Quantum scarring from TGD point of view

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June 20, 2019

Abstract

Quantum scarring (QS) and its many-body counterpart MBQS are very interesting challenges for TGD. Eigenstate thermalization hypothesis (ETH) states the time development of energy eigenstate to a superposition of large number of eigenstates with the same energy gives rise to thermalization. In MBQS the thermalization is however very slow for most states and there are states for which it it does not occur at all and the system returns to the original state periodically. Integrable systems for which the energies of the states are rational multiples of a finite number of "fundamentals" have this property. MBQS in the case considered occurs for an array containing ground state atoms and their Rydberg counterparts.

In TGD framework one can consider the possibility that instead of Rydberg atoms one has pseudo Rydberg atoms having non-standard value $h_{eff} = nh_0$ of Planck constant such that $h_{eff} = mh$ is true (also fractional effective principal quantum number is possible and could serve as a test for the proposal). In this framework the exchange force between valence electrons would be scaled by factor $(h_{eff}/h)^2$ and promote localization in turn forcing the periodic orbits. Even if this effect is not involved in the case considered, it could make possible to have dark variant of MBQS at higher temperatures.

Contents

1 Introduction

I learned about very interesting phenomenon serving as a challenge for TGD. In quantum scarring the system does not thermalize as one might expect as the popular article "*Quantum scarring appears to defy universe's push for disorder*" describes (see http://tinyurl.com/y2bo8r8y). The experimental article by Bernien et al with title *Probing many-body dynamics on a 51-atom quantum simulator* [D1] (see http://tinyurl.com/yykagmeu) has the following abstract.

> *Controllable, coherent many-body systems can provide insights into the fundamental properties of quantum matter, enable the realization of new quantum phases and could ultimately lead to computational systems that outperform existing computers based on*

classical approaches. Here we demonstrate a method for creating controlled many-body quantum matter that combines deterministically prepared, reconfigurable arrays of individually trapped cold atoms with strong, coherent interactions enabled by excitation to Rydberg states. We realize a programmable Ising-type quantum spin model with tunable interactions and system sizes of up to 51 qubits. Within this model, we observe phase transitions into spatially ordered states that break various discrete symmetries, verify the high-fidelity preparation of these states and investigate the dynamics across the phase transition in large arrays of atoms. In particular, we observe robust many-body dynamics corresponding to persistent oscillations of the order after a rapid quantum quench that results from a sudden transition across the phase boundary. Our method provides a way of exploring many-body phenomena on a programmable quantum simulator and could enable realizations of new quantum algorithms.

There are many theoretical articles about MBQS. As an example I include the abstract of the article "*Quantum scarred eigenstates in a Rydberg atom chain: entanglement, breakdown of thermalization, and stability to perturbations*" by Turner et al [D2] (see http://tinyurl.com/y54unclz) serving as basis of TGD inspired considerations.

Recent realization of a kinetically constrained chain of Rydberg atoms by Bernien et al., [Nature (London) 551, 579 (2017)] resulted in the observation of unusual revivals in the many-body quantum dynamics. In our previous work [C. J. Turner et al., Nat. Phys. 14, 745 (2018)], such dynamics was attributed to the existence of quantum scarred eigenstates in the many-body spectrum of the experimentally realized model. Here, we present a detailed study of the eigenstate properties of the same model.
We find that the majority of the eigenstates exhibit anomalous thermalization: the observable expectation values converge to their Gibbs ensemble values, but parametrically slower compared to the predictions of the eigenstate thermalization hypothesis (ETH). Amidst the thermalizing spectrum, we identify non-ergodic eigenstates that strongly violate the ETH, whose number grows polynomially with system size. Previously, the same eigenstates were identified via large overlaps with certain product states, and were used to explain the revivals observed in experiment.
Here, we find that these eigenstates, in addition to highly atypical expectation values of local observables, also exhibit sub-thermal entanglement entropy that scales logarithmically with the system size. Moreover, we identify an additional class of quantum scarred eigenstates, and discuss their manifestations in the dynamics starting from initial product states.
We use forward scattering approximation to describe the structure and physical properties of quantum scarred eigenstates. Finally, we discuss the stability of quantum scars to various perturbations. We observe that quantum scars remain robust when the introduced perturbation is compatible with the forward scattering approximation. In contrast, the perturbations which most efficiently destroy quantum scars also lead to the restoration of canonical thermalization.

