Some Questions Related to the Twistor Lift of TGD

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Abstract

In this chapter I consider questions related to both classical and quantum aspects of twistorialization.

1. The first group of questions relates to the twistor lift of classical TGD. What does the induction of the twistor structure really mean? Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications? How does gravitational coupling emerge at fundamental level? Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space?

   During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift can be criticized and allows an alternative. This option led to a much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant: cosmological term would correspond to the *entire* 4-D action and the cancellation of Kähler action and cosmological term would lead to the small value of the effective cosmological constant.

2. Second group of questions relates to the construction of scattering amplitudes. The idea is to generalize the usual construction for massless states. In TGD all single particle states are massless in 8-D sense and this gives excellent hopes about the applicability of 8-D twistor approach. $M^8 - H$ duality turns out to be the key to the construction. Also the holomorphy of twistor amplitudes in helicity spinors λ_i and independence on $\tilde{\lambda}_i$ is crucial. The basic vertex corresponds to 4-fermion vertex for which the simplest expression can be written immediately. $n > 4$-fermion scattering amplitudes can be also written immediately.

   If scattering diagrams correspond to computations as number theoretic vision suggests, the diagrams should be reducible to tree diagrams by moves generalizing the old-fashioned hadronic duality. This condition reduces to the vanishing of loops which in terms of BCFW recursion formula states that the twistor diagrams correspond to closed objects in what might be called WCFW homology.

Contents

1 Introduction

During last couple years (I am writing this in the beginning of 2017) a kind of palace revolution has taken place in the formulation and interpretation of TGD. The notion of twistor lift and 8-D generalization of twistorialization have dramatically simplified and also modified the view about what classical TGD and quantum TGD are.

The notion of adelic physics suggests the interpretation of scattering diagrams as representations of algebraic computations with diagrams producing the same output from given input are equivalent. The simplest possible manner to perform the computation corresponds to a tree diagram [L2]. As will be found, it is now possible to even propose explicit twistorial formulas for scattering formulas since the horrible problems related to the integration over WCW might be circumvented altogether.

From the interpretation of p-adic physics as physics of cognition, $h_{eff}/h = n$ could be interpreted dimension of extension dividing the the order of its Galois group. Discrete coupling constant evolution would correspond to phase transitions changing the extension of rationals and its Galois group. TGD inspired theory of consciousness is an essential part of TGD and the crucial Negentropy Maximization Principle in statistical sense follows from number theoretic evolution as increase of the order of Galois group for extension of rationals defining adeles.

In the sequel I consider the questions related to both classical and quantum aspects of twistorialization.

1.1 Questions related to the classical aspects of twistorialization

Classical aspects are related to the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces realized as extremals of 6-D analog of Kähler action in the product $T(M^4) \times T(CP_2)$ of twistor space of M^4 and CP_2 such that twistor structure is induced. The outcome is 4-D Kähler action with volume term having interpretation in terms of cosmological constant. Hence the twistorialization has profound physical content rather than being mere alternative formulation for TGD.

1. What does the induction of the twistor structure really mean? What is meant with twistor space. For instance, is the twistor sphere for M^4 time-like or space-like. The induction procedure involves dimensional reduction forced by the condition that the projection of the sum of Kähler forms for the twistor spaces $T(M^4)$ and $T(CP_2)$ gives Kähler form for the twistor sphere of X^4. Better understanding of the details is required.

2. Can the analog of Kähler form $J(M^4)$ assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory? What would be the physical implications?

   The basic objection is the loss of Poincare invariance. This can be however avoided by introducing the moduli space for Kähler forms. This moduli space is actually the moduli space of causal diamonds (CDs) forced in any case by zero energy ontology (ZEO) and playing central role in the generalization of quantum measurement theory to a theory of consciousness and in the explanation of the relationship between geometric and subjective time [K5].

   Why $J(M^4)$ would be needed? $J(M^4)$ corresponds to parallel constant electric and magnetic fields in given direction. Constant E and $B = E$ fix directions of quantization axes for energy (rest system) and spin. One implication is transversal localization of imbedding space spinor modes: imbedding space spinor modes are products of harmonic oscillator Gaussians in transversal degrees of freedom very much like quarks inside hadrons.

   Also CP breaking is implied by the electric field and the question is whether this could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$. The estimate for the mass splitting of neutral kaon and anti-kaon is of correct order of magnitude.

   Whether stationary spherically symmetric metric as minimal surface allows a sensible physical generalization is a killer test for the hypothesis that $J(M^4)$ is covariantly constant. The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.

3. How does gravitational coupling emerge at fundamental level? The first naive guess is obvious: string area action is scaled by $1/G$ as in string models. The objection is that p-adic mass calculations suggest that string tension is determined by CP_2 size R: the analog of string tension appearing in mass formula given by p-adic mass calculations would be by a factor about 10^{-8} smaller than that estimated from string tension. The discrepancy evaporates by noticing that p-adic mass calculations rely on p-adic thermodynamics at imbedding space level whereas string world sheets appear at space-time level. Furthermore, if the action assignable to string world sheets is effective action expressing 4-D action in 2-D form as strong form of holography (SH) suggests string tension is expected to be function of the parameters appearing in the 4-D action.

4. Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface having by definition vanishing induced Kähler form: $J(M^4)+J(CP_2)=0$. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket? Could string world sheets be minimal surfaces satisfying $J(M^4)+J(CP_2)=0$. The Lagrangian condition allows also more general solutions - even 4-D space-time surfaces and one obtains analog of brane hierarchy. Could one allow spinor modes also at these analogs of branes. Is Lagrangian condition equivalent with the original condition that induced W boson fields making the em charge of induced spinor modes ill-defined vanish and allowing also solution with other dimensions. How Lagrangian property relates to the idea that string world sheets correspond to complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift [L3] contained an error. This led to much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant.

1.2 Questions related to the quantum aspects of twistorialization

Also the questions related to the quantum aspects of twistorialization of TGD are discussed.

1. There are several notions of twistor. Twistor space for M^4 is $T(M^4) = M^4 \times S^2$ [B20] (see `http://arxiv.org/pdf/1308.2820.pdf`) having projections to both M^4 and to the standard twistor space $T_1(M^4)$ often identified as CP_3. $T(M^4) = M^4 \times S^2$ is necessary for the twistor lift of space-time dynamics. CP_2 gives the factor $T(CP_2) = SU(3)/U(1) \times U(1)$ to the classical twistor space $T(H)$. The quantal twistor space $T(M^8) = T_1(M^4) \times T(CP_2)$ assignable to momenta. The possible way out is $M^8 - H$ duality relating the momentum space M^8 (isomorphic to the tangent space H) and H by mapping space-time associative and co-associative surfaces in M^8 to the surfaces which correspond to the base spaces of in H: they construction would reduce to holomorphy in complete analogy with the original idea of Penrose in the case of massless fields.

2. The standard twistor approach has problems. Twistor Fourier transform reduces to ordinary Fourier transform only in signature (2,2) for Minkowski space: in this case twistor space is real RP_3 but can be complexified to CP_3. Otherwise the transform requires residue integral to define the transform (in fact, p-adically multiple residue calculus could provide a nice manner to define integrals and could make sense even at space-time level making possible to define action).

   Also the positive Grassmannian requires (2,2) signature. In $M^8 - H$ relies on the existence of the decomposition $M^2 \subset M^2 = M^2 \times E^2 \subset M^8$. M^2 could even depend on position but $M^2(x)$ should define an integrable distribution. There always exists a preferred M^2, call it M_0^2, where 8-momentum reduces to light-like M^2 momentum. Hence one can apply 2-D variant of twistor approach. Now the signature is (1,1) and spinor basis can be chosen to be real! Twistor space is RP_3 allowing complexification to CP_3 if light-like complex momenta are allowed as classical TGD suggests!

3. A further problem of the standard twistor approach is that in M^4 twistor approach does not work for massive particles. In TGD all particles are massless in 8-D sense. In M^8 M^4-mass squared corresponds to transversal momentum squared coming from $E^4 \subset M^4 \times E^4$ (from CP_2 in H). In particular, Dirac action cannot contain anyo mass term since it would break chiral invariance.

   Furthermore, the ordinary twistor amplitudes are holomorphic functions of the helicity spinors λ_i and have no dependence on $\tilde{\lambda}_i$: no information about particle masses! Only the momentum conserving delta function gives the dependence on masses. These amplitudes would define as such the M^4 parts of twistor amplitudes for particles massive in TGD sense. The simplest 4-fermion amplitude is unique.

Twistor approach gives excellent hopes about the construction of the scattering amplitudes in ZEO. The construction would split into two pieces corresponding to the orbital degrees of freedom in "world of classical worlds" (WCW) and to spin degrees of freedom in WCW: that is spinors, which correspond to second quantized induced spinor fields at space-time surface (actually string world sheets- either at fundamental level or for effective action implied by strong form of holography (SH)).

1. At WCW level there is a perturbative functional integral over small deformations of the 3-surface to which space-time surface is associated. The strongest assumption is that this 3-surface corresponds to maximum for the real part of action and to a stationary phase for its imaginary part: minimal surface extremal of Kähler action would be in question. A more general but number theoretically problematic option is that an extremal for the sum of Kähler action and volume term is in question.

   By Kähler geometry of WCW the functional integral reduces to a sum over contributions from preferred extremals with the fermionic scattering amplitude multiplied by the ration X_i/X, where $X = \sum_i X_i$ is the sum of the action exponentials for the maxima. The ratios of exponents are however number theoretically problematic.

   Number theoretical universality is satisfied if one assigns to each maximum independent zero energy states: with this assumption $\sum X_i$ reduces to single X_i and the dependence on action exponentials becomes trivial! ZEO allow this. The dependence on coupling parameters of the action essential for the discretized coupling constant evolution is only via boundary conditions at the ends of the space-time surface at the boundaries of CD.

   Quantum criticality of TGD [K16, K17, K18] demands that the sum over loops associated with the functional integral over WCW vanishes and strong form of holography (SH) suggests that the integral over 4-surfaces reduces to that over string world sheets and partonic 2-surfaces corresponding to preferred extremals for which the WCW coordinates parametrizing them belong to the extension of rationals defining the adele [L4]. Also the intersections of the real and various p-adic space-time surfaces belong to this extension.

2. Second piece corresponds to the construction of twistor amplitude from fundamental 4-fermion amplitudes. The diagrams consists of networks of light-like orbits of partonic two surfaces, whose union with the 3-surfaces at the ends of CD is connected and defines a boundary condition for preferred extremals and at the same time the topological scattering diagram.

   Fermionic lines correspond to boundaries of string world sheets. Fermion scattering at partonic 2-surfaces at which 3 partonic orbits meet are analogs of 3-vertices in the sense of Feynman and fermions scatter classically. There is no local 4-vertex. This scattering is assumed to be described by simplest 4-fermion twistor diagram. These can be fused to form more complex diagrams. Fermionic lines runs along the partonic orbits defining the topological diagram.

3. Number theoretic universality [K18] suggests that scattering amplitudes have interpretation as representations for computations. All space-time surfaces giving rise to the same computation wold be equivalent and tree diagrams corresponds to the simplest computation. If the action exponentials do not appear in the amplitudes as weights this could make sense but would require huge symmetry based on two moves. One could glide the 4-vertex at the end of internal fermion line along the fermion line so that one would eventually get the analog of self energy loop, which should allow snipping away. An argument is developed stating that this symmetry is possible if the preferred M_0^2 for which 8-D momentum reduces to light-like M^2-momentum having unique direction is same along entire fermion line, which can wander along the topological graph.

   The vanishing of topological loops would correspond to the closedness of the diagrams in what might be called BCFW homology. Boundary operation involves removal of BCFW bridge and entangled removal of fermion pair. The latter operation forces loops. There would be no BCFW bridges and entangled removal should give zero. Indeed, applied to the proposed four fermion vertex entangled removal forces it to correspond to forward scattering for which the proposed twistor amplitude vanishes.

To sum up, the twistorial approach leads to a proposal for an explicit construction of scattering amplitudes for the fundamental fermions. Bosons and fermions as elementary particles are bound states of fundamental fermions assignable to pairs of wormhole contacts carrying fundamental fermions at the throats. Clearly, this description is analogous to a quark level description of hadron. Yangian symmetry with multilocal generators is expected to crucial for the construction of the many-fermion states giving rise to elementary particles. The problems of the standard twistor approach find a nice solution in terms of $M^8 - H$ duality, 8-D masslessness, and holomorphy of twistor amplitudes in λ_i and their indepence on $\tilde{\lambda}_i$.

2 More details about the induction of twistor structure

The notion of twistor lift of TGD [L2] [L7] has turned out to have powerful implications concerning the understanding of the relationship of TGD to general relativity. The meaning of the twistor lift really has remained somewhat obscure. There are several questions to be answered. What does one mean with twistor space? What does the induction of twistor structure of $H = M^4 \times CP_2$ to that of space-time surface realized as its twistor space mean?

2.1 What does one mean with twistor space?

The notion of twistor space has been discussed in [L2] from TGD point of view.

1. In the case of twistor space of M^4 the starting point of Penrose was the isomorphism between the conformal group of Spin(4,2) of 6-D Minkowski space $M^{4,2}$ and the group SU(2,2) acting on 2+2 complex spinors.

   6-D twistor space could be identified as 6-D coset space $SU(2,2)/SU(2,1)\times U(1)$. For E^6 this would give projective space $CP_3 = SU(4)/SU(3) \times U(1)$ and in twistor Grassmann approach this definition is indeed used. It is thought that the problems caused by Euclidization are not serious.

2. One can think $SU(2,2)$ as 4×4 complex matrices with orthogonal complex row vector $Z_i = (Z_{i1}, ..., Z_{i4})$, and norms $(1, 1, -1 - 1)$ in the metric $s^2 = \sum \epsilon_i |z_i|^2$, $\epsilon_i \leftrightarrow (1, 1, -1, -1)$. The sub-matrices defined by (Z_{k2}, Z_{k3}, Z_{k4}), $k = 2, 3, 4$, can be regarded apart from normalization elements of $SU(1,2)$. The column vector with components Z_{i1} with $Z_{11} = \sqrt{1 + \rho^2}$, $\rho^2 = |Z_{21}|^2 - |Z_{31}|^2 - |Z_{41}|^2$ corresponds to a point of the twistor space. The S^2 fiber for given values of ρ and (Z_{31}, Z_{41}) could be identified as the space spanned by the values of Z_{21}. Note that S^2 would have time-like signature and the signature of twistor space would be (3,3), which conforms with the existence of complex structure. There would be dimensional democracy at this level.

3. The identification of 4-D base of the twistor space is unclear to me. The base space of the this twistor space should correspond to the conformal compactification M^4_c of M^4 having metric defined only apart from conformal scaling. The concrete realization M^4_c would be in terms of $M^{4,2}$ light-cone with points projectively identified. As a metric object this space is ill-defined and can appear only at the level of scattering amplitudes in conformally invariant quantum field theories in M^4.

4. Mathematicians define also a second variant of twistor space with S^2 fiber and this space is just $M^4 \times S^2$ [B20] (see `http://tinyurl.com/yb4bt74l`). This space has a well-defined metric and seems to be the only possible one for the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces. Whether the signature of S^2 is time-like or space-like has remained an open question but time-like signature looks natural. The radius R_P of S^2 has been proposed to be apart from a numerical constant equal to Planck length l_P. Note that the isometry group is 9-D $SO(3,1) \times SU(2)$ rather than 15-D $SU(2,2)$. In TGD light-likeness in 8-D sense replaces light-likeness in 4-D sense: does this somehow replace the conformal symmetry group $SO(4,2)$ with $SO(3,1) \times SO(3)$? Could $SU(2)$ rotate the direction of spin quantization axis.

I must confess that I have found the notions of twistor and twistor sphere very difficult to understand. Perhaps this is not solely due to my restricted mathematical skills. Also the physics of twistors looks confusing to me.

The twistor space assignable to Minkowski space and corresponding twistor sphere have several meanings. Consider first the situation in standard framework.

1. One can define twistor space as complex 8-D space C^4. Given four-momentum corresponds however to projective line so that one can argue that twistor space is 6-D space $T_1(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$ of projective lines of C^4 in C^4. One could also argue that one must take the signature of Minkowski space into account. $SU(2,2)$ acts as symmetries of twistor bilinear form and one would have $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$. In this case twistor sphere could correspond to the projective line in C^4.

2. Incidence relations $\mu^{\dot{a}} = m^{a\dot{a}}\lambda_a$ relate M^4 points to those of twistor space. In the usual twistor formalism twistor sphere corresponds to the projective line of 8-D C^4. When m is not light-like, it corresponds to a matrix which is invertible and one can solve μ from λ and vice versa. The twistor spheres associated with m_1 and m_2 are said to intersect if $m_1 - m_2$ is a complex light-like vector defining a complexified light ray. One could identify twistor sphere of $T_1(M^4)$ as the Riemann sphere defined by these complex points and going to CP_3 one actually eliminates it altogether, which is somewhat unsatisfactory.

3. When m is light-like and thus expressible as $\mu = \lambda \otimes \tilde{\lambda}$ one has $\mu = \mu_0 + t\tilde{\lambda}$, t a complex number. One can say that one has a full Riemann sphere S^2 of solutions. There is also additional degeneracy due to the scaling of both λ and μ. For light-like M^4 points (say momenta) one obtains a Riemann sphere in 6-D twistor space. Which twistor sphere is the correct one: the sphere associated with all points of M^4 and 8-D twistor space or the sphere associated with light-like points of M^4 and 6-D twistor space?

Consider now the situation in TGD.

1. For the twistor lift of Kähler action lifting the dynamics of space-time surfaces to the dynamics of their twistor spaces, the twistor lift of M^4 corresponds to $T(M^4) = M^4 \times CP_2$. This might look strange but the proper mathematical definition of twistor space relies on double fibration involving both views about twistor space discussed in [B20] (see `http://tinyurl.com/yb4bt74l`). This double fibration would be crucially involved with $M^8 - H$ duality. The fiber space is $T(M^4) = M^4 \times CP_1$, where CP_1 corresponds to the projective sphere assignable to complex spinors λ. This fiber is trivially projected both to M^4 and less trivially to a subset of 6-dimensional complex projective space $T_1(M^4) = CP_3$.

   At space-time level $T(M^4)$ is the only correct choice since twistor space must have isometries of M^4. This choices brings into the dynamics Planck length essentially as the radius of

S^2 and cosmological constant as volume term resulting in the dimensional reduction of 6-D Kähler action forced by twistor space property of 6-surface.

At the level of momentum space - perhaps the M^8 appearing in M^8-H duality identifiable as tangent space of H - the twistor space would correspond to twistor space assignable to momentum space and should relate to the ordinary twistor space $T_1(M^4)$ - whatever it is!

2. In M^8 picture the twistor space is naturally associated with preferred $M^2 \subset M^4$, where M^4 is quaternionic space. The moduli space of $M^2 \subset M^4$ for time direction assigned with real octonion, is parametrized by S^2 and a possible interpretation is as twistor sphere of $M^2 \times CP_1$. Interestingly, $M^2 \subset M^4$ is characterized by light-like vector together with its unique dual light-like vector.

   By restricting 4-D conformal invariance to 2-D situation, one finds that the twistor space becomes RP_3 but can be complexified to CP_3 to allowing complexified M^2 momenta. The signature (1,1) of M^2 and reality of spinor basis gives hopes of resolving the conceptual problems of the ordinary twistor approach. For the real spinor spinor pair (λ,μ) the solutions to the co-incidence relations real M^2 spinors but one can allowing their complex multiples.

3. M^8-H correspondence allows to map M^4 points to each other: this involves a choice of $M^4 \subset M^8$. M^8-H correspondence maps quaternionic (and co-quaternionic) surfaces in M^8 to preferred extremals of Kähler in H proposed to correspond to the base bases of of twistor bundles $T(X^4) \subset T(M^4) \times T(CP_2)$ constructible using holomorphic maps. One can thus argue that there should be also a correspondence between the twistor spaces $T(M^4)$ and $T_1(M^4)$ - the correspondence between the twistor spheres would be enough.

   The two M^4:s correspond to each other naturally. What is required is a map of twistorial spheres S^2 to each other. Suppose that the twistorial sphere of H corresponds to that assignable to the choice of $M^2 \subset M^8$ by a choice of quaternionic imaginary unit in M^4 of equivalently by a choice of a light-like vector n of M^2 plane. But by incidence relations the light-like vector n has twistor sphere CP_1 as a pre-image in complexified $T_1(M^2)=CP_3$ characterized by the shifts $\mu \rightarrow \mu+\tilde{\lambda}$. Therefore the two twistor spheres can be identified by mapping n of $S^2(T(M^4)$ to its counterpart of $T_1(M^2)$ isometrically.

It therefore seems that the double fibration is essential in TGD framework and the usual twistor space is assignable to the M^8 interpreted asthe space of complexified octonion momenta subject to the quaternionicity condition. Sharply defined transversed quaternionic momentum eigenstates in $E^2 \times E^4$ are replaced with wave functions in $T(CP_2)$ reducing locally to $CP_2 \times U(2)/U(1) \times U(1)$ with em charge identifiable as the analog of angular momentum for the wave functions in $CP_1 = U(2)/U(1) \times U(1)$. In $M^4 \times CP_2$ picture one has spinor modes labelled by electroweak quantum numbers.

2.2 Twistor lift of TGD

In TGD one replaces imbedding space $H = M^4 \times CP_2$ with the product $T = T(M^4) \times T(CP_2)$ of their 6-D twistor spaces, and calls $T(H)$ the twistor space of H. For CP_2 the twistor space is the flag manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ consisting of all possible choices of quantization axis of color isospin and hypercharge.

1. The basic idea is to generalize Penrose's twistor program by lifting the dynamics of space-time surfaces as preferred extremals of Kähler action to those of 6-D Kähler action in twistor space $T(H)$. The conjecture is that field equations reduce to the condition that the twistor structure of space-time surface as 4-manifold is the twistor structure induced from $T(H)$.

   Induction requires that dimensional reduction occurs effectively eliminating twistor fiber $S^2(X^4)$ from the dynamics. Space-time surfaces would be preferred extremals of 4-D Kähler action plus volume term having interpretation in terms of cosmological constant. Twistor lift would be more than an mere alternative formulation of TGD.

2. The reduction would take place as follows. The 6-D twistor space $T(X^4)$ has S^2 as fiber and can be expressed locally as a Cartesian product of 4-D region of space-time and of S^2. The

signature of the induced metric of S^2 should be space-like or time-like depending on whether the space-time region is Euclidian or Minkowskian. This suggests that the twistor sphere of M^4 is time-like as also standard picture suggests.

3. Twistor structure of space-time surface is induced to the allowed 6-D surfaces of $T(H)$, which as twistor spaces $T(X^4)$ must have fiber space structure with S^2 as fiber and space-time surface X^4 as base. The Kähler form of $T(H)$ expressible as a direct sum

$$J(T(H)) = J(T(M^4)) \oplus J(T(CP_2))$$

induces as its projection the analog of Kähler form in the region of $T(X^4)$ considered.

There are physical motivations (CP breaking, matter antimatter symmetry, the well-definedness of em charge) to consider the possibility that also M^4 has a non-trivial symplectic/Kähler form of M^4 obtained as a generalization of ordinary symplectic/Kähler form [L7]. This requires the decomposition $M^4 = M^2 \times E^2$ such that M^2 has hypercomplex structure and E^2 complex structures.

This decomposition might be even local with the tangent spaces $M^2(x)$ and $E^2(x)$ integrating to locally orthogonal 2-surfaces. These decomposition would define what I have called Hamilton-Jacobi structure [K12]. This would give rise to a moduli space of M^4 Kähler forms allowing besides covariantly constant self-dual Kähler forms with decomposition (m^0, m^3) and (m^1, m^2) also more general self-dual closed Kähler forms assignable to integrable local decompositions. One example is spherically symmetric stationary self-dual Kähler form corresponding to the decomposition (m^0, r_M) and (θ, ϕ) suggested by the need to get spherically symmetric minimal surface solutions of field equations. Also the decomposition of Robertson-Walker coordinates to (a, r) and (θ, π) assignable to light-cone M^4_+ can be considered.

The moduli space giving rise to the decomposition of WCW to sectors would be finite-dimensional if the integrable 2-surfaces defined by the decompositions correspond to orbits of subgroups of the isometry group of M^4 or CD. This would allow planes of M^4, and radial half-planes and spheres of M^4 in spherical Minkowski coordinates and of M^4_+ in Robertson-Walker coordinates. These decomposition could relate to the choices of measured quantum numbers inducing symmetry breaking to the subgroups in question. These choices would chose a sector of WCW [K5] and would define quantum counterpart for a choice of quantization axes as distinct from ordinary state function reduction with chosen quantization axes.

4. The induced Kähler form of S^2 fiber of $T(X^4)$ is assumed to reduce to the sum of the induced Kähler forms from S^2 fibers of $T(M^4)$ and $T(CP_2)$. This requires that the projections of the Kähler forms of M^4 and CP_2 to $S^2(X^4)$ are trivial. Also the induced metric is assumed to be direct sum and similar conditions holds true.These conditions are analogous to those occurring in dimensional reduction.

Denote the radii of the spheres associated with M^4 and CP_2 as $R_P = kl_P$ and R and the ratio R_P/R by ϵ. Both the Kähler form and metric are proportional to R_p^2 resp. R^2 and satisfy the defining condition $J_{kr}g^{rs}J_{sl} = -g_{kl}$. This condition is assumed to be true also for the induced Kähler form of $J(S^2(X^4)$.

