Could ZEO provide a new approach to the quantization of fermions?

M. Pitkänen
Email: matpitka6@gmail.com.
`http://tgdtheory.com/`.

April 22, 2020

Abstract

The exact details of the quantization of fermions have remained open in TGD framework. The basic problem is the possibility of divergences coming from anti-commutators of fermions expected to involve delta functions in the continuum case. In standard framework normal ordering saves from these divergences for the "free" part of the action but higher order terms give the usual divergences of quantum field theories. In supersymmetric theories the normal ordering divergences however cancel.

In TGD the bosonic divergenges are absent due to the generalization of the notion of point-like particle to 3-surface. In fermionic sector normal ordering divergences cancel in unique number theoretic discretization based on what I call cognitive representations but in continuum case the situation is unclear.

Induction procedure plays a key role in the construction of classical TGD. The longstanding question has been whether the induction of spinor structure could be generalized to the induction of second quantization of free fermions at the level of 8-D imbedding space to the level of space-time. The problem is that the anticommutators are 8-D delta functions in continuum case and could induce rather horrible divergences. It will be found that zero energy ontology (ZEO) and new view about space-time and particles allow to modify the standard quantization procedure by making modified Dirac action bi-local so that one gets rid of divergences. Also the multi-local Yangian algebras proposed on basis of physical intuition to be central in TGD emerge automatically.

Contents

1 Introduction

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What happens in TGD?

1. The replacement of point like particles with 3-surfaces replaces the dynamics of fields with that of surfaces. The resulting non-locality in the scale of 3-surfaces gives excellent hopes about the cancellation of divergences in the bosonic sector. The situation is very similar to that in super-string models.

2. What about fermions? The TGD counterpart of Dirac action - modified Dirac action - is dictated uniquely by the bosonic action which is induced from twistor lift of TGD as sum of Kähler action analogous to Maxwell action and of volume term [K3, K6]. Supersymmetry in TGD sense is proposed in [L14] overcomes the problems of standard SUSY - in particular Majorana spinors are avoided. The key idea is theta parameters are replaced with fermionic creation operatos and the spartners correspond to states created by the local composites of these.

   In the second quantization based on cognitive representations [L4, L7] as unique discretization of the space-time surface for an adele defined by extension of rationals superpartners would correspond to local composites of quarks and anti-quarks. TGD variant of super-space of SUSY approach so that space-time as 4-surface is replaced with its super-variant identified as union of surfaces associated with the components of super coordinates. Fermions are correlates of quantum variant of Boolean logic which can be seen as square root of Riemann geometry. There is no need for Majorana fermions.

   This approach replaced the earlier view in which right-handed neutrinos served a role as generators of $\mathcal{N} = 2$ SUSY [K9, K5]. In the approach to be discussed one is forced to ask whether their counterparts as local 3-quarks composites could make comeback in a more precise formulation of the picture first discussed in [L14]. The answer turns out to be negative (see Appendix).

   The simplest option involves only quarks as fundamental fermions and leptons would be local composites of 3 quarks: this is possibly by the TGD based view about color. Quark oscillator operators are enough for the construction of gamma matrices of "world of classical worlds" (WCW [K6]) and they inherit their anti-commutators from those of fermionic oscillator operators. Even the super-variant of WCW can be considered. The challenge is to fix these anti-commutation relations for oscillator operator basis at 3-D surface: the modified Dirac equation would dictate the commutation relations later. This is not a trivial problem. One can also wonder whether one avoid the normal ordering divergences.

3. In a discretization the anti-commutators of fermions and antifermions by cognitive representations [L6, L10, L12, L7] do not produce problems but in the continuum variant of this approach one obtains normal ordering divergences. Adelic approach [L4] suggests that also continuum variant of the theory must exists as also that of WCW so that one should find a manner to get rid of the divergences by defining the quantization of fermions in such a manner that one gets rid of divergences.

One can start by collecting a list of reasonable looking conditions possibly leading to the understanding of the fermionic quantization, in particular anticommutation relations.

1. The quantization should be consistent with the number theoretic vision implying discretization in terms of cognitive representations [L6, L10, L12, L7]. Could one assume that anti-commutators for the quark field for discretization is just Kronecker delta so that the troublesome squares of delta function could be avoided already in Dirac action and expressions of conserved quantities unless one performs normal ordering which is somewhat ad hoc procedure.

   The anti-commutators of induced spinor fields located at opposite boundaries of CD and quite generally, at points of $H = M^4xCP_2$ (or in M^8 by M^H duality) with non-space-like separation should be determined by the time evolution of induced spinor fields given by modified Dirac equation.

   In the case of cognitive representation could fix the anti-commutators for given time slice in $M^4 \times CP_2$ as usual Kronecker delta for the set of points with algebraic coordinates so that if anti-commutators of fermionic operators between opposite boundaries of CD were not needed, everything would be well-defined. By solving the modified Dirac equation for the induced spinors one can indeed express the induced spinor field at the opposite boundary of CD in terms of its values at given boundary. Doing this in practice is however difficult.

2. Situation gets more complex if one requires that also the continuum variant of the theory exists. One encounters problems with fermionic quantization since one expects delta function singularities giving rise to at least normal ordering singularities. The most natural manner to quantize quarks fields is as a free field in $H = M^4 \times CP_2$ expanded as harmonics of H. This however implies 7-D delta functions and bad divergences from them. Can one get rid of these divergences by changing the standard quantization recipes based on ordinary ontology in which one has initial value problem in time= constant snapshot of space-time to a quantization more appropriate in zero energy ontology (ZEO)?

