

# Introduction to "Hyper-finite Factors and Hierarchy of Planck Constants"

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September 14, 2018

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# 1 Basic Ideas Of Topological Geometroynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

## 1.1 Basic Vision Very Briefly

*T(opological) G(eometro)D(ynamics)* is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space  $H = M^4 \times CP_2$ , where  $M^4$  is 4-dimensional (4-D) Minkowski space and  $CP_2$  is 4-D complex projective space (see Appendix).
2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of  $H$  to the space-time surface. Electroweak gauge potentials are identified as projections of the components of  $CP_2$  spinor connection to the space-time surface, and color gauge potentials as projections of  $CP_2$  Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of  $H$  and induced spinor fields just  $H$  spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in imbedding space metric and parallel translation using spinor connection of imbedding space.

The induction procedure applies to octonionic structure and the conjecture is that for preferred extremals the induced octonionic structure is quaternionic: again one just projects the octonion units. I have proposed that one can lift space-time surfaces in  $H$  to the Cartesian product of the twistor spaces of  $M^4$  and  $CP_2$ , which are the only 4-manifolds allowing twistor space with Kähler structure. Now the twistor structure would be induced in some sense, and should co-incide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of  $M^4$  and  $CP_2$  must allow identification: this 2-sphere defines the  $S^2$  fiber of the twistor space of space-time surface. This poses constraint on the imbedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of  $CP_2$  codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of  $CP_2$  geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in  $CP_2$  scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

$M^4$  and  $CP_2$  are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure.  $M^4$  light-cone boundary allows

a huge extension of 2-D conformal symmetries. Imbedding space  $H$  has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure.  $M^4$  and  $CP_2$  allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogeneity in background space-time surface which looks like the space-time of general relativity in long length scales.

One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about  $10^4$  Planck lengths ( $CP_2$  size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. A possible resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics. A possible objection is that the Kähler-Dirac gamma matrices do not define an integrable distribution of 2-planes defining string world sheet.

An even stronger condition would be that the induced classical gauge fields at string world sheet vanish: this condition is allowed by the topological description of particles. The  $CP_2$  projection of string world sheet would be 1-dimensional. Also the number theoretical condition that octonionic and ordinary spinor structures are equivalent guaranteeing that fermionic dynamics is associative leads to the vanishing of induced gauge fields.

The natural action would be given by string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant would be present as a fundamental constant in string action and the ratio  $\hbar/G/R^2$  would be determined by quantum criticality condition. The hierarchy of Planck constants  $\hbar_{eff}/\hbar = n$  assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by  $T = 1/\hbar_{eff}G$  apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of super string theories. This option allows the natural interpretation of  $M^4$  type vacuum extremals with  $CP_2$  projection, which is Lagrange manifold as good approximations

for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether also induced spinor fields associated with Kähler-Dirac action and de-localized inside entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using imbeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces serve are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [?, ?, ?]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.
4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the imbedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.
5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are

introduced: Kähler geometry and associated spinor structure are enough. “Topological” in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [?]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of everyday world represent non-trivial topology of space-time in TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerges as a technical tool, and its Kähler structure is possible only for  $H = M^4 \times CP_2$ . What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchies of Planck constants  $h_{eff} = n \times h$  reducing to the quantum criticality of TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

## 1.2 Two Vision About TGD And Their Fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

### 1.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space  $H = M^4 \times CP_2$ , where  $M^4$  denotes Minkowski space and  $CP_2 = SU(3)/U(2)$  is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of  $M^4 \times CP_2$  leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of  $CP_2$  explains electro-weak and color quantum numbers. The different H-chiralities of  $H$ -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the  $CP_2$  spinor connection, Killing vector fields of  $CP_2$  and of  $H$ -metric to four-surface define classical electro-weak, color gauge fields and metric in  $X^4$ .

The choice of  $H$  is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects  $H = M^4 \times CP_2$  uniquely.  $M^4$  and  $CP_2$  are also unique spaces allowing twistor space with Kähler structure.

### 1.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs

of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models. Scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

### 1.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a “topological condensate” containing matter as particle like 3-surfaces “glued” to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the “topological condensate” there could be “vapor phase” that is a “gas” of particle like 3-surfaces and string like objects (counterpart of the “baby universes” of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see **Fig.** <http://tgdtheory.fi/appfigures/manysheeted.jpg> or **Fig.** ?? in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell’s theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of  $CP_2$  and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of  $CP_2$  size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at

which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

### 1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially  $CP_2$  coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of  $CP_2$  metric define a natural starting point and  $CP_2$  indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

#### 1.3.1 Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other things this leads to models for cell membrane, nerve pulse, and EEG.

### 1.4 P-Adic Variants Of Space-Time Surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. One might say that TGD space-time is adelic. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.



A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

## 1.5 The Threads In The Development Of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.
2. The discussions with Tony Smith initiated a fourth thread which deserves the name “TGD as a generalized number theory”. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the “physics as generalized number theory” thread.
3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called “physics as a generalized number theory”. In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K21, K13, K9, K27, K17, K26, K25, K16] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K19, K3, K10, K2, K5, K6, K7, K15, K24] are warmly recommended to the interested reader.

### 1.5.1 *Quantum TGD as spinor geometry of World of Classical Worlds*

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was “Do not quantize”. The basic ingredients to the new

approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space  $CH$  (“world of classical worlds”, WCW) consisting of all possible 3-surfaces in  $H$ . “All possible” means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay  $A \rightarrow B + C$ . Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.
2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word “world of classical worlds” (WCW) instead of rather formal “configuration space”. I hope that “WCW” does not induce despair in the reader having tendency to think about the technicalities involved!
3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory <sup>1</sup>
4. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of  $M^4$  Dirac operator appearing in propagators. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense.

In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the  $CP_2$  time.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer  $n$  are naturally proportional to a representation matrix of

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<sup>1</sup>There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as Kähler action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also Kähler Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

scaling:  $S(n) = S^n$ , where  $S$  is unitary S-matrix associated with the minimal CD [K22]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of  $S$  and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

U-matrix elements between M-matrices for various CDs are proportional to the inner products  $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$ , where  $\lambda$  represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and  $H^i$  form an orthonormal basis of Hermitian square roots of density matrices.  $\circ$  tells that  $S$  acts at the active boundary of CD only. It turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix. S-matrix has interpretation as exponential of the Virasoro generator  $L_{-1}$  of the Virasoro algebra associated with super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The Kähler-Dirac gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.
6. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical  $W$  boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also  $Z^0$  field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that  $\sqrt{g_4}$  vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time

(recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the  $\sqrt{g_4}$  factor coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only “almost” since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

### 1.5.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space (“world of classical worlds”, WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name “TGD as a generalized number theory”. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

#### 1. *p-Adic TGD and fusion of real and p-adic physics to single coherent whole*

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduce the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of

this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, imbedding space, and WCW.

The notion of p-adic manifold [K28] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheet as chart map provided a possible solution of the basic challenge of relating real and p-adic classical physics. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, “thought bubbles” ). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach and this leads to problems with symmetries: canonical identification does not commute with symmetries.

It is now clear that much more elegant approach based on abstraction exists [K29]. The map of real preferred extremals to p-adic ones is not induced from a local correspondence between points but is global. Discretization occurs only for the parameters characterizing string world sheets and partonic 2-surfaces so that they belong to some algebraic extension of rationals. Restriction to these 2-surfaces is possible by strong form of holography. Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** <http://tgdtheory.fi/appfigures/cat.jpg> or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces

(imaginings) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K8].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

