The Recent View about Twistorialization in TGD Framework

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Abstract

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to

the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by M^8-H correspondence.

Contents

1 Introduction

The construction of scattering amplitudes is a dream that I have had since the birth of TGD for four decades ago. Various ideas have gradually emerged, some of them have turned out to be wrong, and some of them have survived. At this age I must admit that the dream about explicit algorithms that any graduate student could apply to construct the scattering amplitudes, would require a collective effort and probably will not be realized during my lifetime.

I have however identified a set of general powerful principles leading to a generalization of the recipes for constructing twistorial amplitudes and already now these principles suggest the possibility of rather concrete realizations. In the sequel several additional insights are developed in more detail. Some of them are discussed already earlier in the formulation of M^8-H duality [L2] in adelic framework [L3, L4] and in the chapters developing the TGD based generalization of twistor Grasmannian approach [K10, K17, K16, K18].

1. A proposal made already earlier [K18] is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization [L3] in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure [K4, K1, K13].

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture [L2] implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

2 General view about the construction of scattering amplitudes in TGD framework

Before twistorial considerations a general vision about the basic principles of TGD and construction of scattering amplitudes in TGD framework is in order.

2.1 General principles behind S-matrix

Although explicit formulas for scattering amplitudes are probably too much to hope, one can try to develop a convincing general view about principles behind the S-matrix.

2.1.1 World of Classical Worlds

The first discovery was what I called the “world of classical worlds” (WCW) [K4, K1, K13] as a generalization of loop space allowing to replace path integral approach failing in TGD work. This led to a generalization of Einstein’s geometrization program to an attempt to geometrize entire quantum physics. The geometry of WCW would be essentially unique from its mere existence since the existence of Riemann connection requires already in the case of loop spaces maximal isometries. Super-symplectic and super-conformal symmetries generalizing the 2-D conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces (metrically 2-D!) would define the isometries.

Physical states would be classical spinor fields in the infinite-dimensional WCW and spinors at given point of WCW would be fermionic Fock states. Gamma matrices would be linear combinations of fermionic oscillator operators associated with the analog of massless Dirac equation at space-time surface determined by the variational principle whose preferred extremals the space-time surfaces are. Strong form of holography implied by strong form of general coordinate invariance would imply that it is enough to consider the restrictions of the induced spinor fields at string world sheets and partonic 2-surfaces (actually at discrete points at them defining the ends of boundaries of string world sheets) [K12, K13].

2.1.2 Zero Energy Ontology and generalization of quantum measurement theory to a theory of consciousness

The attempts to understand S-matrix led to the question about what does state function reduction really mean. This eventually led to the discovery of Zero Energy Ontology (ZEO) in which time=constant snapshot as a physical state is replaced with preferred extremal satisfying infinite number of additional gauge conditions [L5]. Temporal pattern becomes the fundamental entity: this conforms nicely with the view neuroscientists and computational scientists for whom behavior and program are basic notions. One can say that non-deterministic state function reduction replaces this kind time evolution with new one. One gets rid of the basic difficulty of ordinary quantum measurement theory.

Causal diamond (CD) is the basic geometric object of ZEO. The members of the state pair defining zero energy state - the analog of physical event characterized by initial and final states - have opposite total conserved quantum numbers and reside at the opposite light-like boundaries of CD being associated with 3-surfaces connected by a space-time surface, the preferred extremal. CDs form a fractal hierarchy ordered by their discrete size scale.

One ends up to a quite radical prediction: the arrow of time changes in “big” state function reduction changing the roles of active and passive boundaries of CD. The state function reductions occurring in elementary reactions represent an example of “big” state function reduction. The sequence of “small” state function reductions - analogs of so called weak measurements - defines self as a conscious entity having CD as imbedding space correlate [L5].

In ZEO based view about WCW 3-surfaces X^3 are pairs of 3-surfaces at boundaries of CD connected by preferred extremals of the action principle. WCW spinors are pairs of fermionic Fock states at these 3-surfaces and WCW spinor fields are WCW spinors depending on X^3 . They satisfy the analog of massless Dirac equation which boils down to the analogs of Super Virasoro conditions including also gauge conditions for a sub-algebra of super-symplectic algebra. S-matrix describing time evolution followed by “small” state function reduction relates two WCW spinor fields of this kind.

2.1.3 Generalization of twistor Grassmannian approach to TGD framework

Twistorial approach generalizes from M^4 to $H = M^4 \times CP_2$. One possible motivation could be the fact that ordinary twistor approach describes only scattering of massless particles. In the proposed generalization particles are massless in 8-D sense and in general massive in 4-D sense [K10, K17, K16, K18].

1. The existence of twistor lift of Kähler action as 6-D analog of Kähler action fixes the choice of H uniquely: only M^4 and CP_2 allow twistor space with Kähler structure. The 12-D product of the twistor spaces of M^4 and CP_2 induces twistor structure for 6-D surface X^6 under additional conditions guaranteeing that the X^6 is twistor space of 4-D surface X^4 (S^2 bundle over X^4) - its twistor lift. The conjecture that 6-D Kähler action indeed gives rise to twistor spaces of X^4 as preferred extremals.

2. This conjecture is the analog for Penrose's original twistor representation of Maxwellian fields reducing dynamics of massless fields to homology. There is also an analogy with massless fields. Dimensional reduction of Kähler action occurs for 6-surfaces, which represent twistor spaces and the external particles entering CD would be minimal surfaces defining simultaneous preferred extremals of Kähler action satisfying infinite number of additional gauge conditions. Minimal surfaces indeed satisfy generalization of massless field equations. In the interior of CD defining interaction region there is a coupling to Kähler 4-force and one has analog of massless particle coupling to Maxwellian field.

3. 6-D Kähler action would give the preferred extremals via the analog of dimensional reduction essential for the twistor space property requiring that one has S^2 bundle over space-time surface. I have considered the generalization of the standard twistorial construction of scattering amplitudes of $\mathcal{N} = 4$ SUSY to TGD context. In particular, the crucial Yangian invariance of the amplitudes holds true also now in both M^4 and CP_2 sectors.

4. Skeptic could argue that TGD generalization of twistors does not tell anything about the origin of the Yangian symmetry. During writing of this contribution I however realized that the hierarchy of Grassmannians realizing the Yangian symmetries could be seen as a hierarchy of reduced WCWs associated with the hierarchy of adeles defined by the hierarchy of extensions of rationals. The isometries of Grassmannian would emerge in the reduction of the isometry group of WCW to a finite-D isometry group of Grassmannian and would be caused by finite measurement resolution described number theoretically. Of course, one can consider also more general flag manifolds with Kähler property as candidates for the analogs of Grassmannians. I will represent the argument in more detail later.

   This could also relate to the postulated infinite hierarchy of hyper-finite factors of type II_1 (HFFs) [K11, K3] as a correlate for the finite measurement resolution with included sub-factor inducing transformations which act trivially in the measurement resolution used.

Remark: There is an amusing connection with empiria. Topologist Barbara Shipman observed that honeybee dance allows a description in terms of flag manifold $F = SU(3)/U(1) \times U(1)$, which is the space for the choices of quantization axes of color quantum numbers and also the twistor space in CP_2 degrees of freedom [A1]. This suggest that QCD type physics might make sense in macroscopic length scales. p-Adic length scale hypothesis and the predicted long range classical color gauge fields suggest a hierarchy of QCD type physics. One can indeed construct a TGD based model of honeybee dance with aconcrete interpretation and representation for the points of F at space-time level [L6].

2.1.4 $M^8 - H$ duality

$M^8 - H$ duality provides two equivalent manners to see the dynamics with either M^8 or $H = M^4 \times CP_2$ as imbedding space [L2]. One might speak of number theoretic compactification which is a completely non-dynamical analog for spontaneous compactification.

1. In M^8 picture the space-time corresponds to a zero locus for either imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial ($RE(o)$ and $IM(o)$ are defined by the decomposition $o = RE(o) + I_4 IM(o)$, where I_4 is octonion unit orthogonal to quaternionic subalgebra). The dynamics is purely algebraic and ultra-local.

2. At the level of H the dynamics is dictated by variational principle and partial differential equations. Space-time surfaces are preferred extremals of the twistor lift of Kähler action reduced to a sum of 4-D Kähler action and volume term analogous to cosmological term in GRT. The equivalence of these descriptions gives powerful constraints and should follow

from the infinite number of gauge conditions at the level of H associated with a sub-algebra of supersymplectic algebra implying the required dramatic reduction of degrees of freedom [K1, K13]. One has a hierarchy of these sub-algebras, which presumably relates to the hierarchy of HFFs and hierarchy of extensions of rationals.

H picture works very nicely in applications. For instance, the notions of field body and magnetic body are crucial in all applications.

2.1.5 Adelic physics

The adelization of ordinary physics fusing real number based physics and various p-adic variants of physics in order to describe cognition.

1. Adelic physics [L3, L4] gives powerful number theoretic constraints when combined with $M^8 - H$ duality and leads to the vision about evolutionary hierarchy defined by extensions of rationals. The higher the level in the hierarchy, the higher the dimension n of the extension identified in terms of Planck constant $h_{eff}/h = n$ labelling the levels of dark matter hierarchy.

2. Adelic hypothesis allows to sharpen the strong form of holography to a statement that discrete cognitive representations consisting of a finite number of points identified as points of space-time surface with M^8 coordinates in the extension of rationals fixes the space-time surface itself. This dramatic reduction would be basically due to finite measurement resolution realized as an inherent property of dynamics. Cognitive representation in fact gives the WCW coordinates of the space-time surface in WCW! WCW reduces to a number theoretic discretization of a finite-dimensional space with Kähler structure and presumably maximal isometries.

3. In ZEO space-time surface becomes analogous to a computer program determined in terms of finite net of numbers! Of course, at the QFT limit of TGD giving standard model and GRT space-time is locally much more complex since one approximates the many-sheeted space-time with single slightly curved region of M^4. This is the price paid for getting rid (or losing) the topological richness of the many-sheeted space-time crucial for the understanding living matter and even physics in galactic scales.

4. Skeptic can argue that this discretization of WCW leads to the loss of WCW geometry based on real numbers. One can however consider also continuous values for the points of cognitive representations and assigning metric to the points of cognitive representation. Metric could be defined as kind of induced metric. One slices CD by parallel CDs by shift the CD along the axis connecting its tips. This allows to see the point of cognitive representation as point at one particular CD. One shifts slightly the point along its CD. Imbedding space metric allows to deduce the infinitesimal line element ds^2 and to deduce the metric components. This allows a definition of differential geometry so that the analog of WCW metric makes sense as a hierarchy of finite-dimensional metrics for space-time surfaces characterize by the cognitive representations.

   The interpretation in real context would be in terms of finite measurement resolution and the hierarchy would correspond to a hierarchy of hyper-finite factors (HFFs) [K11, K3], whose defining property is that they allow arbitrarily precise finite-dimensional approximations. What would be new is that the hierarchy of extensions of rationals would define a hierarchy of discretizations and hierarchy of HFFs.

Thabove list involves several unproven conjectures, which I can argue to be intuitively obvious with the experience of four decades: I cannot of course expect that a colleague reading for the first time about TGD would share these intuitions.

2.2 Classical TGD

Classical TGD is now rather well understood both in both $H = M^4 \times CP_2$ and M^8 pictures. Applications of classical TGD are in H picture and rather detailed phenomenology has emerged. M^8 picture has led to a rather precise vision about adelic physics and to understanding of finite measurement resolution.

2.2.1 Classical TGD in M^8 picture

Classical TGD in M^8 picture is discussed in [L2].