Let us introduce the following shorthand notations

$$\begin{array}{lll} S_1^2 = S^2(X^4) \ , & S_2^2 = S^2(CP_2) \ , & S_3^2 = S^2(M^4) \ , \\ J_i = \frac{J(S_i^2)}{R^2} \ , & g_i = \frac{g(S_i^2))}{R^2} \ . & \end{array} \quad (2.1)$$

This gives the following equations.

$$J_1 = J_2 + \epsilon J_3 \ , \quad g_1 = g_2 + \epsilon g_3 \ , \quad J_1 g_1 J_1 = -g_1 \ . \quad (2.2)$$

Projections to $S^2_1 = S^2(X^4)$ are assumed at r.h.s.. The product of the third equation is defined as tensor contraction and involves contravariant form of g.

2.3 Solutions to the conditions defining the twistor lift

Consider now solutions to the conditions defining the twistor lift.

1. The simplest solution type corresponds to the situation in which either S^2_2 (S^2_3) equals to S^2_1) and S^2_3 (S^2_2) projection of $T(X^4)$ is single point. In this case the conditions of Eq. are trivially satisfied. These two solutions could correspond to Euclidian and Minkowskian space-time regions. Also the solution for which twistor sphere degenerates to a point must be considered and form $J(M^4) = 0$ this would correspond to the reduction of dimensionally reduced action to Kähler action defining the original variant of TGD. Note that preferred extremals are conjectured to be minimal surfaces extremals of Kähler action always [L1].

2. One can consider also more general solutions. Depending on situation, one can use for $S^2(X^4)$ either the coordinates of S^2_2 or S^2_3. Let us choose S^2_2. One can of course change the roles of the spheres.

   Consider an ansatz for which the projections of J_2 and J_3 to S^2_1 are in constant proportionality to each other. This is guaranteed if the spherical coordinates ($u = cos(\Theta), \Phi$) of S^2_2 and S^2_3 are related by $(u(M^4), \Phi(M^4)) = (u(CP_2), n\Phi(CP_2))$ so that the map between the two spheres has winding number n. With this assumption one has

$$\begin{aligned} J_1 &= (1+\epsilon n)J_2 \ , \\ g_1 &= (1+\epsilon n^2)g_2 \ , \end{aligned} \tag{2.3}$$

   The third condition of Eq. 1 equation gives

$$(1+n\epsilon)^2 \quad = \quad (1+n^2\epsilon)^2 \ . \tag{2.4}$$

   This in turn gives

$$1+n\epsilon = \delta(1+n^2\epsilon) \ , \quad \delta = \pm 1 \ . \tag{2.5}$$

   The only solution for $\delta = +1$ is $n = 0$ or $n = 1$. For $\delta = -1$ there are no solutions.

   One has 3+1 different solutions corresponding to the degenerate solution $(n_1, n_2) = (0, 0)$ and 3 solutions with (n_1, n_2) equal $(1, 0)$, $(0, 1)$ or $(1, 1)$. The conditions are very stringent and it is not clear whether there are any other solutions.

3. The further conditions implying locally direct sum for g and J pose strong restrictions on space-time surfaces. The conjecture that the solutions of these conditions correspond to preferred extremals of 6-D Kähler action leads by dimensional reduction to the conclusion that the 4-D action contains besides 4-D Kähler action also a volume term coming from S^2 Kähler actions and giving rise to cosmological constant.

   What is of special interest is that for the degenerate solution the volume term vanishes, and one has mere 4-D Kähler action with induced Kähler form possibly containing also $J(M^4)$, which leads to a rather sensible cosmology having interpretation as infinite volume limit for causal diamond (CD) inside which space-time surfaces exist. This limit could be appropriate for QFT limit of TGD, which indeed corresponds to infinite-volume limit at which cosmological constant approaches zero.

What could be the physical interpretation of the solutions?

1. Physical intuition suggests that S^2_1 must be space-like for Euclidian signature of space-time region $[(n_1, n_2) = (1, 0)]$ and time-like for Minkowskian signature $[(n_1, n_2) = (0, 1)]$.

2. By quantum classical correspondence one can argue that the non-vanishing of space-time projection of $J(M^4)$ resp. $J(CP_2)$ is necessary to fix local quantization axis of spin resp. weak isospin. If so, then $n_1 = 1/0$ resp. $n_2 = 1/0$ would tell that the projection of $J(CP_2)$ resp. $J(M^2)$ is non-vanishing/vanishes. If both contributions vanish $[(n_1, n_2) = (0, 0)]$ one has generalized Lagrangian 4-surface, which would be vacuum extremal. The products of 2-D Lagrangian manifolds for M^4 and CP_2 would be vacuum extremals. One can wonder whether there exist 4-surfaces representable as a graph of a map $M^4 \to CP_2$ such that the induced Kähler form vanishes. This picture allows only the imbeddings of trivial Robertson-Walker cosmology as vacuum extremal of Kähler action since both M^4 contribution to Kähler action and volume term would be non-vanishing $[(n_1, n_2) = (0, 1)]$.

2.4 Twistor lift and the reduction of field equations and SH to holomorphy

It has become clear that twistorialization has very nice physical consequences. But what is the deep mathematical reason for twistorialization? Understanding this might allow to gain new insights about construction of scattering amplitudes with space-time surface serving as analogs of twistor diatrams.

Penrose's original motivation for twistorilization was to reduce field equations for massless fields to holomorphy conditions for their lifts to the twistor bundle. Very roughly, one can say that the value of massless field in space-time is determined by the values of the twistor lift of the field over the twistor sphere and helicity of the massless modes reduces to cohomology and the values of conformal weights of the field mode so that the description applies to all spins.

I want to find the general solution of field equations associated with the Kähler action lifted to 6-D Kähler action. Also one would like to understand strong form of holography (SH). In TGD fields in space-time are are replaced with the imbedding of space-time as 4-surface to H. Twistor lift imbeds the twistor space of the space-time surface as 6-surface into the product of twistor spaces of M^4 and CP_2. Following Penrose, these imbeddings should be holomorphic in some sense.

Twistor lift $T(H)$ means that M^4 and CP_2 are replaced with their 6-D twistor spaces.

1. If S^2 for M^4 has 2 time-like dimensions one has 3+3 dimensions, and one can speak about hyper-complex variants of holomorphic functions with time-like and space-like coordinate paired for all three hypercomplex coordinates. For the Minkowskian regions of the space-time surface X^4 the situation is the same.

2. For $T(CP_2)$ Euclidian signature of twistor sphere guarantees this and one has 3 complex coordinates corresponding to those of S^2 and CP_2. One can also now also pair two real coordinates of S^2 with two coordinates of CP_2 to get two complex coordinates. For the Euclidian regions of the space-time surface the situation is the same.

Consider now what the general solution could look like. Let us continue to use the shorthand notations $S^2_1 = S^2(X^4); S^2_2 = S^2(CP_2); S^2_3 = S^2(M^4)$.

1. Consider first solution of type $(1, 0)$ so that coordinates of S^2_2 are constant. One has holomorphy in hypercomplex sense (light-like coordinate $t - z$ and $t + z$ correspond to hypercomplex coordinates).

   (a) The general map $T(X^4)$ to $T(M^4)$ should be holomorphic in hyper-complex sense. $S^2_1$is in turn identified with S^2_3 by isometry realized in real coordinates. This could be also seen as holomorphy but with different imaginary unit. One has analytical continuation of the map $S^2_1 \to S^2_3$ to a holomorphic map. Holomorphy might allows to achieve this rather uniquely. The continued coordinates of S^2_1 correspond to the coordinates assignable with the integrable surface defined by $E^2(x)$ for local $M^2(x) \times E^2(x)$ decomposition of the local tangent space of X^4. Similar condition holds true for $T(M^4)$. This leaves only $M^2(x)$ as dynamical degrees of freedom. Therefore one has only one holomorphic function defined by 1-D data at the surface determined by the integrable distribution of $M^2(x)$ remains. The 1-D data could correspond to the boundary of the string world sheet.

(b) The general map $T(X^4)$ to $T(CP_2)$ cannot satisfy holomorphy in hyper-complex sense. One can however provide the integrable distribution of $E^2(x)$ with complex structure and map it holomorphically to CP_2. The map is defined by 1-D data.

(c) Altogether, 2-D data determine the map determining space-time surface. These two 1-D data correspond to 2-D data given at string world sheet: one would have SH.

2. What about solutions of type $(0,1)$ making sense in Euclidian region of space-time. One has ordinary holomorphy in CP_2 sector.

   (a) The simplest picture is a direct translation of that for Minkowskian regions. The map $S^2_1 \to S^2_2$ is an isometry regarded as an identification of real coordinates but could be also regarded as holomorphy with different imaginary unit. The real coordinates can be analytically continued to complex coordinates on both sides, and their imaginary parts define coordinates for a distribution of transversal Euclidian spaces $E^2_2(x)$ on X^4 side and $E^2(x)$ on M^4 side. This leaves 1-D data.

   (b) What about the map to $T(M^4)$? It is possible to map the integrable distribution $E^2_2(x)$ to the corresponding distribution for $T(M^4)$ holomorphically in the ordinary sense of the word. One has 1-D data. Altogether one has 2-D data and SH and partonic 2-surfaces could carry these data. One has SH again.

3. The above construction works also for the solutions of type $(1,1)$, which might make sense in Euclidian regions of space-time. It is however essential that the spheres S^2_2 and S^2_3 have real coordinates.

SH thus would thus emerge automatically from the twistor lift and holomorphy in the proposed sense.

1. Two possible complex units appear in the process. This suggests a connection with quaternion analytic functions [L2] suggested as an alternative manner to solve the field equations. Space-time surface as associative (quaterionic) or co-associate (co-quaternionic) surface is a further solution ansatz.

   Also the integrable decompositions $M^2(x) \times E^2(x)$ resp. $E^2_1(x) \times E^2_2(x)$ for Minkowskian resp. Euclidian space-time regions are highly suggestive and would correspond to a foliation by string wold sheets and partonic 2-surfaces. This expectation conforms with the number theoretically motivated conjectures [K18].

2. The foliation gives good hopes that the action indeed reduces to an effective action consisting of an area term plus topological magnetic flux term for a suitably chosen stringy 2-surfaces and partonic 2-surfaces. One should understand whether one must choose the string world sheets to be Lagrangian surfaces for the Kähler form including also M^4 term. Minimal surface condition could select the Lagrangian string world sheet, which should also carry vanishing classical W fields in order that spinors modes can be eigenstates of em charge.

   The points representing intersections of string world sheets with partonic 2-surfaces defining punctures would represent positions of fermions at partonic 2-surfaces at the boundaries of CD and these positions should be able to vary. Should one allow also non-Lagrangian string world sheets or does the space-time surface depend on the choice of the punctures carrying fermion number (quantum classical correspondence)?

3. The alternative option is that any choice produces of the preferred 2-surfaces produces the same scattering amplitudes. Does this mean that the string world sheet area is a constant for the foliation - perhaps too strong a condition - or could the topological flux term compensate for the change of the area?

   The selection of string world sheets and partonic 2-surfaces could indeed be also only a gauge choice. I have considered this option earlier and proposed that it reduces to a symmetry identifiable as U(1) gauge symmetry for Kähler function of WCW allowing addition to it of a real part of complex function of WCW complex coordinates to Kähler action. The additional term in the Kähler action would compensate for the change if string world sheet action in SH. For complex Kähler action it could mean the addition of the entire complex function.

A couple of questions remain to be pondered.

1. In TGD the induced spinor structure need not be equivalent with the ordinary spinor structure. For instance, induced gamma matrices are not covariantly constant and spinors are imbedding space spinors. Induced spinor structure saves also from problems. Induced spinor structure exists even when standard twistor structure fails to do so. Induced spinor structure is also unique unlike the ordinary spinor structure. A practical example relates to the difficulty of the lattice QCD as thermodynamics with periodic boundary conditions in a box: there are $2^4 = 16$ spinor structures.

   In the same way, there is no need to expect or require that the induced twistor structure reduces to ordinary one: it is enough to require that the S^2 bundle structure implied by the proposed dimensional reduction of 6-D surfaces to S^2 bundles having space-time surface as a base space takes place. This would simplify the construction in an essential manner.

2. Space-time surface can be identified as a section of twistor bundle. For physical reasons this section should not only exist but be global and unique. For general bundles this need not be the case. For non-trivial principal bundles one cannot find any sections. The tangent bundle of sphere does not allow a global everywhere non-vanishing section. Could some additional condition guarantee that the section exists and is unique? In algebraic geometry additional conditions such as holomorphy can fix the global section highly uniquely.

   Now the variational principle reducing the construction to finding of space-time surfaces as an extremal of dimensionally reduced Kähler action guarantees both the existence and uniqueness. This also gives the reason why for the twistor lift of Kähler action: one cannot only assume that the 6-surface equals to ordinary twistor bundle of some 4-surface since in this case the section need not be unique.

3 How does the twistorialization at imbedding space level emerge?

An objection against twistorialization at imbedding space level is that M^4-twistorialization requires 4-D conformal invariance and massless fields. In TGD one has towers of particle with massless particles as the lightest states. The intuitive expectation is that the resolution of the problem is that particles are massless in 8-D sense as also the modes of the imbedding space spinor fields are.

To explain the idea, let us select a fixed decomposition $M^8 = M_0^4 \times E_0^4$ and assume that the momenta are complex - for motivations see below.

1. With inspiration coming from M^8-H duality [K10] suppose that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed, and corresponding to real octonionic unit and preferred imaginary unit. Obviously 8-D light-likeness for $M^8 = M_0^4 \times E_0^4$ reduces to 4-D light-likeness for a preferred choice of $M^8 = M^4 \times CP_2$ decomposition.

2. This suggests that in the case of massive M_0^4 momenta one can apply twistorialization to the light-like M^4-momentum and code the information about preferred M^4 by a point of CP_2 and about 8-momentum in $M^8 = M_0^4 \times E_0^4$ by an $SU(3)$ transformation taking M_0^4 to M^4. Pairs of twistors and $SU(3)$ transformations would characterize arbitrary quaternionic 8-momenta. 8-D masslessness gives however 2 additional conditions for the complex 8-momenta probably reducing $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2! This would also solve the basic problem of twistor approach created by the existence of massive particles.

The assumption of complex momenta in previous considerations might raise some worries. The space-time action of TGD is however complex if Kähler coupling strength is complex, and there are reasons to believe that this is the case. Both four-momenta and color quantum numbers -

all Noether charges in fact - could be complex. A possible physical interpretation for complex momenta could be in terms of the natural width of states induced by the finite size of CD. Also in twistor Grassmannian approach one encounters complex but light-like four-momenta. Note that complex light-like space-time momenta correspond in general to massive real momenta. It is not clear whether it makes sense to speak about width of color quantum numbers: their reality would give additional constraint. The emergence of M^4 mass in this manner could be involved with the classical description for the emergence of the third helicity.

The observation that octonionic twistors make sense and their restriction to quaternionic twistors produce ordinary M^4 twistors provides an alternative view point to the problem. Also $M^8 - H$ duality proposed to map quaternionic 4-D surfaces in octonionic M^8 to (possibly quaternionic) 4-D surfaces in $M^4 \times CP_2$ is expected to be relevant. The twistor lift of $M^8 - H$ duality would give $T(M^8) - T(H)$ duality.

Twistor Grassmann approach [B8, B5, B4, B12, B13, B3] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ whereas the twistor lift of classical TGD uses $M^4 \times S^2$. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identified as surfaces in $T(M^4) \times T(CP_2)$ requires the mapping of these twistor spaces to each other - the incidence relations of Penrose indeed realize this map.

3.1 $M^8 - H$ duality at space-time level

Twistors emerge as a description of massless particles with spin [B19] but are not needed for spin zero particles. Therefore one can consider first mere momenta.

1. Consider first space-time surfaces of M^8 with Minkowskian signature of the induced metric so that the tangent space is M^4. $M^8 - H$ duality [K10] implies that CP_2 points parameterize quaternionic sub-spaces M^4 of octonions containing fixed $M_0^2 \subset M^4$. Using the decomposition $1+1+3+\overline{3}$ of complexified octonions to representations of $SU(3)$, it is easy to see that this space is indeed CP_2. M^4 correspond to the sub-space $1+1+2$ where 2 is $SU(2) \subset SU(3)$ doublet.

   CP_2 spinor mode would be spinor mode in the space of quaternionic sub-spaces $M^4 \subset M^8$ with $M_0^2 \subset M^4$ with real octonionic unit defining preferred time like direction and imaginary unit defining preferred spin quantization axis. $M^8 - H$ duality allows to map quaternionic 4-surfaces of $M^4 \supset M_0^2$ to 4-surfaces in H. The latter could be quaternionic but need not to.

2. For Euclidian signature of the induced metric tangent space is E^4. In this case co-associative surfaces are needed since the above correspondence make sense only if the tangent space corresponds to M^4. For instance, for CP_2 type exremals tangent space corresponds to E^4. M^4 and E^4 change roles. Also now the space of co-associative tangent spaces is CP_2 since co-associative tangent space is the octonionic orthogonal complement of the associative tangent space. One would have Euclidian variant of the associative case.

$M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the level, which deserves to be called fundamental. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4? In any case, one should demonstrate that the spectrum of states with $M^4 \times E^4$ with quaternionic light-like 8-momenta is equivalent with the spectrum of states for $M^4 \times CP_2$

3.2 Parametrization of light-like quaternionic 8-momenta in terms of $T(CP_2)$

The following argument shows that the twistor space $T(CP_2)$ emerges naturally from $M^8 - H$ correspondence for quaternionic light-like M^8 momenta.

1. Continue to assume a fixed decomposition $M^8 = M_0^4 \times E_0^4$, and that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed. Light-like quaternionic

8-momentum in $M^8 = M_0^4 \times E_0^4$ can be reduced to light-like M^4 momentum and vanishing E^4 momentum for some preferred $M^8 = M^4 \times E^4$ decomposition.

One can therefore describe the situation in terms of light-like M^4-momentum and $U(2)$ transformation (as it turns out) mapping this momentum to 8-D momentum in given frame and giving the M_0^4 and E_0^4 momenta. The alternative description is in terms M_0^4 massive momentum and the E_0^4 momentum. The space of light-like complex M^4 momenta with fixed M_0^2 part and non-vanishing E^2 part is given by CP_2 as also the space of quaternionic planes. Given quaternionic plane is in turn characterized by massless M^4-momentum.

2. The description of M^4-massive momentum should be based on twistor associated with the light-like M^4 momentum plus something describing the $SU(3)$ transformation leaving the preferred imaginary unit of M_0^2 un-affected. The transformations leaving unaffected the M^4 part of M^8-momentum coded by the $SU(2)$ doublet 2 of color triplet 3 in the color decomposition of complex 8-momentum $1+1+3+\overline{3}$ but acting on E^4 part $1+\overline{3}$ non-trivially correspond to $U(2)$ subgroup. $U(2)$ element thus codes for the E^4 part of the light-like momentum and $SU(3)$ code for quaternionic 8-momenta, which can be also massive. Massless and complex M^4 momenta are coded by $SU(3)/U(2) = CP_2$ as also the tangent spaces of Minkowskian space-time regions (by $M^8 - H$ duality).

   The complexity of particle 8-momenta -and more generally Noether charges - is not in conflict with the hermiticity of quantal Noether charges if total classical and quantal Noether charges are real (and equal by QCC). This would give rise to a kind of confinement condition applying to many-particle states. I have earlier proposed that single particle conformal weights are complex but that conformal confinement holds in the sense that the total conformal weights are real.

3. General complex quaternionic momenta with fixed M^4 part are parameterized by $SU(3)$. Complex light-like 8-momenta satisfy two additional constraints from light-likenes condition, and one expects the reduction of $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2. Therefore the light-like 8-momentum is coded by a twistor assignable to massless M^4-momentum by an point of $SU(3)/U(1) \times U(1)$ giving $T(M^4) \times T(CP_2)$.

By the previous arguments, the inclusion of helicities and electroweak charges gives twistor lift of $M^8 - H$ correspondence.

1. In the case of E^4 the helicities would correspond to two $SO(4)$ spins to be mapped to right and left-handed electroweak spins or weak spin and weak charges. Twistor space $T(CP_2)$ gives hopes about a unified description of color - and electro-weak quantum numbers in terms of partial waves in the space $SU(3)/U(1) \times U(1)$ for selections of quantization axes for color quantum numbers.

2. A possible problem relates to the particles massive in M^4 sense having more helicity states than massless particles. How can one describe the presence of additional helicities. Should one introduce the analog of Higgs mechanism providing the missing massless helicities? Quantum view about twistors describes helicity as a quantum number - conformal weight - of a wave function in the twistor sphere S^2. In the case of massive gauge bosons which would require the introduction of zero helicity as a spin 0 wave function in twistor space.

3. One should relate the description in terms of M^8 momenta to the description in terms of $M^4 \times CP_2$ color partial waves massless in 8-D sense. The number of partial waves for given CP_2 mass squared is finite and this should be the case for quaternionic E^4 momenta. How color quantum numbers determining the M^4 mass relate to complex E^4 momenta parameterized by $U(2)$ plus two constraints coming from complex light-likeness. The number of degrees of freedom is 2 for given $U(2)$ orbit and the quantization suggests dramatic reduction in the number of 8-momenta. This strongly suggests that it is only possible to talk about wave functions in the space of allowed E^4 momenta - that is in the twistor space $T(CP_2)$. Fixing the M^4-part of 8-momentum parameterized by a point of CP_2 leaves only a wave function in the fiber S^2.

The discussion leaves some questions to ponder.

1. $M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the fundamental level. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4?

2. What about more general $SO(1,7)$ transformations? Are they needed? One could consider the possibility that $SO(1,7)$ acts in the moduli space of octonion structures of M^8. If so, then these additional moduli must be included. Otherwise given 8-D momenta have M_0^2 part fixed and orbit of given M^4 momentum is the smaller, the smaller the E^2 part of M^4 momentum is. It reduces to point if M^4 momentum reduces to M_0^2.

3.3 A new view about color, color confinement, and twistors

To my humble opinion twistor approach to the scattering amplitudes is plagued by some mathematical problems. Whether this is only my personal problem is not clear.

1. As Witten shows in [B8], the twistor transform is problematic in signature (1,3) for Minkowski space since the the bi-spinor μ playing the role of momentum is complex. Instead of defining the twistor transform as ordinary Fourier integral, one must define it as a residue integral. In signature (2,2) for space-time the problem disappears since the spinors μ can be taken to be real.

2. The twistor Grassmannian approach works also nicely for (2,2) signature, and one ends up with the notion of positive Grassmannians. Could it be that something is wrong with the ordinary view about twistorialization rather than only my understanding of it?

3. For M^4 the twistor space should be non-compact $SU(2,2)/SU(2,1) \times U(1)$ rather than $CP_3 = SU(4)/SU(3) \times U(1)$, which is taken to be. I do not know whether this is only about short-hand notation or a signal about a deeper problem.

4. Twistorilizations does not force SUSY but strongly suggests it. The super-space formalism allows to treat all helicities at the same time and this is very elegant. This however forces Majorana spinors in M^4 and breaks fermion number conservation in $D = 4$. LHC does not support $\mathcal{N} = 1$ SUSY. Could the interpretation of SUSY be somehow wrong? TGD seems to allow broken SUSY but with separate conservation of baryon and lepton numbers.

In number theoretic vision something rather unexpected emerges and I will propose that this unexpected might allow to solve the above problems and even more, to understand color and even color confinement number theoretically. First of all, a new view about color degrees of freedom emerges at the level of M^8.

1. One can always find a decomposition $M^8 = M_0^2 \times E^6$ so that the possibly complex light-like quaternionic 8-momentum restricts to M_0^2. The preferred octonionic imaginary unit represent the direction of imaginary part of quaternionic 8-momentum. The action of G_2 to this momentum is trivial. Number theoretic color disappears with this choice. For instance, this could take place for hadron but not for partons which have transversal momenta.

2. One can consider also the situation in which one has localized the 8-momenta only to $M^4 = M_0^2 \times E^2$. The distribution for the choices of $E^2 \subset M_0^2 \times E^2 = M^4$ is a wave function in CP_2. Octonionic $SU(3)$ partial waves in the space CP_2 for the choices for $M_0^2 \times E^2$ would correspond ot color partial waves in H. The same interpretation is also behind $M^8 - H$ correspondence.

3. The transversal quaternionic light-like momenta in $E^2 \subset M_0^2 \times E^2$ give rise to a wave function in transversal momenta. Intriguingly, the partons in the quark model of hadrons have only precisely defined longitudinal momenta and only the size scale of transversal momenta can be specified. This would of course be a profound and completely unexpected connection! The introduction of twistor sphere of $T(CP_2)$ allows to describe electroweak charges and brings in CP_2 helicity identifiable as em charge giving to the mass squared a contribution proportional to Q_{em}^2 so that one could understand electromagnetic mass splitting geometrically.

The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4) + J(CP_2)$. Em charge is the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere is natural.

4. In general case the M^2 component of momentum would be massive and mass would be equal to the mass assignable to the E^6 degrees of freedom. One can however always find $M_0^2 \times E^6$ decomposition in which M^2 momentum is light-like. The naive expectation is that the twistorialization in terms of M^2 works only if M^2 momentum is light-like, possibly in complex sense. This however allows only forward scattering: this is true for complex M^2 momenta and even in M^4 case.

   The twistorial 4-fermion scattering amplitude is however *holomorphic* in the helicity spinors λ_i and has no dependence on $\tilde{\lambda}_i$. Therefore carries no information about M^2 mass! Could M^2 momenta be allowed to be massive? If so, twistorialization might make sense for massive fermions!

M_0^2 momentum deserves a separate discussion.

1. A sharp localization of 8-momentum to M_0^2 means vanishing E^2 momentum so that the action of $U(2)$ would becomes trivial: electroweak degree of freedom would simply disappear, which is not the same thing as having vanishing em charge (wave function in $T(CP_2)$ twistorial sphere S^2 would be constant). Neither M_0^2 localization nor localization to single M^4 (localization in CP_2) looks plausible physically - consider only the size scale of CP_2. For the generic CP_2 spinors this is impossible but covariantly constant right-handed neutrino spinor mode has no electro-weak quantum numbers: this would most naturally mean constant wave function in CP_2 twistorial sphere.

