The vanishing of super-conformal charges as a gauge conditions selecting preferred extremals of Kähler action

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Abstract

Quantum classical correspondence suggests that the generalized super-conformal invariance should have concrete realization already at classical level that is for the extremals of Kähler action. This indeed can be the case. The realization is actually completely obvious: one requires that various classical Noether charges associated with super-conformal invariance vanish at the ends of space-time surfaces located at light-like boundaries of causal diamond. Similar conditions would hold true at partonic orbits which are light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. These conditions can be regarded as fixing conformal gauge and are natural if the Kähler metric of "world of classical worlds" (WCW) has these symmetries as isometries. These conditions define precisely also the notion of preferred extremal, Bohr orbitology in the framework of Zero Energy Ontology (ZEO) and also realize concretely strong form of holography implied by the strong form of General Coordinate Invariance. In TGD framework classical isometry charges are complex: a fascinating possibility inspired by twstor approach is that the states are massless in complex sense but massive in the real sense.

1 Introduction

Classical TGD [K1] involves several key questions waiting for clearcut answers.

1. The notion of preferred extremal emerges naturally in positive energy ontology, where Kähler metric assigns a unique (apart from gauge symmetries) preferred extremal to given 3-surface at M^4 time= constant section of imbedding space $H = M^4 \times CP_2$. This would quantize the initial values of the time derivatives of imbedding coordinates and this could correspond to the Bohr orbitology in quantum mechanics.

2. In zero energy ontology (ZEO) initial conditions are replaced by boundary conditions. One fixes only the 3-surfaces at the opposite boundaries of CD and in an ideal situation there would exist a unique space-time surface connecting them. One must however notice that the existence of light-like wormhole throat orbits at which the signature of the induced metric changes ($det(g_4) = 0$) its signature might change the situation. Does the attribute "preferred" become obsolete and does one lose the beautiful Bohr orbitology, which looks intuitively compelling and would realize quantum classical correspondence?

3. Intuitively it has become clear that the generalization of super-conformal symmetries by replacing 2-D manifold with metrically 2-D but topologically 3-D light-like boundary of causal diamond makes sense. Generalized super-conformal symmetries should apply also to the wormhole throat orbits which are also metrically 2-D and for which conformal symmetries respect $detg(g_4) = 0$ condition. Quantum classical correspondence demands that the generalized super-confornal invariance has a classical counterpart. How could this classical counterpart be realized?

4. Holography is one key aspect of TGD and mean that 3-surfaces dictate everything. In positive energy ontology the content of this statement would be rather obvious and reduce to Bohr orbitology but in ZEO situation is different. On the other hand, TGD strongly suggests strong form of holography based stating that partonic 2-surfaces (the ends of wormhole throat orbits at boundaries of CD) and tangent space data at them code for quantum physics of TGD. General coordinate invariance would be realized in strong sense: one could formulate the theory either in terms of space-like 3-surfaces at the ends of CD or in terms of light-like wormhole throat orbits. This would realize Bohr orbitology also in ZEO by reducing the boundary conditions to those at partonic 2-surfaces. How to realize this explicitly at the level of field equations? This has been the challenge.

Answering questions is extremely useful activity. During last years Hamed has posed continually questions related to the basic TGD. At this time Hamed asked about the derivation of field equations of TGD. In "simple" field theories involving some polynomial non-linearities the deduction of field equations is of course totally trivial process but in the extremely non-linear geometric framework of TGD situation is quite different.

While answering the questions I made what I immediately dare to call a breakthrough discovery in the mathematical understanding of TGD. To put it concisely: one can assume that the variations at the light-like boundaries of CD vanish for all conformal variations which are not isometries. For isometries the contributions from the ends of CD cancel each other so that the corresponding variations need not vanish separately at boundaries of CD! This is extremely simple and profound fact. This would be nothing but the realisation of the analogs of conformal symmetries classically and give precise content for the notion of preferred external, Bohr orbitology, and strong form of holography. And the condition makes sense only in ZEO!