1. In M^8 picture one ends to an extremely simple number theoretic construction of space-time surfaces fixing only discrete or even finite number of space-time points to obtain space-time surface for a given extension of rationals. The reason is that space-time surfaces are zero loci for $RE(P)$ or $IM(P)$ of octonionic polynomials obtained by continuing real polynomial with coefficients in an extension of rationals to an octonionic polynomial.

   Needless to say, the hierarchy of algebraic extensions of rationals is what makes the dynamics at given level so simple. The coordinates of space-time surface as a point of WCW must be in the extension of rationals. As noticed, the points of space-time surface defining the cognitive representation determining the space-time surface serve as its natural WCW coordinates.

2. The highly non-trivial point is that no variational principle is involved with M^8 construction. Therefore it seems that neither WCW metric nor Kähler function is needed. If this is the case, the exponential of Kähler function definable as action exponential does not appear in scattering amplitudes and must disappear also at H-side from the scattering amplitudes.

3. Skeptic could argue that one loses general coordinate invariance in this approach. This is not true. Linear M^8 coordinates are the only possible option and forced already by symmetries. The choice octonionic and quaternionic structures fixes the linear M^8 coordinates almost uniquely since time direction is associated with real octonion unit and one spatial direction to special imaginary unit defining spin quantization axis. In algebraic approach identifying space-time surface as a zero locus of $RE(P)$ or $IM(P)$ these coordinates define space-time coordinates highly uniquely.

   Skeptic could also argue that number theoretic discretization implies reduction of the basic symmetry groups to their discrete sub-groups. This is true and one can argue that this loss of symmetry is due to the use of cognitive representations with finite resolution. Points with algebraic coordinates could be seen as a choices of representatives from a set of points, which are equivalent as far as measurement resolution is considered.

4. A physically important complication related to M^8 dynamics is the possibility of different octonionic and quaternionic structures. For instance, external particles arriving into CD correspond to different octonionic and quaternionic structures in general since Lorentz boost affects the octonionic structure changing the direction of time axis, which corresponds to the real octonionic unit. In color degrees of freedom one has wave function over different quaternionic structures: essentially color partial waves labelled by color quantum numbers [K5].

   One can apply Poincare transformations and color rotations (or transformation in sub-groups of these groups if one requires that the image points belong to the same extension) to the discrete cognitive representation defining space-time surface. The moduli spaces for these structures are essential for the understanding the standard Poincare and color quantum numbers and standard conservation laws in M^8 picture. Also the size scales of CDs define moduli as also Lorentz boosts leaving either boundary of CD unaffected.

2.2.2 Classical TGD in H picture

At the H side one action principle has partial differential equations and infinite number of gauge conditions associated with a sub-algebra of super-symplectic algebra selecting only extremely few preferred extremals of the action principle in terms of gauge conditions for a sub-algebra of super-symplectic algebra. This dynamics is conjectured to follow from the assumption that 6-D lift of space-time surface X^4 to a CP_1 bundle over X^4 is twistor space of X^4. This condition requires the analog of dimensional reduction since S^2 fiber is dynamically trivial.

For 6-D preferred extremals identifiable as twistor spaces of space-time surfaces the 6-D Kähler action in the product of twistor spaces of M^4 and CP_2 is assumed to dimensionally reduce to 4-D Kähler action plus volume term identifiable as the analog of cosmological constant term. This picture reproduces a description of scattering events highly analogous to that emerging in

M^8. External particles correspond to minimal surfaces as analogs of free massless fields and all couplings disappear from the value of the action. The interior of CD corresponds to non-trivial coupling to Kähler 4-force which does not vanish. In M^8 picture one has associative and non-associative regions as counterparts of these regions.

What is remarkable is that the dynamics determined by partial differential equations plus gauge conditions would be equivalent with the number theoretic dynamics determined in terms of zero loci for real or imaginary parts of octonionic polynomials.

2.3 Scattering amplitudes in ZEO

The construction of scattering amplitudes even at the level of principle is far from well-understood. I have discussed rather concrete proposals for the twistorial construction but the feeling is that something is still missing [K10, K17, K16, K18]. This feeling might well reflect my quite too limited mathematical understanding of twistors and experience about practical construction of the scattering amplitudes. Later I will discuss possible identification of the missing piece of puzzle.

Consider first the general picture about the construction of scattering amplitudes suggested by ZEO inspired theory of quantum measurement theory defining also a theory of consciousness.

1. The portions of space-time surfaces outside CD correspond to external particles. They satisfy associativity conditions at M^8 side making possible to map them to minimal surfaces in $H = M^4 \times CP_2$ satisfying various infinite number of gauge conditions for a sub-algebra of super-symplectic algebra isomorphic with it.

   Remark: There is an additional condition requiring that associative tangent space or normal space contains fixed complex subspace of quaternions. It is not quite clear whether this condition can be generalized so that the distribution of these spaces is integrable.

   At both sides the dynamics of external particles is in a well-defined sense critical at both sides and does not depend at all on coupling constants.

2. Inside CDs associativity conditions break down in M^8 and one cannot map this spacetime region - call it X^4 - to H [L2]. It is however possible to construct counterpart of X^4 in H as a preferred extremal for the twistor lift of Kähler action by fixing the 3-surfaces at the boundaries of CD (boundary conditions). The dependence on couplings at the level of H would come from the vanishing conditions for classical Noether charges, which depend on coupling parameters.

3. If the two descriptions of the scattering amplitudes are equivalent, the dependence on coupling parameters in H should have a counterpart in M^8. Coupling constants making sense only at H side are expected to depend on the size scale of CD and on the extension of rationals defining the adele [L3, L4]. Coupling constants should be determined completely by the boundary values of Noether charges at the ends of space-time surface, and therefore by the 3-D ends of associative space-time regions representing external particles at M^8 side. This would suggest that coupling constants are functions of the coefficients of the polynomials and the points of cognitive representation.

2.3.1 Zero energy ontology and the life cycle of self

ZEO meant a decisive step in the understanding of quantum TGD since it solved the basic paradox of quantum measurement problem by forcing to realize that subjective and geometric time are not the same thing [L5].

1. Both the passive boundary of CD and the members of state pairs at it are unaffected during the sequence of state reductions analogous to weak measurements (see `http://tinyurl.com/zt36hpb`) defining self as a generalized Zeno effect. The members of state pairs associated with the active boundary change and the active boundary itself drifts farther away from the passive one in the sequence of "small" state function reductions.

   Also the space-time surfaces connecting passive and active boundaries change during the sequence of weak measurements. Only the 3-surfaces at the passive boundary are unaffected.

Hence the geometric past relative to the active boundary changes during the life cycle of self. In positive energy ontology (PEO) this is not possible.

2. In "big" state function reduction the roles of passive and active boundary are changed and the arrow of time identifiable as the direction in which CD grows changes. In consciousness theory "big" state function reduction corresponds to the death of self and subsequent re-incarnations as a self with an opposite arrow of geometric time.

3. In ZEO the life cycle of self corresponds to a sequence of steps. Single step begins with a unitary time evolution in which a superposition of states associated with CDs larger than the original CD emerges. Then follows the analog of weak measurement leading to a localization to a CD in the moduli space of CDs so that it has a fixed and in general larger size. A measurement of geometric time occurs and gives rise to an experience about the flow of time.

   This option would allow to identify the total S-matrix as a product of the S-matrices associated with various steps in spirit with the interpretation as a generalized Zeno effect.

   Remark: In the usual description one fixes the time interval to which one assigns the S-matrix. There is no division to steps giving rise to the experience of time flow.

4. The measurement of geometric time would be a partial measurement reducing more general unitary time evolution to a unitary time evolution in the standard sense. Can one generalize the notion of partial measurement to other observables so that one would still have unitary time evolution albeit in more restricted sense? Or should one consider giving up the unitary time evolution?

   These observables should commute with the observables having the states at passive boundary as eigenstates: otherwise the state at passive boundary would change. If this picture makes sense, the "big" reduction to the opposite boundary meaning the death of self would necessarily occur when all observables commuting with the eigen observables at the passive boundary have been measured. It could of course occur already earlier.

   Should one allow measurements of all observables commuting with the eigen observables at the passive boundary. This would lead to partial de-coherence of the zero energy state. In TGD inspired quantum biology this could allow to understand ageing as an unavoidable gradual loss of the quantum coherence.

2.3.2 More detailed interpretation of ZEO

There are several questions related to the detailed interpretation of ZEO. The intuitive picture is that inside CD representing self one has collection of sub-CDs representing sub-selves identified as mental images of self. On can loosely say, that sub-CDs represent mind. The sub-CDs are connected by on mass shell lines, which correspond to external particles - matter. Sub-CDs can also have sub-CDs and the hierarchy can have several levels.

The states at the boundaries of CD have opposite total quantum numbers. One can consider two interpretations.

1. In positive energy ontology (PEO) the notion of zero energy state could be seen only as an elegant manner to express conservation laws. This is done in QFT quite generally - also in twistor approach. Also the largest CD would have external particles emanating from its boundaries travelling to the geometric past and future. One would have however have only information about the interior of the CD possessed by conscious entity for which CD plus its sub-CDs (mental images) serve as correlates.

   In this picture the arrow of time is fixed since it must be same for all sub-CDs in order to void inconsistency with the basic idea about self as generalized Zeno effect realized as a sequence of weak measurements.

2. ZEO suggest a more radical interpretation. Zero energy state defines an event. There would be the largest CD defining self and sub-CDs would correspond to mental images. There would be no external particles emanating from the boundaries of the largest CD. In this framework it becomes possible to speak about the death of self as the first state function reduction to the opposite boundary changing the roles of active and passive boundaries of self.

This picture should be consistent with what we know about arrow of time and in TGD framework with the idea that the arrow of time can also change - in particular in living matter.

1. How would the standard arrow of time emerge in ZEO? One could see the emergence of the global arrow of geometric time as a process in which the size of the largest CD increases: the sub-CDs are forced to have the same arrow of time as the largest CD and cannot make state function reductions on opposite boundary (die) independently of it. During evolution the size of the networks with the same arrow of geometric time increases and fixed arrow of geometric time is established in longer scales.

2. This picture cannot be quite correct. The applications of TGD inspired consciousness require that the mental images of self can have arrow of geometric time opposite to that of self. For instance, motor actions could be sensory perceptions in non-standard arrow of time. Memory could be communications with brain of geometric past - seeing in time direction - involving signals to geometric past requiring temporary reversals of the arrow of time at some level of self-hierarchy. Hence space-time regions with different arrows of time but forming a connected space-time surface ought to be possible.

   Many-sheeted space-time means a hierarchy of space-time sheets connected by what I call wormhole contacts having Euclidian signature of the induced metric. Space-time sheets at different levels of the hierarchy are not causally connected in the sense that one cannot speak of signal propagation in the regions of Euclidian signature. This suggests that the space-time sheets connected by wormhole contacts can have different arrows of geometric time and are associated with their own CDs.

   In this manner one would avoid the paradox resulting when sub-self - mental image - dies so that its passive boundary becomes active and the particles emanating from it end up to the passive boundary of CD, where no changes are allowed during the life cycle of self. If the particles emanating from time-reversed sub-self and up to boundaries of parallel CD, the problem is circumvented.

3. Wormhole contacts induce an interaction between Minkowskian space-time sheets that they connect. The interaction is not mediated by classical signals but by boundary conditions at the boundaries between Minkowskian regions and Euclidian wormhole contact. These two boundaries are light-like orbits of opposite wormhole throats (partonic 2-surfaces).