The systems exhibiting quantum scarring (QS) thermalize very slowly or do not thermalize at all. Instead, the system returns to its original state periodically. This behavior does not conform with ergodicity stating that the system goes through all possible state during time evolution.

There are a lot of systems, which fail to be ergodic.

1. In integrable systems - for which TGD is an excellent candidate - all states starting from energy eigenstate have this recurrence property as isolated systems if the energies are commeasurate (rational multiples of same unit of energy). In the recent case only preferred states have this recurrence property.

   In the experimental situation one considers a quenched system: the initial state can be modelled as energy eigenstate of some Hamiltonian H_0 which is replaced with $H = H_0 + H_1$ so that the state is not energy eigenstate anymore. Periodic behavior requires that the state is superposition of finite number of state with commensurate energies in the resolution considered. In the ideal situation the eigenstates of H are integer spaced so that they have the period of the ground state as common periodicity. Period increases if there are states with energies close to each other since states E and $E + \Delta E$ must satisfy $ET = n \times 2\pi$ and $\Delta E \times T = m \times 2\pi$ giving $T = m2\pi/\Delta E = (m/n) \times (E/\Delta E$.

2. For spin glass [B1] the energy landscape is a fractal with valleys inside valleys, and the system ends down to some valley as it dissipates. The mountains of the energy landscape force the localization and the thermalization is prevented.

   Some kind of dynamical localization is expected place in the situations in which only preferred states give rise to a quantum scar. Dynamical localization could due to genuinely quantal state repulsive exchange forces depending on the relative direction of spins of valence electrons of Rydberg atoms.

One can distinguish between quantum scarring (QS) and quantum many-body scarring (MBQS).

1. In QS the wave function of the particle concentrates along unstable periodic classical orbit. The less unstable the orbit is, the stronger the scarring is. The classical orbit makes itself visible as a quantum scar.

2. In MBQS scarring is a generalization of quantum scarring and the state of many-particle system returns to the original one. In principle one can describe many-particle system wave-mechanically as single particle state in a higher-D configuration space so that in principle this does not bring anything new. MBQS has been observed in a 1-D lattice formed by Rydberg atoms and ordinary atoms so that configuration space is effectively discrete. Some atoms of this system at very low temperature are excited to what are believed to be Rydberg atoms with large value of principal quantum number n_P and therefore large radius. This requires energy because bound states energies are proportional to $1/n_P^2\hbar^2$.

   Remark: "Believed to be" sounds strange but in TGD framework atoms for which valence electrons have nonstandard value of Planck constant $h_{eff} = nh_0$ can look like Rydberg atoms. For $h = 6h_0$ suggested by experiments of Randell Mills [L1] one would have $h_{eff} = nh/6$ so that one could have one can have fractional principal quantum number $n_{P,eff} = (nn_P)/6$: this provides a test for h_{eff} hypothesis using irradiation with corresponding frequencies. For $n = 6n_1$ one might fail to distinguish these states from Rydberg states since the radii of the states scale like $n_{P,eff}^2$. Large value of h_{eff} would make possible quantum coherence in long length scales and this could be highly relevant for integrability.

Eigenstate thermalization (EST) is an important notion. Eigenstate thermalization takes place by unitary time evolution, which usually generates a superposition of large number of states with same total quantum numbers, in particular energy. Single particle states have however varying energies and in the superposition single particle states get entangled. For sub-systems the density matrix is assumed to develop to a thermal density matrix. In particular, entanglement entropy is identified as thermal entropy. For QS and MBQS EST would occur very slowly or not at all.

