From Principles To Diagrams

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Abstract

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

1 Introduction

The generalization of twistor diagrams to TGD framework has been very inspiring (and also frightening) mission impossible and allowed to gain deep insights about what TGD diagrams could be mathematically. I of course cannot provide explicit formulas but the general structure for the construction of twistorial amplitudes in $\mathcal{N}=4$ SUSY suggests an analogous construction in TGD thanks to huge symmetries of TGD and unique twistorial properties of $M^4 \times CP_2$. The twistor program in TGD framework has been summarized in [K5].

Contrary to the original expectations, the twistorial approach is not a mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

There are some new results forcing a profound modification of the recent view about TGD but consistent with the general picture. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. The fiber F of Minkowskian twistor space must be identified with sphere S^2 with signature $(-1,-1)$ and would be a variant of the complex space with complex coordinates associated with S^2 and transversal space E^2 in the decomposition $M^4 = M^2 \times E^2$ and one hyper-complex coordinate associated with M^2.

The action principle in 6-D context is also Kähler action, which dimensionally reduces to Kähler action plus cosmological term. This brings in the radii of spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistors space of M^4 and CP_2. For $S(CP_2)$ the radius is of order CP_2 radius R. $R(S^2(M^4))$ could be of the order of Planck length l_P, which would thus become purely classical parameter contrary the expectations. An alternative option is $R(S^2(M^4)) = R$ The radius of S^2 associated with space-time surface is determined by the induced metric and is emergent length scale. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared brings in a further length scale closely related to cosmological constant which is also dynamical and has correct sign to explain accelerated expansion of the Universe. The order of magnitude for L must be radius of the $S^2(X^4)$ and therefore small. This could mean a gigantic cosmological constant. Just as in GRT based cosmology!

This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$ 6-D Kähler action is a sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action and for it the contribution from $S^2(CP_2)$ fiber is assumed to be absent: this could be due to the imbedding of $S^2(X^4)$ reducing to identification $S^2(M^4)$ and is not true generally. Second term in action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$. The independency implies that couplings strengths are independent for them.

The analog for Kähler coupling strength (analogous to critical temperature) associated with $S^2(M^4)$ must be extremely large - so large that one has $\alpha_K(M^4) \times R(M^4)^2 \sim L^2$, L size scale of the recent Universe. This makes possible the small value of cosmological constant assignable to the volume term given by this part of the dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ comes essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. In fact, it turns that one can assumed that the entire 6-D Kähler action contributes if one assumes that the winding numbers (w_1, w_2) for the map $S^2(X^4) \rightarrow S^2(M^4) \times S^2(CP_2)$ satisfy $(w_1, w_2) = (n, 0)$ in cosmological scales. The identification of w_1 as $h_{eff}/h = n$ is highly suggestive.

The dimensionally reduced dynamics is a highly non-trivial modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure. Strong constraints come also from the condition that induced spinor structure coming from that for twistor space $T(H)$ is essentially that coming from that of H.

Second new element is the fusion of the twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

In the sequel I will discuss the recent understanding of twistorizalization, which is considerably improved from that in the earlier formulation. I formulate the dimensional reduction of 6-D Kähler action and consider the physical interpretation. There are considerable uncertainties at the level of details I dare believe that basically the situation is understood. After that I proceed to discuss the basic principles behind the recent view about scattering amplitudes as generalized Feynman diagrams.

2 twistor lift of Kähler action

First I will try to clarify the mathematical details related to the twistor spaces and how they emerge in the recent context. I do not regard myself as a mathematician in technical sense and I can only hope that the representation based on physical intuition does not contain serious mistakes.

2.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique. Space-times are surfaces in $H = M^4 \times CP_2$. M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept. Strictly speaking, it is E^4 and S^4 allow twistor space with Kähler structure [A2] : in the case of M^4 signature could cause problems. The standard identification for the twistor space of M^4 would be Minkowskian variant $PT = P_3 = SU(2,2)/SU(2,1) \times U(1)$ of 6-D twistor space $PT = CP_3 = SU(4)/SU(3) \times U(1)$ of E^4. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

The case of M^4 is however problematic. It is often stated that the twistor space is $PT = CP_3 = SU(4)/SU(3) \times U(1)$. The metric of twistor space does not appear in the construction of twistor amplitudes. Already the basic structure of PT suggests that this identification cannot be correct.

As if the situation were not complicated enough, there are two notions of twistor space: the twistor space identified as P_3 and as a trivial sphere bundle $M^4 \times CP_1$ having Kähler structure - what Kähler structure actually means in case of M^4 is hower not quite clear.

These considerations lead to a proposal - just a proposal - for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4) \times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$. In the following $T(M^4)$ therefore denotes the trivial sphere bundle $M^4 \times CP_1$ over M^4 and twistorialization of scattering amplitudes would involve the projection from $T(M^4)$ to P_3. What is nice in this formulation is that one could use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds).

2.2 Some basic definitions

What twistor structure in Minkowskian signature does really mean geometrically has remained a confusing question for me. The problems associated with the Minkowskian signature of the metric are encountered also in twistor Grassmann approach to the scattering amplitudes but are circumvented by performing Wick rotation that is using E^4 or S^4 instead of M^4 and applying algebraic continuation. Also complexification of Minkowksi space for momenta is used. These tricks do not apply now.

To make this more concrete, let us sum up the basic definitions.

1. Bi-spinors in representations (1/2,0) and (0,1/2) of Lorentz group are the building bricks of twistors. Bi-spinors v^a and their conjugates $v^{a'}$ have the following inner products:

$$\begin{array}{l} \langle vw\rangle = \epsilon_{ab}v^a w^b \ \ , \qquad [vw] = \epsilon_{a'b'}v^{a'}w^{b'} \ \ , \\ \epsilon_{ab} = (0,1;-1,0) \ \ , \ \ \ \epsilon_{a'b'} = (0,1;-1,0) \ \ . \end{array} \tag{2.1}$$

   Unprimed spinor and its primed variant of the spinor are related by complex conjugation. Index raising is by the inverse ϵ^{ab} of ϵ_{ab}.

2. Twistors are identified as pairs of 2-spinor and its conjugate

$$Z^\alpha = (\lambda_a, \mu^{a'}) \ \ , \ \ \ \overline{Z}_\alpha = (\overline{\mu}^a, \lambda_{a'}) \tag{2.2}$$

   The norm for Z^α is defined as

$$Z^\alpha \overline{Z}^\alpha = \langle \lambda\overline{\mu}\rangle + \left[\overline{\lambda}\mu\right] \ \ . \tag{2.3}$$

   One can write the metric explicitly as direct sum of terms of form $dudv$ (metric of M^2) and each of the can be taken to diagonal form (1,-1). Hence the metric can be written as $diag(1,1,1,1,-1,-1,-1,-1)$.

3. This norm allows to decompose PT to 3 parts PT_+,PT_- and PN in a projectively invariant manner depending on whether the sign of the norm is negative, positive, or whether it vanishes. PT_+ and PT_- serve as loci for the twistor lifts of positive and negative energy modes of massless fields. PN corresponds to the 5-D boundary of the lightcone of $M(2,4)$. By projective identification along light-like radial coordinate it reduces to what is known as conformal compactification of M^4, whose metric is defined only apart from a conformal factor. The natural metric of $PT = P_3$ does not seem to play any role in the construction of the amplitudes relying on projective invariants. The signature of M^4 metric however makes itself visible in the structure of PT: for the Euclidian variant of twistor space one would not have this decomposition to three parts.

Another definition of twistor space - to be used in the geometrization of twistor approach to be proposed - is as a trivial S^2 bundle $M^4 \times CP_1$ over M^4. Since the twistor spheres associated with the points of M^4 with light-like separation intersect, these two definitions cannot be equivalent. In fact, the proper definition of twistor space relies on double fibration involving both views about twistor space discussed in [**?**] (see `http://tinyurl.com/yb4bt74l`).

1. The twistor bundle denoted as PS is the product $M^4 \times CP_1$ with CP_1 realized as projective space and having coordinates $(x^{aa'}, \lambda_a)$, $\{x^{aa'}\} \leftrightarrow x^\mu \sigma_\mu$, where the spinor λ_a is projective 2-spinor in $(1/2, 0)$ representation.

2. The twistors defined in this manner have a trivial projection q to M^4 and non-trivial projection p to P_3 with local projective coordinates $(\lambda_a, \mu^{a'})$. The projection p is defined by the projectively invariant incidence relation

$$\mu^{a'} = ix^{aa'}\lambda_a$$

If $y^{aa'}$ and $a^{aa'}$ differ by light-like vector there exists spinor λ annihilated by the difference vector and there exists twistor $(\lambda_a, \mu^{a\prime})$ to which both (x, λ) and (y, λ) are mapped by the incidence relation. Thus the images of twistor spheres associated for points with light-like separation intersect so that one does not have a proper CP_1 bundle structure.

3. The trivial twistor bundle $T(M^4) = M^4 \times CP_1$ would define the twistor space of M^4 in geometric sense. For this space the metric matters and the radius of CP_1 turns out to allow identification in terms of Planck length. Gravitational interaction would bring in Planck length as a basic scale in this manner. PT in turn would define the twistor space in which the twistor lifts of imbedding space-spinor fields are defined. For this space the metric, which is degenerate and seems to be only projectively defined should not be relevant as the construction of twistorial amplitudes suggests. Note however that the identification as the Minkowskian variant of P_3 allows also the introduction of metric.

This picture has an important immediate implication for the construction of quantum TGD. Positive and negative energy parts of zero energy states are defined at light-like boundaries of $CD \times CP_2$, where CD is the intersection of future and past directed light-cones. The twistor lifts of the amplitudes from $\delta CD \times CP_2$ must be single valued. The strongest condition guaranteeing this is that they do not depend on the radial light-like coordinate at δCD. Super-symplectic symmetry implying the analog of conformal gauge symmetry for the radial light-like coordinate could guarantee this. There is however a hierarchy of conformal gauge symmetry breakings corresponding to the inclusion hierarchy of isomorphic sub-algebras so that this condition is too strong. A weaker condition is that the amplitude $F(m, \lambda)$ in $T(M^4)$ is constant along the light-like ray for the λ associated with the m along this ray. An even stronger condition is that $F(m, \lambda)$ vanishes along the ray. Particle would not propagate along δCD and would avoid remaining at the boundary of CD, a condition which is perfectly sensible physically.

2.3 What does twistor structure in Minkowskian signature really mean?

The following considerations relate to $T(M^4)$ identified as trivial bundle $M^4 \times CP_1$ with natural coordinates $(m^{aa'}, \lambda_a)$, where λ_a is projective spinor. The challenge is to generalize the complex structure of twistor space of E^4 to that for M^4. It turns out that the assumption that twistor space has ordinary complex structure fails. The first guess was that the fiber of twistor space is hyperbolic sphere with metric signature $(1, -1)$ having infinite area so that the 6-D Kähler action would be infinite. This makes no sense. The only alternative, which comes in mind is a hypercomplex generalization of the Kähler structure for M^4 lifted to twistor space, which locally means only adding of S^2 fiber with metric signature $(-1, -1)$.

1. To proceed one must make an explicit the definition of twistor space. The 2-D fiber S^2 consists of antisymmetric tensors of X^4 which can be taken to be self-dual or anti-self-dual by taking any antisymmetric form and by adding to its plus/minus its dual. Each tensor of this kind defines a direction - point of S^2. These points can be also regarded as quaternionic imaginary units. One has a natural metric in S^2 defined by the X^4 inner product for antisymmetric tensors: this inner product depends on space-time metric. Kähler action density is example of a norm defined by this inner product in the special case that the antisymmetric tensor is induced Kähler form. Induced Kähler form defines a preferred imaginary unit and is needed to define the imaginary part $\omega(X, Y) = ig(X, -JY)$ of hermitian form $h = h + i\omega$.

2. To define the analog of Kähler structure for M^4, one must start from a decomposition of $M^4 = M^2 \times E^2$ (M^2 is generated by light-like vector and its dual) and E^2 is orthogonal to it. M^2 allows hypercomplex structure, which light-like coordinates $(u = t - z, v = t + z)$ and E^2 complex structure and the metric has form $ds^2 = dudv + dzd\bar{z}$. Hypercomplex numbers can

be represented as $h = t + iez$, $i^2 = -1, e^2 = -1$ $i^2 = -1, e^2 = -1$. Hyper-complex numbers do not define number field since for light-like hypercomplex numbers $t + iez$, $t = \pm z$ do not have finite inverse. Hypercomplex numbers allow a generalization of analytic functions used routinely in physics. Kähler form representing hypercomplex imaginary unit would be replaced with eJ. One would consider sub-spaces of complexified quaternions spanned by real unit and units eI_k, $k = 1, 2, 3$ as representation of the tangent space of space-time surfaces in Minkowskian regions. This is familiar already from M^8 duality [K11].

$M^4 = M^2 \times E^2$ decomposition can depend on point of M^4 (polarization plane and light-like momentum direction depend on point of M^4. The condition that this structure allows global coordinates analogous to $(u, v, z, \overline{z})$ requires that the distributions for M^2 and E^2 are integrable and thus define 2-D surfaces. I have christened this structure Hamilton-Jacobi structure. It emerges naturally in the construction of extremals of Kähler action that I have christened massless extremals (MEs, [K1]) and also in the proposal for the generalization of complex structure to Minkowskian signature [K8].

One can define the analog of Kähler form by taking sum of induced Kähler form J and its dual $*J$ defined in terms of permutation tensor. The normalization condition is that this form integrates to the negative of metric $(J \pm *J)^2 = -g$. This condition is possible to satisfy.

3. How to lift the Hamilton Jacobi structure of M^4 to Kähler structure of its twistor space? The basic definition of twistors assumes that their exists a field of time-like directions, and that one considers projections of 4-D antisymmetric tensors to the 3-space orthogonal to the time-like direction at given point. One can say that the projection yields magnetic part of the antisymmetric tensor (say induced Kähler form J) with positive norm with respect to natural metric induced to the twistor fiber from the inner product between two-forms. This unique time direction would be defined the light-like vector defining M^2 and its dual. Therefore the signature of the metric of S^2 would be $(-1, -1)$. In quaternionic picture this direction corresponds to real quaternionic unit.

4. To sum up, the metric of the Minkowskian twistor space has signature $(-1, -1, 1, -1, -1, -1)$. The Minkowskian variant of the twistor space would give 2 complex coordinates and one hyper-complex coordinate. Cosmological term would be finite and the sign of the cosmological term in the dimensionally reduced action would be positive as required. Also metric determinant would be imaginary as required. At this moment I cannot invent any killer objection against this option.

It must be made clear that the proposed definition of twistor space of M^4 does not seem to be equivalent with the twistor space assignable to conformally compactified M^4. One has trivial S^2 bundle and Hamilton-Jacobi structure, which is hybrid of complex and hyper-complex structure.

2.4 What does the induction of the twistor structure to space-time surface really mean?

Consider now what the induction of the twistor structure to space-time surface X^4 could mean.

