Why Mersenne Primes Are So Special?

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June 20, 2019

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Abstract

Mersenne primes and their Gaussian variants play a key role in TGD based model of hadrons and elementary particles. In this article quantum information theoretic arguments for the special role of Mersenne primes are discussed. The main idea is that information storage as states of prime-dimension state space is stable about self-reductions of state since these spaces cannot be decomposed to a tensor product so that the notion of entanglement does not even make sense. Mersenne primes $M_k = 2^k - 1$ are nearest to a power of two. Quantum computation would take place in 2^k-dimensional state space of k qubits and storage in $2^k - 1$ dimensional state space to which outcome of computation would be mapped isometrically.

1 Introduction

Mersenne primes are central in TGD based world view. p-Adic thermodynamics combined with p-adic length scale hypothesis stating that primes near powers of two are physically preferred provides a nice understanding of elementary particle mass spectrum. Mersenne primes $M_k = 2^k - 1$, where also k must be prime, seem to be preferred. Mersenne prime labels hadronic mass scale (there is now evidence from LHC for two new hadronic physics labelled by Mersenne and Gaussian Mersenne), and weak mass scale. Also electron and tau lepton are labelled by Mersenne prime. Also Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ seem to be important. Muon is labelled by Gaussian Mersenne and the range of length scales between cell membrane thickness and size of cell nucleus contains 4 Gaussian Mersennes!

What gives Mersenne primes so special physical status in TGD Universe? I have considered this problem many times during years. The key idea is that natural selection is realized in much more general sense than usually thought, and has chosen them and corresponding p-adic length scales. Particles characterized by p-adic length scales should be stable in some well-defined sense.

Since evolution in TGD corresponds to generation of information, the obvious guess is that Mersenne primes are information theoretically special. Could the fact that $2^k - 1$ represents almost k bits be of significance? Or could Mersenne primes characterize systems, which are information theoretically especially stable?

In the following a more refined TGD inspired quantum information theoretic argument based on stability of entanglement against state function reduction, which would be fundamental process governed by Negentropy Maximization Principle (NMP) and requiring no human observer, will be discussed.

2 How to achieve stability against state function reductions?

TGD provides actually several ideas about how to achieve stability against state function reductions. This stability would be of course marvellous fact from the point of view of quantum computation since it would make possible stable quantum information storage. Also living systems could apply this kind of storage mechanism.

1. p-Adic physics leads to the notion of negentropic entanglement (NE) for which number theoretic entanglement entropy is negative and thus measures genuine, possibly conscious information assignable to entanglement (ordinary entanglement entropy measures the lack of information about the state of either entangled system). NMP [K3] favors the generation of NE. NE can be however transferred from system to another (stolen using less diplomatically correct expression!), and this kind of transfer is associated with metabolism. This kind of transfer would be the most fundamental crime: biology would be basically criminal activity! Religious thinker might talk about original sin.

   In living matter NE would make possible information storage. In fact, TGD inspired theory of consciousness constructed as a generalization of quantum measurement theory in Zero Energy Ontology (ZEO) identifies the permanent self of living system (replaced with a more negentropic one in biological death, which is also a reincarnation) as the boundary of CD, which is not affected in subsequent state function reductions and carries NE. The changing part of self - sensory input and cognition - can be assigned with opposite changing boundary of CD [K1].

2. Also number theoretic stability can be considered. Suppose that one can assign to the system some extension of algebraic numbers characterizing the WCW coordinates ("world of classical worlds") parametrizing the space-time surface (by strong form of holography (SH) the string world sheets and partonic 2-surfaces continuable to 4-D preferred extremal) associated with it.

   This extension of rationals and corresponding algebraic extensions of p-adic numbers would define the number fields defining the coefficient fields of Hilbert spaces (in the case of p-adic Hilbert spaces well-definedness might require restriction of coefficients to an algebraic extension of rationals although the coefficients would be regarded as p-adic numbers). Assume that you have an entangled system with entanglement coefficients in this number field. Suppose you want to diagonalize the corresponding density matrix. The eigenvalues belong in general case to a larger algebraic extension since they correspond to roots of a characteristic polynomials assignable to the density matrix. Could one say, that this kind of entanglement is stable (at least to some degree) against state function reduction since it means going to an eigenstate which does not belong to the extension used? Reader can decide!