Induction procedure plays a key role in the construction of classical TGD. The longstanding question has been whether the induction of spinor structure could be generalized to the induction of second quantization of free fermions at the level of 8-D imbedding space to the level of space-time so that induced spinor field $\Psi(x)$ would be identified as $\Psi(h(x))$, where $h(x)$ corresponds to the imbedding space coordinates of the space-time point. One would have restrictions of free fermion theory from imbedding space H to space-time surface.

The problem is that the anticommutators are 8-D delta functions in continuum case and could induce rather horrible divergences. It will be found that zero energy ontology (ZEO) [K4] [L13] and the new view about space-time and particles allow to modify the standard quantization procedure by making modified Dirac action bi-local so that one gets rid of divergences. The rule is simple: given partonic 2-surface contains either creation operators or annihilation operators but not both. Also the multi-local Yangian algebras proposed on basis of physical intuition to be central in TGD [K8] emerge naturally.

2 Induction of quantum spinor structure in ZEO and second quantization of fermions

In the following it will be argued that the quantization based on ZEO as induction of second quantization at the level of imbedding space $M^4 \times CP_2$ (or M^8 by$M^8 - H$ duality [L11, L8, L9]) could provide a solution to the divergence problem for fermions.

2.1 How could ZEO help?

Consider first what ZEO is.

1. In ZEO one replaces time=constant snapshots with pairs of 3-surfaces at opposite boundaries of causal diamond (CD $=cd \times CP_2$, where cd is the intersection of future and past directed light-cones of M^4). In ordinary classical ontology these snapshots correspond to initial and final states of classical deterministic time evolution. Note that 3-surfaces can be seen as unions of 3-surfaces (with several disjoint components) at the opposite boundaries of CD.

General Coordinate Invariance (GCI) implies holography so that one can equivalently talk about space-time surfaces as preferred extremals analogous to Bohr orbits. Strong form of holography (SHF) requires that either the data provided 3-D light-like parton orbits or by 3-surfaces at the ends of CD is enough to code for quantum states.

The action is given by the dimensional reduction of the 6-D Kähler action for the twistor lift of TGD. The product of twistor spaces of M^4 and CP_2 replaces imbedding space H. The existence of 6-D Kähler action requires that these twistor spaces allow Kähler structure and this is indeed the case and actually fixes the theory completely [**?**]: standard model symmetries lead to the same outcome as also $M^8 - H$ duality based on classical number fields.

Dimensional reduction is required to get 6-D surface as an analog of twistor space of space-time surface as a sphere bundle with space-time surface as base space. [**?**, **?**] A central physical prediction is length scale dependent cosmological constant as coefficient of volume term in the dimensionally reduced action. Quantum states are replaced by superpositions of pairs of 3-surfaces or equivalently of space-time surfaces.

2. The analog of massless Dirac action - modified Dirac action - is needed and is dictated by supersymmetry from the bosonic action [K3, K6]. This means tha the gamma matrices appearing in modified Dirac operator D are obtained as contractions of with imbedding space gamma matrices with canonical momentum currents defined by the bosonic action. This is required by hermiticity of the D. The volume term in the bosonic action guarantees that the 4 gamma matrices are in general linearly independent.

3. Dirac action can be also super-symmetrized as also imbedding space-time coordinates using local quark multi-linears [L14]. I have discussed this at the level of cognitive representations, which are unique number theoretical discretizations of space-time surfaces consisting of points of imbedding space with coordinates in extension of rationals defining the adele in question [L4, L5]. The p-adic number fields in the fundamental rational adele are replaced with the their extensions induced by the extensions of rationals and the hierarchy of adeles defines an evolutionary hierarchy.

How ZEO could help in attempt to make quantization of fermions precise and computable and free of divergences? It is best to proceed by making questions.

1. Could fermionic bi-linear in the Dirac action be replaced by bi-linears of form

   $$\overline{\Psi}_1 D_2^{\rightarrow} \Psi_2 - \overline{\Psi}_1 D^{\leftarrow} \Psi_2 \ \ ,$$

   where Ψ_1 would be spinor field at the 3-surface associated passive boundary CD and Ψ_2 induced spinor field at the opposite active boundary of CD? Bi-locality would allow to get rid of divergences.

   One can also construct a slicings of CD by light-cones parallel to passive and active boundaries respectively. One would obtain Dirac equation in the interior of space-time surface.

   The generalized Dirac action would involve two 4-D integrals over space-time surface so that integration is 8-D. The slicing of M^4 by light-cones would allow to define time ordering for the slices, and the definition of action would naturally involve hermitian conjugation of action permuting the roles of Ψ_i as in time order product of spinor fields in quantum field theories. The Dirac equation would be obtained in the same manner as usually by varying with respect to $\overline{\Psi_1}$ and Ψ_2.

2. This is not yet the most general definition of the bilinearity and modified Dirac action. The cancellation of normal ordering divergences allows also to consider the bi-linears associated with different components of 3-surface at either boundary. Even more generally, by strong form of holography one could allow pairing of partonic 2-surfaces for given connected 3-surface. In particular opposite throats of wormhole contacts can be paired.

   These terms would have interpretation in term of bi-linears of fermion and anti-fermion oscillators whereas the pairing with members associated with opposite boundaries of CD would correspond to a pairing of fermionic (anti-fermionic) creation operators and anti-fermionic (fermionic) annihilation operators. One would obtains geometric counterpart for the decomposition of Dirac action and canonical momentum currenrs to various oscillator operator bi-linears.