1. The induction procedure for Kähler structure of 12-D twistor space T requires that the induced metric and Kähler form of the base space X^4 of X^6 obtained from T is the same as that obtained by inducing from $H = M^4 \times CP_2$. Since the Kähler structure and metric of T is lift from H this seems obvious. Projection would compensate the lift.

2. This is not yet enough. The Kähler structure and metric of S^2 projected from T must be same as those lifted from X^4. The connection between metric and ω implies that this condition for Kähler form is enough. The antisymmetric Kähler forms in fiber obtained in these two manners co-incide. Since Kähler form has only one component in 2-D case, one obtains single constraint condition giving a commutative diagram stating that the direct projection to S^2 equals with the projection to the base followed by a lift to fiber. The resulting induced Kähler form is not covariantly constant but in fiber S^2 one has $J^2 = -g$.

   As a matter of fact, this condition might be trivially satisfied as a consequence of the bundle structure of twistor space. The Kähler form from $S^2 \times S^2$ can be projected to S^2 associated with X^4 and by bundle projection to a two-form in X^4. The intuitive guess - which might be of course wrong - is that this 2-form must be same as that obtained by projecting the Kähler form of CP_2 to X^4. If so then the bundle structure would be essential but what does it really mean?

3. Intuitively it seems clear that X^6 must decompose locally to a product $X^4 \times S^2$ in some sense. This is true if the metric and Kähler form reduce to direct sums of contributions from the tangent spaces of X^4 and S^2. This guarantees that 6-D Kähler action decomposes to a sum of 4-D Kähler action and Kähler action for S^2.

   This could be however too strong a condition. Dimensional reduction occurs in Kaluza-Klein theories and in this case the metric can have also components between tangent spaces of the fiber and base being interpreted as gauge potentials. This suggests that one should formulate the condition in terms of the matrix $T \leftrightarrow g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}$ defining the norm of the induced Kähler form giving rise to Kähler action. T maps Kähler form $J \leftrightarrow J_{\alpha\beta}$ to a contravariant tensor $J_c \leftrightarrow J^{\alpha\beta}$ and should have the property that $J_c(X^4)$ ($J_c(S^2)$) does not depend on $J(S^2)$ ($J(X^4)$).

   One should take into account also the self-duality of the form defining the imaginary unit. In X^4 the form $S = J \pm *J$ is self-dual/anti-self dual and would define twistorial imaginary unit since its square equals to $-g$ representing the negative of the real unit. This would suggest that 4-D Kähler action is effectively replaced with $(J \pm *J) \wedge (J \pm *J) = J^*J \pm J \wedge J$, where $*J$ is the Hodge dual defined in terms of 4-D permutation tensor ϵ. The second term is topological term (Abelian instanton term) and does not contribute to field equations. This in turn would mean that it is the tensor $T \pm \epsilon$ for which one can demand that $S_c(X^4)$ ($S_c(S^2)$) does not depend on $S(S^2)$ ($S(X^4)$).

4. The preferred quaternionic imaginary unit should be represented as a projection of Kähler form of 12-D twistor space $T(H)$. The preferred imaginary unit defining twistor structure as sum of projections of both $T(CP_2)$ and $T(M^4)$ Kähler forms would guarantee that vacuum extremals like canonically imbedded M^4 for which $T(CP_2)$ Kähler form contributes nothing have well-defined twistor structure. $T(M^4)$ or $T(CP_2)$ are treated completely symmetrically but the maps of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers induce symmetry breaking.

   For Kähler action $M^4 - CP_2$ symmetry does not make sense. 4-D Kähler action to which 6-D Kähler action dimensionally reduces can depend on CP_2 Kähler form only. I have also considered the possibility of covariantly constant self-dual M^4 term in Kähler action but given it up because of problems with Lorentz invariance. One should couple the gauge potential of M^4 Kähler form to induced spinors. This would mean the existence of vacuum gauge fields coupling to sigma matrices of M^4 so that the gauge grop would be non-compact $SO(3,1)$ leading to a breakdown of unitarity.

There is still one difficulty to be solved.

1. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared, brings in a further length scale. The first guess is that $1/L^2$ is closely related to cosmological constant, which is also dynamical and $1/L^2$has indeed correct sign to explain accelerated expansion of the Universe. Unfortunately, if $1/L^2$ is of order cosmological constant, the value of the ordinary Kähler coupling strength α_K would be enormous. As a matter of fact, the order of magnitude for L^2 must be equal to the area of $S^2(X^4)$ and in good approximation equal to $L^2 = 4\pi R^2(S^2(M^4))$ and therefore in the same range as Planck length l_P and CP_2 radius R. This would imply a gigantic value of cosmological constant. Just as in GRT based cosmology!

2. This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$, 6-D Kähler action is sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action. For it the contribution from $S^2(CP_2)$ fiber is absent if the imbedding of $S^2(X^4)$ to $S^2(M^4) \times S^2(CP_2)$ reduces to identification with $S^2(M^4)$ so that $S^2(CP_2)$ is effectively absent: this is not true generally. Second term in the

action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$, which can indeed contribute without breaking of Lorentz symmetry. In fact, one can assume that also the Kähler form of M^4 contributes as will be found.

3. The independency implies that Kähler couplings strengths are independent for them. If one wants that cosmological constant has a reasonable order of magnitude, $L \sim R(S^2(M^4))$ must hold true and the analog $\alpha_K(S^2(M^4))$ of the ordinary Kähler coupling strength (analogous to critical temperature) must be extremely large - so large that one has

$$\alpha_K(M^4) \times 4\pi R(M^4)^2 \sim L^2 \ ,$$

   where L is the size scale of the recent Universe.

   This makes possible the small value of cosmological constant assignable to the volume term given by this part of dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ would be essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. One can criticize this identification of 6-D Kähler action as artificial but it seems to be the only option that works. Interestingly also the contribution from M^4 Kähler form can be allowed since it is also extremely small. For canonically imbedded M^4 this contribution vanishes by self-duality of M^4 Kähler form and is extremely small for the vacuum extremals of Kähler action.

4. For general winding numbers of the map $S^2(X^4) \rightarrow S^2(M^4) \times S^2(CP_2)$ also $S^2(CP_2)$ Kähler form contributes and cosmological constant is gigantic. It would seem that only the winding numbers $(w_1, w_2) = (n, 0)$ are consistent with the observed value of cosmological constant. Hence it seems that there is no need to pose any additional conditions to the Kähler action if one uses the fact that $T(M^4)$ and $T(CP_2)$ parts are independent!

It is good to list the possible open issues related to the precise definition of the twistor structure and of M^4 Kähler action.

1. The proposed definition of M^4 twistor space a Cartesian product of M^4 and $S^2(M^4)$ parts involving Hamilton-Jacobi structure does not seem to be equivalent with the twistor identification as $SU(2,2)/SU(2,1) \times U(1)$ having conformally compactified M^4 as base space. There exists an entire moduli space of Hamilton-Jacobi structures. If the M^4 part of Kähler form participates in dynamics, one must include the specification of the Hamilton-Jacobi structure to the definition of CD and integrate over Hamilton Jacobi-structures as part of integral over WCW in order to gain Lorentz invariance. Note that Hamilton-Jacobi structure enters to dynamics also through the construction of massless extremals [K1].

2. The presence of M^4 part of Kähler form in action implies breaking of Lorentz invariance for extremals of lifted Kähler action. The same happens at the level of induced spinors if this Kähler form couples to imbedding space spinors. If $T(M^4)$ is trivial bundle, one can include only the $T(S^2(M^4))$ part of Kähler form to Kähler action and couple only this to the spinors of $T(H)$. The integration over Hamilton-Jacobi structures becomes un-necessary.

3. If one includes M^4 part of Kähler form to 6-D Kähler action, one has several options. One can have sum of the Kähler actions for $T(M^4)$ and $T(CP_2)$ or Kähler action defined by the sum $J(T(M^4)/g_K$ and $J(T(CP_2)/\alpha_K$ with $\alpha_K(M^4) = g_K^2(M^4)/4\pi\hbar$ and $\alpha_K = g_K^2/4\pi\hbar$ with a proper normalization to guarantee that the squares of induced Kähler forms give sum of Kähler actions as in the first option. In this case one obtains interference term proportional to $Tr(J(M^4)J(CP_2)$. For the proposed value of α_K also the interference term is extremely small as compared to Kähler action in recent cosmology.

2.5 Could M^4 Kähler form introduce new gravitational physics?

The introduction of M^4 Kähler form could bring in new gravitational physics.

1. As found, the twistorial formulation of TGD assigns to M^4 a self dual Kähler form whose square gives Minkowski metric. It can (but need not if M^4 twistor space is trivial as bundle) contribute to the 6-D twistor counterpart of Kähler action inducing M^4 term to 4-D Kähler action vanishing for canonically imbedded M^4.

2. Self-dual Kähler form in empty Minkowski space satisfies automatically Maxwell equations and has by Minkowskian signature and self-duality a vanishing action density. Energy momentum tensor is proportional to the metric so that Einstein Maxwell equations are satisfied for a non-vanishing cosmological constant! M^4 indeed allows a large number of self dual Kähler fields (I have christened them as Hamilton-Jacobi structures). These are probably the simplest solutions of Einstein-Maxwell equations that one can imagine!

3. There however exist quite a many Hamilton-Jacobi structures. However, if this structure is to be assigned with a causal diamond (CD) it must satisfy additional conditions, say $SO(3)$ symmetry and invariance under time translations assignable to CD. Alternatively, covariant constancy and $SO(2) \subset SO(3)$ symmetry might be required.

This raises several questions. Could M^4 Kähler form replace CP_2 Kähler form in the picture for how gravitational interaction is mediated at quantal level? Could one speak of flux tubes of the magnetic part of this Kähler form? Or should one consider the Kähler field as a sum of the two Kähler forms weighted by the inverses $1/g_K$ of corresponding Kähler couplings. If so then M^4 contribution would be negligible except for canonically imbedded M^4 in the recent cosmology. Note that α_K and $\alpha_K(M^4)$ have interpretation as analogs of quantum critical temperatures but can depend on the p-adic lengths scale defining the cosmology.

1. The natural expectation is that Kähler form characterizes CD having preferred time direction suggested strongly by number theoretical considerations involving quaternionic structure with preferred direction of time axis assignable to real unit quaternion.

   Self-duality gives rise to Kähler magnetic and electric fields in the same spatial direction identifiable as a local quantization axis for spin assignable to CD assignable to observer. CD indeed serves as a correlate for conscious entity in TGD inspired theory of consciousness. Flux tube would connect mass M to mass m assignable to observer and flux tube direction would define spin quantization axes for the CD of the observer. Spin quantization axis would be naturally in the direction of magnetic field, which is direction of the flux tube.

2. The self-dual Kähler form could be spherically symmetric for CDs and represent self dual magnetic monopole field (dyon) with monopole charge at the line connecting the tips of CD and have non-vanishing components $J^{tr} = \epsilon^{tr\theta\phi} J_{\theta\phi}$, $J_{\theta\phi} = sin(\theta)$. One would have genuine monopole, which is somewhat questionable feature. Only the entire radial flux would be quantized. CD could be associated with the mass M of the central object. The gauge potential associated with J could be chosen to be $A_\mu \leftrightarrow (1/r, 0, 0, cos(\theta)$. I have considered this kind of possibility earlier in context of TGD inspired model of anyons but gave up the idea.

   The moduli space for CDs with second tip fixed would be hyperbolic space $H^3 = SO(3,1)/SO(3)$ or a space obtained by identifying points at the orbits of some discrete subgroup of $SO(3,1)$ as suggested by number theoretic considerations. This induced Kähler field could make the blackholes with center at this line to behave like M^4 magnetic monopoles if the M^4 part of Kähler form is induced into the 6-D lift of Kähler action with extremely small coefficients of order of magnitude of cosmological constant. Cosmological constant and the possibility of CD monopoles would thus relate to each other.

3. The self-dual M^4 Kähler form could be also covariantly constant ($J_{tz} = J_{xy} = 1$) and represent electric and magnetic fluxes in a fixed direction identifiable as a quantization axes for spin and characterizing CD. In this case the CD would be associated with the mass m of observer. The moduli space of CDs would be now $SO(3,1)/SO(1,1) \times SO(2)$ which is completely analogous to the twistor space $SU(3)/U(1) \times U(1)$.

4. Boundary conditions (allowing no boundaries!) demand that the flux tubes have closed cross section - say sphere S^2 - rather than disk: stability is guaranteed if the S^2 cross section is

mapped to homologically non-trivial surface of CP_2 or is projection of it. This would give monopole flux also for CP_2 Kähler form so that the original hypothesis would be correct.

5. Radial flux tubes are possible both spherically symmetric and covariantly constant Kähler form possibly mediating gravitational interaction but the flux is not quantized unless preferred extremal property implies this: in any case M^4 flux would be very small unless one has large value of gravitational Planck constant implying n-sheeted covering of M^4 and flux is scale up by n since every sheet gives a contribution. For spherically symmetric M^4 Kähler form the flux tubes would have naturally conical structure spanning a constant solid angle. For covariantly constant Kähler form the flux tubes would be cylindrical.

There are further interpretational problems.

1. The classical coupling of M^4 Kähler gauge potential to induced spinors is not small. Can one really tolerate this kind of coupling equivalent to a coupling to a self dual monopole field carrying electric and magnetic charges? One could of course consider the condition that the string world sheets carrying spinor modes are such that the induced M^4 Kähler form vanishes and gauge potential become pure gauge. M^4 projection would be 2-D Lagrange manifold whereas CP_2 projection would carry vanishing induce $W$and possibly also Z^0 field in order that em charge is well defined for the modes. These conditions would fix the string world sheets to a very high degree in terms of maps between this kind of 2-D sub-manifolds of M^4 and CP_2. Spinor dynamics would be determined by the avoidance of interaction!

   Recall that one could interpret the localization of spinor modes to 2-surfaces in the sense of strong form of holography: one can continued induced spinor fields to the space-time interior as indeed assumed but the continuation is completely determined by the data at 2-D string world sheets.

   It must be emphasized that the imbedding space spinor modes characterizing the ground states of super-symplectic representations would not couple to the monopole field so that at this level Poincare invariance is not broken. The coupling would be only at the space-time level and force spinor modes to Lagrangian sub-manifolds.

2. At the static limit of GRT and for $g_{ij} \simeq \delta_{ij}$ implying $SO(3)$ symmetry there is very close analogy with Maxwell's equations and one can speak of gravi-electricity and gravi-magnetism with 4-D vector potential given by the components of $g_{t\alpha}$. The genuine $U(1)$ gauge potential does not however relate to the gravimagnetism in GRT sense. Situation would be analogous to that for CP_2, where one must add to the spinor connection $U(1)$ term to obtain respectable spinor structure. Now the $U(1)$ term would be added to trivial spinor connection of flat M^4: its presence would be justified by twistor space Kähler structure. If the induced M^4 Kähler form is present as a classical physical field it means genuinely new contribution to U(1) electroweak of standard model. If string world sheets carry vanishing M^4 Kähler form, this contribution vanishes classically.