3. Hilbert spaces behave like natural numbers with respect to direct sum and tensor product. The dimension of the tensor product is product mn of the dimensions of the tensor factors. Hilbert space with dimension n can be decomposed to a tensor product of prime Hilbert spaces with dimensions which are prime factors of n. In TGD Universe state function reduction is a dynamical process, which implies that the states in state spaces with prime valued dimension are stable against state function reduction since one cannot even speak about tensor product decomposition, entanglement, or reduction of entanglement. These state spaces are quantum indecomposable and would be thus ideal for the storage of quantum information!

   Interestingly, the system consisting of k qubits have Hilbert space dimension $D = 2^k$ and is thus maximally unstable against decomposition to $D = 2$-dimensional tensor factors! In TGD Universe NE might save the situation. Could one imagine a situation in which Hilbert space with dimension $M_k = 2^k - 1$ stores the information stably? When information is processed this state space would be mapped isometrically to 2^k-dimensional state space making possible quantum computations using qubits. The outcome of state function reduction halting the computation would be mapped isometrically back to M_k-D space. Note that isometric maps generalizing unitary transformations are an essential element in the proposal for the tensor net realization of holography and error correcting codes [K4].

Can one imagine any concrete realization for this idea? This question will be considered in the sequel.

3 How to realize $M_k = 2^k - 1$-dimensional Hilbert space physically?

One can imagine at least three physical realizations of $M_k = 2^k - 1$-dimensional Hilbert space.

1. The set with k elements has 2^k subsets. One of them is empty set and cannot be physically realized. Here the reader might of course argue that if they are realized as empty boxes, one can realize them. If empty set has no physical realization, the wave functions in the set of non-empty subsets with $2^k - 1$ elements define $2^k - 1$-dimensional Hilbert space. If $2^k - 1$ is Mersenne prime, this state state space is stable against state function reductions since one cannot even speak about entanglement!

   To make quantum computation possible one must map this state space to 2^k dimensional state space by isometric imbedding. This is possible by just adding a new element to the set and considering only wave functions in the set of subsets containing this new element. Now also the empty set is mapped to a set containing only this new element and thus belongs to the state space. One has 2^k dimensions and quantum computations are possible. When the computation halts, one just removes this new element from the system, and the data are stored stably!

2. Second realization relies on k bits represented as spins such that $2^k - 1$ is Mersenne prime. Suppose that the ground state is spontaneously magnetized state with $k + l$ parallel spins, with the l spins in the direction of spontaneous magnetization and stabilizing it. $l > 1$ is probably needed to stabilize the direction of magnetization: $l \leq k$ suggests itself as the first guess. Here thermodynamics and a model for spin-spin interaction would give a better estimate.

   The state with the k spins in direction opposite to that for l spins would be analogous to empty set. Spontaneous magnetization disappears, when a sufficient number of spins is in direction opposite to that of magnetization. Suppose that k corresponds to a critical number of spins in the sense that spontaneous magnetization occurs for this number of parallel spins. Quantum superpositions of $2^k - 1$ states for k spins would be stable against state function reduction also now.

   The transformation of the data to a processable form would require an addition of $m \geq 1$ spins in the direction of the magnetization to guarantee that the state with all k spins in direction opposite to the spontaneous magnetization does not induce spontaneous magnetization in opposite direction. Note that these additional stabilizing spins are classical and their direction could be kept fixed by a repeated state function reduction (Zeno effect). One would clearly have a critical system.

3. Third realization is suggested by TGD inspired view about Boolean consciousness [K2]. Boolean logic is represented by the Fock state basis of many-fermion states. Each fermion mode defines one bit: fermion in given mode is present or not. One obtains 2^k states. These states have different fermion numbers and in ordinary positive energy ontology their realization is not possible.

   In ZEO situation changes. Fermionic zero energy states are superpositions of pairs of states at opposite boundaries of CD such that the total quantum numbers are opposite. This applies to fermion number too. This allows to have time-like entanglement in which one has superposition of states for which fermion numbers at given boundary are different. This kind of states might be realized for super-conductors to which one at least formally assigns coherent state of Cooper pairs having ill-defined fermion number.

   Now the non-realizable state would correspond to fermion vacuum analogous to empty set. Reader can of course argue that the bosonic degrees of freedom assignable to the space-time surface are still present. I defend this idea by saying that the purely bosonic state might be unstable or maybe even non-realizable as vacuum state and remind that also bosons in TGD framework consists of pairs of fundamental fermions.