I attach below the answers to the questions of Hamed almost as such apart from slight editing and little additions, re-organization, and correction of typos.

2 Field equations for Kähler action

Hamed made some questions relating to the derivation of field equations for the extremals of Kähler action which led to the recent progress. I comment first these questions since they lead naturally to the basic new idea.

2.1 The physical interpretation of the canonical momentum current

Hamed asked about the physical meaning of $T_k^n \equiv \partial L/\partial(\partial_n h^k)$ - normal components of canonical momentum labelled by the label k of imbedding space coordinates - it is good to start from the physical meaning of a more general vector field

$$T_k^\alpha \equiv \frac{\partial L}{\partial(\partial_\alpha h^k)}$$

with both imbedding space indices k and space-time indices α - canonical momentum currents. L refers to Kähler action.

1. One can start from the analogy with Newton's equations derived from action principle (Lagrangian). Now the analogs are the partial derivatives $\partial L/\partial(dx^k/dt)$. For a particle in potential one obtains just the momentum. Therefore the term canonical momentum current/density: one has kind of momentum current for each imbedding space coordinate.

2. By contracting with generators of imbedding space isometries (Poincare and color) one indeed obtains conserved currents associated with isometries by Noether's theorem:

$$j^{A\alpha} = T^{\alpha}_{k} j^{Ak} \ .$$

By field equations the divergences of these currents vanish and one obtains conserved charged- classical four-momentum and color charges:

$$D_{\alpha} T^{A\alpha} = 0 \ .$$

3. The normal component of conserved current must vanish at boundaries with one time-like direction if one has such:

$$T^{An} = 0.$$

Now one has wormhole throat orbits which are not genuine boundaries albeit analogous to them and one must be very careful. The quantity T^{n}_{k} determines the values of normal components of currents and must vanish at possible space-like boundaries.

Note that in TGD field equations reduce to the conservation of isometry currents as in hydrodynamics where basic equations are just conservation laws.

2.2 The basic steps in the derivation of field equations

First a general recipe for deriving field equations from Kähler action - or any action as a matter of fact.

1. At the first step one writes an expression of the variation of the Kähler action as sum of variations with respect to the induced metric g and induced Kähler form J. The partial derivatives in question are energy momentum tensor and contravariant Kähler form.

2. After this the variations of g and J are expressed in terms of variations of imbedding space coordinates, which are the primary dynamical variables.

3. The integral defining the variation can be decomposed to a total divergence plus a term vanishing for extremals for all variations: this gives the field equations. Total divergence term gives a boundary term and it vanishes by boundary conditions if the boundaries in question have time-like direction.

   If the boundary is space-like, the situation is more delicate in TGD framework: this will be considered in the sequel. In TGD situation is also delicate also because the light-like 3-surfaces which are common boundaries of regions with Minkowskian or Euclidian signature of the induced metric are not ordinary topological boundaries. Therefore a careful treatment of both cases is required in order to not to miss important physics.

Expressing this summary more explicitly, the variation of the Kähler action with respect to the gradients of the imbedding space coordinates reduces to the integral of

$$T_k^\alpha \partial_\alpha \delta h^k + \frac{\partial K}{\partial h^k}\delta h^k \ .$$

The latter term comes only from the dependence of the imbedding space metric and Kähler form on imbedding space coordinates. One can use a simple trick. Assume that they do not depend at all on imbedding space coordinates, derive field equations, and replaced partial derivatives by covariant derivatives at the end. Covariant derivative means covariance with respect to both space-time and imbedding space vector indices for the tensorial quantities involved. The trick works because imbedding space metric and Kähler form are covariantly constant quantities.

The integral of the first term $T_k^\alpha \partial_\alpha \delta h^k$ decomposes to two parts.

