More about physical interpretation of algebraic extensions of rationals

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Abstract

The number theoretic vision has begun to show its power. The basic hierarchies of quantum TGD would reduce to a hierarchy of algebraic extensions of rationals and the parameters - such as the degrees of the irreducible polynomials characterizing the extension and the set of ramified primes - would characterize quantum criticality and the physics of dark matter as large h_{eff} phases. The identification of preferred p-adic primes as ramified primes of the extension and generalization of p-adic length scale hypothesis as prediction of NMP are basic victories of this vision. In this article a more detailed view about emergence of preferred p-adic primes as ramified primes is discussed. All boils down to quantum criticality. Classically it corresponds to appearance of space-time surface whose ends at the boundaries of CD collapse to single 3-surface (or partonic 2-surface) to give singular covering.

In the parameter space characterized by the algebraic extension of rational criticality corresponds toa prime ideal corresponding to ramified prime - ramification is indeed analogous to criticality. For other than ramified primes this collapse does not occur and the sheets define separate 3-surfaces related to each other by Galois group serving as a number theoretical symmetry group- the connection with the Langlands program is obvious. The remaining discrete degrees are however physical for ramified space-time surfaces although the action of Galois group is trivial. Whether the degree of the polynomial defining

algebraic extension and the ramified primes are completely independent physical parameters or whether they are dependent is an interesting question. The TGD inspired generalization of AdS/CFT duality suggests that they are not. The outcome would be that also electron would possess quite an impressive number of these discrete degrees of freedom (given by Mersenne prime M_{127}).

1 Introduction

The number theoretic vision has begun to show its power. The basic hierarchies of quantum TGD would reduce to a hierarchy of algebraic extensions of rationals and the parameters - such as the degrees of the irreducible polynomials characterizing the extension and the set of ramified primes (`http://en.wikipedia.org/wiki/Ramification_(mathematics)`) - would characterize quantum criticality and the physics of dark matter as large h_{eff} phases. The identification of preferred p-adic primes as ramified primes of the extension and generalization of p-adic length scale hypothesis as prediction of NMP [K1, K2] are basic victories of this vision.

By strong form of holography the parameters characterizing string world sheets and partonic 2-surfaces serve as WCW coordinates. By various conformal invariances, one expects that the parameters correspond to conformal moduli, which means a huge simplification of quantum TGD since the mathematical apparatus of superstring theories becomes available and number theoretical vision can be realized. Scattering amplitudes can be constructed for a given algebraic extension and continued to various number fields by continuing the parameters which are conformal moduli and group invariants characterizing incoming particles.

There are many un-answered and even un-asked questions.

1. How the new degrees of freedom assigned to the n-fold covering defined by the space-time surface pop up in the number theoretic picture? How the connection with preferred primes emerges?

2. What are the precise physical correlates of the parameters characterizing the algebraic extension of rationals? Note that the most important extension parameters are the degree of the defining polynomial and ramified primes.

2 Some basic notions

Some basic information about extensions are in order. I emphasize that I am not a specialist.

2.1 Basic facts

The algebraic extensions of rationals are determined by roots of polynomials. Polynomials be decomposed to products of irreducible polynomials, which by definition do not contain factors which are polynomials with rational coefficients. These polynomials are characterized by their degree n, which is the most important parameter characterizing the algebraic extension.

One can assign to the extension primes and integers - or more precisely, prime and integer ideals. Integer ideals correspond to roots of monic polynomials $P_n(x) = x^n + ..a_0$ in the extension with integer coefficients. Clearly, for $n = 0$ (trivial extension) one obtains ordinary integers. Primes as such are not a useful concept since roots of unity are possible and primes which differ by a multiplication by a root of unity are equivalent. It is better to speak about prime ideals rather than primes.

Rational prime p can be decomposed to product of powers of primes of extension and if some power is higher than one, the prime is said to be ramified and the exponent is called ramification index. Eisenstein's criterion (`http://en.wikipedia.org/wiki/Eisenstein's_criterion` states that any polynomial $P_n(x) = a_nx^n + a_{n-1}x^{n-1} + ...a_1x + a_0$ for which the coefficients a_i, $i < n$ are divisible by p and a_0 is not divisible by p^2 allows p as a maximally ramified prime. mThe corresponding prime ideal is n:th power of the prime ideal of the extensions (roughly n:th root of p). This allows to construct endless variety of algebraic extensions having given primes as ramified primes.

