More about the construction of scattering amplitudes in TGD framework

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Abstract

During years I have considered several proposals for S-matrix in TGD framework - perhaps the most realistic proposal relies on the generalization of twistor Grassmann approach to TGD context. There are several questions waiting for an answer. How to achieve unitarity? What it is to be a particle in classical sense? Can one identify TGD analogs of quantum fields? Could scattering amplitudes have interpretation as Fourier transforms of n-point functions for the analogs of quantum fields?

Unitarity is certainly the issue #1 and in the sequel almost trivial solution to the unitarity problem based on the existence of super-symplectic transformations acting as isometries of "world of classical worlds" implying infinite number of conserved Noether charges in turn guaranteeing unitarity.

Also quantum classical correspondence and the role of string world sheets for strong form of holography (SH) are discussed. What is found that number theoretic view justifies the assignment of 2-D action to string world sheets. Does the 2-D action appear as a primary term in the action or emerge dynamically as SH encourages to think? The latter option turns out to be the correct one. String world sheets can be regarded as 2-D edges/folds of 4-D space-time. 2-D edges would be minimal surfaces and carry a singular part of 4-D action. Also partonic 2-surfaces would form analog of foam with network defined by nodes connected by edges (geodesic lines), as the proposal for symplectic QFT as part of quantum TGD indeed suggests. This picture has interpretation as a (generalized) calibration that I suggested for long time ago.

The notion of discrete coupling constant evolution, quantum criticality, and number theoretical arguments lead in twistor Grassmann approach to an extremely simple proposal: the loop corrections to scattering amplitudes must vanish so that the twistorial recursion formulas for the scattering amplitudes trivialize. This requires that unitarity cuts are replaced with poles of finite width: scattering amplitude would reduce to a sum over resonance poles just as was assumed in the dual resonance models based on stringy picture, which later led to super string models. Superstring models failed: could it be that dual resonance models were much close to reality than superstrings and M-theory? TGD suggests that this might be the case.

Contents

1 Introduction

The construction of scattering amplitudes in TGD framework has been a longstanding problem, and I have considered several proposals - perhaps the most realistic proposal relies on the generalization of twistor Grassmann approach to TGD context [L9]. These approaches have however suffered from their ad hoc character.

One reason for the slow progress might be the fact that I have not conditioned Feynman diagrams into my spine: I have intentionally avoided this in the fear that it would prevent genuine thinking. Second reason is that TGD is really different and my mathematical skills are rather limited. For instance, in TGD classical theory is an exact part of quantum theory and particles are replaced with 3-surfaces: there is no hope of starting from Lagrangian with simple non-linearities and writing Feynman rules and deducing beta functions.

There are several questions waiting for an answer. How to achieve unitarity? What it is to be a particle in classical sense? Can one identify TGD analogs of quantum fields? Could scattering amplitudes have interpretation as Fourier transforms of n-point functions for the analogs of quantum fields?

Unitarity is certainly the issue #1 and in the sequel almost trivial solution to unitarity problem is proposed. Also quantum classical correspondence is discussed.

2 Some background

2.1 Supersymplectic algebra

Let us collect what I think is known in TGD framework.

1. The "world of classical worlds" (WCW) [K9] geometry does not exist without maximal group of isometries and WCW is assumed to possess super-symplectic algebra (SSA) assignable to light-cone boundary (boundaries of causal diamonds (CDs)) as isometries. Also Kac-Moody algebras for isometries of imbedding space realized at the light-like partonic orbits serving as boundaries between Euclidian and Minkowskian regions of space-time surface are expected to be of key importance (for p-adic mass calculations applying these symmetries see [K4].

   SSA has a fractal hierarchy of isomorphic sub-algebras and the proposal is that one has hierarchy of criticalities such that sub-SSA and its commutator with SSA annihilate the physical states so that SSA effectively reduces to a finite-D Lie-algebra generating the physical states. Sub-SSA takes the role of gauge algebra and one could say that it represents finite measurement resolution. This hierarchy would correspond to a hierarchies of inclusions of von Neumann algebras known as hyper-finite factors of type II_1 [K7, K3].

   It seems obvious to me that the scattering amplitudes should allow a formulation in terms of SSA effectively reducing to finite-D Lie-algebra of corresponding Kac-Moody algebra plus Kac-Moody algebras associated with imbedding space isometries.

   Remark: Conformal weights of SSA associated with the radial light-like coordinate are non-negative so that one has analogy with Yangian algebra. The TGD variant of twistor

Grassmann approach [K11] [L9] strongly suggests that SSA extends to Yangian having multi-local generators with locus corresponding to partonic 2-surface.

2. There are both classical and fermionic Noether charges associated with SSA and the Kac-Moody algebras [K2, K8, K9]. Quantum-classical correspondence (QCC) suggests that the eigenvalues for Cartan algebra Noether charges in the fermionic representation correspond to bosonic charges assignable to the dimensionally reduced Kähler action. One obtains also fermionic super-charges in 1-1 correspondence with the modes of the induced spinor field. Super-charges are very much like oscillator operators creating or annihilating fermions and there is a temptation to think that these fermionic SSA and Kac-Moody charges take the role of operators creating fermionic and bosonic states.

   One could think of constructing many-particle states at both boundaries of causal diamond (CD) by decomposing SSA to Cartan algebra and to parts acting like creation and annihilation operators. States would be created by the generators acting like oscillator operators.