### 2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces  $X^4$  be regarded as associative or co-associative (“quaternionic” is equivalent with “associative” ) surfaces of  $H$  endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of  $X^4$  [K18]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.
2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of  $M^8$  regarded as octonionic linear space to surfaces in  $M^4 \times CP_2$  [K18] ? This conjecture -  $M^8 - H$  duality - would give for  $M^4 \times CP_2$  deep number theoretic meaning.  $CP_2$  would parametrize associative planes of octonion space containing fixed complex plane  $M^2 \subset M^8$  and  $CP_2$  point would thus characterize the tangent space of  $X^4 \subset M^8$ . The point of  $M^4$  would be obtained by projecting the point of  $X^4 \subset M^8$  to a point of  $M^4$  identified as tangent space of  $X^4$ . This would guarantee that the dimension of space-time surface in  $H$  would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.
3.  $M^8 - H$  duality can be generalized to a duality  $H \rightarrow H$  if the images of the associative surface in  $M^8$  is associative surface in  $H$ . One can start from associative surface of  $H$  and assume that it contains the preferred  $M^2$  tangent plane in 8-D tangent space of  $H$  or integrable distribution  $M^2(x)$  of them, and its points to  $H$  by mapping  $M^4$  projection of  $H$  point to itself and associative tangent space to  $CP_2$  point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.
4.  $G_2$  defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of  $G_2$  could produce new associative/co-associative surfaces. The action of  $G_2$  would be analogous to that of gauge group.
5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either  $M^8$  or  $M^4 \times CP_2$ . As surfaces of  $M^8$  identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of  $M^8$

or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in  $M^4 \times CP_2$  [K18] provided one can assign to each point of tangent space a hyper-complex plane  $M^2(x) \subset M^4 \subset M^8$ . One can also speak about  $M^8 - H$  duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane  $M^2(x) \subset M^4$ . As a consequence, the  $M^4$  projection of space-time surface at each point contains  $M^2(x)$  and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets  $Y^2$  and partonic 2-surfaces  $X^2$ . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of  $M^2(x)$  is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type  $II_1$  about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the “free” gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension  $D = 8$ .
2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.
3. For gamma matrix option induced rather than Kähler-Dirac gamma matrices must be in question since Kähler-Dirac gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

### 3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields.

The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of  $n > 1$  variables.

## 1.6 Hierarchy Of Planck Constants And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

### 1.6.1 Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant  $\hbar$  replaced with what might be called gravitational Planck constant  $\hbar_{gr} = \frac{GmM}{v_0}$  ( $\hbar = c = 1$ ).  $v_0$  is a velocity parameter having the value  $v_0 = 144.7 \pm .7$  km/s giving  $v_0/c = 4.6 \times 10^{-4}$ . This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of  $v_0$  seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of  $h_{gr}$ . Equivalence Principle and the independence of gravitational Compton length on mass  $m$  implies however that one can restrict the values of mass  $m$  to masses of microscopic objects so that  $h_{gr}$  would be much smaller. Large  $h_{gr}$  could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K14].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification  $h_{eff} = n \times h_{gr}$ . The large value of  $h_{gr}$  can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values  $h_{eff}/h = n$  can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of  $n$ . Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that fermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with  $h_{eff}/h = n > 1$ . One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ( $E = hf_{high} = h_{eff}f_{low}$ ) of bunch of  $n$  low energy gravitons.



### 1.6.2 Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about  $10^{-10}$  times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis  $h_{eff} = h_{gr}$  - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by  $h_{eff}$  reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K11, K12, K23] ) support the view that dark matter might be a key player in living matter.

### 1.6.3 Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple  $\hbar = n\hbar_0$  of the ordinary Planck constant  $\hbar_0$  is assigned with a multiple singular covering of the imbedding space [K4]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invari-

ance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer  $n$  is highly suggestive notion and would imply that  $n$  sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see **Fig.** <http://tgdtheory.fi/appfigures/planckhierarchy.jpg> or **Fig. ??** the appendix of this book).  $n$  would naturally correspond the value of  $h_{eff}$  and its factors negentropic entanglement with unit density matrix would be between the  $n$  sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of  $n$ .

#### 1.6.4 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken  $U(2)_{ew}$  invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical  $W$  boson fields vanish at these surfaces and also classical  $Z^0$  field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like  $h_{eff}$ .

## 1.7 Twistors And TGD

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K20]. The reason is that  $M^4$  and  $CP_2$  are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of  $M^4 \times CP_2$  is Cartesian product of those of  $M^4$  and  $CP_2$ . The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in  $H$  such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric. This condition would define the dynamics, and the conjecture is that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

The condition that the basic formulas for the twistors in  $M^8$  serving as tangent space of imbedding space generalize. This is the case if one introduces octonionic sigma matrices allowing twistor representation of 8-momentum serving as dual for four-momentum and color quantum numbers. The conditions that octonionic spinors are equivalent with ordinary requires that the induced gamma matrices generate quaternionic sub-algebra at given point of string world sheet. This is however not enough: the charge matrices defined by sigma matrices can also break associativity and induced gauge fields must vanish: the  $CP_2$  projection of string world sheet would be one-dimensional at most. This condition is symplectically invariant. Note however that for the interior dynamics of induced spinor fields octonionic representations of Clifford algebra cannot be equivalent with the ordinary one.

One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of superconformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this is nothing but a concretization of Equivalence Principle. Also a connection with string model emerges.

Twistor approach developed rapidly during years. Witten's twistor string theory generalizes: the most natural counterpart of Witten's twistor strings is partonic 2-surface. The notion of positive Grassmannian has emerged and TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the

only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings. Braid interpretation gives further support for the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

## 2 Bird's Eye Of View About The Topics Of The Book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type  $II_1$  about which the spinors of the world of classical worlds represent a canonical example.

### 1. Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space  $M^4 \times CP_2$  glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.
3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.
4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.
5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights.  $CP_2$  type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle

carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and  $Z^0$  fields are ideal for communication purposes and charged  $W$  MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy  $W$  MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

### 2. *Physics as infinite-dimensional geometry in the "world of classical worlds"*

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of  $\delta M_+^4 \times CP_2$ , where  $\delta M_+^4$  is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type  $II_1$  having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

### 3. *Physics as a generalized number theory*

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

### 4. *The organization of the book*

The first part of the book is devoted to hyper-finite factors and hierarchy of Planck constants.

1. Configuration space spinors indeed define a canonical example about hyper-finite factor of type  $II_1$ . The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type  $II_1$  could provide the mathematics needed to develop a more explicit view about the construction of M-matrix. This has turned out to be the case to the extent that a general master formula for M-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.
2. The idea about hierarchy of Planck constants emerged from anomalies of biology and the strange finding that planetary orbits could be regarded as Bohr orbits but with a gigantic value of Planck constant. This lead to the vision that dark matter corresponds to ordinary

particles but with non-standard value of Planck constant and to a generalization of the 8-D imbedding space to a book like structure with pages partially characterized by the value of Planck constant. Using the intuition provided by the inclusions of hyper-finite factors of type  $II_1$  one ends up to a prediction for the spectrum of Planck constants associated with  $M^4$  and  $CP_2$  degrees of freedom. This inspires the proposal that dark matter could be in quantum Hall like phase localized at light-like 3-surfaces with macroscopic size and behaving in many respects like black hole horizons.

### 3 Sources

The eight online books about TGD [K21, K13, K27, K17, K9, K26, K25, K16] and nine online books about TGD inspired theory of consciousness and quantum biology [K19, K3, K10, K2, K5, K6, K7, K15, K24] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (<http://tinyurl.com/ybv8dt4n>) contains a lot of material about TGD. In particular, a TGD glossary at <http://tinyurl.com/yd6jf3o7>.

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (<http://tinyurl.com/ycyrxj4o> founded by Lian Sidorov and in *Prespacetime Journal* (<http://tinyurl.com/ycvktjhn>), *Journal of Consciousness Research and Exploration* (<http://tinyurl.com/yba4f672>), and *DNA Decipher Journal* (<http://tinyurl.com/y9z52khg>), all of them founded by Huping Hu. One can find the list about the articles published at <http://tinyurl.com/ybv8dt4n>. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

## 4 The contents of the book

### 4.1 Part I: Hyper-Finite Factors of Type II and Hierarchy of Planck Constants

#### 4.1.1 What von Neumann Right After All?

The work with TGD inspired model for topological quantum computation led to the realization that von Neumann algebras, in particular so called hyper-finite factors of type  $II_1$ , seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD. The discussions of this chapter have been restricted to the basic notions are discussed and only short mention is made to TGD applications discussed in second chapter.

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation  $*$  and observables correspond to Hermitian operators. Any measurable function  $f(A)$  of operator  $A$  belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace:  $tr(Id) = 1$ .

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection

probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type  $II_1$ .

The definitions of adopted by von Neumann allow however more general algebras. Type  $I_n$  algebras correspond to finite-dimensional matrix algebras with finite traces whereas  $I_\infty$  associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type  $III$  non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD. It however seems that in TGD framework based on Zero Energy Ontology identifiable as “square root” of thermodynamics a square root of thermodynamical state is needed.

The inclusions of hyper-finite factors define an excellent candidate for the description of finite measurement resolution with included factor representing the degrees of freedom below measurement resolution. This would also give connection to the notion of quantum group whose physical interpretation has remained unclear. This idea is central to the proposed applications to quantum TGD discussed in separate chapter.

