Quantum Measurement and Quantum Computation in TGD Universe

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Abstract

During years I have been thinking how quantum computation could be carried out in TGD Universe. There are considerable deviations from the standard view. Zero Energy Ontology (ZEO), weak form of NMP dictating the dynamics of state function reduction, negentropic entanglement (NE), and hierarchy of Planck constants define the basic differences between TGD based and standard quantum measurement theory. TGD suggests also the importance of topological quantum computation (TQC) like processes with braids represented as magnetic flux tubes/strings along them.

The natural question that popped in my mind was how NMP and Zero Energy Ontology (ZEO) could affect the existing view about TQC. The outcome was a more precise view about TQC. The basic observation is that the phase transition to dark matter phase reduces dramatically the noise affecting quantum quits. This together with robustness of braiding as TQC program raises excellent hopes about TQC in TGD Universe. The restriction to negentropic space-like entanglement (NE) defined by a unitary matrix is something new but does not seem to have any fatal consequences as the study of Shor's algorithm shows.

NMP strongly suggests that when a pair of systems - the ends of braid - suffer state function reduction, the NE must be transferred somehow from the system. How? The model for quantum teleportation allows to identify a possible mechanism allowing to achieve this. This mechanism could be fundamental mechanism of information transfer also in living matter and phosphorylation could represent the transfer of NE according to this mechanism: the transfer of metabolic energy would be at deeper level transfer of negentropy. Quantum measurements could be actually seen as transfer of negentropy at deeper level.

NE defines an excellent candidate for an analog of error correcting code. If only the diagonal form of the unitary entanglement matrix carries information, the quantization of phases as roots of unity provides a scenario in which Nature itself would take care of error correction.

1 ZEO based quantum measurement theory

It is interesting to test how the view about quantum computation must be modified in TGD Universe. There are considerable deviations from the standard view. Zero Energy Ontology (ZEO), weak form of NMP dictating the dynamics of state function reduction [K1], negentropic entanglement, and hierarchy of Planck constants [K3] define the basic differences between TGD based and standard quantum measurement theory. TGD suggests also the importance of topological quantum computation (TQC) like processes with braids represented as magnetic flux tubes/strings along them.

Consider first the quantum measurement theory based on ZEO.

1. Sub-system–complement pair defining larger system defines the counterpart for the pair observer-measured system in standard quantum measurement theory. In TGD framework density matrix for a sub-system–complement pair defines the universal observable. As a matter of fact, for a given system all sub-system-complement pairs defining possible splitting of this kind and the state function reduction is realized for the pair giving rise to maximum of maximal negentropy gain (NMP). A further essential assumption is that the reduction proceeds from a system inside CDE to subsystems as a cascade obeying this basic rule.

2. ZEO implies that state function reductions occur at either boundary of causal diamond (CD) - the active boundary. The sequence of reductions leaving passive boundary and state at it unaffected gives rise to a conscious entity - self. What is new that at the active boundary the state changes. Even the active boundary itself drifts to the geometric future so that the size of CD increases. This gives rise to the experience about flow of time.

   This is the TGD counterpart for the unitary time evolution and its duration corresponds to the increases of the proper time distance between the tips of CD. Eventually NMP forces the first state function reduction to the opposite boundary: this corresponds to a genuine state function reduction. The self dies and re-incarnates at the opposite boundary as time reversed self since CD increases after than at the opposite boundary to the direction of geometric past.

   In the standard quantum models for quantum computation one assumes that measurement can be realized by some interaction Hamiltonian: the state of entangled system-observer pair develops to an eigen state of the interaction Hamiltonian. The time development by this interaction Hamiltonian gives entangled state defined by the density matrix. This description can be seen as an approximation to TGD based description in which one can assign definite duration to the analog of the unitary evolution.

