Zero Energy Ontology and Matrices

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Abstract

During years the basic mathematical and conceptual building bricks of quantum TGD have become rather obvious.

One important building brick is Zero Energy Ontology (ZEO). ZEO forces to generalize the notion of S-matrix by introducing M-matrices and U-matrix and allows a new view about observer based on TGD inspired theory of consciousness.

Second building brick consists of various hierarchies and connections between them. There is the hierarchy of quantum criticalities for super-symplectic algebra and its Yangian extension acting as a spectrum generating algebra. This hierarchy is closely related to the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchies of criticalities correspond also to fractal hierarchies of breakings of super-symplectic gauge conformal symmetry: only the sub-algebra isomorphic to the original gauge algebra acts as gauge algebra after the breaking. At each step one criticality is reduced and the number of physical degrees of freedom increases. There is also a natural connection between these hierarchies with hierarchies of hyperfinite factors of type II_1 (HFFs) and their inclusions providing a description for the notion of measurement resolution. Also the construction of zero energy states using super-symplectic Yangian provides a concrete realization for the notion of finite measurement resolution in the structure of zero energy states and manifesting in the structure of space-time surfaces serving as classical correlates of quantum states.

There are also other important building bricks but in this chapter only ZEO and hyperfinite factors are discussed.

1 Introduction

Zero energy ontology has become gradually one of the corner stones of quantum TGD. Quantum criticality has been the key idea from beginning but its understanding has grown rather slowly. Now it can be understood in terms of several hierarchies: hierarchy of Planck constants, hierarchy of breakings of super-symplectic symmetry represented as gauge symmetry, hierarchy of CDs, even hierarchy of conscious entities. Hyperfinite factors of type II_1 are highly suggestive candidates for the mathematical realization of these hierarchies. This motivate the discussion of ZEO and HFFs in the same chapter. Only general identifications for M and U matrices generalizing S-matrix to TGD framework are given but concrete proposals are left to later chapters.

1.1 Zero Energy Ontology And Interpretation Of Light-Like 3-Surfaces As Generalized Feynman Diagrams

1. Zero energy ontology (ZEO) is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Either space-like 3-surfaces at the boundaries of CDs or light-like 3-surfaces connecting the boundaries of CDs can be seen as the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

3. String word sheets and partonic 2-surfaces however appear also in TGD as carriers of spinor modes: this follows from the condition that em charge is well defined for the modes. The

condition follows also from number theoretic arguments and is assumed quite generally. This has far reaching consequences for the understanding of gravitation in TGD framework and profound deviations from string models are predicted due to the hierarchy of Planck constants absolutely essential for the description of gravitational bound states in terms of strings connecting partonic 2-surfaces. Macroscopic quantum coherence in even astrophysical scales is predicted [K19, K20].

1.2 Identification Of The Counterpart Of M-Matrix As Time-Like Entanglement Coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix- call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several arguments suggesting that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics or at least formalism resembling thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

1.3 Topics Of The Chapter

The goal is to provide some conceptual background for the attempts to identify scattering amplitudes in TGD framework.

First the basic ideas and implications of ZEO are described. I will represent motivations for ZEO in TGD framework, compare ZEO with the positive energy ontology, and try to make clear the implications of ZEO for quantum measurement theory since they relate also directly to the notion of conscious observer as it is understood in TGD inspired theory of consciousness. After that the definitions of M-matrix and U-matrix are discussed.

The notion of hyper-finite factor expected to play central role in the mathematical description of finite measurement resolution, in the realization of the hierarchy of Planck constants [K8, K19], the hierarchy quantum criticalities, and the hierarchy of gauge symmetry breakings for the super-symplectic algebra. This motivates the discussion of the basic results and ideas are about HFFs. The views about M-matrix as a characterizer of time-like entanglement and M-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of M-matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. A more detailed discussion can be found in the book [K18], in particular in chapter [K15].

I do not pretend of having handle about the huge technical complexities and can only recommend the works of von Neumann [A16, A20, A18, A13]. Tomita [A15]. [B2, B1, B3]. the work of Powers and Araki and Woods which served as starting point for the work of Connes [A5, A4]. The work of Jones [A10], and other leading figures in the field. What is may main contribution is fresh physical interpretation of this mathematics which also helps to make mathematical conjectures. The book of Connes [A5] available in web provides an excellent overall view about von Neumann algebras and non-commutative geometry.

In the last section some general speculations about U-matrix are represented. The negative and positive energy parts of zero energy state can contain zero energy parts in shorter scales - quantum field theorist might talk about quantum fluctuations. One can have also U-matrix and M-matrix elements between this kind of states and even between zero energy states and a hierarchy suggests itself. Since fermions could be seen as correlates of Boolean cognition and zero energy states in fermion sectors as quantal Boolean statements, one can ask whether these matrices could define Boolean hierarchies: statements about statements about...

2 Zero Energy Ontology

Zero energy ontology has changed profoundly the views about the construction of S-matrix and forced to introduce the separate notions of M-matrix and U-matrix. M-matrix generalizes the notion of S-matrix as used in particle physics. The unitary U-matrix is something new having a natural place in TGD inspired theory of consciousness. Therefore it it best to begin the discussion with a brief summary of zero energy ontology.

2.1 Motivations For Zero Energy Ontology

Zero energy ontology was first forced by the finding that the imbeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals. The interpretation is that positive and negative energy parts of states compensate each other so that all quantum states have vanishing net quantum numbers. One can however assign to state quantum numbers as those of the positive energy part of the state. At space-time level zero energy state can be visualized as having positive energy part in geometric past and negative energy part in geometric future. In time scales shorter than the temporal distance between states positive energy ontology works. In longer time scales the state is analogous to a quantum fluctuation.

Zero energy ontology gives rise to a profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

2.2 Zero Energy Ontology

Zero energy ontology (ZEO) is one of the cornerstones of TGD and has become part of TGD during last six years. Zero energy states are identified as superpositions of pairs of positive and negative energy states assigned with the future and past boundaries of causal diamonds (CDs) and correspond in ordinary ontology to physical events with positive and negative energy parts of the state identified as counterparts for the initial and final states of the event. Effective 2-dimensionality allows a further reduction to the level of partonic 2-surfaces: also their 4-D tangent space data matter. Symmetry considerations lead to a beautiful view about generalizations S-matrix to U-matrix in terms o forthogonal M-matrices which in turn are expressible as products of orthogonal basis of hermitian square roots of density matrices and unitary S-matrix [K17]. One can say that quantum theory is “complex” square root of thermodynamics.

Therefore one should try to find tests for ZEO.

2.2.1 The hierarchy of CDs

The basic assumption is that the sizes of CDs come as integer multiples of CP_2 scale R and for prime multiples of R correspond to secondary p-adic length scales $L_{p,2} = L_{p,1}\sqrt{p}$, $L_{p_1} = R\sqrt{p}$, where R denotes CP_2 scale. For electron with $p = M_{127} = 2^{127} - 1$ one has $T_{p_2} = .1$ seconds and defines a fundamental bio-rhythm. This time scale should have preferred role in physics. More generally the secondary p-adic time scales assignable to elementary particles should define time scales relevant to macroscopic physics. The corresponding size scale can be assigned to the magnetic body of the elementary particle. Also it should be possible to assign to quark mass scales special biological time scales as has been indeed done [K2]. h predictions could be tested.