#### 4.1.2 Evolution of Ideas about Hyper-finite Factors in TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology (ZEO). In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework.

##### 1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type  $III_1$  appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type  $II_1$ . Therefore also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is HFF of type  $II_1$ . If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type  $II_1$ . Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type  $II_\infty$  results.
2. WCW is a union of sub-WCWs associated with causal diamonds ( $CD$ ) defined as intersections of future and past directed light-cones. One can allow also unions of  $CD$ s and the proposal is that  $CD$ s within  $CD$ s are possible. Whether  $CD$ s can intersect is not clear.
3. The assumption that the  $M^4$  proper distance  $a$  between the tips of  $CD$  is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that  $a$  can have all possible values. Since  $SO(3)$  is the isotropy group of  $CD$ , the  $CD$ s associated with a given value of  $a$  and with fixed lower tip are parameterized by the Lobatchevski space  $L(a) = SO(3,1)/SO(3)$ . Therefore the  $CD$ s with a free position of lower tip are parameterized by  $M^4 \times L(a)$ . A possible interpretation is in terms of quantum cosmology with  $a$  identified as cosmic time. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type  $III_1$ . If one allows all values of  $a$ , one ends up with  $M^4 \times M^4_+$  as the space of moduli for WCW.
4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of

unit matrix 1 and 7-D gamma matrices  $\gamma_k$  and Pauli sigma matrices by replacing 1 and  $\gamma_k$  by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. One can start from a local octonionic Clifford algebra in  $M^8$ . Associativity (co-associativity) condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of  $M^8$ . This means that the induced gamma matrices associated with the Kähler action span a complex quaternionic (complex co-quaternionic) sub-space at each point of the sub-manifold. This associative (co-associative) sub-algebra can be mapped a matrix algebra. Together with  $M^8 - H$  duality this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative (co-associative) algebra and thus to HFF of type  $II_1$ .

### 2. Hyper-finite factors and M-matrix

HFFs of type  $III_1$  provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism  $\Delta^{it}$  (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.
2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its “complex square root” abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.
3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology (ZEO): the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.
4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing “complex square roots”. Physically they would correspond to different measurement interactions meaning the analog of state function collapse in zero modes fixing the classical conserved charges equal to the quantal counterparts. Classical charges would be parameters characterizing zero modes.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator  $L_0$  would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

### 3. Connes tensor product as a realization of finite measurement resolution

The inclusions  $\mathcal{N} \subset \mathcal{M}$  of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In ZEO  $\mathcal{N}$  would create states experimentally indistinguishable from the original one. Therefore  $\mathcal{N}$  takes the role of complex numbers in non-commutative quantum theory. The space  $\mathcal{M}/\mathcal{N}$  would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative  $\mathcal{N}$ -valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their  $\mathcal{N}$  “averaged” counterparts. The “averaging” would be in terms of the complex square root of  $\mathcal{N}$ -state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that  $\mathcal{N}$  acts like complex numbers on M-matrix elements as far as  $\mathcal{N}$ -“averaged” probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in  $\mathcal{M}(\mathcal{N}$  interpreted as finite-dimensional space with a projection operator to  $\mathcal{N}$ ). The condition that  $\mathcal{N}$  averaging in terms of a complex square root of  $\mathcal{N}$  state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

#### 4. *Analogs of quantum matrix groups from finite measurement resolution?*

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if  $q$  is a root of unity. For  $q = \pm 1$  (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

#### 5. *Quantum spinors and fuzzy quantum mechanics*

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to  $q = 1$ . The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with “true” and “false”. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently



fuzzy. State function reduction would occur only after a transition to  $q=1$  phase and decoherence is not a problem as long as it does not induce this transition.

### 4.1.3 Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic theme of TGD since 2005. The basic idea was stimulated by the suggestion of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by  $\hbar_{gr} = GM_1M_2/v_0$ , where the velocity parameter  $v_0$  has the approximate value  $v_0 \simeq 2^{-11}$  for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

#### 1. Basic physical ideas

The basic phenomenological rules are simple.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Effective imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.
2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order  $CP_2$  size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain:  $E = hf$  implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconstructions of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

In astrophysics and cosmology the implications are even more dramatic. The interpretation of  $\hbar_{gr}$  introduced by Nottale in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses  $M$  and  $m$ . The huge value of  $\hbar_{gr}$  means that the integer  $\hbar_{gr}/\hbar_0$  interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in astronomical scales. The gravitational Compton length  $GM/v_0 = r_S/2v_0$  does not depend on  $m$  so that all particles around say Sun say same gravitational Compton length.

By the independence of gravitational acceleration and gravitational Compton length on particle mass, it is enough to assume that only microscopic particles couple to the dark gravitons

propagating along flux tubes mediating gravitational interaction. Therefore  $h_{gr} = h_{eff}$  could be true in microscopic scales and would predict that cyclotron energies have no dependence on the mass of the charged particle meaning that the spectrum ordinary photons resulting in the transformation of dark photons to ordinary photons is universal. An attractive identification of these photons would be as bio-photons with energies in visible and UV range and thus inducing molecular transitions making control of biochemistry by dark photons. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation. The energy of the graviton is gigantic unless the emission is assumed to take place from a microscopic systems with large but not gigantic  $h_{gr}$ .

3. Why Nature would like to have large - maybe even gigantic - value of effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths  $\alpha = g^2/4\pi\hbar$ . If the effective value of  $\hbar$  replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle,  $\alpha$  is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter  $GMm/\hbar$  has gigantic value. Replacing  $\hbar$  with  $h_{gr} = GMm/v_0$  the coupling strength becomes  $v_0 < 1$ .

#### 2. Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of  $M^4$  and  $CP_2$  with numbers of sheets given by integers  $n_a$  and  $n_b$  and  $\hbar = n\hbar_0$ .  $n = n_a n_b$ .

With the advent of zero energy ontology (ZEO), it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. In ZEO 3-surfaces are unions of space-like 3-surface at opposite boundaries of CD. The non-determinism of Kähler action due to the huge vacuum degeneracy would naturally explain the existence of several space-time sheets connecting the two 3-surfaces at the opposite boundaries of CD. Quantum criticality suggests strongly conformal invariance and the identification of  $n$  as the number of conformal equivalence classes of these space-time sheets. Also a connection with the notion of negentropic entanglement emerges.

#### 4.1.4 Mathematical speculations inspired by the hierarchy of Planck constants

This chapter contains the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) as separate from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first speculative question is about possible relationship between Jones inclusions of hyper-finite factors of type  $II_1$  (hyper-finite factors are von Neuman algebras emerging naturally in TGD framework). The basic idea is that the discrete groups assignable to inclusions could correspond to discrete groups acting in the effective covering spaces of imbedding space assignable to the hierarchy of Planck constants.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of it survives.

### 4.1.5 Negentropy Maximization Principle

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The proposal is that the dynamics of consciousness is governed by Negentropy Maximization Principle (NMP), which states the information content of conscious experience is maximal. The formulation of NMP is the basic topic of this chapter.

NMP codes for the dynamics of standard state function reduction and states that the state function reduction process following  $U$ -process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies at each step a decomposition of the system to unique unentangled subsystems and the process repeats itself for these subsystems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states. The natural assumption is that self loses consciousness when it entangles via bound state entanglement.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement (NE), which is not bound state entanglement in standard sense since the condition that state function reduction leads to an eigenstate of density matrix requires the final state density matrix to be a projection operator.

NE might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of NE the outcome of the reduction is not random: the prediction is that second law is not a universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and  $p$ -adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism. A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

There are two options to consider. Strong form of NMP, which would demand maximal negentropy gain: this would not allow morally responsible free will if ethics is defined in terms of evolution as increase of NE resources. Weak form of NMP would allow self to choose also lower-dimensional sub-space of the projector defining the final state sub-space for strong form of NMP. Weak form turns out to have several highly desirable consequences: it favours dimensions of final state space coming as powers of prime, and in particular dimensions which are primes near powers of prime: as a special case,  $p$ -adic length scale hypothesis follows. Weak form of NMP allows also quantum computations, which halt unlike strong form of NMP.

Besides number theoretic negentropies there are also other new elements as compared to the earlier formulation of NMP.