3. Negentropic entanglement (NE) is possible for entanglement coefficients in algebraic extension of rationals since in this case number theoretic entropy having negative values is well-defined. If the density matrix does not belong to the same algebraic extension, state function reduction requires a phase transition extending the algebraic extension of rationals used and could be seen as kind of evolutionary jump. This kind of NE could be therefore rather stable and could be interpreted as a kind of cognitive entanglement representing a rule with instances represented as state pairs in the superposition. If the state function reduction occurs it leads to a ray of state space if density matrix is non-degenerate.

   If the density matrix contains as a direct summand a higher-dimensional projector, a reduction giving rise to a projector to this sub-space is allowed by the interpretation as measurement of density matrix producing its eigen space. The state remains negentropically entangled by the unitary matrix giving rise to the projector. Weak form of NMP [K1] however allows reductions also to the subspaces of this sub-space assuming preferred state basis so that also the reduction to a ray of state space is possible as a special case. In this case any state basis is eigenbasis for the sub-space and this suggests an interpretation in terms of meditative states in which distinctions disappear.

2 TQC in TGD

How could (topological) quantum computation be realized in TGD framework?

1. In standard quantum theory unitary time evolution realizes the quantum computation. Unitary time evolution is engineered in terms of gates performing standardized operations for qubits. For TQC braiding defines the space-time entanglement between the systems A and B at the ends of the braid. Call this system $A \otimes B$. One can speak about evolution a kind of "space-like" topological quantum computer program with negentropically entangled "initial"

and “final” states at the ends of the braid. Basic braiding operation defines the basic gate in terms of so called R-matrix and the desired NE can be build using an appropriate braiding. For the sake of concreteness the following considerations assume TQC. In fact, if there is entanglement between ends, it must be unitary entanglement since only this entanglement is respected by NMP.

2. In TQC the program is defined by braid and is robust against perturbations. The quantum states at the ends of the braid are however sensitive to noise and this requires complex error correction procedures to eliminate the errors, which are basically spin flip changing the value of qubit and change of its phase. If only phase $\pm$ 1 is allowed phase change actually reduces to spin flip in suitable basis.

   In standard quantum computation the small value of Planck constant is the basic problem. Coherence times tend to be very short and the control of external noise is a tough challenge. In TGD quantum criticality gives rise to phases of matter with effective value $h_{eff} = n \times h$ of Planck constant identified as dark matter. These phases are involved also with NE. Only systems with same value of $h_{eff} = n \times h$ have direct interactions with each other. This should dramatically reduce the noise since visible matter particle must transform to dark matter particle to interact directly with dark matter to produce noise. Also the scaling up of interaction time scales gives hopes that quantum coherence times are long enough to perform TQC.

3. The value of h_{eff} is expected to correlate with the duration of self defined as the increase ΔT of the temporal distance during the sequence of state function reductions to the same passive boundary of CD. ΔT could be interpreted as quantum coherence time. Coherence time for classical fields could be identified as the temporal distance between the tips of CD increasing during quantum computation.

4. TGD promises to guarantee the reduction of noise in terms of darkness of the particles involved with the computation: this instability is the weakness of TQC although TQC program itself is robust. TGD also promises the understanding of the role of quantum criticality in quantum measurement. The very fact that quantum measurements necessarily involve the amplification of small quantum effects to macroscopic “classical” effect, indeed strongly suggests quantum criticality.

5. The key challenge is to prepare a desired kind of negentropically entangled state - say a dark many-particle state associated with a braid system. One should be able to manipulate of dark matter, which we are not yet able to even detect! That dark matter appears at quantum criticality could be extremely helpful in the attempts to get grasp on the dark matter. A simple clue is that the disappearance of visible matter could serve as a signature for the emergence of dark matter.

One should somehow be able to perform state function reduction of the negentropically entangled system to one of the eigenstates of the density matrix associated with unitary entanglement matrix. This requires TGD counter part of time evolution. One can imagine two options.

1. One can couple the negentropically entangled system pair AB to a measurement apparatus C, whose function is to develop ordinary entanglement with both systems during the repeated sequence of state function reductions at fixed boundary. In the state function reduction to the opposite boundary a time reversed reduced state results and gives rise to rays of state space for both A and B. One can however argue that the situation cannot be so simple: NMP requires that entanglement negentropy increases so that NE should be transferred somewhere. This will be discussed below.