   For the preferred extremals of twistor lift of TGD either M^4 or CP_2 twistor sphere can effectively collapse to a point. This would mean disappearence of the degrees of freedom associated with M^4 helicity or electroweak quantum numbers.

2. The localization to $M^4 \supset M_0^2$ is possible for the tangent space of quaternionic space-time surface in M^8. This could correlate with the fact that neither leptonic nor quark-like induced spinors carry color as a spin like quantum number. Color would emerge only at the level of H and M^8 as color partial waves in WCW and would require de-localization in the CP_2 cm coordinate for partonic 2-surface. Note that also the integrable local decompositions $M^4 = M^2(x) \times E^2(x)$ suggested by the general solution ansätze for field equations are possible.

3. Could it be possible to perform a measurement localization the state precisely in fixed M_0^2 always so that the complex momentum is light-like but color degrees of freedom disappear? This does not mean that the state corresponds to color singlet wave function! Can one say that the measurement eliminating color degrees of freedom corresponds to color confinement. Note that the subsystems of the system need not be color singlets since their momenta need not be complex massless momenta in M_0^2. Classically this makes sense in many-sheeted space-time. Colored states would be always partons in color singlet state.

4. At the level of H also leptons carry color partial waves neutralized by Kac-Moody generators, and I have proposed that the pion like bound states of color octet excitations of leptons explain so called lepto-hadrons [K11]. Only right-handed covariantly constant neutrino is an exception as the only color singlet fermionic state carrying vanishing 4-momentum and living in all possible M_0^2:s, and might have a special role as a generator of supersymmetry acting on states in all quaternionic subs-spaces M^4.

5. Actually, already p-adic mass calculations performed for more than two decades ago [K4, K2, K6], forced to seriously consider the possibility that particle momenta correspond to their projections o $M_0^2 \subset M^4$. This choice does not break Poincare invariance if one introduces moduli space for the choices of $M_0^2 \subset M^4$ and the selection of M_0^2 could define quantization axis of energy and spin. If the tips of CD are fixed, they define a preferred time direction

assignable to preferred octonionic real unit and the moduli space is just S^2. The analog of twistor space at space-time level could be understood as $T(M^4) = M^4 \times S^2$ and this one must assume since otherwise the induction of metric does not make sense.

What happens to the twistorialization at the level of M^8 if one accepts that only M_0^2 momentum is sharply defined?

1. What happens to the conformal group $SO(4,2)$ and its covering $SU(2,2)$ when M^4 is replaced with $M_0^2 \subset M^8$? Translations and special conformational transformation span both 2 dimensions, boosts and scalings define 1-D groups $SO(1,1)$ and R respectively. Clearly, the group is 6-D group $SO(2,2)$ as one might have guessed. Is this the conformal group acting at the level of M^8 so that conformal symmetry would be broken? One can of course ask whether the 2-D conformal symmetry extends to conformal symmetries characterized by hyper-complex Virasoro algebra.

2. Sigma matrices are by 2-dimensionality real (σ_0 and σ_3 - essentially representations of real and imaginary octonionic units) so that spinors can be chosen to be real. Reality is also crucial in signature $(2,2)$, where standard twistor approach works nicely and leads to 3-D real twistor space.

   Now the twistor space is replaced with the real variant of $SU(2,2)/SU(2,1) \times U(1)$ equal to $SO(2,2)/SO(2,1)$, which is 3-D projective space RP^3 - the real variant of twistor space CP_3, which leads to the notion of positive Grassmannian: whether the complex Grassmannian really allows the analog of positivity is not clear to me. For complex momenta predicted by TGD one can consider the complexification of this space to CP_3 rather than $SU(2,2)/SU(2,1) \times U(1)$. For some reason the possible problems associated with the signature of $SU(2,2)/SU(2,1) \times U(1)$ are not discussed in literature and people talk always about CP_3. Is there a real problem or is this indeed something totally trivial?

3. SUSY is strongly suggested by the twistorial approach. The problem is that this requires Majorana spinors leading to a loss of fermion number conservation. If one has $D = 2$ only effectively, the situation changes. Since spinors in M^2 can be chosen to be real, one can have SUSY in this sense without loss of fermion number conservation! As proposed earlier, covariantly constant right-handed neutrino modes could generate the SUSY but it could be also possible to have SUSY generated by all fermionic helicity states. This SUSY would be however broken.

   There is an delicacy involved. If $J(M^4)$ is present, the action of the gauge commutator $[D_k, D_l] = J_{kl}(M^4)$ on right-handed neutrino is non-vanishing and gives rise to the constant term $J^{kl}(M^4)\Sigma_{kl}$ appearing in the square of Dirac equation at imbedding space level. Neutrino would become massive at imbedding space level and also other states receive an additional contribution to mass squared. String world sheets can be however analogs of Lagrangian sub-manifolds so that $J(M^4)$ projected to them vanishes, and one can have massless right-handed neutrino. Also the right- or left M^4-handedness of operator $J^{kl}(M^4)\Sigma_{kl}$ makes it possible to annihilate the spinor mode at string world sheet. The physical interpretation of this picture is still unclear.

4. The selection of M_0^2 could correspond at space-time level to a localization of spinor modes to string world sheets. Could the condition that the modes of induced spinors at string world sheets are expressible using real spinor basis imply the localization? Whether this localization takes place at fundamental level or only for effective action being due to SH, is a question to be settled. The latter options looks more plausible.

To sum up, these observation suggest a profound re-evaluqtion of the beliefs related to color degrees of freedom, to color confinement, and to what twistors really are.

3.4 How do the two twistor spaces assignable to M^4 relate to each other?

Twistor Grassmann approach [B8, B5, B4, B12, B13, B3] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$. Twistor lift of classical TGD uses $M^4 \times S^2$: this seems to be necessary since $T_1(M^4)$ does not allow M^4 as space-space. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identifed as surfaces in $T(M^4) \times T(CP_2)$ is an attractive idea suggesting a very close correspondence with twistor string theory of Witten and construction of scattering amplitudes in twistor Grassmann approach.

One should be able to relate these two twistor spaces and map the twistor spaces $T(X^4)$ identified as surfaces in $T(H) = T(M^4) \times T(CP_2)$ to those in $T_1(H) = T_1(M^4) \times T(CP_2)$. This map is strongly suggested also by twistor string theory. This map raises hopes about the analogs of twistor Grassmann amplitudes based on introduction of $T(CP_2)$.

At least the projections of 2-surfaces to $T(M^4)$ should be mappable to those in $T_1(M^4)$. A stronger condition is that $T(M^4)$ is mappable to $T_1(M^4)$. Incidence relations for twistors $Z = (\lambda, \mu)$ assigning to given M^4 coordinates twistor sphere, are given by

$$\mu_{\dot{\alpha}} = m_{\alpha\dot{\alpha}}\lambda^{\alpha} \quad .$$

This condition determines a 2-D sub-space - complex light ray - of complexified Minkowski space M^4_c. Also complex scaling of Z determines the same sub-space. Therefore twistor sphere corresponds to a complex light ray M^4_c, whose points differ by a shift by a complex light-like vector (λ is null bi-spinor annihilated by light-like m).

Since twistor line (projective sphere) determines a point of M^4_c, two points of twistor sphere labelled by A and B are needed to determined m:

$$m_{\alpha\dot{\alpha}} = \frac{\lambda_{A,\alpha}\mu_{B,\dot{\alpha}}}{\langle\lambda_A\lambda_B\rangle\rangle} + \frac{\lambda_{B,\alpha}\mu_{A,\dot{\alpha}}}{\langle\lambda_B\lambda_A\rangle} \quad .$$

The solutions are invariant under complex scalings $(\lambda, \mu) \to k(\lambda, \mu)$. Therefore co-incidence relations allow to assign projective line - sphere S^2 - to a point of M^4 in $T(M^4)$. This sphere naturally corresponds to S^2 in $T(M^4) = M^4 \times S^2$. This allows to assign pairs $(m \times S^2)$ in $T(M^4)$ to spheres of $T_1(M^4)$ and one can map the projections of 2-surfaces to $T(M^4)$ to $T_1(M^4)$.

Thus one cannot assign M^4 point to single twistor but can map any pair of points at twistor sphere of $T_1(M^4)$ to the same point of M^4 in $T(M^4) = M^4 \times S^2$ and also identify the twistor sphere with S^2. Twistor spheres are labelled by the base space of $T_1(M^4)$ and therefore base space can be mapped to M^4.

Two M^4 points separated by light-like distance correspond to twistor spheres intersecting at one point as is clear from the fact that the difference $m_1 - m_2$ of the points annihilates the twistor λ. $T_1(M^4)$ is singular as fiber bundle over M^4 since the same point of fiber is projected to two different points of M^4.

Could one replace $T(M^4)$ with $T_1(M^4)$ by modifying the induction procedure suitable?

1. $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ has $SU(2,2)$ invariant metric and $SU(2,2)$ corresponds to the 15-D spin covering group of $SO(4,2)$ having $SO(3,1)$ as sub-group. What does one obtain if one induces the metric of the base space of $T_1(M^4)$ to M^4 via the above identification?

   The induced metric would depend on the choice of the base space, and one would have analog of gauge invariance since for a given point of the base the point of the fiber sphere can be chosen freely. A reasonable guess is that the induced metric is determined apart from conformal scaling. One could fix the gauge by - say - assuming that the S^2 point is constant but it is not clear whether this allows to get the flat M^4 metric with any choice.

2. If the twistor sphere of $T_1(M^4)$ has radius of order Planck length l_P, the overall scaling factor of the metric of $T_1(M^4)$ is of order l_P^2. Also the induced M^4 metric would have this scaling factor. For $T_1(M^4)$ one could not perform this scaling. This need not be a problem in $T(M^4)$ since one scale up the flat metric of M^4 by scaling the coordinates. This kind of scaling would in fact smooth out the possible deviations from flat M^4 metric very effectively. In any case, it seems that one must assume that imbedding space corresponds to $T(M^4)$.

3.5 Can the Kähler form of M^4 appear in Kähler action?

I have already earlier considered the question whether the analog of Kähler form assignable to M^4 could appear in Kähler action. Could one replace the induced Kähler form $J(CP_2)$ with the sum $J = J(M^4) + J(CP_2)$ such that the latter term would give rise to a new component of Kähler form both in space-time interior at the boundaries of string world sheets regarded as point-like particles? This could be done both in the Kähler action for the interior of X^4 and also in the topological magnetic flux term $\int J$ associated with string world sheet and reducing to a boundary term giving couplings to U(1) gauge potentials $A_\mu(CP_2)$ and $A_\mu(M^4)$ associated with $J(CP_2)$ and $J(M^4)$. The interpretation of this coupling is an interesting challenge.

3.5.1 Conditions on $J(M^4)$

What conditions one can pose on $J(M^4)$?

1. The simplest possibility is that $J(M^4)$ is covariantly constant and self-dual and satisfies $J^2(M^4) = -g(M^4)$ meaning that $J(M^4)$ *resp.* $g(M^4)$ represents imaginary *resp.* real unit. Hypercomplexity for M^2 would suggest the restriction $J^2(M^2) = g(M^2)$ and $J^2(E^2) = -g(E^2)$. Since complexified octonions are used, it is convenient to include imaginary unit to $J(M^2)$ so that one indeed obtains $J^2(M^4) = -g(M^4)$. $J(M^4)$ would define a global decomposition $M^4 = M^2 \times E^2$ in terms of parallel constant electric and magnetic fields of equal magnitude. CD with this variant of $J(M^4)$ would be naturally associated with planewave like radiative solutions.

2. One could however give up the covariant constancy. In this case spherically symmetric variants of $J(M^4)$ naturally associated with spherically symmetric stationary metric and possible analogs of Robertson-Walker metrics. $J(M^4)$ would be closed except at the world line connecting the tips of CD and carry identical magnetic and electric charges.

3. $J(M^4)$ would define Hamilton Jacobi-structure and an attractive idea is that the orthogonal 2-surfaces associated with the foliation of M^4 are orbits of a subgroup of Poincare group. This structure would characterize quantum measurement at the level of WCW and quantum measurement would involve selection of a sector of WCW characterized by $J(M^4)$ [K5].

The most plausible assumption is that $J(M^4)$ is covariantly constant.

3.5.2 Objections against $J(M^4)$

Consider now the objections against introducing $J(M^4)$ to the Kähler action at imbedding space level.

1. $J(M^4)$ would would break translational and Lorentz symmetries at the level of imbedding space since $J(M^4)$ cannot be Lorentz invariant. For imbedding space spinor modes this term would bring in coupling to the self-dual Kähler form in M^4. The simplest choice is $A = (A_t = z, A_z = 0, A_x = y, A_y = 0)$ defining decomposition $M^4 = M^2 \times E^2$. For Dirac equation in M^4 one would have free motion in preferred time-like (t,z)-plane plane M^2 in whereas in x- and y-directions (E^2 plane) would one have harmonic oscillator potentials due to the gauge potentials of electric and magnetic fields. One would have something very similar to quark model of hadron: quark momenta would have conserved longitudinal part and non-conserved transversal part. The solution spectrum has scaling invariance $\Psi(m^k) \to \Psi(\lambda m^k)$ so that there is no preferred scale and the transversal scales scale as $1/E$ and $1/k_x$.

2. Since $J(M^4)$ is not Lorentz invariant, Lorentz boosts would produce new $M^2 \times E^2$ decomposition (or its local variant). If one assumes above kind of linear gauge as gauge invariance suggests, the choices with fixed second tip of causal diamond (CD) define finite-dimensional moduli space $SO(3,1)/SO(1,1) \times SO(2)$ having in number theoretic vision an interpretation as a choice of preferred hypercomplex plane and its orthogonal complement. This is the moduli space for hypercomplex structures in M^4 with the choices of origins parameterized by M^4. The introduction of the moduli space would allow to preserve Poincare invariance.

3. If one generalizes the condition for Kähler metric to $J^2(M^4) = -g(M^4)$ fixing the scaling of J, the coupling to $A(M^4)$ is also large and suggests problems with the large breaking of Poincare symmetry for the spinor modes of the imbedding space for given moduli. The transversal localization by the self-dual magnetic and electric fields for $J(M^4)$ would produce wave packets in transversal degrees of freedom: is this physical?

   This moduli space is actually the moduli space introduced for causal diamonds (CDs) in zero energy ontology (ZEO) forced by the finite value of volume action: fixing of the line connecting the tips of CD the Lorentz boost fixing the position for the second tip of CD parametrizes this moduli space apart from division with the group of transformations leaving the planes M^2 and E^2 having interpretation a plane defined by light-like momentum and polarization plane associated with a given CD invariant.

4. Why this kind of symmetry breaking for Poincare invariance? A possible explanation proposed already earlier is that quantum measurement involves a selection of quantization axis. This choice necessarily breaks the symmetries and $J(M^4)$ would be an imbedding space correlate for the selection of rest frame and quantization axis of spin. This conforms with the fact that CD is interpreted as the perceptive field of conscious entity at imbedding space level: the contents of consciousness would be determined by the superposition of space-time surfaces inside CD. The choice of $J(M^4)$ for CD would select preferred rest system (quantization axis for energy as a line connecting tips of CD) via electric part of $J(M^4)$ and quantization axis of spin (via magnetic part of $J(M^4)$. The moduli space for CDs would be the space for choices of these particular quantization axis and in each state function reduction would mean a localization in this moduli space. Clearly, this reduction would be higher level reduction and correspond to a decision of experimenter.

To summarize, for $J(M^4) = 0$ Poincare symmetries are realized at the level of imbedding space but obviously broken slightly by the geometry of CD. The allowance of $J(M^4) \neq 0$ implies that both translational and rotational symmetries are reduced for a given CD: the interpretation would be in terms of a choice of quantization axis in state function reduction. They are however lifted to the level of moduli space of CDs and exact in this more abstract sense. This is nothing new: already the introduction of ZEO and CDs force by volume term in action forced by twistor lift of TGD implies the same. Also the view about state function reduction requires wave functions in the moduli space of CDs. This is also essential for understanding how the arrow of geometric time is inherited from that of subjective time in TGD inspired theory of consciousness [K1, K22].

3.5.3 Situation at space-time level

What about the situation at space-time level?

1. The introduction of $J(M^4)$ part to Kähler action has nice number theoretic aspects. In particular, J selects the preferred complex and quaternionic sub-space of octonionic space of imbedding space. The simplest possibility is that the Kähler action is defined by the Kähler form $J(M^4) + J(CP_2)$.

   Since M^4 and CP_2 Kähler geometries decouple it should be possible to take the counterpart of Kähler coupling strength in M^4 to be much larger than in CP_2 degrees of freedom so that M^4 Kähler action is a small perturbation and slowly varying as a functional of preferred extremal. This option is however not in accordance with the idea that entire Kähler form is induced.

2. Whether the proposed ansätze for general solutions make still sense is not clear. In particular, can one still assume that preferred extremals are minimal surfaces? Number theoretical vision strongly suggests - one could even say demands - the effective decoupling of Kähler action and volume term. This would imply the universality of quantum critical dynamics. The solutions would not depend at all on the coupling parameters except through the dependence on boundary conditions. The coupling between the dynamics of Kähler action and volume term would come also from the conservation conditions at light-like 3-surfaces at which the signature of the induced metric changes.

3. At space-time level the field equations get more complex if the M^4 projection has dimension $D(M^4) > 2$ and also for $D(M^4) = 2$ if it carries non-vanishing induced $J(M^4)$. One would obtain cosmic strings of form $X^2 \times Y^2$ as minimal surface extremals of ordinary Kähler action or X^2 Lagrangian manifold of M^4 as also CP_2 type vacuum extremals and their deformations with M^4 projection Lagrangian manifold. Thus the differences would not be seen for elementary particle and string like objects. Simplest string worlds sheet for which $J(M^4)$ vanishes would correspond to a piece of plane M^2.

   M^4 is the simplest minimal surface extremal of Kähler action necessarily involving also $J(M^4)$. The action in this case vanishes identically by self-duality (in Euclidian signature self-duality does not imply this). For perturbations of M^4 such as spherically symmetric stationary metric the contribution of M^4 Kähler term to the action is expected to be small and the come mainly from cross term mostly and be proportional to the deviation from flat metric. The interpretation in terms of gravitational contribution from M^4 degrees of freedom could make sense.

4. What about massless extremals (MEs)? How the induced metric affects the situation and what properties second fundamental form has? Is it possible to obtain a situation in which the energy momentum tensor T^α and second fundamental form $H^k_{\alpha\beta}$ have in common components which are proportional to light-like vector so that the contraction $T^{\alpha\beta}H^k_{\alpha\beta}$ vanishes?

   Minimal surface property would help to satisfy the conditions. By conformal invariance one would expect that the total Kähler action vanishes and that one has $J^\alpha_\gamma J^{\gamma\beta} \propto ag^{\alpha\beta} + bk^\alpha k^\beta$. These conditions together with light-likeness of Kähler current guarantee that field equations are satisfied.

   In fact, one ends up to consider a generalization of MEs by starting from a generalization of holomorphy. Complex CP_2 coordinates ξ^i would be functions of light-like M^2 coordinate $u_+ = k \cdot m$, k light-like vector, and of complex coordinate w for E^2 orthogonal to M^2. Therefore the CP_2 projection would 3-D rather than 2-D now.

   The second fundamental form has only components of form $H^k_{u_+w}$, $H^k_{u_+\overline{w}}$ and H^k_{ww}, $H^k_{\overline{w}\overline{w}}$. The CP_2 contribution to the induced metric has only components of form Δg_{u_+w}, $\Delta g_{+\overline{w}}$, and $g_{\overline{w}w}$. There is also contribution $g_{u_+u_-} = 1$, where v is the light-like dual of u in plane M^2. Contravariant metric can be expanded as a power series for in the deviation (Δg_{u_+w}, $\Delta g_{u_+\overline{w}}$) of the metric from ($g_{u_+u_-}, g_{w\overline{w}}$). Only components of form g^{u_+,u_i} and $g^{w,\overline{w}}$ are obtained and their contractions with the second fundamental form vanish identically since there are no common index pairs with simultaneously non-vanishing components. Hence it seems that MEs generalize!

   I have asked earlier whether this construction might generalize for ordinary MEs. One can introduce what I have called Hamilton-Jacobi structure for M^4 consisting of locally orthogonal slicings by integrable 2-surfaces having tangent space having local decomposition $M^2_x \times E^2_x$ with light-like direction depending on point x. An objection is that the direction of light-like momentum depends on position: this need not be inconsistent with momentum conservation but would imply that the total four-momentum is not light-like anymore. Topological condensation for MEs and at MEs could imply this kind modification.

5. There is also a topological magnetic flux type term for string world sheet. Topological term can be transformed to a boundary term coupling classical particles at the boundary of string world sheet to CP_2 Kähler gauge potential (added to the equation for a light-like geodesic line). Now also the coupling to M^4 gauge potential would be obtained. The condition $J(M^4) + J(CP_2) = 0$ at string world sheets [L2] is very attractive manner to identify string world sheets as analogs of Lagrangian manifolds but does not imply the vanishing of the net $U(1)$ couplings at boundary since the induce gauge potentials are in general different.

   Also topological term including also M^4 Kähler magnetic flux for string world sheet contributes also to the modified Dirac equation since the gamma matrices are modified gamma matrices required by super-conformal symmetries and defined as contractions of canonical momentum densities with imbedding space gamma matrices [K13]. This is true both in space-time interior, at string world sheets and at their boundaries. CP_2 (M^4) term gives a contribution proportional to CP_2 (M^4) gamma matrices.

At imbedding space level transversal localization would be the outcome and a good guess is that the same happens also now. This is indeed the case for M^4 defining the simplest extremal. The general interpretation of M^4 Kähler form could be as a quantum tool for transversal dynamical localization of wave packets in Kähler magnetic and electric fields of M^4. Analog for decoherence occurring in transversal degrees of freedom would be in question. Hadron physics could be one application.

3.5.4 Testing the existence of $J(M^4)$

How to test the idea about $J(M^4)$?

1. It might be possible to kill the assumption that $J(M^4)$ is covariantly constant by showing that one does not obtain spherically symmetric Schwartschild type metric as a minimal surface extremal of generalized Kähler action: these extremals are possible for ordinary Kähler action [L1] [K20]. For the canonical imbedding of M^4 field equations are satisfied since energy momentum tensor vanishes identically. For the small deformations the presence of $J(M^4)$ would reduce rotational symmetry to cylindrical symmetry.

   The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.An attractive proposal is that the pairs of orthogonal 2-surface correspond to Hamilton-Jacobi structures for which the two surfaces are orbits of subgroups of Poincare group.

2. $J(M^4)$ could make its presence manifest in the physics of right-handed neutrino having no direct couplings to electroweak gauge fields. Mixing with left handed neutrino is however induced by mixing of M^4 and CP_2 gamma matrices. The transversal localization of right-handed neutrino in a background, which is a small deformation of M^4 could serve as an experimental signature.

3. CP breaking in hadronic systems is one of the poorly understood aspects of fundamental physics and relates closely to the mysterious matter-antimatter asymmetry. The constant electric part of self dual $J(M^4)$ implies CP breaking. I have earlier consider that Kähler electric fields could cause this breaking but now the electric field is not constant. Second possibility is that matter and antimatter correspond to different values of h_{eff} and are dark relative to each other. The question is whether $J(M^4)$ could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$ and whether this breaking could explain hadronic CP breaking and matter anti-matter asymmetry. Could M^4 part of Kähler electric field induce different $h_{eff}/h = n$ for particles and antiparticles.

3.5.5 Kerr effect, breaking of T symmetry, and Kähler form of M^4

I encountered in Facebook a link to a very interesting article [?] (see `http://tinyurl.com/h5lmplw`). Here is the abstract of the article.

We prove an instance of the Reciprocity Theorem that demonstrates that Kerr rotation, also known as the magneto-optical Kerr effect, may only arise in materials that break microscopic time reversal symmetry. This argument applies in the linear response regime, and only fails for nonlinear effects. Recent measurements with a modified Sagnac Interferometer have found finite Kerr rotation in a variety of superconductors. The Sagnac Interferometer is a probe for nonreciprocity, so it must be that time reversal symmetry is broken in these materials.

Magneto-optic Kerr effect (see `http://tinyurl.com/hef8xgv`) occurs when a circularly polarized light beam (plane wave) (often with normal incidence) reflects from a sample. For instance, reflected circular polarized beams suffers a phase change in the reflection: as if they would spend some time at the surface before reflecting. Linearly polarized light reflects as elliptically polarized light. In magneto-optic Kerr effect there are many options depending on the relative directions of the reflection plane (incidence is not normal in the general case so that one can talk about reflection plane) and magnetization.

Kerr angle θ_K is defined as 1/2 of the difference of these phase angle increments caused by reflection for oppositely circularly polarized plane wave beams. As the name tells, magneto-optic Kerr effect is often associated with magnetic materials. Kerr effect has been however observed also for high Tc superconductors and this has raised controversy. As a layman in these issues I can safely wonder whether the controversy is created by the expectation that there are no magnetic fields inside the super-conductor. Anti-ferromagnetism is however important for high Tc superconductivity. In TGD based model for high Tc superconductors the supracurrents would flow along pairs of flux tubes with the members of $S = 0$ ($S = 1$) Cooper pairs at parallel flux tubes carrying magnetic fields with opposite (parallel) magnetic fluxes. Therefore magneto-optic Kerr effect could be in question after all.

The author claims to have proven that Kerr effect in general requires breaking of microscopic time reversal symmetry. Time reversal symmetry breaking (TRSB) caused by the presence of magnetic field and in the case of unconventional superconductors is explained nicely at `http://tinyurl.com/jbabcjt`. Magnetic field is required. Magnetic field is generated by a rotating current and by right-hand rule time reversal changes the direction of the current and also of magnetic field. For spin 1 Cooper pairs the analog of magnetization is generated, and this leads to T breaking.