2.2.2 Generalization of standard conservation laws in ZEO

ZEO together with sub-manifold geometry provides a new view about conservation laws and resolves the problem posed by the fact that gravitational interactions do not seem to respect energy conservation in cosmological time scales. Conservation laws holds true only in the scale associated with given CD, not universally (this would allow only single infinitely large CD).

Superconducting coherent states involve quantum superposition of states with different numbers of Cooper pairs and therefore break the super-selection rule associated with fermion number in ordinary ontology. In ZEO they could be understood without giving up the superselection rule associated with fermion number.

Experimental tests should try to prove that quantum number conservation is a length scale dependent notion. For instance, creation of matter from vacuum is possible in ZEO, and one might hope that its occurrenece could be in some scale for CDs aritficially.

2.2.3 Breaking of second law in standard form

In standard physics second law states that all systems are entropic but a system can reduce its entropy by feeding its entropy to the environment. Negentropic entanglement carries genuine information and life can be seen as islands of negentropy in the sea of entropy. This forces to generalized second law. The proposed generalization [L1] [K9] can be characterized as maximally pessimistic.

The generation of negentropic entanglement is assumed to be accompanied by generation of compensating entropic entanglement. The modified form of second law is suggested by the mechanism of directed attention based on negentropic entanglement assignable to magnetic flux tube connecting self and and target. Negentropic entanglement prevails during the attention but disappears after state function reduction giving rise to entropy at the level of ensemble. Second law would hold true above time scale assignab le to the duration of negentropic entanglement.

There are also other reasons to reconsider second law. The breaking of second law in standard form since the arrow of geometric time can change locally. Living systems are indeed accompanied by syntropic effects as realized by Italian quantum physicist Fantappie [J2, J3]. These effects could be understood as entropic effects but with a reversed arrow of geometric time. The mechanism would be based on negative energy signals. Phase conjugate laser waves are known to obey second law in reversed direction of geometric time. Cooling effects due to the absorption of negative energy signals inducing the breaking of the standard form of the second law are predicted to be possible. One can also imagine a spontaneous excitation of atoms generating radiation in the return to ground state in a situation when there is a target able to receive negative energy signals emitted in spontaneous excitation.

Standard form of second law assumes that quantum coherence is absent in the scales in which it is applied. Both the hierarchy of Planck constants and negentropic entanglement however make possible macroscopic quantum coherence characterized by the scale involved and the natural guess is that the time scale associated with causal diamond in question defines the scale above which one can expect second law to hold. There is evidence for the breaking of second law in time scale of.1 seconds [?].

2.2.4 Negative energy signals

Zero energy ontology allows to assign to zero energy states an arrow of time naturally since one can require that states have well defined single particle quantum numbers at either upper or lower boundary of CD. Also the spontaneous change of the arrow of geometric time is possible. The simplest possible description for U-process is that U-matrix relates to each other these two kinds of states and state function reductions can occur at upper and lower boundaries of CD meaning reduction to single particle states with well defined quantum numbers. The precise correlates for the generation of geometric arrow of time are not completely understood.

Negative energy signals to geometric past would serve as counterparts for time reversed states in the case of radiation and phase conjugate laser waves are natural counterparts for them. The signal property requires a dissipative process proceeding in preferred time direction and this kind of process has been assigned to sub-CDs and should proceed as state function reduction sequence in preferred direction of time determined by the quantum arrow of time for the zero energy state.

This process would be essential for the experience of flow of time in preferred direction and for generation of arrow of geometric time as explain in previous chapter and also in [K1]. For phase conjugate laser beams the reversed time direction for dissipation is observed.

Negative energy signals make possible remote metabolism as sucking of energy from remote energy source provided resonance conditions for transitions are satisfied. The counterpart of population inverted laser could serve as ideal source and the the negative energy signal could serve as a control switch inducing phase transition like process taking the excited atom like systems to ground state (induce emission). This process should occur in living matter. Anomalous excitation of atomic state by absorbing energy by remote metabolism and subsequent generation of radiation could also serve as a signature. It could also lead to cooling effects breaking second law.

Negative energy signals would also make possible realization of intentional action by initiating the activity already in geometric past. This would be very desirable in rapidly changing circumstances. The time anomalies of Libet for active aspect of consciousness could be interpreted in terms of time mirror mechanism [J1] and further experiments in longer time scales might be perhaps carried out.

Negative energy signals could be also essential for the mechanism of long term memory. They would induce a breakdown for a system analogous to population reversed laser via induced emission meaning generation of strong positive energy signal [K10].

2.2.5 Definition of energy in zero energy ontology

The approach relying on the two super conformal structures of quantum TGD gives hopes of defining the notion of energy for positive and negative energy parts of the state.

1. CD allows translational invariance only in its interior and since partonic two surfaces are located to the boundary of CD, one can argue that translations assigned to them lead out from CD. One can however argue that if it is enough to assign eigenstates of four-momentum to partons and require that only the total four-momentum generators acts on the physical state by shifting CD. Since total four-momentum vanishes for CD this would mean that wave function in cm degrees of CD is just constant plane wave. Super-conformal invariance would indeed allow to assign momentum eigenstates to the super-conformal representations.

2. A more stringent condition would be that four-momentum generators act as translation like operators on partons themselves. Since light-like 3-surfaces assignable to incoming and outgoing legs of the generalized Feynman diagrams are the basic objects, one can hope of having enough translational invariance to define the notion of energy. If translations are restricted to time-like translations acting in the direction of the future (past) then one has local translation invariance of dynamics for classical field equations inside $\delta M^4_\pm$ as a kind of semigroup. Also the M^4 translations leading to interior of X^4 from the light-like 2-surfaces surfaces act as translations. Classically these restrictions correspond to non-tachyonic momenta defining the allowed directions of translations realizable as particle motions. These two kinds of translations can be assigned to super-symplectic conformal symmetries at $\delta M^4_\pm \times CP_2$ and and super Super-Kac-Moody type conformal symmetries acting as super-symplectic isometries. Super-symplectic algebra is realized in terms of second quantized spinor fields and covariantly constant modes of right-handed neutrino. Symplectic group has as sub-group symplectic isometries and the Super-Kac-Moody algebra associated with this group and represented in terms of spinor modes localized to string world sheets plays also a key role in TGD.

   Finite M^4 translations to the interior of CD do not respect the shape of the partonic 2-surface. Local M^4 translations vanishing at the boundary of CD however act as Kac-Moody symmetries of the light-like 3-surfaces and reduce physically to gauge transformations: hence one could allow also the deformations of the partonic 2-surface in the interior of the light-like 3-surface. This corresponds to the effective metric 2-dimensionality stating that all information both about the geometry of WCW and quantum physics is carried by the partonic 2-surfaces X^2 resulting as intersections of the light-like 3-surfaces X^3_l and space-like 3-D surfaces X^3 at the boundaries of CD and the distribution of 4-D tangent planes of X^2.

3. The condition selecting preferred extremals of Kähler action is induced by a global selection of $M^2 \subset M^4$ as a plane belonging to the tangent space of X^4 at all its points [K5] and interpreted as a plane of nonphysical polarizations so that direct connection with number theory and gauge symmetries emerges. The M^4 translations of X^4 as a whole in general respect the form of this condition in the interior. Furthermore, if M^4 translations are restricted to M^2, also the condition itself - rather than only its general form - is respected. This observation, the earlier experience with p-adic mass calculations, and also the treatment of quarks and gluons in QCD encourage to consider the possibility that translational invariance should be restricted to M^2 translations so that mass squared, longitudinal momentum and transversal mass squared would be well defined quantum numbers. This would be enough to realize zero energy ontology. Encouragingly, M^2 appears also in the generalization of the causal diamond to a book-like structure forced by the realization of the hierarchy of Planck constant at the level of the imbedding space.