   In number theoretic picture the presence of wormhole contact is reflected in the properties set of points in extension of rationals defining the cognitive representation in turn defining the space-time surface. In particular, the points associated with wormhole contact have space-like distance although they are at opposite boundaries of CD and have time-like distance in the metric of imbedding space. This kind of point pairs associated with wormhole contacts serve serve as a tell-tale signature for them.

3 The counterpart of the twistor approach in TGD

The analogs of twistor diagrams could emerge in TGD [K17, K18] in the following manner in ZEO.

1. Portions of space-time surfaces inside CDs would appear as analogs of vertices and the spacetime surfaces connecting them as analogs of propagator lines. The "lines" connecting sub-CDs would carry massless on mass shell states but possibly with complex momenta analogous to those appearing in twistor diagrams. This is true also classically at level of H: the coupling constants appearing in the action defining classical dynamics - at least Kähler coupling strength - are complex so that also conserved quantities have also imaginary parts.

   Remark: At the level of M^8 one does not have action principle and cannot speak of Noether charges. Here the conserved charged are associated with the symmetries of the moduli spaces such as the moduli spaces for octonion and quaternion structures [L2]. The identification of the classical charges in Cartan algebra at H level with the quantum numbers labeling wave functions in moduli space at M^8 level could be seen as a realization of quantum classical correspondence.

2. At space-time level the vertices of twistor diagrams correspond to partonic 2-surfaces in the interior of given CD. In H description fermionic lines along the light-like orbits of partonic 2-surfaces scatter at partonic 2-surfaces. If each partonic 2-surface defining a vertex is surrounded by a sub-CD, these two views about TGD variants of twistor diagrams are unified. Sub-CD can of course contain more complex structures such as pair of wormhole contacts assignable to an elementary particle.

3.1 Could the classical number theoretical dynamics define the hard core of the scattering amplitudes?

The natural hope is that the simple picture about classical dynamics at the level of M^8 should have similar counterpart at the level of scattering amplitudes in M^8. The above arguments suggest that the scattering diagrams correspond to CDs connected by external particle lines representing on mass shell particles. These surfaces are associative at the level of M^8 and minimal surfaces at the level of H. This suggests that scattering amplitude for single CD serves as a building brick for scattering amplitudes: the rest would be "just kinematics" dictated by the enormous symmetries of WCW.

1. Everything in the construction should reduce to a hard core around which one would have integrations (or sums for number theoretic realization of finite measurement resolution) over various moduli characterizing the standard quantum numbers. Twistors for M^4 and CP_2 and the moduli for the choices of CDs should correspond to essentially kinematic contribution involving no genuine dynamics.

2. The scattering amplitudes should make sense in all sectors of adele. This poses powerful constraints on them. The exponential of Kähler function reducing to action exponential can in principle appear in the description at H-side but cannot be present at M^8 side. Therefore it should disappear also at the level of H.

   If the scattering amplitude at the level of H is sum over contributions with the same value of the action exponential, the exponentials indeed cancel and I have proposed that this condition holds true. In perturbative quantum field theory it holds practically always and in integrable theories is exact. This would mean enormous simplification since all information about the action principle in H would appear in the vanishing conditions for the Noether charges of the subalgebra of super-symplectic algebra at the ends of the space-time surface. These Noether changes indeed depend on the action principle and thus on coupling constants.

3. Could the hard core in the construction of the scattering amplitudes be just the choice of the cognitive representation as points if M^8 belonging to the algebraic extension defining the adele and determining space-time surface in terms of octonionic polynomial inside this CD defining the interaction region?

   The set of points of extension of rationals in the cognitive representation defines space-time surface and also its WCW coordinates. The restriction to a cognitive representation with given number of points in given extension of rationals would mean a reduction of WCW to a finite-dimensional sub-space.

   The first wild guess is that this space is Kähler manifold with maximal symmetries - just as WCW is. A further wild guess is that these reduced WCWs are Grassmannians and correspond to those appearing in the twistor Grassmannian approach. A more general conjecture is inspired by the vision that super-symplectic gauge conditions effectively reduce the super-symplectic algebra to a Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to ADE hierarchy. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

4. One must allow the action of Galois group and this gives several options for given set X of points in algebraic extension.

   (a) One can construct $X^4(X)$ in terms of octonionic polynomial and construct a representation of Galois group as superposition of space-time surfaces obtained from space-time surface by the action of Galois group on X giving rise to new sets $X_g = g(X)$.

(b) One can also consider the action of Galois group on X and get larger set Y of points and construct single multi-sheeted surface $X^4(Y)$. This surface corresponds to Planck constant $h_{eff}/h = n$, where n is the dimension of algebraic extension.

(c) One can also consider the actions of sub-groups of $H \subset Gal$ to X to get space-time surface with $h_{eff}/h = m$ dividing n. There are several options corresponding to representations for all sub-groups of Galois group. A hierarchy of symmetry breakings seems to be involved with unbroken symmetry associated with the largest value of h_{eff}/h.

5. In this picture the hard core would reduce to the classical number theoretical dynamics of space-time surface in M^8. The additional degrees of freedom would be due to the possibility of different octonionic and quaternionic structures and choices of size scales and Lorentz boosts and translations of CDs. The symmetries would dictate the S-matrix in the moduli degrees of freedom: the dream is that this part of the dynamics reduces to kinematics, so to say.

   The discrete coupling constant evolution would be determined by the hierarchy of extensions of rationals and by the hierarchy of p-adic length scales. The cancellation of radiative corrections in the sense of sub-CDs inside CDs could be achieved by replacing coupling constant evolution with its discrete counterpart.

If this dream has something to do with reality, the construction of scattering amplitudes would reduce to their construction in moduli degrees of freedom and here the generalization of twistorial approach relying on Yangian symmetry allowing to identify scattering amplitudes as Yangian invariants might "trivialize" the situation. It will be found that the Yangian symmetry could corresponds to general coordinate transformations for the reduced WCW forced by the restriction of the spacetime surfaces to those allowed by octonionic polynomials with coefficients in the extensiom of rationals.

3.2 Do loop contributions to the scattering amplitudes vanish in TGD framework?

In TGD scattering amplitudes interpreted as zero energy states would correspond at imbedding space level to collections of space-time surfaces inside CDs analogous to vertices and connected by lines defined by the space-time surfaces representing on-mass-shell particles. One would have massless particles in 8-D sense. The quaternionicity of 8-momentum leads to $M^4 \times CP_2$ picture and CP_2 twistors should replace E^4 twistors of M^8 approach.

3.2.1 Why loop corrections should vanish?

There are several arguments suggesting that the loop contributions should vanish in TGD framework. This would give rise to a discrete coupling constant evolution analogous to a sequence of phase transitions between different critical coupling parameters. Amplitudes would be obtained as tree diagrams.

1. In ZEO it is far from clear what the basic operation defining the loop contribution could even mean. One would have zero energy state for which the members of added particle pair have opposite but momenta but the amplitude is superposition of states with varying momenta. Why should one allow zero energy states containing one particle which is not an eigenstate of momentum? This suggests that ZEO does not allow loop contributions at all: the distinction between PEO and ZEO would make itself visible in rather dramatic manner.

2. The restriction of the BCFW to tree diagrams is internally consistent since the loop term is identically vanishing in this case. The first term in the BCFW for diagram with l loops involves a factor with $l > 0$ loops which vanishes. In $l = 1$ case the second term is obtained from $(n + 2, l - 1 = 0)$ diagram by generating loop but this vanishes by assumption.

3. Number theoretic vision does not favor the decomposition of the amplitude to an infinite sum of amplitudes since this is expected to lead to the emergence of transcendental numbers and functions in the amplitude in conflict with the number theoretical universality.

Loops indeed give logarithms and poly-logarithms of rational functions of external momenta in Grassmannian approach. This violates the number theoretical universality since the p-adic counterpart of logarithm exist only for the argument of form $x = 1 + O(p)$. This condition cannot hold true for all primes simultaneously.

Discrete coupling constant evolution suggests the vanishing of loops. One can imagine two alternative mechanisms for the vanishing of loop contributions. Either the loop contributions do not make sense at all in ZEO, or the sum of loop contributions for the critical values of coupling constants vanishes. The summing up of loop contributions to zero for critical values of couplings should happen for all values of external momenta and other quantum numbers: this does not look plausible.

3.2.2 General number theoretic ideas about coupling constant evolution

The discrete coupling constant evolution would be associated with the scale hierarchy for CDs and the hierarchy of extensions of rationals.

1. Discrete p-adic coupling constant evolution would naturally correspond to the dependence of coupling constants on the size of CD. For instance, I have considered a concrete but rather ad hoc proposal for the evolution of Kähler couplings strength based on the zeros of Riemann zeta [K2]. Number theoretical universality suggests that the size scale of CD identified as the temporal distance between the tips of CD using suitable multiple of CP_2 length scale as a length unit is integer, call it l. The prime factors of the integer could correspond to preferred p-adic primes for given CD.

2. I have also proposed that the so called ramified primes of the extension of rationals correspond to the physically preferred primes. Ramification is algebraically analogous to criticality in the sense that two roots understood in very general sense co-incide at criticality. Could the primes appearing as factors of l be ramified primes of extension? This would give strong correlation between the algebraic extension and the size scale of CD.

In quantum field theories coupling constants depend in good approximation logarithmically on mass scale, which would be in the case of p-adic coupling constant evolution replaced with an integer n characterizing the size scale of CD or perhaps the collection of prime factors of n (note that one cannot exclude rational numbers as size scales). Coupling constant evolution could also depend on the size of extension of rationals characterized by its order and Galois group.

In both cases one expects approximate logarithmic dependence and the challenge is to define "number theoretic logarithm" as a rational number valued function making thus sense also for p-adic number fields as required by the number theoretical universality.

1. Coupling constant evolution with respect to CD size scale

Consider first the coupling constant as a function of the length scale $l_{CD}(n)/l_{CD}(1) = n$.

1. The number $\pi(n)$ of primes $p \leq n$ behaves approximately as $\pi(n) = n/log(n)$. This suggests the definition of what might be called "number theoretic logarithm" as $Log(n) \equiv n/\pi(n)$. Also iterated logarithms such $log(log(x))$ appearing in coupling constant evolution would have number theoretic generalization.

2. If the p-adic variant of $Log(n)$ is mapped to its real counterpart by canonical identification involving the replacement $p \rightarrow 1/p$, the behavior can very different from the ordinary logarithm. $Log(n)$ increases however very slowly so that in the generic case one can expect $Log(n) < p_{max}$, where p_{max} is the largest prime factor of n, so that there would be no dependence on p for p_{max} and the image under canonical identification would be number theoretically universal.

   For $n = p^k$, where p is small prime the situation changes since $Log(n)$ can be larger than small prime p. Primes p near primes powers of 2 and perhaps also primes near powers of 3 and 5 - at least - seem to be physically special. For instance, for Mersenne prime $M_k = 2^k - 1$ there would be dramatic change in the step $M_k \rightarrow M_k + 1 = 2^k$, which might relate to its special physical role.

3. One can consider also the analog of $Log(n)$ as

$$Log(n) = \sum_p k_p Log(p) \ ,$$

where p^{k_i} is a factor of n. $Log(n)$ would be sum of number theoretic analogs for primes factors and carry information about them.

One can extend the definition of $Log(x)$ to the rational values $x = m/n$ of the argument. The logarithm $Log_b(n)$ in base $b = r/s$ can be defined as $Log_b(x) = Log(x)/Log(b)$.