Ramification is analogous to criticality. When the gradient potential function $V(x)$ depending on parameters has multiple roots, the potential function becomes proportional a higher power of $x - x_0$. The appearance of power is analogous to appearance of higher power of prime of extension in ramification. This gives rise to cusp catastrophe. In fact, ramification is expected to be number theoretical correlate for the quantum criticality in TGD framework. What this precisely means at the level of space-time surfaces, is the question.

2.2 Galois group as symmetry group of algebraic physics

I have proposed long time ago that Galois group `http://en.wikipedia.org/wiki/Splitting_of_prime_ideals_in_Galois_extensions` acts as fundamental symmetry group of quantum TGD and even made clumsy attempt to make this idea more precise in terms of the notion of number theoretic braid. It seems that this notion is too primitive: the action of Galois group must be realized at more abstract level and WCW provides this level.

First some facts (I am not a number theory professional, as the professional reader might have already noticed!).

1. Galois group acting as automorphisms of the field extension (mapping products to products and sums to sums and preserves norm) characterizes the extension and its elements have maximal order equal to n by algebraic n-dimensionality. For instance, for complex numbers Galois group acs as complex conjugation. Galois group has natural action on prime ideals of extension mapping them to each other and preserving the norm determined by the determinant of the linear map defined by the multiplication with the prime of extension. For instance, for the quadratic extension $Q(\sqrt{5})$ the norm is $N(x + \sqrt{5}y) = x^2 - 5y^2$: not that number theory leads to Minkowkian metric signatures naturally. Prime ideals combine to form orbits of Galois group.

2. Since Galois group leaves the rational prime p invariant, the action must permute the primes of extension in the product representation of p. For ramified primes the points of the orbit of ideal degenerate to single ideal. This means

that primes and quite generally, the numbers of extension, define orbits of the Galois group.

Galois group acts in the space of integers or prime ideals of the algebraic extension of rationals and it is also physically attractive to consider the orbits defined by ideals as preferred geometric structures. If the numbers of the extension serve as parameters characterizing string world sheets and partonic 2-surfaces, then the ideals would naturally define subsets of the parameter space in which Galois group would act.

The action of Galois group would leave the space-time surface invariant if the sheets co-incide at ends but permute the sheets. Of course, the space-time sheets permuted by Galois group need not co-incide at ends. In this case the action need not be gauge action and one could have non-trivial representations of the Galois group. In Langlands correspondence these representation relate to the representations of Lie group and something similar might take place in TGD as I have indeed proposed.

Remark: Strong form of holography supports also the vision about quaternionic generalization of conformal invariance implying that the adelic space-time surface can be constructed from the data associated with functions of two complex variables, which in turn reduce to functions of single variable.

If this picture is correct, it is possible to talk about quantum amplitudes in the space defined by the numbers of extension and restrict the consideration to prime ideals or more general integer ideals.

1. These number theoretical wave functions are physical if the parameters characterizing the 2-surface belong to this space. One could have purely number theoretical quantal degrees of freedom assignable to the hierarchy of algebraic extensions and these discrete degrees of freedom could be fundamental for living matter and understanding of consciousness.

2. The simplest assumption that Galois group acts as a gauge group when the ends of sheets co-incide at boundaries of CD seems however to destroy hopes about non-trivial number theoretical physics but this need not be the case. Physical intuition suggests that ramification somehow saves the situation and that the non-trivial number theoretic physics could be associated with ramified primes assumed to define preferred p-adic primes.

3 How new degrees of freedom emerge for ramified primes?

How the new discrete degrees of freedom appear for ramified primes?

1. The space-time surfaces defining singular coverings are n-sheeted in the interior. At the ends of the space-time surface at boundaries of CD however the ends co-incide. This looks very much like a critical phenomenon.

   Hence the idea would be that the end collapse can occur only for the ramified prime ideals of the parameter space - ramification is also a critical phenomenon

- and means that some of the sheets or all of them co-incide. Thus the sheets would co-incide at ends only for the preferred p-adic primes and give rise to the singular covering and large h_{eff}. End-collapse would be the essence of criticality! This would occur, when the parameters defining the 2-surfaces are in a ramified prime ideal.

2. Even for the ramified primes there would be n distinct space-time sheets, which are regarded as physically distinct. This would support the view that besides the space-like 3-surfaces at the ends the full 3-surface must include also the light-like portions connecting them so that one obtains a closed 3-surface. The conformal gauge equivalence classes of the light-like portions would give rise to additional degrees of freedom. In space-time interior and for string world sheets they would become visible.

   For ramified primes n distint 3-surfaces would collapse to single one but the n discrete degrees of freedom would be present and particle would obtain them. I have indeed proposed number theoretical second quantization assigning fermionic Clifford algebra to the sheets with n oscillator operators. Note that this option does not require Galois group to act as gauge group in the general case. This number theoretical second quantization might relate to the realization of Boolean algebra suggested by weak form of NMP [K1, K2].