If this state is effectively decoupled from the rest of the Universe, one has 2^k-1-dimensional state space and states are stable against state function reduction. Information processing becomes possible by adding some positive energy fermions and corresponding negative energy fermions at the opposite boundaries of CD. Note that the added fermions do not have time-like quantum entanglement and do not change spin direction during time evolution.

The proposal is that Boolean consciousness is realized in this manner and zero energy states represents quantum Boolean thoughts as superposition of pairs $(b_1 \otimes b_2)$ of positive and negative energy states and having identification as Boolean statements $b_1 \rightarrow b_2$. The mechanism would allow both storage of thoughts as memories and their processing by introducing the additional fermion(s).

4 So: why Mersenne primes would be so special?

Returning to the original question "Why Mersenne primes are so special?". A possible explanation is that elementary particle or hadron characterized by a p-adic length scale $p = M_k = 2^k - 1$ both stores and processes information with maximal effectiveness. This would not be surprising if p-adic physics defines the physical correlates of cognition assumed to be universal rather than being restricted to human brain.

In adelic physics p-dimensional Hilbert space could be naturally associated with the p-adic adelic sector of the system. Information storage could take place in $p = M_k = 2^k - 1$ phase and information processing (cognition) would take place in 2^k-dimensional state space. This state space would be reached in a phase transition $p = 2^k - 1 \rightarrow 2$ changing effective p-adic topology in real sector and genuine p-adic topology in p-adic sector and replacing padic length scale $\propto \sqrt{p} \simeq 2^{k/2}$ with k-nary 2-adic length scale $\propto 2^{k/2}$.

Electron is characterized by the largest not completely super-astrophysical Mersenne prime M_{127} and corresponds to $k = 127$ bits. Intriguingly, the secondary p-adic time scale of electron corresponds to .1 seconds defining the fundamental biorhythm of 10 Hz.

This proposal suffers from deficiencies. It does not explain why also Gaussian Mersennes are special. Gaussian Mersennes correspond ordinary primes near power of 2 but not so near as Mersenne primes are. Neither does it explain why also more general primes $p \simeq 2^k$ seem to be preferred. Furthermore, p-adic length scale hypothesis generalizes and states that primes near powers of at least small primes q: $p \simeq q^k$ are special at least number theoretically. For instance, $q = 3$ seems to be important for music experience and also $q = 5$ might be important (Golden Mean).

Could the proposed model relying on criticality generalize? There would be $p < 2^k$-dimensional state space allowing isometric imbedding to 2^k-dimensional space such that the bit configurations orthogonal to the image would be unstable in some sense. Say against a phase transition changing the direction of magnetization. One can imagine the variants of above described mechanism also now. For $q > 2$ one should consider pinary digits instead of bits but the same arguments would apply (except in the case of Boolean logic).

5 Brain and Mersenne integers

I received a link to an interesting the article "Brain Computation Is Organized via Power-of-Two-Based Permutation Logic" by Kun Xie, Grace E. Fox, Jun Liu, Cheng Lyu, Jason C. Lee, Hui Kuang, Stephanie Jacobs, Meng Li, Tianming Liu, Sen Song and Joe Z. Tsien in Frontiers in Systems Neuroscience [?]see `http://tinyurl.com/zfymqrq`).

The proposed model is about how brain classifies neuronal inputs. The following represents my attempt to understand the model of the article.

1. One can consider a situation in which one has n inputs identifiable as bits: bit could correspond to neuron firing or not. The question is however to classify various input combinations.

The obvious criterion is how many bits are equal to 1 (corresponding neuron fires). The input combinations in the same class have same number of firing neurons and the number of subsets with k elements is given by the binomial coefficient $B(n,k) = n!/k!(n-k)!$. There are clearly $n-1$ different classes in the classification since no neurons firing is not a possible observation. The conceptualization would tell how many neurons fire but would not specify which of them.

2. To represent these bit combinations one needs 2^n-1 neuron groups acting as unit representing one particular firing combination. These subsets with k elements would be mapped to neuron cliques with k firing neutrons. For given input individual firing neurons ($k=1$) would represent features, lowest level information. The n cliques with $k=2$ neurons would represent a more general classification of input. One obtains $M_n = 2^n-1$ combinations of firing neurons since the situations in which no neurons are firing is not counted as an input.