2. The measurement interaction must be able to achieve ordinary state function reduction by generating entanglement with the system formed by negentropically entangled system. One must have interaction between ordinary and dark matter and this requires transformation of ordinary matter to dark matter with the same value of h_{eff}. Quantum criticality allows the transformation of ordinary matter to dark matter so that the measuring system should be quantum critical [K3].

3. Could one do without a third system? Weak form of NMP allows also a reduction to the lower dimensional sub-spaces of the N-D sub-space considered and also 1-D ray is possible. This process corresponds to a duration of single self, which dies when the first reduction to the opposite boundary of its CD occurs. If the braid system is not changed in the state function to the opposite boundary one can hope that a reduction to a 1-D ray can occur with some probability. By waiting long enough one can obtain state function reductions which determine the probabilities for the reduction to a given ray or sub-space. The important difference to the standard picture would be that the system does it itself. No external measurements at the end of braid would be carried out. This is however too good to be true. Only one of the two quantum measurements required by Shor algorithm can be both carried out in his manner.

The interpretation in terms of consciousness theory allows also to consider the possibility that the measurement corresponds at deeper level to transfer of negentropic entanglement.

1. One has besides AB also the third system C. The NE for AB is transferred to NE for AC and can be transferred further - say to entanglement to NE for CD. In TGD framework the iteration of this process makes possible a transfer of conscious information associated with NE for AB to that of conscious observer.

2. If the state of C is eigenstate of spin in the basis used, the final state of B is also an eigenstate of spin. Hence the transfer of NE could be thus interpreted as a measurement of the state of B or as the measurement of state of AC in Bell basis. This conforms with the fact that state function reduction for a subsystem can be interpreted as a state function reduction for its complement. Could the deeper interpretation of quantum measurement be as a transfer of NE so that essentially quantum information theory would be in question.

3. The measurement is performed for the negentropically entangled Bell states for the pair AC and performs the transfer of entanglement inducing a unitary rotation. Since in the case of NE defined by a unitary matrix any state basis is allowed, one could ask whether the outcomes are equivalent from the point of view of consciousness theory at least. The knowledge of the final state of B allows to deduce the unitary rotation needed to rotate AC state to the original AB state so that this information is enough to realize a faithful NE transfer. Since the conscious experience is dictated both by the bit telling the state of B and by the state of AC one can ask whether the conscious experience and is same for all four outcomes.

3 Where and how the NE could be transferred?

NMP demands that entanglement negentropy increases. An interesting question is, where and how the entanglement negentropy is transferred.

1. Does NE correspond to information transferred to the performer of quantum measurement? If so, the quantum measurement process would be basically transfer of information realized as NE. Living systems would be carrying out this all the time and ATP-ADP transformation defining the basic step of energy metabolism would be just this kind of transfer. The transfer corresponds at the level of space-time geometry the transfer of the end of magnetic flux tubes plus particles from a donor to the acceptor.

2. A possible manner to carry out the transfer of negentropic entanglement is inspired by the quantum teleportation protocol (`https://en.wikipedia.org/wiki/Quantum_teleportation`). In the simplest situation this protocol is as follows. Alice wants to send qubit C to Bob. A Bell state (`https://en.wikipedia.org/wiki/Bell_state`) is shared between Alice and Bob by mutual agreement in advance so that both know it. Alice can achieve the teleportation by a quantum measurement in the tensor product of the qubit C with the AB Bell state.

   Alice reduces the system AC to one of the four Bell states and communicates the result classically to Bob. The factored out state of B is the original state or one of three states related to it by unitary rotation. Alice sends classically two bits telling what the measurement outcome was. If the outcome was the original state to be sent, Bob does nothing. If it was one of the three remaining states, Bob performs a unitary rotation giving as a result the original state.