   The time evolution dictated by preferred extremals and corresponding modified Dirac equation would transform initial states at boundary A of CD to final states at boundary B. This time evolution is determined by preferred extremal property and by modified Dirac equation [K8]. Time evolution is not obtained by exponentiating quantum Hamiltonian as in QFT approach. The existence of infinite-D SSA of Noether changes should make it possible to prove unitarity.

2.2 General argument for unitarity

The argument for unitarity is very general and based on zero energy ontology (ZEO). Causal diamond (CD) containing space-time surfaces having ends at its opposite boundaries is central for ZEO. Zero energy states are quantum superpositions of space-time surfaces, which are preferred extremals of dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term. CD has two boundaries: the active boundary (B) and passive boundary (A) and space-time surfaces as preferred extremals have ends at these boundaries [L5].

In ZEO one has two kinds of state function reductions.

1. At the active boundary (B) one has "small" state function reductions as counterparts of weak measurements following unitary time evolutions shifting the active boundary B farther from passive boundary A in statistical sense. During each unitary time evolution there is a de-localization with respect to the distance between the tips of CD followed by localization serving also as time measurement. This would yield the correlation between experienced time as sequence of these weak measurements and geometric time identified as distance between the tips of CD.

   Also measurements of observables commuting with the observables, whose eigenstates the states at boundary A are, are possible. Passive boundary (A) and the members of zero energy states associated with it do not change, and this gives rise to what one might call generalized Zeno effect.

   S-matrix would correspond to the evolution between two weak measurements for the states at the active boundary of CD and expected to be unitary. At passive boundary of CD and states at it would not be affected. The time evolution in the fermionic sector would be induced by the modified Dirac equation. Now one can express the states at new active boundary in terms of those at old active boundary and one would obtain unitary S-matrix by expressing the final states in terms of the state basic for the original boundary.

2. In "big" state function reduction the roles of passive boundary A and active boundary B are changed. The states at B are superpositions of states in the state basis for SSA. Unitary S-matrix would be obtained by expressing these states in terms of SSA basis.

Unitarity does not seem to be a problem since the conservation of Cartan charges for SSA in the fermionic representation would not allow breaking of unitarity. The time evolution would be induced by the preferred extremal property and modified Dirac equation.

Scattering amplitudes would involve an integration over positions of particles meaning that instead of single 4-surface one would have large number of them contributing to single scattering amplitude. Different position would correspond to different values of zero modes not contributing to WCW metric. Number theoretical vision [L3, L4] demands that the exponent of action is same for all of these surfaces: with inspiration coming from the idea about quantum TGD as square root of thermodynamics, I have indeed proposed [L7] this quantum analog of micro-canonical ensemble (for which energy is constant) as a manner to get rid of difficulties in the realization of number theoretical universality. The number theoretically cumbersome action exponents would cancel out from the scattering amplitudes.

3 Does 4-D action generate lower-dimensional terms dynamically?

The original proposal was that the action defining the preferred extremals is 4-D Kähler action. Later it became obvious that there must be also 2-D string world sheet term present and probably also 1-D term associated with string boundaries at partonic 2-surfaces. The question has been whether these lower-D terms in the action are primary of generated dynamically. By super-conformal symmetry the same question applies to the fermionic part of the action. The recent formulation based on the twistor lift of TGD contains also volume term but the question remains the same.

Quantum criticality would be realized as a minimal surface property realized by holomorphy in suitably generalized sense [L10, L8]. The reason is that the holomorphic solutions of minimal surface equations involve no coupling parameters as the universality of the dynamics at quantum criticality demands.

Minimal surface equation would be true apart from possible singular surfaces having dimension $D = 2, 1, 0$. $D = 2$ corresponds to string world sheets and partonic 2-surfaces. If there are 0-D singularities they would be associated with the ends of orbits of partonic 2-surfaces at boundaries of causal diamond (CD). Minimal surfaces are solutions of non-linear variant of massless d'Alembertian having as effective sources the singular surfaces at which d'Alembertian equation fails. The analogy with gauge theories is highly suggestive: singular surfaces would act as sources of massless field.

Strings world sheets seem to be necessary. The basic question is whether the singular surfaces are postulated from the beginning and there is action associated with them or whether they emerge dynamical from 4-D action. One can consider two extreme options.

Option I: There is an explicit assignment of action to the singular surfaces from the beginning. A transfer of Noether charges between space-time interior and string world sheets is possible. This kind of transfer process can take place also between string world sheets and their light-like boundaries and happens if the normal derivatives of imbedding space coordinates are discontinuous at the singular surface.

Option II: No separate action is assigned with the singular surfaces. There could be a transfer of Noether charges between 4-D Kähler and volume degrees of freedom at the singular surfaces causing the failure of minimal surface property in 4-D sense. But could singular surfaces carry Noether currents as 2-D delta function like densities?

This is possible if the discontinuity of the normal derivatives generates a 2-D singular term to the action. Conservation laws require that at string world sheets energy momentum tensor should degenerate to a 2-D tensor parallel to and concentrated at string world sheet. Only 4-D action would be needed - this was actually the original proposal. Strings and particles would be essentially edges of space-time - this is not possible in GRT. Same could happen also at its boundaries giving rise to point like particles. Super-conformal symmetry would make this possible also in the fermionic sector.

For both options the singular surfaces would provide a concrete topological picture about the scattering process at the level of single space-time surface and telling what happens to the initial state. The question is whether Option I actually reduces to Option II. If the 2-D term is generated to 4-D action dynamically, there is no need to postulate primary 2-D action.

3.1 Can Option II generate separate 2-D action dynamically?

The following argument shows that Option II with 4-D primary action can generate dynamically 2-D term into the action so that no primary action need to be assigned with string world sheets.