3. If all neurons are firing then all the however level cliques are also activated. Set theoretically the subsets of set partially ordered by the number of elements form an inclusion hierarchy, which in Boolean algebra corresponds to the hierarchy of implications in opposite direction. The clique with all neurons firing correspond to the most general statement implying all the lower level statements. At k:th level of hierarchy the statements are inconsistent so that one has $B(n,k)$ disjoint classes.

The $M_n = 2^n - 1$ (Mersenne number) labelling the algorithm is more than familiar to me.

1. For instance, electron's p-adic prime corresponds to Mersenne prime $M_{127} = 2^{127} - 1$, the largest not completely super-astrophysical Mersenne prime for which the mass of particle would be extremely small. Hadron physics corresponds to M_{107} and M_{89} to weak bosons and possible scaled up variant of hadron physics with mass scale scaled up by a factor 512 ($= 2^{(107-89)/2}$). Also Gaussian Mersennes seem to be physically important: for instance, muon and also nuclear physics corresponds to $M_{G,n} = (1+i)^n - 1$, $n = 113$.

2. In biology the Mersenne prime $M_7 = 2^7 - 1$ is especially interesting. The number of statements in Boolean algebra of 7 bits is 128 and the number of statements that are consistent with given atomic statement (one bit fixed) is $2^6 = 64$. This is the number of genetic codons which suggests that the letters of code represent 2 bits. As a matter of fact, the so called Combinatorial Hierarchy $M(n) = M_{M(n-1)}$ consists of Mersenne primes $n = 3, 7, 127, 2^{127}-1$ and would have an interpretation as a hierarchy of statements about statements about ... It is now known whether the hierarchy continues beyond M_{127} and what it means if it does not continue. One can ask whether M_{127} defines a higher level code - memetic code as I have called it - and realizable in terms of DNA codon sequences of 21 codons [?] (see `http://tinyurl.com/jukyq6y`).

3. The Gaussian Mersennes $M_{G,n}$ $n = 151, 157, 163, 167$, can be regarded as a number theoretical miracles since the these primes are so near to each other. They correspond to p-adic length scales varying between cell membrane thickness 10 nm and cell nucleus size 2.5 μm and should be of fundamental importance in biology. I have proposed that p-adically scaled down variants of hadron physics and perhaps also weak interaction physics are associated with them.

I have made attempts to understand why Mersenne primes M_n and more generally primes near powers of 2 seem to be so important physically in TGD Universe.

1. The states formed from n fermions form a Boolean algebra with 2^n elements, but one of the elements is vacuum state and could be argued to be non-realizable. Hence Mersenne number $M_n = 2^n-1$. The realization as algebra of subsets contains empty set, which is also physically non-realizable. Mersenne primes are especially interesting as sine the reduction of statements to prime nearest to M_n corresponds to the number M_n-1 of physically representable Boolean statements.

2. Quantum information theory suggests itself as explanation for the importance of Mersenne primes since M_n would correspond the number of physically representable Boolean statements of a Boolean algebra with n-elements. The prime $p \leq M_n$ could represent the number of elements of Boolean algebra representable p-adically [?] (see http://tinyurl.com/gp9mspa).

3. In TGD Fermion Fock states basis has interpretation as elements of quantum Boolean algebra and fermionic zero energy states in ZEO expressible as superpositions of pairs of states with same net fermion numbers can be interpreted as logical implications. WCW spinor structure would define quantum Boolean logic as "square root of Kähler geometry". This Boolean algebra would be infinite-dimensional and the above classification for the abstractness of concept by the number of elements in subset would correspond to similar classification by fermion number. One could say that bosonic degrees of freedom (the geometry of 3-surfaces) represent sensory world and spinor structure (many-fermion states) represent that logical thought in quantum sense.

4. Fermion number conservation would seem to represent an obstacle but in ZEO it can circumvented since zero energy states can be superpositions of pair of states with opposite fermion number F at opposite boundaries of causal diamond (CD) in such a manner that F varies. In state function reduction however localization to single value of F is expected to happen usually. If superconductors carry coherent states of Cooper pairs, fermion number for them is ill defined and this makes sense in ZEO but not in standard ontology unless one gives up the super-selection rule that fermion number of quantum states is well-defined.

One can of course ask whether primes n defining Mersenne primes (see https://en.wikipedia.org/wiki/Mersenne_prime) could define preferred numbers of inputs for subsystems of neurons. This would predict $n = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257, ..$ as favoured numbers of inputs. $n = 127$ would correspond to memetic code.

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