1. ZEO modifies dramatically the formulation of NMP since  $U$ -matrix acts between zero energy states and can be regarded as a collection of orthonormal  $M$ -matrices, which generalize the ordinary  $S$ -matrix and define what might be called a complex square root of density matrix so that kind of a square root of thermodynamics at single particle level justifying also  $p$ -adic mass calculations based on  $p$ -adic thermodynamics is in question.
2. The hierarchy of Planck constants labelling a hierarchy of quantum criticalities is a further new element having important implications for consciousness and biology.
3. Hyper-finite factors of type  $II_1$  represent an additional technical complication requiring separate treatment of NMP taking into account finite measurement resolution realized in terms of inclusions of these factors.

NMP has wide range of important implications.

1. In particular, one must give up the standard view about second law and replace it with NMP taking into account the hierarchy of CDs assigned with ZEO and dark matter hierarchy labelled by the values of Planck constants, as well as the effects due to NE. The breaking of

second law in standard sense is expected to take place and be crucial for the understanding of evolution.

2. Self hierarchy having the hierarchy of CDs as imbedding space correlate leads naturally to a description of the contents of consciousness analogous to thermodynamics except that the entropy is replaced with negentropy.
3. In the case of living matter NMP allows to understand the origin of metabolism. NMP demands that self generates somehow negentropy: otherwise a state function reduction to the opposite boundary of CD takes place and means death and re-incarnation of self. Metabolism as gathering of nutrients, which by definition carry NE is the manner to avoid this fate. This leads to a vision about the role of NE in the generation of sensory qualia and a connection with metabolism. Metabolites would carry NE and each metabolite would correspond to a particular qualia (not only energy but also other quantum numbers would correspond to metabolites). That primary qualia would be associated with nutrient flow is not actually surprising!
4. NE leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed.
5. NMP implies continual generation of NE. One might refer to this ever expanding universal library as “Akaschic records”. NE could be experienced directly during the repeated state function reductions to the passive boundary of CD - that is during the life cycle of sub-self defining the mental image. Another, less feasible option is that interaction free measurement is required to assign to NE conscious experience. As mentioned, qualia characterizing the metabolite carrying the NE could characterize this conscious experience.
6. A connection with fuzzy qubits and quantum groups with NE is highly suggestive. The implications are highly non-trivial also for quantum computation allowed by weak form of NMP since NE is by definition stable and lasts the lifetime of self in question.

#### 4.1.6 Criticality and dark matter

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large  $h_{eff}$  phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

#### 4.1.7 About the Nottale’s formula for $h_{gr}$ and the possibility that Planck length $l_P$ and $CP_2$ length $R$ are identical giving $G = R^2/h_{eff}$ ?

Nottale’s formula for the gravitational Planck constant  $\hbar_{gr} = GMm/v_0$  involves parameter  $v_0$  with dimensions of velocity. I have worked with the quantum interpretation of the formula but the physical origin of  $v_0$  - or equivalently the dimensionless parameter  $\beta_0 = v_0/c$  (to be used in the sequel) appearing in the formula has remained open hitherto. In this chapter a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed. In ZEO the non-changing parts of zero energy states are assigned to the passive boundary of CD and  $\beta_0$  should be assigned to it.

There are two measures for the size of the system. The  $M^4$  size  $L_{M^4}$  is identifiable as the maximum of the radial  $M^4$  distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size  $L_{ind}$  defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has  $L_{ind} < L_H$ . The identification  $\beta_0 = L_{M^4}/L_H$  does not allow the identification

of  $L_H = L_{M^4}$ .  $L_H$  would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for  $\beta_0$  by approximating the space-time surface as Robertson-Walker cosmology expected to be a good approximation near the passive light-like boundary of CD. The resulting formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$  is analogous to gravitational fine structure constant for  $h_{eff} = h_{gr}$ . Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of  $G$  does not appear at all in the perturbative expansion in this region! Could  $G$  have several values? This suggests the generalization  $G = l_P^2/\hbar \rightarrow G = R^2/\hbar_{eff}$  so that  $G$  would indeed have a spectrum and that Planck length  $l_P$  would be equal to  $CP_2$  radius  $R$  so that only one fundamental length would be associated with twistorialization. Ordinary Newton's constant would be given by  $G = R^2/h_{eff}$  with  $h_{eff}/h_0$  having value in the range  $10^7 - 10^8$ .

The second topic of the chapter relates to the the fact that the measurements of  $G$  give differing results with differences between measurements larger than the measurement accuracy. This suggests that there might be some new physics involved. In TGD framework the hierarchy of Planck constants  $h_{eff} = nh_0$ ,  $h = 6h_0$  together with the condition that theory contains  $CP_2$  size scale  $R$  as only fundamental length scale, suggest the possibility that Newtons constant is given by  $G = R^2/\hbar_{eff}$ , where  $R$  replaces Planck length ( $l_P = \sqrt{\hbar G} \rightarrow l_P = R$ ) and  $\hbar_{eff}/h$  is in the range  $10^6 - 10^7$ . The spectrum of Newton' constant is consistent with Newton's equations if the scaling of  $\hbar_{eff}$  inducing scaling  $G$  is accompanied by opposite scaling of  $M^4$  coordinates in  $M^4 \times CP_2$ : dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of scale invariance. In the special case  $h_{eff} = h_{gr} = GMm/v_0$  quantum critical dynamics as gravitational fine structure constant  $(v_0/c)/4\pi$  as coupling constant and it has no dependence of the value of  $G$  or masses  $M$  and  $m$ .

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of  $G$  differing by 47 ppm in terms of varying  $h_{eff}$ . Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration  $g$  at surface of Earth induced by a change of  $h_{eff}$  due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of  $G_{eff}$  at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

#### 4.1.8 Holography and Quantum Error Correcting Codes: TGD View

Preskill et al suggest a highly interesting representation of holography in terms of quantum error correction codes The idea is that time= constant section of AdS, which is hyperbolic space allowing tessellations, can define tensor networks. So called perfect tensors are building bricks of the tensor networks providing representation for holography and at the same time defining error correcting codes by mapping localized interior states (logical qubits) to highly entangled non-local boundary states (physical qubits).

There are three observations that put bells ringing and actually motivated this article.

1. Perfect tensors define entanglement which TGD framework corresponds negentropic entanglement playing key role in TGD inspired theory of consciousness and of living matter.
2. In TGD framework the hyperbolic tessellations are realized at hyperbolic spaces  $H_3(a)$  defining light-cone proper time hyperboloids of  $M^4$  light-cone.
3. TGD replaces AdS/CFT correspondence with strong form of holography.

A very attractive idea is that in living matter magnetic flux tube networks defining quantum computational networks provide a realization of tensor networks realizing also holographic error

correction mechanism: negentropic entanglement - perfect tensors - would be the key element. As I have proposed, these flux tube networks would define kind of central nervous system make it possible for living matter to experience consciously its biological body using magnetic body.

These networks would also give rise to the counterpart of condensed matter physics of dark matter at the level of magnetic body: the replacement of lattices based on subgroups of translation group with infinite number of tessellations means that this analog of condensed matter physics describes quantum complexity.

## 4.2 Part II: Applications of p-adic length scale hypothesis and dark matter hierarchy

### 4.2.1 Recent status of lepto-hadron hypothesis

TGD suggests strongly the existence of lepto-hadron physics. Lepto-hadrons would be bound states of color excited leptons and the anomalous production of  $e^+e^-$  pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level  $k = 127$  and having typical mass scale of one  $MeV$ . The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orthopositronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and  $e^+e^-$  pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis ( $Z^0$  decay width and production of colored lepton jets in  $e^+e^-$  annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for  $e^+e^-$  production cross section is of correct order of magnitude only provided one assumes that lepto-pions decay to lepto-nucleon pair  $e_{ex}^+e_{ex}^-$  first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing  $n > 2$  particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored  $\mu$  has emerged. Towards the end of 2008 CDF anomaly gave support for the colored excitation of  $\tau$ . The lifetime of the light long lived state identified as a charged  $\tau$ -pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral  $\tau$ -pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral  $\tau$ -pion to 3  $\tau$ -pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

### 4.2.2 TGD and Nuclear Physics

This chapter is devoted to the possible implications of TGD for nuclear physics. In the original version of the chapter the focus was in the attempt to resolve the problems caused by the incorrect interpretation of the predicted long ranged weak gauge fields. What seems to be a breakthrough in this respect came around 2005, more than a decade after the first version of this chapter, and is based on TGD based view about dark matter inspired by the developments in the mathematical understanding of quantum TGD. In this approach condensed matter nuclei can be either ordinary, that is behave essentially like standard model nuclei, or be in dark matter phase in which case they generate long ranged dark weak gauge fields responsible for the large parity breaking effects in living matter. This approach resolves trivially the objections against long range classical weak fields.

