Why the non-trivial zeros of Riemann zeta should reside at critical line?

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Abstract

It is shown that the troublesome looking "1/2" in the non-trivial zeros of Riemann zeta can be understood as being necessary to allow a hermitian realization of the radial scaling generator rd/dr at light-cone boundary, which in the radial light-like radial direction corresponds to half-line R^+. Its presence allows unitary inner product and reduces the situation to that for ordinary plane waves on real axis. For preferred extremals strong form of holography poses extremely strong conditions expected to reduce the scaling momenta $s = 1/2 + iy$ to the zeros of zeta at critical line. RH could be also seen as a necessary condition for the existence of super-symplectic representations and thus for the existence of the "World of Classical Worlds" as a mathematically well-defined object. We can thank the correctness of Riemann's hypothesis for our existence!

1 What is the origin of the troublesome 1/2 in non-trivial zeros of zeta?

Riemann Hypothess (RH) states that the non-trivial (critical) zeros of zeta lie at critical line $s = 1/2$. It would be interesting to know how many physical justifications for why this should be the case has been proposed during years. Probably this number is finite, but very large it certainly is. In Zero Energy Ontology (ZEO) forming one of the cornerstones of the ontology of quantum TGD, the following justification emerges naturally.

1. The "World of Classical Worlds" (WCW) consisting of space-time surfaces having ends at the boundaries of causal diamond (CD), the intersection of future and past directed light-cones times CP_2 (recall that CDs form a fractal hierarchy). WCW thus decomposes to sub-WCWs and conscious experience for the self associated with CD is only about space-time surfaces in the interior of CD: this is a trong restriction to epistemology, would philosopher say.

   Also the light-like orbits of the partonic 2-surfaces define boundary like entities but as surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian. By holography either kinds of 3-surfaces can be taken as basic objects, and if one accepts strong form of holography, partonic 2-surfaces defined by their intersections plus string world sheets become the basic entities.

2. One must construct tangent space basis for WCW if one wants to define WCW Kähler metric and gamma matrices. Tangent space consists of allowed deformations of 3-surfaces at the ends of space-time surface at boundaries of CD, and also at light-like parton orbits

extended by field equations to deformations of the entire space-time surface. By strong form of holography only very few deformations are allowed since they must respect the vanishing of the elements of a sub-algebra of the classical symplectic charges isomorphic with the entire algebra. One has almost 2-dimensionality: most deformations lead outside WCW and have zero norm in WCW metric.

3. One can express the deformations of the space-like 3-surface at the ends of space-time using a suitable function basis. For CP_2 degrees of freedom color partial waves with well defined color quantum numbers are natural. For light-cone boundary $S^2 \times R^+$, where R^+ corresponds to the light-like radial coordinate, spherical harmonics with well defined spin are natural choice for S^2 and for R^+ analogs of plane waves are natural. By scaling invariance in the light-like radial direction they look like plane waves $\psi_s(r) = r^s = exp(us)$, $u = log(r/r_0)$, $s = x + iy$. Clearly, u is the natural coordinate since it replaces R^+ with R natural for ordinary plane waves.

4. One can understand why $Re[s] = 1/2$ is the only possible option by using a simple argument. One has super-symplectic symmetry and conformal invariance extended from 2-D Riemann surface to metrically 2-dimensional light-cone boundary. The natural scaling invariant integration measure defining inner product for plane waves in R^+ is $du = dr/r = dlog(r/r_0)$ with u varying from $-\infty$ to $+\infty$ so that R^+ is effectively replaced with R. The inner product must be same as for the ordinary plane waves and indeed is for $\psi_s(r)$ with $s = 1/2 + iy$ since the inner product reads as

$$\langle s_1, s_2 \rangle \equiv \int_0^\infty \overline{\psi_{s_1}} \psi_{s_2} dr = \int_0^\infty exp(i(y_1 - y_2) r^{-x_1 - x_2} dr \quad .$$