2.6 A connection with the hierarchy of Planck constants?

A connection with the hierarchy of Planck constants is highly suggestive. Since also a connection with the p-adic length scale hierarchy suggests itself for the hierarchy of p-adic length scales it seems that both length scale hierarchies might find first principle explanation in terms of twistor lift of Kähler action.

1. Cosmological considerations encourage to think that $R_1 \simeq l_P$ and $R_2 \simeq R$ hold true. One would have in early cosmology $(w_1, w_2) = (1, 0)$ and later $(w_1, w_2) = (0, 1)$ guaranteeing $R_D$grows from l_P to R during cosmological evolution. These situations would correspond the solutions $(w_1 = n, 0)$ and $(0, w_2 = n)$ one has $A = n4\pi R_1^2$ and $A = n \times 4\pi R_2^2$ and both Kähler coupling strengths are scaled down to α_K/n. For $\hbar_{eff}/h = n$ exactly the same thing happens!

   There are further intriguing similarities. $h_{eff}/h = n$ is assumed to correspond *multi-sheeted* (to be distinguished from *many-sheeted*!) covering space structure for space-time surface.

Now one has covering space defined by the lift $S^2(X^4) \to S^2(M4) \times S^2(CP_2)$. These lifts define also lifts of space-time surfaces.

Could the hierarchy of Planck constants correspond to the twistorial surfaces for which $S^2(M^4)$ is n-fold covering of $S^2(X^4)$? The assumption has been that the n-fold multi-sheeted coverings of space-time surface for $h_{eff}/h = n$ are singular at the ends of space-time surfaces at upper and lower boundaries if causal diamond (CD). Could one consider a more precise definition of twistor space in such a manner that CD replaces M^4 and the covering becomes singular at the light-like boundaries of CD - the branches of space-time surface would collapse to single one.

Does this collapse have a clear geometric meaning? Are the projections of various branches of the S^2 lift automatically identical so that one would have the original picture in which one has n identical copies of the same space-time surface? Or can one require identical projections only at the light-like boundaries of CD?

2. $w_1 = w_2 = w$ is essentially the first proposal for conditions associated with the lifting of twistor space structure. $w_1 = w_2 = n$ gives $ds^2 = (R_1^2 + R_2^2)(d\theta^2 + w^2 d\phi^2)$ and $A = n \times 4\pi(R_1^2 + R_2^2)$. Also now Kähler coupling strength is scaled down to α/n. Again a connection with the hierarchy of Planck constants suggests itself.

3. One can consider also the option $R_1 = R_2$ option giving $ds^2 = R_1^2(2d\theta^2 + (w_1^2 + w_2^2)d\phi^2$. If the integers w_i define Pythagorean square one has $w_1^2 + w_2^2 = n^2$ and one has $R_1 = R_2$ option that one has $A = n \times 4\pi R^2$. Also now the connection with the hierarchy of Planck constants might make sense.

2.7 Twistorial variant for the imbedding space spinor structure

The induction of the spinor structure of imbedding space is in key role in quantum TGD. The question arises whether one should lift also spinor structure to the level of twistor space. If so one must understand how spinors for $T(M^4)$ and $T(CP_2)$ are defined and how the induced spinor structure is induced.

1. In the case of CP_2 the definition of spinor structure is rather delicate and one must add to the ordinary spinor connection U(1) part, which corresponds physically to the addition of classical U(1) gauge potential and indeed produces correct electroweak couplings to quarks and leptons. It is assumed that the situation does not change in any essential manner: that is the projections of gauge potentials of spinor connection to the space-time surface give those induced from $M^4 \times CP_2$ spinor connection plus possible other parts coming as a projection from the fiber $S^2(M^2) \times S^2(CP_2)$. As a matter of fact, these other parts should vanish if dimensional reduction is what it is meant to be.

2. The key question is whether the complications due to the fact that the geometries of twistor spaces $T(M^4)$ and $T(CP_2)$ are not quite Cartesian products (in the sense that metric could be reduced to a direct sum of metrics for the base and fiber) can be neglected so that one can treat the sphere bundles approximately as Cartesian products $M^4 \times S^2$ and $CP_2 \times S^2$. This will be assumed in the following but should be carefully proven.

3. Locally the spinors of the twistorspace $T(H)$ are tensor products of imbedding spinors and those for of $S^2(M^4) \times S^2(CP_2)$ expressible also as tensor products of spinors for $S^2(M^4)$ and $S^2(CP_2)$. Obviously, the number of spinor components increases by factor $2 \times 2 = 4$ unless one poses some additional conditions taking care that one has dimensional reduction without the emergence of any new spin like degrees of freedom for which there is no physical evidence. The only possible manner to achieve this is to pose covariant constancy conditions already at the level of twistor spaces $T(M^4)$ and $T(CP_2)$ leaving only single spin state in these degrees of freedom.

4. In CP_2 covariant constancy is possible for right-handed neutrino so that CP_2 spinor structure can be taken as a model. In the case of CP_2 spinors covariant constancy is possible for right-handed neutrino and is essentially due to the presence of U(1) part in spinor connection

forced by the fact that the spinor structure does not exist otherwise. Ordinary S^2spinor connection defined by vielbein exists always. One can however add a coupling to a suitable multiple of Kähler potential satisfying the quantization of magnetic charge (the magnetic flux defined by $U(1)$ connection is multiple of 2π so that its imaginary exponential is unity).

S^2 spinor connections must must have besides ordinary vielbein part determined by S^2 metric also $U(1)$ part defined by Kähler form coupled with correct coupling so that the curvature form annihilates the second spin state for both $S^2(M^4)$ and $S^2(CP_2)$. $U(1)$ part of the spinor curvature is proportional to Kähler form $J \propto sin(theta) d\theta d\phi$ so that this is possible. The vielbein and $U(1)$ parts of the spinor curvature ear proportional Pauli spin matrix $\sigma_z = (1,0;0,-1)/2$ and unit matrix $(1,0;0,1)$ respectively so that the covariant constancy is possible to satisfy and fixes the spin state uniquely.

5. The covariant derivative for the induced spinors is defined by the sum of projections of spinor gauge potentials for $T(M^4)$ and $T(CP_2)$. With above assumptions the contributions gauge potentials from $T(M^4)$ and $T(CP_2)$ separately annihilate single spinor component. As a consequence there are no constraints on the winding numbers w_i, $i = 1,2$ of the maps $S^2(X^4) \to S^2(M^4)$ and $S^2(X^4) \to S^2(CP_2)$. Winding number w_i corresponds to the imbedding map $(\Theta_i = \theta, \Phi_i = w_i\phi)$.

6. If the square of the Kähler form in fiber degrees of freedom gives metric to that its square is metric, one obtains just the area of S^2 from the fiber part of action. This is given by the area $A = 4\pi\sqrt{2(w_1^2R_1^2 + w_2^2R_2^2)}$ since the induced metric is given by $ds^2 = (R_1^2 + R_2^2)d\theta^2 + (w_1^2R_1^2 + w_2^2R_2^2)d\phi^2$ for $(\Theta_1 = \theta, \Phi = n_1\phi, \Phi_2 = n_2\phi)$.

2.8 Twistor googly problem transforms from a curse to blessing in TGD framework

There was a nice story with title "Michael Atiyahs Imaginative State of Mind" about mathematician Michael Atyiah in Quanta Magazine (see `http://tinyurl.com/jta2va8`). The works of Atyiah have affected profoundly the development of theoretical physics. What was pleasant to hear that Atyiah belongs to those scientists who do not care what others think. As he tells, he can afford this since he has got all possible prices. This is consoling and encouraging even for those who have not cared what others think and for this reason have not earned any prizes. Nor even a single coin from what they have been busily doing their whole lifetime!

In the beginning of the story "twistor googly problem" was mentioned. I had to refresh my understanding about googly problem. In twistorial description the modes of massless fields (rather than entire massless fields) in space-time are lifted to the modes in its 6-D twistor-space and dynamics reduces to holomorphy. The analog of this takes place also in string models by conformal invariance and in TGD by its extension.

One however encounters what is known as googly problem: one can have twistorial description for circular polarizations with well-defined helicity +1/-1 but not for general polarization states - say linear polarizations, which are superposition of circular polarizations. This reflects itself in the construction of twistorial amplitudes in twistor Grassmann program for gauge fields but rather implicitly: the amplitudes are constructed only for fixed helicity states of scattered particles. For gravitons the situation gets really bad because of non-linearity.

Mathematically the most elegant solution would be to have only +1 or -1 helicity but not their superpositions implying very strong parity breaking and chirality selection. Parity parity breaking occurs in physics but is very small and linear polarizations are certainly possible! The discusion of Penrose with Atyiah has inspired a possible solution to the problem known as "palatial twistor theory" (see `http://tinyurl.com/hr7hmh2`). Unfortunately, the article is behind paywall too high for me so that I cannot say anything about it.

What happens to the googly problem in TGD framework? There is twistorialization at space-time level and imbedding space level.

1. One replaces space-time with 4-surface in $H = M^4 \times CP_2$ and lifts this 4-surface to its 6-D twistor space represented as a 6-surface in 12-D twistor space $T(H) = T(M^4) \times T(CP_2)$. The twistor space has Kähler structure only for M^4 and CP_2 so that TGD is unique. This

Kähler structure is needed to lift the dynamics of Kähler action to twistor context and the lift leads to the a dramatic increase in the understanding of TGD: in particular, Planck length and cosmological constant with correct sign emerge automatically as dimensional constants besides CP_2 size.

2. Twistorialization at imbedding space level means that spinor modes in H representing ground states of super-symplectic representations are lifted to spinor modes in T(H). M^4 chirality is in TGD framework replaced with H-chirality, and the two chiralities correspond to quarks and leptons. But one cannot superpose quarks and leptons! "Googly problem" is just what the superselection rule preventing superposition of quarks and leptons requires in TGD!

One can look this in more detail.

1. Chiral invariance makes possible for the modes of massless fields to have definite chirality: these modes correspond to holomorphic or antiholomorphic amplitudes in twistor space and holomorphy (antiholomorphy is holomorphy with respect to conjugates of complex coordinates) does not allow their superposition so that massless bosons should have well-defined helicities in conflict with experimental facts. Second basic problem of conformally invariant field theories and of twistor approach relates to the fact that physical particles are massive in 4-D sense. Masslessness in 4-D sense also implies infrared divergences for the scattering amplitudes. Physically natural cutoff is required but would break conformal symmetry.

2. The solution of problems is masslessness in 8-D sense allowing particles to be massive in 4-D sense. Fermions have a well-defined 8-D chirality - they are either quarks or leptons depending on the sign of chirality. 8-D spinors are constructible as superpositions of tensor products of M^4 spinors and of CP_2 spinors with both having well-defined chirality so that tensor product has chiralities (ϵ_1, ϵ_2), $\epsilon_i = \pm 1$, $i = 1, 2$. H-chirality equals to $\epsilon = \epsilon_1\epsilon_2$. For quarks one has $\epsilon = 1$ (a convention) and for leptons $\epsilon = -1$. For quark states massless in M^4 sense one has either $(\epsilon_1, \epsilon_2) = (1, 1)$ or $(\epsilon_1, \epsilon_2) = (-1, -1)$ and for massive states superposition of these. For leptons one has either $(\epsilon_1, \epsilon_2) = (1, -1)$ or $(\epsilon_1, \epsilon_2) = (-1, 1)$ in massless case and superposition of these in massive case.

3. The twistor lift to $T(M^4) \times T(CP_2)$ of the ground states of super-symplectic representations represented in terms of tensor products formed from H-spinor modes involves only quark and lepton type spinor modes with well-defined H-chirality. Superpositions of amplitudes in which different M^4 helicities appear but M^4 chirality is always paired with completely correlating CP_2 chirality to give either $\epsilon = 1$ or $\epsilon = -1$. One has never a superposition of of different chiralities in either M^4 or CP_2 tensor factor. I see no reason forbidding this kind of mixing of holomorphicities and this is enough to avoid googly problem. Linear polarizations and massive states represent states with entanglement between M^4 and CP_2 degrees of freedom. For massless and circularly polarized states the entanglement is absent.

4. This has interesting implications for the massivation. Higgs field cannot be scalar in 8-D sense since this would make particles massive in 8-D sense and separate conservation of B and L would be lost. Theory would also contain a dimensional coupling. TGD counterpart of Higgs boson is actually CP_2 vector, and one can say that gauge bosons and Higgs combine to form 8-D vector. This correctly predicts the quantum numbers of Higgs. Ordinary massivation by constant vacuum expectation value of vector Higgs is not an attractive idea since no covariantly constant CP_2 vector field exists so that Higgsy massivation is not promising except at QFT limit of TGD formulated in M^4. p-Adic thermodynamics gives rise to 4-D massivation but keeps particles massless in 8-D sense. It also leads to powerful and correct predictions in terms of p-adic length scale hypothesis.

Anonymous reader gave me a link to the paper of Penrose and this inspired further more detailed considerations of googly problem.

1. After the first reading I must say that I could not understand how the proposed elimination of conjugate twistor by quantization of twistors solves the googly problem, which means that both helicities are present (twistor Z and its conjugate) in linearly polarized classical modes so that holomorphy is broken classically.

2. I am also very skeptic about quantizing of either space-time coordinates or twistor space coordinates. To me quantization is natural only for linear objects like spinors. For bosonic objects one must go to higher abstraction level and replace superpositions in space-time with superpositions in field space. Construction of "World of Classical Worlds" (WCW) in TGD means just this.

3. One could however think that circular polarizations are fundamental and quantal linear combination of the states carrying circularly polarized modes give rise to linear and elliptic polarizations. Linear combination would be possible only at the level of field space (WCW in TGD), not for classical fields in space-time. If so, then the elimination of conjugate of Z by quantization suggested by Penrose would work.

4. Unfortunately, Maxwell's equations allow classically linear polarisations! In order to achieve classical-quantum consistency, one should modify classical Maxwell's equations somehow so that linear polarizations are not possible. Googly problem is still there!

What about TGD?

1. Massless extremals representing massless modes are very "quantal": they cannot be superposed classically unless both momentum and polarisation directions for them (they can depend space-time point) are exactly parallel. Optimist would guess that the classical local classical polarisations are circular. No, they are linear! Superposition of classical linear polarizations at the level of WCW can give rise to local linear but not local circular polarization! Something more is needed.

2. The only sensible conclusion is that only gauge boson quanta (not classical modes) represented as pairs of fundamental fermion and antifermion in TGD framework can have circular polarization! And indeed, massless bosons - in fact, all elementary particles- are constructed from fundamental fermions and they allow only two M^4, CP_2 and $M^4 \times CP_2$ helicities/-chiralities analogous to circular polarisations. B and L conservation would transform googly problem to a superselection rule as already described.