3.1.1 Dimensional hierarchy of surfaces and strong form of holography

String world sheets having light-like boundaries at the light-like orbits of partonic 2-surfaces are certainly needed to realize strong form of holography [K8]. Partonic 2-surfaces emerge automatically as the ends of the orbits of wormhole contacts.

1. There could (but neet not) be a separate terms in the primary action corresponding to string world sheets and their boundaries. This hierarchy bringing in mind branes would correspond to the hierarchy of classical number fields formed by reals, complex numbers, quaternions (space-time surface), and octonions (imbedding space in M^8-side of M^8 duality). The tangent - or normal spaces of these surfaces would inherit real, complex, and quaternionic structures as induced structure. The number theoretic interpretation would allow to see these surfaces as images of those surfaces in M^8 mapped to H by $M^8 - H$ duality. Therefore it would be natural to assign action to these surfaces.

2. This makes in principle possible the transfer of classical and quantum charges between space-time interior and string world sheets and between from string world sheets to their light-like boundaries. TGD variant of twistor Grassmannian approach [K11, L9] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers. Quantum criticality realized in terms of minimal surface property realized holomorphically is central for TGD and one can ask whether it could play a role in the definition of S-matrix and identification of particles as geometric objects.

3. For preferred extremals string world sheets (partonic 2-surfaces) would be complex (co-complex) manifolds in octonionic sense. Minimal surface equations would hold true outside string world sheets. Conservation of various charges would require that the divergences of canonical momentum currents at string world sheet would be equal to the discontinuities of the normal components of the canonical momentum currents in interior. These discontinuities would correspond to discontinuities of normal derivatives of imbedding space coordinates and are acceptable. Similar conditions would hold true at the light-like boundaries of string world sheets at light-like boundaries of parton orbits. String world sheets would not be minimal surfaces and minimal surface property for space-time surface would fail at these surfaces.

   Quantum criticality for string world sheets would also correspond to minimal surface property. If this is realized in terms of holomorphy, the field equations for Kähler and volume parts at string world sheets would be satisfied separately and the discontinuities of normal components for the canonical momentum currents in the interior would vanish at string world sheets.

4. The idea about asymptotic states as free particles would suggest that normal components of canonical momentum currents are continuous near the boundaries of CD as boundary conditions at least. The same must be true at the light-like boundaries of string world sheets. Minimal surface property would reduce to the property of being light-like geodesics at light-like partonic 2-surface. If this is not assumed, the orbit is space-like. Even if these conditions are realized, one can imagine the possibility that at string world sheets 4-D minimal surface equation fails and there is transfer of charges between Kähler and volume degrees of freedom (Option II) and therefore breaking of quantum criticality.

   If the exchange of Noether charges vanishes everywhere at string world sheets and boundaries, one could argue that they represent independent degrees of freedom and that TGD reduces to string model. The proposed equation for coupling constant evolution however contains a coefficients depending on the total action so that this would not be the case.

5. Assigning action to the lower-D objects requires additional coupling parameters. One should be able to express these parameters in terms of the parameters appearing in 4-D action (α_K and cosmological constant). For string sheets the action containing cosmological term is 4-D

and Kähler action for $X^2 \times S^2$, where S^2 is non-dynamical twistor sphere is a good guess. Kähler action gets contributions from X^2 and S^2. If the 2-D action is generated dynamically as a singular term of 4-D action its coupling parameters are those of 4-D action.

6. There is a temptation to interpret this picture as a realization of strong form of holography (SH) in the sense that one can deduce the space-time surfaces by using data at string world sheets and partonic 2-surfaces and their light-like orbits. The vanishing of normal components of canonical momentum currents would fix the boundary conditions.

   If double holography $D = 4 \rightarrow D = 2 \rightarrow D = 1$ were satisfied it might be even possible to reduce the construction of S-matrix to the proposed variant of twistor Grassmann approach. This need not be the case: p-adic mass calculations rely on p-adic thermodynamics for the excitions of massless particles having CP_2 mass scale and it would seem that the double holography can makes sense for massless states only.

   In M^8-picture [L2] the information about space-time surface is coded by a polynomial defined at real line having coefficients in an extension of rationals. This real line for octonions corresponds to the time axis in the rest system rather than light-like orbit as light-like boundary of string world sheet.

3.1.2 Stringy quantum criticality?

The original intuition [L10] was that there are canonical momentum currents between Kähler and volume degrees of freedom at singular surfaces but no transfer of canonical momenta between interior and string world sheets nor string world sheets and their boundaries. Also string world sheets would be minimal surfaces as also the intuition from string models suggests. Could also the stringy quantum criticality be realized?

1. Some imbedding space coordinates h^k must have discontinuous partial derivatives in directions normal to the string world sheet so that 3-surface has 1-D edge along fermionic string connecting light-like curves at partonic 2-surfaces in both Minkowskian and Euclidian regions. A closed highly flattened rectangle with long and short edges would be associated with closed monopole flux tube in the case of wormhole contact pairs assigned with elementary particles. 3-surfaces would be "edgy" entities and space-time surfaces would have 2-D and 1-D edges. In condensed matter physics these edges would be regarded as defects.

2. Quantum criticality demands that the dynamics of string world sheets and of interior effectively decouple. Same must take place for the dynamics of string world sheets and their boundaries. Decoupling allows also string world sheets to be minimal surfaces as analogs of complex surfaces whereas string world sheet boundaries would be light-like (their deformations are always space-like) so that one obtains both particles and string like objects.