One could allow both creation and annihilation operators at the same boundary of CD provided they are not associated with the same 3-surface. If the action can have terms associated with string world sheets, partonic 2-surfaces and 3-D their light-like orbits, even the condition that they do not correspond to same partonic 2-surfaces or string world sheet is enough to get rid of divergences. One could go even further and allow creation and annihilation operators at different points of partonic orbit if they reside at 1-D light-like boundaries of different string world sheets.

3. What could be the minimum condition for avoiding divergences. The super-symmetrization to be discussed in more details below allows even local composites of quark and antiquark creation (annihilation) operators in the super-field since these have vanishing anti-commutators.

4. The modified Dirac action would not involve anti-commutators of $\Psi_1^\dagger$ and Ψ_2 at the same point so that the action would be finite even if one identifies the anti-commutations relations as those induced by the identification $\Psi(x) = \Psi(h(x))$, where $Psi(h)$ is second quantized imbedding space spinor field written as superposition over all its modes with oscillator operators obeying standard anti-commutation relations in $H = M^4 \times CP_2$. ZEO would make everything unique , finite, and calculable! One would obtain also direct connection with the p-adic mass calculations in which spinor modes of H-spinor fields define ground states of the super-symplectic representations.

Consider now what conditions that super-symmetrization in terms of local composites of fermionic oscillator operators [L14] gives.

1. In the proposed discretization based on cognitive representations super-symmetrization replaces imbedding space coordinates h with super-coordinates h_S having a local expansion in powers of hermitian local composites of Ψ with vanishing fermion number and same quantum numbers as tangent space coordinates of H. In the case of super quark field all super components have same quantum numbers as quark itself. Leptonic super-field would consist of local composites of 3-quarks with leptonic quantum numbers and could be regarded as superpartner of quark field.

2. The local composites contain in the general case both Ψ and its conjugate $\overline{\Psi}$ and here one encounters a problem if the anti-commutations are induced from those for second quantized imbedding space spinor field. This is the case even if the quantization is carried out using discretized 4-momenta ,which is of course number theoretically well-motivated, the problem remains.

3. Quark creation operators and antiquark annihilation operators cannot be allowed at the same point. This would however allow local composites involving both types of creation operators at given boundary of CD. The minimum option would be this. The quantization at imbedding space level indeed allows unique decomposition of positive and negative energy parts unlike quantization in general curved space-time. For the most general option avoiding divergences the only condition is that annilation and creation operators do not appear at the same point of light-like curve as boundary of string world sheet at the light-like partonic orbit. It is however essential that modified Dirac action contains terms assignable to 2-D and possibly also 3-D surfaces. This point will be discussed later.

4. This approach forces to generalize the view about fermionic conserved quantities. Usually conserved quantities have leading term given by a term bilear in creation and annihilation operators acting on fermionic vacuum. Accepting the proposed bi-linearity, the conserved quantities would contain only nonlocal bi-linear terms of various kinds. How does one define the notion of eigenstate?

Assume that both kinds of oscillator operators at both boundaries of CD but that given point of partonic 2-surface carries only creation or annihilation type operators. This allows to get rid of divergences and define the notion of eigenstate at given boundary of CD by assuming that given boundary of CD corresponds to same fermionic vacuum. Bi-local composites of creation and annihilation operators appearing in the hermiatian operator representing observable would shift fermions between points of the partonic 2-surface but delocalization would allow to consruct eigenstates.

2.2 Fermionic quantization as induced quantization

Consider now the details of fermionic quantization as induced quantization.

1. The fermionic propagators for a pair of points of space-time surface would be induced from those for free fermions in $H = M^4 \times CP_2$. Induced spinor field at space-time surface would be a restriction of imbedding space spinor field and fermionic oscillator operators at 3-surface at the boundary of CD would be Fourier components of H spinor fields at 4-surface with respect to the basic of spinor spinor modes at 4-surface restricted to 3-surface.

   For 4-D surfaces the generalization of Dirac action involves 2 4-D integrals corresponding to Ψ_1 and Ψ_2. Since modified gamma matrices Γ^α have dimension $d = -1$, Ψ_1 and Ψ_2 must have dimension $d = -7/2$. Remarkably, also imbedding space spinor fields have this dimension! Therefore it is natural to identify the space-time propagators as induced propagators. This works only for 4-D space-time surface for 8-D H so that space-time dimension would be fixed.

2. There is however an objection. Does one need modified Dirac equation for induced spinor fields at all? Could one do using only second quantized imbedding space spinor fields to define correlation functions needed in S-matrix? Could one construct fermionic multi-linears using just these.

   One would lose all the nice theory related to the induced spinor structure and the supersymmetry based connection with the bosonic action. One would lose also quantum classical correspondence between eigenvalues of fermionic conserved charges in Cartan algebra and classical conserved quantities defined by the bosonic part of the action.

   For what purposes the modified Dirac action would be needed? Could one see space-time description and imbedding space description as duals of each other. Could space-time description give interpretation for classical Noether charges for bosonic action in terms of fermionic charges given basically by modified Dirac action and imbedding space anti-commutation relations?

2.3 How to generalize the Dirac action for string world sheets and partonic 2-surfaces

The description of 4-D Dirac action is not yet the whole story. One must assign modified Dirac action also to string world sheets and partonic 2-surfaces and avoid divergences and justify the proposed picture about second quantization. Consider first the physical picture.

1. One has also string world sheets, partonic 2-surfaces, and their light-like orbits as regions at which the signature of the induced metric changes from Minkowskian to Euclidian. Wormhole contacts have interpretation as building bricks of particles and if the wormhole contact carries homology charge (Kähler magnetic charge) also second wormhole contact is necessary to give closed flux lines traversing along first Minkowskian sheet going through second wormhole contact and returning back along second space-time sheet and returning through first wormhole contact.