4. For $p \in \{2, 3, 5\}$ one has $Log(p) > log(p)$, where for larger primes one has $Log(p) < log(p)$. One has $Log(2) = 2 > log(2) = .693...$, $Log(3) = 3k/2 > log(3) = 1.099$, $Log(5) = 5/3 = 1.666.. > log(5) = 1.609$. For $p = 7$ one has $Log(7) = 7/4 \simeq 1.75 < log(7) \simeq 1.946$. Hence these primes and CD size scales n involving large powers of $p \in \{2, 3, 5\}$ ought to be physically special as indeed conjectured on basis of p-adic calculations and some observations related to music and biological evolution [K7, K8, K9, K15].

In particular, for Mersenne primes $M_k = 2^k - 1$ one would have $Log(M_k) \simeq klog(2)$ for large enough k. For $Log(2^k)$ one would have $k \times Log(2) = 2k > log(2^k) = klog(2)$: there would be sudden increase in the value of $Log(n)$ at $n = M_k$. This jump in p-adic length scale evolution might relate to the very special physical role of Mersenne primes strongly suggested by p-adic mass calculations [K5].

5. One can wonder whether one could replace the $log(p)$ appearing as a unit in p-adic negentropy [K6] with a rational unit $Log(p) = p/\pi(p)$ to gain number theoretical universality? One could therefore interpret the p-adic negentropy as real or p-adic number for some prime. Interestingly, $|Log(p)|_p = 1/p$ approaches zero for large primes p (eye cannot see itself!) whereas $|Log(p)|_q = 1/|\pi(p)|_q$ has large values for the prime power factors q^r of $\pi(p)$.

2. The dependence of $1/\alpha_K$ on the extension of rationals

Consider next the dependence on the extension of rationals. The natural algebraization of the problem is to consider the Galois group of the extension.

1. Consider first the counterparts of primes and prime factorization for groups. The counterparts of primes are simple groups, which do not have normal subgroups H satisfying $gH = Hg$ implying invariance under automorphisms of G. Simple groups have no decomposition to a product of sub-groups. If the group has normal subgroup H, it can be decomposed to a product $H \times G/H$ and any finite group can be decomposed to a product of simple groups.

All simple finite groups have been classified (see `http://tinyurl.com/jn44bxe`). There are cyclic groups, alternating groups, 16 families of simple groups of Lie type, 26 sporadic groups. This includes 20 quotients G/H by a normal subgroup of monster group and 6 groups which for some reason are referred to as pariahs.

2. Suppose that finite groups can be ordered so that one can assign number $N(G)$ to group G. The roughest ordering criterion is based on $ord(G)$. For given order $ord(G) = n$ one has all groups, which are products of cyclic groups associated with prime factors of n plus products involving non-Abelian groups for which the order is not prime. $N(G) > ord(G)$ thus holds true. For groups with the same order one should have additional ordering criteria, which could relate to the complexity of the group. The number of simple factors would serve as an additional ordering criterion.

If its possible to define $N(G)$ in a natural manner then for given G one can define the number $\pi_1(N(G))$ of simple groups (analogs of primes) not larger than G. The first guess is that that the number $\pi_1(N(G))$ varies slowly as a function of G. Since Z_i is simple group, one has $\pi_1(N(G)) \geq \pi(N(G))$.

3. One can consider two definitions of number theoretic logarithm, call it Log_1.

$$\begin{array}{ll} a) & Log_1(N(G)) = \frac{N(G)}{\pi_1(N(G))} \ , \\ b) & Log_1(G) = \sum_i k_i Log_1(N(G_i)) \ , \ \ Log_1(N(G_i)) = \frac{N(G_i)}{\pi_1(N(G_i))} \ . \end{array} \tag{3.1}$$

Option a) does not provide information about the decomposition of G to a product of simple factors. For Option b) one decomposes G to a product of simple groups G_i: $G = \prod_i G_i^{k_i}$ and defines the logarithm as Option b) so that it carries information about the simple factors of G.

4. One could organize the groups with the same order to same equivalence class. In this case the above definitions would give

$$\begin{array}{ll} a) & Log_1(ord(G)) = \frac{ord(G)}{\pi_1(ord(G))} < Log(ord(G)) \ , \\ b) & Log_1(ord(G)) = \sum_i k_i Log(ord(G_i)) \ , \ \ Log_1(ord(G_i)) = \frac{ord(G_i)}{\pi_1(ord(G_i))} \ . \end{array} \tag{3.2}$$

Besides groups with prime orders there are non-Abelian groups with non-prime orders. The occurrence of same order for two non-isomorphic finite simple groups is very rare (see `http://tinyurl.com/ydd6uomb`). This would suggests that one has $\pi_1(ord(G)) < ord(G)$ so that $Log_1(ord(G))/ord(G) < 1$ would be true.

5. For orders $n(G) \in \{2, 3, 5\}$ one has $Log_1(n(G)) = Log(n(G)) > log(n(G))$ so that the ordes $n(G)$ involving large factors of $p \in \{2, 3, 5\}$ would be special also for the extensions of rationals. S_3 with order 6 is the first non-abelian simple group. One has $\pi(S_3) = 4$ giving $Log(6) = 6/4 = 1.5 < log(6) = 1.79$ so that S_3 is different from the simple groups below it.

To sum up, number theoretic logarithm could provide answer to the long-standing question what makes Mersenne primes and also other small primes so special.

3.2.3 Considerations related to coupling constant evolution and Riemann zeta

I have made several number theoretic peculations related to the possible role of zeros of Riemann zeta in coupling constant evolution. The basic problem is that it is not even known whether the zeros of zeta are rationals, algebraic numbers or genuine transcendentals or belong to all these categories. Also the question whether number theoretic analogs of ζ defined for p-adic number fields could make sense in some sense is interesting.

1. Is number theoretic analog of ζ possible using $Log(p)$ instead of $log(p)$?

The definition of $Log(n)$ based on factorization $Log(n) \equiv \sum_p k_p Log(p)$ allows to define the number theoretic version of Riemann Zeta $\zeta(s) = \sum n^{-s}$ via the replacement $n^{-s} = exp(-log(n)s) \rightarrow exp(-Log(n)s)$.

1. In suitable region of plane number-theoretic Zeta would have the usual decomposition to factors via the replacement $1/(1 - p^{-s}) \rightarrow 1/(1 - exp(-Log(p)s)$. p-Adically this makes sense for $s = O(p)$ and thus only for a finite number of primes p for positive integer valued s: one obtains kind of cut-off zeta. Number theoretic zeta would be sensitive only to a finite number of prime factors of integer n.

2. This might relate to the strong physical indications that only a finite number of cognitive representations characterized by p-adic primes are present in given quantum state: the ramified primes for the extension are excellent candidates for these p-adic primes. The size scale n of CD could also have decomposition to a product of powers of ramified primes. The finiteness of cognition conforms with the cutoff: for given CD size n and extension of rationals the p-adic primes labelling cognitive representations would be fixed.

3. One can expand the regions of converge to larger p-adic norms by introducing an extension of p-adics containing e and some of its roots (e^p is automatically a p-adic number). By introducing roots of unity, one can define the phase factor $exp(-iLog(n)Im(s))$ for suitable values of $Im(s)$. Clearly, $exp(-ipIm(s))/\pi(p))$ must be in the extension used for all primes p involved. One must therefore introduce prime roots $exp(i/\pi(p))$ for primes appearing in cutoff. To define the number theoretic zeta for all p-adic integer values of $Re(s)$ and all integer values of $Im(s)$, one should allow all roots of unity $(ep(i2\pi/n))$ and all roots $e^{1/n}$: this requires infinite-dimensional extension.

4. One can thus define a hierarchy of cutoffs of zeta: for this the factorization of Zeta to a finite number of "prime factors" takes place in genuine sense, and the points $Im(s) = ik\pi(p)$ give rise to poles of the cutoff zeta as poles of prime factors. Cutoff zeta converges to zero for $Re(s) \to \infty$ and exists along angles corresponding to allowed roots of unity. Cutoff zeta diverges for $(Re(s) = 0, Im(s) = ik\pi(p))$ for the primes p appearing in it.

Remark: One could modify also the definition of ζ for complex numbers by replacing $exp(log(n)s)$ with $exp(Log(n)s)$ with $Log(n) = \sum_p k_p Log(p)$ to get the prime factorization formula. I will refer to this variant of zeta as modified zeta ($\tilde{\zeta}$) below. $\tilde{\zeta}$ would carry explicit number theoretic information via the dependence of its "prime factors" $1/(1 - exp(-Log(p)s))$.

2. Could the values of $1/\alpha_K$ be given as zeros of ζ or of $\tilde{\zeta}$

In [K2] I have discussed the possibility that the zeros $s = 1/2 + iy$ of Riemann zeta at critical line correspond to the values of complex valued Kähler coupling strength α_K: $s = i/\alpha_K$. The assumption that p^{iy} is root of unity for some combinations of p and y $[log(p)y = (r/s)2\pi]$ was made. This does not allow s to be complex rational. If the exponent of Kähler action disappears from the scattering amplitudes as $M^8 - H$ duality requires, one could assume that s has rational values but also algebraic values are allowed.

1. If one combines the proposed idea about the Log-arithmic dependence of the coupling constants on the size of CD and algebraic extension with $s = i/\alpha_K$ hypothesis, one cannot avoid the conjecture that the zeros of zeta are complex rationals. It is not known whether this is the case or not. The rationality would not have any strong implications for number theory but the existence irrational roots would have (see `http://tinyurl.com/y8bbnhe3`). Interestingly, the rationality of the roots would have very powerful physical implications if TGD inspired number theoretical conjectures are accepted.

   The argument discussed below however shows that complex rational roots of zeta are not favored by the observations [A2] about the Fourier transform for the characteristic function for the zeros of zeta. Rather, the findings suggest that the imaginary parts [L1] should be rational multiples of 2π, which does not conform with the vision that $1/\alpha_K$ is algebraic number. The replacement of $log(p)$ with $Log(p)$ and of 2π with is natural p-adic approximation in an extension allowing roots of unity however allows $1/\alpha_K$ to be an algebraic number. Could the spectrum of $1/\alpha_K$ correspond to the roots of ζ or of $\tilde{\zeta}$?

2. A further conjecture discussed in [K2] was that there is 1-1 correspondence between primes $p \simeq 2^k$, k prime, and zeros of zeta so that there would be an order preserving map $k \to s_k$. The support for the conjecture was the predicted rather reasonable coupling constant evolution for α_K. Primes near powers of 2 could be physically special because $Log(n)$ decomposes to sum of $Log(p)$:s and would increase dramatically at $n = 2^k$ slightly above them.

   In an attempt to understand why just prime values of k are physically special, I have proposed that k-adic length scales correspond to the size scales of wormhole contacts whereas particle space-time sheets would correspond to $p \simeq 2^k$. Could the logarithmic relation between L_p and L_k correspond to logarithmic relation between p and $\pi(p)$ in case that $\pi(p)$ is prime and could this condition select the preferred p-adic primes p?

3. The argument of Dyson for the Fourier transform of the characteristic function for the set of zeros of ζ

Consider now the argument suggesting that the roots of zeta cannot be complex rationals. On basis of numerical evidence Dyson [A2] (http://tinyurl.com/hjbfsuv) has conjectured that the Fourier transform for the characteristic function for the critical zeros of zeta consists of multiples of logarithms $log(p)$ of primes so that one could regard zeros as one-dimensional quasi-crystal.

This hypothesis makes sense if the zeros of zeta decompose into disjoint sets such that each set corresponds to its own prime (and its powers) and one has $p^{iy} = U_{m/n} = exp(i2\pi m/n)$ (see the appendix of [L1]). This hypothesis is also motivated by number theoretical universality [K14, L3].