4. That the cm degrees of freedom for CD would be gauge like degrees of freedom sounds strange. The paradoxical feeling disappears as one realizes that this is not the case for sub-CDs, which indeed can have non-trivial correlation functions with either upper or lower tip of the CD playing a role analogous to that of an argument of n-point function in QFT description. One can also say that largest CD in the hierarchy defines infrared cutoff.

2.2.6 Objection against zero energy ontology and quantum classical correspondence

The motivation for requiring geometry and topology of space-time as correlates for quantum states is the belief that quantum measurement theory requires the representability of the outcome of quantum measurement in terms of classical physics -and if one believes in geometrization- one ends up with generalization of Einstein's vision.

There is however a counter argument against this view and second one against zero energy ontology in which one assigns eigenstates of four-momentum with causal diamonds (CDs).

1. One can argue that momentum eigenstates for which particle regarded as a topological inhomogenuity of space-time surface, which is non-localized cannot allow a space-time correlate.

2. Even worse, CDs have finite size so that strict four-momentum eigenstates strictly are not possible.

On the other hand, the paradoxical fact is that we are able to perceive momentum eigenstates and they look localized to us. This cannot be understood in the framework of standard Poincare symmetry.

The resolution of the objections and of the apparent paradox could rely on conformal symmetry assignable to light-like 3-surfaces implying a generalization of Poincare symmetry and other symmetries with their Kac-Moody variants for which symmetry transformations become local.

1. Poincare group is replaced by its Kac-Moody variant so that all non-constant translations act as gauge symmetries. Translations which are constant in the interior of CD and trivial at the boundaries of CDs are physically equivalent with constant translations. Hence the latter objection can be circumvented.

2. The same argument allows also a localization of momentum eigenstates at the boundaries of CD. In the interior the state is non-local. Classically the momentum eigenstate assigned with the partonic 2-surface is characterized by its 4-D tangent space data coding for momentum classically. The Kähler-Dirac equation and Kähhler action indeed contain and additional term representing coupling to four-momenta of particles. Formally this corresponds only to a gauge transform linear in momentum but Kahler gauge potential has U(1) gauge symmetry only as a spin glass like degeneracy, not as a gauge symmetry so that space-time surface depends on momenta.

3. Conscious observer corresponds in TGD inspired theory of consciousness to CD and the sensory data of the observer come from partonic 2-surfaces at the boundaries of CD and its sub-CDs. This implies classicality of sensory experience and momentum eigenstates look classical for conscious perceiver.

The usual argument resolving the paradox is based on the notion of wave packet and also this notion could be involved. The notion of finite measurement resolution is key notion of TGD and it is quite possible that one can require the localization of momentum eigenstates at the boundaries of CDs only modulo finite measurement resolution for the position of the partonic 2-surfaces.

2.3 The Anatomy Of Quantum Jump In Zero Energy Ontology (ZEO)

Zero energy ontology (ZEO) emerged around 2005 and has had profound consequences for the understanding of quantum TGD. The basic implication is that state function reductions occur at the opposite light-like boundaries of causal diamonds (CDs) forming a hierarchy, and produce zero energy states with opposite arrows of time. Also concerning the identification of quantum jump as moment of consciousness ZEO encourages rather far reaching conclusions. In ZEO the only difference between motor action and sensory representations is that the arrows of imbedding space time (CDs) are opposite for them. Furthermore, sensory perception followed by motor action corresponds to a basic structure in the sequence of state function reductions and it seems that these processes occur fractally for CDs of various size scales.

1. State function reduction can be performed to either boundary of CD but not both simultaneously. State function reduction at either boundary is equivalent to state preparation giving rise to a state with well defined quantum numbers (particle numbers, charges, four-momentum, etc...) at this boundary of CD. At the other boundary single particle quantum numbers are not well defined although total conserved quantum numbers at boundaries are opposite by the zero energy property for every pair of positive and negative energy states in the superposition. State pairs with different total energy, fermion number, etc.. for other boundary are possible: for instance, the coherent states of super-conductor for which fermion number is ill defined are possible in zero energy ontology and do not break the super-selection rules.

2. The basic objects coding for physics are U-matrix, M-matrices and S-matrix. M-matrices correspond to hermitian square roots of density matrices multiplied by a universal S-matrix which depends on the scale n of CD in very simple manner: $S(n) = S^n$ giving thus a unitary representation for scalings. The explicit construction of a unitary U-matrix in terms of M-matrices is carried out in [K17]: U-matrix elements are essentially inner products of M-matrices associated with CDs with various size scales. One can say that quantum theory is formally a square root of thermodynamics. The thermodynamics in question would however relate more naturally to NMP rather than second law, which at ensemble level and for ordinary entanglement can be seen as a consequence of NMP.

   The non-triviality of M-matrix requires that for given state reduced at say the "lower" boundary of CD there is entire distribution of states at "upper boundary" (given initial state can lead to a continuum of final states). Even more, all size scales of CDs are possible since the position of only the "lower" boundary of CD is localized in quantum jump whereas the location of upper boundary of CD can vary so that one has distribution over CDs with different size scales and over their Lorentz boots and translates.

3. The quantum arrow of time follows from the asymmetry between positive and negative energy parts of the state: the other is prepared and the other corresponds to the superposition of the final states resulting when interactions are turned on: also quantum superposition over CDs of different sizes with second boundary belonging to the same fixed $\delta M^4_{\pm}$ is possible. What is remarkable that the arrow of time at imbedding space level (at least) changes direction as quantum jump occurs to opposite boundary.

   It is however possible to have sequences of quantum jumps occurring at the same boundary: these periods are counterparts for repeated state function reductions, which do not change the state at all in standard quantum measurement theory. During these periods the superposition of opposite boundaries of CDs and states at them change, and the average distance between the tips of CDs tends to increase, hence the flow of subjective time and its arrow.

   NMP dictates when the first quantum jumps to the opposite boundary of CD takes place. The sequence of state function reduction at the same boundary defines self as a conscious entity and the increase of the average distance between the tips of CD defines the life-time of self.

This brings strongly in mind the old proposal of Fantappie [J2] that in living matter the arrow of time is not fixed and that entropy and its diametric opposite syntropy apply to the two arrows of the imbedding space time. The arrow of subjective time assignable to second law would hold true but the increase of syntropy would be basically a reflection of second law since only the arrow of the geometric time at imbedding space level has changed direction. The arrow of geometric at space-time level which conscious observer experiences directly could be always the same if quantum classical correspondence holds true in the sense that the arrow of time for zero energy states corresponds to arrow of time for preferred extremals. The failure of strict non-determinism making possible phenomena analogous to multi-furcations makes this possible.

4. This picture differs radically from the standard view and if quantum jump represents a fundamental algorithm, this variation of the arrow of geometric time should manifest itself in the functioning of brain and living organisms. The basic building brick in the functioning of brain is the formation of sensory representation followed by motor action/volition realized as the first reduction at the opposite boundary.

   These processes look very much like temporal mirror images of each other such as the state function reductions to opposite boundaries of CD look like. The fundamental process could correspond to a sequences of these two kinds of state function reductions at opposite boundaries of CDs and maybe independently for CDs of different size scales in a "many-particle" state defined by a union of CDs.

How the formation of cognitive and sensory representations could relate to quantum jump?