3. By field equations the sums for the divergences of stringy canonical momentum currents and the corresponding singular parts of these currents in the interior must vanish. By quantum criticality in interior the divergencespf Kähler and volume terms vanish separately. Same must happen for the sums in case of string world sheets and their boundaries. The discontinuity of normal derivatives implies that the contribution from the normal directions to the divergence reduces to the sum of discontinuities in two normal directions multiplied by 2-D delta function. Thid contribution is in the general case equal to the divergence of corresponding stringy canonical momentum current but must vanish if one has quantum criticality also at string world sheets and their boundaries.

   The separate continuity of Kähler and volume parts of canonical momentum currents would guarantee this but very probably implies the continuity of the induced metric and Kähler form and therefore of normal derivatives so that there would be no singularity. However, the condition that total canonical momentum currents are continuous makes sense, and indeed implies a transfer of various conserved charges between Kähler action and volume degrees of freedom at string world sheets and their boundaries in normal directions as was conjectured in [L10].

4. What about the situation in fermionic degrees of freedom? The action for string world sheet X^2 would be essentially of Kähler action for $X^2 \times S^2$, where S^2 is twistor sphere. Since the modified gamma matrices appearing in the modified Dirac equation are determined in terms of canonical momentum densities assignable to the modified Dirac action, there could be similar transfer of charges involved with the fermionic sector and the divergences of Noether charges and super-charges assignable to the volume action are non-vanishing at the singular surfaces. The above mechanism would force decoupling between interior spinors and string world sheets spinors also for the modified Dirac equation since modified gamma matrices are determined by the bosonic action.

   Remark: There is a delicacy involved with the definition of modified gamma matrices, which for volume term are proportional to the induced gamma matrices (projections of the imbedding space gamma matrices to space-time surface). Modified gamma matrices are proportional to the contractions $T^{\alpha}_{k}\Gamma^k$ of canonical momentum densities $T^{\alpha_k} = \partial L/\partial(\partial_\alpha h^k)$ with imbedding space gamma matrices Γ^k. To get dimension correctly in the case of volume action one must divide away the factor $\Lambda/8\pi G$. Therefore fermionic super-symplectic currents do not involve this factor as required.

It remains an open question whether the string quantum criticality is realized everywhere or only near the ends of string world sheets near boundaries of CD.

3.1.3 String world sheet singularities as infinitely sharp edges and dynamical generation of string world sheet action

The condition that the singularities are 2-D string world sheets forces 1-D edges of 3-surfaces to be infinitely sharp.

Consider an edge at 3-surface. The divergence from the discontinuity contains contributions from two normal coordinates proportional to a delta function for the normal coordinate and coming from the discontinuity. The discontinuity must be however localized to the string rather than 2-surface. There must be present also a delta function for the second normal coordinate. Hence the value of also discontinuity must be infinite. One would have infinitely sharp edge. A concrete example is provided by function $y = |x|^\alpha$ $\alpha < 1$. This kind of situation is encountered in Thom's catastrope theory for the projection of the catastrophe: in this case one has $\alpha = 1/2$. This argument generalizes to 3-D case but visualization is possible only as a motion of infinitely sharp edge of 3-surface.

Kähler form and metric are second degree monomials of partial derivatives so that an attractive assumption is that $g_{\alpha\beta}$, $J_{\alpha\beta}$ and therefore also the components of volume and Kähler energy momentum tensor are continuous. This would allow $\partial_{n_i} h^k$ to become infinite and change sign at the discontinuity as the idea about infinitely sharp edge suggests. This would reduce the continuity conditions for canonical momentum currents to rather simple form

$$T^{n_i n_j}\Delta\partial_{n_j} h^k = 0 \quad . \tag{3.1}$$

which in turn would give

$$T^{n_i n_j} = 0 \tag{3.2}$$

stating that canonical momentum is conserved but transferred between Kähler and volume degrees of freedom. One would have a condition for a continuous quantity conforming with the intuitive view about boundary conditions due to conservation laws. The condition would state that energy momentum tensor reduces to that for string world sheet at the singularity so that the system becomes effectively 2-D. I have already earlier proposed this condition.

The reduction of 4-D locally to effectively 2-D system raises the question whether any separate action is needed for string world sheets (and their boundaries)? The generated 2-D action would be similar to the proposed 2-D action. By super-conformal symmetry similar generation of 2-D action would take place also in the fermionic degrees of freedom. I have proposed also this option already earlier. This would mean that Option II is enough.

The following gives a more explicit analysis of the singularities. The vanishing on the discontinuity for the sum of normal derivative gives terms with varying degree of divergence. Denote by n_i resp. t_i the coordinate indices in the normal resp. tangent space. Suppose that some derivative $\partial_{n_i} h^k$ become infinite at string. One can introduce degree n_D of divergence for a quantity appearing as part of canonical momentum current as the degree of the highest monomial consisting of the diverging derivatives $\partial_{n_i} h^k$ appearing in quantity in question. For the leading term in continuity conditions for canonical momentum currents of total action one should have $n_D = 2$ to give the required 2-D delta function singularity.