   One can also consider the possibility that wormhole contacts carry no homology charge but in this case they are not stable. Whether all elementary particles have two homologically charged wormhole contacts as building brick or whether only quarks carry homology charges is not yet settled in the general case. If leptons are local 3-quark composites, one could assume non-vanishing homology charge for them.

2. M^8-picture [L11] predicts 6-D surfaces having topology of 6-sphere S^6 and analogous to branes in string theory. S^6 has 3-ball B^3 represented as $t = constant$ intersection of $cd \subset M^4$

as M^4 projection so that radial M^4 coordinate r_M satisfies $r_M \leq t$ (t is linear time of M^4 and corresponds to octonionic real coordinate). t corresponds to a root of a polynomial P having rational (or possibly even algebraic) coefficients so that one obtains connection with number theory.

These 6-sphere S^6 is a special solution of the algebraic equations stating vanishing of either real or imaginary part of the octonionic polynomial obtained as an algebraic continuation of P: both real and imaginary part in quaternionic sense vanish for these roots. The other solutions are 4-D regions of space-time surface.

In $M^8 = M^4 \times E^4$ the point of B^3 with distance $r_M \leq t$ from origin corresponds to the sphere of E^4 with radius r_M since point of 8-D light-cone boundary is in question.

The physical picture is that the space-time surfaces consist of 4-D roots of octonionic polynomials glued together at partonic 2-surfaces which appear as 2-D intersection of 6-spheres and 4-D surfaces. These partonic 2-surfaces would define a generalization of vertices appearing in topological diagrams having light-like partonic orbits as lines. Space-time surfaces need not intersect along 3-D B^3 although this cannot be excluded. String world sheets would have their light-like boundaries at these orbits and strings would thus connect partonic 2-surfaces.

In TGD inspired theory of consciousness these 6-spheres represent "very special moments in the life of self" [L8].

2.3.1 Modified Dirac action for 2-surfaces, partonic orbits and string world sheet boundaries involving self-pairing

How to identify the counterpart of Dirac action for string world sheets and their 1-D light-like boundaries and partonic 2-surfaces and their light-like orbits? Self-pairing in the sense that dimensions of paired objects are same, comes first in mind.

1. If the bi-local Dirac action couples two 2-surfaces the action must contain dimensional constants if one wants spinors to be induced from imbedding space spinors withe dimension $d = -7/2$. Since the modified Dirac action is determined by the 2-D part of corresponding bosonic action argued to emerge automatically at the 2-D singularities, one can argue that the emerging dimensional scaling factor guarantees that the action is dimensionless. For $d = 2$ dimensional objects the scaling factor would have length scale dimension 4 and scaling factor would be apart from numerical constant R^4, R CP_2 radius.

2. This works also for the pairing of two partonic orbits $d = 3$: now the scaling factor proportional to R^2. Chern-Simons action is rather plausible possibility. For $d = 1$ the scale parameter would be proportional R^6. In both cases these parts of actions would be related to singularities.

These action would have rather satisfactory features. In Appendix also the possibility that 2-D surfaces pair with special 6-D solution of polynomial equations in M^8 with topology of S^6 is discussed. This surface has 5-D sub-manifold with which partonic orbits could pair. It turns however that these pairings lead to non-physical predictions.

2.4 This is not enough!

In optimistic mood one might think that the reduction of fermion propagation at space-time level to the level of imbedding space could be enough. Unfortunately this is not the case: a nice advance in quantitative understanding has occurred but is not enough. One must consider two pictures for the propagation of physical (elementary) particles to be distinguished from fundamental fermions.

2.4.1 About the massivation of elementary particles in H-picture

Consider first the existing ideas about propagation of elementary particles - as opposed to fundamental fermions - when space-time surface is regarded as a surface in $H = M^4 \times CP_2$.

1. p-Adic mass calculations carried out for about 25 years ago were based on this picture. One has super-conformal invariance and super-Kac-Moody representations accompanied by Super-Virasoro representations. Ground states would be characterized in the case of fermion states by spinor harmonics of H [K2]. The masses of these harmonics can be calculated and the mass scale is determined by CP_2 radius $R \sim 10^{-4} l_{Planck}$ and is huge. There is electroweak symmetry breaking and only the right-handed neutrino is massless.

2. To get physical masses, being extremely small in CP_2 mass scale and depending only slightly on electroweak quantum numbers, one must have massless ground states as the first approximation. This requires tachyonic ground state with large tachyonic mass - this is highly analogous to Higgs mechanism. The conformal weight of ground state would be negative and equal to $h_{vac} = -5/2$ and would be dictated the tachyonic ground state mass. Physical particles would be Kac-Moody excitations of the tachyonic ground states and massless in the first approximation. The origin of the tachyonic ground state conformal weight has remained poorly understood in H-picture.

   The idea is that the thermal mixing of states with different mass squared - conformal weight - gives rise to the observed mass squared. p-Adic thermodynamics for mass squared- essentially Virasoro generator L_0 representing scaling - leads to excellent predictions for the masses of elementary particles. The only free parameters are integer valued. p-Adic temperature quantized by number theoretical conditions to $T = 1/n$ and having value $T = 1$. The p-adic prime p, which by p-adic length scale hypothesis is near power 2^k of 2 with Mersenne primes and ordinary primes associated with Gaussian Mersennes favored. The masses are exponentially sensitive to the value of k and family replication phenomenon explained in terms of genus of partonic 2-surface is also essential and predicts mass ratios of leptons correctly [K1].