This result is very interesting from the point of TGD. The reason is that twistorial lift of TGD requires that imbedding space $M^4 \times CP_2$ has Kähler structure in generalized sense [L3, L5]. M^4 has the analog of Kähler form, call it $J(M^4)$. $J(M^4)$ is assumed to be self-dual and covariantly constant as also CP_2 Kähler form, and contributes to the Abelian electroweak U(1) gauge field (electroweak hypercharge) and therefore also to electromagnetic field. By definition it satisfies $J^2(M^4) = -g(M^4)$ saying that it represents imaginary unit geometrically.

$J(M^4)$ implies breaking of Lorentz invariance since it defines decomposition $M^4 = M^2 \times E^2$ implying preferred rest frame and preferred spatial direction identifiable as direction of spin quantization axis. In zero energy ontology (ZEO) one has moduli space of causal diamonds (CDs) and therefore also moduli space of Kähler forms and the breaking of Lorentz invariance cancels. Note that a similar Kähler form is conjectured in quantum group inspired non-commutative quantum field theories and the problem is the breaking of Lorentz invariance.

What is interesting that the action of P,CP, and T on Kähler form transforms it from self-dual to anti-self-dual form and vice versa. If $J(M^4)$ is self-dual as also $J(CP_2)$, all these 3 discrete symmetries are broken in arbitrarily long length scales. On basis of tensor property of $J(M^4)$ one expects P: $(J(M^2), J(E^2) \rightarrow (J(M^2), -J(E^2)$ and T: $(J(M^2), J(E^2) \rightarrow (-J(M^2), J(E^2)$. Under C one has $(J(M^2), J(E^2) \rightarrow (-J(M^2), -J(E^2)$. This gives CPT: $(J(M^2), J(E^2) \rightarrow (J(M^2), J(E^2)$ as expected.

One can imagine several consequences at the level of fundamental physics.

1. One implication is a first principle explanation for the mysterious CP violation and matter antimatter asymmetry not predicted by standard model (see below).

2. A new kind of parity breaking is predicted. This breaking is separate from electroweak parity breaking and perhaps closely related to the chiral selection in living matter.

3. The breaking of T might in turn relate to Kerr effect if the argument of authors is correct. It could occur in high Tc superconductors in macroscopic scales. Also large $h_{eff}/h = n$ scaling up quantum scales in high Tc superconductors could be involved as with the breaking of chiral symmetry in living matter. Strontium ruthenate for which Cooper pairs are in $S = 1$ state is is indeed found to exhibit TRSB (for references and explanation see `http://tinyurl.com/jbabcjt`).

   In TGD based model of high Tc superconductivity [K7, K8] the members of the Cooper pair are at parallel magnetic flux tubes with the same spin direction of magnetic field. The magnetic fields and thus the direction of spin component in this direction changes under T causing TRSB. The breaking of T for $S = 1$ Cooper pairs is not spontaneous but would occur at the level of physics laws: the time reversed system finds itself experiences in the original self-dual $J(M^4)$) rather than in $(-J(M^2), J(E^2))$ demanded by T symmetry.

3.6 What causes CP violation?

CP violation and matter antimatter asymmetry involving it represent white regions in the map provided by recent day physics. Standard model does not predict CP violation necessarily accompanied by the violation of time reflection symmetry T by CPT symmetry assumed to be exact. The violation of T must be distinguished from the emergence of time arrow implies by the randomness associated with state function reduction.

CP violation was originally observed for mesons via the mixing of neutral kaon and antikaon having quark content $n\overline{s}$ and $\overline{n}s$. The lifetimes of kaon and antikaon are different and they transform to each other. CP violation has been also observed for neutral mesons of type $n\overline{b}$. Now it has been observed also for baryons Λ_b with quark composition u-d-b and its antiparticle (see `http://tinyurl.com/zyk8w44`). Standard model gives the Feynman graphs describing the mixing in standard model in terms of CKM matrix (see `http://tinyurl.com/hvpz2su`).

The CKM mixing matrix associated with weak interactions codes for the CP violation. More precisely, the small imaginary part for the determinant of CKM matrix defines the invariant coding for the CP violation. The standard model description of CP violation involves box diagrams in which the coupling to heavy quarks takes place. b quark gives rise to anomalously large CP violation effect also for mesons and this is not quite understood. Possible new heavy fermions in the loops could explain the anomaly.

Quite generally, the origin of CP violation has remained a mystery as also CKM mixing. In TGD framework CKM mixing has topological explanation in terms of genus of partonic 2-surface assignable to quark (sphere, torus or sphere with two handles). Topological mixings of U and D type quarks are different and the difference is not same for quarks and antiquarks. But this explains only CKM mixing, not CP violation.

Classical electric field - not necessary electromagnetic - prevailing inside hadrons could cause CP violation. So called instantons are basic prediction of gauge field theories and could cause strong CP violation since self-dual gauge field is involved with electric and magnetic fields having same strength and direction. That this strong CP violation is not observed is a problem of QCD. There are however proposals that instantons in vacuum could explain the CP violation of hadron physics (see `http://tinyurl.com/zptbd4j`).

What says TGD? I have considered this in [L7] and earlier blog posting (see `http://tinyurl.com/hvzqjua`).

1. M^4 and CP_2 are unique in allowing twistor space with Kähler structure (in generalized sense for M^4). If the twistor space $T(M^4) = M^4 \times S^2$ having bundle projections to both M^4 and to the conventional twistor space CP_3, or rather its non-compact version) allows Kähler structure then also M^4 allow the generalized Kähler structure and the analog symplectic structure.

   This boils down to the existence of self-dual and covariantly constant U(1) gauge field $J(M^4)$ for which electric and magnetic fields E and B are equal and constant and have the same direction. This field is not dynamical like gauge fields but would characterize the geometry of M^4. $J(M^4)$ implies violation Lorentz invariance. TGD however leads to a moduli space for causal diamonds (CDs) effectively labelled by different choices of direction for these self-dual Maxwell fields. The common direction of E and B could correspond to that for spin quantization axis. $J(M^4)$ has nothing to do with instanton field. It should be noticed that also the quantum group inspired attempts to build quantum field theories for which space-time geometry is non-commutative introduce the analog of Kähler form in M^4, and are indeed plagued by the breaking of Lorentz invariance. Here there is no moduli space saving the situation.

2. The choice of quantization axis would therefore have a correlate at the level of "world of classical worlds" (WCW). Different choices would correspond to different sectors of WCW. The moduli space for the choices of preferred point of CP_2 and color quantization axis corresponds to the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ of WCW. One could interpret also the twistor space $T(M^4) = M^4 \times S^2$ as the space with given point representing the position of the tip of CD and the direction of the quantization axis of angular momentum. This choice requires a characterization of a unique rest system and the directions of quantization axis and time axes defines plane M^2 playing a key role in TGD approach to twstorialization [L3, L5].

3. The prediction would be CP violation for a given choice of $J(M^4)$. Usually this violation would be averaged out in the average over the moduli space for the choices of M^2 but in some situation this would not happen. Why the CP violation does not average out when there is CKM mixing of quarks? Why the parity violation due to the preferred direction is not compensated by C violation meaning that the directions of E and B fields would be exactly opposite for quarks and antiquarks. Could the fact that quarks are not free but inside hadron induce CP violation? Could a more abstract formulation say that the wave function in the moduli space for $J(M^4)$ (wave function for the choices of spin quantization axis!) is not CP symmetric and this is reflected in the CKM matrix.

4. An important delicacy is that $J(M^4)$ can be both self-dual and anti-self-dual depending on whether the magnetic and electric field have same or opposite directions. It will be found that reflection P and CP transform self-dual $J(M^4)$ to anti-self-dual one. If only self-dual $J(M^4)$ is allowed, one has both parity breaking and CP violations.

Can one understand the emergence of CP violation in TGD framework?

1. Zero energy state is pair of two positive and negative energy parts. Let us assume that positive energy part is fixed - one can call corresponding boundary of CD passive. This state corresponds to the outcome of state function reduction fixing the direction of quantization axes and producing eigenstates of measured observables, for instance spin. Single system at passive boundary is by definition unentangled with the other systems. It can consists of entangled subsystems hadrons are basic example of systems having entanglement in spin degrees of freedom of quarks: only the total spin of hadron is precisely defined.

   The states at the active boundary of CD evolve by repeated unitary steps by the action of the analog of S-matrix and are not anymore eigenstates of single particle observables but entangled. There is a sequence of trivial state function reductions at passive boundary inducing sequence of unitary time evolutions to the state at the active boundary of CD and shifting it. This gives rise to self as a generalized Zeno effect.

   Classically the time evolution of hadron corresponds to a superposition of space-time surfaces inside CD. The passive ends of the space-time surface or rather, the quantum superposition of them - is fixed. At the active end one has a superposition of 3-surfaces defining classical correlates for quantum states at the active end: this superposition changes in each unitary step during repeated measurements not affecting the passive end. Also time flows, which means that the distance between the tips of CD defining clock-time increases as the active boundary of CD shifts farther away.

2. The classical field equations for space-time surface follow from an action, which at space-time level is sum of Kähler action and volume term. If Kähler form at space-time surface is induced (projected to space-time surface) from $J = J(M^4) + J(CP_2)$, the classical time evolution is CP violating. CKM mixing is induced by different topological mixings for U and D type quarks (recall that 3 particle generations correspond to different genera for partonic 2-surfaces: sphere, torus, and sphere with two handles). $J(M^4) + J(CP_2)$ defines the electroweak $U(1)$ component of electric field so that $J(M^4)$ contributes to $U(1)$ part of em field and is thus physically observable.

3. Topological mixing of quarks corresponds to a superposition of time evolutions for the partonic 2-surfaces, which can also change the genus of partonic 2-surface defined as the number of handles attached to 2-sphere. For instance, sphere can transform to torus or torus to a sphere with two handles. This induces mixing of quantum states. For instance, one can say that a spherical partonic 2-surface containing quark would develop to quantum superposition of sphere, torus, and sphere with two handles. The sequence of state function reductions leaving the passive boundary of CD unaffected (generalized Zeno effect) by shifting the active boundary from its position after the first state function reduction to the passive boundary could but need not give rise to a further evolution of CKM matrix.

4. The determinant of CKM matrix is equal to phase factor by unitarity ($UU^\dagger = 1$) and its imaginary part characterizes CP breaking. The imaginary part of the determinant should be

proportional to the Jarlskog invariant $J = \pm Im(V_{us}V_{cb}\overline{V}_{ub}\overline{V}_{cs})$ characterizing CP breaking of CKM matrix (see `http://tinyurl.com/kakxwl8`).

If the topological mixings are different for U and D type quarks, one obtains CKM mixing. How could the classical time evolution for quarks and for antiquarks as their CP transforms differ? To answer the question one must look how $J(M^4)$ transforms under C, P, T and CP.

1. $J(M^4) = (J_{0z}, J_{xy} = \epsilon J_{0z})$, $\epsilon = \pm 1$, characterizes hadronic space-time sheet (all space-time sheets in fact). Since $J(M^4)$ is tensor, P changes only the sign of J_{0z} giving $J(M^4) \rightarrow (-J_{0z}, J_{xy})$. Since C changes the signs of charges and therefore the signs of fields created by them, one expects $J(M^4) \rightarrow -J(M4)$ under C. CP would give $J(M^4) \rightarrow (J_{0z}, -J_{xy})$ transforming selfdual $J(M^4)$ to anti-selfdual $J(M^4)$. If WCW has no anti-self-dual sector, CP is violated at the level of WCW.

2. If CPT leaves $J(M^4)$ invariant, one must have $J(M^4) \rightarrow (J_{0z}, -J_{xy})$ under T rather than $J(M^4) \rightarrow (-J_{0z}, J_{xy})$. The anti-unitary character of T could correspond for additional change of sign under T. Otherwise CPT should act as $J(M^4) \rightarrow -J(M^4)$ and only $(CPT)^2$ would correspond to unity.

3. Same considerations apply to $J(CP_2)$ but the difference would be that induced $J(M^4)$ for space-time surfaces, which are small deformations of M^4 covariantly constant in good approximation. Also for string world sheets corresponding to small cosmological constant $J(M^4) \times J(M^4) - 2 \simeq 0$ holds true in good approximation and induced $J(M^4)$ at string world sheet is in good approximation covariantly constant. If the string world sheet is just M^2 characterizing $J(M^4)$ the condition is exact and was has Kähler electric field induced by $J(M^4)$ but no corresponding magnetic field. This would make the CP breaking effect large.

If CP is not violated, particles and their CP transforms correspond to different sectors of WCW with self dual and anti-self dual $J(M^4)$. If only self-dual sector of WCW is present then CP is violated. Also P is violated at the level of WCW and this parity breaking is different from that associated with weak interactions and could relate to the geometric parity breaking manifesting itself via chiral selection in living matter. Classical time evolutions induce different CKM mixings for quarks and antiquarks reflecting itself in the small imaginary part of the determinant of CKM matrix. CP breaking at the level of WCW could explain also matter-antimatter asymmetry. For instance, antimatter could be dark with different value of $h_{eff}/h = n$.

What is interesting that P is badly broken in long length scales as also CP. The same could be true for T. Could this relate to the thermodynamical arrow of time? In ZEO state function reductions to the opposite boundary change the direction of clock time. Most physicist believe that the arrow of thermodynamical time and thus also clock time is always the same. There is evidence that in living matter both arrows are possible. For instance, Fantappie has introduced the notion of syntropy as time reversed entropy [J1]. This suggests that thermodynamical arrow of time could correspond to the dominance of the second arrow of time and be due to self-duality of $J(M^4)$ leading to breaking of T. For instance, the clock time spend in time reversed phase could be considerably shorter than in the dominant phase. A quantitative estimate for the ratio of these times might be given some power of the ratio $X = l_P/R$.

3.7 Quantitative picture about CP breaking in TGD

One must specify the value of α_1 and the scaling factor transforming $J(CD)$ having dimension length squared as tensor square root of metric to dimensionless $U(1)$ gauge field $F = J(CD)/S$. This leads to a series of questions.

How to fix the scaling parameter S?

1. The scaling parameter relating $J(CD)$ and F is fixed by flux quantization implying that the flux of $J(CD)$ is the area of sphere S^2 for the twistor space $M^4 \times S^2$. The gauge field is obtained as $F = J/S$, where $S = 4\pi R^2(S^2)$ is the area of S^2.

2. Note that in Minkowski coordinates the length dimension is by convention shifted from the metric to linear Minkowski coordinates so that the magnetic field B_1 has dimension of inverse

length squared and corresponds to $J(CD)/SL^2$, where L is naturally be taken to the size scale of CD defining the unit length in Minkowski coordinates. The $U(1)$ magnetic flux would the signed area using L^2 as a unit.

How $R(S^2)$ relates to Planck length l_P? l_P is either the radius $l_P = R$ of the twistor sphere S^2 of the twistor space $T = M^4 \times S^2$ or the circumference $l_P = 2\pi R(S^2)$ of the geodesic of S^2. Circumference is a more natural identification since it can be measured in Riemann geometry whereas the operational definition of the radius requires imbedding to Euclidian 3-space.

How can one fix the value of $U(1)$ coupling strength α_1? As a guideline one can use CP breaking in K and B meson systems and the parameter characterizing matter-antimatter symmetry.

1. The recent experimental estimate for so called Jarlskog parameter characterizing the CP breaking in kaon system is $J \simeq 3.0 \times 10^{-5}$. For B mesons CP breading is about 50 times larger than for kaons and it is clear that Jarlskog invariant does not distinguish between different meson so that it is better to talk about orders of magnitude only.

2. Matter-antimatter asymmetry is characterized by the number $r = n_B/n_\gamma \sim 10^{-10}$ telling the ratio of the baryon density after annihilation to the original density. There is about one baryon 10 billion photons of CMB left in the recent Universe.

Consider now the identification of α_1.

1. Since the action is obtained by dimensional reduction from the 6-D Kähler action, one could argue $\alpha_1 = \alpha_K$. This proposal leads to unphysical predictions in atomic physics since neutron-electron $U(1)$ interaction scales up binding energies dramatically.

   $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$, the ratio of the areas of twistor spheres of $T(M^4)$ and $T(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$ so that the coupling constant evolution for α_1 and α_K would be identical.

2. ϵ indeed serves in the role of coupling constant strength at classical level. α_K disappears from classical field equations at the space-time level and appears only in the conditions for the super-symplectic algebra but ϵ appears in field equations since the Kähler forms of J resp. CP_2 Kähler form is proportional to $R^2(S^2)$ resp. $R^2(CP_2)$ times the corresponding $U(1)$ gauge field. $R(S^2)$ appears in the definition of 2-bein for $R^2(S^2)$ and therefore in the modified gamma matrices and modified Dirac equation. Therefore $\sqrt{\epsilon} = R(S^2)/R(CP_2)$ appears in modified Dirac equation as required by CP breaking manifesting itself in CKM matrix.

   NTU for the field equations in the regions, where the volume term and Kähler action couple to each other demands that ϵ and $\sqrt{\epsilon}$ are rational numbers, hopefully as simple as possible. Otherwise there is no hope about extremals with parameters of the polynomials appearing in the solution in an arbitrary extension of rationals and NTU is lost. Transcendental values of ϵ are definitely excluded. The most stringent condition $\epsilon = 1$ is also unphysical. $\epsilon = 2^{2r}$ is favoured number theoretically.

Concerning the estimate for ϵ it is best to use the constraints coming from p-adic mass calculations.

1. p-Adic mass calculations [K4] predict electron mass as

$$m_e = \frac{\hbar}{R(CP_2)\sqrt{5+Y}} \quad .$$

   Expressing m_e in terms of Planck mass m_P and assuming $Y = 0$ ($Y \in (0,1)$) gives an estimate for $l_P/R(CP_2)$ as

$$\frac{l_P}{R(CP_2)} \simeq 2.0 \times 10^{-4} \quad .$$

2. From $l_P = 2\pi R(S^2)$ one obtains estimate for ϵ, α_1, $g_1 = \sqrt{4\pi\alpha_1}$ assuming $\alpha_K \simeq \alpha \simeq 1/137$ in electron length scale.

$$\begin{aligned} \epsilon &= 2^{-30} \simeq 1.0 \times 10^{-9} \ , \\ \alpha_1 &= \epsilon\alpha_K \simeq 6.8 \times 10^{-12} \ , \\ g_1 &= \sqrt{4\pi\alpha_1} \simeq 9.24 \times 10^{-6} \ . \end{aligned}$$

There are two options corresponding to $l_P = R(S^2)$ and $l_P = 2\pi R(S^2)$. Only the length of the geodesic of S^2 has meaning in the Riemann geometry of S^2 whereas the radius of S^2 has operational meaning only if S^2 is imbedded to E^3. Hence $l_P = 2\pi R(S^2)$ is more plausible option.

For $\epsilon = 2^{-30}$ the value of $l_P^2/R^2(CP_2)$ is $l_P^2/R^2(CP_2) = (2\pi)^2 \times R^2(S^2)/R^2(CP_2) \simeq 3.7 \times 10^{-8}$. $l_P/R(S^2)$ would be a transcendental number but since it would not be a fundamental constant but appear only at the QFT-GRT limit of TGD, this would not be a problem.

One can make order of magnitude estimates for the Jarlskog parameter J and the fraction $r = n(B)/n(\gamma)$. Here it is not however clear whether one should use ϵ or α_1 as the basis of the estimate

1. The estimate based on ϵ gives

$$J \sim \sqrt{\epsilon} \simeq 3.2 \times 10^{-5} \ , \quad r \sim \epsilon \simeq 1.0 \times 10^{-9} \ .$$

The estimate for J happens to be very near to the recent experimental value $J \simeq 3.0 \times 10^{-5}$. The estimate for r is by order of magnitude smaller than the empirical value.

2. The estimate based on α_1 gives

$$J \sim g_1 \simeq 0.92 \times 10^{-5} \ , \quad r \sim \alpha_1 \simeq .68 \times 10^{-11} \ .$$

The estimate for J is excellent but the estimate for r by more than order of magnitude smaller than the empirical value. One explanation is that α_K has discrete coupling constant evolution and increases in short scales and could have been considerably larger in the scale characterizing the situation in which matter-antimatter asymmetry was generated.

There is an intriguing numerical co-incidence involved. $h_{eff} = \hbar_{gr} = GMm/v_0$ in solar system corresponds to $v_0 \simeq 2^{-11}$ and appears as coupling constant parameter in the perturbative theory obtained in this manner [K9]. What is intriguing that one has $\alpha_1 = v_0^2/4\pi^2$ in this case. Where does the troublesome factor $(1/2\pi)^2$ come from? Could the p-adic coupling constant evolutions for v_0 and α_1 correspond to each other and could they actually be one and the same thing? Can one treat gravitational force perturbatively either in terms of gravitational field or $J(CD)$? Is there somekind of duality involved?

Atomic nuclei have baryon number equal the sum $B = Z + N$ of proton and neutron numbers and neutral atoms have $B = N$. Only hydrogen atom would be also $U(1)$ neutral. The dramatic prediction of $U(1)$ force is that neutrinos might not be so weakly interacting particles as has been thought. If the quanta of $U(1)$ force are not massive, a new long range force is in question. $U(1)$ quanta could become massive via $U(1)$ super-conductivity causing Meissner effect. As found, $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$.

Quantal $U(1)$ force must be also consistent with atomic physics. The value of the parameter α_1 consistent with the size of CP breaking of K mesons and with matter antimatter asymmetry is $\alpha_1 = \epsilon\alpha_K = 2^{-30}\alpha_K$.

1. Electrons and baryons would have attractive interaction, which effectively transforms the em charge Z of atom $Z_{eff} = rZ$, $r = 1 + (N/Z)\epsilon_1$, $\epsilon_1 = \alpha_1/\alpha = \epsilon \times \alpha_K/\alpha \simeq \epsilon$ for $\alpha_K \simeq \alpha$ predicted to hold true in electron length scale. The parameter

$$s = (1 + (N/Z)\epsilon)^2 - 1 = 2(N/Z)\epsilon + (N/Z)^2\epsilon^2$$

would characterize the isotope dependent relative shift of the binding energy scale.

The comparison of the binding energies of hydrogen isotopes could provide a stringent bounds of the value of α_1. For $l_P = 2\pi R(S^2)$ option one would have $\alpha_1 = 2^{-30}\alpha_K \simeq .68 \times 10^{-11}$ and $s \simeq 1.4 \times 10^{-10}$. s is by order of magnitude smaller than $\alpha^4 \simeq 2.9 \times 10^{-9}$ corrections from QED (see `http://tinyurl.com/kk9u4rh`). The predicted differences between the binding energy scales of isotopes of hydrogen might allow to test the proposal.

2. $B = N$ would be neutralized by the neutrinos of the cosmic background. Could this occur even at the level of single atom or does one have a plasma like state? The ground state binding energy of neutrino atoms would be $\alpha_1^2 m_\nu/2 \sim 10^{-24}$ eV for $m_\nu = .1$ eV! This is many many orders of magnitude below the thermal energy of cosmic neutrino background estimated to be about 1.95×10^{-4} eV (see `http://tinyurl.com/ldu95o9`). The Bohr radius would be $\hbar/(\alpha_1 m_\nu) \sim 10^6$ meters and same order of magnitude as Earth radius. Matter should be $U(1)$ plasma. $U(1)$ superconductor would be second option.

4 About the interpretation of the duality assignable to Yangian symmetry

The $D = 4$ conformal generators acting on twistors have a dual representation in which they act on momentum twistors: one has dual conformal symmetry, which becomes manifest in this representation. These two separate symmetries extend to Yangian symmetry providing a powerful constraint on the scattering amplitudes.

In TGD the conformal Yangian extends to super-symplectic Yangian - actually, all symmetry algebras have a Yangian generalization with multi-locality generalized to multi-locality with respect to partonic 2-surfaces. The generalization of the dual conformal symmetry has remained obscure. In the following I describe what the generalization of the two conformal symmetries and Yangian symmetry would mean in TGD framework. I also propose an information theoretic duality between Euclidian and Minkowskian regions of space-time surface. I am not algebraist and apologize for the unavoidable inaccuracies.

4.1 Formal definition associated with Yangian

The notion of Yangian appears as two very different looking variants. The first variant can be found from Wikipedia (see `goo.gl/q1twRZ`) and second variant assignable to gauge theories can be found from [B6, B7].

Consider first the Wikipedia definition. The definition is in terms of quantum group notion in which the elements of matrix representing group element are made non-commuting operators.

1. The generators of Yangian algebra are labelled by an integer $n \geq -1$ with $n = -1$ generator identified as unit matrix. $n \geq 1$ generators generate the algebra and commutators with $n = 1$ generators preserving the weight allow to assign quantum numbers to them. From the Wikipedia article one learns that Yangian is generated by elements $t_{ij}^{(p)}$, $1 \leq i, j \leq N$, $p \geq 0$ of quantum matrices satisfy the relations

$$\left[t_{ij}^{(p+1)}, t_{kl}^{(q)}\right] - \left[t_{ij}^{(p)}, t_{kl}^{(q+1)}\right] = -(t_{kj}^{(p)} t_{il}^{(q)} - t_{kj}^{(q)} t_{il}^{(p)}) \ . \tag{4.1}$$

Note there are two operations involved: commutator and operator product. The formula here is not consistent with the formula used in Yang-Mills theories for the commutators between $m = 0$ generators and generators with generators having $n \in \{0, 1\}$, and it seems that this formula suggesting $m, n \to m + n - 1$ in commutator cannot hold true for the commutators with $m = 0$ generators.