- $\partial_{n_i} h^k$ has $n_D \leq 1$. If it is also discontinuous - say changes sign - one has $n_D = 2$ for $\Delta\partial_{n_i} h^k$ in direction n_i.
- One has $n_D(g_{t_i t_j}) = 0$, $n_D(g_{t_i n_j}) = 1$, $n_D(g_{n_i n_i}) = 2$ and $n_D(g_{n_i n_j}) = 1$ or 2 for $i \neq j$. One has $n_D(g) = 4$ $(g = det(g_{\alpha\beta}))$. For contravariant metric one gas $n_D(g^{t_i t_j}) = 0$ and $n_D(g^{n_i j}) = n_D(g^{n_i n_j}) = -2$ as is easy to see from the formula for $g^{\alpha\beta}$ in terms of cofactors.
- Both Kähler and volume terms in canonical momentum current are proportional to $\sqrt{g}$ with $n_D(\sqrt{g}) = 2$ having leading term proportional to 2-determinant $\sqrt{det(g_{n_i n_j})}$. In Kähler action the leading term comes from tangent space part J_{ij} and has $n_D = -1$ coming from the partial derivative. The remaining parts involving $J_{t_i n_j}$ or $J_{n_i n_j}$ have $n_D < 0$.
- Consider the behavior of the contribution of volume term to the canonical momentum currents. For $g^{n_i t_j}\partial_{t_j} h^k \sqrt{g}$ one has $n_D = 0$ so that this term is finite. For $g^{n_i n_j}\partial_{n_j} h^k \sqrt{g}$ one has $n_D \leq 1$ and this term can be infinite as also its discontinuity coming solely from the change of sign for $\partial_{n_j} h^k$. If $\partial_{n_j} h^k$ is infinite and changes sign, one can have $n_D = 2$ as required by 2-D delta function singularity.

  The continuity condition for the canonical momentum current would state the vanishing of $n_D = 2$ discontinuity but would not imply separate vanishing of discontinuity for Kähler and volume parts of canonical momentum currents - this in accordance with the idea about canonical momentum transfer. If the sign of partial derivative only changes the coefficient of the partial derivative must vanish so that the condition reduces to the condition $T^{n_i n_j} = 0$ already given for the components of the total energy momentum tensor, which would be continuous by the above assumption.

3.1.4 A connection with Higgs vacuum expectation?

What about the physical interpretation of the singular divergences of the isometry currents J_A of the volume action located at string world sheet?

1. The divergences of J_A are proportional to the trace of the second fundamental form H formed by the covariant derivatives of gradients $\partial_\alpha h^k$ of H-coordinates in the interior and vanish. The singular contribution at string world sheets is determined by the discontinuity of the isometry current J_A and involves only the first derivatives $\partial_\alpha h^k$.
2. One of the first questions after ending up with TGD for 41 years ago was whether the trace of H in the case of CP_2 coordinates could serve as something analogous to Higgs vacuum expectation value. The length squared for the trace has dimensions of mass squared. The discontinuity of the isometry currents for $SU(3)$ parts in $h = u(2)$ and its complement t, whose complex coordinates define $u(2)$ doublet. $u(2)$ is in correspondence with electroweak algebra and t with complex Higgs doublet. Could an interpretation as Higgs or even its vacuum expectation make sense?
3. p-Adic thermodynamics explains fermion masses elegantly (understanding of boson masses is not in so good shape) in terms of thermal mixing with excitations having CP_2 mass scale and assignable to short string associated with wormhole contacts. There is also a contribution from long strings connecting wormhole contacts and this could be important for the understanding of weak gauge boson masses. Could the discontinuity of isometry currents in t determine this contribution to mass. Edges/folds would carry mass.

4. The non-singular part of the divergence multiplying 2-D delta function has dimension 1/length squared and the square of this vector in CP_2 metric has dimension of mass squared. Could the interpretation of the discontinuity as Higgs expectation make sense? If so, Higgs expectation would vanish in the space-time interior.

   Could the interior modes of the induced spinor field - or at least the interior mode of right-handed neutrino ν_R having no couplings to weak or color fields - be massless in 8-D or even 4-D sense? Could ν_R and $\overline{\nu}_R$ generate an unbroken $\mathcal{N}=2$ SUSY in interior whereas inside string world sheets right-handed neutrino and antineutrino would be eaten in neutrino massivation and the generators of $\mathcal{N}=2$ SUSY would be lost somewhat like charged components of Higgs!

   If so, particle physicists would be trying to find SUSY from wrong place. Space-time interior would be the correct place. Would the search of SUSY be condensed matter physics rather than particle physics?

3.2 Summarizing the recent view about elementary particles

It is interesting to see how elementary particles and their basic interaction vertices could be realized in this framework.

1. In TGD framework particle would correspond to pair of wormhole contact associated with closed magnetic flux tube carrying monopole flux. Strongly flattened rectangle with Minkowskian flux tubes as long edges with length given by weak scale and Euclidian wormhole contacts as short edges with CP_2 radius as lengths scale is a good visualization. 3-particle vertex corresponding to the replication of this kind of flux tube rectangle to two rectangles would replace 3-vertex of Feynman graph. There is analogy with DNA replication. Similar replication is expected to be possible also for the associated closed fermionic strings.

2. Denote the wormhole contacts by A and B and their opposite throats by A_i and B_i, $i=1,2$. For fermions A_1 can be assumed to carry the electroweak quantum numbers of fermion. For electroweak bosons A_1 and A_2 (for instance) could carry fermion and anti-fermion, whose quantum numbers sum up to those of ew gauge boson. These "corner fermions" can be called *active*.

   Also other distributions of quantum numbers must be considered. Fermion and anti-fermion could in principle reside at the same throat - say A_1. One can however assume that second wormhole contact, say A has quantum numbers of fermion or weak boson (or gluon) and second contact carries quantum numbers screening weak isospin.