3. One can consider the situation also classically [K7]. Classical particles are assumed to have wormhole throats of wormhole contacts as building bricks. Wormhole contact has Euclidian signature of metric but the model as CP_2 type extremal predicts light-like geodesic line as M^4 projection. The boundary of the wormhole contact is light-like 3-surface and analogous to massless particle. QCC encourages suggests that physical particles are massless in good approximation. This gives justification for the idea about negative ground state conformal weight.

   String world sheets are also present as singularities of minimal surfaces representing space-time surfaces and are themselves minimal surfaces [L11] Strings generate correlations between fundamental fermions at wormhole throats. This could give a justification for the p-adic thermodynamics for Super Virasoro representations.

   A possible physical picture is that the p-adic thermodynamics is associated with the strings connecting opposite wormhole throats. Should one assume that the fermion is de-localized along this short string having Euclidian signature inside the wormhole contact? Could the Euclidian signature explaining tachyonic ground state weight and tachyonic mass squared and could the mass of fermion give rise to the positive contributions allowing massless states?

2.4.2 Massivation and propagation in M^8 picture

What about massivation in M^8 picture?

1. In M^8 picture one has decomposition $M^8 \times M^4 \times CP_2$. This decomposition is highly non-unique and the Lorentz group $SO(1,7)$ gives new decomposition having interpretation as a particular octonionic sructure involving choice of real axis as time axis and preferred imaginary axis defining quantization axis for angular momentum and meaning choices of preferred M^2 crucial for $M^8 - H$ duality. Therefore the choice of M^4 is not unique [L1, L2, L3, L11]. The decomposition $H = M^4 \times CP_2$ however does not involve this non-uniqueness of M^4.

   $M^8 - H$ duality suggests that the space-time surfaces in H could somehow represent the non-uniqueness of the choice of $M^4 \subset M^8$. M^4 allows warped imbeddings to $M^4 \times CP_2$ with metric components related by scalings to the metric of canonically imbedded M^4. Warped imbeddings correspond to maps of M^4 to a geodesic circle of CP_2 given by $\Phi = k \cdot m$, k 4-D wave vector, which can be light-like or space-like. The metric is given by $g_{kl} = m_{kl} + R^2 k_k k_l$. Could these warped imbeddings correspond to the non-standard choices of M^4?

2. The mass squared in M^8 vanishes but M^4 mass squared is non-vanishing and equal to the E^4 mass squared. By a suitable choice of the rest frame in M^8 M^4 momentum equals to M^8 momentum and the mass squared vanishes [L9]. It seems that this choice gives mass squared which corresponds to the H-mass squared, which would vanish in excellent approximation.

3. The superposition of states with different M^8-momenta such that the choice of M^4 giving masslessness is different for these states forces massivation and p-adic thermodynamics would describe it. The natural condition is that the choice of M^4 is such that dominant contribution to the state is massless the other states give thermal contribution to the mass squared [L9].

2.4.3 Common features of H- and M^8 pictures

M^8 - and H-pictures share several features.

1. CD as product $cd \times CP_2$ is replaced with 8-D CD cd_8 identified as intersection of future and past directed light-cone. S^6 is preserved and can be regarded as $t = constant$ section of cd as in case of H.

2. Also now both Euclidian and Minkowskian regions are expected to be possible and CP_2 type extremal should correspond to 4-surface having 4-D E^4 projection and 1-D projection light-like geodesic as M^4 projection. Here one must however notice that M^4 projection is not light-like for a general choice of M^4!

3. Space-time surfaces in M^8 0are analogous to complex manifolds which are minimal surfaces: what is done is that complex numbers are replaced by complexified octonions so that one has analogs of polynomials of complex variable replaced with complexified octonions.

   This encourages the conjecture that these surfaces are minimal surfaces in E^4. Closed minimal surfaces are not in general possible because minimal surfaces have vanishing external curvatures. Rather, this surface should be analogous to a soap film spanned by frame. The frames would naturally correspond to the 3-D ends of this surface at the boundaries of CD_8. This would give one further motivation for ZEO. Also the gluing of regions of space-time surfaces together along partonic surfaces at 6-D branes S^6 could have interpretation in terms of frames.

   The counterpart for the orbit of wormhole contact connecting two Minkowskian space-time sheets as deformed CP_2 type extremal would be minimal surface with Euclidian signature glued to Minkowskian regions of space-time surface along 3-D light-like surfaces. Also the wormhole throats would play the role of soap film frames.

4. The bi-local picture about modified Dirac action in H is preserved. The situation however simplifies dramatically since now the electroweak splitting of the masses for the modes of modified Dirac operator is absent.

2.5 Connection with the Yangian symmetries

There is a connection also with the Yangian picture proposed on basic of twistorialization and symmetry arguments.

1. For Yangian algebra the generators are multi-local. This picture strongly suggests Yangian algebras since fermionic oscillator operators are superpositions of contributions from several 3-surface at given boundary of CD [K8]. Allowing operators, which are multi-local having fermions at several components of 3-surface such that quarks and antiquarks are at opposite boundaries of CD.

   One could say that quarks and leptons as local composites of 3- quarks are fundamental fermions if particle corresponds to partonic 2-surface. If it corresponds to 3-surface then also spartners are possible as composites of partonic 2-surfaces - in particular pairs of wormhole throats. These conditions conditions are consistent with the phenomenological picture that has guided the development of TGD based view about elementary particles. This would allow also to understand SUSY breaking.

2. Yangian picture could also generalize to the level of WCW an even super variant of WCW since 3-surface would be pair of 3-surfaces consisting of several components at opposite boundaries of CD. WCW gamma matrices and their conjugates would be associated with disjoint partonic 2-surface of CD and all anti-commutators and commutators would be well-defined and expressible in terms of anti-commutators of second quantized free spinor fields in H. This is just the original dream about generalization of induction of spinor structure to quantum realm that has waited its realization for four decades.