1. The earlier view was based on the idea that p-adic space-time sheets can transform to real ones and vice versa in quantum jump and these process correspond to a realization of intention as action and formation of though. This view is mathematically awkward and has been replaced with the adelic vision in which all systems have both sensory (real space-time sheets) and cognitive (p-adic space-time sheets) space-time correlates. The real and p-adic number fields form a book like structure - adele- with an algebraic extension of rationals as its back. Same applieds at the level of imbedding space, space-time surfaces, and WCW. In this framedwork holography makes it possible to understand real and p-adic space-time surfaces as continuations of string world sheets and partonic 2-surfaces to space-time surfaces, either real or p-adic. The string world sheets themselves are in the intersection of reality and various p-adicities in the sense that the parameters characterizing them belong to an extension of rational numbers.

2. Self having the mental image about intention can be be seen as the agent transforming intention to action. By NMP negentropy is typically generated in this transition tending to increase the value of Planck constant $h_{eff} = n \times h$ and thus reducing quantum criticality and occurring therefore spontaneously. Negentropy Maximization Principle eventually forces the occurrence of volitional action - self experiences the urge to perform the action so strong that cannot resist. Subself representing the mental image about intention tries to prevent it as long as possible because it means death: all living systems try to stay at the existing level of criticality and avoid the fatal final state function reduction by practicing homeostasis and using metabolic energy. Weak form of NMP states that self has freedom to decide whether it performs the reduction producing maximal entanglement negentropy. It can also perform ordinary quantum jump reducing entanglement entropy to zero and destroying entanglement. The outcome is isolation from the external world. The motivation for the weak form of NMP is that we do not live in the best possible world and have free will to choose between Good and Evil. Strong form of NMP would produce always mazimal negentropy gain and would mean best possible world.ur in various length scales in fractal manner.

2.4 Conscious Entities And Arrow Of Time In TGD Universe

"Fractality from your blog" posed an interesting question about possible asymmetry between boundaries of causal diamond CD. The answer to the question led to recall once again the incomplete understanding of details about how the arrow of time emerges in zero energy ontology (ZEO).

The basic vision is following.

1. CDs form a fractal scale hierarchy. Zero energy states possess a wave function in moduli degrees of freedom characterizing sizes of CDs as well telling what Lorentz boost leaving boundary invariant are allowed for them. Boosts form by number theoretic constraints a discrete subgroup of Lorentz group defining analogs of lattices generated by boosts instead of translations.

2. The arrow of subjective time maps to that of geometric time somehow. The origin of arrow comes from the fact that state function reductions can occur to either boundary of given CD and reduction creates time-asymmetric state since second boundary of CD is in a quantum superposition of different sizes and there is a superposition of many-particle states with different particles numbers and quantum number distributions. It is possible that each state function reduction leaving the passive boundary intact, involves localization in the moduli space of CDs with second boundary fixed.

3. Subjective existence corresponds to a sequence *of moments of consciousness*: state function reductions at opposite boundaries of CDs. State function reduction localizes either boundary but the second boundary is in a quantum superposition of several locations and size scales for CD. This predicts that the arrow of time is not constant. In fact, there is considerable evidence for the variation of the arrow of time in living systems and Fantappie [J2] introduced long time ago the notion of syntropy to describe his view about the situation.

4. The first very naive proposal was that state function reductions occur *alternately* to the two boundaries of CD. This assumption would be indeed natural if one considered single fixed CD rather than superposition CDs with different size and state function reduction localizing their either boundary: restriction to single CD was what I indeed did first.

5. This assumption leads to the question about why do we do not observe this alternation of the arrow of time all the time in our personal experience. Some people actually claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. But why do we experience the arrow of time as stable in the standard state of consciousness?

One possible way to solve the problem - perhaps the simplest one - is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately!

I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs change in these reductions. Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions do not affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time? This option would allow the size scale of CD associated with human consciousness be rather short, say.1 seconds. It would allow to understand why we do not observe continual change of arrow of time.

Maybe living systems are working hardly to keep the personal arrow of time un-changed - living creatures try to prevent kettle from boiling by staring at it intensely. Maybe it would be extremely difficult to live against the collective arrow of time.

An objection against this picture as compared to the original one assuming alternate reductions to the opposite boundaries of CD is that is that one can understand state preparation as state function reduction to the opposite boundary. This interpetation makes sense almost as such also in the new picture if the average time period for which the reductions occur to a given boundary is shorter in elementary particles scales than in macroscopic scales characteristic for human consciousness. The approximate reversibility in elementary particle scales can be understood as summing up of the two arrows of time to no arrow at all.

This picture allows also to identify self as a continuous entity as the sequence of state function reductions occurring at the same boundary of CD. The average increase of the temporal distance between the tips of cD defines the life-time of self. The number of reductions would give a measure for the subjectively experienced of life-time of self.

In elementary particle time scales reversibility is a good approximation and this suggests that in elementary particle scales the number of state function reductions at the same boundary of CD is small so that the effects due to the change of the arrow of time cancel on the average.

NMP would eventually force "death" of self since the state function reduction at opposite boundary would generate more negentropy. "Death" of self would mean birth of self asssociated with the opposite boundary of CD. The age of self identified as the proper time distance between the tips would increase in statistical sense even when its arrow can change. The act of volition would have a natural identification as the first state function reduction at the opposite boundary of CD.

This picture raises a series of questions. Do our wake-up periods correspond to sequences of state function reductions for self and are sleeping periods wake-up periods of the self at the opposite boundary of CD? The arrow of geometric time should change at some space-time sheet associated with the self hierarchy. How could one demonstrate this? Are the memories of the "other" self predictions of future from our point of view? Do we sleep in order to get information from future, to remember what the future will be?

How the hierarchy of Planck constants defining a hierarchy of quantum criticalities does relate to this picture? The ageing of self having has as a correlate the increase of the size scale of CD. Could this increase be due to the increase of h_{eff} expected to occur spontaneously since it corresponds to a reduction of criticality and therefore to the appearance of new physical degrees of freedom as symplectic gauge degrees of freedom transform to physical ones in gauge symmetry breaking. This is not the case. The time evolution must be analogous to shift in time rather than scaling. This of course corresponds to the QFT view about time evolution.

In the first state function reduction to the opposite boundary of CD however scaling of CD is possible and would correspond to the scaling of CD represented by exponent of infinitesimal scaling operator as in conformal field theories. The emergence of bew physical degrees of freedom suggest increasing perceptive and cognitive capabilities. The increase of h_{eff} could be seen as evolution as also the associated increase of resources of negentropic entanglement suggests. The total increase of h_{eff} measured by the ratio $h_{eff}(final)/h_{eff}(initial)$ could be seen as a measure for the progress per single life period of self.

3 A Vision About The Role Of HFFs In TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

3.1 Basic Facts About Factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

3.1.1 *Basic notions*

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A3] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n

correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0, 1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X, μ) and vice versa.