To sum up, both the extreme non-linearity of Kähler action, the representability of all elementary particles in terms of fundamental fermions and antifermions, and the generalization of conserved M^4 chirality to conservation of H-chirality would be essential for solving the googly problem in TGD framework.

3 Surprise: Twistorial Dynamics Does Not Reduce to a Trivial Reformulation of the Dynamics of Kähler Action

I have thought that twistorialization classically means only an alternative formulation of TGD. This is definitely not the case as the explicit study demonstrated. Twistor formulation of TGD is in terms of of 6-D twistor spaces $T(X^4)$ of space-time surfaces $X^4 \subset M^4 \times CP_2$ in 12-dimensional product $T = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of $T(M^4)$ of M^4 and $T(CP_2)$ of CP_2. The induced Kähler form in X^4 defines the quaternionic imaginary unit defining twistor structure: how stupid that I realized it only now! I experienced during single night many other "How stupid I have been" experiences.

Classical dynamics is determined by 6-D variant of Kähler action with coefficient $1/L^2$ having dimensions of inverse length squared. Since twistor space is bundle, a dimensional reduction of 6-D Kähler action to 4-D Kähler action plus a term analogous to cosmological term - space-time volume - takes place so that dynamics reduces to 4-D dynamics also now. Here one must be careful: this happens provided the radius of $S2$ associated with X^4 does not depend on point of X^4. The emergence of cosmological term was however completely unexpected: again "How stupid I have been" experience. The scales of the spheres and the condition that the 6-D action is dimensionless bring in 3 fundamental length scales!

3.1 New scales emerge

The twistorial dynamics gives to several new scales with rather obvious interpretation. The new fundamental constants that emerge are the radii of the spheres associated with $T(M^4)$ and $T(CP_2)$. The radius of the sphere associated with X^4 is not a fundamental constant but determined by the induced metric. By above argument the fiber is sphere for both Euclidian signature and Minkowskian signatures.

1. For CP_2 twistor space the radius of $S^2(CP_2)$ must be apart from numerical constant equal to CP_2 radius R. For $S^2(M^4)$ one an consider two options. The first option is that also now the radius for $S^2(M^4)$ equals to $R(M^4) = R$ so that Planck length would not emerge from fundamental theory classically as assumed hitherto. Second imaginable option is that it does and one has $R(M^4) = l_P$.

2. If the signature of $S^2(M^4)$ is $(-1, -1)$ both Minkowskian and Euclidian regions have $S^2(X^4)$ with the same signature $(-1, -1)$. The radius R_D of $S^2(X^4)$ is dynamically determined.

Recall first how the cosmological constant emerges from TGD framework. The key point is that the 6-D Kähler action contains two terms.

1. The first term is essentially the ordinary Kähler action multiplied by the area of $S^2(X^4)$ which is compensated by the length scale, which can be taken to be the area $4\pi R^2(M^4)$ of $S^2(M^4)$. This makes sense for winding numbers $(w_1, w_2) = (1, 0)$ meaning that $S^2(CP_2)$ is effectively absent but $S^2(M^4)$ is present.

2. Second term is the analog of Kähler action assignable assignable to the projection of $S^2(M^4)$ Kähler form. The corresponding Kähler coupling strength $\alpha_K(M^4)$ is huge - so huge that one has $\alpha_K(M^4)4\pi R^2(M^4) \equiv L^2$, where $1/L^2$ is of the order of cosmological constant and thus of the order of the size of the recent Universe. $\alpha_K(M^4)$ is also analogous to critical temperature and the earlier hypothesis that the values of L correspond to p-adic length scales implies that the values of come as $\alpha_K(M^4) \propto p \simeq 2^k$, p prime, k prime.

   The assignment of different value of α_K to M^4 and CP_2 degrees of freedom can be criticized as ad hoc assumption. In [K13] a scenario in which the value of α_K is universal. This option has very nice properties and one can overcome the problem associated with cosmological constant by assuming that it the *entire* 4-D action corresponds to the effective cosmological constant. The cancellation between Kähler action and volume term would give rise to very small cosmological constant and also its p-adic evolution could be understood.

3. One can get an estimate for the relative magnitude of the Kähler action $S(CP_2) = \pi/8\alpha_K$ assignable to CP_2 type vacuum extremal and the corresponding cosmological term. The magnitude of the volume term is of order $1/4\pi\alpha_K(M^4)$ with $\alpha_K(M^4)$ given by $\alpha_K(M^4) = L^2/4\pi R^2(M^4)$. The sequel the magnitude of L is estimated to be $L = (2^{3/2}\pi l_P/R_D) \times R_U$, where R_U is the recent size of the Universe. This estimate follows from the identification of the volume term as cosmological constant term.

   For $R_D = R_M = l_P$ this gives $\alpha_K(M^4) = 2\pi(R_U/l_P)^2 \sim 2 \times 10^{18}$. For $\alpha_K \simeq 1/137$ the ratio of the two terms is of order 10^{-20}. The cosmological terms is completely negligible in elementary particle scales. For vacuum extremals the situation changes and the overall effect is presumably the transformation of 4-D spin glass degeneracy so that the potentials wells in the analog spin glass energy landscape do not correspond to vacuum extremal anymore and perturbation theory around them is in principle possible. The huge value of $\alpha_K(M^4)$ implies that the system corresponds mathematically to an extremely strongly interacting system so that perturbation theory fails to converge. The geometry of "world of classical worlds" (WCW) provides the needed non-perturbative approach and leads to to strong form of holography.

4. One could argue that the Kähler form assignable to M^4 cannot contribute to the action since it does not contribute to spinor connection of M^4 - an assumption that can be challenged. For canonically imbedded M^4 self-duality implies that this contribution to action vanishes. For vacuum extremals of ordinary Kähler action the contribution to the action density is

proportional to the CP_2 part of induced metric and to $1/\alpha_K(M^4)$, and therefore extremely small.

The breaking of Lorentz invariance can be seen as a possible problem for the induced spinor fields coupling to the self-dual Kähler potential. This corresponds to coupling to constant magnetic field and constant electric field, which are duals of each other. This would give rise to the analogs of cyclotron energy states in transversal directions and to the analogs of states in constant electric field in longitudinal directions. Could this extremely small effect serve as a seed for the generation of Kähler magnetic flux tubes carrying longitudinal electric fields in various scales? Note also that the value of $\alpha_K(M^4)$ is predicted to decrease as p-adic length scale so that the effect would be larger in early cosmology and in short length scales.

Hence one can consider the possibility that the action is just the sum of full 6-D Kähler actions assignable to $T(M^4)$ and $T(CP_2)$ but with different values of α_K if one has $(w_1, w_2) = (n, 0)$. Also other $w_2 \neq 0$ is possible but corresponds to gigantic cosmological constant.

Given the parameter L^2 as it is defined above, one can deduce an expression for cosmological constant Λ and show that it is positive.

1. 6-D Kähler action has dimensions of length squared and one must scale it by a dimensional constant: call it $1/L^2$. L is a fundamental scale and in dimensional reduction it gives rise to cosmological constant. Cosmological constant Λ is defined in terms of vacuum energy density as $\Lambda = 8\pi G\rho_{vac}$ can have two interpretations. Λ can correspond to a modification of Einstein-Hilbert action or - as now - to an additional term in the action for matter. In the latter case positive Λ means negative pressure explaining the observed accelerating expansion. It is actually easy to deduce the sign of Λ.

   $1/L^2$ multiplies both Kähler action - $F^{ij}F_{ij}$ ($\propto E^2 - B^2$ in Minkowskian signature). The energy density is positive. For Kähler action the sign of the multiplier must be positive so that $1/L^2$ is positive. The volume term is fiber space part of action having same form as Kähler action. It gives a positive contribution to the energy density and negative contribution to the pressure.

   In $\Lambda = 8\pi G\rho_{vac}$ one would have $\rho_{vac} = \pi/L^2R_D^2$ as integral of the $-F^{ij}F_{ij}$ over S^2 given the π/R_D^2 (no guarantee about correctness of numerical constants). This gives $\Lambda = 8\pi^2 G/L^2R_D^2$. Λ is positive and the sign is same as as required by accelerated cosmic expansion. Note that super string models predict wrong sign for Λ. Λ is also dynamical since it depends on R_D, which is dynamical. One has $1/L^2 = k\Lambda$, $k = 8\pi^2 G/R_D^2$ apart from numerical factors.

   The value of L of deduced from Euclidian and Minkowskian regions in this formal manner need not be same. Since the GRT limit of TGD describes space-time sheets with Minkowskian signature, the formula seems to be applicable only in Minkowskian regions. Again one can argue that one cannot exclude Euclidian space-time sheets of even macroscopic size and blackholes and even ordinary concept matter would represent this kind of structures.

2. L is not size scale of any fundamental geometric object. This suggests that L is analogous to α_K and has value spectrum dictated by p-adic length scale hypothesis. In fact, one can introduce the ratio of $\epsilon = R^2/L^2$ as a dimensionless parameter analogous to coupling strength what it indeed is in field equations. If so, L could have different values in Minkowskian and Euclidian regions.

3. I have earlier proposed that $R_U \equiv 1/\sqrt{1/\Lambda}$ is essentially the p-adic length scale $L_p \propto \sqrt{p} = 2^{k/2}$, $p \simeq 2^k$, $k$prime, characterizing the cosmology at given time and satisfies $R_U \propto a$ meaning that vacuum energy density is piecewise constant but on the average decreases as $1/a^2$, a cosmic time defined by light-cone proper time. A more natural hypothesis is that L satisfies this condition and in turn implies similar behavior or R_U. p-Adic length scales would be the critical values of L so that also p-adic length scale hypothesis would emerge from quantum critical dynamics! This conforms with the hypothesis about the value spectrum of α_K labelled in the same manner [L1].

4. At GRT limit the magnetic energy of the flux tubes gives rise to an average contribution to energy momentum tensor, which effectively corresponds to negative pressure for which the

expansion of the Universe accelerates. It would seem that both contributions could explain accelerating expansion. If the dynamics for Kähler action and volume term are coupled, one would expect same orders of magnitude for negative pressure and energy density - kind of equipartition of energy.

Consider first the basic scales emerging also from GRT picture. $R_U \sim \sqrt{1/\Lambda} \sim 10^{26}$ m = 10 Gly is not far from the recent size of the Universe defined as $c \times t \sim 13.8$ Gly. The derived size scale $L_1 \equiv (R_U \times l_P)^{1/2}$ is of the order of $L_1 = .5 \times 10^{-4}$ meters, the size of neuron. Perhaps this is not an accident. To make life of the reader easier I have collected the basic numbers to the following table.

$$\begin{array}{lll} m(CP_2) \simeq 5.7 \times 10^{14}\ \ GeV\ \ , & m_P = 2.435 \times 10^{18}\ \ GeV\ \ , & \frac{R(CP_2)}{l_P} \simeq 4.1 \times 10^3\ \ , \\ R_U = 10\ \ Gy\ \ , & t = 13.8\ \ Gy\ \ , & L_1 = \sqrt{l_P R_U} = .5 \times 10^{-4}\ \ m\ \ . \end{array} \tag{3.1}$$

Let us consider now some quantitative estimates. $R(X^4)$ depends on homotopy equivalence classes of the maps from $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$ - that is winding numbers w_i, $i = 1, 2$ for these maps. The simplest situations correspond to the winding numbers $(w_1, w_2) = (1, 0)$ and $(w_1, w_2) = (0, 1)$. For $(w_1, w_2) = (1, 0)$ M^4 contribution to the metric of $S^2(X^4)$ dominates and one has $R(X^4) \simeq R(M^4)$. For $R(M^4) = l_P$ so Planck length would define a fundamental length and Planck mass and Newton's constant would be quantal parameters. For $(w_1, w_2) = (0, 1)$ the radius of sphere would satisfy $R_D \simeq R$ (CP_2 size): now also Planck length would be quantal parameter.

Consider next additional scales emerging from TGD picture.

1. One has $L = (2^{3/2}\pi l_P/R_D) \times R_U$. In Minkowskian regions with $R_D = l_P$ this would give $L = 8.9 \times R_U$: there is no obvious interpretation for this number in recent cosmology. For $(R_D = R)$ one obtains the estimate $L = 29$ Mly. The size scale of large voids varies from about 36 Mly to 450 Mly (see `http://tinyurl.com/jyqcjhl`).
2. Consider next the derived size scale $L_2 = (L \times l_P)^{1/2} = \sqrt{L/R_U} \times L_1 = \sqrt{2^{3/2}\pi l_P/R_D} \times L_1$. For $R_D = l_P$ one has $L_2 \simeq 3L_1$. For $R_D = R$ making sense in Euclidian regions, this is of the order of size of neutrino Compton length: 3 μm, the size of cellular nucleus and rather near to the p-adic length scale $L(167) = 2.6$ m, corresponds to the largest miracle Gaussian Mersennes associated with $k = 151, 157, 163, 167$ defining length scales in the range between cell membrane thickness and the size of cellular nucleus. Perhaps these are co-incidences are not accidental. Biology is something so fundamental that fundamental length scale of biology should appear in the fundamental physics.

The formulas and predictions for different options are summarized by the following table.

$$\begin{array}{ccc} Option & L = \frac{2^{3/2}\pi l_P}{R_D} \times R_U & L_2 = \sqrt{L l_P} = \sqrt{\frac{2^{3/2}\pi l_P}{R_D}} \times L_1 \\ R_D = R\ \ , & 29\ \ Mly\ \ , & \simeq 3\ \ \mu m\ \ , \\ R_D = l_P\ \ , & 8.9 R_U\ \ , & \simeq 3L_1 = 1.5 \times 10^{-4}\ \ m\ \ , \end{array} \tag{3.2}$$

In the case of M^4 the radius of S^2 cannot be fixed it remains unclear whether Planck length scale is fundamental constant or whether it emerges.

3.2 Estimate for the cosmic evolution of R_D

One can actually get estimate for the evolution of R_D as function of cosmic time if one accepts Friedman cosmology as an approximation of TGD cosmology.

1. Assume critical mass density so that one has

$$\rho_{cr} = \frac{3H^2}{8\pi G} \quad .$$

2. Assume that the contribution of cosmological constant term to the mass mass density dominates. This gives $\rho \simeq \rho_{vac} = \Lambda/8\pi G$. From $\rho_{cr} = \rho_{vac}$ one obtains

$$\Lambda = 3H^2 \quad .$$

3. From Friedman equations one has $H^2 = ((da/dt)/a)^2$, where a corresponds to light-cone proper time and t to cosmic time defined as proper time along geodesic lines of space-time surface approximated as Friedmann cosmology. One has

$$\Lambda = \frac{3}{g_{aa}a^2}$$

in Robertson-Walker cosmology with $ds^2 = g_{aa}da^2 - a^2 d\sigma_3^2$.