4 About the physical interpretation of the parameters characterizing algebraic extension of rationals in TGD framework

It seems that Galois group is naturally associated with the hierarchy $h_{eff}/h = n$ of effective Planck constants defined by the hierarchy of quantum criticalities. n would naturally define the maximal order for the element of Galois group. The analog of singular covering with that of $z^{1/n}$ would suggest that Galois group is very closely related to the conformal symmetries and its action induces permutations of the sheets of the covering of space-time surface.

Without any additional assumptions the values of n and ramified primes are completely independent so that the conjecture that the magnetic flux tube connecting the wormhole contacts associated with elementary particles would not correspond to very large n having the p-adic prime p characterizing particle as factor ($p = M_{127} = 2^{127} - 1$ for electron). This would not induce any catastrophic changes.

TGD based physics could however change the situation and reduce number theoretical degrees of freedom: the intuitive hypothesis that p divides n might hold true after all.

1. The strong form of GCI implies strong form of holography. One implication is that the WCW Kähler metric can be expressed either in terms of Kähler function or as anti-commutators of super-symplectic Noether super-charges defining WCW gamma matrices. This realizes what can be seen as an analog of Ads/CFT correspondence. This duality is much more general. The following argument supports this view.

(a) Since fermions are localized at string world sheets having ends at partonic 2-surfaces, one expects that also Kähler action can be expressed as an effective stringy action. It is natural to assume that string area action is replaced with the area defined by the effective metric of string world sheet expressible as anti-commutators of Kähler-Dirac gamma matrices defined by contractions of canonical momentum currents with imbedding space gamma matrices. It string tension is proportional to h_{eff}^2, string length scales as h_{eff}.

(b) AdS/CFT analogy inspires the view that strings connecting partonic 2-surfaces serve as correlates for the formation of - at least gravitational - bound states. The distances between string ends would be of the order of Planck length in string models and one can argue that gravitational bound states are not possible in string models and this is the basic reason why one has ended to landscape and multiverse non-sense.

2. In order to obtain reasonable sizes for astrophysical objects (that is sizes larger than Schwartschild radius $r_s = 2GM$) For $h_{eff} = h_{gr} = GMm/v_0$ one obtains reasonable sizes for astrophysical objects. Gravitation would mean quantum coherence in astrophysical length scales.

3. In elementary particle length scales the value of h_{eff} must be such that the geometric size of elementary particle identified as the Minkowski distance between the wormhole contacts defining the length of the magnetic flux tube is of order Compton length - that is p-adic length scale proportional to $\sqrt{p}$. Note that dark physics would be an essential element already at elementary particle level if one accepts this picture also in elementary particle mass scales. This requires more precise specification of what darkness in TGD sense really means.

   One must however distinguish between two options.

   (a) If one assumes $n \simeq \sqrt{p}$, one obtains a large contribution to classical string energy as $\Delta \sim m_{CP_2}^2 L_p/\hbar_{eff}^2 \sim m_{CP_2}/\sqrt{p}$, which is of order particle mass. Dark mass of this size looks un-feasible since p-adic mass calculations assign the mass with the ends wormhole contacts. One must be however very cautious since the interpretations can change.

   (b) Second option allows to understand why the minimal size scale associated with CD characterizing particle correspond to secondary p-adic length scale. The idea is that the string can be thought of as being obtained by a random walk so that the distance between its ends is proportional to the square root of the actual length of the string in the induced metric. This would give that the actual length of string is proportional to p and n is also proportional to p and defines minimal size scale of the CD associated with the particle. The dark contribution to the particle mass would be $\Delta m \sim m_{CP_2}^2 L_p/\hbar_{eff}^2 \sim m_{CP_2}/p$, and completely negligible suggesting that it is not easy to make the dark side of elementary visible.

4. If the latter interpretation is correct, elementary particles would have huge number of hidden degrees of freedom assignable to their CDs. For instance, electron would have $n = 2^{127} - 1 \simeq 10^{38}$ hidden discrete degrees of freedom and would be rather intelligent system - 127 bits is the estimate- and thus far from a point-like idiot of standard physics. Is it a mere accident that the secondary p-adic time scale of electron is .1 seconds - the fundamental biorhythm - and the size scale of the minimal CD is slightly large than the circumference of Earth?

   Note however, that the conservation option assuming that the magnetic flux tubes connecting the wormhole contacts representing elementary particle are in $h_{eff}/h = 1$ phase can be considered as conservative option.

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