3. The model assumes that the weak isospin is neutralized in length scales longer than the size of the flux tube structure given by electro-weak scale. The screening fermions can be called *passive*. If the weak isospin of $W^{\pm}$ boson is neutralized in the scale of flux tube, 2 $\nu_L\overline{\nu}_R$ pairs are needed (lepton number for these pairs must vanish) for W^-. For Z $\nu_L\overline{\nu}_R$ and $\overline{\nu}_L nu_R$ are needed. The pairs of passive fermions could reside in the interior of flux tube, at string world sheet or at its corners just like active fermions. The first extreme is that the neutralizing neutrino-antineutrino pairs reside in interior at the opposite long edges of the rectangular *flux tube*. Second extreme is that they are at the corners of rectangular *closed string*.

4. Rectangular closed string containing active fermion at wormhole A (say) and with members of isospin neutralizing neutrino-antineutrino pair at the throats of B serves as basic units. In scales shorter than string length the end A would behave like fermion with weak isospin. At longer scales physical fermion would be hadron like entity with vanishing isospin and one could speak of confinement of weak isospin.

   From these physical fermions one can build gauge bosons as bound states. Weak bosons and also gluons would be pairs of this kind of fermionic closed strings connecting wormhole contacts A and B. Gauge bosons (and also gravitons) could be seen as composites of string like physical fermions with vanishing net isospin rather than those of point like fundamental fermions.

5. The decay of weak boson to fermion-antifermion pair would be flux tube replication in which closed strings representing physical fermion and anti-fermion continue along different copies of flux tube structure. The decay of boson to two bosons - say $W \to WZ$ - by replication of flux tube would require creation of a pair of physical fermionic closed strings representing Z. This would correspond to a V-shaped vertex with the edge of V representing closed fermionic closed string turning backwards in time. In decays like $Z \to W^+W^-$ two closed fermion strings would be created in the replication of flux tube. Rectangular fermionic string would turns backwards in time in the replication vertex and the rectangular strings of Z would be shared between W^+ and W^-.

This mesonlike picture about weak bosons as bound states of fermions sounds complex as compared with standard model picture. On the other hand only the spinor fields assignable to single fermion family are present.

A couple of comments concerning this picture are in order.

1. M^8 duality provides a different perspective. In M^8 picture these vertices could correspond to analogs of local 3 particle vertices for octonionic superfield, which become nonlocal in the map taking $M^8 = M^4 \times CP_2$ surfaces to surfaces in $H = M^4 \times CP_2$. The reason is that M^4 point is mapped to M^4 point but the tangent space at E^4 point is mapped to a point of CP_2. If the point in M^8 corresponds to a self-intersection point the tangent space at the point is not unique and point is mapped to two distinct points. There local vertex in M^8 would correspond to non-local vertex in H and fermion lines could just begin. This would mean that at H-level fermion line at moment of replication and V-shaped fermion line pair beginning at different point of throat could correspond to 3-vertex at M^8 level.

2. The 3-vertex representing replication could have interpretation in terms of quantum criticality: in reversed direction of time two branches of solution of classical field equations would co-incide.

3.3 Gravitation as a square of gauge interaction

I encountered in FB a link to an interesting popular article (see `http://tinyurl.com/y5r4glgg`) about theoretical physicist Henrik Johansson who has worked with supergravity in Wallenberg Academy. He has found strong mathematical evidence for a new duality. Various variants of super quantum gravity support the view that supersymmetric quantum theories of gravitation can be seen as a double copy of a gauge theory. One could say that spin 2 gravitons are gluons with color charge replaced with spin. Since the information about charges disappears, gluons can be understood very generally as gauge bosons for given gauge theory, not necessarily QCD.

The article of C. D. White [B1] (see `https://arxiv.org/pdf/1708.07056.pdf`) entitled "The double copy: gravity from gluons" explains in more detail the double copy duality and also shows that it relates in many cases also exact classical solutions of Einsteins equations and YM theories. One starts from L-loop scattering amplitude involving products of kinematical factors n_i and color factors c_i and replaces color factors with extra kinematical factors $\tilde{n}_i$. The outcome is an L-loop amplitude for gravitons.

As if gravitation could be regarded as a gauge theory with polarization and/or momenta identified giving rise to effective color charges. This is like taking gauge potential and giving it additional index to get metric tensor. This naive analogy seems to hold true at the level of scattering amplitudes and also for many classical solutions of field equations. Could one think that gravitons as states correspond to gauge singlets formed from two gluons and having spin 2? Also spin 1 and spin 0 states would be obtained and double copies involve also them.

TGD view about elementary particles indeed predicts that gravitons be regarded in certain sense pairs of gauge bosons. Consider now gravitons and assume for simplicity that spartners of fundamental fermions - identifiable as local multi-fermion states allowed by statistics - are not involved: this does not change the situation much [L11]. Graviton's spin 2 requires 2 fermions and 2 anti-fermions: fermion or anti-fermion at each throat. For gauge bosons fermion and anti-fermion at two throats is enough. One could therefore formally see gravitons as pairs of two gauge bosons in accordance with the idea about graviton is a square of gauge boson.

The fermion contents of the monopole flux tube associated with elementary particle determines quantum numbers of the flux tube as particle and characterizes corresponding interaction. The interaction depends also on the charges at the ends of the flux tube. This leads to a possible interpretation for the formation of bound states in terms of flux tubes carrying quantum numbers of particles.

1. These long flux tubes can be arbitrarily long for large values of $\hbar_{eff} = n \times \hbar_0$ assigned to the flux tube. A plausible guess for for the expression of $\hbar$ in terms of $\hbar_0$ is as $\hbar = 6 \times \hbar_0$ [L1, L6]. The length of the flux tube scales like $\hbar_{eff}$.