By defining $t_{ij}^{(-1)} = \delta_{ij}$ and setting

$$T(z) \quad = \quad \sum_{p\geq -1} t_{ij}^{(p)} z^{-p+1} \quad . \tag{4.2}$$

$T(z)$ is thus a quantum matrix depending on the point of 2-D space.

2. Introduce R-matrix $R(z) = 1 + z^{-1}P$ acting on $C^N \otimes C^N$, where P is the operator permuting the tensor factors. This allows to write the defining relations as Yang-Baxter equation (see `http://tinyurl.com/gogn75s`):

$$R_{12}(z-w)T_1(z)T_2(w) = T_2(w)T_1(z)R_{12}(z-w) \quad . \tag{4.3}$$

R_{12}, which depends only on the difference $z - w$, performs the permutation of the generators $T_1(z)$ and $T_2(w)$.

Yangian is a Hopf algebra with co-multiplication Δ mapping $T(z)$ acting in V to operator acting in $V \otimes V$, co-unit ϵ and antipode s given by

$$(\Delta \otimes id)T(z) = T_{12}(z)T_{13}(z) \quad , \quad (\epsilon \otimes id)T(z) = I \quad , \quad (s \otimes id)T(z) = T(z)^{-1} \quad . \tag{4.4}$$

Δ taking generator $T(z)$ acting in V to generator $\Delta(T) = T_{12}(z)$ acting in $V \otimes V$. Δ transforms a generator acting on single-particle states to a generator acting on 2-particles states.

3. The Yangian weight of the commutator of elements with weights m and n is $m + n - 1$ rather than $m + n$ as for Virasoro and Kac-Moody algebras. This means that generators with conformal weight 1 do not affect the conformal weight and Cartan algebra elements defining quantum numbers of generators have weight 1. For conformal algebras the Cartan algebra defining quantum numbers has conformal weight 0.

   For Virasoro algebra having integer valued conformal weights the scaling $L_0 = zd/dz$ appears as basic derivative operation and generators are products $L_n = z^n zd/dz$. By taking translation operator $T = d/dz$ as the derivative operator and writing $K_n = z^n d/dz$, the weight of commutator becomes $m + n - 1$. This is a trivial change. The map $u = exp(z)$ relates these two representations. That $n \leq 2$ appear in generators distinguishes the representations from Virasoro and Kac-Moody representations - note however that also for these algebras the generators with positive weight generate physical states.

What bothers me in this definition is that only the action of the generators with $p = 1$ leaves the weight unaffected whereas for the dual conformal symmetry generators with both $p = 0$ and $p = 1$ do this and define conformal symmetry and its dual.

4.2 Dual conformal symmetry in $\mathcal{N} = 4$ SUSY

Yangian symmetry appears also in gauge theories and the definition looks very different from the Wikipedia definition. In $\mathcal{N} = 4$ SUSY conformal symmetry (in 4-D sense) has two representations. There is a duality between two representations of conformal generators crucial for twistor Grassmannian approach [B6, B7] (see `http://tinyurl.com/n22lwuy`).

1. In the first representation conformal symmetry generators $J_a^{(0)}$ are local and act in the space of external momenta. This induces a local and linear action in twistor space.

2. The generators $J_a^{(1)}$ of the dual conformal symmetry act in a local manner in the space of region momenta and associated momentum twistor space whereas the action of $J_a^{(1)}$ is bi-local in the momentum space and corresponding twistor space.

Region momenta can be assigned with a twistor diagram defined by a closed polygon of Minkowski space having region momenta (, which need not be light-like) as edges having external light-like momenta emitted at the corners. The dual of this representation is the representation in which the light-like external momenta summing up to zero form a closed polygon.

Yangian is generated by ordinary generators $J_a^{(0)}$ and bi-local dual generators $J_a^{(1)}$.

1. They satisfy the commutations

$$\left[J_a^{(0)}, J_b^{(1)}\right] = f_{ab}{}^c J_c^{(1)} \quad . \tag{4.5}$$

   This condition is perfectly sensible physically but is not consistent with the above general consistency condition pf Eq. 4.1 from R-matrix requiring that the commutator has vanishing weight. Now the weights are additive in commutator.

2. The generators $J_a^{(1)}$ have an easy-to-guess representation:

$$J_a^{(1)} = f_a{}^{cb} \sum_{0\leq i<j\leq n} J_{ib}^{(0)} J_{jc}^{(0)} \tag{4.6}$$

   making explicit the bi-locality. The commutators of these generators have also weight 1. This is consistent with the above general formula unlike the formula the commutators of generators with vanishing weight. Both generators form a closed sub-algebra of Yangian and this must be behind the possibility to represent $J_a^{(1)}$ locally.

3. Also so called Serre relations are satisfied. They look rather complex and look different from the relations associated with R-matrix.

$$\begin{aligned} X(a,b,c) &+ \epsilon(a,b,c)X(b,c,a) + \epsilon(c,a,b)X(c,a,b) = h\epsilon_{rm,tn} Y(l,m,n) f^l_{ar} f^m_{bs} f^n_{ct} f^{rst} \quad , \\ X(a,b,c) &= \left[J_a^{(1)}, \left[J_b^{(1)}, J_c^{(0)}\right\}\right\} \quad , \quad Y(l,m,n) = \left\{J_l^{(0)}, J_m^{(0)}, J_n^{(0)}\right] \\ \epsilon(a,b,c) &= (-1)^{|a|(|b|+|c|)} \quad , \quad \epsilon_{rm,tn} = (-1)^{|r|m|+|t|n|} \quad . \end{aligned} \tag{4.7}$$

   Here the mixed brackets the $[.,\}$ denote the graded commutator, and $\{.,..]$ denotes the graded symmetrizer. h is a parameter characterizing the Yangian and should correspond to the parameter characterizing quantum group.

   These conditions are sufficient to give a representation of graded Yangian if the tensor product $\mathcal{R}\otimes\overline{\mathcal{R}}$ of the representation $\mathcal{R}$ and its conjugate $\overline{\mathcal{R}}$ contains adjoint representation only once. The higher generators can be generate by applying co-product operation to the generators.

4. Both local and bi-local generators form two closed sub-algebras. This is not consistent with the consistency conditions of appearing in Wikipedia definition. The Wikipedia definition seems to be wrong for commutators of generators $[J_A^{(m)}, J_B^{(n)}]$ with weights $(m,n) \in \{(0,0),(0,1),(1,0)\}$.

5. Co-product Δ has representation

$$\begin{aligned} \Delta(J^A) &= J^A\otimes 1 + 1\otimes J^A \quad , \\ \Delta(Q^A) &= Q^A\otimes 1 + 1\otimes Q^A + f^A_{BC} J^B\otimes J^C \quad . \end{aligned} \tag{4.8}$$

The first formula is obvious. Single particle generator lifted to a tensor product is sum of the single particle generators acting on the tensor factors. When Q^A annihilates single spin representations, one obtains just the defining formula for the bi-local generators.

One could have a situation in which single particle states are actually many-particle states annihilated by Q^A and satisfying the condition that adjoint is contained only once in $\mathcal{R}\otimes\overline{\mathcal{R}}$. In TGD framework one might argue that this kind of effective single particle states could quite generally define bound states behaving like single particle states physically. One would obtain infinite hierarchy of this kind of states realizing concretely the vision about fractal hierarchy.

4.3 Possible TGD based interpretation of Yangian symmetries

In TGD partonic 2-surfaces replace point-like objects and multi-locality is with respect to these. The proposal is that the TGD counterpart of the Yangian algebra [B7] of gauge theories could act as symmetries of many-parton states characterized by n partonic 2-surfaces assignable to the same 3-D surface at the boundary of causal diamond (CD). What is remarkable that this symmetry would relate particle states with different particle numbers to each other unlike the usual single particle symmetries.

1. This condition forces the partons to form a bound state with partonic 2-surfaces having *space-like* separations. Note that the separations along orbits of wormhole throats at opposite ends of CD are space-like or light-like. This must be taken into account when correlation functions are calculated. In QFT there is no description of this kind and this could explain the general failure of QFT in the description of bound states already in QED, where Bethe-Salpeter equation predicts large numbers of non-existing states.

2. Yangian algebra involves complex (hypercomplex) coordinate z which could be associated with the boundaries of string world sheets connecting partonic surfaces at the same boundary (at opposite boundaries) of CD. One can also assign complex coordinate with partonic 2-surfaces and the braiding of fermionic lines would be described by the matrix R assignable to the Yangian. The Cartan algebra of local and bi-local string like operators define quantum numbers for states. That point-like and string-like operators generate the algebra conforms with the idea about tensor networks with nodes connected by edges.

   On can think that partonic 2-surfaces form a single connected unit consisting of partonic surfaces connected by boundaries of string world sheets assignable to the topological Feynman diagram defined by the light-like 3-surface defining the boundary between Euclidian and Minkowskian regions of the space-time surface.

3. The operation Δ for Yangian would assign to the generators acting on single parton states generators acting on 2-parton states. R_{12} would act as an exchange operation for parton states, which could reduces to many-fermion states at partonic 2-surfaces.

4. R_{12} can appear in many contexts in TGD. It can be associated with braiding of fermionic lines inside partonic orbits or magnetic flux tubes at the ends of space-time surfaces. It can be also associated with the fermionic lines in the preferred plane M^2 associated with twistor scattering amplitudes.

   From the twistorial point of view the preferred M^2 defined by light-like quaterionic 8-momentum is of special interest. M^2 identified as octonionic complex plane and its complexification brings in mind integrable field theories in M^2 allowing Yangian symmetry characterized by R-matrix. The scattering matrix is trivial for these field theories: scattering involves only a phase shift. In twistorial approach to TGD scattering is non-trivial. The R-matrix would be present also now and exchange the momentum projections in preferred M^2 plane. If the entire scattering diagram -apart from external lines corresponds to the same M^2, the braiding operation permutes also fermions at different partonic 2-surfaces located at the ends of string.

The possibility to localize the action of generators $J^{(1)}$ in momentum twistor representation leads to ask whether the stringy generators appearing TGD framework could allow local action using the analog of the space of region momenta. Could M^8-H duality [K10, K17] make this possible? At M^8 level the light-like momenta (in 8-D sense) would correspond to differences of region momenta assignable to strings connecting the partonic 2-surfaces. The 8-D region momenta should be quaternionic. They cannot be light-like as is easy to see.

The notion of region momentum and thus localization would make sense only in M^8, where the wave functions are completely localizable to quaternionic light-like momenta in M^8, whereas in H one has localization to light-like momenta only in preferred M^2 plus wave functions in the space of planes M^4 and in the space of transverse momenta in $E^2 \subset M^4$. This would suggest that M^8-H duality corresponds to the duality of twistor and momentum twistor representations.

What would be new that this duality would be realized also at the level of space-time surfaces. One would have associative/quaternionic space-time surfaces in M^8 and preferred extremals of dimensionally reduced Kähler action in H identifiable as 6-D holomorphic surfaces representing twistor spaces of space-time surfaces.

Note that M^8-H duality could be seen as a number-theoretic analog of spontaneous compactification. Non-perturbative effects would force a delocalization in the space of light-like 8-momenta in M^8 to give states having interpretation as wave functions in H. Nothing would happen to the topology of M^8. Only the state space would be compactified.

4.4 A new kind of duality of old duality from a new perspective?

M^8-H duality [K10, K17] maps the preferred extremals in H to those $M^4 \times CP_2$ and vice versa. The tangent spaces of an associative space-time surface in M^8 would be quaternionic (Minkowski) spaces.

In M^8 one can consider also co-associative space-time surfaces having associative *normal* space [K10]. Could the co-associative normal spaces of associative space-time surfaces in the case of preferred extremals form an integrable distribution therefore defining a space-time surface in M^8 mappable to H by M^8-H duality? This might be possible but the associative tangent space and the normal space correspond to the same CP_2 point so that associative space-time surface in M^8 and its possibly existing co-associative companion would be mapped to the same surface of H.

This dead idea however inspires an idea about a duality mapping Minkowskian space-time regions to Euclidian ones. This duality would be analogous to inversion with respect to the surface of sphere, which is conformal symmetry. Maybe this inversion could be seen as the TGD counterpart of finite-D conformal inversion at the level of space-time surfaces. There is also an analogy with the method of images used in some 2-D electrostatic problems used to reflect the charge distribution outside conducting surface to its virtual image inside the surface. The 2-D conformal invariance would generalize to its 4-D quaterionic counterpart. Euclidian/Minkowskian regions would be kind of Leibniz monads, mirror images of each other.

1. If strong form of holography (SH) holds true, it would be enough to have this duality at the informational level relating only 2-D surfaces carrying the holographic information. For instance, Minkowskian string world sheets would have duals at the level of space-time surfaces in the sense that their 2-D normal spaces in X^4 form an integrable distribution defining tangent spaces of a 2-D surface. This 2-D surface would have induced metric with Euclidian signature.

   The duality could relate either a) Minkowskian and Euclidian string world sheets or b) Minkowskian/Euclidian string world sheets and partonic 2-surfaces common to Minkowskian and Euclidian space-time regions. a) and b) is apparently the most powerful option information theoretically but is actually implied by b) due to the reflexivity of the equivalence relation. Minkowskian string world sheets are dual with partonic 2-surfaces which in turn are dual with Euclidian string world sheets.

   (a) Option a): The dual of Minkowskian string world sheet would be Euclidian string world sheet in an Euclidian region of space-time surface, most naturally in the Euclidian "wall neighbour" of the Minkowskian region. At parton orbits defining the light-like boundaries between the Minkowskian and Euclidian regions the signature of 4-metric is $(0,-1,-1,-1)$ and the induced 3-metric has signature $(0,-1,-1)$ allowing light-like curves. Minkowskian and Euclidian string world sheets would naturally share these light-like curves aas common parts of boundary.

(b) Option b): Minkowskian/Euclidian string world sheets would have partonic 2-surfaces as duals. The normal space of the partonic 2-surface at the intersection of string world sheet and partonic 2-surface would be the tangent space of string world sheets so that this duality could make sense locally. The different topologies for string world sheets and partonic 2-surfaces force to challenge this option as global option but it might hold in some finite region near the partonic 2-surface. The weak form of electric-magnetic duality [K15] could closely relate to this duality.

In the case of elementary particles regarded as pairs of wormhole contacts connected by flux tubes and associated strings this would give a rather concrete space-time view about stringy structure of elementary particle. One would have a pair of relatively long (Compton length) Minkowskian string sheets at parallel space-time sheets completed to a parallelepiped by adding Euclidian string world sheets connecting the two space-time sheets at two extremely short (CP_2 size scale) Euclidian wormhole contacts. These parallelepipeds would define lines of scattering diagrams analogous to the lines of Feynman diagrams.

This duality looks like new but as already noticed is actually just the old electric-magnetic duality [?]een from number-theoretic perspective.

5 TGD view about construction of twistor amplitudes

In the following TGD view about twistorialization and its relation to other visions about TGD is discussed. I start with a brief summary of twistor approach to scattering amplitudes and then describe the application of this approach TGD.

5.1 Some key ideas of the twistor Grassmann approach

In the following I summarize the basic technical ideas of twistor Grassmann approach. I am not a specialist. On the other hand, my views about twistorialization of TGD differ in many aspects about those applied in the twistorialization of gauge theories, and my own attention is directed towards the physical interpretation and mathematical consistency rather than calculational techniques.

5.1.1 Variants of twistor formalism

The reader can find details about twistors in the article of Witten [B8] and in the thesis of Trnka [B21] (see `http://tinyurl.com/zbj9ad7`).

1. Helicity spinor formalism assigns to light-like momentum pair of conjugate spinors $(\lambda_a, \tilde{\lambda}_{\dot{a}})$ transforming in conjugate representations of Lorentz group $SL(2,C)$. Light-like momentum is expressible as $p^k\sigma_k$ using Pauli sigma matrices and this gives the representation as matrix components $p^{a\dot{a}} = \lambda^a\lambda^{\dot{a}}$. The determinant of the matrix equals to $p^kp_k = 0$ since its rows are linearly dependent.

   One can introduce the bilinears $[\tilde{\lambda}_1, \tilde{\lambda}_2] = -[\tilde{\lambda}_2, \tilde{\lambda}_1]$ and $\langle\lambda_1, \lambda_2\rangle = -\langle\lambda_2, \lambda_1\rangle$ using the antisymmetric Lorentz invariant bilinear defined by permutation symbols ϵ^{ab} and $\epsilon^{\dot{a}\dot{b}}$. The inner product $p_1 \cdot p_2$ is expressible as $p_1 \cdot p_2 = \langle\lambda_1, \lambda_2\rangle[\tilde{\lambda}_1, \tilde{\lambda}_2]$.

   One could express also polarization vectors of massless bosons using pair $(\lambda, \tilde{\mu})$ of helicity spinors. There is however a more elegant approach available. The spinors $(t\lambda, \tilde{\lambda}/t)$ correspond to same momentum for all non-vanishing complex values of t. t represents an element of little group of Lorentz group leaving the helicity state invariant. The helicity dependence of the scattering amplitude is fixed by the transformation property under little group and coded to the weight under the scalings by t: $A(t_a\lambda, t_a^{-1}\tilde{\lambda}_a) = t_a^{-2h_a}A(\lambda, \tilde{\lambda})$. Thus the formalism allows very elegant description of spin and can be applied in SUSYs.

For Minkowski signature (2,2) the spinors are real and this makes this signature preferred. Personally I see this as a basic problem of twistorialization. A possible TGD inspired solution of the problem is provided by the effective replacement of M^4 with M^2 with signature $(1,1)$ and thus allowing real spinors.

2. Twistors $(\lambda_a, \mu_{\dot{a}})$ are obtained by performing a twistor Fourier transform of scattering amplitude $A(\lambda, \tilde{\lambda})$ with respect to $\tilde{\lambda}$.

   At local level [B8] the twistor transform corresponds to Fourier transform

$$\begin{array}{c} \tilde{\lambda}_{\dot{a}} \rightarrow i\frac{\partial}{\mu^{\dot{a}}} \ , \\ -i\frac{\partial}{\tilde{\lambda}^{\dot{a}}} \rightarrow \mu_{\dot{a}} \ . \end{array}$$

   The action of little group corresponds now to the scaling $(\lambda, \mu) \rightarrow t(\lambda, \mu)$ and does not affect the helicity state. For this reason twistors differing by complex scaling can be identified. The proper twistor space is CP_3 rather than C^4.

   The twistor transform of the amplitude transforms as $A(t_a\lambda, t_a\tilde{\lambda}_a) = t_a^{-2h_a-2}A(\lambda, \mu)$.

   In signature (2,2) the helicity spinors $(\lambda, \tilde{\lambda})$ are real so that the twistor Fourier transform reduces to an ordinary Fourier transform. In signature (1,3) the rigorous definition is rather challenging and is discussed by Penrose [B19]. One manner to define the transform is by using residue integral. Residue integral is also p-adically attractive.

   The incidence relation of Penrose given by

$$\mu_{\dot{a}} = -x_{a\dot{a}}\lambda^a$$

   relates M^4 coordinates to λ, μ. By little group invariance entire complex twistor line corresponds to a given point of M^4.

   The twistor transform of plane wave allows to construct the twistor transform of momentum space wave function, and is given by $\delta^2(\mu_{\dot{a}} + x_{a\dot{a}}\lambda^a)$, which is non-vanishing at complex light ray. Twistor Fourier transform in real Minkowski space is therefore non-vanishing at light ray and maps light rays to twistors.

   If the incidence relation for given (λ, μ) is satisfied at two space-time points m_1, m_2, the difference $m_1 - m_2$ is a light-like vector since corresponding matrix has vanishing determinant. Two intersecting twistor lines correspond to M^4 points with light-like distance. This allows to develop geometric picture about twistor diagrams in which the external light-like momenta correspond to intersections of twistor lines assignable to the internal lines of graph.

3. Momentum twistors define a third basic notion. It is convenient to describe particle scattering with external light-like momenta in terms of a diagram in which the external momenta are assigned with the vertices of a polygon such that the lines carry possibly complex momenta. Clearly, the polygon like object is obtained by repeatedly adding light-like momenta to the polygon and since the sum of the external momenta vanishes, the polygon closes.

   The vertices of polygon correspond to intersections of twistor lines defining light-like momenta as differences of the momenta associated with the lines meeting at the vertex. One can assign to the complex momenta of internal lines twistors known as momentum twistors.

   Dual momentum twistor is a further variant of twistor concept being defined in terms of three adjacent momentum twistors contracting them with the 4-D permutation symbol defined in the representation of twistor as a point of C^4 [B21].

5.1.2 Leading singularities

Twistor Grassmann approach to planar loop amplitudes relies on the idea that the discontinuities associated with the singularities of the scattering amplitudes carry all information about the amplitudes. This of course holds true already for the tree diagrams having only poles as singularities.

The idea is same as in the case of analytic continuation: 1-D data at poles and cuts allows to construct the functions. This idea generalizes to functions of several variables and leads to a generalization of residue calculus. At space-time level strong form of holography (SH) relies on the same idea: the 3-D data determine 4-D dynamics and in TGD allowing strong form of holography 2-D data is almost enough.

The discontinuities assignable to singularities can have lower-dimensional singularities so that a hierarchical structure is obtained. The leading singularities are those for which maximal number of propagators are on mass shell and the diagram decomposes to a product of diagrams with virtual particle on mass shell. For one loop diagrams the maximal number of propagators is $N=4$ corresponding to the fixing of four components of loop momentum. For L loops it is $N=4L$.

Non-leading singularities have less than the maximal number of propagators on shell and this leaves integral over a subset of loop momenta. If the number of propagator is larger than $4L$, one can have kinematical singularities for some combinations of external momenta.

In the case of scattering amplitudes in twistor Grassmann formulation one encounters a similar situation. In twistor Grassmann approach one defines also the loop integrals in momentum space as residue integrals in the space of complexified momenta. If the functions involved are rational functions the residue integrals are well-defined.

One of the surprising findings is that the leading singularities of MHV loop amplitudes always proportional to tree amplitudes. Second finding is that for $\mathcal{N}=4$ theory the leading singularities determine completely the scattering amplitudes [B21].

In TGD framework quantum criticality suggests that locally all loop corrections vanish and coupling constant evolution is discrete. This would mean that the only singularities correspond to poles of propagators and this indeed leads to diagrams in which internal lines have complex on mass shell momenta. If this vision is correct, this part of twistor Grassmann approach does not look relevant from TGD point of view.

5.1.3 BCFW recursion formula

The original form of BCFW recursion formula [B4] was derived for tree diagrams. The finding was that the diagrams can be decomposed to two pieces containing with a propagator line connecting them.

1. The proof of this result was rather simple in spinor helicity formalism and based on modification of two momenta p_k and p_n by BCFW shift:

$$\begin{array}{l} p_k(z)=\lambda_k(\tilde{\lambda}_k - z\tilde{\lambda}_n) \ , \\ p_n(z)=(\lambda_n + z\lambda_k)\tilde{\lambda}_n) \ , \end{array} \tag{5.1}$$

   Obviously, the modification is induced by modifications $\tilde{\lambda}_k$ and λ_n. With some assumptions about asymptotic behaviour of scattering amplitude A, one can express the original amplitude $A=A(z=0)$ as residue integral

$$A(z=0) = \frac{1}{2\pi}\oint_C dz\frac{A(z)}{z} \ . \tag{5.2}$$

   Here C does not close any other poles than $z=0$. This integral is the negative of the residue integral around the complement of the region closed by C.

2. It is assumed that poles are the only singularities in this region. Hence one can express $A(z)$ as sum of its poles

$$A(z) = \sum_i \frac{c_i}{z-z_i} \ . \tag{5.3}$$

3. With these assumptions the residue integral gives

$$A = A(0) = \frac{1}{2\pi}\sum_i \frac{c_i}{z_i} \ . \tag{5.4}$$

This leads to the desired factorization with c_i reducing to a product of amplitudes and z_i identifiable as a complex pole for the propagator connecting the sub-diagrams in the decomposition.

In [B9] details of the BCFW shift in the general case are given. One assumes a more general shift $p_i \rightarrow \hat{p}_i = p_i + zr_i$ such that r_i are light-like, mutually orthogonal, orthogonal to p_i, and sum up to zero. The modified momenta are complex massless and sum up to zero. One can define $P_I = \sum_{i<I} p_i$ and $R_I = \sum_{i<I} r_i$. The shifted variant $\hat{P_I}^2 = P_I^2 + 2zP \cdot R_I$ is linear in z and vanishes for $z = z_I = -P_I^2/P_I \cdot R_I$. Z_I define the counterparts z_i. Performing the residue integral one obtains $A(0) = \frac{1}{2\pi}\sum_I \frac{c_I}{z_I}$.

This formula allows a recursive construction of tree diagrams by starting from the basic vertices of YM theory. BCFW recursion formula was later generalized to a recursion for the sum planar loops diagrams in terms of diagrams with lower number of loops [B9, B21].

5.1.4 Scattering amplitudes in terms of Yangian invariants defined as multiple residue integrals in Grassmannian manifolds

The generators of Yangian are ordinary conformal generators with conformal weight 0 and dual generators with conformal weight 1. The latter generators act in simple manner in momentum twistor space.

Twistor Grassmannian approach utilizing either twistors or momentum twistors allows to demonstrate that these both conformal symmetry and its dual are present.

The construction of Yangian invariants is summarize in [B21]. Grassmannian residues are Yangian invariants. Yangian transformation introduces total divergence and is exact if its integral vanishes. The operations producing new Yangian invariant can change n or k or both.

1. There are several relatively trivial manners to construct Yangian invariants. One can take the integrand of n-1-D invariant and formally interpret it as integrand of n-D invariant. One can integrate over one twistor variable so that n decreases by one unit.

   Invariants can be multiplied. One can a merge invariants by identifying the twistors in the factors of the product. For instance, one can take the fundamental invariants defining 3-vertices and multiply them to build twistor box giving rise to four particles. One can also merge invariants by integrating over the identified invariants.