3.1.2 *Weights, states and traces*

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

2. A positive linear functional is weight with $\omega(1)$ finite.

3. A state is a weight with $\omega(1) = 1$.

4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$. For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

3.1.3 *Tomita-Takesaki theory*

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{3.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*^{\ *} = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \quad , J\mathcal{M}J = \mathcal{M}' \quad .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A8, A14] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

3.1.4 *Modular automorphisms*

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

3.1.5 *Crossed product as a manner to construct factors of type III*

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

3.1.6 *Inclusions and Connes tensor product*

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K15] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A1] and those of factors of type III by Alain Connes [A4] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

3.1.7 *Basic findings about inclusions*

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A1] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a)\ \ \mathcal{M}:\mathcal{N} = 4cos^2(\pi/h)\ \ ,\quad h \geq 3\ \ , \\ &b)\ \ \mathcal{M}:\mathcal{N} \geq 4\ \ . \end{aligned} \tag{3.2}$$

the numbers at right hand side are known as Beraha numbers [A11] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B4] , for $\mathcal{M}:\mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M}:\mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A19] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M}:\mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M}:\mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

3.1.8 *Connes tensor product*

The inclusions The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} \ \ = \ \ \mathcal{M}/\mathcal{N} \otimes \mathcal{N}\ \ . \tag{3.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M} \otimes \mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn} W_{nr} e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

3.1.9 *Factors in quantum field theory and thermodynamics*

Factors arise in thermodynamics and in quantum field theories [A17, A8, A14] . There are good arguments showing that in HFFs of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A14] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ \ , \ \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ \ .$$

An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

3.1.10 *Factors in quantum field theory and thermodynamics*

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An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

3.2 TGD And Factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

3.2.1 *The problems*

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

3.2.2 *ZEO and factors*

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.

3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

3.2.3 *Zero modes and factors*

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K16] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two superconformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

3.2.4 *Crossed product construction in TGD framework*

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L^*_0$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between

tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

3.2.5 *Quantum criticality and inclusions of factors*

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K8] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

3.3 Can One Identify M-Matrix From Physical Arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

3.3.1 *A proposal for M-matrix*

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of imbedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K12].

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface the vertices would be represented by partonic 2-surfaces. In [K12] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

   The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K3] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

   The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??** (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K12] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

3.3.2 *Quantum TGD as square root of thermodynamics*

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A17] [K15]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A17], [K15] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

3.3.3 *Quantum criticality and hierarchy of inclusions*

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K8]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h = n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K12]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

3.3.4 *Summarizing*

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

3.4 Finite Measurement Resolution And HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

3.4.1 *About the notion of observable in ZEO*

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton

orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.

5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

3.4.2 *Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix*

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{3.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{3.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M} : \mathcal{N} \times \langle r_1 | Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger}) | r_2 \rangle \quad . \tag{3.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M} : \mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \quad . \tag{3.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

3.4.3 *Quantum M-matrix*

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

3.4.4 *Quantum fluctuations and inclusions*

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

3.4.5 *M-matrix in finite measurement resolution*

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III$_1$.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II$_1$ this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A2] .

5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A2] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

3.4.6 *Is universal M-matrix possible?*

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{3.8}$$

   for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{3.9}$$

   for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

3.4.7 *Connes tensor product and space-like entanglement*

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

3.4.8 *2-vector spaces and entanglement modulo measurement resolution*

John Baez and collaborators [A12] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

3.5 Questions About Quantum Measurement Theory In Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K9, K13, K1].

3.5.1 *Fractal hierarchy of state function reductions*

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

3.5.2 *How quantum classical correspondence is realized at parton level?*

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

3.5.3 *Quantum measurements in ZEO*

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

3.6 Miscellaneous

The following considerations are somewhat out-of-date: hence the title "Miscellaneous".

3.6.1 *Connes tensor product and fusion rules*

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A19] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A6] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A7] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_\mathcal{N} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K4] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

3.6.2 *Connection with topological quantum field theories defined by Chern-Simons action*

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A9] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K14] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K7] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A9] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

4 The Relation Between U-Matrix And M-Matrices

S-matrix is the key notion in quantum field theories. In Zero Energy Ontology (ZEO) this notion must be replaced with the triplet U-matrix, M-matrix, and S-matrix. U-matrix realizes unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed.

This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect) [K9]. In TGD inspired theory of consciousness self corresponds to the sequence of these state function reductions [K13, K1, K11]. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood.

The original view about the relationship was a purely formal guess: M-matrices would define the orthonormal rows of U-matrix. This guess is not correct physically and one must consider in detail what U-matrix really means.

1. First about the geometry of CD [K17]. The boundaries of CD will be called passive and active: passive boundary correspond to the boundary at which repeated state function reductions take place and give rise to a sequence of unitary time evolutions U followed by localization in the moduli of CD each. Active boundary corresponds to the boundary for which U induces delocalization and modifies the states at it.

   The moduli space for the CDs consists of a discrete subgroup of scalings for the size of CD characterized by the proper time distance between the tips and the sub-group of Lorentz boosts leaving passive boundary and its tip invariant and acting on the active boundary only. This group is assumed to be represented unitarily by matrices Λ forming the same group for all values of n.

   The proper time distance between the tips of CDs is quantized as integer multiples of the minimal distance defined by CP_2 time: $T = nT_0$. Also in quantum jump in which the size scale n of CD increases the increase corresponds to integer multiple of T_0. Using the logarithm of proper time, one can interpret this in terms of a scaling parametrized by an integer. The possibility to interpret proper time translation as a scaling is essential for having a manifest Lorentz invariance: the ordinary definition of S-matrix introduces preferred rest system.

2. The physical interpretation would be roughly as follows. M-matrix for a given CD codes for the physics as we usually understand it. M-matrix is product of square root of density matrix and S-matrix depending on the size scale of CD and is the analog of thermal S-matrix. State function at the opposite boundary of CD corresponds to what happens in the state function reduction in particle physics experiments. The repeated state function reductions at same boundary of CD correspond to TGD version of Zeno effect crucial for understanding consciousness. Unitary U-matrix describes the time evolution zero energy states due to the increase of the size scale of CD (at least in statistical sense). This process is dispersion in the moduli space of CDs: all possible scalings are allowed and localization in the space of moduli of CD localizes the active boundary of CD after each unitary evolution.

In the following I will proceed by making questions. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_1 of the Virasoro algebra associated with the super-symplectic algebra.

4.1 What One Can Say About M-Matrices?

1. The first thing to be kept in mind is that M-matrices act in the space of zero energy states rather than in the space of positive or negative energy states. For a given CD M-matrices are products of hermitian square roots of hermitian density matrices acting in the space of zero energy states and universal unitary S-matrix $S(CD)$ acting on states at the active end of CD (this is also very important to notice) depending on the scale of CD:

$$M^i = H^i \circ S(CD) \ .$$

Here "$\circ$" emphasizes the fact that S acts on zero energy states at active boundary only. H^i is hermitian square root of density matrix and the matrices H^i must be orthogonal for given CD from the orthonormality of zero energy states associated with the same CD. The zero energy states associated with different CDs are not orthogonal and this makes the unitary time evolution operator U non-trivial.

2. Could quantum measurement be seen as a measurement of the observables defined by the Hermitian generators H^i? This is not quite clear since their action is on zero energy states. One might actually argue that the action of this kind of observables on zero energy states does not affect their vanishing net quantum numbers. This suggests that H^i carry no net quantum numbers and belong to the Cartan algebra. The action of S is restricted at the active boundary of CD and therefore it does not commute with H^i unless the action is in a separate tensor factor. Therefore the idea that S would be an exponential of generators H^i and thus commute with them so that H^i would correspond to sub-spaces remaining invariant under S acting unitarily inside them does not make sense.