For $x_1 + x_2 = 1$ one obtains standard delta function normalization for ordinary plane waves:

$$\langle s_1, s_2 \rangle \int \int_{-\infty}^\infty exp[i(y_1 - y_2)u] du \propto \delta(y_1 - y_2) \quad .$$

If one requires that this holds true for all pairs (s_1, s_2), one obtains $x_i = 1/2$ for all s_i. Preferred extremal condition gives extremely powerful additional constraints and leads to a quantisation of $s = -x - iy$: the first guess is that non-trivial zeros of zeta are obtained: $s = 1/2 + iy$. This identification would be natural by generalised conformal invariance. Thus RH is physically extremely well motivated but this of course does not prove it.

5. The presence of the real part $Re[s] = 1/2$ in the eigenvalues of scaling operator apparently breaks hermiticity of the scaling operator. There is however a compensating breaking of hermiticity coming from the fact that real axis is replaced with half-line and origin is pathological. What happens that real part $1/2$ effectively replaces half-line with real axis and obtains standard plane wave basis. Note also that the integration measure becomes scaling invariant - something very essential for the representations of super-symplectic algebra. For $Re[s] = 1/2$ the hermicity conditions for the scaling generator rd/dr in R^+ reduce to those for the translation generator d/du in R.

2 Relation to number theoretical universality and existence of WCW

This relates also to the number theoretical universality and mathematical existence of WCW in an interesting manner.

1. If one assumes that p-adic primes p correspond to zeros $s = 1/2 + y$ of zeta in 1-1 manner in the sense that $p^{iy(p)}$ is root of unity existing in all number fields (algebraic extension of p-adics is allowed) one obtains that the plane wave exists for p at points $r = p^n$. p-Adically wave function is discretized to a delta function distribution concentrated at $(r/r_0) = p^n$- a logarithmic lattice. This can be seen as space-time correlate for p-adicity for light-like

momenta to be distinguished from that for massive states where length scales come as powers of $p^{1/2}$. Something very similar is obtained from the Fourier transform of the distribution of zeros at critical line (Dyson's argument), which led to a the TGD inspired vision about number theoretical universality [L2] (see `http://tgdtheory.fi/public_html/articles/ntu.pdf`).

2. My article "Strategy for Proving Riemann Hypothesis" (`http://www.emis.math.ca/EMIS/journals/AMUC/`) [L1] written for 12 years ago ((for a slightly improved version see `http://arxiv.org/abs/math/0111262`) relies on coherent states instead of eigenstates of Hamiltonian. The above approach in turn absorbs the problematic 1/2 to the integration measure at light cone boundary and conformal invariance is also now central.

3. Quite generally, I believe that conformal invariance in the extended form applying at metrically 2-D light-cone boundary (and at light-like orbits of partonic 2-surfaces) could be central for understanding why physics requires RH and maybe even for proving RH assuming it is provable at all in existing standard axiomatic system. For instance, the number of generating elements of the extended supersymplectic algebra is infinite (rather than finite as for ordinary conformal algebras) and generators are labelled by conformal weights defined by zeros of zeta (perhaps also the trivial conformal weights). $s = 1/2 + iy$ guarantees that the real parts of conformal weights for all states are integers. By conformal confinement the sum of ys vanishes for physical states. If some weight is not at critical line the situation changes. One obtains as net conformal weights all multiples of x shifted by all half odd integer values. And of course, the realisation as plane waves at boundary of light-cone fails and the resulting loss of unitary makes things too pathological and the mathematical existence of WCW is threatened.

4. The existence of non-trivial zeros outside the critical line could thus spoil the representations of super-symplectic algebra and destroy WCW geometry. RH would be crucial for the mathematical existence of the physical world! And the physical worlds exist only as mathematical objects in TGD based ontology: there are no physical realities behind the mathematical objects (WCW spinor fields) representing the quantum states. TGD inspired theory of consciousness tells that quantum jumps between the zero energy states give rise to conscious experience, and this is in principle all that is needed to understand what we experience.

REFERENCES

Articles about TGD

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