In TGD framework one can consider two approaches to MBQS and QS. The general approach starting from the key ideas of TGD and the approach starting directly from the special properties of Rydberg atoms and their possible analogs with non-standard value of $h_{eff} > h$. The key question is whether MBQS is analogous to the periodicity in integrable systems with commeasurate energies.

2 General TGD based considerations

In the sequel I will briefly discuss some aspects of the basic principles of TGD with some associations to MBQS. Reader can however skip directly to the concrete proposal if this looks easier.

2.1 TGD as generalization of Wheeler's superspace approach and as geometrization of quantum physics

2.1.1 The world of classical worlds (WCW)

One could see TGD as a generalization of Wheeler's superspace approach and generalization of Einstein's geometrization program for physics. Integrability, quantum criticality, quantum classical correspondence, zero energy ontology, and hierarchy of Planck constants are the aspects of TGD, which seem to be relevant for MBQS.

1. There are excellent reasons to believe that TGD Universe is integrable and quantum critical system [K4, K3] in very general sense. Also MBQS are conjectured to possess these properties. Quantum criticality would be responsible for the ground state degeneracy characterizing the model Hamiltonian of Turner et al [D2].

2. TGD generalizes Einstein's vision about the geometrization of physics to the level of quantum physics. The basic geometric object is the "world of classical worlds" (WCW) consisting of pairs of 3-surfaces with members at opposite boundaries of a causal diamond (CD) and connected by preferred extremal of the action which for the twistor lift of TGD decomposes to a sum of so called Kähler action analogous to Maxwell action and a volume term, whose coefficient corresponds to cosmological constant.

   General Coordinate invariance implies holography in the sense that the these pairs of 3-surfaces as analogs of Bohr orbits are equivalent with the 4-D preferred extremals connecting them. Classical theory is an exact part of quantum TGD. Preferred extremals are minimal surfaces which fail to be such only at 2-D singular surfaces having identification as string world sheets and representing orbits of folds of a 3-surface [L6, L7].

   In zero energy ontology (ZEO) quantum state - called zero energy state - is a superposition of deterministic preferred extremals. Simplest zero energy states are superpositions with same eigenvalues of observables and total quantum numbers are conserved.

   Remark: Wave functions concentrated along periodic unstable classical orbits is central to QS. Superposition should be along unstable classical orbit. One could imagine that the state is superposition of 3-surfaces along classical orbit defined as slices obtained by intersecting with translate of either boundary of CD.

3. Zero modes are a key element of TGD and correspond to the degrees of freedom, which do not contribute to WCW metric, which is thus degenerate. There would be states with the same total quantum numbers but different values of zero modes so that ground state degeneracy of the model of Turner et al [D2] could correspond to wave function in zero modes.

2.1.2 Many-fermion states as modes of classical WCW spinor field

Also fermionic degrees of freedom are geometrized.

1. Fermions are geometrized in terms of WCW spinor structure [K2] with WCW gamma matrices expressible as linear combinations of fermionic oscillator operators for second quantized induced spinor fields. Many-fermion states correspond to the modes of WCW spinor field. This implies what I call super-symplectic symmetry as an extension of the symplectic symmetry acting as isometries of WCW necessary for the existence of Riemann connection in infinite-D context [K1, **?**] (for loop spaces this was shown by Freed [A1]). Formally many-fermion states are just modes of classical spinor field in WCW.

2. Quantum-classical correspondence (QCC) implies that classical conserved Cartan charges and total fermionic charges are identical. Each particle in many-particle state corresponds near the boundaries of CD to "free particle" having single particle preferred extremal as correlate. One would have superposition of the collections of preferred extremals in the initial state. Superposition in entangled many-fermion state would correspond to a superposition of unions of corresponding 3-surfaces differing by translation and by properties correlating with other single particle quantum numbers.