About 7 years later (2012) it became clear that the condition that induced spinor fields have well defined em charge localizes their modes in the generic case to 2-surfaces carrying vanishing induced  $W$  gauge fields. It is quite possible that this localization is consistent with Kähler-Dirac

equation only in the Minkowskian regions were the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional.

One can pose the additional condition that also classical  $Z^0$  field vanishes - at least above weak scale. Fundamental fermions would experience only em field so that the worries related to large parity breaking effects would disappear. The proportionality of weak scale to  $h_{eff} = n \times h$  however predicts that weak fields are effectively massless below weak scale. Therefore worries about large parity breaking effects in nuclear physics can be forgotten.

The basic criterion for the transition to dark matter phase having by definition large value of  $\hbar$  is that the condition  $\alpha Q_1 Q_2 \simeq 1$  for appropriate gauge interactions expressing the fact that the perturbation series does not converge. The increase of  $\hbar$  makes perturbation series converging since the value of  $\alpha$  is reduced but leaves lowest order classical predictions invariant.

This criterion can be applied to color force and inspires the hypothesis that valence quarks inside nucleons correspond to large  $\hbar$  phase whereas sea quark space-time sheets correspond to the ordinary value of  $\hbar$ . This hypothesis is combined with the earlier model of strong nuclear force based on the assumption that long color bonds with p-adically scaled down quarks with mass of order MeV at their ends are responsible for the nuclear strong force.

1. *Is strong force due to color bonds between exotic quark pairs?*

The basic assumptions are following.

1. Valence quarks correspond to large  $\hbar$  phase with p-adic length scale  $L(k_{eff} = 129) = L(107)/v_0 \simeq 2^{11}L(107) \simeq 5 \times 10^{-12}$  m whereas sea quarks correspond to ordinary  $\hbar$  and define the standard size of nucleons.
2. Color bonds with length of order  $L(127) \simeq 2.5 \times 10^{-12}$  m and having quarks with ordinary  $\hbar$  and p-adically scaled down masses  $m_q(dark) \simeq v_0 m_q$  at their ends define kind of rubber bands connecting nucleons. The p-adic length scale of exotic quarks differs by a factor 2 from that of dark valence quarks so that the length scales in question can couple naturally. This large length scale as also other p-adic length scales correspond to the size of the topologically quantized field body associated with system, be it quark, nucleon, or nucleus.
3. Valence quarks and even exotic quarks can be dark with respect to both color and weak interactions but not with respect to electromagnetic interactions. The model for binding energies suggests darkness with respect to weak interactions with weak boson masses scaled down by a factor  $v_0$ . Weak interactions remain still weak. Quarks and nucleons as defined by their  $k = 107$  sea quark portions condense at scaled up weak space-time sheet with  $k_{eff} = 111$  having p-adic size  $10^{-14}$  meters. The estimate for the atomic number of the heaviest possible nucleus comes out correctly.

The wave functions of the nucleons fix the boundary values of the wave functionals of the color magnetic flux tubes idealizable as strings. In the terminology of M-theory nucleons correspond to small branes and color magnetic flux tubes to strings connecting them.

2. *General features of strong interactions*

This picture allows to understand the general features of strong interactions.

1. Quantum classical correspondence and the assumption that the relevant space-time surfaces have 2-dimensional  $CP_2$  projection implies Abelianization. Strong isospin group can be identified as the  $SU(2)$  subgroup of color group acting as isotropies of space-time surfaces. and the  $U(1)$  holonomy of color gauge potential defines a preferred direction of strong isospin. Dark color isospin corresponds to strong isospin. The correlation of dark color with weak isospin of the nucleon is strongly suggested by quantum classical correspondence.
2. Both color singlet spin 0 pion type bonds and colored spin 1 bonds are allowed and the color magnetic spin-spin interaction between the exotic quark and anti-quark is negative in this case. p-p and n-n bonds correspond to oppositely colored spin 1 bonds and p-n bonds to colorless spin 0 bonds for which the binding energy is free times higher. The presence of colored bonds forces the presence of neutralizing dark gluon condensate favoring states with  $N - P > 0$ .

3. Shell model based on harmonic oscillator potential follows naturally from this picture in which the magnetic flux tubes connecting nucleons take the role of springs. Spin-orbit interaction can be understood in terms of the color force in the same way as it is understood in atomic physics.

3. *Nuclear binding energies*

1. The binding energies per nucleon for  $A \leq 4$  nuclei can be understood if they form closed string like structures, nuclear strings, so that only two color bonds per nucleon are possible. This could be understood if ordinary quarks and exotic quarks possessing much smaller mass behave as if they were identical fermions. p-Adic mass calculations support this assumption. Also the average behavior of binding energy for heavier nuclei is predicted correctly.
2. For nuclei with  $P = N$  all color bonds can be pion type bonds and have thus largest color magnetic spin-spin interaction energy. The increase of color Coulombic binding energy between colored exotic quark pairs and dark gluons however favors  $N > P$  and explains also the formation of neutron halo outside  $k = 111$  space-time sheet.
3. Spin-orbit interaction provides the standard explanation for magic numbers. If the maximum of the binding energy per nucleon is taken as a criterion for magic, also  $Z=N=4,6,12$  are magic. The alternative TGD based explanation for magic numbers  $Z = N = 4, 6, 8, 12, 20$  would be in terms of regular Platonic solids. Experimentally also other magic numbers are known for neutrons. The linking of nuclear strings provides a possible mechanism producing new magic nuclei from lighter magic nuclei.

4. *Stringy description of nuclear reactions*

The view about nucleus as a collection of linked nuclear strings suggests stringy description of nuclear reactions. Microscopically the nuclear reactions would correspond to re-distribution of exotic quarks between the nucleons in reacting nuclei.

5. *Anomalies and new nuclear physics*

The TGD based explanation of neutron halo has been already mentioned. The recently observed tetra-neutron states are difficult to understand in the standard nuclear physics framework since Fermi statistics does not allow this kind of state. The identification of tetra-neutron as an alpha particle containing two negatively charged color bonds allows to circumvent the problem. A large variety of exotic nuclei containing charged color bonds is predicted.

The proposed model explains the anomaly associated with the tritium beta decay. What has been observed is that the spectrum intensity of electrons has a narrow bump near the endpoint energy. Also the maximum energy  $E_0$  of electrons is shifted downwards. I have considered two explanations for the anomaly. The original models are based on TGD variants of original models involving belt of dark neutrinos or antineutrinos along the orbit of Earth. Around 2008)I realized that nuclear string model provides much more elegant explanation of the anomaly and has also the potential to explain much more general anomalies.

Cold fusion has not been taken seriously by the physics community but the situation has begun to change gradually. There is an increasing evidence for the occurrence of nuclear transmutations of heavier elements besides the production of  ${}^4\text{He}$  and  ${}^3\text{H}$  whereas the production rate of  ${}^3\text{He}$  and neutrons is very low. These characteristics are not consistent with the standard nuclear physics predictions. Also Coulomb wall and the absence of gamma rays and the lack of a mechanism transferring nuclear energy to the electrolyte have been used as an argument against cold fusion. TGD based model relying on the notion of charged color bonds could explain the anomalous characteristics of cold fusion. The basic mechanism making possible to circumvent Coulomb wall could be large  $h_{eff}$  phase for weak bosons scaling the weak length scale to atomic length scale so that proton could transform to neutron by the exchange of dark  $W$  boson with target nucleus.

### 4.2.3 Nuclear String Hypothesis

Nuclear string hypothesis is one of the most dramatic almost-predictions of TGD. The hypothesis in its original form assumes that nucleons inside nucleus form closed nuclear strings with neighboring



nuclei of the string connected by exotic meson bonds consisting of color magnetic flux tube with quark and anti-quark at its ends. It is also possible that neutrons and protons form their own strings. The lengths of flux tubes correspond to the p-adic length scale of electron and therefore the mass scale of the exotic mesons is around 1 MeV in accordance with the general scale of nuclear binding energies. The long lengths of em flux tubes increase the distance between nucleons and reduce Coulomb repulsion. A fractally scaled up variant of ordinary QCD with respect to p-adic length scale would be in question and the usual wisdom about ordinary pions and other mesons as the origin of nuclear force would be simply wrong in TGD framework as the large mass scale of ordinary pion indeed suggests.

1. *A > 4 nuclei as nuclear strings consisting of A ≤ 4 nuclei*

In this article a more refined version of nuclear string hypothesis is developed.

