic space-time sheet without valence quarks.
3. The discovery that the charge radius of proton deduced from the muonic version of hydrogen atom is about 4 per cent smaller than from the radius deduced from hydrogen atom is in complete conflict with the cherished belief that atomic physics belongs to the museum of science. The title of the article *Quantum electrodynamics-a chink in the armour?* of the article published in Nature expresses well the possible implications, which might actually go well extend beyond QED. TGD based model for the findings relies on the notion of color magnetic body carrying both electromagnetic and color fields and extends well beyond the size scale of the particle. This gives rather detailed constraints on the model of the magnetic body.
4. The soft photon production rate in hadronic reactions is by an average factor of about four higher than expected. In the article soft photons assignable to the decays of Z^0 to quark-antiquark pairs. This anomaly has not reached the attention of particle physics which seems to be the fate of anomalies quite generally nowadays: large extra dimensions and black-holes at LHC are much more sexy topics of study than the anomalies about which both existing and speculative theories must remain silent. TGD based model is based on the notion of electric flux tube.

2. Quark gluon plasma

QCD predicts that at sufficiently high collision energies de-confinement phase transitions for quarks should take place leading to quark gluon plasma. In heavy ion collisions at RHIC something

like this was found to happen. The properties of the quark gluon plasma were however not what was expected. There are long range correlations and the plasma seems to behave like perfect fluid with minimal viscosity/entropy ratio. The lifetime of the plasma phase is longer than expected and its density much higher than QCD would suggest. The experiments at LHC for proton proton collisions suggest also the presence of quark gluon plasma with similar properties.

TGD suggests an interpretation in terms of long color magnetic flux tubes containing the plasma. The confinement to color magnetic flux tubes would force higher density. The preferred extremals of Kähler action have interpretation as defining a flow of perfect incompressible fluid and the perfect fluid property is broken only by the many-sheeted structure of space-time with smaller space-time sheets assignable to sub-*CDs* representing radiative corrections. The phase in question corresponds to a non-standard value of Planck constant: this could also explain why the lifetime of the phase is longer than expected.

3. Breaking of discrete symmetries

Zero energy ontology provides a fresh approach to discrete symmetries and provides also a general mechanism for their breaking. A general vision about breaking of discrete symmetries relies on quantum measurement theory: the quantum jump selecting the quantization axes induces localization to a single *CD* and therefore induces breaking of discrete symmetries due to the choice of quantization axes. The time scale of *CD* is excellent candidate for defining mass and time scales characterizing the symmetry breaking. Entropic gravity idea has a variant in TGD framework resulting from the fact that in ZEO quantum theory is a square root of thermodynamics in a well-defined sense. Thermodynamical stability could force the generation of the arrow of time and also force it to be different for matter and antimatter inducing in this manner matter antimatter asymmetry and breaking of discrete symmetries like CP. Also CPT could be broken spontaneously and there are experimental indications that this takes place for top quark with mass difference which is surprisingly large- few per cent of top mass.

4. Are exotic Super Virasoro representations relevant for hadron physics?

In p-adic context exotic representations of Super Virasoro with $M^2 \propto p^k$, $k = 1, 2, \dots, m$ are possible. For $k = 1$ the states of these representations have same mass scale as elementary particles although in real context the masses would be gigantic. This inspires the question whether non-perturbative aspects of hadron physics could be assigned to the presence of these representations. Some intriguing numerical co-incidences suggest that the exotic representations of Super-Virasoro should be assigned with hadron and whereas ordinary Virasoro representations would be assigned with the quark-gluon plasma or possibly sea quarks.

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