2.6 Connection with quantum classical correspondence

The notion of quantum classical correspondence (QCC) deserves some comments.

1. What QCC means is not completely clear. A rather stringent form of QCC would state that the classical expressions for the conserved quantities from the classical action defining space-time surfaces are equal to the eigenvalues of their fermionic counterparts in Cartan algebra of symmetries. This condition might make sense for fermionic conserved quantities identified as bi-linears of fermionic oscillator operators defined by the bi-linears formed from spinor fields at opposite boundaries of CD or more generally - at different partonic 2-surfaces. Everything would be finite and expressible in terms of anti-commutators at the level of imbedding space.

2. The charges in the complement of Cartan algebra should vanish classically corresponding to the vanishing of matrix elements of non-Cartan algebra generators for eigenstates: this would generalize the definition of the rest system as system in which 3-momentum vanishes to all conserved charges. This condition would be analogous to Einstein's equations.

3. What could be the interpretation of these condition? Could one interpret that as stating the condition that the sums of classical conserved quantities and fermionic conserved quantities vanish for eigenstates. One encounters conservation laws also for WCW spinor fields to which Noether theorem for isometries of WCW applies. This gives expressions for conserved quantities in terms of super-symplectic algebra of isometries and includes also the algebra of imbedding space isometries. These operators act on WCW points and WCW spinor fields.

4. Super-symmetrization of the bosonic action by introducing super-coordinates suggests a weaker form of QCC. Both bosonic action and modified Dirac action contribute to the total action and the quantal Noether charges in Cartan algebra can contain also c-number term. If this term is no-vanishing, one can speak about central extension analogous to that induced by addition of constant term to the Hamiltonians of Cartan algebra of symplectic algebra of isomehries of CP_2. The condition that central extension term vanishes would give the strong form of QCC.

3 Appendix: Could the pairing $d = 2, 3$ objects with $d = 6, 5$ objects make sense?

The emergence of 6-sphere S^6 as a brane like objects forces to ask whether a generalized pairing in which 2-surfaces would pair with S^6 so that the modified bi-local Dirac action would involve no dimensional constants. S^6 has also 5-D object as $\delta cd \times S^2$, where $S^2 \subset CP_2$ if homologically non-trivial geodesic sphere, and this could pair with the $d = 3$ light-like partonic orbits.

Bosonic action is needed to define the gamma matrices in the modified Dirac action and 4-D action cannot induce this action. Therefore this option might fail. The pairing idea turns out to have physically questionable implications. The considerations showing this however led to a possible identification of the counterpart of $S^6 \in M^8$ as an object in H.

A further objection is that the pairing of 1-D light-like boundaries of string world sheets is not possible since no 7-D object exists as special solution in M^8. String world sheet boundaries however play a key role as carriers of fermion quantum numbers.

Since self-pairings are well-defined and do not require introduction of new dimensional constants, it seems that they are the correct choice.

3.1 Pairings of 2-D and 3-D object

3.1.1 Pairing of 2-D objects with S^6

These pairings looked at first highly interesting since S^6 as analog of brane would gain further good reason for its existence. Again the induction idea and the condition that the action has no dimensionless parameters can be used as a constraint.

1. The pairing at the level of M^8 would easy to understand. One should assign to these 2-surfaces - string world sheets and partonic 2-surfaces - modified Dirac action as a bi-linear pairing given 2-surface with some other surface. One would use same formulas as for 4+4 pairing. If one does not want to introduce dimensional constants, the bi-linear action involving these 2-D surface pairing with 6-D object. It would be very natural to pair string world sheets and partonic 2-surfaces with brane-like 6-spheres in the modified Dirac action at the level of M^8.

2. What about the pairing at the level of $H = M^4 \times CP_2$? The challenge is to map the 6-sphere of M^8 to 6-D surfaces in H. What is clear that the projection to B^3 of S^6 to cd must be mapped to H as such. Also a group-theoretically natural proposal is that the radial B^3 coordinate r_M is mapped to a radial CP_2 coordinate r_{CP_2} labelling $U(2)$ 3-spheres of CP_2 and define as $r^2_{CP_2} = |\xi_1|^2 + |\xi_2|^2$ so that rotational symmetries of B^3 and CP_2 correspond to each other. Therefore one would have $r_{CP_2} = f(r_M)$ and one should be able to identify the function f.

   At the limit $r \rightarrow t$ one must have light-like 3-surfaces in H as orbit of partonic 2-surfaces. This requires that CP_2 projection becomes 2-D. This is achieved for $r_{CP_2} \rightarrow \infty$ at this limit. This formal 3-sphere is actually homologically charged geodesic 2-sphere of CP_2 at which two patches of CP_2 coordinate covering are glued together. At the level of space-time surfaces this gluing would correspond to the gluing of wormhole contact with Euclidian signature of induced metric and its complement with Minkowskian signature of metric.

3. The modified Dirac operator must annihilate the S^6 solution. The modified Dirac operator has 2 parts corresponding to metric part and Kähler parts from M^4 and from CP_2. Since the induced metric has Euclidian signature a good guess is that the solutions are covariantly constant.

   The condition $r_{CP_2} = r_M$ means that taking r_M as coordinate one has spherical coordinates for $S^2 \subset B^3$ and $S^3 \subset CP_2$ as coordinates plus radial coordinate which can be taken as radial coordinate r_{CP_2} of CP_2, call it R to minimize notational complexity. g_{RR} is sum of M^4 and CP_2 contributions: $g_{RR} = -(\partial_{r_M} f)^2 + s_{RR}$. The condition is that covariantly constant mode expected to have trivial dependence on the coordinates of $S^2 \times S^2$ exists. What is required is that the component A_R of the induced spinor connection vanishes for the mode. This poses a condition on the function $R = f(r_M)$ and might fix it.