1. It is assumed  ${}^4\text{He}$  nuclei and  $A < 4$  nuclei and possibly also nucleons appear as basic building blocks of nuclear strings.  $A \leq 4$  nuclei in turn can be regarded as strings of nucleons. Large number of stable lightest isotopes of form  $A = 4n$  supports the hypothesis that the number of  ${}^4\text{He}$  nuclei is maximal. Even the weak decay characteristics might be reduced to those for  $A < 4$  nuclei using this hypothesis.
2. One can understand the behavior of nuclear binding energies surprisingly well from the assumptions that total *strong* binding energy associated with  $A \leq 4$  building blocks is *additive* for nuclear strings.
3. In TGD framework tetra-neutron is interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants. For heavier nuclei tetra-neutron is needed as an additional building brick.

2. *Bose-Einstein condensation of color bonds as a mechanism of nuclear binding*

The attempt to understand the variation of the nuclear binding energy and its maximum for  $Fe$  leads to a quantitative model of nuclei lighter than  $Fe$  as color bound Bose-Einstein condensates of pion like colored states associated with color flux tubes connecting  ${}^4\text{He}$  nuclei. The color contribution to the total binding energy is proportional to  $n^2$ , where  $n$  is the number of color bonds. Fermi statistics explains the reduction of  $E_B$  for the nuclei heavier than  $Fe$ . Detailed estimate favors harmonic oscillator model over free nucleon model with oscillator strength having interpretation in terms of string tension.

Fractal scaling argument allows to understand  ${}^4\text{He}$  and lighter nuclei as strings of nucleons with nucleons bound together by color bonds. Three fractally scaled variants of QCD corresponding  $A > 4$ ,  $A = 4$ , and  $A < 4$  nuclei are involved. The binding energies of also  $A \leq 4$  are predicted surprisingly accurately by applying simple p-adic scaling to the model of binding energies of heavier nuclei.

3. *Giant dipole resonance as de-coherence of Bose-Einstein condensate of color bonds*

Giant resonances and so called pygmy resonances are interpreted in terms of de-coherence of the Bose-Einstein condensates associated with  $A \leq 4$  nuclei and with the nuclear string formed from  $A \leq 4$  nuclei. The splitting of the Bose-Einstein condensate to pieces costs a precisely defined energy. For  ${}^4\text{He}$  de-coherence the model predicts singlet line at 12.74 MeV and triplet at  $\sim 27$  MeV spanning 4 MeV wide range.

The de-coherence at the level of nuclear string predicts 1 MeV wide bands 1.4 MeV above the basic lines. Bands decompose to lines with precisely predicted energies. Also these contribute to the width. The predictions are in rather good agreement with experimental values. The so called pygmy resonance appearing in neutron rich nuclei can be understood as a de-coherence for  $A = 3$  nuclei. A doublet at  $\sim 8$  MeV and MeV spacing is predicted. The prediction for the position is correct.

4. *Dark nuclear strings as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code*

A speculative picture proposing a connection between homeopathy, water memory, and phantom DNA effect is discussed and on basis of this connection a vision about how the hardware for topological quantum computation (TQC) represented by the genome is actively developed by subjecting it to evolutionary pressures represented by a virtual world representation of the physical environment. The speculation inspired by this vision is that genetic code as well as DNA-, RNA- and amino-acid sequences should have representation in terms of nuclear strings. The model for dark baryons indeed leads to an identification of these analogs and the basic numbers of genetic code including also the numbers of amino-acids coded by a given number of codons are predicted correctly. Hence genetic code would be universal rather than being an accidental outcome of the biological evolution.

#### 4.2.4 Cold Fusion Again

During years I have developed two models of cold fusion and in this article these models are combined together. The basic idea of TGD based model of cold is that cold fusion occurs in two steps. First dark nuclei (large  $h_{eff} = n \times h$ ) with much lower binding energy than ordinary nuclei are formed at magnetic flux tubes possibly carrying monopole flux. These nuclei can leak out the system along magnetic flux tubes. Under some circumstances these dark nuclei can transform to ordinary nuclei and give rise to detectable fusion products.

An essential additional condition is that the dark protons can decay to neutrons rapidly enough by exchanges of dark weak bosons effectively massless below atomic length scale. Also beta decays in which dark W boson decays to dark electron and neutrino can be considered. This allows to overcome the Coulomb wall and explains why final state nuclei are stable and the decay to ordinary nuclei does not yield only protons. Thus it seems that this model combined with the TGD variant of Widom-Larsen model could explain nicely the existing data.

In this chapter I will describe the steps leading to the TGD inspired model for cold fusion combining the earlier TGD variant of Widom-Larsen model with the model inspired by the TGD inspired model of Pollack's fourth phase of water using as input data findings from laser pulse induced cold fusion discovered by Leif Holmlid and collaborators. I consider briefly also alternative options (models assuming surface plasma polariton and heavy electron). After that I apply TGD inspired model in some cases (Pons-Fleischman effect, bubble fusion, and LeClair effect). The model explains the strange findings about cold fusion - in particular the fact that only stable nuclei are produced - and suggests that also ordinary nuclear reactions might have more fundamental description in terms of similar model.

#### 4.2.5 Dark Nuclear Physics and Condensed Matter

In this chapter the possible effects of dark matter in nuclear physics and condensed matter physics are considered. The spirit of the discussion is necessarily rather speculative. The most general form of the hierarchy would involve both singular coverings and factor spaces of  $CD$  (causal diamond of  $M^4$ ) defined as intersection of future and past directed light-cones) and  $CP_2$ . There are grave objections against the allowance of factor spaces. In this case Planck constant could be smaller than its standard value and there are very few experimental indications for this. Quite recently came the realization that the hierarchy of Planck constants might emerge from the basic quantum TGD as a consequence of the extreme non-linearity of field equations implying that the correspondence between the derivatives of imbedding space coordinates and canonical momentum is many-to-one. This makes natural to the introduction of covering spaces of  $CD$  and  $CP_2$ .

Planck constant would be effectively replaced with a multiple of ordinary Planck constant defined by the number of the sheets of the covering. The space-like 3-surfaces at the ends of the causal diamond and light-like 3-surfaces defined by wormhole throats carrying elementary particle quantum numbers would be quantum critical in the sense of being unstable against decay to many-sheeted structures. Charge fractionization could be understood in this scenario. Biological evolution would have the increase of the Planck constant as one aspect. The crucial scaling of the size of  $CD$  by Planck constant can be justified by a simple argument. Note that primary p-adic length scales would scale as  $\sqrt{\hbar}$  rather than  $\hbar$  as assumed in the original model.

Recently the hierarchy of Planck constants have been traced to the non-determinism of Kähler action predicting in zero energy ontology (ZEO) that two space-like 3-surfaces at the ends of

causal diamonds (CD) can be connected by several space-time surfaces. As a matter fact, by infinite number of them related by quantum critical deformations identifiable as conformal transformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes. The number of conformal equivalence classes of space-time sheets would be integer  $n$  defining the effective Planck constant  $\hbar_{eff} = n \times \hbar$ .

1. *What darkness means?*

Dark matter is identified as matter with non-standard value of Planck constant. The weak form of darkness states that only some field bodies of the particle consisting of flux quanta mediating bound state interactions between particles become dark. One can assign to each interaction a field body (em,  $Z^0$ ,  $W$ , gluonic, gravitational) and p-adic prime and the value of Planck constant characterize the size of the particular field body. One might even think that particle mass can be assigned with its em field body and that Compton length of particle corresponds to the size scale of em field body.

Nuclear string model suggests that the sizes of color flux tubes and weak flux quanta associated with nuclei can become dark in this sense and have size of order atomic radius so that dark nuclear physics would have a direct relevance for condensed matter physics. If this happens, it becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore.

2. *What dark nucleons are?*

The basic hypothesis is that nuclei can make a phase transition to dark phase in which the size of both quarks and nuclei is measured in Angstroms. For the less radical option this transition could happen only for the color, weak, and em field bodies. Proton connected by dark color bonds super-nuclei with inter-nucleon distance of order atomic radius might be crucial for understanding the properties of water and perhaps even the properties of ordinary condensed matter. Large  $\hbar$  phase for weak field body of  $D$  and  $Pd$  nuclei with size scale of atom would explain selection rules of cold fusion.

3. *Anomalous properties of water and dark nuclear physics*

A direct support for partial darkness of water comes from the  $H_{1.5}O$  chemical formula supported by neutron and electron diffraction in attosecond time scale. The explanation could be that one fourth of protons combine to form super-nuclei with protons connected by color bonds and having distance sufficiently larger than atomic radius.