1. The first term, whose vanishing gives rise to field equations, is integral of

$$D_\alpha T_k^\alpha \delta h^k \ .$$

2. The second term is integral of

$$\partial_\alpha (T_k^\alpha \delta h^k) \ .$$

   This term reduces as a total divergence to a 3-D surface integral over the boundary of the region of fixed signature of the induced metric consisting of the ends of CD and wormhole throat orbits (boundary of region with fixed signature of induced metric). This term vanishes if the normal components T_k^n of canonical momentum currents vanishes at the boundary like region.

In the sequel the boundary terms are discussed explicitly and it will be found that their treatment indeed involves highly non-trivial physics.

2.3 Complex isometry charges and twistorialization

TGD space-time contains regions of both Minkowskian and Euclidian signature of metric. This has some highly non-trivial consequences.

1. Should one assume that $\sqrt{det(g_4)}$ is imaginary in Minkowskian and real in Euclidian region? For Kähler action this is sensible and Euclidian region would give a real negative contribution giving rise to exponent of Kähler function of WCW ("world of classical worlds") making the functional integral convergent. Minkowskian regions would give imaginary contribution to the exponent causing interference effects absolutely essential in quantum field theory. This contribution would correspond to Morse function for WCW.

   The implication would be that the classical four-momenta in Euclidian/Minkowskian regions are imaginary/real. What could the interpretation be? Should one accept as a fact that four-momenta are complex.

2. Twistor approach to TGD is now in quite good shape [?, K5]. $M^4 \times CP_2$ is the unique choice is one requires that the Cartesian factors allow twistor space with Kähler structure and classical TGD allows twistor formulation.

   In the recent formulation the fundamental fermions are assumed to propagate with light-like momenta along wormhole throats. At gauge theory limit particles must have massless or massive four-momenta. One can however also consider the possibility of complex massless momenta and in the standard twistor approach on mass shell massless particles appearing in graphs indeed have complex momenta. These complex momenta should by quantum classical correspondence correspond directly to classical complex momenta.

3. A funny question popping in mind is whether the massivation of particles could be such that the momenta remain massless in complex sense! The complex variant of light-likeness condition would be

$$p_{re}^2 = p_{Im}^2 \ , \ p_{re} \cdot p_{Im} = 0 \ .$$

   Could one interpret p_{Im}^2 as the mass squared of the particle? Or could p_{Im}^2 code for the decay width of an unstable particle? This option does not look feasible.

4. The complex momenta could provide an elegant 4-D space-time level representation for the isometry quantum numbers at the level of imbedding space. The ground states of the super-conformal representations have as building bricks the spinor harmonics of the imbedding space which correspond to the analogs of massless particles in 8-D sense [K3]. Indeed, the condition giving mass squared eigenvalues for the spinor harmonics is just massless condition in $M^4 \times CP_2$.

   At the space-time level these conditions must be replaced by 4-D conditions and complex masslessness would be the elegant manner to realizes this. Also the massivation of massless states by p-adic thermodynamics could have similar description.

   This interpretation would also conform with $M^8 - M^4 \times CP_2$ duality [K4] at the level of momentum space.

3 Boundary conditions at boundaries of CD

In positive energy ontology one would formulate boundary conditions as initial conditions by fixing both the 3-surface and associated canonical momentum densities at either end of CD (positions and momenta of particles in mechanics). This would bring asymmetry between boundaries of CD. In ZEO the basic boundary condition is that space-time surfaces have as their ends the members of pairs of surfaces at the ends of CD. Besides this one can have additional boundary conditions and the notion of preferred extremal suggests this.

3.1 Do boundary conditions realize quantum classical correspondence?

In TGD framework one must carefully consider the boundary conditions at the boundaries of CDs. What is clear that the time-like boundary contributions from the boundaries of CD to the variation must vanish.