4. The existence of covariantly constant mode suggests large isometry group. Maximal isometry group would be that of metric S^6 having also interpretation as octonionic 6-sphere. Covariantly constant modes are assigned with super-symmetries and the existence of this mode would give rise to the analog of supersymmetry. Thus the existence of this supersymmetry in generalized $M^8 - H$ correspondence would make possible to assign modified Dirac action to 2-surfaces.

   Local 3-quark composite with quantum numbers of right-handed neutrino could correspond to the covariantly constant mode at S^6 for the super variant of modified Dirac equation [L14]. What would happen that the coupling to both CP_2 and M^4 spinor connections would reduce to couplings to Kähler form and vanish for a proper choice of the couplings and appropriate choice of function f.

The question whether a counterpart of S^6 surface in H exists and what it is, is rather natural and the considerations indeed lead to its identification. The trivialization of the fermionic dynamics at S^6 however suggests that they are not needed.

3.1.2 The pairing for light-like 3-surfaces with 5-D objects

Consider next the pairing 3-D light-like partonic orbits with possibly existing 5-D objects.

The first question is whether the 3-D modes of Dirac equation needed at all The physical picture is that fermions at partonic orbits are at *boundaries* of string world sheets at light-like orbits of partonic 2-sheets and the modified Dirac action for string world sheets dictates the dynamics at these boundaries. One might do without spinors restricted to the partonic orbits but this is not obvious.

Are these 3-D spinor modes possible?

1. The 5-D object should be sub-manifold of S^6. At light-like 3-D surfaces the signature of the induced metric changes. This region would be naturally $X_5 = \delta cd \times S^2$ at which the CP_2 projection reduces to homologically charged geodesic sphere $s^2 \subset CP_2$.

2. What about the induced metric of X_5? The induced metric degenerates at δCD to 2-D metric effectively and this means that contravariant modified gamma matrices Γ^α can be ill-defined. The general manner to overcome the problem is to use modified gamma matrices associated with the action defined by Chern-Simons term associated with CP_2 Kähler form and M^4 Kähler form, which is required by twistor lift of TGD. The induced metric appearing in the 3-D permutation symbol and in volume element cancel each other and one obtain a finite result.

   Note also that the contribution from Kähler actions of M^4 and CP_2 to the induced metric are non-vanishing and could be enough: as a matter fact they could reduce to the Chern-Simons terms by boundary conditions.

3. There is however a delicate problem. The Chern-Simons form of CP_2 (M^4) is non-vanishing only if the CP_2 (M^4) projection is 3-D: this is not possible for the partonic 2-surface or string world sheet. Therefore the modified Dirac operator can act only at the paired 5-surface. CP_2 projection is however 2-D homologically non-trivial geodesic sphere and Chern-Simons term vanishes identically leaving only the modified Dirac operator defined by induced metric.

   The M^4 projection of the 5-surface is 3-D δcd and the corresponding Chern-Simons form is non-vanishing. The modified Dirac operator of S^2 couples to M^4 Kähler form. If the coupling is correct, one expects that covariantly constant modes are possible. A coupling of fermionic degrees of freedom at partonic 2-surfaces and boundary of CD would emerge.

4. For $S^2 \subset CP_2$ covariantly constant right-neutrino solutions are possible but not for quarks. Should one allow also leptons as fundamental fermions? The construction of WCW spinor structure does not require both leptons and quarks and the construction of quarks from leptons does not look plausible idea although I have considered also this option. Quarks seems to be the only elegant option.

   There is however a loophole. The local 3-quark composites have electroweak quantum numbers of leptons and they should couple to electroweak gauge fields like leptons so that covariantly constant right-handed neutrino like mode would be possible for super-variant of modified Dirac equation! A long-standing proposal has indeed been that right-handed neutrino and its antineutrino generate $\mathcal{N} = 2$ SUSY [K5]. This picture could realize this proposal also when quarks are the only fundamental fermions.

3.1.3 Summary about pairings

The fact that the modified Dirac action is bi-local means that the propagation of physical particles is more complex than propagation of free fermions in H. One must distinguish between different dimension $d \in \{4, 3, 2\}$ for the geometric objects carrying induced spinor fields.

1. For $d = 4$ there would be pairing of 3-D surfaces such as wormhole contacts. The notion of eigenstates would require de-localization of fermions: wave function would be superposition of contribution at throats. Situation would be 3-dimensional.

   Self-pairing with objects of same dimension seems the most plausible option.also for $d = 2, 3$ but one can consider also the pairing between different dimensions.

2. For quarks at $d = 2$ string world sheets and partonic 2-surfaces there would be only pairing with 6-sphere S^6. For string world sheets one obtains propagating states and one expects stringy mass spectrum for these states in the manner suggested by p-adic mass calculations. This is of course true also for self-pairing.

   Does the state at S^6 or $\delta CD \times S^2$ correspond to a local 3-quark composite representing covariantly constant right handed neutrino.

3. For $d = 3$ one would have 3-D propagation at light-like partonic 2-surface. The modes are now massless and also now the contribution of the state at $\delta cd \times S^2$ has vanishing momentum.

3.2 Could the analog of standard SUSY makes sense in TGD?

For a long time I tried to find whether right-handed neutrinos could give rise to the analog of standard SUSY [K9, K5]. I also proposed that many fermion states at partonic 2-surface give rise to larger but badly broken SUSY. Last summer (2019) I made a breakthrough in the understanding of SUSY in TGD frame.