4. Combining this equations with the TGD based equation

$$\Lambda = \frac{8\pi^2 G}{L^2 R_D^2}$$

one obtains

$$\frac{8\pi^2 G}{L^2 R_D^2} = \frac{3}{g_{aa}a^2} \quad . \tag{3.3}$$

5. Assume that quantum criticality applies so that L has spectrum given by p-adic length scale hypothesis so that one discrete p-adic length scale evolution for the values of L. There are two options to consider depending on whether p-adic length scales are assigned with light-cone proper time a or with cosmic time t

$$T = a \ \ (Option \ \ I) \ \ , \quad T = t \ \ (Option \ \ II) \tag{3.4}$$

Both options give the same general formula for the p-adic evolution of $L(k)$ but with different interpretation of $T(k)$.

$$\frac{L(k)}{L_{now}} = \frac{T(k)}{T_{now}} \ \ , \quad T(k) = L(k) = 2^{(k-151)/2} \times L(151) \ \ , \quad L(151) \simeq 10 \ \ nm \ \ . \tag{3.5}$$

Here $T(k)$ is assumed to correspond to primary p-adic length scale. An alternative - less plausible - option is that $T(k)$ corresponds to secondary p-adic length scale $L_2(k) = 2^{k/2}L(k)$ so that $T(k)$ would correspond to the size scale of causal diamond. In any case one has $L \propto L(k)$. One has a discretized version of smooth evolution

$$L(a) = L_{now} \times \frac{T}{T_{now}} \quad . \tag{3.6}$$

6. Feeding into this to Eq. 3.3 one obtains an expression for $R_D(a)$

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{L(a)} \times g_{aa}^{1/2} \ . \tag{3.7}$$

Unless the dependences on cosmic time compensate each other, R_D is dynamical and becomes very small at very early times since g_{aa} becomes very small. $R(M^4) = l_P$ however poses a lower boundary since either of the maps $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$ must be homotopically non-trivial. For $R(M^4) = l_P$ one would obtain $R_D/l_P = 1$ at this limit giving also lower bound for g_{aa}. For $T = t$ option $a/L(a)$ becomes large and g_{aa} small.

As a matter of fact, in very early cosmic string dominated cosmology g_{aa} would be extremely small constant [K4]. In late cosmology $g_{aa} \rightarrow 1$ holds true and one obtains at this limit

$$\frac{R_D(now)}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a_{now}}{L_{now}} \times l_P \simeq 4.4\frac{a_{now}}{L_{now}} \ . \tag{3.8}$$

7. For $T = t$ option R_D/l_P remains constant during both matter dominated cosmology, radiation dominated cosmology, and string dominated cosmology since one has $a \propto t^n$ with $n = 1/2$ during radiation dominated era, $n = 2/3$ during matter dominated era, and $n = 1$ during string dominated era [K4]. This gives

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{t}\sqrt{g_{aa}}\frac{t(end)}{L(end)} = (\frac{8}{3})^{1/2}\frac{\pi}{n}\frac{t(end)}{L(end)} \ .$$

Here "end" refers the end of the string or radiation dominated period or to the recent time in the case of matter dominated era. The value of n would have evolved as $R_D/l_P \propto (1/n)(t_{end}/L_{end})$, $n \in \{1, 3/2, 2\}$. During radiation dominated cosmology $R_D \propto a^{1/2}$ holds true. The value of R_D would be very nearly equal to $R(M^4)$ and $R(M^4)$ would be of the same order of magnitude as Planck length. In matter dominated cosmology would would have $R_D \simeq 2.2(t(now)/L(now)) \times l_P$.

8. For $R_D(now) = l_P$ one would have

$$\frac{L_{now}}{a_{now}} = (\frac{8}{3})^{1/2}\pi \simeq 4.4 \ .$$

In matter dominated cosmology $g_{aa} = 1$ gives $t_{now} = (2/3) \times a_{now}$ so that predictions differ only by this factor for options I and II. The winding number for the map $S^2(X^4) \rightarrow S^2(CP_2)$ must clearly vanish since otherwise the radius would be of order R.

9. For $R_D(now) = R$ one would obtain

$$\frac{a_{now}}{L_{now}} = (\frac{8}{3})^{1/2} \times \frac{R}{l_P} \simeq 2.1 \times 10^4 \ .$$

One has $L_{now} = 10^6$ ly: this is roughly the average distance scale between galaxies. The size of Milky Way is in the range $1 - 1.8 \times 10^5$ ly and of an order of magnitude smaller.

10. An interesting possibility is that $R_D(a)$ evolves from $R_D \sim R(M^4) \sim l_P$ to $R_D \sim R$. This could happen if the winding number pair $(w_1, w_2) = (1, 0)$ transforms to $(w_1, w_2) = (0, 1)$ during transition to from radiation (string) dominance to matter (radiation) dominance. R_D/l_P radiation dominated cosmology would be related by a factor

$$\frac{R_D(rad)}{R_D(mat)} = (3/4)\frac{t(rad, end)}{L(rad, end)} \times \frac{L(now)}{t(now)}$$

to that in matter dominated cosmology. Similar factor would relate the values of R_D/l_P in string dominated and radiation dominated cosmologies. The condition $R_D(rad)/R_D(mat) = l_P/R$ expressing the transformation of winding numbers would give

$$\frac{L(now)}{L(rad,end)} = \frac{4}{3}\frac{l_P}{R}\frac{t(now)}{t(rad,end)} \quad .$$

One has $t(now)/t(rad,end) \simeq .5 \times 10^6$ and $l_P/R = 2.5 \times 10^{-4}$ giving $L(now)/L(rad,end) \simeq 125$, which happens to be near fine structure constant.

11. For the twistor lifts of space-time surfaces for which cosmological constant has a reasonable value , the winding numbers are equal to $(w_1, w_2) = (n, 0)$ so that $R_D = \sqrt{n}R(S^2(M^4))$ holds true in good approximation. This conforms with the observed constancy of R_D during various cosmological eras, and would suggest that the ratio $\frac{t(end)}{L(end)}$ characterizing these periods is same for all periods. This determines the evolution for the values of $\alpha_K(M^4)$.

$R(M^4) \sim l_P$ seems rather plausible option so that Planck length would be fundamental classical length scale emerging naturally in twistor approach. Cosmological constant would be coupling constant like parameter with a spectrum of critical values given by p-adic length scales.

3.3 What about the extremals of the dimensionally reduced 6-D Kähler action?

It seems that the basic wisdom about extremals of Kähler action remains unaffected and the motivations for WCW are not lost in the case that M^4 Kähler form does not contribute to 6-D Kähler action (the case to be considered below): otherwise the predicted effects are extremely small in the recent Universe. What is new is that the removal of vacuum degeneracy is forced by twistorial action.

1. All extremals, which are minimal surfaces remain extremals. In fact, all the known extremals except vacuum extremals. For minimal surfaces the dynamics of the volume term and 4-D Kähler action separate and field equations for them are separately satisfied. The vacuum degeneracy motivating the introduction of WCW is preserved. The induced Kähler form vanishes for vacuum extremals and the imaginary unit of twistor space is ill-defined. Hence vacuum extremals cannot belong to WCW. This correspond to the vanishing of WCW metric for vacuum extremals.

2. For non-minimal surfaces Kähler coupling strength does not disappear from the field equations and appears as a genuine coupling very much like in classical field theories. Minimal surface equations are a generalization of wave equation and Kähler action would define analogs of source terms. Field equations would state that the total isometry currents are conserved. It is not clear whether other than minimal surfaces are possible, I have even conjectured that all preferred extremals are always minimal surfaces having the property that being holomorphic they are almost universal extremals for general coordinate invariant actions.

3. Thermodynamical analogy might help in the attempts to interpret. Quantum TGD in zero energy ontology (ZEO) corresponds formally to a complex square root of thermodynamics. Kähler action can be identified as a complexified analog of free energy. Complexification follows both from the fact that $\sqrt{g}$ is real/imaginary in Euclidian/Minkowskian space-time regions. Complex values are also implied by the proposed identification of the values of Kähler coupling strength in terms of zeros and pole of Riemann zeta in turn identifiable as poles of the so called fermionic zeta defining number theoretic partition function for fermions [K11] [L1, L3]. The thermodynamical for Kähler action with volume term is Gibbs free energy $G = F - TS = E - TS + PV$ playing key role in chemistry.

4. The boundary conditions at the ends of space-time surfaces at boundaries of CD generalize appropriately and symmetries of WCW remain as such. At light-like boundaries between

Minkowskian and Euclidian regions boundary conditions must be generalized. In Minkowkian regions volume can be very large but only the Euclidian regions contribute to Kähler function so that vacuum functional can be non-vanishing for arbitrarily large space-time surfaces since exponent of Minkowskian Kähler action is a phase factor.

5. One can worry about almost topological QFT property. Although Kähler action from Minkowskian regions at least would reduce to Chern-Simons terms with rather general assumptions about preferred extremals, the extremely small cosmological term does not. Could one say that cosmological constant term is responsible for "almost"?

   It is interesting that the volume of manifold serves in algebraic geometry as topological invariant for hyperbolic manifolds, which look locally like hyperbolic spaces $H_n = SO(n,1)/SO(n)$ [A1] [K7]. See also the article "Volumes of hyperbolic manifolds and mixed Tate motives" (see `http://tinyurl.com/yargy3uw`). Now one would have $n = 4$. It is probably too much to hope that space-time surfaces would be hyperbolic manifolds. In any case, by the extreme uniqueness of the preferred extremal property expressed by strong form of holography the volume of space-time surface could also now serve as topological invariant in some sense as I have earlier proposed. What is intriguing is that AdS_n appearing in AdS/CFT correspondence is Lorentzian analogue H_n.

6. $\alpha(M^4)$ is extremely large so that there is no hope of quantum perturbation theory around canonically imbedded M^4 although the propagator for CP_2 coordinate exists. In the new framework WCW can be seen as a solution to how to construct non-perturbative quantum TGD.

To sum up, I have the feeling that the final formulation of TGD has now emerged and it is clear that TGD is indeed a quantum theory of gravitation allowing to understand standard model symmetries. The existence of twistorial formulation is all that is needed to fix the theory completely. It makes possible gravitation and predicts standard model symmetries. This cannot be said about any competitor of TGD.

4 Basic Principles Behind Construction of Amplitudes

Basic principles of the construction summarized in this section could be seen as axioms trying to abstract the essentials. The explicit construction of amplitudes is too heavy challenge at this stage and at least for me.

4.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique.

1. As already explained, M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept as one might guess from the fact that the projection of Kähler form naturally defines the preferred quaternionic imaginary unit defining the twistor structure for space-time surface. Both M^4 and its Euclidian variant E^4 allow twistor space. The first guess is that the twistor space of M^4 is Minkowskian variant $T(M^4) = SU(2,2)/SU(2,1)\times U(1)$ of 6-D twistor space $CP_3 = SU(4)/SU(3)\times U(1)$ of E^4. This is sensible assumption at the level of momentum space but the second candidate, which is simply $T(M^4) = M^4 \times CP_1$, is the only sensible option at space-time level. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

2. This leads to a proposal for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4)\times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$.

   What is nice in this formulation is that one might be able to use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds) provided one can generalize the notion of Kähler structure from Euclidian to Minkowskian signature. It has been already described how this approach leads to a profound understanding of the relationship between TGD and GRT. Planck length emerges whereas fundamental constant as also cosmological constant emerges dynamically from the length scale parameter appearing in 6-D Kähler action. One can say, that twistor extension is absolutely essential for really understanding the gravitational interactions although the modification of Kähler action is extremely small due to the huge value of length scale defined by cosmological constant.

3. Masslessness (masslessness in complex sense for virtual particles in twistorialization) is essential condition for twistorialization. In TGD massless is masslessness in 8-D sense for the representations of superconformal algebras. This suggests that 8-D variant of twistors makes sense. 8-dimensionality indeed allows octonionic structure in the tangent space of imbedding space. One can also define octonionic gamma matrices and this allows a possible generalization of 4-D twistors to 8-D ones using generalization of sigma matrices representing quaternionic units to to octonionic sigma "matrices" essential for the notion of twistors. These octonion units do not of course allow matrix representation unless one restricts to units in some quaternionic subspace of octonions. Space-time surfaces would be associative and thus have quaternionic tangent space at each point satisfying some additional conditions.

4.2 Strong form of holography

Strong form of holography (SH) following from general coordinate invariance (GCI) for space-times as surfaces states that the data assignable to string world sheets and partonic 2-surfaces allows to code for scattering amplitudes. The boundaries of string world sheets at the space-like 3-surfaces defining the ends of space-time surfaces at boundaries of causal diamonds (CDs) and the fermionic lines along light-like orbits of partonic 2-surfaces representing lines of generalized Feynman diagrams become the basic elements in the generalization of twistor diagrams (I will not use the attribute "Feynman" in precise sense, one could replace it with "twistor" or even drop away). One can assign fermionic lines massless in 8-D sense to flux tubes, which can also be braided. One obtains a fractal hierarchy of braids with strands, which are braids themselves. At the lowest level one has braids for which fermionic lines are braided. This fractal hierarchy is unavoidable and means generalization of the ordinary Feynman diagram. I have considered some implications of this hierarchy in [L2].

The precise formulation of strong form of holography (SH) is one of the technical problems in TGD. A comment in FB page of Gareth Lee Meredith led to the observation that besides the purely number theoretical formulation based on commutativity also a symplectic formulation in the spirit of non-commutativity of imbedding space coordinates can be considered. One can however use only the notion of Lagrangian manifold and avoids making coordinates operators leading to a loss of General Coordinate Invariance (GCI).

4.3 The existence of WCW demands maximal symmetries

Quantum TGD reduces to the construction of Kähler geometry of infinite-D "world of classical worlds" (WCW), of associated spinor structure, and of modes of WCW spinor fields which are purely classical entities and quantum jump remains the only genuinely quantal element of quantum TGD. Quantization without quantization, would Wheeler say.

By its infinite-dimensionality, the mere mathematical existence of the Kähler geometry of WCW requires maximal isometries. Physics is completely fixed by the mere condition that its mathematical description exists. Super-symplectic and other symmetries of "world of classical worlds" (WCW) are in decisive role. These symmetry algebras have conformal structure and generalize and extend the conformal symmetries of string models (Kac-Moody algebras in particular). These symmetries give also rise to the hierarchy of Planck constants. The super-symplectic symmetries extend to a Yangian algebra, whose generators are polylocal in the sense that they involve products of generators associated with different partonic surfaces. These symmetries leave scattering amplitudes invariant. This is an immensely powerful constraint, which remains to be understood.

4.4 Quantum criticality

Quantum criticality (QC) of TGD Universe is a further principle. QC implies that Kähler coupling strength is mathematically analogous to critical temperature and has a discrete spectrum. Coupling constant evolution is replaced with a discrete evolution as function of p-adic length scale: sequence of jumps from criticality to a more refined criticality or vice versa (in spin glass energy landscape you at bottom of well containing smaller wells and you go to the bottom of smaller well). This implies that either all radiative corrections (loops) sum up to zero (QFT limit) or that diagrams containing loops correspond to the same scattering amplitude as tree diagrams so that loops can eliminated by transforming them to arbitrary small ones and snipping away moving the end points of internal lines along the lines of diagram (fundamental description).