   Quantum state with given total quantum numbers such as energy as for (ETH) is superposition of several many-particle states in general since total quantum numbers are sums of those with varying single particle quantum numbers. At fundamental level this would hold true in fermionic degrees of freedom (bosons are composites of fermions and antifermions in TGD Universe). For MBQS there would be only 2 different orbits corresponding to ground state of atom and Rydberg atom: the electronic Bohr orbits as pieces of space-time surface would be different for these. Therefore the situation would be rather simple classically.

3. The space-time surface - as opposed to 3-surfaces at the ends of CD - associated with many-particle system would be connected as analog of connected Feynman diagram and correspond to a formation of magnetic flux tubes between atoms as correlates of entanglement. The periodicity of the entanglement would correspond to periodic generation and disappearance of entanglement and flux tube - kind of breathing consisting of phase transitions between gas phase and liquid phase. Somewhat similar situation is encountered in simple systems consisting of plastic balls exhibiting basic aspects of life [L3].

2.2 Number theoretical vision

Number theoretical vision is second thread of TGD besides the vision about geometrization of physics.

1. p-Adic physics and their fusion to form a hierarchy of adelic physics characterized by a hierarchy of extensions of rational numbers inducing in turn extensions of various p-adic number fields [L4, L5]. Classical number fields represent second key aspect of number theoretical vision [L2].

   Adelic physics predicts a hierarchy $h_{eff} = nh_0$ ($h = 6h_0$ is a good guess [L1]) of effective values of Planck constant assumed to label a hierarchy of phases behaving like dark matter and having an interpretation as a dimension for extension of rationals.

2. One can ask whether non-standard value of h_{eff} guaranteeing quantum coherence in scales longer than expected is involved with MBQS. One can ask whether Rydberg atoms be actually atoms with valence electrons, which are dark for some value of h_{eff} and have scaled orbits with scaling factor $(h_{eff}/h)^2 = (n/6)^2$. If n is not a multiple of 6, one can speak of fractional principal quantum number $n_P = n/6$ and this might allow to test the hypothesis. For $h_{eff} > h$ pseudo Rydberg electrons could form a nanoscopic quantum system.

   MBQS is observed in very low temperatures and one can argue that the ordinary value of Planck constant is enough. One can however wonder whether MBQS is possible at higher temperatures for non-standard value of h_{eff} just like high Tc superconductivity if it is due to large h_{eff}.

3. If the presence of flux tube connections is necessary for large scale quantum coherence in the scale of the entire system needed and serves also as a correlate for entanglement, one can argue $h_{eff} > h$ is needed. Otherwise one expects thermalization to occur since the system decomposes to smaller quantum-coherent systems.

2.3 ZEO and generalization of quantum measurement theory

ZEO forces to generalize quantum measurement theory. One could also say that the need to solve the basic paradox of quantum measurement theory forces ZEO.

1. In ZEO state function reduction is replaced with the counterpart of ordinary state function reduction- "big state function reduction" (BSR) and the counterpart of weak measurement - "small state function reduction" (SSR) . The unitary evolution of state corresponds in TGD sequence of unitary evolutions followed by SSR affecting only the states at the active boundary of CD and also de-localizing the active boundary whereas passive boundary and members of state pairs at it would remain unaffected.

   SSR would localize the active boundary so that one has only single CD in superposition and mean also time measurement with time defined as the distance between the tips of CD. BSRs would change the roles of passive and active boundaries of CD and change the arrow of time assignable to the state by passive-active characterization.

2. Are SSRs or BSRs associated with the reduction of entanglement and return to the initial state in MBQS? SSR looks a more plausible interpretation. BSR would reduce the entanglement at the active boundary making it passive and change the arrow of time and next BSR would bring back the original arrow of time and CD boundary would be slightly shifted towards future. It is not clear whether the entanglement is small in the beginning of sequence of SSRs.

3 A concrete TGD inspired model for MBQS

The fact that MBQS occurs only for special initial states forces to ask whether it reflects the special properties of the system considered or some general properties such as integrability for a system with commeasurate energies. Or is MBQS something between these two cases: could the property of having energy spectrum with energies coming as rational multiples of a fundamental be dynamically generated (localization)?