1. One can re-write the discrete Fourier transform over zeros of ζ at critical line as

$$f(x) = \sum_y exp(ixy)) \ , \ y = Im(s) \ .$$

The alternative form reads as

$$f(u) = \sum_s u^{iy} \ , \ u = exp(x) \ .$$

$f(u)$ is located at powers p^n of primes defining ideals in the set of integers.

For $y = p^n$ one would have $p^{iny} = exp(inlog(p)y)$. Note that $k = nlog(p)$ is analogous to a wave vector. If $exp(inlog(p)y)$ is root of unity as proposed earlier for some combinations of p and y, the Fourier transform becomes a sum over roots of unity for these combinations: this could make possible constructive interference for the roots of unity, which are same or at least have the same sign. For given p there should be several values of $y(p)$ with nearly the same value of $exp(inlog(p)y(p))$ whereas other values of y would interfere deconstructively.

For general values $y = x^n$ $x \neq p$ the sum would not be over roots of unity and constructive interference is not expected. Therefore the peaking at powers of p could take place. This picture does not support the hypothesis that zeros of zeta are complex rational numbers so that the values of $1/\alpha_K$ correspond to zeros of zeta and would be therefore complex rationals as the simplest view about coupling constant evolution would suggest.

Remark: Mumford has argued (http://tinyurl.com/zemw27o) that the Fourier transform should include also the trivial zeros at $s = -2, -4, -6...$ giving and exponentially small contributions and providing a slowly varying background to the Fourier transform.

2. What if one replaces $log(p)$ with $Log(p) = p/\pi(p)$, which is rational and thus ζ with $\tilde{\zeta}$? For large enough values of p $Log(p) \simeq log(p)$ finite computational accuracy does not allow distinguish $Log(p)$ from $log(p)$. For $Log(p)$ one could thus understand the finding in terms of constructive interference for the roots of unity if the roots of zeta are of form $s = 1/2 + i(m/n)2\pi$. The value of y cannot be rational number and $1/\alpha_K$ would have real part equal to y proportional to 2π which would require infinite-D extension of rationals. In p-adic sectors infinite-D extension does not conform with the finiteness of cognition.

Remark: It is possible to check by numerical calculations whether the locus of complex zeros of $\tilde{\zeta}$ is at line $Res(2) = 1/2$. If so, then Fourier transform would make sense. One can also check whether the peaks at $nlog(p)$ are shifted to $nLog(p)$: for $p = 2$ one would have $Log(2) = 2 > log(2)$. The positions of peaks should shift to the right for $p = 2, 3, 5$ and to the left for $p > 5$. This should be easy to check by numerical calculations.

3. Numerical calculations have however finite accuracy, and allow also the possibility that y is algebraic number approximating rational multiple of 2π in some natural manner. In p-adic sectors would obtain the spectrum of y and $1/\alpha_K$ as algebraic numbers by replacing 2π in the formula $is = \alpha_K = i/2 + q \times 2\pi$, $q = r/s$, with its approximate value:

$$2\pi \rightarrow sin(2\pi/n)n = i\frac{n}{2}(exp(i2\pi/n) - exp(-i2\pi/n))$$

for an extension of rationals containing n:th of unity. Maximum value of n would give the best approximation. This approximation performed by fundamental physics should appear in the number theoretic scattering amplitudes in the expressions for $1/\alpha_K$ to make it algebraic number.

y can be approximated in the same manner in p-adic sectors and a natural guess is that $n = p$ defines the maximal root of unity as $exp(i2\pi/p)$. The phase $exp(ilog(p)y)$ for $y = qsin(2\pi/n(y))$, $q = r/s$, is replaced with the approximation induced by $log(p) \rightarrow Log(p)$ and $2\pi \rightarrow sin(2\pi/n)n$ giving

$$exp(ilog(p)y) \rightarrow exp(iq(y)sin(2\pi/n(y))\frac{p}{\pi(p)}) \ .$$

If s in $q = r/s$ does not contain higher powers of p, the exponent exists p-adically for this extension and can can be expanded in positive powers of p as

$$\sum_n i^n q^n sin(2\pi/p)^n (p/\pi(p))^n \ .$$

This makes sense p-adically.

Also the actual complex roots of ζ could be algebraic numbers:

$$s = i/2 + q \times sin(\frac{2\pi}{n(y)})n(y) \ .$$

If the proposed correlation between p-adic primes $p \simeq 2^k$, k prime and zeros of zeta predicting a reasonable coupling constant evolution for $1/\alpha_K$ is true, one can have naturally, $n(y) = p(y)$, where p is the p-adic prime associated with y: the accuracy in angle measurement would increase with the size scale of CD. For given p there could be several roots y with same $p(y)$ but different $q(y)$ giving same phases or at least phases with same sign of real part.

Whether the roots of $tilde\zeta$ are algebraic numbers and at critical line $Re(s) = 1/2$ is an interesting question.

Remark: This picture allows many variants. For instance, if one assumes standard zeta, one could consider the possibility that the roots y_p associated with p and giving rise to constructive interference are of form $y = q \times (Log(p)/log(p)) \times sin(2\pi/p)p$, $q = r/s$.

4. Could functional equation and Riemann hypothesis generalize?

It is interesting to list the elementary properties of the $\tilde{\zeta}$ before trying to answer to the questions of the title.

1. The replacement $log(n) \rightarrow Log(n) \equiv sum_p k_p Log(p)$ implies that $\tilde{\zeta}$ codes explicitly number theoretic information. Note that $Log(n)$ satisfies the crucial identity $Log(mn) = Log(m) + Log(n)$. $\tilde{\zeta}$ is an analog of partition function with rational number valued $Log(n)$ taking the role of energy and $1/s$ that of a complex temperature. In ZEO this partition function like entity could be associated with zero energy state as a "square root" of thermodynamical partition function: in this case complex temperatures are possible. $|\tilde{\zeta}|^2$ would be the analog of ordinary partition function.

2. Reduction of $\tilde{\zeta}$ to a product of "prime factors" $1/[1 - exp(-Log(p)s)]$ holds true by $Log(n) \equiv sum_p k_p Log(p)$, $Log(p) = p/\pi(p)$.

3. $\tilde{\zeta}$ is a combination of exponentials $exp(-Log(n)s)$, which converge for $Re(s) > 0$. For ζ one has exponentials $exp(-log(n)s)$, which also converge for $Re(s) > 0$: the sum $\sum n^{-s}$ does not however converge in the region $Re(s) < 1$. Presumably $\tilde{\zeta}$ fails to converge for $Re(s) \leq 1$. The behavior of terms $exp(-Log(n)s)$ for large values of n is very similar to that in ζ.

4. One can express ζ o in terms of η function defined as

$$\eta(s) = \sum (-1)^n n^{-s} \ .$$

The powers $(-1)^n$ guarantee that η converges (albeit not absolutely) inside the critical strip $0 < s < 1$.

By using a decomposition of integers to odd and even ones, one can express ζ in terms of η:

$$\zeta = \frac{\eta(s)}{(-1 + 2^{-s+1})} \quad .$$

This definition converges inside critical strip. Note the pole at $s = 1$ coming from the factor. One can define also $\tilde{\eta}$:

$$\tilde{\eta}(s) = \sum(-1)^n e^{-Log(n)s} \quad .$$

The formula relating $\tilde{\zeta}$ and $\tilde{\eta}$ generalizes: 2^{-s} is replaced with $exp(-2s)$ ($Log(2) = 2$):

$$\tilde{\zeta} = \frac{\tilde{\eta}(s)}{-1 + 2exp^{-2s}} \quad .$$

This definition $\tilde{\zeta}$ converges in the critical strip $Re(s) \in (0, 1)$ and also for $Re(s) > 1$. $\tilde{\zeta}(1-s)$ converges for $Re(s) < 1$ so that in $\tilde{\eta}$ representation both converge.

Note however that the poles of ζ at $s = 1$ has shifted to that at $s = log(2)/2$ and is below $Re(s) = 1/2$ line. If a symmetrically posioned pole at $s = 1 - log(2)/2$ is not present in $\tilde{\eta}$, functional equation cannot be true.

5. $Log(n)$ approaches $log(n)$ for integers n not containing small prime factors p for which $\pi(n)$ differs strongly from $p/log(p)$. This suggests that allowing only terms $exp(-Log(n)s)$ in the sum defining $\tilde{\zeta}$ not divisible by primes $p < p_{max}$ might give a cutoff $\tilde{\zeta}^{cut,p_{max}}(s)$ behaving very much like ζ from which "prime factors" $1/(1 - exp(-Log(p)s)$, $p < p_{max}$ are dropped of. This is just division of $\tilde{\zeta}$ by these factors and at least formally, this does not affect the zeros of $\tilde{\zeta}$. Arbitrary number of factors can be droped. Could this mean that $\tilde{\zeta}^{cut}$ has same or very nearly same zeros as ζ at critical line? This sounds paradoxical and might reflect my sloppy thinking: maybe the lack of the absolute implies that the conclusion is incorrect.

The key questions are whether $\tilde{\zeta}$ allows a generalization of the functional equation $\xi(s) = \xi(1-s)$ with $\xi(s) = \frac{1}{2}s(s-1)\Gamma(s/2)\pi^{-s/2}\zeta(s)$ and whether Riemann hypothesis generalizes. The derivation of the functional equation is quite a tricky task and involves integral representation of ζ .

1. One can start from the integral representation of ζ true for $s > 0$.

$$\zeta(s) = \frac{1}{(1 - 2^{1-s})\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{e^t + 1} dt \quad , \quad Re(s) > 0 \quad .$$

deducible from the expression in terms of $\eta(s)$. The factor $1/(1 + e^t)$ can be expanded in geometric series $1/(1 + e^t) = \sum(-1)^n exp(nt)$ converning inside the critical strip. One formally performs the integrations by taking nt as an integration variable. The integral gives the result $\sum(-1)^n/n^z)\Gamma(s)$.

The generalization of this would be obtained by a generalization of geometric series:

$$1/(1 + e^t) = \sum(-1)^n exp(nt) \rightarrow \sum(-1)^n e^{exp(Log(n))t}$$

in the integral representation. This would formally give $\tilde{\zeta}$: the only difference is that one takes $u = exp(Log(n))t$ as integration variable.

One could try to prove the functional equation by using this representation. One proof (see `http://tinyurl.com/yak93hyr`) starts from the alternative expression of ζ as

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_1^\infty \frac{t^{s-1}}{e^t - 1} dt \quad , \quad Re(s) > 1 \quad .$$

One modifies the integration contour to a contour C coming from $+\infty$ above positive real axis, circling the origin and returning back toc$+\infty$ below the real axes to get a modified representation of ζ:

$$\zeta(s) = \frac{1}{2isin(\pi s)\Gamma(s)} \int_1^{\infty} \frac{(-w)^{s-1}}{e^w - 1} dw \quad , \quad Re(s) > 1 \quad .$$

One modifies the C further so that the origin is circled around a square with vertices at $\pm(2n+1)\pi$ and $\pm i(2n+1)\pi$.

One calculates the integral the integral along C as a residue integral. The poles of the integrand proportional to $1/(1-e^t)$ are at imaginary axis and correspond to $w = ir2\pi$, $r \in Z$. The residue integral gives the other side of the functional equation.

2. Could one generalize this representation to the recent case? One must generalize the geometric series defined by $1/(e^w - 1)$ to $-\sum e^{exp(Log(n))w}$. The problem is that one has only a generalization of the geometric series and not closed form for the counterpart of $1/(exp(w)-1)$ so that one does not know what the poles are. The naive guess is that one could compute the residue integrals term by term in the sum over n. An equally naive guess would be that for the poles the factors in the sum are equal to unity as they would be for Riemann zeta. This would give for the poles of n:th term the guess $w_{n,r} = r2\pi/exp(Log(n)$, $r \in Z$. This does not however allow to deduce the residue at poles.Note that the poles of $\tilde{\eta}$ at $s = log(2)/2$ suggests that functional equation is not true.