The crucial property of water is the presence of molecular clusters. Tetrahedral clusters allow an interpretation in terms of magic  $Z=8$  protonic dark nuclei. The icosahedral clusters consisting of 20 tetrahedral clusters in turn have interpretation as magic dark dark nuclei: the presence of the dark dark matter explains large portion of the anomalies associated with water and explains the unique role of water in biology. In living matter also higher levels of dark matter hierarchy are predicted to be present. The observed nuclear transmutation suggest that also light weak bosons are present.

4. *Implications of the partial darkness of condensed matter*

The model for partially dark condensed matter inspired by nuclear string model and the model of cold fusion inspired by it allows to understand the low compressibility of the condensed matter as being due to the repulsive weak force between exotic quarks, explains large parity breaking effects in living matter, and suggests a profound modification of the notion of chemical bond having most important implications for bio-chemistry and understanding of bio-chemical evolution.

4.2.6 Dark Forces and Living Matter

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether electro-weak charges of elementary particles are screened in electro-weak length scale or not. The TGD based view about dark matter assumes that weak charges are indeed screened for ordinary matter in electro-weak length scale but that dark electro-weak bosons correspond to much longer symmetry breaking length scale. The localization of the modes of Kähler-Dirac action

to 2-D surfaces at which  $W$  fields vanish realizes this idea concretely. Also  $Z^0$  fields can vanish and are expected to do so above weak scale.

The large value of  $\hbar$  in dark matter phase implies that Compton lengths and -times are scaled up. In particular, the sizes of nucleons and nuclei become of order atom size so that dark nuclear physics would have direct relevance for condensed matter physics. It becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore. This view forces a profound re-consideration of the earlier ideas in nuclear and condensed physics context. It however seems that most of the earlier ideas related to the classical  $Z^0$  force and inspired by anomaly considerations survive in a modified form.

The weak form of electric-magnetic duality led to the identification of the long sought for mechanism causing the weak screening in electroweak scales. The basic implication of the duality is that Kähler electric charges of wormhole throats representing particles are proportional to Kähler magnetic charges so that the  $CP_2$  projections of the wormhole throats are homologically non-trivial. The Kähler magnetic charges do not create long range monopole fields if they are neutralized by wormhole throats carrying opposite monopole charges and weak isospin neutralizing the axial isospin of the particle's wormhole throat. One could speak of confinement of weak isospin. The weak field bodies of elementary fermions would be replaced with string like objects with a length of order  $W$  boson Compton length. Electro-magnetic flux would be feeded to electromagnetic field body where it would be feeded to larger space-time sheets. Similar mechanism could apply in the case of color quantum numbers. Weak charges would be therefore screened for ordinary matter in electro-weak length scale but dark electro-weak bosons correspond to much longer symmetry breaking length scale for weak field body. Large values of Planck constant would make it possible to zoop up elementary particles and study their internal structure without any need for gigantic accelerators.

One can still worry about large parity breaking effects - say in nuclear physics- since the couplings of spinors to classical weak fields are there. Around 2012 it became clear that the condition that induced spinor fields have well defined em charge localizes their modes in the generic case to 2-surfaces carrying vanishing induced  $W$  gauge fields. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional.

One can pose the additional condition that also classical  $Z^0$  field vanishes - at least above weak scale. Fundamental fermions would experience only em field so that the worries related to large parity breaking effects would disappear. The proportionality of weak scale to  $h_{eff} = n \times \hbar$  however predicts that weak fields are effectively massless belong scaled up weak scale. Therefore worries about large parity breaking effects in ordinary nuclear physics can be forgotten.

In this chapter possible implications of the dark weak force for the understanding of living matter are discussed. The basic question is how classical  $Z^0$  fields could make itself visible. Large parity breaking effects in living matter suggests which direction one should look for the answer to the question. One possible answer is based on the observation that for vacuum extremals classical electromagnetic and  $Z^0$  fields are proportional to each other and this means that the electromagnetic charges of dark fermions standard are replaced with effective couplings in which the contribution of classical  $Z^0$  force dominates. This modifies dramatically the model for the cell membrane as a Josephson junction and raises the scale of Josephson energies from IR range just above thermal threshold to visible and ultraviolet. The amazing finding is that the Josephson energies for biologically important ions correspond to the energies assigned to the peak frequencies in the biological activity spectrum of photoreceptors in retina suggesting. This suggests that almost vacuum extremals and thus also classical  $Z^0$  fields could be in a central role in the understanding of the functioning of the cell membrane and of sensory qualia. This would also explain the large parity breaking effects in living matter.

A further conjecture is that EEG and its predicted fractally scaled variants which same energies in visible and UV range but different scales of Josephson frequencies correspond to Josephson photons with various values of Planck constant. The decay of dark ELF photons with energies of visible photons would give rise to bunches of ordinary ELF photons. Biophotons in turn could correspond to ordinary visible photons resulting in the phase transition of these photons to photons with ordinary value of Planck constant. This leads to a very detailed view about the role of dark electromagnetic radiation in biomatter and also to a model for how sensory qualia are realized. The general conclusion might be that most effects due to the dark weak force are associated with

almost vacuum extremals.

#### 4.2.7 Super-Conductivity in Many-Sheeted Space-Time

In this chapter a model for high  $T_c$  super-conductivity as quantum critical phenomenon is developed. The relies on the notions of quantum criticality, dynamical quantized Planck constant requiring a generalization of the 8-D imbedding space to a book like structure, and many-sheeted space-time. In particular, the notion of magnetic flux tube as a carrier of supra current of central concept.

With a sufficient amount of twisting and weaving these basic ideas one ends up to concrete model for high  $T_c$  superconductors as quantum critical superconductors consistent with the qualitative facts that I am personally aware. The following minimal model looks the most realistic option found hitherto.

1. The general idea is that magnetic flux tubes are carriers of supra currents. In anti-ferromagnetic phases these flux tube structures form small closed loops so that the system behaves as an insulator. Some mechanism leading to a formation of long flux tubes must exist. Doping creates holes located around stripes, which become positively charged and attract electrons to the flux tubes.
2. The basic mechanism for the formation of Cooper pairs is simple. Magnetic flux tubes would be carriers of dark particles and magnetic fields would be crucial for super-conductivity. Two parallel flux tubes carrying magnetic fluxes in opposite directions is the simplest candidate for super-conducting system. This conforms with the observation that antiferromagnetism is somehow crucial for high temperature super-conductivity. The spin interaction energy is proportional to Planck constant and can be above thermal energy: if the hypothesis that dark cyclotron energy spectrum is universal is accepted, then the energies would be in bio-photon range and high temperature super-conductivity is obtained. If fluxes are parallel spin  $S = 1$  Cooper pairs are stable.  $L = 2$  states are in question since the members of the pair are at different flux tubes.
3. The higher critical temperature  $T_{c1}$  corresponds to a formation local configurations of parallel spins assigned to the holes of stripes giving rise to a local dipole fields with size scale of the order of the length of the stripe. Conducting electrons form Cooper pairs at the magnetic flux tube structures associated with these dipole fields. The elongated structure of the dipoles favors angular momentum  $L = 2$  for the pairs. The presence of magnetic field favors Cooper pairs with spin  $S = 1$ .
4. Stripes can be seen as 1-D metals with delocalized electrons. The interaction responsible for the energy gap corresponds to the transversal oscillations of the magnetic flux tubes inducing oscillations of the nuclei of the stripe. These transverse phonons have spin and their exchange is a good candidate for the interaction giving rise to a mass gap. This could explain the BCS type aspects of high  $T_c$  super-conductivity.
5. Above  $T_c$  supra currents are possible only in the length scale of the flux tubes of the dipoles which is of the order of stripe length. The reconnections between neighboring flux tube structures induced by the transverse fluctuations give rise to longer flux tubes structures making possible finite conductivity. These occur with certain temperature dependent probability  $p(T, L)$  depending on temperature and distance  $L$  between the stripes. By criticality  $p(T, L)$  depends on the dimensionless variable  $x = TL/\hbar$  only:  $p = p(x)$ . At critical temperature  $T_c$  transverse fluctuations have large amplitude and makes  $p(x_c)$  so large that very long flux tubes are created and supra currents can run. The phenomenon is completely analogous to percolation.
6. The critical temperature  $T_c = x_c \hbar / L$  is predicted to be proportional to  $\hbar$  and inversely proportional to  $L$  (, which is indeed to be the case). If flux tubes correspond to a large value of  $\hbar$ , one can understand the high value of  $T_c$ . Both Cooper pairs and magnetic flux tube structures represent dark matter in TGD sense.