1. System could be an integrable system for which the evolution is periodic if energies are commensurate. The spectrum should not differ too much from harmonic oscillator spectrum since small energy differences tend to spoil the periodicity. There are excellent reasons to expect that TGD is integrable theory but the behavior resembling harmonic oscillator is not obvious.

   The system is unstable and should be therefore critical and possess zero modes generating long range quantum fluctuations for which large h_{eff} phases can serve as correlates. This is achieved if ground state has a large degeneracy with respect to energy. Small perturbations can be always described in terms of harmonic oscillators. The frequencies of harmonic oscillators should be expressible as multiples of fundamentals whose ratios are rational numbers.

2. In TGD framewor the large value of h_{eff} makes possible quantum coherence in longer length scales and commeasurate integrability in such a manner that eigen-energies resemble harmonic oscillator spectrum coming as integer multiples of rather few rationally related fundamentals.

3. Space-time sheet is a natural candidate for a quantum coherent structure and if the space-time sheet decomposes into smaller disjoint sheets also coherence would be lost. Magnetic flux tubes connecting smaller space-time sheets to larger units would be natural correlates of quantum coherence and carry large h_{eff} phases. One could perhaps speak of dynamically generated quantum coherence and integrability with small number of fundamental energies.

4. Dynamical localization should occur and could be due to interatomic forces. Exchange forces due to the Fermi statistics generate spin-dependent interactions, which are short ranged and repulsive for parallel spins. The exchange forces are excellent candidates for inducing the localization.

   Dark valence electrons with large h_{eff} would have stronger exchange forces. This would promote the localization since one could not have effective Rydberg atoms (ERAs) with too small distance between them. If one has a system consisting of ordinary atoms (OAs) plus ERAs, the dark valence electrons could form a macroscopic quantum having MBQS states for this reason.

   The physical picture is that states in which ERAs have too small mutual distance are not possible. This gives a constraint to the dynamics. Typically the "spin flip" giving rise to an ERA can occur only for atoms with sufficiently large distances to the nearest ERAs. This constraint dynamics forces localization inducing periodicity.

3.1 About intermolecular -, van der Waals -, and exchange forces

Intermolecular forces (see http://tinyurl.com/mmxnctm) include exchange forces due to Pauli exclusion principle, electrostatic interactions between permanent electric and magnetic multipoles, which can be both attractive and repulsive, and attractive interactions between permanent and induced multipoles - induction -, and between induced multipoles - so called dispersion forces.

In standard QFT van der Waals force-London dispersion force comes from interaction with zero point energy and analogous to Casimir force. London dispersion force is proportional to the product of ionization energies of atoms divided by their sum and product of polarizabilities and therefore proportional to $1/h_{eff}^2$ and would weaken for large h_{eff}.

Lennard-Jones potential (see http://tinyurl.com/y9bjcxn5) provides the simplest parameterization of these forces. There is attractive $1/r^6$ term representing dispersion forces and repulsive $1/r^{12}$ term interpreted in terms of exchange forces repulsive/attractive for parallel/opposite spins of electrons. This follows from antisymmetry of the wave function. The dispersion force if proportional to the energy scale of atom and therefore to $1/h_{eff}^2$ so that its scale decreases for large h_{eff}.

The strength of the exchange force is proportional to the inner product of spins and therefore proportional to h_{eff}^2 and increases with h_{eff}^2. This make increase the range of this force and together with the weakening of the dispersion force would make the radius at which the van der Waals force becomes repulsive larger. This would promote dynamical localization.

3.2 Consistency with the model of MBQS of Turner et al

In the model of MBQS discussed by by Turner et al [?] (see `http://tinyurl.com/y54unclz`) the situation is indeed very much like proposed above. One considers a model Hamiltonian H having decomposition $H = H_0 + H_1$. Ground state and Rydberg state are formally described as two possible states of spin.