There is however no need for a functional equation if one is only interested in $F(s) \equiv \tilde{\zeta}(s) + \tilde{\zeta}(1-s)$ at the critical line! Also the analog of Riemann hypothesis follows naturally!

1. In the representation using $\tilde{\eta}$ $F(s)$ converges at critical striple and is *real*(!) at the critical line $Re(s) = 1/2$ as follows from the fact that $1 - s = \overline{s}$ for $Re(s) = 1/2$! Hence $F(s)$ is expected to have a large number of zeros at critical line. Presumably their number is infinite, since $F(s)^{cut,p_{max}}$ approaches $2\zeta^{cut,p_{max}}$ for large enough p_{max} at critical line.

2. One can define a different kind of cutoff of $\tilde{\zeta}$ for given n_{max}: $n < n_{max}$ in the sum over $e^{-Log(n)s}$. Call this cutoff $\tilde{\zeta}^{cut,n_{max}}$. This cutoff must be distinguished from the cutoff $\tilde{\zeta}^{cut,p_{max}}$ obtained by dropping the "prime factors" with $p < pmax$. The terms in the cutoff are of the form $u^{\sum k_p p/\pi(p)}$, $u = exp(-s)$. It is analogous to a polymomial but with fractional powers of u. It can be made a polynomial by a change of variable $u \rightarrow v = exp(-s/a)$, where a is the product of all $\pi(p)$:s associated with all the primes involved with the integers $n < n_{max}$.

   One could solve numerically the zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ using program modules calculating $\pi(p)$ for a given p and roots of a complex polynomial in given order. One can check whether also all zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ might reside at critical line.

3. One an define also $F(s)^{cut,n_{max}}$ to be distinguished from $F(s)^{cut,p_{max}}$. It reduces to a sum of terms $exp(-Log(n)/2)cos(-Log(n)y)$ at critical line, $n < n_{max}$. Cosines come from roots of unity. $F(s)$ function is not sum of rational powers of $exp(-iy)$ unlike $\zeta(\tilde{s})$. The existence of zero could be shown by showing that the sign of this function varies as function of y. The functions $cos(-Log(n)y)$ have period $\Delta y = 2\pi/Log(n)$. For small values of n the exponential terms $exp(-Log(n)/2)$ are largest so that they dominate. For them the periods Δy are smallest so that one expected that the sign of both $F(s)$ and $F(s)^{cut,n_{max}}$ varies and forces the presence of zeros.

   One could perhaps interpret the system as quantum critical system. The rather large rapidly varying oscillatory terms with $n < n_{max}$ with small $Log(n)$ give a periodic infinite set of approximate roots and the exponentially smaller slowly varying higher terms induce small perturbations of this periodic structure. The slowly varying terms with large $Log(n)$ become however large near the $Im(s) = 0$ so that here the there effect is large and destroys the period structure badly for small root of $\hat{\zeta}$.

3.2.4 Is the vanishing of the loop corrections consistent with unitarity?

Skeptic could argue that the vanishing of loop corrections is not consistent with unitarity. The following argument however shows that the fact that momenta in TGD framework are 8-D light-like momenta could save the situation. If not only single particle states but also *many-particle states* have light-like 8-momenta, the discontinuity of the amplitude at pole $P^2(M^8) = 0$ implies the discontinuity of the amplitude as function of $s \equiv P^2(M^4)$ along s-axis.

Minkowskian contribution to mass squared would essentially the sum of conformal (stringy) contribution from vibrational degrees of freedom and color contribution from CP_2 degrees of freedom. This suggests a weak form of color confinement: many-particle states could have vanishing color hyper charge and isospin but the eigenvalue value of color Casimir operator would be non-vanishing.

To get more concrete view about the situation the reader is encouraged to study the slides of Jaroslav Trnka explaining BCFW recursion formula [B5] (see `http://tinyurl.com/pqjzffj`) or the article [B3] of Elvang and Huang (see `http://tinyurl.com/y9rhbzhk`).

1. Unitarity condition $SS^\dagger = Id$ for S-matrix $S = 1 + iT$ gives $i(T - T^\dagger) = TT^\dagger$. For forward scattering the physical interpretation is that the discontinuity of $-2Im(T) = i(T - T^\dagger)$ in forward scattering as a function of total mass s above kinematical threshold along real axis is essentially the total scattering rate.

2. For a given tree amplitude, which is rational function, one replaces external momenta p_i with $\hat{p}_i = p_i + zr_i$. r_i real, light-like and orthogonal to each other and their sum vanishes. This gives on mass shell scattering amplitude with complex light-like momenta satisfying conservation conditions.

3. One can consider any non-trivial subset I of momenta and for this set one has $\hat{P}_I^2 = P_i^2 + 2zP \cdot R_I$, where one has $P_I = \sum_i p_i$ and $R_I = \sum_i r_i$. This gives

$$\hat{P}_I^2 = -P_I^2 \frac{(z - z_I)}{z_I} \quad , \quad z_I = \frac{P_I^2}{2P_I \cdot R_I} \quad .$$

   The poles of the modified amplitude $\hat{A}_n(z)$ come from the propagators at $\hat{P}_I^2 = 0$ and correspond to the points $z = z_I$.

4. From the modified scattering amplitude $\hat{A}_n(z)$ one can obtain the original scattering amplitude by performing a residue integral for $\hat{A}_n(z)/z$ along a curve enclosing the poles z_I. This gives

$$A_n = \hat{A}_n(z = 0) + \sum_{z_I} Res_{z=z_I}(\frac{\hat{A}_n(z)}{z}) + B_n \quad .$$

   B_n comes from the possible pole at $z = \infty$ and is often assumed to vanish. If so, the amplitude factorizes into a sum of products

$$Res_{z=z_I} \frac{\hat{A}_n(z)}{z} = \sum_I \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I) \quad .$$

   The amplitudes appearing in the product are for modified complex momenta.

The vanishing of loop corrections thus implies that the product terms $\hat{A}_L(1/P^2)\hat{A}_R$ in the BCFW formula give rational functions having no cuts just as the number theoretical vision demands. The discontinuities of the imaginary part of the amplitude are at poles and reduce to the products $\hat{A}_L\hat{A}_R$ with complex on-mass- shell light-like momenta as unitarity demands.

For forward scattering the discontinuity would be essentially positive definite total scattering rate. It would be however non-vanishing only at $P^2 = 0$ so that scattering rate could be non-vanishing only for $P^2 = 0$! This does not make sense in 4-D physics. Is it possible to overcome this difficulty in TGD framework?

1. The first thing to notice is that classical TGD predicts complex Noether charges since for instance Kähler coupling strength has imaginary part. This would suggest that the momenta of incoming particles could be complex. Could complex value of $P(M^4) \equiv P$ implying

$$P^2 = Re(P)^2 - Im(P)^2 + i2Re(P) \cdot Im(P) = 0$$

save the situation? The condition requires that $Re(P)$ and $Im(P)$ are light-like and parallel so that one would obtain only light-like four-momenta as total M^4 momenta.

2. However, in TGD light-likeness holds true in 8-D sense for single particle states: this led to the proposed generalization of twistor approach allowing particles to be massive in 4-D sense. $M^8 - H$ duality allows to speak about light-like M^8 momenta satisfying quaternionicity condition. The wave functions in CP_2 degrees of freedom emerge from momentum wave functions in M^8 degrees of freedom respecting quaternionicity. The condition $P^2(M^8) = 0$ implies that $Re[P(M^8)]$ and $Im[P(M^8)]$ are light-like and parallel. $Im[P(M^8)]$ can be arbitrarily small. One has also $Re[P(M^4)]^2 = Re[P(E^4)]^2$ and $Im[P(M^4)]^2 = Im[P(E^4)]^2$.

3. Could one pose the condition $P^2(M^8) = 0$ also on *many-particle states*? Kind of strong form of conformal invariance would be in question: not only single-particle states but also many-particle states would be massless in 8-D sense. Now $s = Re[P(M^4))]^2 = Re[P(E^4))]^2$ could have a continuum of values. The discontinuity along s-axis required by unitarity would would emerge from the discontinuity due to the pole at $P^2(M^8) = 0$! Hence 8-dimensional light-likeness in strong sense would be absolutely essential for having vanishing loop corrections together with non-vanishing scattering rates!

4. What could this mean in $M^4 \times CP_2$ picture? The intuitive expectation is that $Re[P(E^4)]^2$ corresponds to the eigenvalue Λ of CP_2 d'Alembertian so that the higher the momentum, the larger the value of Λ. CP_2 d'Alembertian would be essentially the M^4 mass squared of the state. This would allow vanishing color quantum numbers Y and I_3 but force symmetry breaking $SU(3) \rightarrow SU(2) \times U(1)$. This picture is not quite accurate: also the vibrational degrees of freedom contribute to the mass squared what might be called stringy contribution.

   Could the geometry of CP_2 induce this symmetry breaking? For instance, Kähler gauge potential depends on the $U(2)$ invariant "radial" coordinate of CP_2 and is invariant only under $U(2)$ rotations and changes by gauge transformation in other color rotations. Could one assign the symmetry breaking to the choice of color quantization axes boiling down at the classical level to the fixing of CP_2 Kähler function would?

   One would have color confinement in weak sense: in QCD picture physical states correspond to color singlet representations. This is certainly very strong statement in a sharp conflict with the standard view about color confinement. It would make sense in TGD framework, where color as a spin like quantum number is replaced with angular momentum like quantum number. One could say that macroscopic systems perform macroscopic color rotation. The model for the honeybee dance [L6] conforms with this view and actually led to the proposal for a modification of cosmic string type extremals $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$ by putting Y^2 in 2-D rigid body color rotation along both time axis and spatial axis of the string world sheet X^2.

5. This picture raises again the old question about the relationship of color and electroweak quantum numbers in TGD framework. Could one regard electroweak quantum numbers as a spin related to color group $SU(3)$ just as one can relate ordinary spin with Lorentz transformations? Color quantum numbers of say quarks would be analogous to orbital angular momentum. The realization of the action of the electroweak $U(2)_{ew}$ on CP_2 spinors indeed involves also geometric color rotation affecting the gauge potentials in the general case and $U(2)_{ew}$ can be identified as holonomy group of CP_2 spinor connection and sugroup of $SU(3)$. One could also see electroweak symmetry breaking as a further symmetry breaking $U(2) \rightarrow U(1) \times U(1)$ assignable with the flag manifold $SU(3)/U(1) \times U(1)$ parameterizing different choices of color quantization axes and having interpretation as CP_2 twistor space.

Remark: Number theoretic vision means that the quaterionic M^8-momenta are discrete with components having values in the extension of rationals. $P^2(M^4)$ becomes discrete if one poses $P^2(M^8) = 0$ condition for all states. The values of discontinuity of $Im(T)$ correspond now to a discrete sequence of poles along s-axis approximating cut. At the continuum limit this discrete sequence of poles becomes cut. Continuum limit would correspond to a finite measurement resolution in which one cannot distinguish the poles from each other.

3.3 Grassmannian approach and TGD

Grassmannian approach has provided besides technical progress deeper views about twistorialization and also led to the understanding of the Yangian symmetry.

3.3.1 Grassmannian twistorialization - or what I understand about it

The twistorialization of the scattering amplitudes works for planar amplitudes in massless theories and involves the following ingredients.

1. All scattering amplitudes are expressible in terms of on-mass-shell scattering amplitudes with massless on-mass-shell particles in complex sense.