Quantum criticality at the level of super-conformal symmetries leads to the hierarchy of Planck constants $h_{eff} = n \times h$ labelling a hierarchy of sub-algebras of super-symplectic and other conformal algebras isomorphic to the full algebra. Physical interpretation is in terms of dark matter hierarchy. One has conformal symmetry breaking without conformal symmetry breaking as Wheeler would put it.

4.5 Physics as generalized number theory, number theoretical universality

Physics as generalized number theory vision has important implications. Adelic physics is one of them. Adelic physics implied by number theoretic universality (NTU) requires that physics in real and various p-adic numbers fields and their extensions can be obtained from the physics in their intersection corresponding to an extension of rationals. This is also enormously powerful condition and the success of p-adic length scale hypothesis and p-adic mass calculations can be understood in the adelic context.

In TGD inspired theory of consciousness various p-adic physics serve as correlates of cognition and p-adic space-time sheets can be seen as cognitive representations, "thought bubbles". NTU is closely related to SH. String world sheets and partonic 2-surfaces with parameters (WCW coordinates) characterizing them in the intersection of rationals can be continued to space-time surfaces by preferred extremal property but not always. In p-adic context the fact that p-adic integration constants depend on finite number of pinary digits makes the continuation easy but in real context this need not be possible always. It is always possible to imagine something but not always actualize it!

4.6 Scattering diagrams as computations

Quantum criticality as possibility to eliminate loops has a number theoretic interpretation. Generalized Feynman diagram can be interpreted as a representation of a computation connecting given set X of algebraic objects to second set Y of them (initial and final states in scattering) (trivial example: $X = \{3,4\} \rightarrow 3 \times 4 = 12 \rightarrow 2 \times 6 \rightarrow \{2,6\} = Y$. The 3-vertices ($a \times b = c$) and their time-reversals represent algebraic product and co-product.

There is a huge symmetry: all diagrams representing computation connecting given X and Y must produce the same amplitude and there must exist minimal computation. This generalization of string model duality implies an infinite number of dualities unless the finite size of CD allows only a finite number of equivalent computations. These dualities are analogous to the dualities of super-string model, in particular mirror symmetry stating that same quantum physical situation does not correspond to a unique space-time geometry and topology (Calabi-Yau and its mirror represent the same situation). The task of finding this computation is like finding the simplest representation for the formula X=Y and the noble purpose of math teachers is that we should learn to find it during our school days. This generalizes the duality symmetry of old fashioned string models: one can transform any diagram to a tree diagram without loops. This corresponds to quantum criticality in TGD: coupling constants do not evolve. The evolution is actually there but discrete and corresponds to infinite number critical values for Kahler coupling strength analogous to temperature.

4.7 Reduction of diagrams with loops to braided tree-diagrams

1. In TGD pointlike particles are replaced with 3-surfaces and by SH by partonic 2-surfaces. The important implication of 3-dimensionality is braiding. The fermionic lines inside light-like orbits of partonic 2-surfaces can be knotted and linked - that is braided (this is dynamical braiding analogous to dance). Also the fermionic strings connecting partonic 2-surfaces at space-like 3-surfaces at boundaries of causal diamonds (CDs) are braided (space-like braiding).

   Therefore ordinary Feynman diagrams are not enough and one must allow braiding for tree diagrams. One can also imagine of starting from braids and allowing 3-vertices for their strands (product and co-product above). It is difficult to imagine what this braiding could mean. It is better to imagine braid and allow the strands to fuse and split (annihilation and pair creation vertices).

2. This braiding gives rise in the planar projection representation of braids to a generalization of non-planar Feynman diagrams. Non-planar diagrams are the basic unsolved problem of twistor approach and have prevented its development to a full theory allowing to construct exact expressions for the full scattering amplitudes (I remember however that Nima Arkani-Hamed et al have conjectured that non-planar amplitudes could be constructed by some procedure: they notice the role of permutation group and talk also about braidings (describable using covering groups of permutation groups)). In TGD framework the non-planar Feynman diagrams correspond to non-trivial braids for which the projection of braid to plane has crossing lines, say a and b, and one must decide whether the line a goes over b or vice versa.

3. An interesting open question is whether one must sum over all braidings or whether one can choose only single braiding. Choice of single braiding might be possible and reflect the failure of string determinism for Kähler action and it would be favored by TGD as almost topological quantum field theory (TQFT) vision in which Kähler action for preferred extremal is topological invariant.

4.8 Scattering amplitudes as generalized braid invariants

The last big idea is the reduction of quantum TGD to generalized knot/braid theory (I have talked also about TGD as almost TQFT). The scattering amplitude can be identified as a generalized braid invariant and could be constructed by the generalization of the recursive procedure transforming in a step-by-step manner given braided tree diagram to a non-braided tree diagram: essentially what Alexander the Great did for Gordian knot but tying the pieces together after cutting. At each step one must express amplitude as superposition of amplitudes associated with the different outcomes of splitting followed by reconnection. This procedure transforms braided tree diagram to a non-braided tree diagrams and the outcome is the scattering amplitude!

5 Tensor Networks and S-matrices

The concrete construction of scattering amplitudes has been the toughest challenge of TGD and the slow progress has occurred by identification of general principles with many side tracks. One of the key problems has been unitarity. The intuitive expectation is that unitarity should reduce to a local notion somewhat like classical field equations reduce the time evolution to a local variational principle. The presence of propagators have been however the obstacle for locally realized unitarity in which each vertex would correspond to unitary map in some sense.

TGD suggests two approaches to the construction of S-matrix.

1. The first approach is generalization of twistor program [K5]. What is new is that one does not sum over diagrams but there is a large number of equivalent diagrams giving the same outcome. The complexity of the scattering amplitude is characterized by the minimal diagram. Diagrams correspond to space-time surfaces so that several space-time surfaces give rise to the same scattering amplitude. This would correspond to the fact that the dynamics breaks

classical determinism. Also quantum criticality is expected to be accompanied by quantum critical fluctuations breaking classical determinism. The strong form of holography would not be unique: there would be several space-time surfaces assignable as preferred extremals to given string world sheets and partonic 2-surfaces defining "space-time genes".

2. Second approach relies on the number theoretic vision and interprets scattering amplitudes as representations for computations with each 3-vertex identifiable as a basic algebraic operation [K5]. There is an infinite number of equivalent computations connecting the set of initial algebraic objects to the set of final algebraic objects. There is a huge symmetry involved: one can eliminate all loops moving the end of line so that it transforms to a vacuum tadpole and can be snipped away. A braided tree diagram is left with braiding meaning that the fermion lines inside the line defined by light-like orbit are braided. This kind of braiding can occur also for space-like fermion lines inside magnetic flux tubes and defining correlate for entanglement. Braiding is the TGD counterpart for the problematic non-planarity in twistor approach.

Third approach involving local unitary as an additional key element is suggested by tensor networks relying on the notion of perfect entanglement discussed by Preskill et al [?].

1. Tensor networks provide an elegant representation of holography mapping interior states isometrically (in Hilbert space sense) to boundary states or vice versa for selected subsets of states defining the code subspace for holographic quantum error correcting code. Again the tensor net is highly non-unique but there is some minimal tensor net characterizing the complexity of the entangled boundary state.

2. Tensor networks have two key properties, which might be abstracted and applied to the construction of S-matrix in zero energy ontology (ZEO): perfect tensors define isometry for any subspace defined by the index subset of perfect tensor to its complement and the non-unique graph representing the network. As far as the construction of Hilbert space isometry between local interior states and highly non-local entangled boundary states is considered, these properties are enough.

One cannot avoid the question whether these three constructions could be different aspects of one and same construction and that tensor net construction with perfect tensors representing vertices could provide and additional strong constraint to the long sought for explicit recipe for the construction of scattering amplitudes.

5.1 Objections

It is certainly clear from the beginning that the possibly existing description of S-matrix in terms of tensor networks cannot correspond to the perturbative QFT description in terms of Feynman diagrams.

1. Tensor network description relates interior and boundary degrees in holography by a isometry. Now however unitary matrix has quite different role. It could correspond to U-matrix relating zero energy states to each other or to the S-matrix relating to each other the states at boundary of CD and at the shifted boundary obtained by scaling. These scalings shifting the second boundary of CD and increasing the distance between the tips of CD define the analog of unitary time evolution in ZEO. The U-matrix for transitions associated with the state function reductions at fixed boundary of CD effectively reduces to S-matrix since the other boundary of CD is not affected.

   The only manner one could see this as holography type description would be in terms of ZEO in which zero energy states are at boundaries of CD and U-matrix is a representation for them in terms of holography involving the interior states representing scattering diagram in generalized sense.

2. The appearance of small gauge coupling constant tells that the entanglement between "states" in state spaces whose coordinates formally correspond to quantum fields is weak and just

opposite to that defined by a perfect tensor. Quite generally, coupling constant might be the fatal aspect of the vertices preventing the formulation in terms of perfect entanglement.

One should understand how coupling constant emerges from this kind of description - or disappears from standard QFT description. One can think of including the coupling constant to the definition of gauge potentails: in TGD framework this is indeed true for induced gauge fields. There is no sensical manner to bring in the classical coupling constants in the classical framework and the inverse of Kähler coupling strength appears only as multiplier of the Kähler action analogous to critical temperature.

More concretely, there are WCW spin degrees of freedom (fermionic degrees of freedom) and WCW orbital degrees of freedom involving functional integral over WCW. Fermionic contribution would not involve coupling constants whereas the functional integral over WCW involving exponential of vacuum functional could give rise to the coupling constants assignable to the vertices in the minimal tree diagram.

3. The decomposition $S = 1 + iT$ of unitary S-matrix giving unitarity as the condition $-i(T - T^\dagger) + T^\dagger T = 0$ reflects the perturbative thinking. If one has only isometry instead of unitary transformation, this decomposition becomes problematic since T and $T^\dagger$ whose some appears in the formula act in different spaces. One should have the generalization of Id as a "trivial" isometry. Alternatively, one should be able to extend the state space H_{in} by adding a tensor factor mapped trivially in isometry.

4. There are 3- and 4-vertices rather than only -say, 3-vertices as in tensor networks. For non-Abelian Chern-Simons term for simple Lie group one would have besides kinetic term only 3-vertex $Tr(A \wedge A \wedge A)$ defining the analog of perfect tensor entanglement when interpreted as co-product involving 3-D permutation symbol and structure constants of Lie algebra. Note also that for twistor Grassmannian approach the fundamental vertices are 3-vertices. It must be however emphasized that QFT description emerges from TGD only at the limit when one identifies gauge potentials as sums of induced gauge potentials assignable to the space-time sheets, which are replaced with single piece of Minkowski space.

5. Tensor network description does not contain propagators since the contractions are between perfect tensors. It is to make sense propagators must be eliminated. The twistorial factorization of massless fermion propagator suggest that this might be possible by absorbing the twistors to the vertices.

These reasons make it clear that the proposed idea is just a speculative question. Perhaps the best strategy is to look this crazy idea from different view points: the overly optimistic view developing big picture and the approach trying to debunk the idea.

5.2 The overly optimistic vision

With these prerequisites on one can follow the optimistic strategy and ask how tensor networks could the allow to generalize the notion of unitary S-matrix in TGD framework.

1. Tensor networks suggests the replacement of unitary correspondence with the more general notion of Hilbert space isometry. This generalization is very natural in TGD since one must allow phase transitions increasing the state space and it is quite possible that S-matrix represents only isometry: this would mean that $S^\dagger S = Id_{in}$ holds true but $SS^\dagger = Id_{out}$ does not even make sense. This conforms with the idea that state function reduction sequences at fixed boundary of causal diamonds defining conscious entities give rise evolution implying that the size of the state space increases gradually as the system becomes more complex. Note that this gives rise to irreversibility understandandable in terms of NMP [K3]. It might be even impossible to formally restore unitary by introducing formal additional tensor factor to the space of incoming states if the isometric map of the incoming state space to outgoing state space is inclusion of hyperfinite factors.

2. If the huge generalization of the duality of old fashioned string models makes sense, the minimal diagram representing scattering is expected to be a tree diagram with braiding

and should allow a representation as a tensor network. The generalization of the tensor network concept to include braiding is trivial in principle: assign to the legs connecting the nodes defined by perfect tensors unitary matrices representing the braiding - here topological QFT allows realization of the unitary matrix. Besides fermionic degrees of freedom having interpretation as spin degrees of freedom at the level of "World of Classical Worlds" (WCW) there are also WCW orbital degrees of freedom. These two degrees of freedom factorize in the generalized unitarity conditions and the description seems much simpler in WCW orbital degrees of freedom than in WCW spin degrees of freedom.

3. Concerning the concrete construction there are two levels involved, which are analogous to descriptions in terms of boundary and interior degrees of freedom in holography. The level of fundamental fermions assignable to string world sheets and their boundaries and the level of physical particles with particles assigned to sets of partonic 2-surface connected by magnetic flux tubes and associated fermionic strings. One could also see the ends of causal diamonds as analogous to boundary degrees of freedom and the space-time surface as interior degrees of freedom.

The description at the level of fundamental fermions corresponds to conformal field theory at string world sheets.

1. The construction of the analogs of boundary states reduces to the construction of N-point functions for fundamental fermions assignable to the boundaries of string world sheets. These boundaries reside at 3-surfaces at the space-like space-time ends at CDs and at light-like 3-surfaces at which the signature of the induced space-time metric changes.

2. In accordance with holography, the fermionic N-point functions with points at partonic 2-surfaces at the ends of CD are those assignable to a conformal field theory associated with the union of string world sheets involved. The perfect tensor is assignable to the fundamental 4-fermion scattering which defines the microscopy for the geometric 3-particle vertices having twistorial interpretation and also interpretation as algebraic operation.

   What is important is that fundamental fermion modes at string world sheets are labelled by conformal weights and standard model quantum numbers. No four-momenta nor color quantum numbers are involved at this level. Instead of propagator one has just unitary matrix describing the braiding.

3. Note that four-momenta emerging in somewhat mysterious manner to stringy scattering amplitudes and mean the possibility to interpret the amplitudes at the particle level.

Twistorial and number theoretic constructions should correspond to particle level construction and also now tensor network description might work.

1. The 3-surfaces are labelled by four-momenta besides other standard model quantum numbers but the possibility of reducing diagram to that involving only 3-vertices means that momentum degrees of freedom effectively disappear. In ordinary twistor approach this would mean allowance of only forward scattering unless one allows massless but complex virtual momenta in twistor diagrams. Also vertices with larger number of legs are possible by organizing large blocks of vertices to single effective vertex and would allow descriptions analogous to effective QFTs.

2. It is highly non-trivial that the crucial factorization to perfect tensors at 3-vertices with unitary braiding matrices associated with legs connecting them occurs also now. It allows to split the inverses of fermion propagators into sum of products of two parts and absorb the halves to the perfect tensors at the ends of the line. The reason is that the inverse of massless fermion propagator (also when masslessness is understood in 8-D sense allowing M^4 mass to be non-vanishing) to be express as bilinear of the bi-spinors defining the twistor representing the four-momentum. It seems that this is absolutely crucial property and fails for massive (in 8-D sense) fermions.