2. Nottale [E1] proposed that it makes sense to speak about gravitational Planck constant h_{gr}. In TGD this idea is generalized and interpreted in framework of generalized quantum theory [K6, K5, K12]. For flux tubes assignable to gravitational bound states along which gravitons propagate, one would have $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is parameter with dimensions of velocity. One could write interaction strength as

$$GMm = v_0 \times \hbar_{gr} \quad .$$

3. $\hbar_{gr}$ obtained from this formula must satisfy $\hbar_{gr} > \hbar$. This generalizes to other interactions. For instance, one has one would have

$$Z_1 Z_2 e^2 = \frac{v_0 \hbar_{em}}{}$$

for electromagnetic flux tubes in the case that ones $h_{em} > \hbar$. The interpretation of the velocity parameter v_0 is discussed in [K12].

One could even turn the situation around and say that the value of $\hbar_{eff}$ fixes the interaction strength. $\hbar_{eff}$ would depend on fermion content and thus of virtual particle and also on the masses or other charges at the ends of the flux tube. The longer the range of the interaction, the larger the typical value of $\hbar_{eff}$.

4. The interpretation could be in terms long length scale quantum fluctuations at quantum criticality. Particles generate U-shaped monopole flux tubes with varying length proportional to $\hbar_{gr}$. If these U-shaped flux tubes from two different particles find each other, they reconnect to flux tube pairs connecting particles and give rise to interaction. What comes in mind is tiny curious and social animals studying their environment.

5. I have indeed proposed this picture in biology: the U-shaped flux tubes would be tentacles with which bio-molecules (in particular) would be scanning their environment. This scanning would be the basic mechanism behind immune system. It would also make possible for bio-molecules to find each in molecular crowd and provide a mechanism of catalysis. Could this picture apply completely generally? Would even elementary particles be scanning their environment with these tentacles?

6. Could one interpret the flux tubes as analogs of virtual particles or could they replace virtual particles of quantum field theories? The objection is that flux tubes would have time-like momenta whereas virtual particle analogs would have space-like momenta. The interpretation makes sense only if the associated momenta are between space-like and time-like that is light-like so that flux tube would correspond to mass shell particle. But this is the case in twistor approach to gauge theories also in TGD [L11] (see `http://tinyurl.com/y62no62a`).

Perhaps the following interpretation is more appropriate. Flux tubes are accompanied by strings and string world sheets can be interpreted as stringy description of gravitation and other interactions.

3.4 Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

   Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

   It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. $M^8 - H$ duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of imbedding space coordinates can be discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K1]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

4 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

4.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K10]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A1]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of imbeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of h^k are discontinuous but canonical momentum densities for the entire action are continuous.

There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

$M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L8].

4.2 Twistor lift at the level of scattering amplitudes and unexpected connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L3]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would

not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD.

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

4.3 Number-theoretic approach to unitarity

Twistorialization leads to the proposal that cuts in the scattering amplitudes are replaced with sums over poles, and that also many-particle states have discrete momentum and mass squared spectrum having interpretation in terms of bound states. Gravitation would be the natural physical reason for the discreteness of the mass spectrum and in string models it indeed emerges as "stringy" mass spectrum. The situation is very similar to that in dual resonance models, which were predecessors of super string theories.

Number theoretical discretization based on the hierarchy of extensions of rationals defining extensions of p-adic number fields gives rise to cognitive representatations as discrete sets of space-time surface and discretization of 4-momenta and S-matrix with discrete momentum labels. In number theoretic discretization cuts reduce automatically to sequences of poles. Whether this discretization is an approximation reflecting finite cognitive resolution or whether finite cognitive representation is a property of physical states reflecting itself as a condition that various parameters characterizing them belong to the extension considered, remains an open question.

One can approach the unitarity conditions also number theoretically. In the discretization forced by the extension of rationals the amplitudes are defined between states having a discrete spectrum of 4-momenta. Unitarity condition reduces to a purely algebraic condition involving only sums. In these conditions the Dirac delta functions associated with the mass squared of the resonances are replaced with Kronecker deltas.

1. For given extension of rationals the unitary conditions are purely algebraic equations

$$i(T_{mn}+\overline{T}_{nm})=\sum_r T_{mr}\overline{T}_{nr}=T_{mn}\overline{T}_{nn}+T_{mm}\overline{T}_{mn}+\sum_{r\neq m,n} T_{mr}\overline{T}_{nr} \quad .$$

   where T_{mn} belongs the extension. Complex imaginary unit i corresponds to that appearing in the extension of octonions in M^8-H duality [L2].

2. In the forward direction $m=n$ one obtains

$$2Im(T_{mm})=Re(T_{mm})^2+Im(T_{mm})^2+P_m \quad , \quad P_m=\sum_{r\neq m} T_{mr}\overline{T}_{mr} \quad .$$

   P_m represents total probability for non-forward scattering.

3. One can think of solving $Im(T_{mm})$ algebraically from this second order polynomial in the lowest order approximation in which $T_{mn}=0$ for $m\neq n$. This gives

$$2Im(T_{mm})=1+\sqrt{1-P_m-Re(T_{mm})^2} \quad .$$

   Reality requires $1-Re(T_{mm})^2-P_m\geq 0$ giving

$$Re(T_{mm})^2+P_m\leq 1 \quad .$$

   This condition is identically true by unitarity since probability for scattering cannot be larger than 1.