2. The scattering amplitude is sum over contributions with varying number of loops. BCFW recursion relation allows to construct scattering amplitudes from their singularities using 3-particle amplitudes as building brick amplitudes. There are two types of singularities.

   For the first type of singularity one has on-shell internal line and one obtains a sum over all possible decompositions of the scattering amplitude to a product of on-mass-shell scattering amplitudes multiplied by delta function for momentum squared of the internal line. Second type of singularity corresponds to the so called forward limit and is obtained from $(n + 2, k)$ amplitude by contracting two added adjacent particles to form a loop so that their momenta are opposite and integrating over the momentum.

3. The singular term is algebraically analogous to an exterior derivative of the scattering amplitude and can be integrated explicitly: the integration adds BCFW bridge to the both terms such that the forward limit loop in the second term is under the bridge. The outcome is BCFW formula for l-loop amplitude with n external particles with k negative helicities consisting of these two terms.

Twistor Grassmannian approach expresses the on mass shell scattering amplitudes appearing as building bricks as residue integrals over Grassmannian $Gr(n, k)$, where n is the number of particles and k is the the number of negative helicities. The Grassmannian approach is described in a concise form in the slides by Jaroslav Trnka [B5] (see `http://tinyurl.com/pqjzffj`).

1. The construction of the on-mass-shell scattering amplitudes appearing in BCFW formula as residue integrals in Grassmannians follows by expressing the momentum conserving delta functions in twistor description in terms of auxiliary variables serving as coordinates of Grassmannian $G(n, k, C)$ for the on mass shell tree amplitude with n external particles having k negative helicities. Grassmannian has dimension $d = (n - k)k$ and can be identified as the space of k-planes - or equivalently $n - k$-planes in C^N. Grasmannian has a representation as homogenous space $G(n, k, C) = U(n)/U(n - k) \times U(k)$ having $SU(n)$ as the group of isometries. For $k = 1$ one obtains projective space which is also symmetric space (allowing reflection along geodesic lines as isometries).

2. Grassmannians emerge as an auxiliary construct, and the multiple residue integral over Grassmannian gives sum of residues so that the introduction of Grassmannians might look like un-necessary complication. The selection of points of Grassmannian for given external quantum numbers by residue integral given at the same time the value of the amplitude might however have some deeper meaning.

   The construction involves standard mathematics, which is however new for physicists. For instance, notions such as Plücker coordinates, Schubert cells and cell decomposition appear.

One can relate to each other various widely different looking expressions for the amplitudes as being associated with different cell decompositions of Grassmannian. The singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k,n)$ define a hierarchy of Schubert cells.

3. The so called positive Grassmannian [B4] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N}=4$ SUSY. The points of positive Grassmannian $Gr_+(k,n)$ are representable as $k\times n$ matrices with positive $k\times k$ determinants. The singularities correspond to the boundaries of $Gr_+(k,n)$ with some $k\times k$ determinants vanishing. For tree diagrams the singularities correspond to poles appearing in the factorized term of the BCFW decomposition of the scattering amplitude. The positivity conditions hold true also for the twistors representing external particles.

4. Positivity conditions guarantee the convexity of the integration region determined by the C-matrix as point of $Gr_+(k,n)$ appearing in the conditions dictating the integration region.

   To better understand the meaning of positivity one can first consider triangle call it T - as a representation of positive Grassmannian $Gr_+(1,3)=P^2_+$. Any interior points of T can be regarded as center of mass for suitable positive masses at the vertices of the triangle. These conditions generalizes to the case of general polygons, which must be convex. If the number of vertices of the polygon is larger than 3, convexity is not automatically satisfied, and requires additional conditions.

   This description generalizes to Grassmannians $Gr_+(k,n)$. Masses define the analog of C-matrix as element of $Gr_+(k,n)$ appearing in the twistor approach and the vertices of the triangle are analogous to the twistors associated with external particles combining to form a point of $Gr(4,n)$. Positivity condition is generalized to the condition that $k\times k$ minors of the $k\times n$ matrix are positive.

5. Also the twistors associated with the external particles must satisfy analogs of the positivity conditions. This involves the replacement of $Gr(4,n)$ associated with twistors of the external particles with $Gr_+(k+4,n)$. The additional k components of the twistors are Grassman numbers and determined by the superparts of the twistors (see the slides of Trnka at `http://tinyurl.com/pqjzffj`. I must admit that I did not understand this.

6. Residue integral can be defined in terms of what is called canonical form Ω - analog of volume form - having logarithmic singularities at the boundaries of the $Gr_+(k,n)$. Hence one can perform a reduction of the residue integral to a sum of integrals over $G(k,k+4)$ instead of $G(k,n)$ (actually not so surprising since the residue integrals give as outcome the residues at discrete points!).

   This leads to a reduction of the residue integral over $Gr_+(k,n)$ to a sum of lower dimensional residue integrals over triangulation defined by $Gr_+(k,k+4)$ represented as surfaces of $Gr_+(k,n)$ glued together along sides. The geometric analog would be decomposition of polygon to a union of triangles.

   This simplifies the situation dramatically [B6, B5, B4] and leads to the notion of amplituhedron [B2, B1]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N}=4$ SUSY.

7. It should be possible to construct Ω explicitly having the desired singularities which would be in TGD framework poles with $P^2(M^8)=P^2(M^4\times CP_2)=0$ if the proposed realization of unitary makes sense? Could one just assumes that Ω vanishes for that part of the boundary of $Gr_+(k,n)$, which gives loop singularities? Could these points $Gr_+(k,n)$ be transcendental and excluded for this reason?

If loop corrections are vanishing as ZEO strongly suggests, only tree amplitudes are needed. Therefore it is appropriate to summarize what I have managed to understand about the construction of the tree amplitudes with general value of k in the amplituhedron approach.

1. The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n\geq k+m$. Actually a map from $G(k,n)\times G(k+4,n)\rightarrow G_+(k,k+m)$ is in question. $m=4$ identifiable

as the apparent dimension of twistor space without projective identification giving the actual dimension $d = 3$. n is the number of external particles and k the number of negative helicities.

The value of m is $m = 4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y = C \cdot Z$, where Y corresponds to point of $G_+(k, k+4)$ and Z to the point of $G(k+4, n)$ performs this mapping, which is clearly many-to one. One can decompose integral over $G_+(k, n)$ to integrals over positive regions $G_+(k, k+4)$ intersecting only along their common boundary portions. The decomposition of a convex polygon in plane to triangles represent the basic example of this kind of decomposition. Obviously there are several decompositions of this kind.

2. Each decomposition defines a sum of contributions to the scattering amplitude involving integration of a projectively invariant volume form over the positive region in question. The form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k, k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

3. In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w = exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz = dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

4. One must extend the bosonic twistors Z_a of external particles by adding k coordinates. This extension looks very difficult to understand intuitively. Somewhat surprisingly, these coordinates are anti-commutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron. An interesting question is whether the addition of k-dimensional anti-commutative parts to Z_a expressible in terms of super-coordinates is only a trick or whether it could have some physical interpretation.

3.3.2 Grassmannians as reduced WCWs?

Grassmannians appear as auxiliary spaces in twistor approach. Could Grassmannians and the procedure assigning to external momenta and helicities discrete set of points of Grassmannian and scattering amplitude have some concrete interpretation in TGD framework?

1. The points of cognitive representation define WCW coordinates for space-time surface. For a fixed number of points in cognitive representation WCW is effectively replaced with a finite-dimensional reduced WCW. These points would naturally correspond to the points defining ends of fermionic lines at partonic 2-surfaces. WCW has Kähler metric with Euclidian signature. This could be true also for its reduction.

2. The experience with twistorialization suggests that these spaces could be simply Grassmannians $Gr(n, r, C)$ consisting or r-dimensional complex planes of n-dimensional complex space representable as coset spaces $U(n)/U(n-r) \times U(r)$ appearing as auxiliary spaces in the construction of twistor amplitudes.

   Note that the correlation between quantum states and geometry would be present since n corresponds to the number of external particles and r to those with negative helicity in ordinary twistor Grassmann approach. In TGD framework discretized variants of these spaces corresponding to the extension of rationals used would appear. Yangian symmetries could correspond to general coordinate transformations for the reduced WCW acting as gauge symmetry. These transformations act as diffeomorphisms for so called positive Grassmannians also in the standard twistorialization. If the reduced WCWs indeed correspond to twistor Grassmannians, one would have a completely unexpected connection with supersymmetric QFTs.

3. The reduction of WCW to a finite dimensional Kähler manifold suggests that also WCW spinors become ordinary spinors for Kähler manifold so that gamma matrices form a finite-D fermionic oscillator operator algebra. WCW has maximal symmetries and it would not be surprising if also the finite-D Kähler manifold would possess maximal symmetries. Note that WCW gamma matrices together with isometry generators of WCW give rise to a super-symplectic algebra involving a generalization of 2-D conformal invariance replacing 2-D surfaces with light-like 3-surfacs.

4. The interpretation of supersymmetry would be different from the standard one. Kähler structure implies that $\mathcal{N}$ is even and Majorana spinors are absent and both baryon and lepton number can be conserved separately. The ordinary fermionic oscillator algebra is a Clifford algebra and could be interpreted in terms of a broken supersymmetry.

Also more general flag manifolds than Grassmannians can be considered. If these spaces are homogenous spaces they have maximal isometries. They should have also Kähler structure. Compactness looks also a highly desirable property. The gauge conditions for the subalgebra of super-symplectic algebra state that the sub-algebra and its commutator with the entire algebra annihilate physical states and give rise to vanishing classical Noether charges. This would effectively reduce the super-symplectic algebra to a finite-D Lie group or Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to the ADE hierarchy as the hierarchy of inclusions of HFFs as an alternative correlate for the realization of finite measurement resolution suggests. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

3.3.3 Interpretation for Grassmannian residue integrations

The identification of Grassmannians (or possibly more general spaces) as reduced WCWs would give a genuine physical interpretation for the Grassmannian integrations as residue integrations over reduced WCW. What looks mysterious and maybe even frustrating is that the outcome of the entire process is sum over discrete residues: what does this mean?

1. The residue integration is only over a surface of reduced WCW with dimension equal to one half of that of WCW. One has integrand, which depends on the external quantum numbers coded in terms of twistors and on coordinates of reduced WCW. The residue integration is analogous to summation over amplitude associated with space-time surfaces coded by different cognitive representations.

2. One can argue that a continuous residue integral over Grassmannian is not consistent with the number theoretic discretization. The outcome is however discrete set of space-time surfaces labelled by cognitive representations as points of Grassmannian. Of the points in question are in the extension and if this is equivalent with the corresponding property for the coordinates of Grassmannian, there should be no problems. The restriction of external momenta to the extension of rationals might guarantee this.

3. The full multiple residue integral leaves only pole contributions, which correspond to a discrete collection of space-time surfaces (at least the set of space-time surfaces obtained by the action of Galois group), that is discrete set of points of reduced WCW. It seems that the entire residue integration is just a manner to realize quantum classical correspondence by associating to the external quantum numbers space-time surfaces and corresponding cognitive representations - and of course, also the scattering amplitude.

4. One can also ask whether the positivity of Grassmannian might relate to the fact that p-adic numbers as ordinary integers are always non-negative (most of them infinite). The positivity

might be necessary in order to have number theoretic universality. If the minors associated with the C-matrix serve as coordinates for $Gr_+(k,n)$ they could be interpreted also as p-adic numbers. If they are allowed to be negative, one encounters problems since p-adic numbers are not well-ordered and one cannot say whether p-adic number is negative or positive.