5.3 Twistorial and number theoretic visions

Both twistorial and number theoretical ideas have given a strong boost to the development of ideas.

1. With experience coming from twistor Grassmannian approach, twistor approach is conjectured to allow an extension of super-symplectic and other superconformal symmetry algebras to Yangian algebras by adding a hierarchy of multilocal generators [K5]. The twistorial diagrams for $\mathcal{N} = 4$ SUSY can be reduced to a finite number and there is large number of equivalent diagrams. One expects that this is true also in TGD framework.

   Twistorial approach is extremely general and quite too demanding to my technical skills but its is a useful guideline. An important outcome of twistor approach is that the intermediate states are massless on-mass-shell states but with complex momenta. Does this generalize and could each vertex define unitary scattering event with complex four-momenta in possibly complexified Minkowski space? Or could even real momenta be possible for massive particles, which would be massless in 8-D sense thanks to the existence of octonionic tangent space structure of 8-D imbedding space? And what is the role of the unique twistorial properties of M^4 and CP_2?

2. Number theoretical vision suggests that the scattering amplitudes correspond to sequences of algebraic operations taking inputs and producing outputs, which in turn serve as inputs for a neighboring node [K5]. The vertices form a diagram defining a network like structure defining kind of distributed computations leading from given inputs to given outputs. A computation leading from given inputs to given outputs is suggestive. There exists an infinite number of this kind of computations and there must be the minimal one which defines the complexity of the scattering. The maximally simplifying guess is that this diagram would correspond to a braided tree diagram. At space-time level these diagrams would correspond to different space-time surfaces defining same physics: this is because of holography meaning that only the ends of space-time surfaces at boundaries of CD matter.

   This vision generalizes of the old-fashioned stringy duality. It states that all diagrams can be reduced to minimal diagrams. This is achieved by by moving the ends of internal lines so that loops becomes vacuum tadpoles and can be snipped off. Tree diagrams must be however allowed to braid and outside the vertices the diagrams look like braids. Braids for which threads can split and glue together is the proper description for what the diagrams could be. Braiding would provide the counterpart for the non-planar twistor diagrams.

   The fermion lines inside the light-like 3-surfaces can get braided. Smaller partonic 2-surfaces can topologically condense at given bigger partonic 2-surface (electronic parton surface can topologically condense to nano-scopic parton surface) and the orbits of the condensed partonic 2-surfaces at the light-like orbit of the parton surface can get braided. This gives rise to a hierarchy of braids with braids.

5.4 Generalization of the notion of unitarity

The understanding of unitarity has been the most difficult issue in my attempts to understand S-matrix in TGD framework. When something turns out to be very difficult to understand, it might make sense to ask whether the definition of this something involves un-necessary assumptions. Could unitarity be this kind of notion?

The notion of tensor network suggests that unitarity can generalized and that this generalization allows the realization of unitarity in extremely simple manner using perfect tensors as building bricks of diagrams.

1. Both twistorial and number theoretical approaches define M-matrix and associated S-matrix as a map between the state spaces H_{in} and H_{out} assignable to the opposite boundaries of CD - say positive and negative energy parts of zero energy state. In QFT one has $H_{in} = H_{out}$ and the map would be Hilbert space unitary transformation satisfying $SS^{\dagger} = S^{\dagger}S = Id$.

2. The basic structure of TGD (NMP favoring generation of negentropic entanglement, the hierarchy of Planck constants, length scale hierarchies, and hierarchy of space-time sheets)

suggests that the time evolution leads to an increasingly complex systems with higher-dimensional Hilbert space so that $H_{in} = H_{out}$ need not hold true but is replaced with $H_{in} \subset H_{out}$. This view is very natural since one must allow quantum phase transitions increasing the value of h_{eff} and the value of p-adic prime defining p-adic length scale.

S-matrix would thus define isometric map $H_{in} \subset H_{out}$. Isometry property requires $U^\dagger U = Id_{in}$. If the inclusion of H_{in} to H_{out} is a genuine subspace of H_{out}, the condition $UU^\dagger = Id_{out}$ does not make sense anymore. This means breaking of reversibility and is indeed implied by the quantum measurement theory based on ZEO.

3. It would be at least formally possible to fuse all state spaces to single very large state space by replacing isometry $H_{in} \subset H_{out}$ with unitary map $H_{out} \to H_{out}$ by adding a tensor factor in which the map acts as identity transformation. This is not practical since huge amounts of redundant information would be introduced. Also the information about hierarchical structure essential for the idea of evolution would be lost. This hierarchical of inclusions should also be crucial for understanding the construction of S-matrix or rather, the hierarchy of S-matrices of isometric inclusions including as a special case unitary S-matrices.

4. There is also a further intricacy, which might prevent the formal unitarization by the addition of an inert tensor factor. I have talked a lot about HFFs referring to hyper-finite factors of type II_1 (possibly also of type III_1) and their inclusions [K6]. The reason is that WCW spinors form a canonical representation for these von Neumann algebras.

   Could the isometries replacing unitary S-matrix correspond to inclusions of HFFs? In the recent interpretation the included factor (now H_{in}) corresponds to the degrees of freedom below measurement resolution. Certainly this does not make sense now. The interpretation in terms of finite measurement resolution need not however be the only possible interpretation and the interpretation in terms of measurement resolution might of course be wrong. Therefore one can ask whether the relation between H_{in} and H_{out} could be more complex than just $H_{out} = H_{in} \otimes H_1$ so that formal unitarization would fail.

5.5 Scattering diagrams as tensor networks constructed from perfect tensors

Preskill's tensor network construction [?] realizes isometric maps as representations of holography and as models for quantum error correcting codes. These tensor networks have remarkable similarities with twistorial and number theoretical visions, which suggests that it could be used to construct scattering amplitudes. A further idea inspired by holography is that the description of scattering amplitudes in terms of fundamental fermions and physical particles are dual to each other.

1. In the construction of quantum error codes tensor network defines an isometric imbedding of local states in the interior to strongly entangled non-local states at boundary. Their vertices correspond to tensors, which in the proposal of Preskill et al [?] are perfect tensors such that one can take any m legs of the vertex and the tensor defines isometry from the state space of m legs to that of $n-m$ legs. When the number of indices is $2n$, the entanglement defined by perfect tensor between any n-dimensional subspace and its complement is maximal

   TGD framework maximal entanglement corresponds to negentropic entanglement with density matrix proportional to identity matrix. What is important that the isometry is constructed by composing local isometries associated with a network. Given isometry can be constructed in very many manners but there is some minimal realization.

2. The tensor networks considered in [?] are very special since they are determined by tesselations of hyperbolic space H_2. This kind of tesselations of H_3 could be crucial for understanding the analog of condensed matter physics for dark matter and could appear in biology [K9]. What is crucial is that only the graph property and perfect tensor property matter as far as isometricity is considered so that it is possible to construct very general isometries by using tensor networks.

5.6 Eigenstates of Yangian co-algebra generators as a manner to generate maximal entanglement?

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and also of tensor networks, and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [K12]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K3].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie algebras are generated by ordinary Lie algebra generators and their co-generators subject to constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [?] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see `http://tinyurl.com/y727n8ua`). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see `http://tinyurl.com/y7heufjh`).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

5.7 Two different tensor network descriptions

The obvious question is whether also unitary S-matrix of TGD could be constructed using tensor network built from perfect tensors. In ZEO the role of boundary would be taken by the ends of the space-time at upper and lower light-like boundaries of CD carrying the particles characterized by standard model quantum numbers. Strong form of holography would suggest that partonic surfaces and strings at the ends of CD provide information for the description of zero energy states and therefore of scattering amplitudes. The role of interior would be taken by the space-time surface - in particular the light-like orbits of partonic surfaces carrying the fermion lines identified as boundaries of string world sheets. Conformal field theory description would apply to fermions residing at string world sheets with boundaries at light-like orbits of partonic 2-surfaces.

In QFT Feynman diagrammatics one obtains a sum over diagrams with arbitrary numbers of loops. In both twistorial and number theoretic approach however only a finite number of diagrams with possibly complex on mass shell massless momenta are needed. If the vertices are however such that particles remain on-mass-shell but are allowed to have complex four-momenta then the integration over internal momenta (loops) is not present and tensor network description could make sense. This encourages the conjecture that tensor networks could be used to construct the scattering amplitudes in TGD framework.

What could perfect tensor property mean for the vertices identified as nodes of a tensor network? There are two levels to be considered: the geometric level identifying particles as 3-surfaces with net quantum numbers and the fermion level identifying particles as fundamental fermions at the boundaries of string world sheets.

1. At the geometric level vertices corresponds to light-like orbits of partonic 2-surfaces meeting at common end which is partonic 2-surface. This is 3-D generalization of Feynman diagram as a geometric entity. At the level of fermion lines associated with the light-like 3-surfaces one the basic interaction corresponds to the scattering of 2-fermions leading to re-sharing of fermion lines between outgoing light-like 3-surfaces, which include also representations for virtual particles. One has 4-fermion vertex but not in the sense that it appears in the interaction of weak interactions at low energies.

   Geometrically the basic vertex could be 3-vertex: $n > 3$-vertices are unstable against deformation to lower vertices. For 3-vertex perfect tensor property means that the tensor defining the vertex maps any 1-particle subspaces to 2-particle subspace isometrically. The geometric vertices define a network consisting of 3-D "lines" and 2-D vertices but one cannot tell what is within the 3-D lines and what happens in the 2-D nodes. The lines would consist of braided fundamental fermion lines and in nodes the basic process would be 2+2 scattering for fermions. In the case of 3-vertex momentum conservation would effectively eliminate the four-momentum and the state spaces associated with vertex would be effectively discrete. This is p-adically of utmost importance.

2. At the level of fundamental fermion lines in the interior of particle lines one would have 4-vertices and if a perfect tensor describes it, it gives rise to a unitary map of any 2-fermion subspace to its complement plus isometric maps of 1-fermion subspaces to 3-fermion subspaces. In this case momenta cannot act as labels of fermion lines for rather obvious reasons: the solution of the problem is that conformal weights label fundamental fermion lines

   The conservation of discrete quark and lepton numbers allows only vertices of type $qL \rightarrow qL$ and its variants obtained by crossing. In this case the isometries might allow realization. The isometries must be defined to take into account quark and lepton number conservation by crossing replacing fermion with antifermion. By allowing the states of Hilbert space in node to be both quarks and leptons, difficulties can be avoided.

5.7.1 Tensor network description in terms of fundamental fermions and CFT

Consider first fundamental fermions. What are the labels characterizing the states of fundamental fermions fermions propagating along the lines? There are two options: the labels are either conformal weights or four-momenta.

1. Since fermions corresponds to strings defining the boundaries of string world sheets and since strong form of holography implies effective 2-dimensionality also in fermion sector, the natural guess is that the conformal weights plus some discrete quantum numbers - standard model quantum numbers at least - are in question. The situation would be well-defined also p-adically for this option. In this case one can hope that conformal field theory at partonic 2-surface could define the fermionic 4-vertex more or less completely. There would be no need to assign propagators between different four-fermion vertices. The scattering diagram would define a composite formed from light-like 3-surfaces and one would have single isometry build from 4-fermion perfect tensors. There would be no integrations over internal momenta.

2. Second option is that fundamental fermions are labelled by four-momenta. The outgoing four-momenta in 4-vertices would not be completely fixed by the values of the incoming momenta and this extends the state space. Concerning p-adicization this integral is not desirable and this forces to consider seriously discrete labelling. The unitarity condition for 2+2 scattering would involve integral over 2-sphere. Four-fermion scattering must be unitary process in QFT so that this condition might be possible to satisfy. The problem would be how to fix this fundamental scattering matrix uniquely. This option does not look attractive number theoretically.

The most plausible option is that holography means that conformal field theory describes the scattering of fundamental fermions and QFT type description analogous to twistorial approach describes the scattering of physical fermions. If only 3-vertices are allowed, and if masslessness corresponds to masslessness in 8-D sense, one obtains non-trivial scattering vertices (for ordinary twistor approach all massless momenta would be collinear if real).

5.7.2 Tensor network description for physical particles

Could the twistorial description expected to correspond to the description in terms of particles allow tensor network description?

1. Certainly one must assign four-momenta to incoming *physical* particles - also fermions - but they correspond to pairs of wormhole contacts rather than fundamental fermions at the boundaries of string world sheets. It would be natural to assign four-momenta also to the virtual *physical* fermions appearing in the diagram and the geometric view about scattering would allow only 3-vertices so that momentum conservation would eliminate momentum degrees of freedom effectively. This would be a p-adically good news.

2. At the level of fundamental fermions entanglement is described as a tensor contraction of the CFT vertices. This locality is natural since the vertices are at null distance from each other. At QFT limit the entanglement between the ends of the line is characterized the propagator.

   One must get rid of propagators in order to have tensor network description. The inclusion of propagators to the fundamental tensor diagrams would break the symmetry between the legs of vertex since the propagator cannot be included to its both ends. Situation changes if one can represent the propagator as a bilinear of something more primitive and include the halves to the opposite ends of the line. Twistor representation of four-momentum indeed defines this kind of representation as a bilinear $p^{a\tilde{b}} = \lambda\tilde{\mu}^{\tilde{b}}$ of twistors λ and $\tilde{\mu}$. There is problem due to the diverging $1/p^2$ factor but residue integral eliminates this factor and one can write directly the fermionic propagator factors as $p^{a\tilde{b}}$.

3. In QFT description the perturbative expansion is in powers of coupling constant. If the reduction to braided tree diagrams analogous to twistor diagrams occurs, power g^{N-2} of coupling constant is expected to factorize as a multiplier of a tree diagram with N external legs. One should understand this aspect in the tensor net-work picture.

   For $\mathcal{N} = 4$ SUSY there is coupling constant renormalization. Similar prediction is expected from TGD. Coupling constant evolution is expected to be discrete and induced by the discrete evolution of Kähler coupling strength defined by the spectrum of its critical values. The conjecture is that critical values are naturally labelled by p-adic primes $p \simeq 2^k$, k prime, labelling p-adic length scales. Therefore one might hope that problems could be avoided.

These observations encourage the expectation that twistorial approach involving only 3-vertices allows to realize tensor network idea also at the level of physical particles. It might be essential that twistors can be generalized to 8-D twistors. Octonionic representation of gamma matrices might make this possible. Also the fact twistorial uniqueness of M^4 and CP_2 might be crucial.

Gauge theory follows as QFT limit of TGD so that one cannot in principle require that gauge theory vertices satisfy the isometricity conditions. Nothing however prevents from checking whether gauge theory limit might inherit this property.