3. In TGD framework symplectic algebra actings as isometries of WCW is analogous to a Kac-Moody algebra with finite-dimensional Lie-algebra replaced with the infinite-dimensional symplectic algebra with elements characterized by conformal weights [K6, K5]. There is a temptation to think that the H^i could be seen as a representation for this algebra or its sub-algebra. This algebra allows an infinite fractal hierarchy of sub-algebras of the super-symplectic algebra isomorphic to the full algebra and with conformal weights coming as n-ples of those for the full algebra. In the proposed realization of quantum criticality the elements of the sub-algebra characterized by n act as a gauge algebra. An interesting question is whether this sub-algebra is involved with the realization of M-matrices for CD with size scale n. The natural expectation is that n defines a cutoff for conformal weights relating to finite measurement resolution.

4.2 How Does The Size Scale Of CD Affect M-Matrices?

1. In standard quantum field theory (QFT) S-matrix represents time translation. The obvious generalization is that now scaling characterized by integer n is represented by a unitary S-matrix that is as n:th power of some unitary matrix S assignable to a CD with minimal size: $S(CD) = S^n$. $S(CD)$ is a discrete analog of the ordinary unitary time evolution operator with n replacing the continuous time parameter.

2. One can see M-matrices also as a generalization of Kac-Moody type algebra. Also this suggests $S(CD) = S^n$, where S is the S-matrix associated with the minimal CD. S becomes representative of phase $exp(i\phi)$. The inner product between CDs of different size scales can n_1 and n_2 can be defined as

$$
\begin{aligned}
&\langle M^i(m), M^j(n)\rangle = Tr(S^{-m} \circ H^i H^j \circ S^n) \times \theta(n-m) \ , \\
&\theta(n) = 1 \ \text{ for } \ n \geq 0 \ , \ \theta(n) = 0 \ \text{ for } \ n < 0 \ .
\end{aligned}
\tag{4.1}
$$

   Here I have denoted the action of S-matrix at the active end of CD by "$\circ$" in order to distinguish it from the action of matrices on zero energy states which could be seen as belonging to the tensor product of states at active and passive boundary.

   It turns out that unitarity conditions for U-matrix are invariant under the translations of n if one assumes that the transitions obey strict arrow of time expressed by $n_j - n_i \geq 0$. This simplifies dramatically unitarity conditions. This gives orthonormality for M-matrices associated with identical CDs. This inner product could be used to identify U-matrix.

3. How do the discrete Lorentz boosts affecting the moduli for CD with a fixed passive boundary affect the M-matrices? The natural assumption is that the discrete Lorentz group is represented by unitary matrices λ: the matrices M^i are transformed to $M^i \circ \lambda$ for a given Lorentz boost acting on states at active boundary only.

   One cannot completely exclude the possibility that S acts unitarily at both ends of zero energy states. In this case the scaling would be interpreted as acting on zero energy states rather than those at active boundary only. The zero energy state basis defined by M_i would depend on the size scale of CD in more complex manner. This would not affect the above formulas except by dropping away the "$\circ$".

Unitary U must characterize the transitions in which the moduli of the active boundary of causal diamond (CD) change and also states at the active boundary (paired with unchanging states at the passive boundary) change. The arrow of the experienced flow of time emerges during the period as state function reductions take place to the fixed ("passive") boundary of CD and do not affect the states at it. Note that these states form correlated pairs with the changing states at the active boundary. The physically motivated question is whether the arrow of time emerges statistically from the fact that the size of CD tends to increase in average sense in repeated state function reductions or whether the arrow of geometric time is strict. It turns out that unitarity conditions simplify dramatically if the arrow of time is strict.

4.3 What Can One Say About U-Matrix?

1. Just from the basic definitions the elements of a unitary matrix, the elements of U are between zero energy states (M-matrices) between two CDs with possibly different moduli of the active boundary. Given matrix element of U should be proportional to an inner product of two M-matrices associated with these CDs. The obvious guess is as the inner product between M-matrices

$$
\begin{aligned}
U^{ij}_{m,n} &= \langle M^i(m,\lambda_1), M^j(n,\lambda_2)\rangle \\
&= Tr(\lambda_1^\dagger S^{-m} \circ H^i H^j \circ S^n \lambda_2) \\
&= Tr(S^{-m} \circ H^i H^j \circ S^n \lambda_2 \lambda_1^{-1})\theta(n-m) \ .
\end{aligned}
\tag{4.2}
$$

Here the usual properties of the trace are assumed. The justification is that the operators acting at the active boundary of CD are special case of operators acting non-trivially at both boundaries.

2. Unitarity conditions must be satisfied. These conditions relate S and the hermitian generators H^i serving as square roots of density matrices. Unitarity conditions $UU^\dagger = U^\dagger U = 1$ is defined in the space of zero energy states and read as

$$\sum_{j_1 n_1} U^{ij_1}_{mn_1} (U^\dagger)^{j_1 j}_{n_1 n} = \delta^{i,j}\delta_{m,n}\delta_{\lambda_1,\lambda_2} \tag{4.3}$$

To simplify the situation let us make the plausible hypothesis contribution of Lorentz boosts in unitary conditions is trivial by the unitarity of the representation of discrete boosts and the independence on n.

3. In the remaining degrees of freedom one would have

$$\sum_{j_1, k\geq Max(0,n-m)} Tr(S^k \circ H^i H^{j_1}) Tr(H^{j_1} H^j \circ S^{n-m-k}) = \delta^{i,j}\delta_{m,n} \quad . \tag{4.4}$$

The condition $k \geq Max(0, n-m)$ reflects the assumption about a strict arrow of time and implies that unitarity conditions are invariant under the proper time translation $(n,m) \rightarrow (n+r, m+r)$. Without this condition n back-wards translations (or rather scalings) to the direction of geometric past would be possible for CDs of size scale n and this would break the translational invariance and it would be very difficult to see how unitarity could be achieved. Stating it in a general manner: time translations act as semigroup rather than group.

4. Irreversibility reduces dramatically the number of the conditions. Despite this their number is infinite and correlates the Hermitian basis and the unitary matrix S. There is an obvious analogy with a Kac-Moody algebra at circle with S replacing the phase factor $exp(in\phi)$ and H^i replacing the finite-dimensional Lie-algebra. The conditions could be seen as analogs for the orthogonality conditions for the inner product. The unitarity condition for the analog situation would involve phases $exp(ik\phi_1) \leftrightarrow S^k$ and $exp(i(n-m-k)\phi_2) \leftrightarrow S^{n-m-k}$ and trace would correspond to integration $\int d\phi_1$ over ϕ_1 in accordance with the basic idea of non-commutative geometry that trace corresponds to integral. The integration of ϕ_i would give $\delta_{k,0}$ and $\delta_{m,n}$. Hence there are hopes that the conditions might be satisfied. There is however a clear distinction to the Kac-Moody case since S^n does not in general act in the orthogonal complement of the space spanned by H^i.

5. The idea about reduction of the action of S to a phase multiplication is highly attractive and one could consider the possibility that the basis of H^i can be chosen in such a manner that H^i are eigenstates of of S. This would reduce the unitarity constraint to a form in which the summation over k can be separated from the summation over j_1.

$$\sum_{k\geq Max(0,n-m)} exp(iks_i - (n-m-k)s_j) \sum_{j_1} Tr(H^i H^{j_1}) Tr(H^{j_1} H^j) = \delta^{i,j}\delta_{m,n} \quad . \tag{4.5}$$

The summation over k should gives a factor proportional to δ_{s_i,s_j}. If the correspondence between H^i and eigenvalues s_i is one-to-one, one obtains something proportional to $\delta(i,j)$ apart from a normalization factor. Using the orthonormality $Tr(H^i H^j) = \delta^{i,j}$ one obtains for the left hand side of the unitarity condition

$$exp(is_i(n-m)) \sum_{j_1} Tr(H^i H^{j_1}) Tr(H^{j_1} H^j) = exp(is_i(n-m))\delta_{i,j} \quad . \tag{4.6}$$

Clearly, the phase factor $exp(is_i(n-m))$ is the problem. One should have Kronecker delta $\delta_{m,n}$ instead. One should obtain behavior resembling Kac-Moody generators. H^i should be analogs of Kac-Moody generators and include the analog of a phase factor coming visible by the action of S.