The first part in the Hamilton is sum $H_0 = k\sum X_i$ over single particle terms X_i analogous to paramagnetic spin flip term in the interaction of spins with an external magnetic field. It acts on single particle transforming ordinary atom in ground state to Rydberg atom or vice versa.

Second part $H_1 = \sum i \neq j V_{ij} Q^i Q^j$ of the Hamiltonian describes repulsive interatomic forces and is associated with pairs of particles at different sites. Individual terms are proportional to the projectors Q_i and Q_j to Rydberg states at neighboring sites i and j and the parameter V_{ij} describing interaction strength assumed to behave like $1/|i-j|^n$, $n = 6$, at the limit $i \rightarrow j$. Lennard-Jones potential would suggest $n = 12$ but this is not essential for the model since one considers an approximation in which only nearest neighbour interactions are considered. This part of the Hamiltonian is the large part non-perturbative and spin-flip term is treated as a small perturbation, which suggests that harmonic oscillator type approximation is good.

In nearest neighbour approximation the large part H_1 is proportional to a sum over terms $V_{i,i+1}Q_iQ_{i+1}$ over nearest neighbour pairs. In the states with minimum energy the positive interaction term (somewhat ironically) vanishes: this is guaranteed if all Rydberg sites have ground states as neighbours. One can introduce to the Hamilton this constraint explicitly, and by a scaling ends up to a Hamiltonian which is just the small paramagnetic spin flip term X_i multiplied from left *resp.* right side by projector P_{i-1} *resp.* P_{i+1} to the subspace satisfying the constraint.

The effect of this Hamiltonian is to induce "spin flips" such that the constraint is respected. The outcome is entangled state and the localization caused by the constraint induces the periodic dynamics and failure of ETH for preferred states.

The entanglement between dark and ordinary states makes sense: $h_{eff} = nh_0$ corresponds at space-time level n-sheeted covering of space-time. One must however assume that the entanglement coefficients are in the extension of rationals associated with the smaller value of n (n_1) belonging to that assignable to the larger value of n (n_2): therefore n_1 divides n_2.

If the effective spin-spin interaction is a sensible model for the situation, the value of h_{eff} affects only the parameters determining the spin-spin interaction. The excitation of ERAs requires energy but so does also the excitation of ordinary Rydberg atoms so that this cannot be used as an objection against the model.

REFERENCES

Mathematics

[A1] Freed DS. The Geometry of Loop Groups, 1985.

Theoretical Physics

[B1] Virasoro MA Mezard M, Parisi G. *Spin Glass Theory and Beyond.* World Scientific, 1987.

Condensed Matter Physics

[D1] Bernien H et al. Probing many-body dynamics on a 51-atom quantum simulator. *Nature.*Available at: https://www.nature.com/articles/nature24622, 551:579–584, 2017.

[D2] Turner CJ et al. Quantum scarred eigenstates in a Rydberg atom chain: entanglement, breakdown of thermalization, and stability to perturbations. Available at:https://arxiv.org/pdf/1806.10933.pdf, 2018.

Books related to TGD

[K1] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#compl1, 2006.

[K2] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#cspin, 2006.

[K3] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/neuplanck.html#qcritdark, 2014.

[K4] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#wcwnew, 2014.

Articles about TGD

[L1] Pitkänen M. Hydrinos again. Available at: http://tgdtheory.fi/public_html/articles/Millsagain.pdf, 2016.

[L2] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.

[L3] Pitkänen M. Life-like properties observed in a very simple system. Available at: http://tgdtheory.fi/public_html/articles/plasticballs.pdf, 2017.

[L4] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[L5] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L6] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: http://tgdtheory.fi/public_html/articles/smatrix.pdf, 2018.

[L7] Pitkänen M. Does coupling constant evolution reduce to that of cosmological constant? Available at: http://tgdtheory.fi/public_html/articles/ccevoTGD.pdf, 2019.