2. Inverse soft factor [B16] adds to the diagram expressed in terms of spinor helicity formalism one new particle but keeps k constant. Therefore this operation does cannot be applied in TGD where one has only fermions as external particles. The operation can be formulated as a linear shift for $\tilde{\lambda}_a$ and $\tilde{\lambda}_b$.

3. One can prove the BCFW recursion formula for tree diagrams [B4] by using a deformation of the twistor amplitude in helicity spinor formalism allowing to deduce the factorized formula of the amplitude, two adjacent external lines and deform the twistors λ and $\tilde{\lambda}$ in helicity spinor representation by performing the BCFW shift [B18].

   This deformation describes interaction between the external lines, and is essential in the construction of the scattering amplitudes using BCFW recursion. One takes the sum over the products of diagrams with left and right helicities obtained by putting internal particle on mass shell and adds BCFW bridge. BCFW allows to construct all tree amplitudes by starting from fundamental 3-particle amplitudes.

4. Entangled removal [**?**, B21, B9] removing two external particles producing a loop in the sense of Feynman diagrammatics but residue of the pole of the propagator is possible and appears as part of the boundary operation for the diagrams. The resulting recursion formula allows to deduce loop corrections.

Twistor Grassmann diagrams are known to allow "moves" [B21, B11]. For instance, moves can be used to remove boxes: it is known that apart from scaling factors depending on momenta the diagrams are reducible to ordinary tree diagrams [B21] (`http://tinyurl.com/zbj9ad7`). This allows to consider the possibility that twistor trees could allow to construct all diagrams. Note however that the moves reducing the twistor diagram to a counterpart of tree diagram gives an overall multiplicative factor depending on momenta and helicities.

From TGD point the definition of loop integrals and Grassmannian integrals as residue integrals is of great potential importance. Scattering amplitudes should be number theoretically universal but in p-adic context the definition of definite integral is very difficult. Residue integral provides however a manner to define multiple residue integrals using only holomorphy and the notion of pole. This could be the deep reason for why one should be able to reduce loop integrals to residue integrals.

There is however a potential problem involved related to number theoretic universality. 2π does not exist p-adically in any reasonable sense (if one wants to define it one must introduce infinite-D extension of rationals by powers of 2π. One might hope that 2π cancels from the scattering amplitudes by normalization. Another possibility is that for an extension containing $exp(i2\pi/N)$ as the highest root of unity, one can define π approximately as $i\pi \equiv N \times (exp(i\pi/N) - 1)$. An alternative option is that only the analogs of tree diagrams having only poles as singularities are possible

5.1.5 Linearization of the twistorial representation of overall momentum delta function

An little but not insignificant technical detail [B12] is the linearization of the constraint expressing the overall momentum conservation by interpreting it as a condition in Grassmannian $G(k,n)$, where k is the number of negative helicities and n is the number of particles, and allowing to reduce integrations over $G(k,n)$ to those over $G(k-2,n-4)$.

Spinor helicity diagrams and twistor diagrams are proportional to a delta function expressing overall momentum conservation. Dropping twistor indices this delta function one reads as $\delta(\sum_k P_k) = \delta(\lambda_i\tilde{\lambda}_i)$. One can combine the 2 components of λ_i and $\tilde{\lambda}_i$ to form 2+2 n-component vectors and interpret momentum conservation as orthogonality conditions for the 2-planes spanned by λ_a and $\tilde{\lambda}_{\dot{a}}$ for $k > 2$. These plane spanned by 2 n-component λ vectors can be interpreted as 2 vectors in $G(k, n-k)$ defining rows of $G(k, n-k)$ matrix. $\tilde{\lambda}$ defines a similar plane in $G(n-k, k)$.

These conditions are equivalent with the condition that there exists in $G(k,n)$ a 2-D C and its $n-k$-dimensional orthogonal complement $\tilde{C}$ such that the 2-plane spanned by λ_a is orthogonal to $\tilde{C}$ and the two-plane spanned by $\tilde{\lambda}_{\dot{a}}$ is orthogonal to C. These conditions can be expressed as a product of delta functions $\delta(C\cdot\tilde{\lambda})$ and $\delta(\tilde{C}\cdot\lambda)$.

Since $G(k)$ acts as a "gauge symmetry" for $G(k,n)$, the first $k\times k$ block of the $k\times n$ matrix representing a point of C can be transformed to a unit matrix so that $k\times(n-k)$ variables remain.. Same can be carried out for the last $n\times(n-k)$ block of $\tilde{C}$ by $G(n)$ "gauge invariance" so that $(n-k)\times n$ variables remain. With these gauge choices the orthogonality conditions can be solved explicitly and corresponding integrations can be carried out. The integration over delta functions leaves $(k-2)(n-k-2)$ variables, the dimension of $G(k-2,n-4)$. $G(k,n)$ reduces to $G(k-2,n-4)$ by momentum conservation.

5.2 Basic vision behind scattering amplitudes

It is good to summarize the basic vision about TGD first.

5.2.1 Separation of WCW functional integral and fermionic dynamics

The works of Penrose and Witten have served as inspiration in the attempts to twistorialize TGD and led to the conjecture that the twistor lift of TGD is possible and means that space-time surfaces are replaced with their twistor spaces representable as 6-D surfaces in 12-D product of twistor spaces of M^4 and CP_2. What makes this idea so attractive is that S^4 and CP_2 are the only 4-D compact manifolds with Euclidian signature having twistor space with Kähler structure [?].

TGD would be unique both from the existence of the lift of Kähler action to the product of twistor spaces of M^4 and CP_2!

What the twistor space of M^4 is, is however not at all clear. It can be defined in two manners: as the usual CP_3 very natural at the level of momentum space or as the trivial bundle $T(M^4) = M^4 \times S^2$ natural in the twistorialization at classical space-time level. Standard twistorialization has however problems.

1. There is problem associated with the signature. Twistorialization works best at signature $(2,2)$ for Minkowski space and gives rise to real projective space P^3.

2. Second problem is that CP_3 should be actually $SU(2,2)/SU(2,1) \times U(1)$. There is clearly something not so well understood.

In the number theoretic vision about TGD twistor space would be replaced with commutative hyper-complex $M_2 \subset M^4 \subset M^8$ and this space is just RP^3 and problems wth signature disappear since 2-D spinors can be chosen to have real basis. For complex momenta this extends to CP_3. Number theory would also justify the identification of geometric twistor sphere as $M^4 \times S^2$.

In TGD the dynamics of fields is replaced with that for 4-surfaces. Penrose's idea about generalization of holomorphy of field modes in twistor space generalizes to the holomorphy of the representation of 6-surface representing twistor bundle of space-time leads to a concrete ansatz for space-time surfaces as preferred extremals [L3] [L7].

SH leads to the proposal that the data determining space-surfaces are preferred extremals is given at 2-D surfaces and these 2-D surfaces bring in mind Witten's twistor strings [B8]. By SH the functional integral over them would correspond to that over WCW and twistor amplitudes asignable to given space-time surface would be constructed at fermionic level by the analog of twistor Grassmannian approach. This integral over 2-surfaces corresponds to the deviation of TGD from QFT in fixed background and cannot be equivalent with the introduction of twistor strings.

5.2.2 Adelic physics and scattering diagram as a representation of computation

Adelic physics [L4] suggested to provide quantum physical correlates also for cognition is in a central role. Adelic physics predicts the hierarchy $h_{eff} = n \times h$, where n as dimension of the extension is divisor of the order its Galois group identified in terms of dark matter regarded as a phase of ordinary matter. p-Adic physics and p-adic length scale hypothesis could be also understood.

The number theoretic universality of scattering amplitudes suggests that all loops vanish identically and the evolution of various couplings constants is discrete occurring by phase transitions changing the extension of rationals and values of various coupling parameters.

1. The vanishing of loops at the level of space-time action would mean that the loops associated with the functional integral defined by the action, which is sum of Kähler action and volume term. This vanishing would state essentially local quantum criticality as invariance of coupling parameters under local renormalization group evolution. One would obtain only a sum of action exponentials since Gaussian and metric determinants cancel each other in Kähler metric.

2. Exponents of Kähler action represent a number theoretical nightmare.

   (a) The functional integral expressions for scattering amplitudes are normalized by a functional integral for for the vacuum state. This implies that only the ratios X_i/X of the exponents X_i for the extrema and sum $\sum X_i$ appear in the amplitudes [L4] so that there are slightly better hopes of achieving number theoretic universality.

   (b) Number theoretical universality forces to imagine even more attractive option making sense in ZEO but not in standard ontology. If the amplitude is sum over the contributions normalized by corresponding exponentials X_i rather than $\sum X_i$, exponentials cancel altogether and the couplings constants appear only in boundary conditions. In this case one could speak of a basis of zero energy states assignable to various extrema of the action. The real part of the action is maximum and the the imaginary part of the action saddle point if preferred extrema are minimal surface extremals of Kähler action [L3]. Number-theoretical universality more or less forces this option.

3. An even stronger proposal is based on the idea that that the TGD analogs of stringy diagrams. The lines of these diagrams correspond to light-like parton orbits carrying fermion lines and meeting at vertices which are partonic 2-surfaces. The proposal is that the topological diagrams involving analogs of loops represent algebraic computations so that all diagrams with given initial and final collection of algebraic objects are equivalent.

   If this is the case, all topological diagrams should reduce to topological tree diagrams by a generalization of the duality symmetry of the old-fashioned hadronic string model stating that the sum of s-channel resonances equals to the sum of t-channel exchanges and that these diagrams can be constructed as twistor Grassmann diagrams by allowing on mass shell fermions with complex momenta at internal lines. For external particles the momenta could be real and light-like in 8-D sense. A weaker condition is that real and imaginary parts of complex momenta 8-D momenta are separately light-like and orthogonal.

   One could indeed argue that one cannot allow loops of this kind since it would be impossible to decide which kind graph experimental scattering situation corresponds if all these graphs are different since one observes only the initial and final states. Therefore all scattering diagrams with same real particles in the final states correspond to identical scattering amplitudes.

   These diagrams would correspond to the same amplitude but it might be possible to perform a localization to any of them. p-Adically however the corresponding space-time surface would be different by p-adic non-determinism (the number theoretic discretization - cognitive representation - defined by the common points of reality and p-adicities as space-time surfaces would be different): one might say that the tree representation involves smallest cognitive representation and is therefore the shortest one.

   If the action exponentials X_i cancel from the scattering amplitudes, this option can indeed make sense. Otherwise it is extremely implausible since different contributions would have different vacuum weights.

4. If only the twistor analogs from tree diagrams in Feynman sense are allowed, the scattering amplitudes are rational functions of external momenta as strongly suggested by the number theoretic universality and by the requirement that the diagrams can be interpreted in terms of algebraic computations so that the simplest manner to do the computation corresponds to a tree diagram. Even tree diagrams in Feynman sense are planar so that one would get rid of the basic problem of the twistor approach to SUSY.

Quantum classical correspondence (QCC) states that scattering diagrams have classical counterparts in the sense that fermion lines correspond to the boundaries of string worlds sheets assignable to the light-like orbits of partonic 2-surfaces and topological 3.vertices correspond to 2-surfaces at which the ends of light-like orbits meet. This correlation is extremely restrictive and it is not at all clear whether it leaves room for loops.

In the most general case one would have a superposition of allowed space-time surfaces realizing scattering diagram with given initial and final quantum numbers identified as corresponding classical charges.

The idea about diagram as computation suggests that the simplest possible diagram - tree diagram - is realized together with the corresponding space-time topology. If diagrams with topological loops are possible this requires the existence of moves transforming diagrams to each other. This condition might be not consistent with the condition that the move acts on the space-time surface too. Very simple diagrammatics - even twistor tree diagrammatics - could follow from mere QCC.

5.2.3 Classical number fields and $M^8 - H$ duality

Quaternionicity and octonionicity is second central aspect of number theoretical vision.

1. The key concept is $M^8 - M^4 \times CP_2$ duality allowing to see space-time surfaces quaternionic surface in M^8 or as holomorphic surfaces in the twistor space $T(M^4) \times T(CP_2)$. This would realize SH. Physical states are characterized by quaternionic (possibly complexified-) octonion valued 8-momenta in accordance with the vision that tangent space Minkowskian region of space-time surface is quaternionic and contains preferred hyper-complex M^2, which can

depend on point provided that tangent spaces $M^2(x)$ integrate to 2-D surface. This view leads to a new view about QCD color as octonionic color.

2. Twistor space reduces to that associated with M^2 and 2-D variant of conformal invariance corresponds to SO(2,2) and leads to the identification real projective space P^3 as twistor space. One can however complexify it to CP_3 since momenta are in general complex. The signature is (1,1) so that bi-spinors λ, $\tilde{\lambda}$ have real basis and twistor Fourier transform can be defined as ordinary Fourier transform. The reality of M^2 or induced spinors at string world sheets might allow to have SUSY without Majorana spinors.

   The reduction of external momenta to M^2 implies that real and imaginary parts are parallel and light-like. At classical level this poses strong conditions on preferred extremals. This does not require that color and electroweak quantum numbers are complex. The reason is that they emerge as labels of wave functions in twistor space $T(CP_2)$ representing wave functions in the moduli space of transversal E^2s with corresponding helicity identifiable as em charge.

   Localization of the light-like 8-momentum is possible to preferred M_0^2. Localization does not imply the disappearance of color wave function. The transversal E^2 momentum degrees of freedom however disappear. In the case of leptons and hadrons complete localization could be a good approximation but not in the case of quarks.

5.2.4 Elementary particles have fundamental fermions as building bricks

The assumption that the physics of elementary particles reduces at fundamental level to that of fundamental fermions has strong implications, when combined with the twistor Grassmann approach.

1. In TGD elementary particle would correspond to a pair of wormhole throats of wormhole connecting two space-time sheets with Minkowski signature. Wormhole itself would have Euclidian signature. Wormhole contacts would be connected by monopole flux tube with fermionic quantum numbers at the 4 wormhole throats defining the partonic 2-surfaces.

2. Fundamental vertices are associated with 2-surfaces at which light-like 3-surfaces carrying fermions and antifermions as string world sheet boundaries are glued together along their ends. Note that these surfaces are analogous to vertices of Feynman diagrams and singular as 4-surfaces but 3-surfaces are smooth unlike for stringy vertices.

3. Fermion lines correspond to the boundaries of string world sheets at the light-like orbits of partonic 2-surface at which the signature of the induced metric changes. At momentum space M^8 this picture should also make sense since space-time surfaces in M^8 and H would correspond to each other by $M^8 - H$ duality. At the level of M^8 the orbits of fermion lines could be seen as light-like geodesics along with twistor spheres move. At the edges of string world sheets they would intersect at single point and give rise to external massless particle.

4. The basic vertex is 4-fermion vertex in which fermions scatter classically and assignable to the 2-surface at which the ends of light-like 3-surfaces representing partonic orbits intersect. There would be no local 4-fermion vertex. Fermions would move as free particles in the background and the background would gives rise to the interaction between fermions at partonic vertices analogous to vertices of Feynman diagrams. This would automatically resolve possible problems caused by divergences and would be analogous to the vanishing of bosonic loops from WCW functional integration.

5. FFB couplings could be identified in terms of $FF(F\overline{F})$ couplings, where $F\overline{F}$ is associated with the same partonic orbit. These couplings would not be fundamental.

5.2.5 What could SUSY mean in TGD?

Extended super-conformal invariance is basic symmetry of TGD but it is not whether it possible to have SUSY (space-time supersymmetry) in TGD framework. Certainly the SUSY in question is not $\mathcal{N} = 1$ SUSY since Majorana spinors are definitely excluded. $\mathcal{N} = 2$ SUSY generated by right-handed neutrino and antineutrino can be however considered.

1. If one allows the boundaries of string world sheets carry fermion number bounded only by statistics (all spin-charge states for quarks and leptons would define maximal $\mathcal{N}$ for SUSY). This would allow local vertices for fermions and does not look like an attractive option unless SUSY manages to cancel the divergences.

2. SUSY could mean addition of fermions as separate lines to the orbits of wormhole throat. This SUSY would be broken and only approximately local. The question what the propagator for the many-fermion state at same string line is, is not quite obvious. SUSY would suggest propagator determined by the total spin of the state. I have also considered the possibility that the propagator is just the product of fermionic propagators acting on tensor power of single fermion spaces. The propagator behaves as $1/p^N$ for N fermion state and only for $N = 1, 2$ one would have the usual behavior. This option is not attractive.

3. SUSY could mean addition of right-handed neutrino or its antiparticle to the throat. The short range of weak interactions is explained by assuming that pair of right-handed neutrino and left-handed neutrino compensates the weak isospin at the second wormhole throat carrying quantum numbers of quark or lepton.

   Addition of right-handed neutrino or its antiparticle or both to a given boundary component could give rise to $\mathcal{N} = 2$ SUSY. The breaking of SUSY could correspond to different p-adic length scales for spartners. Mass formula could be exactly the same and provided by p-adic thermodynamics. Why the p-adic mass scale would depend so much on the presence of covariantly constant ν_R having no color and ew interactions nor even gravitational interaction, remains to be understood. If the extensions of rationals are different for the members of SUSY multiplet, the corresponding preferred p-adic primes would be different and this could explain the widely different p-adic mass scales. One can of course ask the covariant constancy means that ν_R does not have any coupling to anything and its presence is undetectable.

5.3 Options for the construction of scattering amplitudes

There are several guidelines in the construction of scattering amplitudes.

1. SH in strongest form would mean that string word sheets and partonic 2-surfaces are all that is needed. In number theoretical vision also fixing the extension of rationals associated with the intersection of realities and p-adicities is needed and leads to a hierarchy of extensions which could realized discrete coupling constant evolution. SH would suggest that hybrids for analogs of string diagrams and Feynman diagrams code for the scattering amplitudes.

2. QCC suggests that the eigenvalues of the Cartan algebra generators of symmetries are equal to classical Noether charges. A weaker condition is that the eigenvalues of fermionic generators not affecting space-time surfaces are equal to the classical Noether charges. The generators have also bosonic parts acting in WCW.

   A prediction following from the condition that there is charge transfer between Euclidian and Minkowskian space-time regions is that the classical charges must be complex valued guaranteed if Kähler coupling strength as a spectrum of complex values. One proposal is that the spectrum of zeros of Riemann zeta determines if [K3]. This supports the twistorial view that momenta in the internal lines can be regarded as complex light-like on mass shell momenta.

3. QCC also suggests that scattering diagrams have space-time correlates. The lines of diagrams correspond to light-like orbits of partons at which the signature of induced metric changes. Vertices correspond to partonic 2-surfaces at which these 3-D lines meet. At fermion level fermion lines at partonic orbits correspond to boundaries of string world sheets.

This however leaves several alternative visions concerning the construction of scattering amplitudes.

5.3.1 What scattering diagrams are?

What does one mean with scattering diagrams is not at alle clear.

1. Are they counterparts of Feynman diagrams so that one would have a superposition of all space-time topologies corresponding to these diagrams? Probably not.

2. Or are they counterparts of twistor Grassmannian diagrams in which all particles are on mass shell but with possibly complex light-like quaternionic 8-momenta in $M^8 = M^4 \times E^4$ with $M^4 = M_0^2 \times E^2$. Why this option is interesting is that twistor Grassmann diagrams allow large number of moves reducing their number.

   This would translate to a conserved and massive longitudinal M^2-momentum; which for a special choice of M^2 is light-like, a wave function in the space of transversal E^2 momenta; color partial wave in the moduli space of E^2 planes for given M_0^2; and em charge describable as CP_2 helicity and allowing twistorialization.

   There is however a problem: the transverse E^6-momentum makes M^2 momentum massive and twistorialization fails. But what if the 8-momenta are real and in twistorial description M^2 momentum becomes complex but light-like. The square for the real part of M^2 momentum would be equal to the square of real E^6 momentum and twistor approach would apply! This map would be define the essence of M^2-twistorialization.

In ZEO one can interpret the construction of preferred extremals as a boundary value problem with ends of space-time surfaces at the boundaries of CD and the light-like orbits of partonic 2-surfaces defining a closed 3-surface and defining the scattering diagram as 3-D boundary. If so, it might be possible to construct rather large number of diagrams, even counterpartz of loop diagrams.

The situation would be analogous to the construction of soap films spanned by wires with wire network analogous to the network formed by the partonic orbits. Also an analogy with 4-D tensor network suggests strongly itself and scattering diagrams representing zero energy states would correxpond to the states of the tensor network.

The basic space-time vertex would be 3-vertex defined by partonic 2-surface.The basic fermionic vertex would be 4-fermion vertex in which fermions do not exchange gauge boson but interact classically at the 2-D vertex. All particles emerge as bound states of fundamental fermions at boundaries of string world sheets.

1. The basic view would be that M^2 momenta, and transversal momenta correspond to M^4-momenta. The moduli space for $M_0^2 \times E^2$ planes corresponds to CP_2 and color quantum numbers. M^2 helicities and electroweak quantum numbers would be coded to the weights twistor wave functions in twistor space if $M^2 \times CP_2$.

2. One approach to scattering amplitudes relies on symmetries. Twistor Grassmannian approach suggest strongly Yangian symmetry. The diagrams should be representations of multi-local Yangian algebra with basic algebra being that of the conformal group of M^4 restricted to M^2.

   This would give nicely real projective space RP^3 allowing to solve some problems of the standard twistor approach. In color degrees of freedom one would have color Yangian: hadrons could correspond to the multilocal generators created by multi-local Yangian generators. The E^2 degrees of freedom would correspond to states generated by Kac-Moody algebra and also now one could have Yangian algebra. The states for the representation of Yangian itself would be singlets.

   Besides fermionic lines there are string world sheets. Infinite-D 2-D conformal group and Kac-Moody symmetries act as symmetries for string world sheets. The super-symplectic group would the isometry group of WCW and would give rise to conditions analogous to Super Virasoro conditions. These conditions would be satisfied by preferred extremals realizing number theoretic variant of SH. Also these symmetries would be extended to their Yangian versions naturally.

3. One can argue that classical field equations do not allow all possible diagrams. More precisely, for a given extension of rationals adelic physics allows only finite number diagrams and the extension induces a natural cutoff as minimal distance between points with coordinates in the extension representing intersection of reality and p-adicities [L4].

   The assumption that the end points of fermionic lines at partonic 2-surfaces at ends of CD and at the vertices carry fermions would give an immediate connection with the adelic physics. As the dimension of the extension increases, the number of the points in the intersection increases and more lines appear in the allowed diagrams. This would give rise to a discrete coupling constant evolution, hierarchy of Planck constants, and p-adic length scale hypothesis.

   Quantum criticality strongly suggests that coupling constant evolution is locally trivial and is discretized with discrete steps realized as phase transitions changing the extension. Galois group would be the fundamental number theoretic symmetry group acting on the intersection and its order would correspond to $h_{eff}/h = n$ allowing to realize the analogs of perturbative phases of gauge theories as perturbative phases.

4. The discreteness of coupling constant evolution demands that loop corrections vanish. This makes perfect sense for the functional integral over WCW. But what about fermionic degrees of freedom and topological counterparts of scattering diagrams, which very probably do not correspond to Feynman diagrams but could be analogous to twistor diagrams? For fermions there is actually no perturbation theory since effective 4-fermion vertices correspond to classical scattering of external fermions at partonic 2-surfaces defining the vertices. This is not a problem since thanks to h_{eff} guaranteeing the existence of perturbative expansion.

5.3.2 Three roads to follow

In ZEO construction of scattering amplitudes is basically a construction of zero energy states and one must be very cautious in applying QFT intuitions relying on positive energy ontology. One ends up to to a road fork.

Option I: Can one interpret the topological space-time diagrams as analogs of Feynman diagrams and assume that by quantum criticality the sum over the topological loops vanish? This option looks rather ad hoc.

Option II: Can one assume - with inspiration coming from adelic physics - that the number of these loops with fixed states at the boundaries of CD is finite and one just sums over these states with weights given by the exponential of the space-time action?

Here one encounters problems with number theoretical universality [L4]. One has superposition of vacuum exponentials over the diagrams and number theoretical universality demands that the ratio of given exponential to the sum is in the extension of rationals involved. This is very tough order - perhaps too tough.

Option III: Can one follow number theoretical vision suggesting that scattering diagrams correspond to computations in some sense [L2]. This leads to a new road fork.

1. Option IIIa): Could one generalize the old-fashioned string duality and require that there exist a huge symmetry allowing to transform the scattering diagrams using basic moves to tree diagrams? The basic moves would allow to shift the end of line past vertex and to remove self energy loop and hence the transformation to tree diagrams would become possible. Originally it was inspired by the idea that the vertices of the scattering diagram correspond to products and co-products in quantum algebra and that the condition involved can be interpreted as algebraic identities.

   Twistor Grassmannian diagrams indeed allow moves allowing surprising simplification allowing to show that all loop corrections with a given number of loops sum up to something proportional to a tree diagram [B21].

   The assumption that the states moving in the internal lines have light-like quaternionic M^8 momenta gives very strong constraints on the moves and it might well be that the moves are not possible in the general case. Even if the move is possible, the value of the action exponential can change so that this option seems to demand mathematical miracles. The proposed manner to achieve number theoretical universality however eliminates action exponentials.

The mathematical miracle might be made possible by the possibility to find preferred M_0^2 in which the 2-momentum of fermion line is light-like. If M_0^2 is constant along entire fermion line, it seems to be possible perform the gliding operation past vertices as will be found. Note that ach fermion can wander around the network formed by the partonic orbits.

Note that the different space-time surface realizing equivalent computations would be cognitively non-equivalent since the cognitive representation defined by the points in extension of rationals would be different. Optimum computation would have smallest number of points and would correspond to tree diagram.

2. Option IIIb): Should one sum over the possible diagrams so that one would have quantum superposition of computations. This is done for loop diagrams in twistor Grassmann approach. Infinite sum is however awkward number theoretically. Adelic vision suggests that the number of loops is finite. The action exponentials would not disappear from the scattering amplitudes and are very problematic from the point of view of number theoretical universality.