4.4 How To Obtain Unitarity Correctly?

It seems that the simple picture is not quite correct yet. One should obtain somehow an integration over angle in order to obtain Kronecker delta.

1. A generalization based on replacement of real numbers with function field on circle suggests itself. The idea is to the identify eigenvalues of generalized Hermitian/unitary operators as Hermitian/unitary operators with a spectrum of eigenvalues, which can be continuous. In the recent case S would have as eigenvalues functions $\lambda_i(\phi) = exp(is_i\phi)$. For a discretized version ϕ would have has discrete spectrum $\phi(n) = 2\pi k/n$. The spectrum of λ_i would have n as cutoff. Trace operation would include integration over ϕ and one would have analogs of Kac-Moody generators on circle.

2. One possible interpretation for ϕ is as an angle parameter associated with a fermionic string connecting partonic 2-surface. For the super-symplectic generators suitable normalized radial light-like coordinate r_M of the light-cone boundary (containing boundary of CD) would be the counterpart of angle variable if periodic boundary conditions are assumed.

   The eigenvalues could have interpretation as analogs of conformal weights. Usually conformal weights are real and integer valued and in this case it is necessary to have generalization of the notion of eigenvalues since otherwise the exponentials $exp(is_i)$ would be trivial. In the case of super-symplectic algebra I have proposed that the generating elements of the algebra have conformal weights given by the zeros of Riemann zeta. The spectrum of conformal weights for the generators would consist of linear combinations of the zeros of zeta with integer coefficients. The imaginary parts of the conformal weights could appear as eigenvalues of S.

3. It is best to return to the definition of the U-matrix element to check whether the trace operation appearing in it can already contain the angle integration. If one includes to the trace operation appearing the integration over ϕ it gives $\delta_{m,n}$ factor and U-matrix has elements only between states assignable to the same causal diamond. Hence one must interpret U-matrix elements as functions of ϕ realized factors $exp(i(s_n - s_m)\phi)$. This brings strongly in mind operators defined as distributions of operators on line encountered in the theory of representations of non-compact groups such as Lorentz group. In fact, the unitary representations of discrete Lorentz groups are involved now.

4. The unitarity condition contains besides the trace also the integrations over the two angle parameters ϕ_i associated with the two U-matrix elements involved. The left hand side of the unitarity condition reads as

$$\begin{aligned} \sum_{k\geq Max(0,n-m)} & I(ks_i)I((n-m-k)s_j) \times \sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j) \\ & = \delta^{i,j}\delta_{m,n} \quad , \\ I(s) = \frac{1}{2\pi} \times \int d\phi exp(is\phi) = \delta_{s,0} \quad . & \end{aligned} \tag{4.7}$$

   Integrations give the factor $\delta_{k,0}$ eliminating the infinite sum obtained otherwise plus the factor $\delta_{n,m}$. Traces give Kronecker deltas since the projectors are orthonormal. The left hand side equals to the right hand side and one achieves unitarity. It seems that the proposed ansatz works and the U-matrix can be reduced by a general ansatz to S-matrix.

5. It should be made clear that the use of eigenstates of S is only a technical trick, the physical states need not be eigenstates. If the active parts of zero energy states where eigenstates of S, U-matrix would not have matrix elements between different H^i and projection operator could not change during time evolution.

4.5 What About The Identification Of S?

1. S should be exponential of time the scaling operator whose action reduces to a time translation operator along the time axis connecting the tips of CD and realized as scaling. In other words, the shift $t/T_0 = m \to m+n$ corresponds to a scaling $t/T_0 = m \to km$ giving $m+n = km$ in turn giving $k = 1+n/m$. At the limit of large shifts one obtains $k \simeq n/m \to \infty$, which corresponds to QFT limit. nS corresponds to $(nT_0) \times (S/T_0) = TH$ and one can ask whether QFT Hamiltonian could corresponds to $H = S/T_0$.

2. It is natural to assume that the operators H^i are eigenstates of radial scaling generator $L_0 = ir_M d/dr_M$ at both boundaries of CD and have thus well-defined conformal weights. As noticed the spectrum for super-symplectic algebra could also be given in terms of zeros of Riemann zeta.

3. The boundaries of CD are given by the equations $r_M = m^0$ and $r_M = T - m_0$, m_0 is Minkowski time coordinate along the line between the tips of CD and T is the distance between the tips. From the relationship between r_M and m_0 the action of the infinitesimal translation $H \equiv i\partial/\partial_{m^0}$ can be expressed as conformal generator $L_{-1} = i\partial/\partial_{rM} = r_M^{-1}L_0$. Hence the action is non-diagonal in the eigenbasis of L_0 and multiplies with the conformal weights and reduces the conformal weight by one unit. Hence the action of U can change the projection operator. For large values of conformal weight the action is classically near to that of L_0: multiplication by L_0 plus small relative change of conformal weight.

4. Could the spectrum of H be identified as energy spectrum expressible in terms of zeros of zeta defining a good candidate for the super-symplectic radial conformal weights. This certainly means maximal complexity since the number of generators of the conformal algebra would be infinite. This identification might make sense in chaotic or critical systems. The functions $(r_M/r_0)^{1/2+iy}$ and $(r_M/r_0)^{-2n}$, $n > 0$, are eigenmodes of r_M/dr_M with eigenvalues $(1/2+iy)$ and $-2n$ corresponding to non-trivial and trivial zeros of zeta.

   There are two options to consider. Either L_0 or iL_0 could be realized as a hermitian operator. These options would correspond to the identification of mass squared operator as L_0 and approximation identification of Hamiltonian as iL_1 as iL_0 making sense for large conformal weights.

   (a) Suppose that $L_0 = r_M d/dr_M$ realized as a hermitian operator would give harmonic oscillator spectrum for conformal confinement. In p-adic mass calculations the string model mass formula implies that L_0 acts essentially as mass squared operator with integer spectrum. I have proposed conformal confinent for the physical states net conformal weight is real and integer valued and corresponds to the sum over negative integer valued conformal weights corresponding to the trivial zeros and sum over real parts of non-trivial zeros with conformal weight equal to 1/2. Imaginary parts of zeta would sum up to zero.

   (b) The counterpart of Hamiltonian as a time translation is represented by $H = iL_0 = ir_M d/dr_M$. Conformal confinement is now realized as the vanishing of the sum for the real parts of the zeros of zeta: this can be achieved. As a matter fact the integration measure dr_M/r_M brings implies that the net conformal weight must be 1/2. This is achieved if the number of non-trivial zeros is odd with a judicious choice of trivial zeros. The eigenvalues of Hamiltonian acting as time translation operator could correspond to the linear combination of imaginary part of zeros of zeta with integer coefficients. This is an attractive hypothesis in critical systems and TGD Universe is indeed quantum critical.

4.6 What About Quantum Classical Correspondence?

Quantum classical correspondence realized as one-to-one map between quantum states and zero modes has not been discussed yet.