3. What makes this protocol so interesting is that in the reduction the NE for AB is transferred to NE for AC as such or modified by a unitary rotation so that four different outcomes are possible. Since the states of C and AB are in 1-1-correspondence it is indeed obvious that the information about the state of B resulting from the measurement of Alice allows the rotation of the Bell state AC to the original state AB. For instance, if the state of B is the original state of C, the state is the original state AB.

   One can apply this procedure by introducing four system D - call it Doris - so that AC NE is transferred to CD NE and AB is now product state. This kind of transfer of negentropic entanglement might be a key event in in phosphorylation and in the utilization of metabolic energy coming from nutrients. The NE between phosphate P of ATP ==B and third system A would be transferred to NE between acceptor molecule and C and A. Also the NE between nutrient B and third system A could be transferred to NE between phosphate and A.

4 Shor's algorithm from TGD point of view

Is the unitary of the entanglement matrix guaranteeing NE too strong an assumption? Just for fun I looked Shor's algorithm (`https://en.wikipedia.org/wiki/Shor's_algorithm`) for the factorization of a given integer, call it N, which has been shown to work for $N = 15$. It turns out that unitary entanglement is not a problem. Furthermore, ordinary quantum measurements are needed for the two systems involved and require interaction coupling negentropically entangled pair of systems to external world so that both negentropically entangled systems generate entanglement with external world.

Consider now the Shor's algorithm. The genuinely quantal step of algorithm is that of finding the period r of the function $f(x) = a^x \ mod \ N$, for integers $1 < a < N$ and $1 < x < N$.

1. According to the Wikipedia article , the computation involves the construction of quantum function $f(x) = a^x$ as

$$\frac{1}{Q}\sum_x |x, f(x)\rangle \ .$$

   Here Q is normalization factor. Since $a^r = 1 \ mod \ N$, $f(x)$ is not a bijection. Unless r divides Q (we do not however know r!), the number $N(z)$ of values of x satisfying $f(x) = z$ varies and the variation is one unit at most. Therefore the entanglement is not unitary and the density matrix of the state is not unit matrix since the norms of states

$$|Z| = \sum_x |x, f(x) = z\rangle$$

   is given by $N(z)$- the number of x mapped to z and varies somewhat. NE would be obtained by normalizing the states $|Z\rangle$ to unit norm and replacing Q by the the number $N(Z)$ of points z to get

$$\frac{1}{\sqrt{N(Z)}}\sum_z \frac{1}{\sqrt{N(z)}}\sum_x |x, f(x) = z\rangle \ .$$

2. Second step in the computation is discrete quantum Fourier transform using as counterparts of plane waves powers of the root of unity defined as $\omega = exp(i2\pi/Q)$, where Q satisfies $N^2 \leq Q < 2N^2$. This operation is unitary and gives rise to unitary entanglement. Since the entire entanglement matrix is product of unitary matrices, it is also unitary. The action of unitary transformation is given for given value of z by the following formula.

$$\sum_x |x, f(x) = z)\rangle \rightarrow \sqrt{1}\sqrt{N} \sum_y \sum_{z=f(x)} \omega^{xy} |y, z = f(x)\rangle \ .$$

The entire state is transformed to

$$\frac{1}{\sqrt{N(Z)}} \frac{1}{\sqrt{N}} \sum_z \frac{1}{\sqrt{N(z)}} \sum_y \omega^{xy} \sum_x |y, z = f(x)\rangle \ .$$

In this expression the state paired $|Z\rangle$ is a superposition of several values of y since the number of different values of z is smaller than those of y by a factor which in ideal situation is the sought four value of r.

3. Quantum measurement should reduce this state to a state with fixed values of y and z. This implies that the normalization factors do not matter. Weak NMP allows a self-reduction a state Z with fixed value of z. The self reduction of the system is however not able to reduce the state Z to $|y, z\rangle$.

   One must couple at least the "y" part of the system to external measurement apparatus generating ordinary or negentropic entanglement with non-degenerate density matrix belonging to the extension used and having $|y\rangle$ as eigenstates. This would force y-reduction. One can of course perform the same for both y and z. The ordinary quantum measurement theory seems to be a necessary part of the picture. In TGD framework additional constraints come from the condition that the measurement involves negentropy transfer. This requires explicit introduce of systems C and D receiving the NE.