3. Option IIIc): Could one regard the light-like partonic orbits as part of the dynamical system - this is what effectively is done if they form part of connected 3-surface defining the topological scattering diagram - and assume that each such diagram corresponds to a different physical situation analogous to a computation?

   One can argue that one must be also able to localize the zero energy state to single computation by state function reduction [L6]! State function reduction to single diagram should be possible. A rather classical picture about space-time would emerge: one would have just a superposition of space-time surfaces with the same topology and same action apart from quantum fluctuations around the point which is maximum with stationary phase. One would also have color wave functions and momentum wave functions in cm degrees of freedom of partonic 2-surfaces as WCW degrees of freedom.

   The action exponential, which is very problematic from the point of view of number theoretic vision, would be cancelled from the functional integral since it is normalized by the action exponential. The dependence on coupling parameters is however visible in the boundary conditions at boundaries of CD stating the vanishing of most supersymplectic charges and identifying the remaining super-symplectic charges and also isometry charge with the fermionic counterparts.

   This picture would be extremely simple and would be analogous to that of integrable quantum field theories in which the integral over small fluctuations gives Gaussian determinant and action exponential (now Gaussian determinant is cancelled by the metric determinant coming the Kähler metric of WCW) [K17].

   One can argue that the absence of loops makes it impossible to have non-perturbative effects. This is not true in adelic physics. Recall that the original motivation for $h_{eff} = n \times h$ was that this phase is generated with perturbation theory ceases to converge [K16]. The large value of h_{eff} scales down the coupling strengths proportional to $1/h_{eff}$ and perturbation theory works again.

It must be admitted that one must accept all these options. Number theoretical universality of scattering amplitudes would select $IIIa)$ and the need to realize given topological diagram using complex enough extension of rationals supports Option IIIc). I believe that the large number of the options reflects my limited mathematical understanding of the situation a careful analysis of the general implications of the options allows to pinpoint the most feasible one.

5.4 About problems related to the construction of twistor amplitudes

The dream is to construct twistorially fermionic scattering amplitudes and this requires the identification of fermionic 4-vertex. There are however several conceptual problems to be solved.

5.4.1 Could M^2 momenta be massive?

The naive objection against massive particles is that one loses the twistorial description both in M^4 sense and M^2 sense. Real quaternionic M^8 momenta are massless but the transversal momentum in E^6 degrees of freedom makes M^2 momenta and M^4 momenta for arbitrary choice of M^4 are massive, and one cannot describe the M^2 and M^4 momenta using the helicity spinor pair $(\lambda, \tilde{lambda})$. The beautiful formalism seems to be lost.

1. The naive argument is however wrong in TGD framework where particles are massless in M^8 sense. This means that mass does not correspond to $\overline{\Psi}\Psi$ in Dirac action but to comes from E^4 momentum (CP_2 "momentum"). 8-D chiral symmetry is unbroken as required by separate conservation of lepton and baryon numbers. In preferred M_0^2 one can indeed make M^2-momentum light-like.

2. Furthermore, 4-fermion twistor amplitudes are *holomorphic* functions of λ_i . There is no dependence of $\tilde{\lambda}$ and therefore no information about light-likeness! Why this amplitude could not describe the scattering of fermions only apparently massive in TGD Universe? Note that the momentum conserving delta function depends on the masses of the particles so that mass-dependence would be purely kinematical and analogous to the dependence on transverse momentum squared. Note that this argument makes sense also for M^4 twistorialization. If this view is correct then twistors are something more profound than momenta.

3. For M^2 twistorialization end would end up to effective (2,2) signature favored by twistorialization. (1,1) signature of real M^2 becomes (2,2) signature for complexified M^2 and real twistor space RP^3 is replaced with CP_3. This looks attractive description. If this picture is correct, all the nice results such as the possibility to assume reduction of amplitudes to positive Grassmannian remain unaffected.

5.4.2 Momentum conservation and mass shell conditions in 4-vertex

What is the exact meaning of the mass shell condition?

1. $H = M^4 \times CP_2$ harmonics would suggest that it mass squared in M^4 is eigenvalue of spinor d'Alembertian plus possible super-conformal contribution from Super Virasoro algebra, which is integer valued in suitable units. M^4-momentum decomposes to longitudinal M_0^2 momentum and transversal E^2 momentum. Super Virasoro algebra in transversal degrees of freedom suggests quantization of E^2 mass squared in integer multiples of a basic unit.

2. The CP_2 part of wave function in H corresponds in M^8 to a wave function in the moduli space of transversal planes E^2 assignable to M_0^2 and is involved only if the deformations of M^4 (or equivalently E^2) are present.

3. In the preferred frame M_0^4 the wave function would be strictly localized in single point of CP_2 and have maximally uncertain color quantum numbers. This kind of localization does look feasible physically. For instance, for color singlet CP_2 wave function of right-handed neutrino there is no localization. For sharp localization of 8-momentum to M_0^2 both color degrees and transvervsal E^2 degrees of freedom would effectively disappear.

4. The wave function in transversal E^2 momentum space with interpretation in terms of transversal momentum distribution - this at least in the case of hadrons.

5. The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4)+J(CP_2)$. Em charge would be the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere looks therefore natural. Note that the contribution to mass squared would be proportional to Q_{em}^2 so that one would obtain the electroweak mass splitting automatically. This is true also for CP_2 spinor harmonics.

5.4.3 How plausible topological loops are?

Topological loops are associated with the networks formed from the orbits of partonic 2-surfaces meeting at their ends (this would define topological 3-vertex containing fermionic 4-vertex). The tree topologies would provide a nice space-time description of particle reactions but loops could be possible? The original vision about construction of WCW geometry indeed was that the space-time surfaces with fixed ends are unique.

In the original vision the non-determinism of Kähler action inspired the hypothesis that loops are possible but volume term removes to high extent this non-determinism. In the recent vision the fusion of 3-surfaces at the ends of CD with light-like parton orbits to single 3-surface as a boundary condition (analogous to a fixing of a frame for soap films) would define the scattering diagram classically. There is no reason why it could not contain topological loops. Option IIIa) assuming that one can transform the diagrams ot tree diagrams, is therefore attractive.

1. There are also conditions from space-time dynamics. Twistor graph topologies correlate with space-time topologies since fermion line are inside the parton orbits and at vertices the ends of the orbits meet. Topological vertices would be basically 3-vertices for partonic 2-surfaces. The fermion and anti-fermion lines associated with the effective boson exchange would be naturally associated with opposite throats of wormhole contact.

   By above argument one can in ZEO pose at space-time level conditions fixing the vertices and identify the graph topology as a topology of the network of light-like 3-surfaces defining the diagram as boundary of 3-surface defined by the union of the ends of space-time and by parton orbits forming a connected surface.

2. There is a further delicacy to be taken into account - measurement resolution coded by the extension of rationals involved. This might allow to interpret addition of loops as in quantum field theories: as a result of increased measurement resolution determined dynamically by the intersection of reality and p-adicities. Different computation yielding the same result would not be cognitively equivalent since these intersections would be different.

3. If this view is correct, one can obtain also loops but non-negativity of energy for a given arrow of time for quantum state would allow only loops resulting from the decay and re-fusion of partonic 2-surfaces. Tadpoles appearing in BCFW recursion formula are impossible if the energy is non-negative. One can of course ask whether the sign of energy could be also negative if complex four-momenta are allowed. If so, one could have also tadpoles classically.

5.4.4 Identification of the fundamental 4-fermion vertex

The fundamental 4-fermion vertex would not be local 4-fermion vertex but correspond to classical scattering at partonic 2-surface. This saves from the TGD counterparts of the problems of QFT approach produced by non-renormalizability.

What would be this 4-fermion vertex? Yangian invariance suggests that the classical interaction between fermions must be expressible in terms of fictive 3-vertex of SUSY theories describing classical interaction as exchange of a fictive boson. This leaves 3 options.

Option I: 4-fermion vertex could be fusion of two 3-vertices with complex massless 8-momenta in M^8 picture. For instance, the exchanged momentum could be complex massless momentum and external momenta real on-mass-shell momenta. This vertex does not have QFT counterpart as such.

Loops could be absent either in the strong sense twistorial loops are absent (Option Ia) or in the sense that corresponding Feynman diagrams contain no loops (Option Ib). In particular, formation of BCFW bridge would not be allowed for Option Ia). Given diagram would be twistorial tree diagram obtained by replacing the vertices of ordinary tree diagram with these 4-vertices with complex massless fermions in 8-D sense.

Option II: 4-fermion could be identified as BCFW bridge associated with a tree Feynman diagram describing an exchange of a fictive boson. This 4-vertex would be analogous to an exchange of ordinary boson and counterpart for a QFT tree diagram. One can even forget the presence of the fictive boson exchange and write the formula for the simplest Yangian invariant as a candidate for four-fermion vertex.

Option III: If one allows higher fermion numbers at the same line, it is also natural to allow branching of lines. This requires allowance of 3-vertex as branching of fermion line as analog of splitting of open string (now strings are actually closed if they continue to another space-time sheet through wormhole contact). The situation would resemble that in SUSY. One cannot completely exclude this possibility.

Consider now the construction of 4-fermion vertex in more detail.

1. The helicities of fermions are $h_i = \pm 1$ and the general conjecture for the 4-fermion twistorial scattering amplitude is the simplest possible holomorphic rational function in λ_i, which does not depend on $\tilde{\lambda}_i$, and satisfies the condition that the scaling $\lambda_i \rightarrow t\lambda_i$ introduces the scaling factor t^{-2}.

2. The rule is that fermions correspond to 2 positive powers of λ_i and antifermions to 2 negative powers in λ_i: schematically the $F_1F_2\bar{F}_3\bar{F}_4$ vertex is of form $\lambda_1^2\lambda_2^1/\lambda_3^2\lambda_4^2$ and constructible from $\langle\lambda_i, \lambda_j$. One can multiply any term in the expression of vertex by a rational function of for which the weights associated with λ_i vanish. Ratios $P_i(f)/P_j(f)$ of functions $P(f)$ obtained by via odd permutations P of the arguments λ_i of function

$$f(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle\lambda_1, \lambda_2\rangle\langle\lambda_2, \lambda_3\rangle\langle\lambda_3, \lambda_4\rangle\langle\lambda_4, \lambda_1\rangle$$

3. invariant under 4 cyclic permutations. The number of these functions would be $4!/4 = 3! = 6$ corresponding to the 6 orbits of an odd permutation under the cyclic group Z_4. The simplest assumption is that these functions are not involved.

   The simplest guess for the 4-fermion scattering amplitude would be following:

$$T(F_1, F_2, \overline{F}_3, \overline{F}_4) = J \times \frac{\langle\lambda_1, \lambda_2\rangle^2}{\langle\lambda_3, \lambda_4\rangle^2} \quad . \tag{5.5}$$

   Charge conjugation would take the function to itse inverse. J is constant.

4. In 4-fermion vertex one has exchange of fictive boson and annihilation to fictive boson and the particles i, j in the vertex should contribute $\langle\lambda_i, \lambda_j\rangle$ to the scattering amplitudes.

Remarkably, this amplitude is holomorphic in λ_i and has no dependence on $\tilde{\lambda}_i$ and therefore carries no information about whether the momenta are light-like or not. It seems that one could allow massive fermions characterized by (λ_i, μ_i) and fermion masses would not be a problem! As already explained in TGD mass is not M^8-scalar and states are massless in 8-D sense: hence twistorialization should work!

One could construct more complex diagrams in very simple manner using these basic diagrams as building bricks just as in the twistor Grassmann approach. One could form product of diagrams A and B using merge operation [B21] identifying twistor variables Z_a and Z_b belonging to the two diagrams A and B to be fused.

For Option Ia) the diagram would represent repeated on mass shell 4-fermion scatterings but with of mass shell particles having complex momenta in 8-D sense. Real on mass shell particles would have massless but real 8-D momenta and physical polarizations.

The conservation of baryon and lepton numbers implies for all options that only $G(m, n = 2{\times}m)$ Grassmannians are needed. This simplifies considerably the twistor Grassmannian approach.

Why fermions as fundamental particles (to be distinguished from elementary particles in TGD) are so special?

1. The mass of the fundamental fermion is not visible in the holomorphic basic amplitude being visible only via momentum conserving delta function $\delta(\sum_i \lambda_i\tilde{\mu}_i)$. This property holds true also for more complex diagrams. Massivation does not require in TGD framework $\bar{\Psi}\Psi$ term in Dirac action since M^4-massive fermions are M^8-massless and have only chiral couplings in 8-D sense. Scalar coupling would also break separate baryon and lepton conservation. Mass term correspond to a momentum in $E^4 \subset M^4 \times E^4 = M^8$ degrees of freedom. Massivation without losing 8-D light-likeness is consistent with conformal symmetry and with 8-D twistor approach.

2. Fermions are exceptional in the sense that the number of helicities is same for both massive and massless fermions. In particular, 4-fermion amplitude has $k = n/2$ and positive Grassmannian $G(n/2, n)$ with special symmetry property that one can take either negative or positive helicities in preferred role, could be important. For massless states with higher spin the number of helicities is 2 and maximal spin is $J_{max} = h_{max}/2$. For M^4 -massive states also the lower helicities $h_{max} - 2k$ are possible. The scattering amplitudes remain holomorphic.

3. For SUSY one would have all helicities $h(k) = h_{max} - k$ and the general form of amplitude could be written from the knowledge of $h(k)$. The number of fermions at the boundary of string world sheets could be maximal allowed by statistics. This would give SUSY in TGD sense but would require splitting of string boundaries: it is not clear whether this can be allowed. For light-like orbits of partonic 2-surface it has been assumed.

   Sparticles could correspond to states with higher fermion number at given partonic orbits. In this case one expects only approximate SUSY: the p-adic primes characterizing different SUSY states could be different. In adelic physics different p-adic prime could correspond to a different extension of rationals: one might say that the particles inside super-multiplets are at different levels in number theoretic evolution!

5.4.5 BCFW recursion formula as a consistency condition: BCFW homology

The basic consistency condition is that the boundary operation in the BCFW recursion formula gives zero so that the recursion formula can be solved without introducing sum over topological loops. The twistorial trees would have no boundaries but would not be boundaries and would be therefore closed in what might be called BCFW homology. Diagrams would correspond to closed forms.

Consider first the proposal assuming that all diagrams are equivalent with twistorial string diagrams with fermionic 4-vertex as the basic vertex. The boundary operation appearing in BCFW formula gives two terms [B12, B21, B9]. Recall that options I, II, and III correspond to twistorial diagrams without loops created by BCFW bridges, to twistor diagrams assignable to Feynman diagrams without loops, and to diagrams analogous to SUSY diagrams for which fermion lines carry also higher fermion number and can split.

1. The first term results as one BCFW bridge by contracting the three lines connecting the external particles to a larger diagram to a point in all possible manners. The non-vanishing of this term does not force loops in the sense of Feynman diagrams. For Option Ia) (no twistorial loops) there are no BCFW boxes to be reduced so that the outcome is zero.

   For option Ib) (no Feynman loops) a BCFW box diagram for which the two outward direct lines of the bridge are fictive, this operation makes sense and reduces the box to that describing the basic 4-fermion vertex. Same is true for the option II. For option III the operation would be essentially the same as in SUSY.

2. Second term corresponds to entangled removal of a fermion and anti-fermion and if it is non-vanishing, loops are unavoidable. This operation creates a closed fermionic loop to which several internal lines couple. By QCC the fermionic loop would be associated with a topological loop. One can argue that the topological tadpole loop must be closed time loop and that this is not possible since the sign of energy must change at the top and bottom of the loop, where the arrow of time changes: actually the energy should vanish. The same result would obtained if one requires that the energy identified as real part of complexified energy is non-negative for all on mass shell particles.

   Consider the 4-fermion vertex to which the fermionic tadpole loop is associated. Entangled removal gives for the members of a pair of external lines opposite momenta and helicities in twistor-diagrammatics. If so, there exist a vertex for which one fermion scatters in forward direction. Momentum conservation implies the same for the second fermion. One would obtain amplitude, which equals to unity rather than vanishing! Integration over four-momenta would give divergence. However, if the 4-momentum in the tadpole vanishes, the corresponding helicity spinor and also the amplitude vanishes. QCC indeed demands that fermionic loop corresponds to a time loop possible only only if the energy and by time-likeness also 3-momentum vanishes.

It seems that only the simplest option - Option Ia) - is consistent with the BCFW reduction formula. One can say that scattering diagrams are closed objects in the BCFW cohomology. Closedness condition might allow also topological loops, which are not tadpole loops: say decay of fermion to 3 fermions fusing back to the fermion.

5.4.6 Under what conditions fermionic self energy loop is removable?

Scattering diagram as a representation of computation demands that the fermionic "self energy" loop involving two external fermions gives free propagator. The situation in which the vertex contains only *light-like* complex momenta in M_0^2 can be considered as an example. In fact, one can always choose in M^8 the frame for given component of state in this manner.

1. The three fermion/antifermion internal lines in the loop would be light-like in complex 2-D sense as also external momentum. For external momenta $Re(p(M^2))$ would be light-like and orthogonal to light-like $Im(p(M^2))$: it is not clear whether $Im(p(M^2))$ vanishes.

   Light-likeness condition gives $Re(k)^2 - Im(k)^2 = 0$ and $Re(k) \cdot Im(k) = 0$, and $Re(k) = \pm Im(k)$ as a solution meaning that $Re(k)$ is proportional to a light-like vector $(1, 1)$ or $(1 - 1)$. This applies to p, k_1,k_2, and $p - k_1 - k_2$. All these vectors are proportional to the same light-like vector in M^2.

   Apart from the degeneracy for sign factors the situation is equivalent with real 2-D case and one has from momentum conservation that the real parts of the virtual momenta are light-like and parallel and one has $Re(k_i) = \lambda_i p$ leaving two real parameters λ_i.

2. The only possible outcome from the integral is proportional to $D_F(p)$. The outcome is non-vanishing if the proportionality constant is proportional to $1/p^2$. This dependence should come from 4-fermion vertices. The integrand is proportional to the product $\lambda_1\lambda_2(1-\lambda_1-\lambda_2)$ and involves times the $D_F(p)$. Vertices give the inverses of these scaling factors. Since the outcome should be proportional to $1/D_F$ and lines are proportional to p^3, the 4- vertices should give a factor $1/p^2$ each.

   Assuming this one obtains integrand $1/(\lambda_1\lambda_2(1 - (\lambda_1 - \lambda_2)^2$. The integral over λ_i is of proportional to

   $$I = \int d\lambda_1 d\lambda_2/\lambda_1\lambda_2(1 - \lambda_1 - \lambda_2) \ .$$

   The ranges of integration are from $(-\infty, \infty)$.

   One can decompose the integral to four parts so that integration ranges are positive. The outcome is

   $$I = \int dlog(\lambda_1) dlog(\lambda 2) \left[\frac{1}{1-\lambda_1-\lambda_2} + \frac{1}{1+\lambda_1+\lambda_2} - \frac{1}{1+\lambda_1-\lambda_2} - \frac{1}{1-\lambda_1+\lambda_2} \right] \ .$$

   The change of variables $(u, v) = (\lambda_1 + \lambda_2, \lambda_1 - \lambda_2)$ transforms the integral to a product of integrals

   $$I = \int dudv \frac{1}{1-u^2} \int dv \frac{1}{1-v^2} \ .$$

   The interpretation as residue integral gives the outcome $I = (4\pi)^2$.

   Residue integration gives finite result for this integrals. One can worry about the singularity of the vertices for M_0^2 on mass shell momenta. The problem is that p is on mass shell so that the outcome from loop diverges. The outcome is $D_F$would be however finite.

5.4.7 Gliding conditions for 4-vertices

One can construct also loop diagrams with loops understood in twistorial sense. The interpretation of twistor diagram as computation requires that there exist moves reducing general loopy diagrams to tree diagrams. This requires that the vertices connected by a fermionic loop lines can be glided along fermion lines such that they become nearest neighbors and that these loops can be removed without affective the diagram.

If these diagrams are acceptable mathematically, moves reducing these loop diagrams to twistorial tree diagrams should exist. Could the basic rule be following?

1. One can glide the vertices past each other along fermion lines and reduce loops connecting points at different part of graph to the analogs of self-energy loops located at single fermion lines. These loops involve decay of fermion to 2 fermions and 1 anfifermion which then fuse to single fermion. All fermions are on mass shell in complex sense. The situation thus reduces to single fermion self energy loop if the gliding is possible always. Mass shell conditions could however prevent this.

2. To single fermion line one can assign D_F - the inverse of massless fermion propagator - having formal interpretation as a density matrix. The loop would not vanish but would give rise to a inverse of fermionic propagator so that the overall outcome should be just D_F. Is it possible to achieve this?

Under what conditions the gliding is possible?

1. Suppose that the 4-vertex V_1 is glided along fermion line past second 4-vertex V_1. V_1 corresponds to momenta $(P_{i,in}, P_{i1,in} - P, P_{i,1}, P_{i,2})$. The momentum $P_i = \sum_{k=1}^{2} P_{i,k}$ of 2 particles emanates from V_i so that the outgoing and incoming momenta are $P_{i,in} - P_i$, and $P_{i,in}$ $i = 1, 2$. Furthermore $P_{1,in} = P_{2,in} - P_2$. These complex momenta are on M^2 mass shell in the proposed sense.

2. Can one perform the gliding without changing the M_0^2-momenta $P_{i,1}$ and $P_{i,2}$? Gliding is possible if the on mass shell condition is satisfied also for $P_{2,in} - P_1 + P_2$ rather than only $P_{2,in} + P_2$. If the mass squared spectrum is integer valued in suitable units the condition reduces to the requirement that $2P_{2,in} \cdot P_1$ is real and integer valued.

   These conditions are independent of the conditions for $2P_{2,in} \cdot P_2$ coming from V_2, the conditions would correlate P_1 and P_2. The construction of the amplitude would involve non-local conditions on vertices rather than only momentum conservation and mass shell conditions at vertices as expected.

   M^2-momentum is however light-like for a special choice$M^2 = M_0^2$. If M_0^2 same along connected fermion lines, the gliding condition would make sense. M_0^2 is constant of motion along fermion line which can wander along the network formed by partonic orbits.

   In fact, M_0^2 must be same for all fermions in given vertex so that its is constant for all connected regions of fermionic part of the graph. Is there any hope of having non-trivial scattering amplitude or must all momenta be light-like and parallel in plane M_0^2? Tree diagrams certainly give rise to non-trivial scattering. One can also assign to all internal lines this kind of networks with M_0^2 that assignable to the internal line. It is quite possible that for general graphs allowing different M_0^2s in internal lines and loops, the reduction to tree graph is not possible.

3. The analogs of these conditions apply also to tree graphs. So that one must either sum over trees with different orderings of vertices or pose additional conditions on the M^2-momenta say the assumption that they are light-like and proportional to the same real momentum $(1, \pm 1)$ along the fermion line.

To conclude: if M_0^2 is constant of motion along the connected networks of fermion lines, the gliding conditions could be satisfied. Action exponentials do not produce trouble if one identifies the basis of zero energy states in such a manner that every maximum of action gives its own separate amplitude (state) as also number theoretic universality demands. The most attractive option number theoretically is the option IIIa) assuming that localization of zero energy state to single computation is possible as quantum measurement: different localizations would have different intersections between reality and p-adicities and would correspond to different computation sequences as cognitive processes. The idea that twistor diagrams are closed forms in the sense that tadpole diagrams vanish is also very attractive and natural in this framework.

5.4.8 Permutation as basic data for a scattering diagram

In twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY the data determining the Yangian invariants defining the basic building bricks of the amplitudes can be constructed using two 3-vertices. For the first (second) kind of vertex the helicity spinors λ_i ($\tilde{\lambda}_i$) are parallel that is $\lambda_1 \propto \lambda_2 \propto \lambda_3$ ($\overline{\lambda}_1 \propto \overline{\lambda}_2 \propto \overline{\lambda}_3$) and can be chosen to be identical by complex scaling invariant: momentum conservation reduces to that for $\tilde{\lambda}_i$ (λ_i). The graphical notation for the two vertices is as a small white *resp.* black disk [B21, B9] (see Fig. 3.3.35 `http://tinyurl.com/zbj9ad7`).

There are two basic moves leaving the amplitude unaffected (see Fig. 3.3.38 at `http://tinyurl.com/zbj9ad7`). Merging symmetry implies that 4-vertices satisfy a symmetry analogous to the duality of old-fashioned hadron physics: an internal line connecting black (white) vertices as exchange in s-channel can be transformed to an exchange in t-channel: $1{+}2 \rightarrow 3{+}4 \equiv 1{+}3 \rightarrow 2{+}4$. Merging symmetry allows to transform the diagram into a form in which neighboring vertices have opposite colors. Square move symmetry follows from the cyclic symmetry of the 4-particle amplitude and means black↔white replacement in 4-vertex.

These two moves do not affect the permutation defining the diagram. A given diagram is represented as a disk with external lines ordered cyclically along its boundary. The permutation of the n external particles associated with the diagram is constructed from the two 3-particle diagrams is defined by the following rule.

Start from k:th point at boundary end and go to the left in each white vertex and to the right in each black vertex (see Fig. 3.3.35 at `http://tinyurl.com/zbj9ad7`*).*

This leads to a particle $P(k)$ and the outcome is a permutation $P : k \rightarrow P(k)$ charactering the twistor diagram.

Moves do not affect the permutation associated with the diagram and leave the amplitude unaffected. BCFW bridge can be interpreted as a permutation of two neighboring external lines and allows to generate non-equivalent diagrams.