1. This is true if the variations are assumed to vanish at the ends of CD. This might be however too strong a condition.

2. One cannot demand the vanishing of T^t_k (t refers to time coordinate as normal coordinate) since this would give only vacuum extremals. One could however require quantum classical correspondence for any Cartan sub-algebra of isometries whose elements define maximal set of isometry generators. The eigenvalues of quantal variants of isometry charge assignable to second quantized induced spinors at the ends of space-time surface are equal to the classical charges. Is this actually a formulation of Equivalence Principle, is not quite clear to me.

3.2 Do boundary conditions realize preferred extremal property as a choice of conformal gauge?

While writing this a completely new idea popped to my mind. What if one poses the vanishing of the boundary terms at boundaries of CDs as additional boundary conditions for *all* variations *except isometries*? Of perhaps for all conformal variations (conformal in TGD sense)? This would *not* imply vanishing of isometry charges since the variations coming from the opposite ends of CD cancel each other! It soon became clear that this would allow to meet all the challenges listed in the beginning!

1. These conditions would realize Bohr orbitology also to ZEO approach and define what "preferred extremal" means.

2. The conditions would be very much like super-Virasoro conditions stating that the superconformal generators with non-vanishing conformal weight annihilate states or create zero norm states but no conditions are posed on generators with vanishing conformal weight (now isometries). One could indeed assume only deformations, which are local isometries assignable to the generalised conformal algebra of the $\delta M^4_+/-\times CP_2$. For arbitrary variations one would not require the vanishing. This could be the long sought for precise formulation of super-conformal invariance at the level of classical field equations!

   It is enough co consider the weaker conditions that the conformal charges defined as integrals of corresponding Noether currents vanish. These conditions would be direct equivalents of quantal conditions.

3. The natural interpretation would be as a fixing of conformal gauge. This fixing would be motivated by the fact that WCW Kähler metric must possess isometries associated with the conformal algebra and can depend only on the tangent data at partonic 2-surfaces as became clear already for more than

two decades ago. An alternative, non-practical option would be to allow all 3-surfaces at the ends of CD: this would lead to the problem of eliminating the analog of the volume of gauge group from the functional integral.

4. The conditions would also define precisely the notion of holography and its reduction to strong form of holography in which partonic 2-surfaces and their tangent space data code for the dynamics.

Needless to say, the modification of this approach could make sense also at partonic orbits.

4 Boundary conditions at parton orbits

The contributions from the orbits of wormhole throats are singular since the contravariant form of the induced metric develops components which are infinite ($det(g_4) = 0$). The contributions are real at Euclidian side of throat orbit and imaginary at the Minkowskian side so that they must be treated as independently.

4.1 Conformal gauge choice, preferred extremal property, hierarchy of Planck constants, and TGD as almost topological QFT

The generalization of the boundary conditions as a classical realization conformal gauge invariance is natural.

1. One can consider the possibility that under rather general conditions the normal components $T^n_k\sqrt{det(g_4)}$ approach to zero at partonic orbits since $det(g_4)$ is vanishing. Note however the appearance of contravariant appearing twice as index raising operator in Kähler action. If so, the vanishing of $T^n_k\sqrt{det(g_4)}$ need not fix completely the "boundary" conditions. In fact, I assign to the wormhole throat orbits conformal gauge symmetries so that just this is expected on physical grounds.

2. Generalized conformal invariance would suggest that the variations defined as integrals of $T^n_k\sqrt{det(g_4)}\delta h^k$ vanish in a non-trivial manner for the conformal algebra associated with the light-like wormhole throats with deformations respecting $det(g_4) = 0$ condition. Also the variations defined by infinitesimal isometries (zero conformal weight sector) should vanish since otherwise one would lose the conservation laws for isometry charges. The conditions for isometries might reduce to $T^n_k\sqrt{det(g_4)} \rightarrow 0$ at partonic orbits. Also now the interpretation would be in terms of fixing of conformal gauge.

3. Even $T^n_k\sqrt{g} = 0$ condition need not fix the partonic orbit completely. The Gribov ambiguity meaning that gauge conditions do not fix uniquely the gauge potential could have counterpart in TGD framework. It could be that there are several conformally non-equivalent space-time surfaces connecting 3-surfaces at the opposite ends of CD.