1. M-matrices would act in the tensor product of quantum fluctuating degrees of freedom and zero modes. The assumption that zero energy states form an orthogonal basis implies that the hermitian square roots of the density matrices form an orthonormal basis. This condition generalizes the usual orthonormality condition.

2. The dependence on zero modes at given boundary of CD would be trivial and induced by 1-1 correspondence $|m\rangle \rightarrow z(m)$ between states and zero modes assignable to the state basis $|m_{\pm}$ at the boundaries of CD, and would mean the presence of factors $\delta_{z_+,f(m_+)} \times \delta_{z_-,f(n_-)}$ multiplying M-matrix $M^i_{m,n}$.

To sum up, it seems that the architecture of the U-matrix and its relationship to the S-matrix is now understood and in accordance with the intuitive expectations the construction of U-matrix reduces to that for S-matrix and one can see S-matrix as discretized counterpart of ordinary unitary time evolution operator with time translation represented as scaling: this allows to circumvent problems with loss of manifest Poincare symmetry encountered in quantum field theories and allows Lorentz invariance although CD has finite size. What came as surprise was the connection with stringy picture: strings are necessary in order to satisfy the unitary conditions for U-matrix. Second outcome was that the connection with super-symplectic algebra suggests itself strongly. The identification of hermitian square roots of density matrices with Hermitian symmetry algebra is very elegant aspect discovered already earlier. A further unexpected result was that U-matrix is unitary only for strict arrow of time (which changes in the state function reduction to opposite boundary of CD).

REFERENCES

Mathematics

[A1] Atyiah-Singer index-theorem. Available at: http://en.wikipedia.org/wiki/Atiyah-Singer_index_theorem.

[A2] KMS state. Available at: http://en.wikipedia.org/wiki/KMS_state.

[A3] Von Neumann algebra. Available at: http://en.wikipedia.org/wiki/Von_Neumann_algebra.

[A4] Connes A. Une classification des facteurs de type III. *Ann Sci Ecole Norm Sup*, 6, 1973.

[A5] Connes A. *Non-commutative Geometry.* Academic Press, San Diego, 1994.

[A6] Wassermann A. Operator algebras and conformal field theory. III. Fusion of positive energy representations of LSU(N) using bounded operators. *Invent Math*, 133(3), 1998.

[A7] Pressley A Chari V. *A Guide to Quantum Groups.* Cambridge University Press, Cambridge, 1994.

[A8] Rovelli C Connes A. Von Neumann algebra automorphisms and time-thermodynamics relation in general covariant quantum theories. Available at: http://arxiv.org/PS_cache/gr-qc/pdf/9406/9406019v1.pdf, 1994.

[A9] Witten E. Quantum field theory and the Jones polynomial. *Comm Math Phys*, 121:351–399, 1989.

[A10] Jones FR. *Braid groups, Hecke algebras and type II_1 factors.* 1983.

[A11] Saleur H. Zeroes of chromatic polynomials: a new approach to the Beraha conjecture using quantum groups. *Comm Math Phys*, 132, 1990.

[A12] Baez J. Higher-dimensional algebra II: 2-Hilbert spaces. Available at: http://arxiv.org/abs/q-alg/9609018, 1997.

[A13] Dixmier J. *Von Neumann Algebras.* North-Holland, Amsterdam, 1981.

[A14] Yngvason J. The role of Type III Factors in Quantum Field Theory. Available at: http://arxiv.org/abs/math-ph/0411058, 2004.

[A15] Takesaki M. *Tomita's Theory of Modular Hilbert Algebras and Its Applications*, volume 128. Springer, Berlin, 1970.

[A16] Neumann von J Murray FJ. On Rings of Operators. *Ann Math*, pages 37116–229, 1936.

[A17] Longo R. Operators algebras and Index Theorems in Quantum Field Theory, 2004.

[A18] Stöltzner M Redei M. *John von Neumann and the Foundations of Quantum Physics. Vol. 8, Dordrecht.* Kluwer, 2001.

[A19] Jones V. In and around the origin of quantum groups. Available at: http://arxiv.org/abs/math/0309199, 2003.

[A20] Neumann von J. Quantum Mechanics of Infinite Systems, 1937.

Theoretical Physics

[B1] Schroer B. Lectures on Algebraic Quantum Field Theory and Operator Algebras. Available at: http://arxiv.org/abs/math-ph/0102018, 2001.

[B2] Kastler D Haag R. An Algebraic Approach to Quantum Field Theory. *J Math Phys*, 5:, 1964.

[B3] Borchers HJ. On Revolutionizing QFT with Tomita's Modular Theory. *J Math Phys* . Available at: http://www.lqp.uni-goettingen.de/papers/99/04/99042900.html, 41:3604–3673, 2000.

[B4] Nakamura I Ito Y. Hilbert schemes and simple singularities. *Proc. of EuroConference on Algebraic Geometry, Warwick.* Available at: http://www.math.sci.hokudai.ac.jp/~nakamura/ADEHilb.pdf, pages 151–233, 1996.

Neuroscience and Consciousness

[J1] Libet B. Readiness potentials preceding unrestricted spontaneous and preplanned voluntary acts. Available at: http://tinyurl.com/jqp1, 1982.

[J2] Fantappie L. *Teoria Unitaria del Mondo Fisico e Biologico.* Di Renzo Editore, Roma, 1942.

[J3] Corpo Di U. The conflict between entropy and syntropy: the vital needs model. 2007.

Books related to TGD

[K1] Pitkänen M. About Nature of Time. In *TGD Inspired Theory of Consciousness.* Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#timenature, 2006.

[K2] Pitkänen M. About the New Physics Behind Qualia. In *Quantum Hardware of Living Matter*. Onlinebook. Available at: http://tgdtheory.fi/public_html/bioware/bioware.html#newphys, 2006.

[K3] Pitkänen M. Appendix A: Quantum Groups and Related Structures. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#bialgebra`, 2006.

[K4] Pitkänen M. Construction of Quantum Theory: M-matrix. In *Towards M-Matrix*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#towards`, 2006.

[K5] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#quthe`, 2006.

[K6] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#compl1`, 2006.

[K7] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#eegdark`, 2006.

[K8] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#Planck`, 2006.

[K9] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#nmpc`, 2006.

[K10] Pitkänen M. Quantum Model of Memory. In *TGD Inspired Theory of Consciousness*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#memoryc`, 2006.

[K11] Pitkänen M. Self and Binding. In *TGD Inspired Theory of Consciousness*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#selfbindc`, 2006.

[K12] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory`, 2006.

[K13] Pitkänen M. Time and Consciousness. In *TGD Inspired Theory of Consciousness*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#timesc`, 2006.

[K14] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Genes and Memes*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/genememe/genememe.html#tqc`, 2006.

[K15] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann`, 2006.

[K16] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#cspin`, 2006.

[K17] Pitkänen M. Construction of Quantum Theory: More about Matrices. In *Towards M-Matrix*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#UandM`, 2012.

[K18] Pitkänen M. *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html`, 2013.

[K19] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#qcritdark, 2014.

[K20] Pitkänen M. Quantum gravity, dark matter, and prebiotic evolution. In *Genes and Memes*. Onlinebook. Available at: http://tgdtheory.fi/public_html/genememe/genememe.html#hgrprebio, 2014.

Articles about TGD

[L1] Pitkänen M. Generalization of thermodynamics allowing negentropic entanglement and a model for conscious information processing. Available at: http://tgdtheory.fi/articles/thermolife.pdf, 2011.