This permutation symmetry generalizes to 4-D SUSY the role of permutations in 1+1-D integrable field theories, where the scattering S-matrix induces only a phase shift of the wave functions of identical particles. The scattering diagram depends only on the permutation of particles induced by the scattering event. Yang-Baxter relation expresses this . Scattering corresponds to particles passing by each other and diagram is drawn in M^2 plane.

1. In 1+1-D integrable theory 3+3 scattering reduces to 2 particle scatterings. This can be illustrated using world lines in M^2 plane (see the illustration of `http://tinyurl.com/gogn75s`). The particle 2 can be taken to be at rest and 1 and 3 move with opposite velocities. There are three 2-particle scatterings of i and j as crossings of world-lines of i and j (pass-by spatially): denote the crossing by ij.

   For the diagram on the left hand side one has crossings 12, 13 and 23 with this time order. For the second case one has crossings 23, 13, and 12 in this time order. Graphically YB relation (see the illustration of `http://tinyurl.com/gogn75s`) says that the scattering amplitude fo 3+3 scattering does not depend on whether the position of the stationary particle 2 is to the left or right from the point at which the second scattering occurs: the time order of scatterings 12 and 23 does not matter.

2. Mathematically the two-particle scatterings are described by operators $R_{12}(u)$, $R_{13}(u+v)$, and $R_{23}(v)$ representing basic braiding operation $ij \rightarrow ji$. u, $u+v$, and v are parameters characterizing the Lorentz boosts determining the velocities of particles. YB equation reads as

$$R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u) \ .$$

For a graphical illustration see `http://tinyurl.com/gogn75s`. The first and third R-matrices are permuted and the outcome is trivial. In pass-by interpretation YB equation states that the two manners to realize $123 \rightarrow 321$ give the same amplitude.

Instead of pass-by one could assume a reconnection of the world lines at the intersection: world lines are split and future pieces are permuted and connected to the past pieces again. With this interpretation one has $123 \rightarrow 123$ (the illustration of Wikipedia article corresponds to this interpretation).

3. At the static limit $u, v \rightarrow 0$ YB equation gives rise to an identity satisfied by braiding matrices. The pass-by at this limit can be interpreted as permutation lifted to braiding (braid groups is covering group of permutation group).

2+2 vertices are fundamental in integrable theories in M^2. Also in TGD 2+2 vertices for fundamental fermions are proposed to be fundamental, and the effective reduction to M^2 is crucial in many respects and reflects $M^8 - CP_2$ duality and 8-D quaternionic light-likeness implying that 2+2 fermion vertices reduce to vertices in M^2. TGD could be an integrable theory able to circumvent the limitations of integrable QFTs in M^2.

1. How could the 2+2-fermionic scattering matrix relate to the R-matrix? In TGD framework the scattering involves momentum transfer even in M_0^2 frame: the parallel light-like M^2 momenta are rescaled in momentum conserving manner. Could R matrix appear as additional factor in the scattering? The earlier picture indeed is that the fermion lines at partonic orbits can experience braiding described by R-matrix at the static limit (string world sheet boundaries would braid!).

2. In TGD the scattering of 2 fermions could occur in two manners by classical interactions at partonic 2-surface. The world lines either cross each other or not. In M^2 the first contribution is planar and second one non-planar. Both options should contribute to the 4-fermion amplitude but this is not be visible in the proposed form of the amplitude. Does the proposed 4-fermion scattering amplitude allow this interpretation?

   In $\mathcal{N} = 4$ SUSY the addition of BCFW bridge would permute the two external particles. In TGD the introduction of BCFW bridge would force to have bosonic lines in the BCFW bridge. This is not possible. The only manner to have BCFW diagram is to allow SUSY perhaps realized as and addition right-handed neutrinos to the fermion lines but this would force to allow splitting of fermion lines requiring splitting of strings.

3. Annihilations of fermion-antifermion pairs to bosons are not possible in 1+1-D QFTs but in TGD topological 3-vertices allow them. Boson would correspond to the final $B \equiv F\overline{F}$ pair at same parton orbit. There are two manners to achieve the annihilation. In s-channel $F\overline{F} \rightarrow vacuum \rightarrow F\overline{F} \equiv B$ is possible. Both F_1 coming from past and F_2 from future scatter classically backwards in time to give $\overline{F}_1$ travelling back to past and $\overline{F}_2$ travelling back to future. In t-channel one can have braiding ($F\overline{F} \rightarrow \overline{F}F \equiv B$.

5.4.9 About unitarity for scattering amplitudes

The first question is what one means with S-matrix in ZEO. I have considered several proposals for the counterparts of S-matrix [K14].In the original U-matrix, M-matrix and S-matrix were introduced but it seems that U-matrix is not needed.

1. The first question is whether the unitary matrix is between zero energy states or whether it characterizes zero energy states themselves as time-like entanglement coefficients between positive and negative energy parts of zero energy states associated with the ends of CD. One can argue that the first option is not sensible since positive and negative energy parts of zero energy states are strongly correlated rather than forming a tensor product: the S-matrix would in fact characterize this correlation partially.

   The latter option is simpler and is natural in the proposed identification of conscious entity - self - as a generalized Zeno effect, that is as a sequence of repeated state function reductions at either boundary of CD shifting also the boundary of CD farther away from the second boundary so that the temporal distance between the tips of CD increases. Each shift of this kind is a step in which superposition of states with different distances of upper boundary from lower boundary results followed by a localization fixing the active boundary and inducing unitary transformation for the states at the original boundary.

2. The proposal is that the the proper object of study for given CD is M-matrix. M-matrix is a product for a hermitian square root of diagonalized density matrix ρ with positive elements and unitary S-matrix $S: M = \sqrt{\rho}S$. Density matrix ρ could be interpreted in this approach as a non-trivial Hilbert space metric. Unitarity conditions are replaced with the conditions $MM^\dagger = \rho$ and $M^\dagger M = \rho$. For the single step in the sequence of reductions at active boundary of CD one has $M \rightarrow MS(\Delta T)$ so that one has $S \rightarrow SS(\Delta T)$. $S(\Delta T)$ depends on the time interval ΔT measured as the increase in the proper time distance between the tips of CD assignable to the step.

What does unitarity mean in the twistorial approach?

1. In accordance with the idea that scattering diagrams is a representation for a computation, suppose that the deformations of space-time surfaces defining a given topological diagram as a maximum of the exponent of Kähler function, are the basic objects. They would define different quantum phases of a larger quantum theory regarded as a square root of thermodynamics in ZEO and analogous to those appearing also in QFTs. Unitarity would hold true for each phase separately.

   The topological diagrams would not play the role of Feynman diagrams in unitarity conditions although their vertices would be analogous to those appearing in Feynman diagrams. This would reduce the unitarity conditions to those for fermionic states at partonic 2-surfaces at the ends of CDs, actually at the ends of fermionic lines assigned to the boundaries of string world sheets.

2. The unitarity conditions be interpreted stating the orthonormality of the basis of zero energy states assignable with given topological diagram. Since 3-surfaces as points of WCW appearing as argument of WCW spinor field are pairs consisting of 3-surfaces at the opposite boundaries of CD, unitarity condition would state the orthonormality of modes of WCW spinor field. If might be even that no mathematically well-defined inner product assignable to either boundary of CD exists since it does not conform with the view provided by WCW geometry. Perhaps this approach might help in identifying the correct form of S-matrix.

3. If only tree diagrams constructed using 4-fermion twistorial vertex are allowed, the unitarity relations would be analogous to those obtained using only tree diagrams. They should express the discontinuity for T in $S = 1 + iT$ along unitary cut as $Disc(T) = TT^\dagger$. T and $T^\dagger$ would be T-matrix and its time reversal.

4. The correlation between the structure of the fermionic scattering diagram and topological scattering diagrams poses very strong restrictions on allowed scattering reactions for given topological scattering diagram. One can of course have many-fermion states at partonic 2-surfaces and this would allow arbitrarily high fermion numbers but physical intuition suggests that for given partonic 2-surface (throat of wormhole contact) the fermion number is only 0, 1, or perhaps 2 in the case of supersymmetry possibly generated by right-handed neutrino.

   The number of fundamental fermions both in initial and final states would be finite for this option. In quantum field theory with only masive particles the total energy in the final state poses upper bound on the number of particles in the final state. When massless particles are allowed there is no upper bound. Now the complexity of partonic 2-surface poses an upper bound on fermions.

   This would dramatically simplify the unitarity conditions but might also make impossible to satisfy them. The finite number of conditions would be in spirit with the general philosophy behind the notion of hyper-finite factor. The larger the number of fundamental fermions associated with the state, the higher the complexity of the topological diagram. This would conform with the idea about QCC. One can make non-trivial conclusions about the total energy at which the phase transitions changing the topology of space-time surface defined by a topological diagram must take place.

5.5 Criticism

One can criticize the proposed vision.

5.5.1 What about loops of QFT?

The idea about cancellation of loop corrections in functional integral and moves allowing to transform scattering diagrams represented as networks of partonic orbits meeting at partonic 2-surfaces defining topological vertices is nice.

Loops are however unavoidable in QFT description and their importance is undeniable. Photon-photon (see `http://tinyurl.com/lqhdujm`) scattering is described by a loop diagram in which fermions appear in box like loop. Magnetic moment of muon see `http://tinyurl.com/p7znfmd`) involves a triangle loop. A further, interesting case is CP violation for mesons (see `http://tinyurl.com/oop4apy`) involving box-like loop diagrams.

Apart from divergence problems and problems with bound states, QFT works magically well and loops are important. How can one understand QFT loops if there are no fundamental loops? How could QFT emerge from TGD as an approximate description assuming lengths scale cutoff?

The key observation is that QFT basically replaces extended particles by point like particles. Maybe loop diagrams can be "unlooped" by introducing a better resolution revealing the non-point like character of the particles. What looks like loop for a particle line becomes in an improved resolution a tree diagram describing exchange of particle between sub-lines of line of the original diagram. In the optimal resolution one would have the scattering diagrams for fundamental fermions serving as building bricks of elementary particles.

To see the concrete meaning of the "unlooping" in TGD framework, it is necessary to recall the qualitative view about what elementary particles are in TGD framework.

1. The fundamental fermions are assigned to the boundaries of string world sheets at the light-like orbits of partonic 2-surfaces: both fermions and bosons are built from them. The classical scatterings of fundamental fermions at the 2-D partonic 2-surface defining the vertices of topological scattering diagrams give rise to scattering amplitudes at the level of fundamental fermions and twistor lift with 8-D light-likeness suggests essentially unique expressions for the 4-fermion vertex.

2. Elementary particle is modelled as a pair of wormhole contacts (Euclidian signature of metric) connecting two space-time sheets with throats at the two sheets connected by monopole flux tubes. All elementary particles are hadronlike systems but at recent energies the substructure is not visible. The fundamental fermions at the wormhole throats at given space-time sheet are connected by strings. There are altogether 4 wormhole throats per elementary particle in the simplest model.

   Elementary boson corresponds to fundamental fermion and antifermion at opposite wormhole throats with very small size (CP_2 size). Elementary fermion has only single fundamental fermion at either throat. There is $\nu_L\overline{\nu}_R$ pair or its CP conjugate at the other end of the flux tube to neutralize the weak isospin. The flux tube has length of order Compton length (or elementary particle or of weak boson) gigantic as compared to the size of the wormhole contact.

3. The vertices of topological diagram involve joining of the stringy diagrams associated with elementary particles at their ends defined by wormhole contacts. Wormhole contacts defining the ends of partonic orbits of say 3 interacting particles meet at the vertex - like lines in Feynman diagram - and fundamental fermion scattering redistributes fundamental fermions between the outgoing partonic orbits.

4. The important point is that there are $2 \times 2 = 4$ manners for the wormhole contacts at the ends of two elementary particle flux tubes to join together. This makes a possible a diagrams in which particle described by a string like object is emitted at either end and glued back at the other end of string like object. This is basically tree diagram at the level of wormhole contacts but if one looks it at a resolution reducing string to a point, it becomes a loop diagram.

5. Improvement of the resolution reveals particles inside particles, which can scatter by tree diagrams. This allows to "unloop" the QFT loops. By increasing resolution new space-time sheets with smaller size emerge and one obtains "unlooped" loops in shorter scales. The space-time sheets are characterized by p-adic length scale and primes near powers of 2 are favored. p-Adic coupling constant evolution corresponds to the gradual "unlooping" by going to shorter and shorter p-adic length scales revealing smaller and smaller space-time sheets.

The loop diagrams of QFTs could thus be seen as a direct evidence of the fractal many-sheeted space-time and quantum criticality and number theoretical universality (NTU) of TGD Universe. Quantum critical dynamics makes the dynamics universal and this explains the unreasonable success of QFT models as far as length scale dependence of couplings constants is considered. The weak point of QFT models is that they are not able to describe bound states: this indeed requires that the extended structure of particles as 3-surfaces is taken into account.

5.5.2 Can action exponentials really disappear?

The disappearance of the action exponentials from the scattering amplitudes can be criticized. In standard approach the action exponentials associated with extremals determine which configurations are important. In the recent case they should be the 3-surfaces for which Kähler action is maximum and has stationary phase. But what would select them if the action exponentials disappear in scattering amplitudes?

The first thing to notice is that one has functional integral around a maximum of vacuum functional and the disappearance of loops is assumed to follow from quantum criticality. This would produce exponential since Gaussian and metric determinants cancel, and exponentials would cancel for the proposal inspired by the interpretation of diagrams as computations. One could in fact *define* the functional integral in this manner so that a discretization making possible NTU would result.

Fermionic scattering amplitudes should depend on space-time surface somehow to reveal that space-time dynamics matters. In fact, QCC stating that classical Noether charges for bosonic action are equal to the eigenvalues of quantal charges for fermionic action in Cartan algebra would bring in the dependence of scattering amplitudes on space-time surface via the values of Noether charges. For four-momentum this dependence is obvious. The identification of $h_{eff}/h = n$ as the dimension of the extension dividing the order of its Galois group would mean that the basic unit for discrete charges depends on the extension characterizing the space-time surface. Also the cognitive representations defined by the set of points for which preferred imbedding space coordinates are in this extension. Could the cognitive representations carry maximum amount of information for maxima? For instance, the number of the points in extension be maximal. Could the maximum configurations correspond to just those points of WCW, which have preferred coordinates in the extension of rationals defining the adele? These 3-surfaces would be in the intersection of reality and p-adicities and would define cognitive representation.

These ideas suggest that the usual quantitative criterion for the importance of configurations could be equivalent with a purely number theoretical criterion. p-Adic physics describing cognition and real physics describing matter would lead to the same result. Maximization for action would correspond to maximization for information.

Irrespective of these arguments, the intuitive feeling is that the exponent of the bosonic action must have physical meaning. It is number theoretically universal if action satisfies $S = q_1 + iq_2\pi$. This condition could actually be used to fix the dependence of the coupling parameters on the extension of rationals [L3]. By allowing sum over several maxima of vacuum functional these exponentials become important. Therefore the above ideas are interesting speculations but should be taken with a big grain of salt.

6 Appendix: Some background about twistors

In the following I try to summarize my view about how the ideas related to the twistor approach to scattering amplitudes evolved. A readable summary of specialist about twistor approach is given in the article *Scattering amplitudes* of Elvang and Huang [B9]. Also the thesis *Grassmannian Origin of Scattering Amplitudes* of Trnka [B21] gives a good summary about the work done in association with

Nima Arkani-Hamed. I am not a specialist and have not been endowed with practical calculations so that my representation considers only the basic ideas and their relationship to TGD. In the following I summarize my very partial view about the development of ideas.

6.1 The pioneering works of Penrose and Witten

The pioneering work of Penrose discussed in *The Central Programme of Twistor Theory* [B19] on twistors initiated the twistor program, which had already had applications in Yang-Mills theories int he description of instantons. The key vision is that massless field equations reduce to holomorphy in twistor formulation.

Witten's *Perturbative Gauge Theory As a String Theory In Twistor Space* [B8] in 2003 initiated the progress leading to dramatic understanding of the planar scattering amplitudes of $\mathcal{N} = 4$ SUSY and eventually to the notion of amplituhedron. The abstract gives some idea about the key ideas.

Perturbative scattering amplitudes in Yang-Mills theory have many unexpected properties, such as holomorphy of the maximally helicity violating amplitudes. To interpret these results, we Fourier transform the scattering amplitudes from momentum space to twistor space, and argue that the transformed amplitudes are supported on certain holomorphic curves. This in turn is apparently a consequence of an equivalence between the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory and the D-instanton expansion of a certain string theory, namely the topological B model whose target space is the Calabi-Yau supermanifold $CP_{3|4}$.

Witten's observation was that the twistor Fourier transform of the scattering amplitudes of YM theories seem to be localized at 2-dimensional complex surfaces of twistor space and this led him to propose that twistor string theory in the twistor space CP_3 could allow to describe the scattering amplitudes. The basic problem of the twistor approach relates to space-time signature: all works nicely in signature (2,2), which suggests that something might be wrong in the basic assumptions.

6.2 BCFW recursion formula

BCFW recursion was first derived for tree amplitudes and later generalized to planar loop diagrams.

1. *Twistor diagram recursion for all gauge-theoretic tree amplitudes* by Hodges [B1] in 2005 and *Direct Proof of Tree-Level Recursion Relation in Yang- Mills Theory* by Britto, Cachazo, Feng, and Witten [B4] in 2005 proposed at tree level a recursion formula for the tree level MHV amplitudes of Yang-Mills theory in twistor space.

2. *Scattering Amplitudes and BCFW Recursion in Twistor Space* By Mason and Skinner [B4] discussed BCFW recursion relations for tree diagrams of YM theories.

3. *The S-Matrix in Twistor Space* by Arkani-Hamed, Cachazo, Cheung and Kaplan [B10] in 2009 discussed NkMHV amplitudes with more than two negative helicities (MHV amplitudes have 2 negative helicities are are extremely simple).

This work is carried out in metric signature (2,2), where the twistor transform reduces to ordinary Fourier transform. The other signatures are problematic. Only planar diagrams are considered. *On-Shell Structures of MHV Amplitudes Beyond the Planar Limit* [B14] in 2014 of Arkani-Hamed et al consider the problem posed by the non-planar diagrams.

6.3 Yangian symmetry and Grassmannian

The discovery of dual super-conformal invariance is one of the key steps of progress. This symmetry means extension of the conformal algebra from space-time level to the level of twistor space so that the dual superconformal invariance acts also on so called momentum twistors assigned with the twistor diagram. These dual conformal symmetries extend to a Yangian algebra containing besides local generators also multilocal generators. The dual conformal generators are bi-local generators and have weight $n = 1$. The Yangian symmetry is completely general and expected to generalize.

In the following I list the abstracts of some important articles.

1. *Magic identities for conformal four-point integrals* by Drummond, Henn, Smirnov, and Sokatchev [B15] in 2006 initiated the development of ideas. The interpretation is as dual conformal invariance generator by the weight 1 generators of Yangian.

   We propose an iterative procedure for constructing classes of off-shell four-point conformal integrals which are identical. The proof of the identity is based on the conformal properties of a sub-integral common for the whole class. The simplest example are the so-called "triple scalar box" and "tennis court" integrals. In this case we also give an independent proof using the method of Mellin-Barnes representation which can be applied in a similar way for general off-shell Feynman integrals.

2. *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [B7] by Drummond, Henn, and Plefka in 2009 continued this work and discussed Yangian algebra as as a symmetry having besides local generators also multilocal generators.

   Tree-level scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory have recently been shown to transform covariantly with respect to a "dual" superconformal symmetry algebra, thus extending the conventional superconformal symmetry algebra psu(2, 2|4) of the theory. In this paper we derive the action of the dual superconformal generators in on-shell superspace and extend the dual generators suitably to leave scattering amplitudes invariant. We then study the algebra of standard and dual symmetry generators and show that the inclusion of the dual superconformal generators lifts the psu(2,2|4) symmetry algebra to a Yangian. The non-local Yangian generators acting on amplitudes turn out to be cyclically invariant due to special properties of psu(2,2|4). The representation of the Yangian generators takes the same form as in the case of local operators, suggesting that the Yangian symmetry is an intrinsic property of planar $\mathcal{N}=4$ super Yang-Mills, at least at tree level.

3. *Dual Superconformal Invariance, Momentum Twistors and Grassmannians* [B17] by Mason and Skinner introduces momentum twistors and Grassmannians.

   Dual superconformal invariance has recently emerged as a hidden symmetry of planar scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory. This symmetry can be made manifest by expressing amplitudes in terms of "momentum twistors", as opposed to the usual twistors that make the ordinary superconformal properties manifest. The relation between momentum twistors and on-shell momenta is algebraic, so the translation procedure does not rely on any choice of space-time signature. We show that tree amplitudes and box coefficients are succinctly generated by integration of holomorphic delta-functions in momentum twistors over cycles in a Grassmannian. This is analogous to, although distinct from, recent results obtained by Arkani-Hamed et al. in ordinary twistor space. We also make contact with Hodges polyhedral representation of NMHV amplitudes in momentum twistor space.

4. *A Duality For The S Matrix* [B12] in 2009 by Arkani-Hamed et al discusses also Yangian invariance and introduces central ideas in algebraic geometry: Grassmannians, higher-dimensional residue theorems, intersection theory, and the Schubert calculus.

   We propose a dual formulation for the S Matrix of $\mathcal{N}=4$ SYM. The dual provides a basis for the leading singularities of scattering amplitudes to all orders in perturbation theory, which are sharply defined, IR safe data that uniquely determine the full amplitudes at tree level and 1-loop, and are conjectured to do so at all loop orders. The scattering amplitude for n particles in the sector with k negative helicity gluons is associated with a simple integral over the space of k planes in n dimensions, with the action of parity and cyclic symmetries manifest. The residues of the integrand compute a basis for the leading singularities. A given leading singularity is associated with a particular choice of integration contour, which we explicitly identify at tree level and 1-loop for all NMHV amplitudes as well as the 8 particle N^2MHV amplitude. We also identify a number of 2-loop leading singularities for up to 8 particles. There are a large number of relations among residues which follow from the multi-variable generalization of Cauchys theorem known as the "global residue theorem". These relations imply highly non-trivial identities guaranteeing the equivalence of many different representa- tions of the same amplitude. They also enforce the cancellation of non-local poles as well as consistent infrared structure at loop level. Our conjecture connects the physics *of scattering amplitudes to a particular subvariety in a Grassmannian; space-time locality is reflected in the topological properties of this space.*

5. *The All-Loop Integrand For Scattering Amplitudes in Planar $\mathcal{N} = 4$ SYM* [B13] by Arkani-Hamed et al in 2010.

   We give an explicit recursive formula for the all L-loop integrand for scattering amplitudes in $\mathcal{N} = 4$ SYM in the planar limit, manifesting the full Yangian symmetry of the theory. This generalizes the BCFW recursion relation for tree amplitudes to all loop orders, and extends the Grassmannian duality for leading singularities to the full amplitude. It also provides a new physical picture for the meaning of loops, associated with canonical operations for removing particles in a Yangian-invariant way. Loop amplitudes arise from the "entangled" removal of pairs of particles, and are naturally presented as an integral over lines in momentum-twistor space. As expected from manifest Yangian-invariance, the integrand is given as a sum over non-local terms, rather than the familiar decomposition in terms of local scalar integrals with rational coefficients. Knowing the integrands explicitly, it is straightforward to express them in local forms if desired; this turns out to be done most naturally using a novel basis of chiral, tensor integrals written in momentum-twistor space, each of which has unit leading singularities. As simple illustrative examples, we present a number of new multi-loop results written in local form, including the 6- and 7-point 2-loop NMHV amplitudes. Very concise expressions are presented for all 2-loop MHV amplitudes, as well as the 5-point 3-loop MHV amplitude. The structure of the loop integrand strongly suggests that the integrals yielding the physical amplitudes are "simple", and determined by IR-anomalies. We briefly comment on extending these ideas to more general planar theories.

6.4 Amplituhedron

The latest development in twistorial revolution was the notion of amplituhedron. Since I do not have intuitive understanding about amplituhedron and since amplituhedron does not have role in the twistorialization of TGD as I understand it now, I provide only abstracts about two articles to it.

1. *The Amplituhedron* [B3] by Arkani-Hamed and Trnka in 2013.

   Perturbative scattering amplitudes in gauge theories have remarkable simplicity and hidden infinite dimensional symmetries that are completely obscured in the conventional formulation of field theory using Feynman diagrams. This suggests the existence of a new understanding for scattering amplitudes where locality and unitarity do not play a central role but are derived consequences from a different starting point. In this note we provide such an understanding for $\mathcal{N} = 4$ SYM scattering amplitudes in the planar limit, which we identify as "the volume" of a new mathematical object–the Amplituhedron–generalizing the positive Grassmannian. Locality and unitarity emerge hand-in-hand from positive geometry.

2. *Positive Amplitudes in the Amplituhedron* [B2] by Arkani-Hamed et al in 2014.

   The all-loop integrand for scattering amplitudes in planar $\mathcal{N} = 4$ SYM is determined by an "amplitude form" with logarithmic singularities on the boundary of the amplituhedron. In this note we provide strong evidence for a new striking property of the superamplitude, which we conjecture to be true to all loop orders: the amplitude form is positive when evaluated inside the amplituhedron. The statement is sensibly formulated thanks to the natural "bosonization" of the superamplitude associated with the amplituhedron geometry. However this positivity is not manifest in any of the current approaches to scattering amplitudes, and in particular not in the cellulations of the amplituhedron related to on-shell diagrams and the positive Grassmannian. The surprising positivity of the form suggests the existence of a "dual amplituhedron" formulation where this feature would be made obvious. We also suggest that the positivity is associated with an extended picture of amplituhedron geometry, with the amplituhedron sitting inside a co-dimension one surface separating "legal" and "illegal" local singularities of the amplitude. We illustrate this in several simple examples, obtaining new expressions for amplitudes not associated with any triangulations, but following in a more invariant manner from a global view of the positive geometry.

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