If so, the boundary values at wormhole throats orbits could matter to some degree: very natural in boundary value problem thinking but new in initial value thinking. This would conform with the non-determinism of Kähler action implying criticality and the possibility that the 3-surfaces at the ends of CD are connected by several space-time surfaces which are physically non-equivalent.

4. The hierarchy of Planck [K2] constants assigned to dark matter, quantum criticality and even criticality indeed relies on the assumption that $h_{eff} = n \times h$ corresponds to n-fold coverings having n space-time sheets which coincide at the ends of CD and that conformal symmetries act on the sheets as gauge symmetries. One would have as Gribov copies n conformal equivalence classes of wormhole throat orbits and corresponding space-time surfaces. Depending on whether one fixes the conformal gauge one has n equivalence classes of space-time surfaces or just one representative from each conformal equivalent class.

5. There is also the question about the correspondence with the weak form of electric magnetic duality [K1]. This duality plus the condition that $j^\alpha A_\alpha = 0$ in the interior of space-time surface imply the reduction of Kähler action to Chern-Simons terms. This would suggest that the boundary variation of the Kähler action reduces to that for Chern-Simons action which is indeed well-defined for light-like 3-surfaces.

   If so, the gauge fixing would reduce to variational equations for Chern-Simons action! A weaker condition is that classical conformal charges vanish. This would give a nice connection to the vision about TGD as almost topological QFT. In TGD framework these conditions do not imply the vanishing of Kähler form at boundaries. The conditions are satisfied if the CP_2 projection of the partonic orbit is 2-D: the reason is that Chern-Simons term vanishes identically in this case.

4.2 Fractal hierarchy of conformal symmetry breakings

A further intuitively natural hypothesis is that there is a fractal hierarchy of breakings of conformal symmetry.

1. Only the generators of conformal sub-algebra with conformal weight multiple of n act as gauge symmetries. This would give infinite hierarchies of breakings of conformal symmetry interpreted in terms of criticality: in the hierarchy n_i divides n_{i+1}.

   Similar degeneracy would be associated with both the parton orbits and the space-like ends at CD boundaries and I have considered the possibility that the integer n appearing in h_{eff} has decomposition $n = n_1 n_2$ corresponding to the degeneracies associated with the two kinds of boundaries. Alternatively, one could have just $n = n_1 = n_2$ from the condition that the two conformal symmetries are 3-dimensional manifestations of single 4-D analog of conformal symmetry.

2. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Could one require that they are conserved that is the contributions of the boundary terms at the ends of CD cancel each other? If so, one would have dynamical conformal symmetry.

What could the proper interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$?

1. Could one interpret the hierarchy in terms of increasing measurement resolution? Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and the conformal hierarchies would correspond to an inclusion hierarchies for hyper-finite factors of type II_1 [?]. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_{\pm} \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

2. Suppose that the Kähler action has vanishing variation under deformations defined by the broken conformal symmetries sot that the corresponding conformal charges are conserve. As a consequence, Kähler function would be critical with respect to the corresponding variations. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories.

   In this kind of situation one could consider the interpretation in terms of criticality: the higher the criticality, the larger then value of h_{eff} and h and the better the resolution. The naive alternative view "the higher the conformal symmetry, the higher the criticality" does not conform with this view. If conformal gauge symmetry tells that the degrees of freedom are below measurement resolution, the situation of course changes.

3. n gives also the number of space-time sheets in the singular covering. Could the interpretation be in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

As should have become clear, the derivation of field equations in TGD framework is not just an application of a formal recipe as in field theories and a lot of non-trivial physics is involved!

REFERENCES

Books related to TGD

[K1] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgclass.html#class, 2006.

[K2] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/neuplanck.html#Planck, 2006.

[K3] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#mless, 2006.

[K4] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#numbervision, 2014.

[K5] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory, 2016.