3.3.4 Posible description of SUSY and its breaking in TGD framework

Although twistor description make sense also in the absence of supersymmetry, super-symmetry is an essential part of the elegance of the Grassmannian approach. For the ordinary SUSY one has gluons and their superpartners characterized in terms of super-twistors. In TGD one has two pictures [L2, K18].

1. At the level H fermions as fundamental particles are described in terms of second quantized induced spinor fields, whose oscillator operators can be used to build gamma matrices for WCW [K12, K13]. In TGD universe all known elementary particles would be composites of fundamental fermions represented as lines at the light-like orbits of partonic 2-surfaces (wormhole throats) and ordinary elementary particles involve a pair of wormhole contacts with throats containing these fermion lines. It is assumed that the fermions are at different points: this allows to avoid problems due to infinities.

   In the proposed generalization of twistor approach $2 \rightarrow 2$ fermion scattering in the classical fields at partonic 2-surface would define the basic $2 \rightarrow 2$-vertex replacing 3-vertices of twistorial SUSY. Essentially one has only two-vertices describing the redistribution of fermions at partonic 2-surface between orbits of the partonic 2-surfaces meeting at it. This is different from $\mathcal{N} = 4$ SUSY [K17]. If one allows completely local multi-fermion states at the level of H one cannot avoid fermionic contact interactions.

   The many-fermion states associated with partonic 2-surfaces would define the analogs of super-multiplets. One can wonder whether a SUSY type description could exist as a limit when the partonic 2-surface is approximated with single point so that also positions of fermions are approximated as single point. SUSY would be only approximate.

2. At the level of M^8 I have proposed the use of polynomials P of super-octonion serving as analogs of super-gluon fields to construct scattering amplitudes [L2]. This allows geometric description of all particles using super-multiplets. Each monomial of theta parameters would give rise to its own space-time surface by the condition that either $IM(P)$ or $RE(P)$ vanishes for the corresponding polynomial P. This condition would reduce the components of super-field to algebraic surfaces.

   There is however an important difference from H picture. The members of super-multiplet defined by P correspond to the coefficients of monomials of theta parameters having interpretation as analogs of oscillator operators. Super-partners would be in this sense point-like objects unlike in H approach, where this can hold true only approximately.

Could H- and M^8 pictures be equivalent and could one understand the breaking of SUSY in this framework?

1. $M^8 - H$ correspondence as a map of associative space-time regions from M^8 to minimal surfaces in H makes sense for the external particles and thus at boundaries of CDs. It assigns to a point of the partonic 2-surface $X^2 \subset X^4 \subset M^8$ the quaternionic tangent space of X^4 at it characterized by a point of CP_2. M^4 point is mapped to itself. There is additional condition requiring that quaternionic tangent space contains fixed complex sub-space but this is not relevant now.

2. Could this map be one-to-many so that super-field component describing purely many-fermion state would be mapped to several points at the image of X^2 in H describing multi-local many-fermion state? This is possible if the points in M^8 are singular in the sense that the action of a normal subgroup H of Galois group Gal leaves the point invariant so that Gal reduces to Gal/H: symmetry breaking takes place.

   The tangent spaces of the degenerate points are however different and are mapped to different points of CP_2 in $M^8 - H$ correspondence making sense at boundaries of CDs but not in their interiors. One would have several fermions with same M^4 coordinates but different CP_2 coordinates and the outcome would be many-fermion state. In the case of 2-fermion state the different values of CP_2 coordinates would be associated with the opposite throats of a wormhole contact whose orbit defines light-like 3-surface. Could light-likeness inducing the reduction of the metric dimension of the tangent space from 4 to 3 somehow induce also this degeneration?

3. Could symmetry breaking as a degeneration of Gal action to that for Gal/H take place for the conditions defining the 4-surfaces associated with the higher components of super-octonion and induce the breaking of SUSY at the level of M^8 manifesting as the non-locality of the fermion state at the level of H? This degeneration would be a typical manifestation of quantum criticality: criticality in general means co-incidence of two roots.

3.3.5 Comments about coupling constant evolution

3.4 Summary

Since the contribution means in well-defined sense a breakthrough in the understanding of TGD counterparts of scattering amplitudes, it is useful to summarize the basic results deduced above as a polished answer to a Facebook question.

There are two diagrammatics: Feynman diagrammatics and twistor diagrammatics.

1. Virtual state is an auxiliary mathematical notion related to Feynman diagrammatics coding for the perturbation theory. Virtual particles in Feynman diagrammatics are off-mass-shell.

2. In standard twistor diagrammatics one obtains counterparts of loop diagrams. Loops are replaced with diagrams in which particles in general have complex four-momenta, which however light-like: on-mass-shell in this sense. BCFW recursion formula provides a powerful tool to calculate the loop corrections recursively.

3. Grassmannian approach in which Grassmannians $Gr(k,n)$ consisting of k-planes in n-D space are in a central role, gives additional insights to the calculation and hints about the possible interpretation.

4. There are two problems. The twistor counterparts of non-planar diagrams are not yet understood and physical particles are not massless in 4-D sense.

In TGD framework twistor approach generalizes.

1. Massless particles in 8-D sense can be massive in 4-D sense so that one can describe also massive particles. If loop diagrams are not present, also the problems produced by non-planarity disappear.

2. There are no loop diagrams- radiative corrections vanish. ZEO does not allow to define them and they would spoil the number theoretical vision, which allows only scattering amplitudes, which are rational functions of data about external particles. Coupling constant evolution - something very real - is now discrete and dictated to a high degree by number theoretical constraints.

3. This is nice but in conflict with unitarity if momenta are 4-D. But momenta are 8-D in M^8picture (and satisfy quaternionicity as an additional constraint) and the problem disappears! There is single pole at zero mass but in 8-D sense and *also many-particle states* have vanishing mass in 8-D sense: this gives all the cuts in 4-D mass squared for all many-particle state. For many-particle states not satisfying this condition scattering rates vanish: these states do not exist in any operational sense! This is certainly the most significant new discovery in the recent contribution.

   BCFW recursion formula for the calculation of amplitudes trivializes and one obtains only tree diagrams. No recursion is needed. A finite number of steps are needed for the calculation and these steps are well-understood at least in 4-D case - even I might be able to calculate them in Grassmannian approach!

4. To calculate the amplitudes one must be able to explicitly formulate the twistorialization in 8-D case for amplitudes. I have made explicit proposals but have no clear understanding yet. In fact, BCFW makes sense also in higher dimensions unlike Grassmannian approach and it might be that the one can calculate the tree diagrams in TGD framework using 8-D BCFW at M^8 level and then transform the results to $M^4 \times CP_2$.

What I said above does yet contain anything about Grassmannians.

1. The mysterious Grassmannians $Gr(k,n)$ might have a beautiful interpretation in TGD: they could correspond at M^8 level to reduced WCWs which is a highly natural notion at $M^4 \times CP_2$ level obtained by fixing the numbers of external particles in diagrams and performing number theoretical discretization for the space-time surface in terms of cognitive representation consisting of a finite number of space-time points.

   Besides Grassmannians also other flag manifolds - having Kähler structure and maximal symmetries and thus having structure of homogenous space G/H - can be considered and might be associated with the dynamical symmetries as remnants of super-symplectic isometries of WCW.

2. Grassmannian residue integration is somewhat frustrating procedure: it gives the amplitude as a sum of contributions from a finite number of residues. Why this work when outcome is given by something at finite number of points of Grassmannian?!

   In M^8 picture in TGD cognitive representations at space-time level as finite sets of points of space-time determining it completely as zero locus of real or imaginary part of octonionic polynomial would actually give WCW coordinates of the space-time surface in finite resolution.

   The residue integrals in twistor diagrams would be the manner to realize quantum classical correspondence by associating a space-time surface to a given scattering amplitude by fixing the cognitive representation determining it. This would also give the scattering amplitude.

   Cognitive representation would be highly unique: perhaps modulo the action of Galois group of extension of rationals. Symmetry breaking for Galois representation would give rise to supersymmetry breaking. The interpretation of supersymmetry would be however different: many-fermion states created by fermionic oscillator operators at partonic 2-surface give rise to a representation of supersymmetry in TGD sense.

REFERENCES

Mathematics

[A1] Shipman B. The geometry of momentum mappings on generalized flag manifolds, connections with a dynamical system, quantum mechanics and the dance of honeybee. Available at: `http://math.cornell.edu/~oliver/Shipman.gif`, 1998.

[A2] Baez J. Quasicrystals and the Riemann Hypothesis. The n-Category Cafe. Available at: `https://golem.ph.utexas.edu/category/2013/06/quasicrystals_and_the_riemann.html`, 2013.

Theoretical Physics

[B1] Trnka J Arkani-Hamed N, Hodges A. Positive Amplitudes In The Amplituhedron. Available at:`http://arxiv.org/abs/1412.8478`, 2014.

[B2] Trnka Y Arkani-Hamed N. The Amplituhedron. Available at: `http://arxiv.org/abs/1312.2007`, 2013.

[B3] Huang Y-T Elvang H. Scattering amplitudes. Available at: http://arxiv.org/pdf/1308.1697v1.pdf, 2013.

[B4] Arkani-Hamed N et al. Scattering amplitides and the positive Grassmannian. Available at: http://arxiv.org/pdf/1212.5605v1.pdf.

[B5] Trnka J. Scattering amplitudes and the positive Grassmannian. Available at: http://tinyurl.com/pqjzffj, 2013.

[B6] Trnka Y. Grassmannian Origin of Scattering Amplitudes. Available at: https://www.princeton.edu/physics/graduate-program/theses/theses-from-2013/Trnka-Thesis.pdf, 2013.

Books related to TGD

[K1] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#compl1, 2006.

[K2] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#fermizeta, 2006.

[K3] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumannnew, 2006.

[K4] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#kahler, 2006.

[K5] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#mless, 2006.

[K6] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#nmpc, 2006.

[K7] Pitkänen M. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#cognic, 2006.

[K8] Pitkänen M. Possible Role of p-Adic Numbers in Bio-Systems. In *Bio-Systems as Self-Organizing Quantum Systems*. Online book. Available at: http://tgdtheory.fi/public_html/bioselforg/bioselforg.html#biopadc, 2006.

[K9] Pitkänen M. Quantum Model for Hearing. In *TGD and EEG*. Online book. Available at: http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html#hearing, 2006.

[K10] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory, 2006.

[K11] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

[K12] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#cspin, 2006.

[K13] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#wcwnew, 2014.

[K14] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#numbervision, 2014.

[K15] Pitkänen M. More Precise TGD View about Quantum Biology and Prebiotic Evolution. In *Genes and Memes*. Online book. Available at: http://tgdtheory.fi/public_html/genememe/genememe.html#geesink, 2015.

[K16] Pitkänen M. About twistor lift of TGD? In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#hgrtwistor, 2016.

[K17] Pitkänen M. From Principles to Diagrams. In *Towards M-Matrix*. Online book.Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#diagrams, 2016.

[K18] Pitkänen M. Questions related to the twistor lift of TGD. In *Towards M-Matrix*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistquestions, 2016.

[K19] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? In *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#ratpoints, 2017.

[K20] Pitkänen M. Philosophy of Adelic Physics. In *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#adelephysics, 2017.

Articles about TGD

[L1] Pitkänen M. Could one realize number theoretical universality for functional integral? Available at: http://tgdtheory.fi/public_html/articles/ntu.pdf, 2015.

[L2] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.

[L3] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[L4] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L5] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: http://tgdtheory.fi/public_html/articles/conscrit.pdf, 2017.

[L6] Pitkänen M. Dance of the honeybee and New Physics. Available at: http://tgdtheory.fi/public_html/articles/Shipmanagain.pdf, 2018.