   Besides this the real root must belong to the original extension of rationals. For instance, if the extension of rationals is trivial, the quantity $1-P_m-Re(T_{mm})^2$ must be a square of rational y giving $1-P_m=y^2+Re(T_{mm})^2$. In the case of extension y is replaced with a number in the extension. I am not enough of number theorist to guess how powerful this kind of number theoretical conditions might be. In any case, the general ansatz for S is a unitary matrix in extension of rationals and this kind of matrices form a group so that there is no hope about unique solution.

4. One could think of iterative solution of the conditions by assuming in the zeroth order approximation $T_{mn} = 0$ for $m \neq n$ giving $Re(T_{mm})^2 + Im(T_{mm})^2 = 1$ reducing to $cos^2(\theta) + sin^2(\theta) = 1$. For trivial extension of rationals θ would correspond to Pythagorean triangle.

   For non-diagonal elements of T_{mn} one would obtain at the next step the conditions

$$i(T_{mn} + \overline{T}_{nm}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{nm} \quad .$$

   This gives a 2 linear equations for T_{mn}.

5. These conditions are not enough to give unique solution. Time reversal invariance gives additional conditions and might help in this respect. T invariance is slightly broken but CPT symmetry could replace T symmetry in the general situation.

   Time reversal operator T (to be not confused with T_{mn} above) is anti-unitary operator and one has $S^\dagger = T(S)$. In wave mechanics one can show that T-invariant S-matrix and thus also T-matrix is symmetric: $S = S^T$. The matrices of this kind do not form a group so that the conditions can be very powerful.

   Combined with the above equations symmetry gives

$$2Im(T_{mn}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn} \quad .$$

   The two conditions for T_{mn} in principle fix it completely in this order.

   One obtains from the real part of the equation

$$2Im(T_{mn}) = Re[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] \quad .$$

   The vanishing of the imaginary part gives

$$Im[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] = 0 \quad .$$

   giving a linear relation between the real and imaginary parts of T_{mn}. No new number theoretical conditions emerge. This relation requires that real and imaginary parts belong to the extension.

6. At higher orders one must feed the resulting ansatz to the unitarity conditions for the diagonal elements T_{nn}. One can hope that the lowest order ansatz leads to rather unique solution by iteration of the unitarity conditions. In higher order conditions the higher order corrections appear linearly so that no new number theoretic conditions emerge at higher orders.

   Physical picture suggests that the S-matrices could be obtained by an iterative procedure. Since infinitely long procedure very probably leads out of the extension, one can ask whether the procedure should stop after finite steps. This property would pose an additional conditions to the S-matrix.

7. Diagonal matrices are solutions to the conditions and for then the diagonal elements are roots of unity in the extension of rationals considered. The automorphisms $S_d \rightarrow US_dU^{-1}$ produce new S-matrices and if the unitary matrix U is orthogonal real matrix in algebraic extension satisfying therefore $UU^T = 1$, the condition $S = S^T$ is satisfied. There are therefore a large number of solutions.

   S-matrices diagonalizable in the extension are not the only solutions. The diagonalization of a unitary matrix $S = S^T$ in general gives a diagonal S-matrix, for which the roots of unity in general do not belong to the extension. Also the diagonalizating matrix fails to be in the extension. This non-diagonalizability might have deep physics content and explain why the physically natural state basis is not the one in which S-matrix is diagonal. In the case of density matrix it would guarantee stability of entanglement.

To sum up, number theoretic conditions could give rise to highly unique discrete S-matrices, when CPT symmetry can be formulated purely algebraically and be combined with unitarity. CPT symmetry might not however allow formulation in terms of automorphisms of diagonal unitary matrices analogous to orthogonal transformations.

5 Summary

It seems that unitarity of S-matrix reduces to the existence of maximal group of WCW isometries. The conservation of charges implies conservation of probability and unitarity.

Disjoint 3-surfaces and also those topologically condensed at larger space-time sheets would have interpretation as topological representations of particles in this approach. The special role of the partonic orbits suggests holography in the sense that these orbits have particle interpretation. Similar holography would make sense true for string world sheets and their boundaries. Action could therefore contain parts associated with $D = 2$ and $D = 1$ surfaces so that oscillator operators associated with these would be involved in the construction of states.

The transfer of quantum numbers from space-time interior to string world sheets could take place in interaction regions for Option I for which one assigns action to singular surfaces identified as surfaces having complex or real tangent space at M^8 level. The transfer would naturally vanish near the boundaries of CD. Same applies to the transfer from string world sheets to their boundaries. For Option II two the string world sheets would not carry Noether currents and only minimal surface property could fail at these surfaces: therefore this option is not realistic. Also for Option I there could be breaking of minimal surface property in this sense and the discontinuity of normal component for Noether currents would imply it automatically.

When this picture is combined with the twistor Grassmannian inspired view about scattering amplitudes using the constraints coming from quantum criticality, discreteness of the coupling constant evolution, and the existence of amplitudes as rational functions with coefficients in a extension of rationals allowing p-adic variants, one ends up to a picture in which amplitudes reduces to sums over resonances - this was just what was assumed in Veneziano model besides s-t duality.

This picture does not conform with QFT picture in superstring framework, where one has single large string tension so that poles cannot be approximated by cuts for low energies. In TGD framework this can be the case since string tension has spectrum reducing to that for cosmological constant. Since momenta are already classically predicted to be complex, resonance poles have finite width and one can in principle understand also unitarity. Therefore twistorialization in TGD framework leads to string models, and strings are indeed an essential part of twistorialization in TGD framework.

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