The members of SUSY multiplets would be replaced with local composites of creation operators and only quarks would be needed to build also leptons. I gave up the idea about right-handed neutrino as generators of SUSY. The pairing of objects with different dimensions in modified Dirac action however forces to reconsider the situation. The conclusion are not however changed.

3.2.1 Breaking of analog of standard SUSY for 4-D fermions

Consider first the analog of standard SUSY and its breaking in TGD framework.

1. In the case of M^8 spinor harmonics there is no symmetry breaking as in the case of CP_2 and M^4 masses assignable to spinor harmonics do not depend on E^4 spin. This seems to be the case in excellent approximation also for the physical particles in H. If $M^8 - H$ duality holds true, the proposed mechanism explaining this would be a description for something which is true from the beginning in M^8 picture.

2. One would have the analog of $\mathcal{N} = 4$ SUSY corresponding to covariantly constant spinors of E^4 having 4 spin states corresponding to right- and left-handed isospin doublets. As a matter of fact, the $SO(4)$ symmetry could be interpreted as the the symmetry of hadron physics not so fashionable nowadays and $SU(3)$ as symmetry for quarks and $SO(4)$ as symmetry for hadrons would have interpretation in terms of $M^8 - H$ duality [L11, L9]. H-picture would correspond to an analog for the breaking of $\mathcal{N} = 4$ SUSY down to $\mathcal{N} = 2$ or even $\mathcal{N} = 1$: the latter option might be due to possibility of only second spin state for $B^3 \subset S^6$ and δCD.

3. What could the breaking of $\mathcal{N} = 4$ SUSY to $\mathcal{N} = 2$ SUSY correspond to? One can have canonically imbedded M^4 as a minimal surface extremal in $M^4 \times CP_2$ and for it the induced Dirac operator would reduce to massless Dirac operator in M^4 have all spinor modes as massless states and also covariantly constant modes would be allowed.

   For deformations of M^4 different M^4 chiralities mix and right- and left-hand chiralities mix. This is a signature for massivation and breaking of SUSY. Also inside Euclidian wormhole contacts representing elementary particles SUSY is broken and only right-handed neutrino is massless in analogy $\mathcal{N} = 2$ SUSY.

3.2.2 Is the analog of standard SUSY possible for lower dimensional fermions?

Pairing with brane like objects suggests strongly the analogs of covariantly constant right-handed neutrino spinors as local 3-quark composites. The earlier proposal was that covariantly constant right-handed neutrino generates analog of $\mathcal{N}=2$ SUSY. The basic objection against the idea is that for the generic space-time surface modified gamma matrices induces mixing of left- and right-handed neutrinos and leads to massivation and loss of these modes.

1. For S^6 and $\delta cd \times S^2$ there are however good reasons to expect that only these modes are possible. Could this make possible the standard SUSY?
2. Could the modes with vanishing M^4 momentum associated with pairings 2-D *resp.* 3-D fermions with 6-D *resp.* 5-D objects be described as analogs of Majorana-like degrees of freedom described as oscillator operators reducing to theta parameters? The non-conservation of fermion number would not be seen in the physics of the momentum carrying degrees of freedom if the transfer of neutrinos to right-handed neutrinos at these surfaces does not happen. SUSY in this sense would not be present for 4-D fermions at wormhole contacts. Could the breaking of $\mathcal{N}=1$ be due to the mixing of 2- or 3-D fermions with 4-D fermions?
3. If this picture is correct, standard SUSY would not have been found because it would have been searched in wrong place. One should be able to study fermionic states with $d=2,3$.

The first objection is that in standard picture the vertices for particle and sparticle couple differently since they have different spins. Vertices as partonic 2-surfaces would be sub-manifolds of S^6. Could the presence of right-handed neutrinos at S^6 affect the vertex and their spin could be seen in the vertex? This does not seem plausible.

Second objection is that the pairing between different dimensions leads also to severe problems with conserved charges for $d \in \{2,3\}$, the reason is that objects with dimension $d=6,5$ have space-like induced metric.

1. Since the bi-local Dirac operator D is hermitian, the notion of eigenstate should make sense for the Noether charges assignable to the bi-local Dirac operator also for $d \in \{2,3\}$. The action of conserved charge creates (annihilates) quark quark with $d=2$ *resp.* $d=3$ and annihilates (creates) ν_R at brane-like surface with $d=6$ *resp.* $d=5$. Suppose that one has a state which is of form

   $$|\Psi\rangle = |q\rangle|vac\rangle + \epsilon|vac\rangle|\nu_R\rangle \ .$$

   This state is superposition of states with quark number one and anti-lepton number 1, which corresponds to quark number 3.
2. Consider a hermitian conserved charge associated with D. The action on $|\Psi\rangle$ is given by

   $$O|\Psi\rangle = o_1|vac\rangle|\nu_R\rangle + \overline{o}_1\epsilon|\rangle|\rangle = o\Psi$$

   .

The condition gives $o=\epsilon\overline{o}_1$ and $o\epsilon = o_1$ solved by $\epsilon = o_1/\overline{o}_1$ and $o=|o_1|$.

The ill-definedness of quark number is analogous to the fact that for Majorana spinors fermion number is conserved only modulo 2. Also the electroweak and color and Poincare quantum numbers are ill-defined in the general case. Only if $d=2,3$ surface contains right-handed neutrino standard model quantum numbers are well-defined. This would reduce the number of allowed states dramatically. This kind of formal eigenstates look unphysical. It seems that self-pairing is the only reasonable option.

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