5 About negentropic entanglement as an analog of an error correction code

In classical computation, the simplest manner to control errors is to take several copies of the bit sequences. In quantum case no-cloning theorem prevents this. Error correcting codes (`https://en.wikipedia.org/wiki/Quantum_error_correction`) code n information qubits to the entanglement of $N > n$ physical qubits. Additional contraints represents the subspace of n-qubits as a lower-dimensional sub-space of N qubits. This redundant representation is analogous to the use of parity bits. The failure of the constraint to be satisfied tells that the error is present and also the character of error. This makes possible the automatic correction of the error is simple enough - such as the change of the phase of spin state or or spin flip.

Negentropic entanglement (NE) obviously gives rise to a strong reduction in the number of states of tensor product. Consider a system consisting of two entangled systems consisting of N_1 and N_2 spins. Without any constraints the number of states in state basis is $2^{N_1} \times 2^{N_2}$ and one as $N_1 + N_2$ qubits. The elements of entanglement matrix can be written as $E_{A,B}$ $A \equiv \otimes_{i=1}^{N_1}(m_i, s_i)$, $B \equiv \otimes_{k=1}^{N_2}(m_k, s_k)$ in order to make manifest the tensor product structure. For simplicity one can consider the situation $N_1 = N_2 = N$.

The un-normalized general entanglement matrix is parametrized by 2×2^{2N} independent real numbers with each spin contributing two degrees of freedom. Unitary entanglement matrix is characterized by 2^{2N} real numbers. One might perhaps say that one has $2N$ real bits instead of almost $2N + 1$ real qubits. If the time evolution according to ZEO respects the negentropic character of entanglement, the sources of errors are reduced dramatically.

The challenge is to understand what kind of errors NE eliminates and how the information bits are coded by it. NE is respected if the errors act as unitary transformations $E \rightarrow UEU^\dagger$ of the unitary entanglement matrix. One can consider two interpretations.

1. The unitary automorphisms leave information content unaffected only if they commute with E. In this case unitary automorphisms acting non-trivially would give rise genuine errors and an error correction mechanism would be needed and would be coded to quantum computer program.

2. One can also consider the possibility that the unitary automorphisms *do not affect* the information content so that the diagonal form of entanglement matrix coded by N phases would carry of information. Clearly, the unitary automorphisms would act like gauge transformations. Nature would take care that no errors emerge. Of course, more dramatic things are in principle allowed by NMP: for instance, the unitary entanglement matrix could reduce to a tensor product of several unitary matrices. Negentropy could be transferred from the system and is indeed transferred as the computation halts.

   By number theoretic universality the diagonalized entanglement matrix would be parametrized by N roots of unity with each having n possible values so that n^N different NEs would be obtained and information storage capacity would be $I = log(n)/log(2) \times N$ bits for $n = 2^k$ one would have $k \times N$ bits. Powers of two for n are favored. Clearly the option for which only the eigenvalues of Ematter, looks more attractive realization of entanglement matrices. If overall phase of E does not matter as one expects, the number of full bits is $k \times N - 1$. This option looks more attractive realization of entanglement matrices.

   In fact, Fermat polygons for which cosine and sine for the angle defining the polygon are expressible by iterating square root besides basic arithmetic operations for rationals (ruler and compass construction geometrically) correspond to integers, which are products of a power of two and of different Fermat primes $F_n = 2^{2^n} + 1$. l

This picture can be related to much bigger picture.

1. In TGD framework number theoretical universality requires discretization in terms of algebraic extension of rationals. This is not performed at space-time level but for the parameters characterizing space-time surfaces at the level of WCW. Strong form of holography is also essential and allows to consider partonic 2-surfaces and string world sheets as basic objects. Number theoretical universality (adelic physics) forces a discretization of phases and number theoretically allowed phases are roots of unity defined by some algebraic extension of rationals. Discretization can be also interpreted in terms of finite measurement resolution. Notice that the condition that roots of unity are in question realizes finite measurement resolution in the sense that errors have minimum size and are thus detectable.