7. The model allows to interpret the characteristic spectral lines in terms of the excitation energy of the transversal fluctuations and gap energy of the Cooper pair. The observed 50 meV threshold for the onset of photon absorption suggests that below  $T_c$  also  $S = 0$  Cooper pairs are possible and have gap energy about 9 meV whereas  $S = 1$  Cooper pairs would have gap energy about 27 meV. The flux tube model indeed predicts that  $S = 0$  Cooper pairs become stable below  $T_c$  since they cannot anymore transform to  $S = 1$  pairs. Their presence could explain the BCS type aspects of high  $T_c$  super-conductivity. The estimate for  $\hbar/\hbar_0 = r$  from critical temperature  $T_{c1}$  is about  $r = 3$  contrary to the original expectations inspired by the model of living system as a super-conductor suggesting much higher value. An unexpected prediction is that coherence length is actually  $r$  times longer than the coherence length predicted by conventional theory so that type I super-conductor could be in question with stripes serving as duals for the defects of type I super-conductor in nearly critical magnetic field replaced now by ferromagnetic phase.

At qualitative level the model explains various strange features of high  $T_c$  superconductors. One can understand the high value of  $T_c$  and ambivalent character of high  $T_c$  super conductors, the existence of pseudogap and scalings laws for observables above  $T_c$ , the role of stripes and doping and the existence of a critical doping, etc...

#### 4.2.8 Quantum Hall effect and Hierarchy of Planck Constants

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of  $\nu$ , and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

1. The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (Kähler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by delocalized electron at the boundary of the vortex. One can raise two questions.

Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

2. Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge fractionalization and fractional braiding statistics. Here the vacuum degeneracy of Kähler action suggests the explanation. Vacuum degeneracy implies that the correspondence between the normal component of the canonical momentum current and normal derivatives of imbedding space coordinates is 1- to- $n$ . These kind of branchings result in multi-furcations induced by variations of the system parameters and the scaling of external magnetic field represents one such variation.
3. At the orbits of wormhole throats, which can have even macroscopic  $M^4$  projections, one has  $1 \rightarrow n_a$  correspondence and at the space-like ends of the space-time surface at light-like boundaries of causal diamond one has  $1 \rightarrow n_b$  correspondence. This implies that at partonic 2-surfaces defined as the intersections of these two kinds of 3-surfaces one has  $1 \rightarrow n_a \times n_b$  correspondence. This correspondence can be described by using a local singular  $n$ -fold covering of the imbedding space. Unlike in the original approach, the covering space is only a convenient auxiliary tool rather than fundamental notion.

4. The fractionalization of charge can be understood as follows. A delocalization of electron charge to the  $n$  sheets of the multi-furcation takes place and single sheet is analogous to a sheet of Riemann surface of function  $z^{1/n}$  and carries fractional charge  $q = e/n$ ,  $n = n_a n_b$ . Fractionalization applies also to other quantum numbers. One can have also many-electron states of these states with several delocalized electrons: in this case one obtains more general charge fractionalization:  $q = \nu e$ .
5. Also the fractional braid statistics can be understood. For ordinary statistics rotations of  $M^4$  rotate entire partonic 2-surfaces. For braid statistics rotations of  $M^4$  (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of  $z^{1/n}$ , the braid rotation by  $\Delta\Phi = 2\pi$ , where  $\Phi$  corresponds to  $M^4$  angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate  $\Phi$  of the  $n$ -branched covering  $\Delta\Phi = 2/\pi$  corresponds to  $\Delta\Phi = n \times 2\pi$ . If one identifies the sheets of multi-furcation and therefore uses  $\Phi$  as angle coordinate, single valued angular momentum eigenstates become in general  $n$ -valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.
6. How to understand the exceptional values  $\nu = 5/2, 7/2$  of the filling fraction? The non-abelian braid group representations can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible. It seems that the minimum number of braids is  $n > 2$  from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives  $n > 3$ . The minimum value is  $n = 4$  consistent with the fractional charge  $e/4$ .

The model introduces  $Z_4$  valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a  $Z_4$  valued momentum characterizing the four delocalized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would correspond to that of 4-fold covering space of imbedding space serving as a convenient auxiliary tool. The more standard explanation is that  $Z_4 = Z_2 \times Z_2$  such that  $Z_2$ 's correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. Electroweak gauge group defined non-abelian braid group in large  $h_{eff}$  phase weak length above atomic length scale so that weak bosons and even fermion behave as effectively massless particles below scaled up weak scale. TGD also predicts the possibility of dynamical gauge groups and maybe this kind of gauge group indeed emerges. Dynamical gauge groups emerge also for stacks of  $N$  branes and the  $n$  sheets of multifurcation are analogous to the  $N$  sheets in the stack for many-electron states.

#### 4.2.9 A Possible Explanation of Shnoll Effect

Shnoll and collaborators have discovered strange repeating patterns of random fluctuations of physical observables such as the number  $n$  of nuclear decays in a given time interval. Periodically occurring peaks for the distribution of the number  $N(n)$  of measurements producing  $n$  events in a series of measurements as a function of  $n$  is observed instead of a single peak. The positions of the peaks are not random and the patterns depend on position and time varying periodically in time scales possibly assignable to Earth-Sun and Earth-Moon gravitational interaction.

These observations suggest a modification of the expected probability distributions but it is very difficult to imagine any physical mechanism in the standard physics framework. Rather, a universal deformation of predicted probability distributions would be in question requiring something analogous to the transition from classical physics to quantum physics.

A possible hint about the nature of the modification comes from the TGD inspired quantum measurement theory proposing a description of the notion of finite measurement resolution in terms of inclusions of so called hyper-finite factors of type II<sub>1</sub> (HFFs) and closely related quantum groups. Also p-adic physics -another key element of TGD- is expected to be involved. A modification of a given probability distribution  $P(n|\lambda_i)$  for a positive integer valued variable  $n$  characterized by

rational-valued parameters  $\lambda_i$  is obtained by replacing  $n$  and the integers characterizing  $\lambda_i$  with so called quantum integers depending on the quantum phase  $q_m = \exp(i2\pi/m)$ . Quantum integer  $n_q$  must be defined as the product of quantum counterparts  $p_q$  of the primes  $p$  appearing in the prime decomposition of  $n$ . One has  $p_q = \sin(2\pi p/m)/\sin(2\pi/m)$  for  $p \neq P$  and  $p_q = P$  for  $p = P$ .  $m$  must satisfy  $m \geq 3$ ,  $m \neq p$ , and  $m \neq 2p$ .

The quantum counterparts of positive integers can be negative. Therefore quantum distribution is defined first as p-adic valued distribution and then mapped by so called canonical identification  $I$  to a real distribution by the map taking p-adic  $-1$  to  $P$  and powers  $P^n$  to  $P^{-n}$  and other quantum primes to themselves and requiring that the mean value of  $n$  is for distribution and its quantum variant. The map  $I$  satisfies  $I(\sum P_n) = \sum I(P_n)$ . The resulting distribution has peaks located periodically with periods coming as powers of  $P$ . Also periodicities with peaks corresponding to  $n = n^+n^-$ ,  $n_q^+ > 0$  with fixed  $n_q^- < 0$ , are predicted. These predictions are universal and easily testable. The prime  $P$  and integer  $m$  characterizing the quantum variant of distribution can be identified from data. The shapes of the distributions obtained are qualitatively consistent with the findings of Shnoll but detailed tests are required to see whether the number theoretic predictions are correct.

The periodic dependence of the distributions would be most naturally assignable to the gravitational interaction of Earth with Sun and Moon and therefore to the periodic variation of Earth-Sun and Earth-Moon distances. The TGD inspired proposal is that the p-dic prime  $P$  and integer  $m$  characterizing the quantum distribution are determined by a process analogous to a state function reduction and their most probably values depend on the deviation of the distance  $R$  through the formulas  $\Delta p/p \simeq k_p \Delta R/R$  and  $\Delta m/m \simeq k_m \Delta R/R$ . The p-adic primes assignable to elementary particles are very large unlike the primes which could characterize the empirical distributions. The hierarchy of Planck constants allows the gravitational Planck constant assignable to the space-time sheets mediating gravitational interactions to have gigantic values and this allows p-adicity with small values of the p-adic prime  $P$ .

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