1. For instance, could 3-vertices of Yang-Mills theory define isometric imbedding of 1-particle states to 2 particle states? For a given gauge boson there should exist always a pair of gauge bosons, which can fuse to it. Consider a basis for Lie-algebra generators of the gauge group. If the generator T is such that there exists no pair $[A, B]$ with the property $[A, B] = T$, Jacobi identity implies that T must commute with all generators and one has direct sum of Lie algebras generated by T and remaining generators.

2. In the case of weak algebra $SU(2) \times U(1)$ the weak mixing of Y and I_3 might allow the isometric imbeddings of type $1 \rightarrow 2$. Does this mean that Weinberg angle must be non-vanishing in order to have consistent theory? A realistic manner to get rid of the problem is to allow at QFT limit the lines to be also fermions so that also U(1) gauge boson can be constructed as fermion pair.

5.7.3 How the two tensor network descriptions would be related?

There are two descriptions for the zero energy states providing representation of scattering amplitudes: the CFT description in terms of fundamental fermions at the boundaries of string world sheets, and the description in terms of physical particles to which one can assign light-like 3-surfaces as virtual lines and total quantum numbers.

1. CFT description in terms of fundamental fermions in some aspects very simple because of its 2-dimensionality and conformal invariance. The description is in terms of physical particles having light-like 3-surfaces carrying some total quantum numbers as correlates and is simpler in different sense. These descriptions should be related by an Hilbert space isometry.

2. The perfect tensor property for 4-fermion vertices makes fundamental fermion states analogous to physical states realizing logical qubits as highly entangled structures. Geometric description in terms of 3-surfaces is in turn analogous to the description in terms of logical qubits.

3. Holography-like correspondence between these descriptions of zero energy states (scattering diagrams) should exist. Physical particles should correspond to the level, at which resolution is smaller and which should be isometrically mapped to the strongly entangled level defined by fundamental fermions and analogous to boundary degrees of freedom (fundamental fermions *are* at the boundaries of string world sheets!).

   The map relating the two descriptions seems to exist. One can assign four-momenta to the legs of conformal four-point function as parameters so that one obtains a mapping from the states labelled by conformal weights to the states labelled by four-momenta! The appearance of 4-momenta from conformal theory is somewhat mysterious looking phenomenon but this duality makes it rather natural.

5.8 Taking into account braiding and WCW degrees of freedom

One must also take intro account braiding and orbital degrees of freedom of WCW. The generalization of tensor network to braided tensor network is trivial. Thanks to the properties of tensor network orbital and spinor degrees of freedom factorize so that also the treatment of WCW degrees of freedom seems to be possible.

5.8.1 What about braiding?

The scattering diagrams would be tree diagrams with braiding of fermionic lines along light-like 3-surfaces - dance of fundamental quarks and leptons at parquette defined by the partonic 2-surface one might say. Also space-like braiding at magnetic flux tubes at the ends of CD is possible and its time evolution between the ends of space-time surfaces defines 2-braiding which is generalization of the ordinary braiding but will not be discussed here. This gives rise to a hierarchy of braidings. One can talk about flux tubes within flux tubes and about light-like 3-surface within light-like 3-surfaces. The smaller light-like 3-surface would be glued by a wormhole contact to the larger one and contact could have Euclidian signature of induced metric.

How can one treat the braiding in the tensor network picture? The answer is simple. Braiding corresponds to an element of braid group and one can represent it by a unitary matrix as one does in topological QFT as one constructs knot invariants. In particular, the trace of this unitary matrix defines a knot invariant. The generalization of the tensor network is simple. One attaches to the links connecting two nodes unitary transformation defining a representation of the braid involved. Local variant of unitarity would mean isometricity at nodes and unitarity at links.

5.8.2 What about WCW degrees of freedom?

The above considerations are about fermions that its WCW spinor degrees of freedom and the space-time surface itself has been regarded as a fixed background. How can one take into account WCW degrees of freedom?

The scattering amplitude involves a functional integral over the 3-surfaces at the ends of CD. The functional integration over WCW degrees of freedom gives an expression depending on Kähler coupling strength α_K and determines the dependence on various gauge coupling strengths expressible in terms of α_K. This makes it possible to have the tensor network description in fermionic degrees of freedom without losing completely the dependence of the scattering amplitudes on gauge couplings. By strong form of holography the functional integral should reduce to that over partonic 2-surfaces and strings connecting them. Number theoretic discretization with a cutoff determined by measurement resolution forces the parameters characterizing the 2-surfaces to belong to an algebraic extension of rationals and is expected to reduce functional integral to a sum over discretized WCW so that it makes sense also in p-adic sectors [K10, K11].

A brief summary of quantum measurement theory in ZEO is necessary. The repeated state function reduction shifts active boundary A of CD and affects the states at it. The passive boundary of CD- call it P - and the states at it - remain unaffected. The repeated state function reductions leaving P unaffected and giving usually rise to Zeno effect, correspond now to the TGD counterpart of unitary time evolution by shifts between subsequent state function reductions. Call A and its shifted version A_{in} and A_{out} and the corresponding state spaces H_{in} and H_{out}. The unitary (or more generally isometric) Smatrix represents this shift. This is the TGD counterpart of a unitary evolution of QFTs. S forms a building brick of a more general unitary matrix U acting in the space of zero energy states but U is not considered now.

Consider now the isometricity conditions.

1. Unitarity conditions generalized to isometricity conditions apply to S. Isometricity conditions $S^\dagger S = Id_{in}$ can be applied at A_{in}. The states appearing in the isometry conditions as initial and final states correspond to A_{in} and A_{out}. There is a trace over WCW spin indices (labels for many-fermion states) of H_{out} in the conditions $S^\dagger S = Id_{in}$. Isometricity conditions involve also an integral over WCW orbital degrees of freedom at both ends: these degrees of freedom are strongly correlated and for a strict classical determinism the correlation between the ends is complete. If the tensor network idea works, the summation over spinor degrees of freedom at A_{out} gives just a unit matrix in the spinor indices at A_{in} and leaves only the WCW orbital degrees of freedom in consideration. This factorization of spinor and orbital WCW degrees of freedom simplifies the situation dramatically.

2. One can express isometricity conditions for modes with $\Psi_{in,M}$ and $\Psi_{out,N}$ at A_{in} and A_{out}: this requires functional integration over 3-surfaces WCW at A_{in} and A_{out}. The conditions are formulated in terms of the labels - call them M_{in}, N_{in} - of WCW spinor modes at A_{in} including standard model quantum numbers and labels characterizing the states of supersymplectic and super-conformal representations. The trace is over the corresponding indices R_{out} at A_{out}. The WCW functional integrals in the generalized unitarity conditions are therefore over A_{in} and A_{out} and should give Kronecker delta $\sum_{R_{out}} S^\dagger_{M_{in}R_{out}} S_{R_{out}N_{in}} = \delta_{M_{in},N_{in}}$.

3. The simplest view would be that Kähler action with boundary conditions implies completely deterministic dynamics. The conditions expressing strong form of holography state that sub-algebras of super-symplectic algebra and related conformal algebras isomorphic to the entire algebra give rise to vanishing Noether charges. Suppose that these conditions posed at the ends of CD are so strong that they fix the time evolution of the space-time surface as preferred extremal completely when posed at either boundary. In this case the isometricity conditions

would be so strong that the double functional integration appearing in the matrix product reduces to that at A_{in} and the isometricity conditions would state just the orthonormality of the basis of WCW spinor modes at A_{in}.

4. Quantum criticality and in particular, the hierarchy of Planck constants providing a geometric description for non-deterministic long range fluctuations, does not support this view. Also the fact that string world sheets connect the boundaries of CD suggests that determinism must be broken. The inner product defining the completeness of the WCW state basis in orbital degrees of freedom can be however generalized to a bi-local inner product involving functional integration over 3-surfaces at both A_{in} and A_{out}. There is however a very strong correlation so that integration volume at A_{out} is expected to be small. This also suggests that one can have only isometricity conditions.

5.9 How do the gauge couplings appear in the vertices?

Reader is probably still confused and wondering how the gauge couplings appear in the vertices from the functional integral over WCW degrees of freedom. In twistorial approach, the vanishing of loops in $\mathcal{N}=4$ SYM theory gives just g^N, N the number of 3-vertices. Each vertex should give gauge coupling. Or equivalently, each propagator line connecting vertices should give α_K. The functional integral should give this factor for each propagator line. Generalization of conformal invariance is expected to give this picture.

To proceed some basic facts about N-point functions of CFTs are needed.

1. In conformal field theory the functional form of two-point function is completely fixed by conformal symmetry:

$$\begin{aligned}
G^{(2)}(z_i,\overline{z}_i) &= \frac{C_{12}}{z_{12}^{2h}\overline{z}_{12}^{2\overline{h}}} \ , \\
z_{ij} &= z_i - z_j \ , \ \ \overline{z}_{ij} = \overline{z}_i - \overline{z}_j \ , \\
h_1 = h_2 = h &= h_a + ih_b \ , \ \ \overline{h} = \overline{h}_a + i\overline{h}_b \ .
\end{aligned} \tag{5.1}$$

$h_1 = h_2 \equiv h$ and its conjugate $\overline{h}$ are conformal weights of conformal field and its conjugate. Note that the conformal weights of conformal fields Φ_1 and Φ_2 must be same. In TGD context C_{12} is expected to be proportional to α_K and this would give to each vertex g_K when couplings are absorbed into vertices.

2. The 3-point function for 3 conformal fields Φ_i, $i=1,2,3$ is dictated by conformal symmetries apart from constant C_{123}:

$$G^{(3)}(z_i,\overline{z}_i) = C_{123}\times\frac{1}{z_{12}^{h_1+h_2-h_3}z_{23}^{h_2+h_3-h_1}z_{31}^{h_3+h_1-h_2}}\times\frac{1}{\overline{z}_{12}^{\overline{h}_1+\overline{h}_2-\overline{h}_3}\overline{z}_{23}^{\overline{h}_2+\overline{h}_3-\overline{h}_1}\overline{z}_{31}^{\overline{h}_3+\overline{h}_1-\overline{h}_2}} \ . \tag{5.2}$$

Here C_{123} should bef fixed by super-symplectic and related symmetries and determined the numerical coefficients various couplings when expressed in terms of g_K.

3. 4-point functions have analogous form

$$\begin{aligned}
G^{(4)}(z_i,\overline{z}_i) &= f_{1234}(x,\overline{x})\prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3}\prod_{i<j}\overline{z}_{ij}^{-(\overline{h}_i+\overline{h}_j)+\overline{h}/3} \ , \\
h &= \sum_i h_i \ ,
\end{aligned} \tag{5.3}$$

but are proportional to an arbitrary function f_{1234} of conformal invariant $x = z_{12}z_{34}/z_{13}z_{24}$ and its conjugate.

If only 3-vertices appear/are needed for physical particles - as both twistorial and number theoretic approaches strongly suggest - the conformal propagators and vertices are fixed apart from constants C_{ijk}, which in turn should be fixed by the huge generalization of conformal symmetries. α_K emerges in the expected manner.

This picture seems to follow from first principles.

1. One can fix the partonic 2-surfaces at the boundaries of CD but there is a functional integral over partonic 2-surfaces defining the vertices: their deformations induce deformations of the legs. One can expand the exponent of Kähler action and in the lowest order the perturbation term is trilinear and non-local in the perturbations. This gives rise to 3-point function of CFT nonlocal in z_i. The functional integral over perturbations gives the propagators in legs proportional to α_K in terms of two point function of CFT. Note that the external propagator legs can be eliminated in S-matrix.

2. The cancellation of higher order perturbative corrections in WCW functional integral is required by the quantum criticality and means trivial coupling constant evolution for α_K and other coupling constants. Coupling constant evolution is discretized with values of α_K analogous to critical temperatures and should correspond to p-adic coupling constant evolution [L1].

3. This picture leaves a lot of details open. An integration over the values of z_i is needed and means a kind of Fourier analysis leading from complex domain. The analog of Fourier analysis would be for deformations of partonic 2-surface labelled by some natural labels. Conformal weights could be natural labels of this kind.

It is easy to get confused since there are several diagrammatics involved: the topological diagrammatics of 3-surfacse assignable to the physical particles with partonic 2-surfaces as vertices, the diagrammatics associated with the perturbative functional integral for the Kähler action, and the fermionic diagrammatics suggested to reduce to tensor network. The conjectures are as follows.

1. The "primary" vertices $G^{(n)}$, $n > 3$ assignable to single partonic 2-surface and coming from a functional integral for Kähler action vanishes. This corresponds to quantum criticality and trivial RG evolution.

2. $G^{(n)}$, $n > 3$ in the sense of topological diagrammatics without loops and involving n partonic 2-surfaces do not vanish. One can construct the analog of $G^{(4)}$ from two $G^{(3)}$:s at different partonic 2-surfaces and propagator defined by 2-point function connecting them as string diagram.

   Also topological variant of $G^{(4)}$ assignable to single partonic 2-surface can be constructed by allowing the 3-D propagator "line" to return back to the partonic 2-surface. This would correspond to an analog of loop. Similar construction applies to "primary" $G^{(n)}$,$n > 4$. In number theoretic vision these loops are eliminated as redundant representations so that one has only braided tree diagrams. Also twistor Grassmann approach supports this view.

To sum up, the tensor network description would apply to fermionic degrees of freedom. In bosonic degrees of freedom functional integral would give CFT picture with 3-vertex as the only "primary" vertex and from this twistorial and number theoretic visions follow via the super-symplectic symmetries of the vertex coefficients C_{ijk} extended to Yangian symmetries.

REFERENCES

Mathematics

[A1] Goncharov A. Volumes of hyperbolic manifolds and mixed Tate motives. http://arxiv.org/abs/alg-geom/9601021, 1996.

[A2] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc.* Available at: http://tinyurl.com/pb8zpqo, 8(43):133–151, 1981.

Books related to TGD

[K1] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#class, 2006.

[K2] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#fermizeta, 2006.

[K3] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#nmpc, 2006.

[K4] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cosmo, 2006.

[K5] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory, 2006.

[K6] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

[K7] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#infmotives, 2011.

[K8] Pitkänen M. What are the counterparts of Einstein's equations in TGD? In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#EinsteinTGD, 2013.

[K9] Pitkänen M. Holography and Quantum Error Correcting Codes: TGD View. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#tensornet, 2014.

[K10] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#wcwnew, 2014.

[K11] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#numbervision, 2014.

[K12] Pitkänen M. From Principles to Diagrams. In *Towards M-Matrix*. Online book.Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#diagrams, 2016.

[K13] Pitkänen M. Questions related to the twistor lift of TGD. In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistquestions, 2016.

Articles about TGD

[L1] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? . Available at: http://tgdtheory.fi/public_html/articles/fermizeta.pdf, 2015.

[L2] Pitkänen M. Is Non-Associative Physics and Language Possible Only in Many-Sheeted Space-time? . Available at: http://tgdtheory.fi/public_html/articles/braidparse.pdf, 2015.

[L3] Pitkänen M. Why the non-trivial zeros of Riemann zeta should reside at critical line? . Available at: http://tgdtheory.fi/public_html/articles/rhagain.pdf, 2015.