2. Hierarchy of quantum criticalities corresponds to a fractal inclusion hierarchy of isomorphic sub-algebras of the super-symplectic algebra acting as conformal gauge symmetries. The generators in the complement of this algebra can act as dynamical symmetries affecting the physical states. Infinite hierarchy of gauge symmetry breakings is the outcome and the weakening of measurement resolution would correspond to the reduction in the size of the broken gauge group. The hierarchy of quantum criticalities is accompanied by the hierarchy of measurement resolutions and hierarchy of effective Planck constants $h_{eff} = n \times h$.

3. These hierarchies are argued to correspond to the hierarchy of inclusions for hyperfinite factors of type II_1 labelled by quantum phases and quantum groups. Inclusion defines finite measurement resolution since included sub-algebra does induce observable effects on the state. By Mac-Kay correspondence the hierarchy of inclusions is accompanied by a hierarchy of simply laced Lie groups which get bigger as one climbs up in the hierarchy. There interpretation as genuine gauge groups does make sense since their sizes should be reduced. An attractive possibility is that these groups are factor groups G/H such that the normal subgroup H (necessarily so) is the gauge group and indeed gets smaller and G/H is the dynamical group identifiable as simply laced group which gets bigger. This would require that both G and H are infinite-dimensional groups. An interesting question is how they relate to the super-symplectic group assignable to "light-cone boundary" $\delta M^4_\pm \times CP_2$. I have proposed this interpretation in the context of WCW geometry earlier.

4. Here I have spoken only about dynamical symmetries defined by discrete subgroups of simply laced groups. I have earlier considered the possibility that discrete symmetries provide a description of finite resolution, which would be equivalent with quantum group description.

Summarizing, these arguments boil down to the conjecture that discrete subgroups of these groups act as effective symmetry groups of entanglement matrices and realize finite quantum measurement resolution. A very deep connection between quantum information theory and these hierarchies would exist.

Gauge invariance has turned out to be a fundamental symmetry principle, and one can ask whether unitary entanglement matrices assuming that only the eigenvalues matter, could give rise to a simulation of discrete gauge theories. The reduction of the information to that provided by the diagonal form be interpreted as an analog of gauge invariance?

1. The hierarchy of inclusions of hyper-finite factors of type II_1 suggests strongly a hierarchy of effective gauge invariances characterizing measurement resolution realized in terms of hierarchy of normal subgroups and dynamical symmetries realized as coset groups G/H. Could these effective gauge symmetries allow to realize unitary entanglement matrices invariant under these symmetries.

2. A natural parametrization for single qubit errors is as rotations of qubit. If the error acts as a rotation on *all* qubits, the rotational invariance of the entanglement matrix defining the analog of S-matrix is enough to eliminate the effect on information processing.

   Quaternionic unitary transformations act on qubits as unitary rotations. Could one assume that complex numbers as the coefficient field of QM is effectively replaced with quaternions? If so, the multiplication by unit quaternion for states would leave the physics and information content invariant just like the multiplication by a complex phase leaves it invariant in the standard quantum theory.

   One could consider the possibility that quaternions act as a discretized version of local gauge symmetry affecting the information qubits and thus reducing further their number and thus also errors. This requires the introduction of the analog of gauge potential and coding of quantum information in terms of SU(2) gauge invariants. In discrete situation gauge potential would be replaced with a non-integrable phase factors along the links of a lattice in lattice gauge theory. In TGD framework the links would correspond the fermionic strings connecting partonic two-surfaces carrying the fundamental fermions at string ends as point like particles. Fermionic entanglement is indeed between the ends of these strings.

3. Since entanglement is multilocal and quantum groups accompany the inclusion, one cannot avoid the question whether Yangian symmetry crucial for the formulation of quantum TGD [K2] could be involved.

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